Published for SISSA by Springer

Received: October 27, 2023
Revised: February 11, 2024
Accepted: March 21, 2024
Published: May 8, 2024

Non-Abelian T-dualities in two dimensional $(0, 2)$ gauged linear sigma models

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Abstract: We consider two dimensional (2D) gauged linear sigma models (GLSMs) with $(0, 2)$ supersymmetry and U(1) gauge group which posses global symmetries. We distinguish between two cases: one obtained as a reduction from the $(2, 2)$ supersymmetric GLSM and another not coming from a reduction. In the first case we find the Abelian T-dual, comparing with previous studies. Then, the Abelian T-dual model of the second case is found. Instanton corrections are also discussed in both situations. We explore the vacua for the scalar potential and we analyse the target space geometry of the dual model. An example with gauge symmetry $\mathrm{U}(1) \times \mathrm{U}(1)$ is discussed, which posses non-Abelian global symmetry. Non-Abelian T-dualization of U(1) $(0, 2)$ $2D$ GLSMs is implemented for models that arise as a reduction from the $(2, 2)$ case; we study a model with U(1) gauge symmetry and SU(2) global symmetry. It is shown that for a positive definite scalar potential, the dual vacua to $\mathbb{P}^1$ constitutes a disk. Instanton corrections to the superpotential are obtained and are shown to be encoded in a shift of the holomorphic function E. We conclude by analyzing an example with $\mathrm{SU}(2) \times \mathrm{SU}(2)$ global symmetry, obtaining that the space of dual vacua to $\mathbb{P}^1 \times \mathbb{P}^1$ consists of two copies of the disk, also for the case of positive definite potential. Here we are able to fully integrate the equations of motion of non-Abelian T-duality, improving the analysis with respect to the studies in $(2, 2)$ models.

Keywords: Duality in Gauge Field Theories, Gauge Symmetry, Sigma Models, Supersymmetry and Duality

ArXiv ePrint: 2310.09456

https://doi.org/10.1007/JHEP05(2024)088

Contents

1 Introduction

The study of suitable features of string theory compactifications leading to more realistic phenomenological scenarios has been always a great deal of interest in physics and mathematics [1–3]. The physical aspects involve the reproduction of important known features of low-energy physics and the possibility of having new predictions of phenomenological interest [4]. Some of these features are relevant, for instance, in the early universe or in microscopic aspects of black hole physics [5, 6], and moreover, in the derivation of the Standard Model of particle physics and beyond [7]. The study of string compactifications with fluxes have played a central role leading to more realistic relations of string theory with phenomenological phenomena at low energies [8]. A very important family of compactifications are described by a two-dimensional non-linear sigma model (NLSM) on target spaces consisting of Calabi-Yau (CY) manifolds. These models are superconformal field theories in 2D with a certain central charge with supersymmetry $(2, 2)$ [9, 10]. They have many interesting features; however, they lead to low energy effective field theories consistent with Grand Unification Theories in the four-dimensional spacetime with exceptional gauge groups. Much more realistic compactifications leading to SU(5) or SO(10) GUTs are the non-linear sigma models with $(0, 2)$ supersymmetry [11]. This family represents a more general kind of compactifications than those of the $(2, 2)$ kind (for some reviews, see [12, 13]).

On the other hand, two-dimensional gauged linear sigma models (GLSMs) with $(2, 2)$ supersymmetry were introduced by Witten in [14], with the aim of studying solutions in string theory with the possibility to implement a change in the spacetime topology (target

JHEP05(2024)088

space) through a simple smooth variation of the parameters in the GLSM. These changes were observed in [15–18]. More importantly these models work as a theory which interpolate among different phases which involve topology change. In some models the GLSMs describe transitions from the Calabi-Yau (CY) phase in the infrared (IR) to the Landau-Ginzburg phase in the ultraviolet (UV) and viceversa. In the IR the CY phase is obtained by studying the supersymmetric space of vacua of the underlying effective scalar potential. This space may have different geometries depending on the specific GLSM that one is considering i.e. one may vary the number of chiral superfields and the corresponding assignments of the charges which these fields carry and the Abelian or non-Abelian gauge groups. The same reference [14] also introduces the $(0, 2)$ GLSMs in 2D. These models have similar properties as the $(2, 2)$ models, but there are many features in which they differ. For instance, the $(0, 2)$ models are chiral. These models have been studied actively and very good treatments can be found in [14, 19–23].

Different aspects of $(2, 2)$ GLSMs have been worked out extensively, one of its prominent applications is the proof of mirror symmetry [24, 25]. The procedure of Hori and Vafa looks to implement the Buscher-Giveon-Roček-Verlinde target space (T-)duality algorithm (for some reviews, see [26–28]) to $(2, 2)$ GLSMs, gauging Abelian global symmetries. This procedure was successful to give a physical proof of the mirror symmetry correspondence. Localization properties of the partition function have been used to test Abelian T-duality in GLSMs that lead to mirror symmetry [29]. Moreover, the duality algorithm can be generalized to consider the gauging of a non-Abelian group in contrast to the gauging of an Abelian group. This is termed the non-Abelian duality and many interesting traditional results have obtained in this direction [30–32]. More recently some interesting results involving the idea of non-Abelian duality can be found in refs. [33–36].

As we mentioned before $(2, 2)$ GLSMs are an important tool to prove mirror symmetry of CY manifolds, in particular for the case of complete intersections of CY manifolds and toric varieties [24, 25], and there have been many studies in GLSMs and their applications [37–49]. However, for GLSMs with $(0, 2)$ supersymmetry the realization of mirror symmetry was least apparent. Certain kind of mirror map can be defined for these models [50–55]. Other notions of the $(0, 2)$ mirror map are discussed in refs. [21–23, 56–58]. In particular, in [56], it was studied the Abelian GLSM with a gauged Abelian global symmetry. The authors follow the duality algorithm mentioned previously and obtained a dual action which is also a $(0, 2)$ GLSM. In particular they found the instanton contributions in the dual action which are compatible with the instanton corrections of the original $(0, 2)$ GLSM. Other developments of $(0, 2)$ GLSMs in different contexts can be found in refs. [59–64]. For a very recent overview of some important results of the GLSMs see [65, 66].

In the context of $(2, 2)$ GLSMs with $U(1)$ gauge symmetry, the possibility of gauging up a non-Abelian global symmetry was explored in [67–69]. In this article, the dual action was given and the instanton corrections of the dual action were determined. Moreover, for the CY phase, there were given some models in which the geometry of the target space was found. One motivation to go beyond the realm of Abelian T-duality in [24] comes from the fact that there is a large set of CY manifolds that do not constitute complete intersections but rather Grassmanians, Pfaffians or determinantal; that can be studied as Non-Abelian GLSMs [70], and a description of the symmetries in these models is of interest [38, 40, 42], in particular the study of mirror symmetry [44, 45, 58]. Nevertheless one can also ask first the question whether the T-dualities leading to mirror symmetry even for Abelian GLSMs can be generalized further, to obtain new geometric identifications. In general the models have multiple global symmetries that can make the group of T-dualities bigger. This is the question that we explore in this paper. Such a search can help to determine relations between apparently different string vacua, serving to explore the landscape.

In the present article we start from the family of $(0,2)$ GLSMs considered in [56], arising from a reduction of the $(2,2)$ theory, and we study the gauging of a non-Abelian global symmetry. We also consider pure $(2,0)$ GLSMs models, not obtained from a supersymmetric reduction.

Let us summarize the main results of our article:

1. We study the non-abelian T-duality of the 2D $(0,2)$ GLSM. We focus in two cases; the first one is the pure $(0,2)$ GLSM, which does not come from a supersymmetry reduction from a $(2,2)$ GLSM. The second case comes from a $(2,2)$ reduction, and this is the one of the most importance in our work.

2. We start with the Abelian T-dualization of the $(0,2)$ GLSM, for models with multiple U(1) gauge fields and multiple chiral superfields. Those models have multiple U(1) global symmetries, which we T-dualize. Then we discuss the example of a single chiral superfield with just a spectator, with a single U(1) global symmetry, performing Abelian-duality. We distinguish two cases, one coming from reduction of $(2,2)$ supersymmetry and the other doesn't. We dualize as well the case of a model with $\mathrm{U}(1) \times \mathrm{U}(1)$ gauge symmetry and $\mathrm{U}(1)^4$ global symmetry, which would lead to its mirror symmetric model as in [56].

3. We perform the non-Abelian T-dualization, for models with multiples U(1) gauge symmetries, and with multiple non-Abelian global symmetries. These $(0,2)$ models come from a reduction of the $(2,2)$ supersymmetric case. The non-Abelian T-duality algorithm is implemented by gauging out this symmetry and introducing Lagrange multipliers; those last impose constraints that fix the resulting local symmetry to have a pure gauge vector field. We take as an example the case of SU(2) dualization. We find the dual action, which is given in terms of the degrees of freedom Y_+^a, where a is the group index, and it contains a scalar potential. We analyze the structure of the whole vacua manifold and, to perform the analysis, we find regions of the space of fields where the potential is positive definite. The result is given by a surface that in function of the values of the parameter $r = \Im(t)$ has the topology of a plane or the topology of the union of the plane and a disk. To describe the vacua, we analyze the three regions: Region 1, Region 2 and Region 3. Region 1 and 2 have the topology of a disk and its boundary, and Region 3 has the topology of the plane. Thus the complete structure of the vacuum space $\mathcal{W}$ is the union of all three regions, where the disk size can be contracted depending on the r parameter. Thus, the whole space of vacua is non-compact.

4. To study the effects of the non-Abelian T-duality in a more interesting example, we consider the case of a model with non-Abelian global symmetry $\mathrm{SU}(2) \times \mathrm{SU}(2)$ and with the product of two Abelian gauge groups. The model has four chiral fields and

JHEP05(2024)088

two Fermi fields. The model is originally $(2,2)$ but it is deformed to obtain a pure $(0,2)$ GLSM which does not come from reduction. The dual action and the scalar potential were obtained. Employing the results of the space of vacua for the case of a single SU(2), the analysis for the complete system was carried out. The space of vacua is the Cartesian product of two copies of $\mathcal{W}$.

The present article is organized as follows: in section 2 a brief overview of $(0,2)$ GLSMs is given. Our aim is not to provide an extensive review but to give the notation and conventions we will follow. We also discuss the field content and interactions of these models. In section 3 we describe Abelian T-duality in $(0,2)$ GLSMs. We presented two examples, both of them with a single U(1) gauge group and a pair of chiral superfields. In the first example we considered the case of a GLSM which is a reduction of a $(2,2)$ GLSM and in the second example we study the case of a pure $(0,2)$ GLSM. We find the equations of motion and the dual Lagrangian in both cases. Moreover, we compute the scalar potential and discuss the geometry of these vacua manifolds. The third example deals with a GLSM with $\mathrm{U}(1)\times\mathrm{U}(1)$ gauge group and $\mathrm{U}(1)^4$ global symmetry group. We presented this model as a preliminary material which will be later generalized to non-Abelian T-duality in section 5. In section 4 we perform non-Abelian dualization for a general global symmetry group G. In order to be specific, we particularize to SU(2) global group and give its dual Lagrangian and study its vacua manifold. Finally, in section 5 we consider an example with a $\mathrm{SU}(2)\times\mathrm{SU}(2)$ global symmetry. We conclude in section 6 with our results and conclusions. At the end of the article we added three appendices. Appendix A is devoted to notation and conventions of supersymmetric 2D representations and appendix B is devoted to show the complete Lagrangian of the original models and the supersymmetric field transformations; appendix C is devoted to carry out the algorithm of T-duality at the level of superfield components. We show that for the simpler example of Abelian duality in this paper, the dual action coincides with the dual action of the reduced GLSM in the superfields language presented in section 3.

2 Field representations of $(0,2)$ supersymmetry

In this section, in order to be as self-contained as possible, we write an overview of the basic ingredients of 2D GLSMs with $(0,2)$ supersymmetry. Moreover, we will write down their corresponding Lagrangians and symmetries, and we will describe the matter content and their interactions. Along the paper we follow the notation and conventions on supersymmetric field theory as the one in the references [71–73]. For reading background material regarding $(0,2)$ GLSMs it is useful to consult the references [14, 19, 20].

We start by reviewing the two-dimensional GLSM with $(0,2)$ supersymmetry and an Abelian gauge group, we follow the original Witten's paper in GLSMs [14]. As usual, the coordinates of the $(0,2)$ superspace are given by $(y^0,y^1,\theta^+,\overline{\theta}^+)$. Where the first two are the space coordinates and the last two the fermionic counterparts. V is the $(0,2)$ vector superfield and in the Wess-Zumino gauge they can be expanded in components as follows:

$$V = v_- - 2i\theta^+\overline{\lambda}_- - 2i\overline{\theta}^+\lambda_- + 2\theta^+\overline{\theta}^+ D, \tag{2.1}$$

$$\Psi = v_+\theta^+\overline{\theta}^+. \tag{2.2}$$

In components, field strength is written as:

$$\begin{aligned}\Upsilon &= i\overline{D}_+(V - i\partial_-\Psi) \\ &= -2\lambda_- + [2iD + (\partial_- v_+ - \partial_+ v_-)]\theta^+ + 2i\partial_+\lambda_-\theta^+\overline{\theta}^+ .\end{aligned} \tag{2.3}$$

There are two kinds of matter fields: the chiral multiplets Φ and the Fermi multiplets Γ. The bosonic covariant chiral fields $\widetilde{\Phi}$ are defined by the following constraint:

$$\overline{\mathcal{D}}_+\widetilde{\Phi} = 0, \tag{2.4}$$

where $\overline{\mathcal{D}}_+$ is the covariant derivative and consequently it has the components expansion:

$$\widetilde{\Phi} = \phi + \sqrt{2}\theta^+\psi_+ - i\theta^+\overline{\theta}^+(\partial_+ + iv_+)\phi, \tag{2.5}$$

which is defined with $\widetilde{\Phi} := \Phi e^{\Psi}$, where the (uncharged) chiral superfield Φ fulfils the relation $\overline{D}_+\Phi = 0$. In order to complete the rest of the matter content let us introduce $\widetilde{\Gamma}$ which constitutes a $(0,2)$ Fermi multiplet. This multiplet satisfies the constraint:

$$\overline{\mathcal{D}}_+\widetilde{\Gamma} = \sqrt{2}\widetilde{E}\,, \qquad \text{then} \qquad \widetilde{E} = \frac{\sqrt{2}}{2}e^{\Psi}\overline{D}_+\Gamma, \tag{2.6}$$

where $\widetilde{E} = E(\widetilde{\Phi})$ is a holomorphic function of the superfield $\widetilde{\Phi}$. Similarly, we can define $\widetilde{\Gamma} := \Gamma e^{\Psi}$ and $\widetilde{E} := Ee^{\Psi}$, where $\overline{D}_+\Gamma = \sqrt{2}E$. Thus, the expansion for this Fermi multiplet and the field E are given by:

$$\widetilde{\Gamma} = \gamma - \sqrt{2}G\theta^+ - i\theta^+\overline{\theta}^+(\partial_+ + iv_+)\lambda - \sqrt{2}\widetilde{E}\overline{\theta}^+\,, \tag{2.7}$$

$$\widetilde{E}(\Phi) = E(\phi) + \sqrt{2}\theta^+\frac{\partial E}{\partial\phi}\psi_+ - i\theta^+\overline{\theta}^+(\partial_+ + iv_+)E(\phi). \tag{2.8}$$

In the present article it will be considered $(0,2)$ GLSMs with a U(1) gauge group (or $\mathrm{U}(1)^n$), and with non-Abelian global symmetries to be gauged. Thus the dynamics of the models studied is given by the addition of all of these Lagrangians, i.e.,

$$L = L_{\texttt{gauge}} + L_{\texttt{chiral}} + L_{\texttt{Fermi}} + L_{D,\theta} + L_J. \tag{2.9}$$

The U(1) gauge theory has a natural Lagrangian given by

$$L_{\texttt{gauge}} = \frac{1}{8e^2}\int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+\overline{\Upsilon}\Upsilon, \tag{2.10}$$

where e is the gauge coupling constant.

The gauge invariant Lagrangian for the chiral fields is given by

$$\begin{aligned}L_{\texttt{chiral}} &= -\frac{i}{2}\int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+\ \widetilde{\Phi}^\dagger(\mathcal{D}_0 - \mathcal{D}_1)\widetilde{\Phi} \\ &= \overline{\phi}\phi D + i\overline{\psi}_+(\partial_- + iv_-)\psi - \sqrt{2}i(\lambda_-\psi_+\overline{\phi} - \overline{\psi}_+\overline{\lambda}_-\phi) \\ &\quad - \frac{1}{2}[\overline{\phi}(\partial_- + iv_-)(\partial_+ + iv_+)\phi - (\partial_+ + iv_+)\overline{\phi}(\partial_- + iv_-)\phi].\end{aligned} \tag{2.11}$$

The dynamics of the Fermi field is given by the Lagrangian:

$$
\begin{aligned}
L_{\texttt{Fermi}} &= -\frac{1}{2}\int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \tilde{\overline{\Gamma}}\widetilde{\Gamma} \\
&= i\overline{\gamma}(\partial_+ + iv_+)\gamma + |G|^2 - |E|^2 - \left(\overline{\gamma}\frac{\partial E}{\partial \phi}\psi_+ + \frac{\partial \overline{E}}{\partial \overline{\phi}}\psi_+\gamma\right).
\end{aligned} \tag{2.12}
$$

In the case of U(1) gauge theories (or in non-Abelian gauge theories with a gauge group with a U(1) factor) we have an additional term in the Lagrangian given by the Fayet-Iliopoulos and Theta term

$$
L_{D,\theta} = \frac{t}{4}\int \mathrm{d}\theta^+ \Upsilon|_{\overline{\theta}^+=0} + \text{h.c.}, \tag{2.13}
$$

where $t = \frac{\theta}{2\pi} + ir$, with $\theta = 2\pi\Re(t)$ being an angular parameter and $r = \Im(t)$ is the Fayet-Iliopoulos parameter.

In (0,2) theories there is a superpotential Lagrangian L_J, which is the $(0,2)$ analog of the superpotential term of the $(2,2)$ model. L_J is of the form

$$
L_J = -\frac{1}{\sqrt{2}}\int \mathrm{d}\theta^+ \left(\Gamma_p J^p(\Phi)|_{\overline{\theta}^+=0}\right) - \text{h.c.}, \tag{2.14}
$$

where $J^p = J^p(\Phi)$ is a holomorphic function of the $(0,2)$ chiral superfield Φ, and Γ_p are Fermi superfields (different from the previous ones). Moreover J^p satisfies the relation $E_p(\Phi)J^p(\Phi) = 0$, where of course $\tilde{\mathcal{D}}_+\Gamma_p = \sqrt{2}E_p$. In the present article we will consider models without superpotential terms and therefore $L_J = 0$.

The scalar potential can be obtained by the usual procedure in supersymmetric theories (integrating in the superspace) and it is given by

$$
\mathrm{U}(\phi_i) = \frac{e^2}{2}\left(\sum_i Q_i|\phi_i|^2 - r\right)^2 + \left(|E(\phi)|^2 + |J(\phi)|^2\right), \tag{2.15}
$$

where it is clear the contributions coming the D-terms from the $(0,2)$ gauge multiplet and from the FI term. The last two terms come from the E field and the last one, corresponds to the contribution from the superpotential.

2.1 Reduction of the $(2,2)$ multiplets to $(0,2)$ superfields

It is known that certain $(0,2)$ GLSMs can be regarded as a supersymmetric reduction of $(2,2)$ GLSMs. The $(2,2)$ GLSM consists of chiral supefield $\Phi^{(2,2)}$, a vector superfield $V^{(2,2)}$ and its twisted field strength $\Sigma^{(2,2)}$.

The $(0,2)$ superfields can be obtained from the $(2,2)$ chiral superfield $\Phi^{(2,2)}$ and the $\Gamma^{(2,2)}$ Fermi superfield as:

$$
\Phi = \Phi^{(2,2)}|_{\theta^-=\overline{\theta}^-=0}, \tag{2.16}
$$

$$
\Gamma = \frac{1}{\sqrt{2}}D_-\Phi^{(2,2)}|_{\theta^-=\overline{\theta}^-=0}. \tag{2.17}
$$

Both matter fields are supersymmetry reductions of the single $(2,2)$ chiral superfield.

The $(2,2)$ vector superfield $V^{(2,2)}$ gives the gauge field strength by:

$$V - i\partial_-\Psi = -\overline{D}_- D_- V^{(2,2)}|_{\theta^-=\bar{\theta}^-=0}. \tag{2.18}$$

And also it gives a twisted chiral superfield Σ, which is identified as: $\overline{\theta}^+\Sigma = -\frac{1}{\sqrt{2}}D_-V|_{\theta^-=\overline{\theta}^-=0}$; which is also simply: $\Sigma = \Sigma^{(2,2)}|_{\theta^-=\overline{\theta}^-=0}$.

It can be verified that if $\Phi^{(2,2)}$ has charge Q, then the E field can be written as $E = Q\sqrt{2}\Sigma\Phi$. The holomorphic function J can be obtained from the $(2,2)$ superpotential W in the form $J = \frac{\partial W}{\partial\widetilde{\Phi}}$.

3 Abelian T-duality in $(0,2)$ GLSMs

In this section we describe the T-duality for GLSMs with a $U(1)^m$ gauge group and a $\mathrm{U}(1)^{k+s}$ global symmetry group to be gauged. We present the original model and find the T-dual model, by solving the equations of motion. For the sake of simplicity we consider the case when the superpotential J of the $(0,2)$ model vanishes, thus the underlying scalar potential consist only of the D-term and the Fayet-Iliopoulos term. We found the dual action and the geometry of the space of dual vacua. We study two separate cases, the first one is the case in which the $(0,2)$ GLSM can be obtained by reduction from a $(2,2)$ model. The second case is the general case of a pure $(0,2)$ GLSM which cannot be obtained from a reduction. In both cases we describe their corresponding instanton corrections. In the rest of the section we describe a particular reduced model with gauge group $\mathrm{U}(1)\times\mathrm{U}(1)$ and an Abelian global symmetry $\mathrm{U}(1)^4$. This model was discussed in [56] and it will be analysed in the context of non-Abelian duality in section 4.

3.1 GLSM with $\mathrm{U}(1)^m$ gauge symmetry and $\mathrm{U}(1)^{k+s}$ global symmetry

Here we describe the Abelian T-dualization for general $(0,2)$ $\mathrm{U}(1)^m$ GLSM with $\mathrm{U}(1)^k$ global symmetry related to the chiral fields and $\mathrm{U}(1)^s$ global symmetry associated to the Fermi fields. We start with a theory with n chiral superfields Φ_i and $\tilde{n}$ Fermi superfields Γ_j, and a given number m of $\mathrm{U}(1)$ gauge symmetries. The chiral superfields Φ_i posses Q_i^a charges and the Fermi superfields Γ_i^a have charges $\widetilde{Q}_j^a$. The Lagrangian of this theory is written in the appendix formula (B.1).

There are m vector superfields V_a, Ψ_a with field strength Υ_a. In principle each kinetic term has a global phase symmetry, under which the chiral or the Fermi fields transform. As all the superfields are distinct, one can employ the m gauge symmetries to absorb m of these phases, giving a total of $k+s$ global symmetries where $k = n - m$ $(n > m)$ $\mathrm{U}(1)$ global symmetries, these transformations are given in the appendix formula (B.2).

In general we would have the possibility to absorb with the m Abelian gauge symmetries not only the global symmetries of the chiral superfields, but the total amount of global symmetries of the chiral and Fermi fields $n+\tilde{n}$. The master Lagrangian will remain the same with some few modifications in the sum's indices. This case will be not considered in this work.

In general one can consider a generic number of Fermi multiplets, this is true because the general (0,2) model, presented here, doesn't come necessarily from a SUSY reduction from the (2,2) theory. Therefore the Fermi multiplets are not necessarily related or coupled to the chiral multiplets. In the opposite case when the chiral superfields and the Fermi superfields come both from the (2,2) reduction, the number of Fermi and chiral fields and their charges need to match.

Starting from (B.1) one can construct the *master Lagrangian* (or also named intermediate Lagrangian) by gauging the global symmetries and adding terms with Lagrange multipliers Λ_b related to field strengths Υ_b

$$\begin{aligned}
L_{\mathtt{master}} =& \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \sum_{a=1}^{m} \frac{1}{8e_a^2} \overline{\Upsilon}_a \Upsilon_a \\
&- \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \Big\{ \sum_{i=1}^{k} \frac{i}{2} \overline{\Phi}_i e^{2\sum_{a=1}^{m} Q_i^a \Psi_a + 2\sum_{b=1}^{k} Q_{1i}^b \Psi_{1b}} \Big(\partial_- + i \sum_{a=1}^{m} Q_i^a V_a + i \sum_{b=1}^{k} Q_{1i}^b V_{1b} \Big) \Phi_i \Big\} \\
&+ \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \Big\{ \sum_{i=1}^{k} \frac{i}{2} \overline{\Phi}_i \Big(\overleftarrow{\partial}_- - i \sum_{a=1}^{m} Q_i^a V_a - i \sum_{a=1}^{k} Q_{1i}^b V_{1b} \Big) e^{2\sum_{a=1}^{m} Q_i^a \Psi_a + 2\sum_{b=1}^{k} Q_{1i}^b \Psi_{1b}} \Phi_i \Big\} \\
&- \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \Big\{ \sum_{j=1}^{s} \frac{1}{2} e^{2\sum_{a=1}^{m} \widetilde{Q}_j^a \Psi_a + 2\sum_{c=1}^{s} \widetilde{Q}_{1j}^c \Psi_{1c}} \overline{\Gamma}_j \Gamma_j + \sum_{j=s+1}^{\widetilde{n}} \frac{1}{2} e^{2\sum_{a=1}^{m} \widetilde{Q}_j^a \Psi_a} \overline{\Gamma}_j \Gamma_j \Big\} \\
&+ \sum_{a=1}^{m} \frac{t_a}{4} \int \mathrm{d}\theta^+ \Upsilon_a |_{\overline{\theta}^+=0} + \sum_{b=1}^{k} \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \Lambda_b \Upsilon_{1b} + \sum_{b=k+1}^{k+s} \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \Lambda_b \Upsilon_{1b} + \text{h.c.} \\
&- \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \sum_{i=k+1}^{n} \Big\{ \frac{i}{2} \overline{\Phi}_i e^{2\sum_{a=1}^{n} Q_i^a \Psi_a} \Big(\partial_- + i \sum_{a=1}^{m} Q_i^a V_a \Big) \Phi_i \\
&- \frac{i}{2} \overline{\Phi}_i \Big(\overleftarrow{\partial}_- - i \sum_{a=1}^{m} Q_i^a V_a \Big) e^{2\sum_{a=1}^{m} Q_i^a \Psi_a} \Phi_i \Big\}.
\end{aligned} \tag{3.1}$$

For simplicity in this expression we are assuming that the chiral superfields are not charged under the global symmetries that the Fermi superfields are charged, and vice-versa. One could choose that each of the chiral superfields to dualize it is charged only under a single U(1) global, such that $Q_{1i}^b = \delta_i{}^b$, as it was done by Hori and Vafa in their fundamental work on mirror symmetry as a T-duality [24]. There are $\mathrm{U}(1)^{k+s}$ global symmetries, where $k + s = n - m + s$. For models coming from supersymmetric reduction s is zero and the Fermi superfield will be gauged with the same global symmetry implemented by the chiral superfields. In the general case there will be additional global symmetries arising due to the Fermi superfields in addition to those due to the chiral superfields in the $(2, 2)$ GLSM.

Let us now analyze the equations of motion from this master Lagrangian when the gauged fields are integrated. Due to the Weiss-Zumino gauge (2.2), $e^{2\Psi} = 1 + 2\Psi$. In this way, the fields Ψ_1, V_1 and Γ_1 are linear and the variation is easy performed. Carrying out the variation of the Lagrangian with respect to ψ_{1b} we obtain for the field V_{1b}:

$$V_{1b} = A_{bd}^{-1} \Big(-\frac{i}{2} \partial_- Y_-^d - R^d \Big), \tag{3.2}$$

where

$$A_{bd} = \sum_{i=1}^{k} |\phi_i|^2 Q_{1i}^d Q_{1i}^b, \tag{3.3}$$

and

$$R^d = \sum_{i=1}^{k} \left(-\frac{i}{2} \overline{\Phi}_i \delta_- \Phi_i Q_{1i}^d + |\Phi_i|^2 \sum_{a=1}^{m} Q_i^a V_a Q_{1i}^d \right). \tag{3.4}$$

Here the new dual variable is defined by: $Y^c_{\pm} \equiv i\overline{D}_+\Lambda^c \pm iD_+\overline{\Lambda}^c$, and for simplicity, it has been used $\delta_- = \partial_- - \overleftarrow{\partial}_-$.

Performing the variation of the Lagrangian with respect to V_{1b}, for the component ψ_{1b} we have

$$\psi_{1b} = A^{-1}_{bd}\Big(-\frac{i}{2}\partial_- Y^d_+ - S^d\Big), \tag{3.5}$$

and

$$S^d = \sum_{i=1}^{k} |\Phi_i|^2 Q^d_{1i} + 2\sum_{a=1}^{m} |\Phi_i|^2 Q^d_{1i} Q^a_{1i}\psi_a. \tag{3.6}$$

The variation with respect the component ψ_{1d} for $d \in \{k+1, \ldots, 2k\}$ yields

$$Q^d_{1j}\overline{\Gamma}_j\Gamma_j = -i\partial_- Y^d_- \to \overline{\Gamma}_j\Gamma_j = -Q^{-1}_{1jd}\partial_- Y^d_-. \tag{3.7}$$

These equations of motion are employed to find the dual model.

3.2 A T-duality algorithm from a model coming from (2,2) reduction

In this subsection we obtain explicit expressions for the equations of motion of a general master Lagrangian of a $(0,2)$ GLSM which is obtained by reduction of a $(2,2)$ model. In this case, there is a Fermi superfield for every chiral superfield and there could be also extra Fermi superfields. These Fermi fields have the same charges under the gauge group than the chiral superfields related to them i.e. $Q_i = \widetilde{Q}_i$ and we consider the case $s = 0$, such that there are no extra Fermi fields. Then all the global symmetries will affect equally the chiral superfields and the Fermi superfields. In this case, the duality procedure will be carry out in the fields Φ and Γ, and there are Lagrange multipliers $\overline{\chi}$ associated to E. So, the new dual fields are $\widetilde{\mathcal{F}} = e^{\Psi}\mathcal{D}_+\chi$ and Y_a.

We start from the following Lagrangian, with n chiral fields and then n Fermi fields (related to them) and without any extra Fermi field. This is $\widetilde{n} = n$ to give:

$$\begin{aligned}
L_{\texttt{master}} =& \int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+ \sum_{a=1}^{m}\frac{1}{8e_a^2}\overline{\Upsilon}_a\Upsilon_a \\
&-\int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+\Big\{\sum_{i=1}^{k}\frac{i}{2}\overline{\Phi}_i e^{2\sum_{a=1}^m Q^a_i\Psi_a + 2\sum_{b=1}^k Q^b_{1i}\Psi_{1b}}\Big(\partial_- + i\sum_{a=1}^{m}Q^a_iV_a + i\sum_{b=1}^{k}Q^b_{1i}V_{1b}\Big)\Phi_i\Big\} \\
&+\int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+\Big\{\sum_{i=1}^{k}\frac{i}{2}\overline{\Phi}_i(\overleftarrow{\partial}_- - i\sum_{a=1}^{m}Q^a_iV_a - i\sum_{b=1}^{k}Q^b_{1i}V_{1b})e^{2\sum_{a=1}^m Q^a_i\Psi_a + 2\sum_{b=1}^k Q^b_{1i}\Psi_{1b}}\Phi_i\Big\} \\
&-\int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+\Big\{\sum_{j=1}^{k}\frac{1}{2}e^{2\sum_{a=1}^m Q^a_j\Psi_a + 2\sum_{b=1}^k Q^b_{1j}\Psi_{1b}}(\overline{\Gamma}_j + \overline{\Gamma}_{1j})(\Gamma_j + \Gamma_{1j})\Big\} \\
&+\sum_{a=1}^{m}\frac{t_a}{4}\int \mathrm{d}\theta^+\Upsilon_a|_{\overline{\theta}^+=0} + \sum_{b=1}^{k}\int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+\Lambda_b\Upsilon_b + \sum_{b=1}^{k}\int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+\bar{\chi}_bE_b + \text{h.c.} \\
&-\frac{i}{2}\int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+\sum_{i=k+1}^{n}\Big\{\overline{\Phi}_i e^{2\sum_{a=1}^m Q^a_i\Psi_a}\Big(\partial_- + i\sum_{a=1}^{m}Q^a_iV_a\Big)\Phi_i \\
&-\overline{\Phi}_i\Big(\overleftarrow{\partial}_- - i\sum_{a=1}^{m}Q^a_iV_a\Big)e^{2\sum_{a=1}^m Q^a_i\Psi_a}\Phi_i\Big\} + \int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+\sum_{j=k+1}^{\widetilde{n}}\frac{1}{2}e^{2\sum_{a=1}^k Q^a_j\Psi_a}\overline{\Gamma}_j\Gamma_j.
\end{aligned} \tag{3.8}$$

JHEP05(2024)088

The terms in the last two lines are not charged under the global (gauged) symmetry so those fields behave as spectators. The main difference with the other case (without reduction) lies on the dualized fields. In this reduced model, the Fermi fields are also gauged and new Lagrange multipliers were added, these terms are located on the 5th line of previous equation. If exists $Q \in \mathrm{GL}(k)$ such that $(Q)_i^c := Q_{1i}^c$, then let be $X := Q^{-1}$ to find the variations, which results in:

$$\begin{aligned} \delta_{V_{1c}} S = 0: \qquad & \left(1+2\sum_{a=1}^{m} Q_j^a \Psi_a + 2\sum_{b=1}^{k} Q_{1j}^b \Psi_{1b}\right) = -\frac{X_c^j Y_+^c}{|\Phi_j|^2}\,, \\ \text{or} \qquad & \Psi_{1d} = \sum_{j=1}^{k} X_d^j \left(\frac{X_c^j Y_+^c}{2|\Phi_j|^2} + \frac{1}{2} + Q_j^a \Psi_a\right), \end{aligned} \tag{3.9}$$

$$\delta_{\Psi_{1c}} S = 0: \qquad -i\overline{\Phi}_j \delta_- \Phi_j + 2\sum_{a=1}^{m} Q_j^a V_a |\Phi_j|^2 + 2\sum_{b=1}^{k} Q_{1j}^b V_{1b} |\Phi_j|^2 = -\frac{i}{2} X_c^j \partial_- Y_-^c, \tag{3.10}$$

$$\delta_{\Gamma_{1j}} S = 0: \qquad \left(1+2\sum_{a=1}^{m} Q_j^a \Psi_a + 2\sum_{b=1}^{k} Q_{1j}^b \Psi_{1b}\right)(\overline{\Gamma}_j + \overline{\Gamma}_{1j}) = -\sqrt{2}\widetilde{\mathcal{F}}_j^\dagger. \tag{3.11}$$

Notice that we have solved the equations for the gauged fields, we denote $\delta_X S = 0$ to the equation of motion obtained for the field X.

From the derived equations of motion, one can obtain the dual Lagrangian for the $(0,2)$ models. In general it is involved to carry out this program, thus we shall consider in this work the simpler models, with some specific values of m, n and $\tilde{n}$. This will be developed in the following section for the case of supersymmetryc reduction and then for the pure $(2,0)$ case.

3.3 GLSMs with a U(1) global symmetry

Now we consider a concrete model described by the GLSM Lagrangian with $m = 1$, two chiral multiplets ($n = 2$) and some Fermi superfields $\tilde{n}$. One of each kind will act as spectator field, besides it will be considered $Q = 1$.

For the implementation of the algorithm of duality we consider only the relevant terms in the Lagrangian; which we call partial Lagrangian $\boldsymbol{\Delta} L_{original}$, where the other terms as the kinetic energies of the gauge fields, the FI terms and the spectator fields are omitted. The Lagrangian is given in (B.4). From this common Lagrangian, the 2 cases: the reduction from $(2,2)$ and the pure $(0,2)$ case are taken.

3.3.1 (0, 2) GLSM from a reduction of a (2, 2) GLSM

As we mentioned before in the case when the $(0,2)$ model is obtained as a reduction from a $(2,2)$ model, all the chiral multiplets have associated an only Fermi field ($s = 0$) and the E field has a special form with the reduced fields given by

$$E = iQ\sqrt{2}\Sigma'\Phi'\,, \tag{3.12}$$

JHEP05(2024)088

where $\Sigma' = \Sigma|_{\theta^- = \bar{\theta}^- = 0}$, and $\Phi' = \Phi|_{\theta^- = \bar{\theta}^- = 0}$. Thus the gauged Lagrangian is written as

$$\begin{aligned}\boldsymbol{\Delta} L_{\texttt{master}} = \int \mathrm{d}\theta^+ \mathrm{d}\bar{\theta}^+ \Big\{ & -\frac{i}{2}\bar{\Phi} e^{2(\Psi+\Psi_1)} \Big(\partial_- + i(V+V_1)\Big)\Phi \\ & + \frac{i}{2}\bar{\Phi}\Big(\overleftarrow{\partial}_- - i(V+V_1)\Big) e^{2(\Psi+\Psi_1)}\Phi \\ & - \frac{1}{2} e^{2(\Psi+\Psi_1)}(\bar{\Gamma}+\bar{\Gamma}_1)(\Gamma+\Gamma_1) + \Lambda\Upsilon_1 + \bar{\Upsilon}_1\bar{\Lambda} + \chi\tilde{E}_1 + \bar{\tilde{E}}_1\chi \Big\}.\end{aligned} \tag{3.13}$$

Thus, the eqs. (3.10)–(3.11) substituted back into (3.13), with the gauge fixing $|\Phi|^2 = 1$, lead to the following Lagrangian:

$$\boldsymbol{\Delta} L_{\texttt{Dual}} = \int \mathrm{d}\theta^+ \mathrm{d}\bar{\theta}^+ \Big\{ -\frac{i}{2}\frac{Y_- \partial_- Y_+}{Y_+} + \frac{\tilde{\mathcal{F}}\bar{\tilde{\mathcal{F}}}}{Y_+} \Big\} - \int \mathrm{d}\bar{\theta}^+ \Big(iY\Upsilon - \bar{E}F \Big) + \text{h.c.}. \tag{3.14}$$

This process can also be realized by components, gauging up each component field to find the scalar potential; this procedure is carried out in appendix A.

To describe the various contributions to the scalar potential coming from the complete dual action (3.14) it is needed the component expansion of the dual fields Y and F which are written in (C.5) and (C.6). Thus as y and H are the bosonic component of each superfield respectively, the scalar potential consist of the complete Lagrangian adding Fayet-Iliopoulos term, gauge action and the spectators fields. The kinetic term of the dual variable Y in the first term of the dual action $-\frac{i}{2}\frac{Y_- \partial_- Y_+}{Y_+}$, does not contribute to the scalar potential. The third term in the Lagrangian $iY\Upsilon_0$ leads to a scalar potential of the form $-2Dy_+ + 2iv_{01}y_- + h.c.$ Moreover, the Fayet-Iliopoulos and Theta term give rise to a potential of the form $D(\frac{it}{2} - \frac{i\bar{t}}{2}) + v_{01}(\frac{t}{2} + \frac{\bar{t}}{2})$. The gauge sector L_{gauge} contributes with a term of the form $\frac{v_{01}^2}{2e^2} + \frac{D^2}{2e^2}$.We have two additional contributions from the terms $\frac{\tilde{\mathcal{F}}\bar{\tilde{\mathcal{F}}}}{Y_+}$ and $-\tilde{\mathcal{F}}E + \text{h.c.}$ which lead to terms of the potential of the form $\frac{-2H\bar{H}}{y_+}$ and $-\sqrt{2}(HE + \overline{EH})$, respectively.

Thus, the scalar potential coming from (3.14) can be written as

$$\begin{aligned} U_{\texttt{dual}} = {} & D\Big(\frac{i}{2}(t-\bar{t}) - 2y_+ + |\phi_2|^2\Big) + \frac{D^2}{2e^2} + \frac{v_{01}^2}{2e^2} + v_{01}\Big(\frac{i}{2}(t+\bar{t}) - 2iy_-\Big) \\ & + \frac{H\bar{H}}{\Re(y)} + \sqrt{2}(HE + \overline{EH}). \end{aligned} \tag{3.15}$$

After eliminating the auxiliary field D and v_{01}, the potential is:

$$\begin{aligned} U_{\texttt{dual}} = {} & \frac{e^2}{2}\Big(-\Im(t) - \Re(y) + |\phi_2|^2\Big)^2 \\ & + \frac{e^2}{2}\Big(\Re(t) + \Im(y)\Big)^2 + \frac{H\bar{H}}{\Re(y)} + \sqrt{2}(HE + \overline{EH}), \end{aligned} \tag{3.16}$$

which minimum condition with respect to H, E and $\Re(y)$ gives $E = 0$, $H = 0$ and:

$$|\phi_2|^2 - \Re(y) = \Im(t), \tag{3.17}$$

while for the original theory, te vacua is:

$$U_{\texttt{original}} = 0 \to |\phi|^2 + |\phi_2|^2 = \Im(t). \tag{3.18}$$

From equation (3.17) one obtains a cone with vertex at $y_+ = -r$. Considering the U(1) gauge symmetry this will lead to the line $\mathbb{R}^+$, such that the dual expected vacua is $\mathbb{R}^+ \times \mathbb{R}$ while for the original model is $\mathbb{P}^1$. Notice that this dual is not the Abelian T-dual leading to the mirror pair obtained in [24, 56].

This T-duality is not mirror symmetry, because we are only performing a dualization in a single field direction. The mirror symmetry can also be obtained by our method, by adding an spectator chiral superfield and gauging two U(1) symmetries, one for each chiral superfield. Although mirror symmetry is obtained by specific dualization of Abelian global symmetries, the whole set of Abelian T-dualities that can be explored is wider. We have discussed these dualities in the (2,2) cases [67].

The superpotential of the original theory is given by [20]:

$$W_{\texttt{original}} = \frac{\Upsilon}{4\pi\sqrt{2}} \ln\left(\frac{\Sigma}{q\mu}\right). \tag{3.19}$$

We propose the following ansatz for superpotential of the dual theory:

$$W_{\texttt{dual}} = iY\Upsilon - \overline{E}F + \beta F e^{\alpha Y}, \tag{3.20}$$

thus we have

$$\begin{aligned} W_{\texttt{dual}} &= \frac{i\Upsilon}{\alpha} \ln\left(\frac{\overline{E}}{\beta}\right) \\ &= \frac{i\Upsilon}{\alpha} \ln\left(\frac{-i\Upsilon_0}{\alpha\beta F}\right). \end{aligned} \tag{3.21}$$

For $(0,2)$ theories coming from a reduction of a (2,2) model with $\overline{E} = -iQ\sqrt{2}\Sigma\Phi$, the non-perturbative dual superpotential is written as [20]

$$W_{\texttt{dual}} = \frac{i\Upsilon}{\alpha} \ln\left(\frac{\Sigma}{\beta/(-iQ\sqrt{2}\Phi)}\right), \tag{3.22}$$

where we can see by comparison with (3.21) the choices of:

$$\alpha = 4i\pi\sqrt{2} \qquad \text{and} \qquad \beta = -iq\mu\sqrt{2}Q\Phi\,. \tag{3.23}$$

3.3.2 The case of a pure $(0,2)$ GLSM

In this section we consider a building block $(0,2)$ model which is not coming from reduction of a $(2,2)$ case, and study the Abelian T-duality on it. In this case we have $m = 1$, $n = 2$, $\tilde{n} = 1$, $k = 1$ and $s = 0$.

For a pure $(0,2)$ Abelian case the model is the same in formula (B.4). But this time the field Γ is not dualized and the gauged fields are only V and Ψ. Thus the dual Lagrangian is given by

$$\boldsymbol{\Delta} L_{\texttt{dual}} = \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \left\{ -\frac{i}{2}\frac{Y_-\partial_- Y_+}{Y_+} + \frac{Y_+\overline{\Gamma}\Gamma}{2|\Phi|^2} \right\} + \int \mathrm{d}\theta^+ (iY\Upsilon) + \frac{t}{4}\int d\theta_+ \Upsilon + \text{h.c.} \tag{3.24}$$

The scalar potential is found to be the same that in the previous case discussed in section 3.1.1, except for the term $\frac{Y_+\overline{\Gamma}\Gamma}{2}$ which contributes to the scalar potential with a term of the form $y_+\overline{G}G - y_+\overline{E}E$. Gathering all that, it results that the scalar potential of the dual theory after eliminating the auxiliary field D is given by:

$$U_{\mathtt{dual}} = \frac{e^2}{2}\Big(-\Im(t) - \Re(y) + |\phi_2|^2\Big)^2 + \frac{e^2}{2}\left[\Re(t) + \Im(y)\right]^2 + 2\Re(y)(E\overline{E} - G\overline{G}), \qquad (3.25)$$

which minimum condition with respect to G, E and $\Re(y)$ gives $E = G = 0$ and:

$$|\phi_2|^2 - \Re(y) = \Im(t). \qquad (3.26)$$

This is precisely the same equation found in the previous case (3.17) and consequently the topology of the manifold of vacua is a also $\mathbb{R}^+ \times \mathbb{R}$. Recall that for the original model the scalar potential reads:

$$|\phi|^2 + |\phi_2|^2 = \Im(t), \qquad (3.27)$$

which together with the U(1) gauge symmetry it constitutes a $\mathbb{P}^1$. Thus, the ansatz for the superpotential in the dual model is

$$\begin{aligned} W_{\mathtt{dual}} &= iY\Upsilon + \beta e^{\alpha(Y+1)} \\ &= \frac{i\Upsilon}{\alpha}\left[\ln\left(\frac{-i\Upsilon}{\alpha\beta}\right)\right]. \end{aligned} \qquad (3.28)$$

Here we have employed an ansatz for the instanton corrections, in order to obtain the same effective potential for the U(1) gauge field as in the $(0,2)$ case coming from a reduction. The symmetries of this non-perturbative ansatz coincide with the case coming from reduction, we will discuss this next. However in this case is not possible to compare to a $(0,2)$ original theory (not coming from reduction), one would have to perform the one-loop computation to correct the D-term; in order to find the effective potential for the U(1) gauge field.

Let us look at the equations of motion for the dual model, for the gauge fields Ψ_1, V_1 in terms of the dual fields Y_+, Y_-, those are:

$$\Psi_1 = \frac{Y_+}{2|\phi|^2} + \frac{1}{2} - \Psi_0, \qquad (3.29)$$

$$V_1 = \frac{i}{2}\frac{\bar{\Phi}\delta_-\Phi}{|\Phi|^2} - \frac{i}{2}\frac{\partial_- Y_-}{|\Phi|^2} - V_0. \qquad (3.30)$$

From here we see that the real part of Y has vev $Y_+ \leq 0$ in the gauge $\Psi_1 + \Psi_0 = 0$. The Lagrangian term of the scalar components of the field strength Υ is given by:

$$\mathcal{L} \supset \frac{v_{01}^2}{2e^2} - iv_{01}\left(\Re(t) + 2y_-\right). \qquad (3.31)$$

From here we see that the Υ field acquires a mass; these will be used as a reference in the following discussion.

We can follow the arguments of Hori and Vafa [24] to propose the non-perturbative correction, based on physical arguments. The arguments are: holomorphicity in t, periodicity in θ, R-symmetry and the asymptotic behaviour. We can arrive to a shape for $W_{\rm dual}$ similar to that of (3.28). The argument of [24] repeated here for our case goes as follows, the superpotential has axial R-symmetry with charge 2. The important objects are the field strength Υ, the energy scale set by the FI and θ parameter $\Lambda = \mu e^{\tilde{t}}, \tilde{t} = 2\pi i t = 2\pi(i\frac{\theta}{2\pi} - r)$ and the dual field $\tilde{Y} = (iY + \frac{t}{4})$. The Langrangian (3.24) has the remnant of following R-symmetry: $\Upsilon \to e^{2i\alpha}\Upsilon$, $\tilde{t} = it/4$ and iY periodic, with period $2\pi i$ and $\Lambda \to e^{2i\alpha}\Lambda$, such that $\alpha = \pi n, n \in \mathbb{Z}$. The non-perturbative correction transforms properly under the R-symmetry if it has the structure $\Delta W = \Upsilon f\left(\frac{\Lambda}{\Upsilon}, \tilde{Y}\right)$ what expanded in Laurent series reads:

$$\begin{aligned}\Delta W &= \Upsilon \sum_{m,n} c_{n,m} \left(\frac{\Lambda}{\Upsilon}\right)^n e^{-im\tilde{Y}} \\ &= \sum_{m,n} c_{n,m} \Upsilon^{1-n} \mu^n e^{(n-m)\tilde{t} + mY}.\end{aligned} \tag{3.32}$$

The dualization implies a massive Υ, coming from the region where the original chiral field satisfies $\phi \neq 0$; thus Υ dependence should be analytic and $n \leq 1$. Then for large r one needs to have a small correction, such that $n - m \geq 0$. Finally since the real part of Y, i.e. $Y_+ \leq 0$, we have $m \geq 0$ to have a small correction. Thus we have $0 \leq m \leq n \leq 1$. The only non trivial solution is $n = m = 1$. Thus the coefficient α in (3.28) is restricted to be 1.

In the $(2,0)$ case coming from reduction the argument to determine which are the non-perturbative corrections to the twisted superpotential W follows the same criteria of symmetry of $(2,2)$ models, explored here for pure $(0,2)$. In addition the models coming from a reduction inherit the arguments of the original $(2,2)$ models. As in the original GLSM there are vertex instantons, several checks of matching are performed; for the example the effective superpotential for the U(1) gauge field is obtained and coincides in the original and the dual model. Second, correlation functions between fermions in both theories coincide, if the non-perturbative corrections are implemented in the dual model [24].

3.3.3 A model with two Abelian gauge symmetries

As an example, let us apply this T-dualization procedure to the case of a $(0,2)$ GLSM coming from a reduction, as discussed in ref. [56]. This is a GLSM with two gauge groups U(1). We have to gauge 4 global symmetries; this is 3 chiral fields Φ and 3 Fermi fields Γ charged under a U(1) gauge symmetry and another 3 chiral $\widetilde{\Phi}$ and Fermi $\widetilde{\Gamma}$ charged under the other U(1). In this case it has to be taken $m = 2$, $n = 6$, $\tilde{n} = 6$, $k = 2$ and $s = 0$. The former example has to give the same dual model of [56], apart from the addition of 2 spectator fields. Let us write the master Lagrangian in general:

$$\begin{aligned}L_{\tt master} = &-\frac{i}{2}e^{2\Psi_1+2\Psi_1'}\overline{\Phi}_1\Big(\partial_- - i(V_1+V_1')\Big)\Phi_1 + \text{h.c.} \\ &-\frac{i}{2}e^{2\Psi_1+2\Psi_2'}\bar{\Phi}_2\Big(\partial_- - i(V_1+V_2')\Big)\Phi_2 + \text{h.c.} \\ &-\frac{i}{2}e^{2\Psi_2+2\Psi_3'}\overline{\widetilde{\Phi}}_1\Big(\partial_- - i(V_2+V_3')\Big)\widetilde{\Phi}_1 + \text{h.c.} \\ &-\frac{i}{2}e^{2\Psi_2+2\Psi_4'}\overline{\widetilde{\Phi}}_2\Big(\partial_- - i(V_2+V_4')\Big)\widetilde{\Phi}_2 + \text{h.c.}\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{2}(\overline{\Gamma}_1+\overline{\Gamma}'_1)e^{2\Psi_1+2\Psi'_1}(\Gamma_1+\Gamma'_1)-\frac{1}{2}(\overline{\Gamma}_2+\overline{\Gamma}'_2)e^{2\Psi_1+2\Psi'_2}(\Gamma_2+\Gamma'_2)\\
&-\frac{1}{2}(\overline{\widetilde{\Gamma}}_1+\overline{\widetilde{\Gamma}}'_1)e^{2\Psi_2+2\Psi'_3}(\widetilde{\Gamma}_1+\widetilde{\Gamma}'_1)-\frac{1}{2}(\overline{\widetilde{\Gamma}}_2+\overline{\widetilde{\Gamma}}'_2)e^{2\Psi_2+2\Psi'_4}(\widetilde{\Gamma}_2+\widetilde{\Gamma}'_2)\\
&+\Lambda_1\Upsilon'_1+\Lambda_2\Upsilon'_2+\Lambda_3\Upsilon'_3+\Lambda_4\Upsilon'_4+\text{h.c.}+\overline{\chi}_1E'_1+\overline{\chi}_2E'_2+\overline{\chi}_3E'_3+\overline{\chi}_4E'_4+\text{h.c.}\\
&-\frac{i}{2}e^{2\Psi_1}\overline{\Phi}_3\Big(\partial_--iV_1\Big)\Phi_3+\text{h.c.}-\frac{i}{2}e^{2\Psi_2}\overline{\widetilde{\Phi}}_3\Big(\partial_--iV_2\Big)\widetilde{\Phi}_3+\text{h.c.}\\
&-\frac{1}{2}e^{2\Psi_1}\overline{\Gamma}_3\Gamma_3-\frac{1}{2}e^{2\Psi_1}\overline{\widetilde{\Gamma}}_3\widetilde{\Gamma}_3+\int d\theta^+d\overline{\theta}^+\Big\{\frac{1}{8e_1^2}\overline{\Upsilon}_1\Upsilon_1+\frac{1}{8e_2^2}\overline{\Upsilon}_2\Upsilon_2\Big\}\\
&+\frac{t_1}{4}\int d\theta^+\Upsilon_1|_{\overline{\theta}^+=0}+\frac{t_2}{4}\int d\theta^+\Upsilon_2|_{\overline{\theta}^+=0}+\text{h.c.}
\end{aligned}\tag{3.33}
$$

The dual fields to the Fermi multiplet are given by $F=\bar{D}_+\chi$, $\mathcal{F}=e^{\psi}F$. The scalar potential, the analysis of the supersymmetric vacua and the instanton corrections will not be discussed here; since for the case of the non-Abelian global symmetry they will be discussed in detail in section 5. The Lagrangian previously obtained constitutes two copies of (3.14), and is exactly the one obtained by [56] excluding the spectator terms. In their work they considered mirror symmetry for $(0,2)$ models coming from a reduction of $(2,2)$.

4 GLSMs with gauge group $\mathrm{U}(1)^m$ and non-Abelian global symmetries

In this section we construct the non-Abelian dual models of $(0,2)$ GLSMs. This can be implemented when there is a non-Abelian global symmetry present. Thus duality algorithm is realized gauging this non-Abelian symmetry and adding Lagrange multipliers which take values in the Lie Algebra of the global group. The only models considered in the present section are assumed to come from a reduction of a $(2,2)$ supersymmetric model thus the number of chiral fields $\mathbf{\Phi}_i$ and the number of Fermi fields $\mathbf{\Gamma}_i$ coincide. They are equally charged under the $\mathrm{U}(1)^m$ gauge group and they are assumed to be also equally charged under the global group. Moreover in order to be as general as possible, we consider models where the total non-Abelian gauged group is $G=G_1\times\cdots\times G_S$. The Lagrangian of this model can be written as

$$
\begin{aligned}
L_{\texttt{master}}=&\int d\theta^+d\overline{\theta}^+\sum_{a=1}^m\frac{1}{8e_a^2}\overline{\Upsilon}_a\Upsilon_a\\
&-\int d\theta^+d\overline{\theta}^+\Big\{\sum_{I=1}^S\frac{i}{2}\mathbf{\Phi}_I^\dagger e^{2\sum_{a=1}^mQ_I^a\Psi_a+2\Psi_{1I}}\Big(\partial_-+i\sum_{a=1}^mQ_I^aV_a+iV_{1I}\Big)\mathbf{\Phi}_I\Big\}\\
&+\int d\theta^+d\overline{\theta}^+\Big\{\sum_{I=1}^S\frac{i}{2}\mathbf{\Phi}_I^\dagger\Big(\overleftarrow{\partial}_--i\sum_{a=1}^mQ_I^aV_a-iV_{1I}\Big)e^{2\sum_{a=1}^mQ_I^a\Psi_a+2\Psi_{1I}}\mathbf{\Phi}_I\Big\}\\
&-\int d\theta^+d\overline{\theta}^+\Big\{\sum_{I=1}^S\frac{1}{2}(\mathbf{\Gamma}_I^\dagger+\mathbf{\Gamma}_{1I}^\dagger)e^{2\sum_{a=1}^mQ_I^a\Psi_a+2\Psi_{1I}}(\mathbf{\Gamma}_I+\mathbf{\Gamma}_{1I})\Big\}\\
&+\int d\theta^+\sum_{a=1}^m\frac{t_a}{4}\Upsilon_a|_{\overline{\theta}^+=0}+\int d\theta^+d\overline{\theta}^+\sum_{I=1}^S\mathrm{Tr}(\Lambda_I\Upsilon_I)+\text{h.c.}\\
&+\int d\theta^+d\overline{\theta}^+\sum_{I=1}^S(\boldsymbol{\chi}_I^\dagger\widetilde{\boldsymbol{E}}_I)+\text{h.c.},
\end{aligned}\tag{4.1}
$$

where $\mathbf{\Phi}_I = (\Phi_{In_1}, \dots, \Phi_{In_I})$, with $I \in \{1, \dots, S\}$, are vectors of chiral superfields, n_I is the dimension of the representation of the Lie algebra of G_I and $V_{1I} = V_{1Ia}\mathcal{T}_a$, $\Psi_{1I} = \Psi_{1Ia}\mathcal{T}_a$ are superfields for each factor gauged group G_I, and it is assumed an inner product. In these definitions, $\mathcal{T}^a$ are the generators of the Lie algebra of G_I. In the notation of the Lagrangian it is understood an inner product on the vector space indexed by the number of factors of the global group, thus we have to sum over the S factors there. For the implementation of the duality algorithm we write the partial Lagrangian given by

$$\begin{aligned}\mathbf{\Delta} L_{\mathtt{master}} = \sum_{I=1}^{S} \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \Big\{ &- \frac{i}{2} e_I \mathbf{\Phi}_I^\dagger \delta_- \mathbf{\Phi}_I + \mathbf{\Phi}^\dagger \mathbf{\Phi} e_I Q_I^\beta V_\beta + V_{1I}^b e_I Z_I^b \\ &+ \Psi_{1I}^a \Big(- i \mathbf{\Phi}_I^\dagger \mathcal{T}^a \delta_- \mathbf{\Phi}_I + 2 Q_I^\beta V_\beta Z_I^a \Big) + \Psi_{1I}^a V_{1I}^b a_I^{ab} \\ &- \frac{1}{2} (\mathbf{\Gamma}_I^\dagger + \mathbf{\Gamma}_I^{a\dagger} \mathcal{T}^a)(e_I + 2\Psi_{1I}^a \mathcal{T}^a)(\mathbf{\Gamma}_I + \mathbf{\Gamma}_I^a \mathcal{T}^a) \\ &+ \Big(V_{1I}^b Y_{+a} + i \Psi_{1I}^b \partial_- Y_{-I}^a \Big) \mathrm{Tr}(\mathcal{T}^a \mathcal{T}^b) \\ &- \frac{\sqrt{2}}{2} \Big(\mathbf{\Gamma}_{1I}^{a\dagger} \mathcal{T}^a \mathcal{T}^b \widetilde{\boldsymbol{\mathcal{F}}}_I^b + \widetilde{\boldsymbol{\mathcal{F}}}_I^{a\dagger} \mathcal{T}^a \mathcal{T}^b \mathbf{\Gamma}_{1I}^b \Big) \Big\}, \end{aligned} \tag{4.2}$$

which is basically the sum of Lagrangians corresponding to each factor of the global group G_I. In the previous Lagrangian we have the following definitions: $a_I^{ab} := \mathbf{\Phi}_I^\dagger \{\mathcal{T}^a, \mathcal{T}^b\} \mathbf{\Phi}_I$, $e_I := 1_I + 2\sum_{\alpha=1}^m Q_I^\alpha \Psi_\alpha$, $Z_I^a := \mathbf{\Phi}_I^\dagger \mathcal{T}^a \mathbf{\Phi}_I$.

The variations with respect to V_{1I}^c, Ψ_{1I}^c and Γ_{1I}^c, give the following equations of motion:

$$\delta_{V_{1I}^c} S = 0: \qquad \Psi_{1I}^a a^{ca} = -Y_{+Ia} \mathrm{Tr}(\mathcal{T}^a \mathcal{T}^c) - e_I Z_I^c := K_I^c, \tag{4.3}$$

$$\begin{aligned}\delta_{\Psi_{1I}^c} S = 0: \qquad & V_{1I}^b a_I^{bc} + 2 Q_I^\beta V_\beta Z_I^c - i \Phi_I^\dagger \mathcal{T}^c \delta_- \Phi_I + i \partial_- Y_{-aI} \mathrm{Tr}(\mathcal{T}^a \mathcal{T}^c) \\ & - (\Gamma_I^\dagger + \Gamma_I^{a\dagger} \mathcal{T}^a) \mathcal{T}^b (\Gamma_I + \Gamma_I^c \mathcal{T}^c) = 0, \end{aligned} \tag{4.4}$$

$$\delta_{\Gamma_{1I}^c} S = 0: \qquad -\frac{1}{2}(\Gamma_I^\dagger + \Gamma_{1I}^{a\dagger} \mathcal{T}^a)(e_I + 2\Psi_{1I}^b \mathcal{T}^b)\mathcal{T}^c - \frac{\sqrt{2}}{2} \widetilde{\mathcal{F}}_I^{a\dagger} \mathcal{T}^a \mathcal{T}^c = 0. \tag{4.5}$$

Thus the corresponding partial dual Lagrangian becomes

$$\begin{aligned}\mathbf{\Delta} L_{\mathtt{dual}} = \sum_{I=1}^{S} \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \Big\{ &- \frac{i}{2} e_I \mathbf{\Phi}_I^\dagger \delta_- \mathbf{\Phi}_I + \mathbf{\Phi}_I^\dagger \mathbf{\Phi}_I e_I Q_I^\beta V_\beta + \widetilde{\boldsymbol{\mathcal{F}}}_I^\dagger X_I^{-1} \widetilde{\boldsymbol{\mathcal{F}}}_I \\ &+ \frac{\sqrt{2}}{2} (\widetilde{\boldsymbol{\mathcal{F}}}_I^\dagger \mathbf{\Gamma}_I + \mathbf{\Gamma}_I^\dagger \widetilde{\boldsymbol{\mathcal{F}}}_I) \Big(- i \mathbf{\Phi}_I^\dagger \delta_- \mathcal{T}^a \mathbf{\Phi}_I + 2 Q_I^\beta V_\beta Z_I^a \Big) \\ &\times \Big(- Y_{+Ib} \mathrm{Tr}(\mathcal{T}^b \mathcal{T}^c) - e_I Z_I^c \Big) b_{ac} \Big\}, \end{aligned} \tag{4.6}$$

where b^{ac} is the inverse of a^{cd}, and $X_I := e_I + 2\mathcal{T}^a K_I^a = e_I - 2\mathcal{T}^a e_I Z_I^c b^{ca} - 2\mathcal{T}^a Y_{+b} \mathrm{Tr}(\mathcal{T}^b \mathcal{T}^c) b^{ca}$. Still, it is necessary to remove the original chiral fields $\mathbf{\Phi}$ from the Lagrangian, step which will be implemented through the process of gauge fixing. Up to this point we have used generic well behaved Lie groups G_i, it has not been necessary to specify them.

4.1 A model with SU(2) global symmetry

Let us consider the case of a model with global symmetry $G = \mathrm{SU}(2)$. Before proceeding, some definitions and conventions related to the SU(2) group are introduced for future reference. Hence, the following relations hold: $\mathrm{Tr}(T^aT^b) = 2\delta^{ab}$, $\{T^a, T^b\} = 2\delta^{ab}\mathbf{I_d}$ for the generators of the Lie algebra of the group SU(2); in this way we have the relations $a^{ab} = 2|\Phi|^2$ and $b^{bc} = \frac{1}{2|\Phi|^2}$. Consequently, with $e_I := 1_I + 2\sum_{\alpha=1}^m Q_I^\alpha \Psi_\alpha$, it is obtained that $X_I := e_I\mathbf{I_d} - T^a \frac{e_I Z_I^a + 2Y_+^a}{|\Phi_I|^2}$.

For the specific model with a U(1) gauge group and an SU(2) global symmetry the partial Lagrangian is given by

$$\begin{aligned}\mathbf{\Delta} L_{\mathtt{master}} = \sum_{I=1}^{s} \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \Bigg\{ \Big(& -i\mathbf{\Phi}_I^\dagger \delta_- \mathcal{T}^a \mathbf{\Phi}_I + Q_\beta V^\beta \Big) \\ \times & \left(e_I |\mathbf{\Phi}_I|^2 - \frac{e_I}{|\mathbf{\Phi}_I|^2} Z^a Z_a - Y_+^a \frac{Z^a}{|\mathbf{\Phi}_I|^2} \right) \\ + & \widetilde{\boldsymbol{\mathcal{F}}}^\dagger X^{-1} \widetilde{\boldsymbol{\mathcal{F}}} + \frac{\sqrt{2}}{2} (\widetilde{\boldsymbol{\mathcal{F}}}_I^\dagger \mathbf{\Gamma}_I + \mathbf{\Gamma}_I^\dagger \widetilde{\boldsymbol{\mathcal{F}}}_I) - \frac{i}{2} e_I \mathbf{\Phi}_I^\dagger \delta_- \mathbf{\Phi}_I \Bigg\}. \end{aligned} \tag{4.7}$$

For future convenience the fields $\mathbf{\Phi} = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$, which are 2 complex fields, can be redefined in terms of new fields Z_0, Z_1, Z_2, Z_3, with the transformation:

$$Z_0 = \overline{\Phi}_1\Phi_1 + \overline{\Phi}_2\Phi_2, \qquad Z_1 = 2\Re(\overline{\Phi}_1\Phi_2), \qquad Z_2 = 2\Im(\overline{\Phi}_1\Phi_2), \qquad Z_3 = \overline{\Phi}_1\Phi_1 - \overline{\Phi}_2\Phi_2. \tag{4.8}$$

Then, the original chiral fields can be eliminated by gauge fixing the Z's, these are 4 real constants; and with the inverse transformation, the products of the original fields are written as sums of these new fields:

$$\overline{\Phi}_1\Phi_1 = \frac{Z_0 + Z_3}{2}, \qquad \overline{\Phi}_1\Phi_2 = \frac{Z_1 + iZ_2}{2}, \qquad \overline{\Phi}_2\Phi_2 = \frac{Z_0 - Z_3}{2}. \tag{4.9}$$

Thus, with the partial gauge fixing: $\Phi_I^\dagger T^b \partial_- \Phi_I = \partial_- \Phi_I^\dagger T^b \Phi_I$, for $b \in \{0, 1, 2, 3\}$, the partial dual Lagrangian has the following form

$$\begin{aligned}\mathbf{\Delta} L_{\mathtt{dual}} = \sum_{I=1}^{s} \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \Bigg\{ & Q_\beta V^\beta \left(e_I - e_I \frac{Z^a Z_a}{Z_0} - \frac{Y_+^a Z^a}{Z_0} \right) \\ & + \widetilde{\boldsymbol{\mathcal{F}}}^\dagger \left(e_I \mathbf{I_d} - \frac{\mathcal{T}^a}{Z_0} (e_I Z_I^a + 2Y_+^a) \right)^{-1} \widetilde{\boldsymbol{\mathcal{F}}} \\ & + \frac{\sqrt{2}}{2} \left(\widetilde{\boldsymbol{\mathcal{F}}}_I^\dagger \mathbf{\Gamma}_I + \mathbf{\Gamma}_I^\dagger \widetilde{\boldsymbol{\mathcal{F}}}_I \right) \Bigg\} + \frac{t}{4} \int \mathrm{d}\theta^+ \Upsilon|_{\overline{\theta}^+ = 0}. \end{aligned} \tag{4.10}$$

To write the scalar potential in a more convenient form, the variable can be defined as $u^a = 2\frac{y_+^a}{Z_0} + \frac{Z_a}{Z_0}$. Therefore, the new dual coordinate is denoted as u^a. After eliminating the auxiliary field D, H and v_- and v_+ we have

$$\begin{aligned} U_{\mathtt{dual}} = & \frac{e^2}{2} \left(r + 1 + \frac{Z^a Z_a}{2Z_0^2} - \frac{u^a Z_a}{2Z_0^2} \right)^2 + \frac{u^c u_c - \frac{2u^c Z_c}{Z_0} + \frac{2Z^a Z_a}{Z_0^2} - 1}{1 - u^c u_c} \\ & \times \left[|E_1|^2 (1 - u_3) + |E_2|^2 (1 + u_3) - E_1 \overline{E}_2 \overline{u}_{12} - E_2 \overline{E}_1 u_{12} \right]. \end{aligned} \tag{4.11}$$

If $A = \frac{-2y^b_+ y^b_+}{1-u_c u^c}\begin{pmatrix} u_3-1 & u_{12} \\ \bar{u}_{12} & -1-u_3 \end{pmatrix}$, then this previous condition is rewritten in the following form

$$U_{\texttt{dual}} = \frac{e^2}{2}\left(\Im(t) - y^a_+ Z_a\right)^2 + (\overline{E}_1 \quad \overline{E}_2)A\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = 0\,. \tag{4.12}$$

This analysis is valid in the IR when the vector fields are integrated.

For future convenience let us write the original scalar potential:

$$U_{\texttt{original}} = \frac{e^2}{2}\left(\sum_i Q_i|\phi_i|^2 - r\right)^2 + \sum_a |E_a|^2. \tag{4.13}$$

Notice that the dependence on the fields E is similar in the original and in the dual model. With the difference that in (4.12) this term is positive definite only in a bounded region of the moduli space. The vacua manifold $\mathcal{W}$ is characterized by the 3 coordinates: y^a_+, and there is one equation for the vacua, thus it is a two-dimensional surface. The Y_- term does not appear on the Lagrangian, then y_- is not a coordinate in the potential. The eigenvalues of the matrix A are: $\lambda_\pm = \frac{2y^a_+ y^a_+}{1\mp\sqrt{u^a u^a}}$. So, because A is Hermitian, there exists a unitary matrix P such that $A = P^\dagger D P$, and D is the diagonal matrix with eigenvalues as entries; therefore:

$$\widetilde{\boldsymbol{E}}^\dagger A \widetilde{\boldsymbol{E}} = \widetilde{\boldsymbol{E}}^\dagger P^\dagger D P \widetilde{\boldsymbol{E}} = (P\widetilde{\boldsymbol{E}})^\dagger D (P\widetilde{\boldsymbol{E}}) = \lambda_+|(P\widetilde{\boldsymbol{E}})_+|^2 + \lambda_-|(P\widetilde{\boldsymbol{E}})_-|^2, \tag{4.14}$$

which is a quadratic form. Thus, the vacua manifold $\mathcal{W}$ is made up of 3 regions depending on whether $y^a_+ y^a_+ + Z^a y^a_+$ is greater than, less than, or equal to 0, these regions are: the inside of a sphere, the outside of it, and the shell of the sphere. Thus, we have three cases:

- Region 1:

$$y^a y^a + Z^a y^a < 0, \qquad \Im(t) = y^a_+ Z^a \quad \text{and} \quad |\widetilde{\boldsymbol{E}}|^2 = 0. \tag{4.15}$$

- Region 2:

$$y^a y^a + Z^a y^a = 0, \qquad \Im(t) = y^a_+ Z^a \quad \text{and} \quad |(P\widetilde{\boldsymbol{E}})_-|^2 = 0. \tag{4.16}$$

- Region 3:

$$y^a y^a + Z^a y^a > 0, \qquad \frac{e}{2}(\Im(t) - y^a_+ Z_a)^2 + \lambda_-|(P\widetilde{\boldsymbol{E}})_-|^2 = -\lambda_+|(P\widetilde{\boldsymbol{E}})_+|^2, \tag{4.17}$$

where:

$$(P\widetilde{\boldsymbol{E}})_\pm = \frac{\mp u_{12}E_1 + (\sqrt{u_c u^c} \pm u_3)E_2}{\sqrt{2\sqrt{u_c u^c}(\sqrt{u_c u^c} \pm u_3)}}. \tag{4.18}$$

As we have three real variables y_1, y_2, y_3, then the vacua $\mathcal{W}$ consist of a two-dimensional surface. For the regions 1 and 2, the potential is semi-definite positive and the surface $y^a_+ Z_a = \Im(t)$ is a plane inside the sphere, thus this is a disk $\mathbf{D}$, where the modulus $r = \Im(t)$ determines the size of the disk (as its relative position inside the sphere). It is important to notice that if $r \notin [-1, 0]$ this disk is empty. However, for the case outside the sphere, one has a surface given by equation (4.17). As a solution of equation (4.12). In figure 1 the geometry of the dual vacua is represented for $r = \Im(t) = -\frac{1}{2}$ while in figure 2 the dual vacua is represented for $r = \Im(t) = 1$; in both cases the rest of the parameters are $e = 5$, $z_1 = 0.700629$, $z_2 = 0.509037$, $z_3 = \frac{1}{2}$, $E_1 = 1 + 1i$, $E_2 = 3 + 4i$ and $z_0 = 1$.

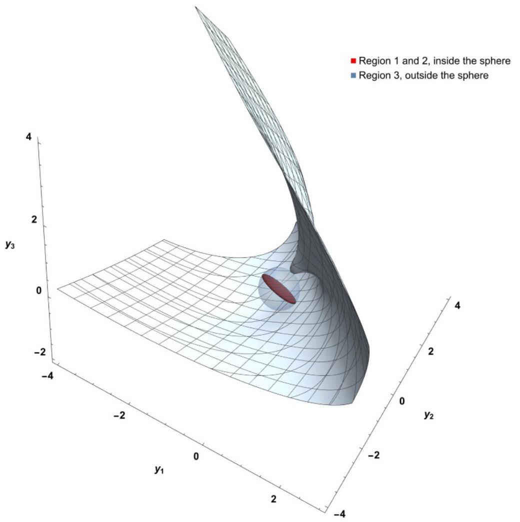

Figure 1. Vacua of the dual model, with parameter $r = \Im(t) = -\frac{1}{2}$. Notice that the change of this parameter changes the topology of the dual space.

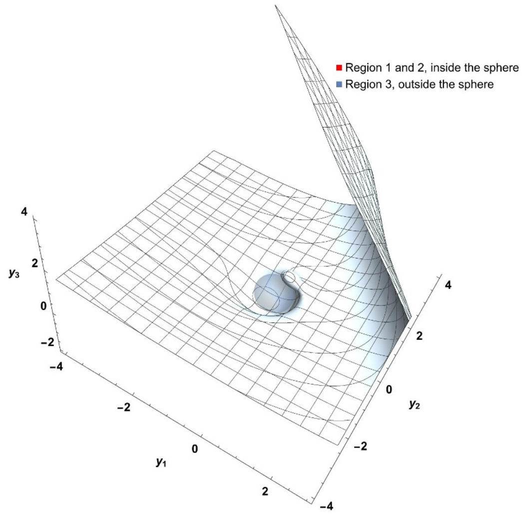

Figure 2. Vacua of the dual model, with $r = \Im(t) = 1$. Notice that the change of this parameter changes the topology of the dual space.

It can be seen that for $r \in [-1, 0]$ the vacua space has the topology of $\mathbb{R}^2 \cup \mathbf{D}$, while for $r \notin [-1, 0]$ the vacua has simply the topology of $\mathbb{R}^2$, although it is geometrically distinct from $\mathbb{R}^2$.

Until this point the superfield E has been arbitrary and it is a function only on Φ (which is gauge fixed) and other parameters; however, it can be chosen a particular form of it that comes from the $(2, 2)$ reduction:

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \Sigma \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} , \tag{4.19}$$

where $\Sigma = \sigma + \sqrt{2}\theta^+\overline{\lambda}_+ - i\theta^+\overline{\theta}^+\partial_+\sigma$, this means for region 1: $|\sigma|^2 = 0$.

It is remarkable that in this case the equations of motion (4.3), (4.4) and (4.5) can be exactly solved, without requiring to project out to an Abelian component (or to particularize to a semichiral vector field) as in the $(2, 2)$ supersymmetric non-Abelian T-duality, as well for the SU(2) group [67].

4.1.1 Instanton correction

The Lagrangian with the instanton correction is given by

$$\begin{aligned} L_{\mathtt{dual}} = \sum_{I=1}^{s} \int d\theta^+ d\overline{\theta}^+ \Big\{ & Q_\beta V^\beta \Big(e_I - e_I \frac{Z^a Z_a}{Z_0} - \frac{Y_+^a Z^a}{Z_0} \Big) \\ & + \widetilde{\boldsymbol{\mathcal{F}}}^\dagger \Big(e_I \mathbf{I_d} - \frac{\mathcal{T}^a}{Z_0} (e_I Z_I^a + 2Y_+^a) \Big)^{-1} \widetilde{\boldsymbol{\mathcal{F}}} + \frac{\sqrt{2}}{2} \Big(\widetilde{\boldsymbol{\mathcal{F}}}_I^\dagger \boldsymbol{\Gamma}_I + \boldsymbol{\Gamma}_I^\dagger \widetilde{\boldsymbol{\mathcal{F}}}_I \Big) \Big\} \\ & + \int d\theta^+ \Big\{ \frac{t}{4} \Upsilon|_{\overline{\theta}^+=0} + \widetilde{\boldsymbol{\mathcal{F}}}^\dagger \beta e^{\alpha^b Y_b} \Big\}, \end{aligned} \tag{4.20}$$

JHEP05(2024)088

where the last term is the instanton correction and its contribution to the bosonic scalar potential is:

$$\int d\theta^+ \widetilde{\boldsymbol{\mathcal{F}}}^\dagger \boldsymbol{\beta} e^{\alpha^b Y_b} = -\sqrt{2}(\overline{H}_0\beta^0 + \overline{H}_1\beta^1)e^{\alpha_b y_+^b}. \tag{4.21}$$

Then, the new vacua equation is:

$$\frac{e}{2}\left(\Im(t) - y_+^a Z_a\right)^2 + \left(\widetilde{\boldsymbol{E}} - e^{\alpha_b y_+^b}\boldsymbol{\beta}\right)^\dagger A\left(\widetilde{\boldsymbol{E}} - e^{\alpha_b y_+^b}\boldsymbol{\beta}\right) = 0\,, \tag{4.22}$$

and similarly, when $0 > y_+^a y_+^a + y_+^a Z_a$ the solution gives:

$$\frac{e}{2}\left(\Im(t) - y_+^a Z_a\right)^2 = 0 \qquad \text{and} \qquad |\varepsilon_1|^2(1-u_3)+|\varepsilon_2|^2(1+u_3)-\varepsilon_1\overline{\varepsilon}_2\overline{u}_{12}-\varepsilon_2\overline{\varepsilon}_1 u_{12}=0, \tag{4.23}$$

where $\boldsymbol{\varepsilon} = \widetilde{\boldsymbol{E}} - e^{\alpha_b y_+^b}\boldsymbol{\beta}$. Notice that the effect of the instanton in the effective potential is just a displacement of the holomorphic function E. Therefore the dual geometry coincides with the analysis performed without instanton corrections. This is a common point with observations of the dualities in the $(2,2)$ GLSMs [67].

5 A model with global symmetry SU(2) × SU(2)

In this section we study a generalization of the model presented in [56] which consist of a GLSM with gauge symmetry $\mathrm{U}(1)_1 \times \mathrm{U}(1)_2$, two chiral fields Φ_1, Φ_2 and two Fermi Γ_1, Γ_2 with charge 1 under the first factor of the gauge symmetry $\mathrm{U}(1)_1$; as well as two chiral fields $\widetilde{\Phi}_1$, $\widetilde{\Phi}_2$ and two Fermi $\widetilde{\Gamma}_1$, $\widetilde{\Gamma}_2$ with charge 1 under the $\mathrm{U}(1)_2$ gauge group. This a deformation of a $(2,2)$ model into a $(0,2)$ model, so the restrictions for the fields E's are:

$$\begin{aligned}
E_1 &= \sqrt{2}\{\Phi_1\Sigma + \widetilde{\Sigma}(\alpha_1\Phi_1 + \alpha_2\Phi_2)\},\\
E_2 &= \sqrt{2}\{\Phi_2\Sigma + \widetilde{\Sigma}(\alpha_1'\Phi_1 + \alpha_2'\Phi_2)\},\\
\widetilde{E}_1 &= \sqrt{2}\{\widetilde{\Phi}_1\widetilde{\Sigma} + \Sigma(\beta_1\widetilde{\Phi}_1 + \beta_2\widetilde{\Phi}_2)\},\\
\widetilde{E}_2 &= \sqrt{2}\{\widetilde{\Phi}_2\widetilde{\Sigma} + \Sigma(\beta_1'\widetilde{\Phi}_1 + \beta_2'\widetilde{\Phi}_2)\},
\end{aligned} \tag{5.1}$$

where α, α', β and β' are real parameters. In the limit when the α's and β's parameters vanish the reduced $(0,2)$ model is recovered. The Lagrangian is given in (B.5) and the scalar potential is given by

$$U_{\texttt{original}} = \frac{e^2}{2}\left(|\phi_1|^2+|\phi_2|^2-r_1\right)^2+\frac{e^2}{2}\left(|\widetilde{\phi}_1|^2+|\widetilde{\phi}_2|^2-r_2\right)^2+|E_1|^2+|E_2|^2+|\widetilde{E}_1|^2+|\widetilde{E}_2|^2. \tag{5.2}$$

The vacuum solution for this model is [56]:

$$|\phi_1|^2 + |\phi_2|^2 = r_1, \qquad |\widetilde{\phi}_1|^2 + |\widetilde{\phi}_2|^2 = r_2, \tag{5.3}$$

i.e., the vacua manifold is a product of $\mathbb{P}^1 \times \mathbb{P}^1$ with Kähler classes r_1 and r_2 respectively, and

$$E_i = \widetilde{E}_i = 0. \tag{5.4}$$

In the SU(2) × SU(2) generalization both chiral fields and Fermi fields are SU(2) multiplets related to a different SU(2) sector. Let us write the master Lagrangian

$$
\begin{aligned}
\boldsymbol{\Delta} L_{\texttt{master}} = & \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \sum_{i=1}^{2} \frac{1}{8e_i^2} \overline{\Upsilon}_i \Upsilon_i + \int \mathrm{d}\theta^+ \sum_{i=1}^{2} \frac{t_i}{4} \Upsilon_i|_{\overline{\theta}^+=0} \\
& - \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \frac{i}{2} \overline{\Phi} e^{2\Psi_1+2\Psi_{1a}T_a} \Big(\partial_- + iV_1 + iV_{1a}T_a\Big) \Phi \\
& - \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \frac{i}{2} \overline{\widetilde{\Phi}} e^{2\Psi_2+2\Psi_{2a}T_a} \Big(\partial_- + iV_1 + iV_{2a}T_a\Big) \widetilde{\Phi} \\
& + \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \left\{ \frac{i}{2} \overline{\Phi} \Big(\overleftarrow{\partial}_- - iV_1 - iV_{1a}T_a\Big) e^{2\Psi_1+2\Psi_{1a}T_a} \Phi \right\} \\
& + \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \left\{ \frac{i}{2} \overline{\widetilde{\Phi}} \Big(\overleftarrow{\partial}_- - iV_2 - iV_{2a}T_a\Big) e^{2\Psi_2+2\Psi_{2a}T_a} \widetilde{\Phi} \right\} \\
& - \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \left\{ \frac{1}{2} (\overline{\Gamma} + \overline{\Gamma}_1) e^{2\Psi_1+2\Psi_{1a}T_a} (\Gamma + \Gamma_1) \right\} \\
& - \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \left\{ \frac{1}{2} (\overline{\widetilde{\Gamma}} + \overline{\widetilde{\Gamma}}_2) e^{2\Psi_2+2\Psi_{2a}T_a} (\widetilde{\Gamma} + \widetilde{\Gamma}_2) \right\} \\
& + \sum_{i=1}^{2} \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \mathrm{Tr}(\Lambda_i \Upsilon_i) + \sum_{i=1}^{2} \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \mathrm{Tr}(\widetilde{\Lambda}_i \Upsilon_i) + \text{h.c.} \\
& + \sum_{i=1}^{2} \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \overline{\chi}_i E_i + \sum_{i=1}^{2} \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \overline{\widetilde{\chi}}_i E_i + \text{h.c.}
\end{aligned}
\tag{5.5}
$$

Let us consider the following ansatz for the deformation of the (2,2) model in which α and β are the parameters of the deformation:

$$
\begin{aligned}
\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} &= \Sigma_0 \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} + \widetilde{\Sigma}_0 \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \alpha_1 + \Sigma \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \alpha_2, \\
\begin{pmatrix} \widetilde{E}_1 \\ \widetilde{E}_2 \end{pmatrix} &= \widetilde{\Sigma}_0 \begin{pmatrix} \widetilde{\Phi}_1 \\ \widetilde{\Phi}_2 \end{pmatrix} + \Sigma_0 \begin{pmatrix} \widetilde{\Phi}_1 \\ \widetilde{\Phi}_2 \end{pmatrix} \beta_1 + \widetilde{\Sigma} \begin{pmatrix} \widetilde{\Phi}_1 \\ \widetilde{\Phi}_2 \end{pmatrix} \beta_2.
\end{aligned}
\tag{5.6}
$$

This implies that (E_1, E_2) and $(\widetilde{E}_1, \widetilde{E}_2)$ are vectors under $\mathrm{SU}(2)_1$ and $\mathrm{SU}(2)_2$ respectively. As well (E_1, E_2) and $(\widetilde{E}_1, \widetilde{E}_2)$ are charged with charges 1 under the $\mathrm{U}(1)_1$ and $\mathrm{U}(1)_2$ respectively. Then, the dual Lagrangian becomes:

$$
\begin{aligned}
\boldsymbol{\Delta} L_{\texttt{dual}} = & \int \mathrm{d}\theta^+ \mathrm{d}\overline{\theta}^+ \Bigg\{ \left[Ve - Ve \frac{Z^a Z_a}{Z_0} - \frac{V Y_+^a Z^a}{Z_0} \right] + \widetilde{\boldsymbol{\mathcal{F}}}^\dagger \left(e\mathbf{I_d} - \frac{\mathcal{T}^a}{Z_0} \left(eZ^a + 2Y_+^a \right) \right)^{-1} \widetilde{\boldsymbol{\mathcal{F}}} \\
& + \left[\widetilde{V}\widetilde{e} - \widetilde{V}\widetilde{e} \frac{\widetilde{Z}^a \widetilde{Z}_a}{\widetilde{Z}_0} - \frac{\widetilde{V} \widetilde{Y}_+^a \widetilde{Z}^a}{\widetilde{Z}_0} \right] + \widetilde{\boldsymbol{\mathcal{F}}}^\dagger \left(\widetilde{e}\mathbf{I_d} - \frac{\mathcal{T}^a}{\widetilde{Z}_0} \left(\widetilde{e}\widetilde{Z}^a + 2\widetilde{Y}_+^a \right) \right)^{-1} \widetilde{\boldsymbol{\mathcal{F}}} \Bigg\} \\
& + \frac{t}{4} \int \mathrm{d}\theta^+ \Upsilon|_{\overline{\theta}^+=0} - \int \mathrm{d}\theta^+ \left[\left(\Sigma_0 + \alpha_1 \widetilde{\Sigma}_0 \right) \widetilde{\boldsymbol{\mathcal{F}}}^\dagger \boldsymbol{\Phi} + \alpha 2 \widetilde{\boldsymbol{\mathcal{F}}}^\dagger \Sigma \boldsymbol{\Phi} \right] \\
& + \frac{\widetilde{t}}{4} \int \mathrm{d}\theta^+ \widetilde{\Upsilon}|_{\overline{\theta}^+=0} - \int \mathrm{d}\theta^+ \left[\left(\widetilde{\Sigma}_0 + \beta_1 \Sigma_0 \right) \widetilde{\boldsymbol{\mathcal{F}}}^\dagger \widetilde{\boldsymbol{\Phi}} + \beta 2 \widetilde{\boldsymbol{\mathcal{F}}}^\dagger \widetilde{\Sigma} \widetilde{\boldsymbol{\Phi}} \right],
\end{aligned}
\tag{5.7}
$$

where the $\boldsymbol{\Phi}$ is fixed (in terms of the Z-parameters) with (4.9).

For the scalar potential we have:

$$\begin{aligned}U_{\mathtt{dual}} =& -e\Big(-y^a_+ Z_a + \Im(t)\Big)^2 - \tilde{e}\Big(-\tilde{y}^a_+ \tilde{Z}_a + \Im(\tilde{t})\Big)^2 \\ &+ \frac{1}{2y^a_+ y_{a+}}\Big[\overline{H}_1 H_1 + \overline{H}_2 H_2 + \Big(\overline{H}_1 H_1 - \overline{H}_2 H_2\Big)\Big(Z^3 + 2y^3_+\Big) \\ &\qquad + \overline{H}_2 H_1\Big(2\overline{w} + \overline{Z}^{12}\Big) + \text{h.c.}\Big] \\ &+ \frac{1}{2\widetilde{y}^a_+ \widetilde{y}_{a+}}\Big[\overline{\widetilde{H}}_1 \widetilde{H}_1 + \overline{\widetilde{H}}_2 \widetilde{H}_2 + \Big(\overline{\widetilde{H}}_1 \widetilde{H}_1 - \overline{\widetilde{H}}_2 \widetilde{H}_2\Big)\Big(\widetilde{Z}^3 + 2\widetilde{y}^3_+\Big) \\ &\qquad + \overline{\widetilde{H}}_2 \widetilde{H}_1\Big(2\overline{w} + \overline{Z}^{12}\Big) + \text{h.c.}\Big] \\ &+ \sqrt{2}\Big[(\sigma_0 + \alpha_1\widetilde{\sigma}_0)(\overline{H}_1\phi_1 + \overline{H}_2\phi_2) + \alpha_2\overline{H}_1(\sigma^{11}\phi_1 + \sigma^{12}\phi_2) \\ &\qquad + \alpha_2\overline{H}_2(\sigma^{21}\phi_1 + \sigma^{22}\phi_2) + \text{h.c}\Big] \\ &+ \sqrt{2}\Big[(\widetilde{\sigma}_0 + \beta_1\sigma_0)\Big(\overline{\widetilde{H}}_1\widetilde{\phi}_1 + \overline{\widetilde{H}}_2\widetilde{\phi}_2\Big) + \beta_2\overline{\widetilde{H}}_1\Big(\widetilde{\sigma}^{11}\widetilde{\phi}_1 + \widetilde{\sigma}^{12}\widetilde{\phi}_2\Big) \\ &\qquad + \beta_2\overline{\widetilde{H}}_2\Big(\widetilde{\sigma}^{21}\widetilde{\phi}_1 + \widetilde{\sigma}^{22}\widetilde{\phi}_2\Big) + \text{h.c}\Big].\end{aligned} \tag{5.8}$$

Thus, the bosonic scalar potential depends of 6 coordinates y^a_+ and $\widetilde{y}^a_+$, and the vacua $U_{\mathtt{dual}} = 0$ after the minimum condition for H's gives:

$$\begin{aligned}U_{\mathtt{dual}} =& \frac{e}{2}\Big(\Im(t) - y^a_+ Z_a\Big)^2 + (\overline{E}_1 \quad \overline{E}_2) A \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \\ &+ \frac{e}{2}\Big(\Im(\bar{t}) - \overline{y}^a_+ \widetilde{Z}_a\Big)^2 + (\overline{\widetilde{E}}_1 \quad \overline{\widetilde{E}}_2)\overline{A}\begin{pmatrix} \widetilde{E}_1 \\ \widetilde{E}_2 \end{pmatrix} = 0,\end{aligned} \tag{5.9}$$

with $A = \frac{-2y^b_+ y^b_+}{1-u_c u^c}\begin{pmatrix} u_3 - 1 & u_{12} \\ \overline{u}_{12} & -1-u_3 \end{pmatrix}$, $\widetilde{A} = \frac{-2\widetilde{y}^b_+ \widetilde{y}^b_+}{1-\widetilde{u}_c \widetilde{u}^c}\begin{pmatrix} \widetilde{u}_3 - 1 & \widetilde{u}_{12} \\ \overline{\widetilde{u}}_{12} & -1-\widetilde{u}_3 \end{pmatrix}$ and

$$\begin{aligned} E_1 &= [\sigma_0 + \alpha_1\widetilde{\sigma}_0 + \alpha_2(\sigma_{11} + \sigma_{12})]\phi_1, \quad E_2 = [\sigma_0 + \alpha_1\widetilde{\sigma}_0 + \alpha_2(\sigma_{21} + \sigma_{22})]\phi_2\,, \\ \widetilde{E}_1 &= [\widetilde{\sigma}_0 + \beta_1\sigma_0 + \beta_2(\widetilde{\sigma}_{11} + \widetilde{\sigma}_{12})]\widetilde{\phi}_1, \quad \widetilde{E}_2 = [\widetilde{\sigma}_0 + \beta_1\sigma_0 + \beta_2(\widetilde{\sigma}_{21} + \widetilde{\sigma}_{22})]\widetilde{\phi}_2\,, \end{aligned} \tag{5.10}$$

which as in the previous case, each one is positive quadratic form when $0 < 1 - u^a u_a$ and $0 < 1 - \widetilde{u}^a\widetilde{u}_a$. If it is this case, the solution is

$$\begin{aligned} r = y^a_+ Z^a, &\qquad \widetilde{r} = \widetilde{y}^a_+ \widetilde{Z}^a, \\ \sigma_0 + \alpha_1\widetilde{\sigma}_0 + \alpha_2(\sigma_{11} + \sigma_{12}) = 0, &\qquad \sigma_0 + \alpha_1\widetilde{\sigma}_0 + \alpha_2(\sigma_{21} + \sigma_{22}) = 0, \\ \widetilde{\sigma}_0 + \beta_1\sigma_0 + \beta_2(\widetilde{\sigma}_{11} + \widetilde{\sigma}_{12}) = 0 \quad \text{and} &\quad \widetilde{\sigma}_0 + \beta_1\sigma_0 + \beta_2(\widetilde{\sigma}_{21} + \widetilde{\sigma}_{22}) = 0, \end{aligned} \tag{5.11}$$

which is simply the Cartesian product $\mathcal{W} \times \mathcal{W}'$ of two copies of the vacua manifold $\mathcal{W}$ found in the exampke of non-Abelian duality in section 4.1. For the instanton correction, it is

$$\int \mathrm{d}\theta^+\Big(\widetilde{\boldsymbol{\mathcal{F}}}^\dagger \boldsymbol{\beta} e^{\alpha^b Y_b} + \widetilde{\widetilde{\boldsymbol{\mathcal{F}}}}^\dagger \widetilde{\boldsymbol{\beta}} e^{\widetilde{\alpha}^b \widetilde{Y}_b}\Big). \tag{5.12}$$

Thus the change for the scalar potential is given by: $\widetilde{\boldsymbol{E}} \to \widetilde{\boldsymbol{E}} - e^{\alpha_b y_+^b}\boldsymbol{\beta}$. This means that the last 4 equations in (5.11) are equal to $|e^{\alpha_b y_+^b}\boldsymbol{\beta}|^2$.

For the analyzed case, when the potential es positive definite, the geometry of the dual model is the one of the product of two disks $\mathbf{D}_1 \times \mathbf{D}_2$, which are the building blocks of the duality in subsection 4.1. Other possible cases involve a not positive definite matrix A or $\tilde{A}$. Notice that the inclusion of instanton corrections preserves the geometry.

6 Discussion and outlook

In this work we describe T-dualities of 2D $(0,2)$ Abelian GLSMs. After a brief review on the basics of $(0,2)$ GLSMs, we started by constructing Abelian T-duality. This is implemented in models with U(1) global symmetries; by gauging them. We analyse two cases: models coming from a $(2,2)$ supersymmetry reduction and pure $(0,2)$ models. The fundamental difference is that in the first case (reduction) the Fermi multiplet is dualized, while in the second case it is not. We study the simple example of a model with two chiral superfields, the first chiral field is charged under the U(1) global symmetry and the other remains as an spectator, which just assists to obtain the global symmetry. A master Lagrangian is obtained by promoting the global symmetry to be local, and adding Lagrange multiplier fields. The equations of motion for the gauge fields are obtained from the master Lagrangian leading to the dual action. The original chiral fields are eliminated by the gauge fixing procedure. We then compute the contributions to the scalar potential for all the terms in the dual Lagrangian. From the potential we determine the geometry of the space of supersymmetric vacua. The geometry of the vacua space for the original model in both cases is $\mathbb{P}^1$. The dual model, under a single U(1) T-duality, has the topology of $\mathbb{R}^+ \times \mathbb{R}$ for both cases. Notice that this is very different to the standard mirror symmetry duality, which will be a T-dualization of both chiral superfields. This model has a single U(1) global symmetry. One can add an spectator superfield in order to have two global U(1)s. Mirror symmetry will be obtained by a T-dualization of both global U(1)s, and our model is obtained by a dualization of a single U(1). In general models, as there are many global symmetries, there are different dualizations that can be realized. The instanton contributions to the superpotential are known for $(0,2)$ models coming from a $(2,2)$ reduction [20, 56]. For the case of a pure $(0,2)$ model, we argue their structure, but to match them to the original theory is a plan for future work. From our results it seems that there is a difference of considering $(0,2)$ models and their dual counterparts, if they come from a reduction or not. Moreover, in section 3 we carried out the duality algorithm for a model with two global Abelian symmetries [56]. This is a model which was later generalized in section 5 to the non-Abelian T-duality case. It consists of a reduction $(0,2)$ GLSM with gauge symmetry $\mathrm{U}(1) \times \mathrm{U}(1)$, six chiral superfields and six Fermi fields. The global Abelian symmetry is given by $\mathrm{U}(1)^4$.

Furthermore we construct T-dualities for Abelian GLSMs which non-Abelian global symmetry. Here we considered only the case coming from the reduction of a $(2,2)$ model. To be as general as possible, we obtain the master Lagrangian of a model with $\mathrm{U}(1)^m$ gauge symmetry and non-Abelian global group $G_1 \times \cdots \times G_S$. Starting from an original $(2,0)$ model with chiral, fermi superfields and a gauge multiplet, we obtain the master Lagrangian by gauging the global symmetries. We find suitable variables to write down the original master Lagrangians as a sum over the S factors of the gauged symmetry group; then we find the equations of motion for the gauge fields. We considered a particular case with just one global $G = \mathrm{SU}(2)$ factor and U(1) gauge symmetry. The dual action is obtained by gauging the global symmetry SU(2). It is observed that under a suitable redefinition of the chiral superfields in terms of new variables (fields) Z's and u^a, the dual action can be rewritten in a simpler form. In these variables also was found the scalar potential (4.12). We also identify the conditions for which the potential is definite positive. This lead us to consider 3 regions depending on whether $y_+^a y_+^a$ is less than, equal or greater to $Z^a y_+^a$. We argued that these regions correspond to the condition with the topology of a open ball, a two-sphere or to the outside part of the sphere, respectively. Thus the vacua manifold for a positive semidefinite scalar potential corresponds to the closed disk $\mathbf{D}$. If the potential is not definite positive the component of the vacua manifold is $\mathbb{R}^2$. Furthermore, we discussed non-perturbative corrections to the superpotential via instantons. We find that if the instanton corrections are incorporated in the potential $U_{\mathtt{dual}}$ the effect is equivalent to shift $\widetilde{\boldsymbol{E}}$ function as $\widetilde{\boldsymbol{E}} - e^{\alpha_b y_+^b}\boldsymbol{\beta}$ in the potential without instanton corrections. This coincides with the observation in the $(2,2)$ GLSMs non-Abelian T-duality were the instanton corrections preserve the dual geometry [67].

In section 5 we present the example of GLSM discussed in [56], which comes from a continuous $(0,2)$ deformation of a $(2,2)$ model. This model is a genuine pure $(0,2)$ GLSM. We worked out this model by gauging the global non-Abelian symmetry $\mathrm{SU}(2) \times \mathrm{SU}(2)$. We find the dual Lagrangian, and analyze the dual geometry of the vacua manifold. For the case of a positive definite potential the manifold is the Cartesian product of the vacua space of the SU(2) simple model already discussed in section 4, i.e. $\mathbf{D}_1 \times \mathbf{D}_2$. There are also instanton corrections affecting both sectors by a similar shifting of $\widetilde{\boldsymbol{E}}$.

Let us comment on the implications of these dualities. For the case of CY target spaces, which are non compact, these manifolds can posses global symmetries. This is the case of the resolved conifold GLSM, whose target space has the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ symmetry [69], which coincides with the global symmetries of the GLSM. We consider that in such cases non-Abelian T-duality on the NLSM could be connected to non-Abelian T-duality in the GLSM; as both dualities have the same source; this is something we would like to explore. However if we consider CY target spaces which are compact, they have a lack of global symmetries. An Abelian T-dualization on the GLSM provides mirror symmetry in the NLSM. It could be that there is a kind of T-duality as the one implemented in the work of Strominger-Yau-Zaslow (SYZ) [74], which connects to non-Abelian T-duality in the GLSM and therefore to a generalization of mirror symmetry. It would be very interesting to explore this possibility.

In the literature T-duality has been applied to diverse physical systems, in string and field theories. For instance in field theory the abelian bosonization was found to be understood in terms of T-duality [75]. Moreover the non-abelian bosonization [76], which encompasses a richer amount of phenomena was described in terms of the non-Abelian T-duality [77]. AdS/CFT solutions have been explored from the non Abelian T-dualities [35, 78–82]. In our case it is well known that mirror symmetry is understood as an Abelian T-duality applied to GLSMs [24, 25]. Thus we believe that the non-Abelian T-duality on GLSMs will correspond to a generalization of mirror symmetry that we still have to understand further. The idea of our study to understand how non-Abelian duality operates in (0,2) GLSMs and provide some examples. It is still an open problem to find what exactly is this generalized mirror symmetry and where it will be relevant.

To summarize we have constructed systematically non-Abelian T-duality in $(0,2)$ gauged linear sigma models in 2D. In the future we would like to analyze more examples, given by realistic CY manifolds. We also are interested in analyzing models with a non-zero superpotential $J \neq 0$, which will lead to compact CY. It would also be interesting to explore the connection of non-Abelian T-dualities in $(0,2)$ models with mirror symmetry in more general CY constructions (as Pfaffians and determinantal varieties). And as a future goal we would like to explore the implications of these GLSMs non-Abelian T-dualities in string theory, as possible extensions of mirror symmetry.

Acknowledgments

We thank Alejandro Cabo Montes de Oca, Kentaro Hori, Hans Jockers, Albrecht Klemm, Stefan Groot Nibbelink, Leopoldo Pando Zayas, Martin Roček, Roberto Santos Silva, Erick Sharpe for useful comments and discussions.

NGCB, JDC and HGC thank the support of the University of Guanajuato grant CIIC 264/2022 “Geometría de dimensiones extras en teoría de cuerdas y sus aplicaciones físicas”. NGCB would like to thank the Grant CONAHCyT A-1-S-37752, “Teorías efectivas de cuerdas y sus aplicaciones a la física de partículas y cosmología” and University of Guanajuato Grants 251/2024 “Teorías efectivas de cuerdas y exploraciones de aprendizaje de máquina” and CIIC 224/202. NGCB would like to acknowledge support from the ICTP through the Associates Programme (2023-2029), and the Isaac Newton Institute for Mathematical Sciences, Cambridge, for support and hospitality during the programme “Black holes: bridges between number theory and holographic quantum information” where work on this paper was undertaken. This work was supported by EPSRC grant no EP/R014604/1. NGCB would like to thank the Simons Center for Geometry and Physics and the organizers of the conference “Gauged Linear Sigma Models @30” for the opportunity to present work related to this research. JDC would like to thank CONAHCyT for the support with a PhD fellowship.

A Covariant derivatives and conventions

The covariant superderivatives are given by

$$D_+ = \partial_{\theta^+} - i\overline{\theta}^+ \partial_+ \,, \qquad \overline{D}_+ = -\partial_{\overline{\theta}^+} + i\theta^+ \partial_+ \,, \tag{A.1}$$

where $\partial_+ := \frac{\partial}{\partial y^0} + \frac{\partial}{\partial y^1}$, $\partial_- := \frac{\partial}{\partial y^0} - \frac{\partial}{\partial y^1}$, $\partial_{\theta^+} := \frac{\partial}{\partial \theta^+}$ and $\partial_{\overline{\theta}^+} := \frac{\partial}{\partial \overline{\theta}^+}$.

The gauge covariant superderivatives $\mathcal{D}_+$, $\overline{\mathcal{D}}_+$, $\mathcal{D}_0$ and $\mathcal{D}_1$ are constructed with the following constraints:

$$\mathcal{D}_0 = D_0 \,, \qquad \mathcal{D}_1 = D_1 \,, \tag{A.2}$$

$$\mathcal{D}_+ = e^{-\Psi} D_+ e^{\Psi} \;=\; (D_+ + D_+\Psi), \tag{A.3}$$

$$\overline{\mathcal{D}}_+ = e^{\Psi} \overline{D}_+ e^{-\Psi} \;=\; (\overline{D}_+ - \overline{D}_+\Psi), \tag{A.4}$$

$$\mathcal{D}_0 - \mathcal{D}_1 \;=\; \partial_- + iV, \tag{A.5}$$

where Ψ and V are real functions, that constitute the gauge degrees of freedom.

The basic gauge invariant field strength Υ is defined as the field strength of V:

$$\begin{aligned}\Upsilon &= [\overline{\mathcal{D}}_+, \mathcal{D}_0 - \mathcal{D}_1]V \\ &= \overline{D}_+(iV + \partial_-\Psi) \\ &= i\overline{D}_+V + \partial_-\overline{D}_+\Psi\,. \end{aligned} \tag{A.6}$$

B Original Lagrangians

The original Lagrangian for a theory with $\mathrm{U}(1)^m$ gauge symmetry, n chiral superfields and $\widetilde{n}$ Fermi superfields is given by

$$\begin{aligned} L =& \int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+\left\{\sum_{a=1}^m \frac{1}{8e_a^2}\overline{\Upsilon}_a\Upsilon_a - \sum_{i=1}^n \frac{i}{2}\overline{\Phi}_i e^{2\sum_{a=1}^m Q_i^a\Psi_a}\left(\partial_- + i\sum_{a=1}^m Q_i^a V_a\right)\Phi_i\right\} \\ &+ \int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+\left\{\sum_{i=1}^n \frac{i}{2}\overline{\Phi}_i\left(\overleftarrow{\partial}_- - i\sum_{a=1}^m Q_i^a V_a\right)e^{2\sum_{a=1}^m Q_i^a\Psi_a}\Phi_i\right\} \\ &- \int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+\left\{\sum_{j=1}^{\widetilde{n}} \frac{1}{2}e^{2\sum_{a=1}^m \widetilde{Q}_j^a\Psi_a}\overline{\Gamma}_j\Gamma_j\right\} + \sum_{a=1}^m \frac{t_a}{4}\int \mathrm{d}\theta^+\Upsilon_a|_{\overline{\theta}^+=0}, \end{aligned} \tag{B.1}$$

where Q_i^a are the charges of the chiral superfields Φ_i and $\widetilde{Q}_j^a$ are the charges of the Fermi superfields Γ_i^a. The fields transformations are given by

$$\begin{aligned} \delta_\zeta V_a &= -\partial_-(\zeta_a + \overline{\zeta}_a)/2, \qquad & \delta_\zeta\Psi &= -i(\zeta_a - \overline{\zeta}_a)/2, \end{aligned} \tag{B.2}$$

$$\Phi_i \to e^{i\sum_{a=1}^m Q_i^a\zeta_a}\Phi_i, \qquad \Gamma_j \to e^{i\sum_{a=1}^m \widetilde{Q}_j^a\zeta_a}\Gamma_j. \tag{B.3}$$

The partial Lagrangian for the case of a single chiral superfield is given by:

$$\begin{aligned} \boldsymbol{\Delta} L_{\mathtt{original}} =& \int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+\left\{\frac{1}{8e^2}\overline{\Upsilon}\Upsilon - \frac{i}{2}\overline{\Phi}e^{2\Psi}\left(\partial_- + iV\right)\Phi + \frac{i}{2}\overline{\Phi}\left(\overleftarrow{\partial}_- - iV\right)e^{2\Psi}\Phi \right.\\ &\left. - \frac{1}{2}e^{2\Psi}\overline{\Gamma}\Gamma\right\} + \frac{t}{4}\int \mathrm{d}\theta^+\Upsilon|_{\overline{\theta}^+=0} + \text{h.c.}\,. \end{aligned} \tag{B.4}$$

The original Lagrangian for a model with $\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetry is given by [56]

$$\begin{aligned} \boldsymbol{\Delta} L_{\mathtt{original}} =& \sum_{i=1}^2 \int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+\left\{-\frac{i}{2}\overline{\Phi}_i\left(e^{2\Psi}\partial_- - \overleftarrow{\partial}_- e^{2\Psi}\right)\Phi_i + Ve^{2\Psi}|\Phi_i|^2 - \frac{1}{2}e^{2\Psi}\overline{\Gamma}_i\Gamma_i\right\} \\ &+ \sum_{i=1}^2 \int \mathrm{d}\theta^+\mathrm{d}\overline{\theta}^+\left\{-\frac{i}{2}\overline{\widetilde{\Phi}}_i\left(e^{2\widetilde{\Psi}}\partial_- - \overleftarrow{\partial}_- e^{2\widetilde{\Psi}}\right)\widetilde{\Phi}_i + Ve^{2\widetilde{\Psi}}|\widetilde{\Phi}_i|^2 - \frac{1}{2}e^{2\widetilde{\Psi}}\overline{\widetilde{\Gamma}}_i\widetilde{\Gamma}_i\right\}. \end{aligned} \tag{B.5}$$

C Abelian T-duality algorithm in superfield components

In this appendix, we implement the Abelian dualization of a $(0,2)$ GLSM coming from a $(2,2)$ reduction in terms of superfield components. This is as an alternative way to the superfield language, to carry out the duality. First, we write down the component expansion of the fields. The gauge superfield is given by

$$V = v_- - 2i\theta^+\overline{\lambda}_- - 2i\overline{\theta}^+\lambda_- + 2\theta^+\overline{\theta}^+D, \qquad \Psi = v_+\theta^+\overline{\theta}^+. \tag{C.1}$$

The fields in the model, including the chiral superfield, the Fermi superfield and the superfield E are written as:

$$\begin{aligned}\widetilde{\Phi} &= \phi + \sqrt{2}\theta^+\psi_+ - i\theta^+\overline{\theta}^+(\partial_+ + iv_+)\phi,\\ \Gamma &= \gamma_- - \sqrt{2}G\theta^+ - i\theta^+\overline{\theta}^+\partial_+\gamma_- - \sqrt{2}E\overline{\theta}^+,\\ E(\Phi) &= E(\phi) + \sqrt{2}\theta^+\frac{\partial E}{\partial\phi}\psi_+ - i\theta^+\overline{\theta}^+\partial_+ E.\end{aligned} \tag{C.2}$$

The field component expansion for the Lagrangian multipliers which have been used in the bulk of the article are:

$$\chi = x + \xi\theta^+ + \rho\overline{\theta}^+ + z\theta^+\overline{\theta}^+, \tag{C.3}$$

$$\Lambda = \omega + k\theta^+ + l\overline{\theta}^+ + \varepsilon\theta^+\overline{\theta}^+. \tag{C.4}$$

The new dual fields are given by

$$Y_\pm = y_\pm + \sqrt{2}(\theta^+\overline{v}_+ + v\overline{\theta}^+) - i\theta^+\overline{\theta}^+\partial_{+y\mp}, \tag{C.5}$$

$$F = \eta_- - \sqrt{2}\theta^+ f - i\theta^+\overline{\theta}^+\partial_+\eta_-. \tag{C.6}$$

Then the appropriate Lagrangian of a single chiral field and a Fermi one with Abelian global symmetry in component fields is written with $\delta_\pm := \partial_\pm - \overleftarrow{\partial}_\pm$ and $I_\pm = \delta_\pm + 2iv_\pm$ as:

$$\begin{aligned}\mathbf{\Delta} L_{\texttt{components}} = &-\frac{1}{2}\bar{\phi}I_-I_+\phi + \frac{i}{2}\bar{\gamma}_-I_+\gamma_- + i\bar{\psi}_+I_-\psi_+ + 2D\bar{\phi}\phi + 2\sqrt{2}i(\bar{\lambda}_-\bar{\psi}_-\phi - \bar{\phi}\psi_+\lambda_-)\\ &+ \bar{G}G - \bar{E}E - \bar{\gamma}_-\frac{\partial E}{\partial\phi}\psi_+ - \bar{\psi}_+\frac{\partial\bar{E}}{\partial\bar{\phi}}\gamma_- \,.\end{aligned} \tag{C.7}$$

To realize the T duality algorithm, gauging the global symmetry we add the fields $v_\pm$, λ_-, D and E (components of the gauged V, Ψ and Γ) as well as the Lagrange multipliers. The original fields will be denoted with a subindex 0, the gauged ones with a subindex 1 and the sum of both without any subindex. Thus, for example: $a := a_0 + a_1$, etc. thus:

$$\begin{aligned}\mathbf{\Delta} L_{\texttt{master}} = &-\frac{1}{2}\bar{\phi}I_-I_+\phi + \frac{i}{2}\bar{\gamma}_-I_+\gamma_- + i\bar{\psi}_+I_-\psi_+ + 2(D_0 + D_1)\bar{\phi}\phi\\ &+ 2\sqrt{2}i((\bar{\lambda}_{-0} + \bar{\lambda}_{-1})\bar{\psi}_-\phi - \bar{\phi}\psi_+(\lambda_{-0} + \lambda_{-1})) - 2i(l - \bar{l})(D_1)\\ &+ \bar{G}G - (\bar{E}_0 + \bar{E}_1)(E_0 + E_1) - \bar{\gamma}_-\frac{\partial(E_0 + E_1)}{\partial\phi}\psi_+ - \bar{\psi}_+\frac{\partial(\bar{E}_0 + \bar{E}_1)}{\partial\bar{\phi}}\gamma_-\\ &+ i(\partial_+\bar{E}_1 x - \bar{x}\partial_+ E_1) + \sqrt{2}\left(\xi\frac{\partial E_1}{\partial\phi}\psi_+ + \bar{\psi}_+\frac{\partial\bar{E}_1}{\partial\bar{\phi}}\xi\right) + \bar{z}E_1 + \bar{E}_1 z\\ &+ 2i(\omega\partial_+\lambda_{-1} - \partial_+\bar{\lambda}_{-1}\bar{\omega}) - 2(\varepsilon\lambda_{-1} + \bar{\lambda}_{-1}\bar{\varepsilon}) - (l + \bar{l})(\partial_- v_{+1} - \partial_+ v_{-1})\,.\end{aligned} \tag{C.8}$$

Taking variations with respect to $v_{\pm,1}$, λ_1, D_1 and E_1 one obtains the corresponding equations of motion:

- For $\delta_{D_1}L$:

$$i(l - \bar{l}) = |\phi|^2. \tag{C.9}$$

- For $\delta_{\lambda_1} L$:

$$\varepsilon + i\partial_+\omega = -\sqrt{2}\overline{\phi}\psi_+. \tag{C.10}$$

- For $\delta_{v_{-1}} L$:

$$2v_+|\phi|^2 = -2\overline{\psi}_+\psi_+ - i(\overline{\phi}\delta_+\phi) - \partial_+(l+\bar{l}). \tag{C.11}$$

- For $\delta_{v_{+1}} L$:

$$2v_-|\phi|^2 = -\overline{\gamma}_-\gamma_- - i(\overline{\phi}\delta_-\phi) + \partial_-(l+\bar{l}). \tag{C.12}$$

- For $\delta_{E_1} L$:

$$\bar{z} + i\partial_+\bar{x} = \bar{E}\,. \tag{C.13}$$

- For $\delta_{\partial_\phi E_1} L$:

$$\sqrt{2}\xi = \bar{\gamma}_-. \tag{C.14}$$

New variables can be defined in the form:

$$y_\pm := il_\mp = i(l \mp \bar{l}), \qquad \sqrt{2}f := \bar{z} + i\partial_+\bar{x}, \qquad \sqrt{2}v = \partial_+\omega - i\varepsilon, \qquad \bar{\eta}_- = \bar{\xi}\,. \tag{C.15}$$

Thus, using eqs. (C.9)–(C.14) in the Lagrangian (C.8) it results the dual Lagrangian:

$$\begin{aligned} L_{\texttt{dual}} = & -\sqrt{2}\left(\bar{f}\bar{E}_0 + E_0 f + \xi\frac{\partial E_0}{\partial\phi}\psi_+ + \bar{\psi}_+\frac{\partial\bar{E}_0}{\partial\bar{\phi}}\bar{\xi} + 2i\bar{\lambda}_{-0}\bar{v} - 2iv\lambda_{-0}\right) \\ & + iy_-\left(\partial_+v_{-0} - \partial_-v_{+0}\right) + 2y_+D_0 - \frac{1}{2}\bar{\phi}\delta_-\delta_+\phi + \frac{i}{2}\bar{\gamma}_-\delta_+\gamma_- + i\bar{\psi}_+\delta_-\psi_+ + \bar{G}G \\ & - \frac{1}{2y_+}\left(-i\bar{\phi}\delta_+\phi - 2\bar{\psi}_+\psi_+ + i\partial_+y_-\right)\left(-i\bar{\phi}\delta_+\phi - \bar{\gamma}_-\gamma_- - i\partial_-y_-\right) + 2\bar{f}f. \end{aligned} \tag{C.16}$$

It is easy to check that this dual Lagrangian coincides with the component field expansion of the dual Lagrangian (3.14). A similar procedure could be carried over in the case of models with non-Abelian T-duality.

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JHEP05(2024)088