Flavor symmetries of six-dimensional $\mathcal{N} = (1, 0)$ theories from AdS/CFT correspondence

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ABSTRACT: We calculate the superconformal indices of a class of six-dimensional $\mathcal{N} = (1, 0)$ superconformal field theories realized on M5-branes at $\mathbb{C}^2/\mathbb{Z}_k$ singularity by using the method developed in previous works of the authors and collaborators. We use the AdS/CFT correspondence, and finite N corrections are included as the contribution of M2-branes wrapped on two-cycles in $S^4/\mathbb{Z}_k$. We confirm that the indices are consistent with the expected flavor symmetries.

KEYWORDS: AdS-CFT Correspondence, Extended Supersymmetry, M-Theory

ARXIV EPRINT: [2103.16094](https://arxiv.org/abs/2103.16094)

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1 Introduction

Recent progress of quantum field theory has enabled us quantitative analyses of strongly coupled field theories. In particular, for supersymmetric theories realized in string/M-theories we can use different powerful methods like dualities and localization. In this paper we will study a class of such theories: six-dimensional $\mathcal{N} = (1,0)$ superconformal field theories realized on a stack of N M5-branes placed at the fixed locus of the $\mathbb{C}^2/\mathbb{Z}_k$ orbifold. A theory in this class has holographic description: m-theory in $AdS_7 \times S^4/\mathbb{Z}_k$. A quiver gauge theory description is also useful to describe the tensor branch of the theory [1–4], and the root of the tensor branch, where the superconformal symmetry restores, corresponds to the strong gauge coupling limit.

An interesting property of such $\mathcal{N} = (1,0)$ theories is non-trivial flavor symmetry depending on k and N [5–11]. A subtle point is that the symmetry may be different from what is read off from the corresponding quiver gauge theory. It was proposed that a

	$N = 2$	$N \geq 3$
$k = 2$	$\text{SO}(7)$	$\text{SU}(2)_a \times \text{SU}(2)_b \times \text{SU}(2)_F$
$k \geq 3$	$\text{SU}(2k)$	$\text{SU}(k)_a \times \text{SU}(k)_b \times \text{U}(1)_F$

Table 1. The flavor symmetries of interacting $\mathcal{N} = (1, 0)$ theories.

certain discrete symmetry of the quiver gauge theory, which is not manifest perturbatively, is gauged in the strong coupling limit and as a result the flavor symmetry is reduced [6]. For example, in the case of $N = k = 2$, although the flavor symmetry of the quiver gauge theory is $\text{SO}(8)$, that of the superconformal theory is $\text{SO}(7)$ [5]. See table 1 for the flavor symmetries for different k and N . The purpose of this paper is to confirm such flavor symmetries on the AdS side.

For generic values of k and N the flavor symmetry is $\text{SU}(k)_a \times \text{SU}(k)_b \times \text{U}(1)_F$. In addition, we also have $\text{SU}(2)_R$ symmetry. These symmetries are manifest on the AdS side; $\text{U}(1)_F \times \text{SU}(2)_R$ is the isometry of $S^4/\mathbb{Z}_k$, and two $\text{SU}(k)$ symmetries are associated with the two A_{k-1} singularities at the fixed points. In the case of $k = 2$ and $N \geq 3$, $\text{U}(1)_F$ is enhanced to $\text{SU}(2)_F$, and this is also understood as the isometry of $S^4/\mathbb{Z}_2$.

The enhancement for $N = 2$ is more interesting. This symmetry enhancement is not manifest on the AdS side, and it is interesting to study how this is realized. We confirm this symmetry enhancement by calculating the superconformal index using the AdS/CFT correspondence. This cannot be seen in the large N limit, and to confirm such symmetry enhancement we need to include finite N corrections.

To calculate the finite N corrections to the superconformal index we use the method developed in the previous work [12] of the authors and collaborators for $(2, 0)$ theories. See also [13–16] for similar analysis of the 4d superconformal index. We calculate the corrections as a contribution from wrapped M2-branes. We focus only on the single-wrapping contribution. For a configuration with more than one wrapped M2-branes its contribution is given in the form of integral over gauge fugacities. We have not yet understood how to choose integration contours in the integrals, and we leave the analysis of such contributions for future work.

This paper is organized as follows. In the next section, we summarize basic properties of the theory we study. In particular, we briefly explain the flavor symmetry. In section 3, we give a formula we use to calculate the index. In section 4 by using the formula we calculate the index for small k and N . For $N = 1$ and arbitrary k the $\mathcal{N} = (1, 0)$ theory is free and we can directly calculate the index without using the holographic description, and we can confirm the formula gives the correct index. This analysis is done for small k in section 4.1. Results for $N = 2$ and $N = 3$ are shown in subsection 4.2 and 4.3, respectively, and the consistency with the flavor symmetries in table 1 is confirmed. The last section is devoted to discussions.

2 6d (1,0) theories

2.1 Setup

An $\mathcal{N} = (1,0)$ theory we discuss is defined as the theory on N M5-branes placed at A_{k-1} singularity. We consider M-theory in the background $\mathbb{R}^{1,5} \times \mathbb{C}^2/\mathbb{Z}_k \times \mathbb{R}_T$. Let X_μ ($\mu = 0, 1, \dots, 5$), z_i ($i = 1, 2$), and x_5 be the coordinates of $\mathbb{R}^{1,5}$, $\mathbb{C}^2$, and $\mathbb{R}_T$, respectively. We also define x_m ($m = 1, 2, 3, 4$) by

$$z_1 = x_1 + ix_2, \quad z_2 = x_3 + ix_4. \quad (2.1)$$

Let R_{ab} ($a, b = 1, \dots, 5$) be the generators of the rotation group $\text{SO}(5)_R$ in the x_a space. We define the orbifold by $\mathbb{Z}_k$ generated by

$$\exp\left(\frac{2\pi i}{k}(R_{12} - R_{34})\right). \quad (2.2)$$

This acts on (z_1, z_2, x_5) as

$$(z_1, z_2, x_5) \rightarrow (e^{2\pi i/k} z_1, e^{-2\pi i/k} z_2, x_5). \quad (2.3)$$

We put N M5-branes at $x_1 = \dots = x_5 = 0$. If it were not for the orbifolding the A_{N-1} -type $\mathcal{N} = (2,0)$ theory would be realized on the worldvolume of the M5-branes. The orbifolding breaks the $\mathcal{N} = (2,0)$ supersymmetry down to $\mathcal{N} = (1,0)$. At the same time, the $\text{SO}(5)_R$ symmetry is broken to $\text{SU}(2)_R \times \text{U}(1)_F$ for $k \geq 3$. $\text{U}(1)_F$ is replaced by $\text{SU}(2)_F$ for $k = 2$. The $\text{SU}(2)_R$ is the R -symmetry of the 6d (1,0) SCFTs, while $\text{U}(1)_F$ or $\text{SU}(2)_F$ does not act on the $\mathcal{N} = (1,0)$ supercharges and is treated as a flavor symmetry. In addition, the orbifold singularity provides $\text{SU}(k)$ flavor symmetry. The singular locus $\mathbb{R}^{1,5} \times \mathbb{R}_T$ is divided by the M5-branes at $x_5 = 0$ into two parts: the $x_5 > 0$ part and the $x_5 < 0$ part. Correspondingly, we have two copies of $\text{SU}(k)$ symmetry which we denote by $\text{SU}(k)_a$ and $\text{SU}(k)_b$. In summary, the bosonic global symmetry is

$$\text{SO}(2,6)_{\text{conf}} \times \text{SU}(2)_R \times G_{\text{flavor}}, \quad (2.4)$$

where $\text{SO}(2,6)_{\text{conf}} \times \text{SU}(2)_R$ is the bosonic subgroup of the 6d $\mathcal{N} = (1,0)$ superconformal symmetry $\text{OSp}(8|2)$ and the flavor symmetry G_{flavor} is generically given by

$$G_{\text{flavor}} = \text{U}(1)_F \times \text{SU}(k)_a \times \text{SU}(k)_b. \quad (2.5)$$

We define the superconformal index as follows. Let H and J_{ij} ($i, j = 1, \dots, 6$) be the generators of $\text{SO}(2)_H \times \text{SO}(6)_{\text{spin}} \subset \text{SO}(2,6)_{\text{conf}}$, and take H , J_{12} , J_{34} , and J_{56} as Cartan generators. H is the Hamiltonian and J_{ij} are spins. We also take R_{12} and R_{34} as $\text{SO}(5)_R$ Cartan generators. To define the index we need to choose one component of the supercharge. We take the component with the following quantum numbers:

$$\mathcal{Q} : (H, J_{12}, J_{34}, J_{56}; R_{12}, R_{34}) = \left(+\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}; +\frac{1}{2}, +\frac{1}{2}\right). \quad (2.6)$$

Note that $\mathcal{Q}$ is invariant under the orbifold group $\mathbb{Z}_k$ generated by (2.2). The anticommutation relation between $\mathcal{Q}$ and its hermitian conjugate $\mathcal{Q}^\dagger$ is

$$\Delta \equiv \{\mathcal{Q}, \mathcal{Q}^\dagger\} = H - (J_{12} + J_{34} + J_{56}) - 2(R_{12} + R_{34}). \quad (2.7)$$

Then we define the superconformal index by

$$\mathcal{I}(q, y_a, u, a_i, b_i) = \text{tr} \left[(-1)^F x^\Delta q^{H + \frac{1}{3}(J_{12} + J_{34} + J_{56})} y_1^{J_{12}} y_2^{J_{34}} y_3^{J_{56}} u^{R_{12} - R_{34}} \prod_{i=1}^{k-1} a_i^{F_{a,i}} b_i^{F_{b,i}} \right]. \quad (2.8)$$

Due to the boson/fermion cancellation only the BPS operators with $\Delta = 0$ contribute to the index, and hence the index does not depend on x . The choice of $\mathcal{Q}$ breaks the $\text{SO}(6)_{\text{spin}}$ to $\text{U}(1)_{\text{spin}} \times \text{SU}(3)_{\text{spin}}$, and y_1, y_2 , and y_3 are the $\text{SU}(3)_{\text{spin}}$ fugacities constrained by $y_1 y_2 y_3 = 1$. $F_{a,i}$ and $F_{b,i}$ are respectively Cartan generators of $\text{SU}(k)_a$ and $\text{SU}(k)_b$. u is the fugacity for $\text{U}(1)_F$ generated by $R_{12} - R_{34}$. Note that for $k = 1$ this index agrees with the $\mathcal{N} = (2, 0)$ superconformal index in [12].

2.2 Flavor symmetries

For the analysis of the operator spectrum and the flavor symmetry of the theory it is convenient to consider the quiver gauge theories realized in the tensor branch. By taking $\text{U}(1)_F$ orbits as M-theory circles we can regard the system as a type IIA brane configuration. N M5-branes become N NS5-branes, and the A_{k-1} singularity becomes a stack of k D6-branes.

The $(2, 0)$ tensor multiplet on an M5-brane separates into a $(1, 0)$ tensor multiplet and a $(1, 0)$ hypermultiplet on the corresponding NS5-brane, and the scalar component in the $(1, 0)$ tensor multiplet corresponds to the location of the NS5-branes in the x_5 direction. At a generic point in the tensor branch all the NS5-branes are separated one by one in the x_5 direction, and a linear quiver gauge theory is realized on the D6-branes.

The worldvolume of the stack of D6-branes is divided into $N + 1$ parts by the NS5-branes. We label the NS5-branes by $i = 1, 2, \dots, N$. The D6-branes suspended between two NS5-branes i and $i + 1$ give $\text{SU}(k)_i$ gauge group while two semi-infinite parts of D6-branes give the flavor symmetries $\text{SU}(k)_a \equiv \text{SU}(k)_0$ and $\text{SU}(k)_b \equiv \text{SU}(k)_N$. Let $(h_i, \tilde{h}_i)$ be the hypermultiplet arising from open strings crossing the i -th NS5-brane. h_i and $\tilde{h}_i$ belong to the bi-fundamental representations $(k, \bar{k})$ and $(\bar{k}, k)$, respectively, of $\text{SU}(k)_{i-1} \times \text{SU}(k)_i$. The $\text{SU}(N)$ groups and the hypermultiplets are depicted as the linear quiver diagram in figure 1. In addition, we also have degrees of freedom that are implicit in the diagram; in each gauge node there exists a tensor multiplet corresponding to the degrees of freedom of the NS5-brane.

Among different gauge invariant operators let us focus on two classes of operators. The first class includes operators defined by

$$S_{ij} = (h_0)_{ia} (\tilde{h}_0)_{aj}, \quad S'_{ij} = (\tilde{h}_N)_{ia} (h_N)_{aj}. \quad (2.9)$$

Figure 1. The linear quiver diagram of the gauge theory realized in the tensor branch is shown.

The other class includes

$$L_{ij} = (h_0)_{ia_1} (h_1)_{a_1 a_2} \cdots (h_N)_{a_N j}, \quad L'_{ij} = (\tilde{h}_N)_{ia_N} \cdots (h_1)_{a_2 a_1} (h_0)_{a_1 j}. \quad (2.10)$$

These operators play an important role when we discuss the flavor symmetry.

The operators S_{ij} and S'_{ij} belong to the adjoint representations of $SU(k)_0$ and $SU(k)_N$, respectively. They have dimension 4 and are the primary operators of the current multiplets of the generic flavor symmetry

$$G_{\text{flavor}} = SU(k)_0 \times SU(k)_N \times U(1)_F. \quad (2.11)$$

(Although there are N classical $U(1)$ symmetries only one of them is anomaly free.)

The operators L_{ij} and L'_{ij} , which belong to the bi-fundamental representations $(k, \bar{k})_{+1}$ and $(\bar{k}, k)_{-1}$ of G_{flavor} in (2.11), respectively, have dimension $2N$. These operators appear in the spectrum only when N is finite, and play a role similar to baryonic operators in four-dimensional quiver gauge theories. They are expected to correspond to wrapped M2-branes on the gravity side. Indeed, their dimension coincide with the mass of an M2-brane wrapped around a large $S^2/\mathbb{Z}_k$ in the unit of the inverse AdS radius.

The flavor symmetry (2.11) is enhanced if one of k or N becomes 2. If $k = 2$ and $N \geq 3$ $U(1)_F$ is enhanced to $SU(2)_F$. Correspondingly, the index is written in terms of $SU(2)_F$ characters. This symmetry is manifest on the gravity side as the isometry of $S^4/\mathbb{Z}_2$.

The enhancement for $N = 2$ is more interesting. If $N = 2$ the operators L and L' have dimension 4 as well as S and S' , and they give additional current multiplets. As the result the flavor symmetry (2.11) is enhanced to $SU(2k)$ for $k \geq 3$. On the gravity side, this enhancement should be realized when we include the contribution of wrapped M2-branes.

The $k = N = 2$ case is most interesting. In this case there are eight $SU(2)$ gauge symmetry doublet in the hypermultiplets, and we can write down 28 gauge invariant dimension 4 operators forming the $SO(8)$ adjoint representation. They correspond to the $SO(8)$ global symmetry of the quiver gauge theory. However, it is known that the symmetry is reduced to $SO(7)$ in a highly non-trivial way [5], and it would be nice if we can reproduce this flavor symmetry on the gravity side by the index calculation.

3 Indices from M-theory in $AdS_7 \times S^4/\mathbb{Z}_k$

3.1 Conjectural formula

Based on the idea explained in detail in [12] we propose the formula of the index for 6d $(1,0)$ theories

$$\mathcal{I}_{N,k}^{(1,0)} = \mathcal{I}^{\text{bulk}} \sum_{n_1, n_2=0}^{\infty} \mathcal{I}_{(n_1, n_2)}^{\text{M2}}. \quad (3.1)$$

This formula gives the index as the combination of contributions from objects in the dual geometry $AdS_7 \times S^4/\mathbb{Z}_k$, where the internal space $\mathcal{S}^4/\mathbb{Z}_k$ is defined by

$$|z_1|^2 + |z_2|^2 + x_5^2 = 1, \quad (3.2)$$

together with the identification by the $\mathbb{Z}_k$ action (2.3). $\mathcal{I}^{\text{bulk}}$ is the contribution of Kaluza-Klein modes in the bulk. We also include in $\mathcal{I}^{\text{bulk}}$ the contribution from localized modes at the fixed points of the orbifold. $\mathcal{I}_{(n_1, n_2)}^{\text{M2}}$ are contributions of wrapped M2-branes in the internal space. n_1 and n_2 are numbers of M2-brane wrapped on the two specific two-cycles $z_1 = 0$ and $z_2 = 0$, respectively. As we will explain in 3.3 the q expansion of $\mathcal{I}_{(n_1, n_2)}$ starts from order $q^{2(n_1+n_2)N}$ terms. In the large N limit all contributions but $\mathcal{I}_{(0,0)} = 1$ decouple and the formula reduces to $\mathcal{I}_{N=\infty, k}^{(1,0)} = \mathcal{I}^{\text{bulk}}$. On the other hand, if N is finite, all sectors labelled by (n_1, n_2) contribute to the index. $\mathcal{I}_{(n_1, n_2)}$ for each (n_1, n_2) is calculated as the index of the theory realized on the wrapped M2-branes by the standard localization formula. If $n_1 + n_2 \geq 2$ the formula includes non-trivial gauge integrals, and unfortunately we have not yet found systematic rules for the integration contours. For this reason we leave the analysis of $n_1 + n_2 \geq 2$ for future work and in this paper we focus only on the single-wrapping sectors $(n_1, n_2) = (1, 0)$ and $(0, 1)$. Namely, we consider the formula

$$\mathcal{I}_{N, k}^{(1,0)} = \mathcal{I}_{N, k}^{\text{grav}} + \mathcal{O}(q^{4N}), \quad (3.3)$$

where $\mathcal{I}_{N, k}^{\text{grav}}$ is defined by

$$\mathcal{I}_{N, k}^{\text{grav}} = \mathcal{I}^{\text{bulk}} \left(1 + \mathcal{I}_{(1,0)}^{\text{M2}} + \mathcal{I}_{(0,1)}^{\text{M2}} \right). \quad (3.4)$$

With the conjectural formula (3.3), we can calculate the index for an arbitrary N and k up to the expected error of order q^{4N} .

3.2 The bulk contribution

Let us first consider $\mathcal{I}^{\text{bulk}}$, which gives the large N index. This is given by the plethystic exponential of the single-particle index, which is the sum of two contributions: the supergravity Kaluza-Klein modes in the internal space $\mathcal{S}^4/\mathbb{Z}_k$ and the vector multiplets localized at the two fixed points of $\mathcal{S}^4/\mathbb{Z}_k$.

The contribution of the Kaluza-Klein modes in $AdS_7 \times \mathcal{S}^4$ without orbifolding has already been studied in [17] and is given by $\text{Pexp } i_{\text{KK}}$ with the single-particle index

$$i_{\text{KK}} = \frac{q^2 \chi_{[1]}^u - q^{\frac{8}{3}} \chi_{[0,1]}^y + q^{\frac{16}{3}} \chi_{[1,0]}^y - q^6 \chi_{[1]}^u}{(1 - uq^2)(1 - u^{-1}q^2)(1 - y_1 q^{\frac{4}{3}})(1 - y_2 q^{\frac{4}{3}})(1 - y_3 q^{\frac{4}{3}})}, \quad (3.5)$$

where $\chi_{[n]}^u$ are the $SU(2)$ characters defined by

$$\chi_{[n]}^u = \frac{u^{n+1} - u^{-(n+1)}}{u - u^{-1}}, \quad (3.6)$$

and $\chi_{[m_1, m_2]}^y$ are the $SU(3)$ characters of the representations with Dynkin labels $[m_1, m_2]$. The Kaluza-Klein modes in the orbifold $\mathcal{S}^4/\mathbb{Z}_k$ is obtained by picking up the $\mathbb{Z}_k$ invariant

modes from the modes in S^4 [18]. Correspondingly, the single-particle index for the orbifold is given by $\mathcal{P}_k i_{KK}$, where $\mathcal{P}_k$ is the projection operator associated with the $\mathbb{Z}_k$ orbifold which acts on a function of the fugacity u as

$$\mathcal{P}_k f(u) = \frac{1}{k} \sum_{l=0}^{k-1} f(e^{2\pi i l/k} u). \quad (3.7)$$

The other contribution we need to include in the single-particle index comes from two A_{k-1} singularities on $\mathbb{C}^2/\mathbb{Z}_k$ at $(z_1, z_2, x_5) = (0, 0, \pm 1)$, where the 7d $SU(k)_a \times SU(k)_b$ vector multiplets are localized. In general, a gauge field in the bulk of AdS corresponds to a flavor symmetry on the boundary, and the corresponding current multiplet contributes to the index. The corresponding single-particle index is $i_F(\chi_{\text{adj.}}^a + \chi_{\text{adj.}}^b)$, where $\chi_{\text{adj.}}^{a/b}$ are characters of adjoint representations of the global $SU(k)_{a/b}$ symmetries and i_F is given by

$$i_F = \frac{q^4}{(1 - q^{\frac{4}{3}} y_1)(1 - q^{\frac{4}{3}} y_2)(1 - q^{\frac{4}{3}} y_3)}. \quad (3.8)$$

Note that i_F is independent of u and we do not have to perform the $\mathbb{Z}_k$ projection.

By combining two contributions, we can calculate the index for the large N limit. For example, for $k = 2$ we obtain

$$\begin{aligned} \text{Pexp}(\mathcal{P}_2 i_{KK} + i_F(\chi_{[2]}^a + \chi_{[2]}^b)) &= 1 - \chi_{[0,1]}^y q^{\frac{8}{3}} + (\chi_{[2]}^u + \chi_{[2]}^a + \chi_{[2]}^b - \chi_{[1,1]}^y) q^4 \\ &\quad + ((2 + \chi_{[2]}^u + \chi_{[2]}^a + \chi_{[2]}^b) \chi_{[1,0]}^y - \chi_{[2,1]}^y) q^{\frac{16}{3}} + \mathcal{O}(q^{\frac{20}{3}}). \end{aligned} \quad (3.9)$$

This includes the contribution of the “center of mass” free tensor multiplet. The existence of such a decoupled free sector is suggested by the coefficient “2” of the term $\chi_{[1,0]}^y q^{\frac{16}{3}}$ in the above expansion, which is identified as the contribution of two copies of stress-tensor multiplets. Such a free tensor multiplet exists for all k and N , and we always remove its contribution in the following calculation. Namely, we define $\mathcal{I}^{\text{bulk}}$ in (3.3) by

$$\mathcal{I}^{\text{bulk}} = \text{Pexp}(\mathcal{P}_k i_{KK} + i_F(\chi_{\text{adj.}}^a + \chi_{\text{adj.}}^b) - i_{\text{tensor}}), \quad (3.10)$$

where the single-particle index i_{tensor} of the free tensor multiplet is given by [17]

$$i_{\text{tensor}} = \frac{-q^{\frac{8}{3}} \chi_{[0,1]}^y + q^4}{(1 - q^{\frac{4}{3}} y_1)(1 - q^{\frac{4}{3}} y_2)(1 - q^{\frac{4}{3}} y_3)}. \quad (3.11)$$

For $k = 2, 3$ (3.10) gives

$$\begin{aligned} \mathcal{I}_{k=2}^{\text{bulk}} &= \text{Pexp}(\mathcal{P}_2 i_{KK} + i_F(\chi_{[2]}^a + \chi_{[2]}^b) - i_{\text{tensor}}) \\ &= 1 + (\chi_{[2]}^u + \chi_{[2]}^a + \chi_{[2]}^b) q^4 + (1 + \chi_{[2]}^u + \chi_{[2]}^a + \chi_{[2]}^b) \chi_{[1,0]}^y q^{\frac{16}{3}} + \mathcal{O}(q^{\frac{20}{3}}), \end{aligned} \quad (3.12)$$

$$\begin{aligned} \mathcal{I}_{k=3}^{\text{bulk}} &= \text{Pexp}(\mathcal{P}_3 i_{KK} + i_F(\chi_{[1,1]}^a + \chi_{[1,1]}^b) - i_{\text{tensor}}) \\ &= 1 + (1 + \chi_{[1,1]}^a + \chi_{[1,1]}^b) q^4 + (2 + \chi_{[1,1]}^a + \chi_{[1,1]}^b) \chi_{[1,0]}^y q^{\frac{16}{3}} + (-\chi_{[1]}^u + \chi_{[3]}^u) q^6 + \mathcal{O}(q^{\frac{20}{3}}). \end{aligned} \quad (3.13)$$

These are interpreted as the indices in the large N limit. The q^4 terms in each index is the contribution of the flavor current multiplets, and we can read off the expected flavor symmetries $SU(2)^3$ for $k = 2$ and $SU(3)^2 \times U(1)$ for $k = 3$. We also confirm that all other terms are consistent with these flavor symmetries.

3.3 Wrapped M2-branes on $S^4/\mathbb{Z}_k$

Next we consider the contribution of M2-branes. The worldvolume of a BPS M2-brane is described by the intersection of S^4 and a holomorphic surface [19, 20]

$$f(z_1, z_2) = 0. \quad (3.14)$$

The consistency with the $\mathbb{Z}_k$ orbifolding require the function f to satisfy

$$f(e^{2\pi i/k} z_1, e^{-2\pi i/k} z_2) = e^{2\pi i w/k} f(z_1, z_2), \quad (3.15)$$

where $w \in \mathbb{Z}/k\mathbb{Z}$ is the topological wrapping number.

In [12], in which the system without $\mathbb{Z}_k$ orbifolding was studied, it was proposed that we can take only M2-brane configurations given by monomials of the form $f(z_1, z_2) = z_1^{n_1} z_2^{n_2}$ and were shown that the formula passes some non-trivial checks. Let us adopt the same assumption. The function $f(z_1, z_2) = z_1^{n_1} z_2^{n_2}$ gives the system with n_1 M2-branes wrapped on $z_1 = 0$ and n_2 M2-branes wrapped on $z_2 = 0$. The total topological wrapping number is $w = n_1 - n_2 \bmod k$. $\mathcal{I}_{(n_1, n_2)}^{\text{M2}}$ in (3.1) is the contribution from the specific wrapping sector with (n_1, n_2) . We focus on the two sectors $(1, 0)$ and $(0, 1)$. In the absence of the $\mathbb{Z}_k$ orbifolding, the contribution of the $(1, 0)$ sector, a single M2-brane wrapped on $z_1 = 0$, is [12]

$$(q^2 u)^N \text{Pexp } i_{z_1=0}^{\text{M2}}, \quad (3.16)$$

with the single-particle index $i_{z_1=0}^{\text{M2}}$ given by

$$i_{z_1=0}^{\text{M2}} = \frac{q^{-2} u^{-1} - q^{\frac{2}{3}} u^{-1} \chi_{[0,1]}^y + q^{\frac{4}{3}} \chi_{[1,0]}^y - q^4}{1 - q^2 u^{-1}}. \quad (3.17)$$

To obtain the index for the $\mathbb{Z}_k$ orbifold, we need two modifications. First, we perform the $\mathbb{Z}_k$ projection on the single-particle index. Second, we insert the character of the $\text{SU}(k)_a \times \text{SU}(k)_b$ bi-fundamental representation because the wrapped M2-brane couples to the localized vector multiplets at the fixed points $(z_1, z_2, x_5) = (0, 0, \pm 1)$. As the result, the contribution of the $(1, 0)$ sector is given by

$$\mathcal{I}_{(1,0)}^{\text{M2}} = (q^2 u)^N \chi_{\text{fund.}}^a \chi_{\text{fund.}}^b \text{Pexp} \left[\mathcal{P}_k i_{z_1=0}^{\text{M2}} \right], \quad (3.18)$$

where $\chi_{\text{fund.}}^{a/b}$ ($\chi_{\text{fund.}}^{a/b}$) are characters of (anti-) fundamental representations of the $\text{SU}(k)_{a/b}$ symmetries. The contribution of the other sector $(0, 1)$ is given from (3.18) by the replacement $u \rightarrow u^{-1}$, $a_i \rightarrow a_i^{-1}$, and $b_i \rightarrow b_i^{-1}$. $u \rightarrow u^{-1}$ is the Weyl reflection of $\text{SO}(5)_R$ exchanging z_1 and z_2 . The inversion of a_i and b_i are necessary because the cycle $z_2 = 0$ has the opposite topological wrapping number to the cycle $z_1 = 0$; the former has $w = +1$ while the latter has $w = -1$. After the replacement we obtain

$$\mathcal{I}_{(0,1)}^{\text{M2}} = (q^2 u^{-1})^N \chi_{\text{fund.}}^a \chi_{\text{fund.}}^b \text{Pexp} \left[\mathcal{P}_k i_{z_2=0}^{\text{M2}} \right], \quad (3.19)$$

where $i_{z_2=0}^{\text{M2}} = i_{z_1=0}^{\text{M2}}|_{u \rightarrow u^{-1}}$.

If $k = 2$ the flavor characters appearing in (3.18) and (3.19) are the same, $\chi_{\text{fund.}}^a \chi_{\text{fund.}}^b = \chi_{\text{fund.}}^a \chi_{\text{fund.}}^b$, and $\mathcal{I}_{(1,0)}^{\text{M2}} + \mathcal{I}_{(0,1)}^{\text{M2}}$ is invariant under the $\text{SU}(2)_R$ Weyl reflection $u \rightarrow u^{-1}$. This is consistent with the symmetry enhancement $\text{U}(1)_F \rightarrow \text{SU}(2)_F$.

4 Results and consistency check

Now we are ready to calculate the index of the 6d $(1, 0)$ theories by using our formula (3.4) for different values of k and N .

4.1 Results for $N = 1$

The theory with $N = 1$ is the free theory consisting of the “center-of-mass” tensor multiplet and hypermultiplets belonging to the bi-fundamental representation of $SU(k)_a \times SU(k)_b$. The index with the tensor multiplet contribution removed is given by

$$\mathcal{I}_{N=1}^{(1,0)} = \text{Pexp} \left[i_{\text{hyper}} (\chi_{\text{fund.}}^a \chi_{\text{fund.}}^b u + \chi_{\text{fund.}}^a \chi_{\text{fund.}}^b u^{-1}) \right], \quad (4.1)$$

where i_{hyper} is given by [17]

$$i_{\text{hyper}} = \frac{q^2}{(1 - q^{\frac{4}{3}} y_1)(1 - q^{\frac{4}{3}} y_2)(1 - q^{\frac{4}{3}} y_3)}. \quad (4.2)$$

Let us compare the index on the gravity side based on the formula (3.4) with the free theory result from (4.1). As we do not include the multiple-wrapping M2-branes, the errors should start at q^4 terms and we check the agreement up to errors of this order.

4.1.1 $k = 2$

On the gravity side (3.4) yields

$$\mathcal{I}_{N=1,k=2}^{\text{grav}} = 1 + \chi_{[1]}^a \chi_{[1]}^b \chi_{[1]}^u q^2 + \chi_{[1]}^a \chi_{[1]}^b \chi_{[1]}^u \chi_{[1,0]}^y q^{\frac{10}{3}} + (\chi_{[2]}^a + \chi_{[2]}^b + \chi_{[2]}^u) q^4 + \mathcal{O}(q^{\frac{14}{3}}). \quad (4.3)$$

Expanding (4.1) with $k = 2$ we obtain

$$\mathcal{I}_{N=1,k=2}^{(1,0)} = \mathcal{I}_{N=1,k=2}^{\text{gr}} + \chi_{[2]}^a \chi_{[2]}^b \chi_{[2]}^u q^4 + \mathcal{O}(q^{\frac{14}{3}}). \quad (4.4)$$

We can see the agreement up to the error of $\mathcal{O}(q^4)$ as expected.

4.1.2 $k = 3$

For $k = 3$ the result on the gravity side is

$$\begin{aligned} \mathcal{I}_{N=1,k=3}^{\text{grav}} = & 1 + \left(u \chi_{[1,0]}^a \chi_{[0,1]}^b + u^{-1} \chi_{[0,1]}^a \chi_{[1,0]}^b \right) q^2 + \left(u \chi_{[1,0]}^a \chi_{[0,1]}^b \chi_{[1,0]}^y + u^{-1} \chi_{[0,1]}^a \chi_{[1,0]}^b \chi_{[1,0]}^y \right) q^{\frac{10}{3}} \\ & + \left(1 + \chi_{[1,1]}^a + \chi_{[1,1]}^b + u^2 \chi_{[0,1]}^a \chi_{[1,0]}^b + u^{-2} \chi_{[1,0]}^a \chi_{[0,1]}^b \right) q^4 + \mathcal{O}(q^{\frac{14}{3}}). \end{aligned} \quad (4.5)$$

On the CFT side we obtain

$$\mathcal{I}_{N=1,k=3}^{(1,0)} = \mathcal{I}_{N=1,k=3}^{\text{grav}} + \left(u^2 \chi_{[2,0]}^a \chi_{[0,2]}^b + u^{-2} \chi_{[0,2]}^a \chi_{[2,0]}^b + \chi_{[1,1]}^a \chi_{[1,1]}^b \right) q^4 + \mathcal{O}(q^{\frac{14}{3}}). \quad (4.6)$$

Again we can find agreement up to expected error terms of order q^4 .

4.2 Results for $N = 2$

When $N = 2$, the generic flavor symmetry $SU(k)_a \times SU(k)_b \times U(1)$ is enhanced to $SU(2k)$ for $k \geq 3$ and $SO(7)$ for $k = 2$ [5, 6, 21]. Then, the indices should be written in terms of the characters of the enhanced symmetries. Let us confirm this for $k = 2$ and $k = 3$. The expected errors due to double-wrapping configurations are of order q^8 , and we show the results below the order.

4.2.1 $k = 2$

We consider the $k = N = 2$ case first. In this case the index should be written in terms of the $SO(7)$ character $\chi_{[l_1, l_2, l_3]}^{SO(7)}$. The last component of the Dynkin labels corresponds to the short root. The formula (3.3) gives

$$\begin{aligned} \mathcal{I}_{N=k=2}^{\text{grav}} = & 1 + \chi_{[0,1;0]}^{SO(7)} q^4 + (1 + \chi_{[0,1;0]}^{SO(7)}) \chi_{[1,0]}^y q^{\frac{16}{3}} \\ & + \left((1 + \chi_{[0,1;0]}^{SO(7)}) \chi_{[2,0]}^y + (1 - \chi_{[1,0;0]}^{SO(7)}) \chi_{[0,1]}^y \right) q^{\frac{20}{3}} + \mathcal{O}(q^8). \end{aligned} \quad (4.7)$$

This is correctly expanded in terms of $SO(7)$ characters. We also confirm that it is not written in terms of characters of $SO(8)$, the symmetry of the corresponding quiver gauge theory.

4.2.2 $k = 3$

For $k = 3$ and $N = 2$ the expected flavor symmetry is $SU(6)$. The formula (3.3) gives

$$\begin{aligned} \mathcal{I}_{N=2,k=3}^{\text{grav}} = & 1 + \chi_{[1,0,0,0,1]}^{SU(6)} q^4 + (1 + \chi_{[1,0,0,0,1]}^{SU(6)}) \chi_{[1,0]}^y q^{\frac{16}{3}} + \chi_{[0,0,1,0,0]}^{SU(6)} q^6 \\ & + \left(1 + \chi_{[1,0,0,0,1]}^{SU(6)} \right) \chi_{[2,0]}^y q^{\frac{20}{3}} + \chi_{[0,0,1,0,0]}^{SU(6)} \chi_{[1,0]}^y q^{\frac{22}{3}} + \mathcal{O}(q^8), \end{aligned} \quad (4.8)$$

and this is correctly written in terms of $SU(6)$ characters.

4.3 Results for $N = 3$

In this subsection we show the results for $N = 3$ calculated on the gravity side. Because we do not have results we can compare, we give the results simply as predictions. The expected errors are of order q^{12} , and we show the results below the order.

4.3.1 $k = 2$

The global symmetry for $N = 3$ and $k = 2$ is $G_{\text{flavor}} = SU(2)_a \times SU(2)_b \times SU(2)_F$. The formula (3.3) gives

$$\begin{aligned} \mathcal{I}_{N=3,k=2}^{\text{grav}} = & 1 + (\chi_{[2]}^a + \chi_{[2]}^b + \chi_{[2]}^u) q^4 + (\chi_{[2]}^a \chi_{[1,0]}^y + \chi_{[2]}^b \chi_{[1,0]}^y + \chi_{[2]}^u \chi_{[1,0]}^y + \chi_{[1,0]}^y) q^{\frac{16}{3}} \\ & + \chi_{[1]}^a \chi_{[1]}^b \chi_{[3]}^u q^6 + (\chi_{[0,1]}^y + \chi_{[2]}^a \chi_{[2,0]}^y + \chi_{[2]}^b \chi_{[2,0]}^y + \chi_{[2,0]}^y + \chi_{[2]}^u (\chi_{[2,0]}^y - \chi_{[0,1]}^y)) q^{\frac{20}{3}} \\ & + \chi_{[1]}^a \chi_{[1]}^b \chi_{[3]}^u \chi_{[1,0]}^y q^{\frac{22}{3}} + (\chi_{[4]}^a + \chi_{[2]}^a \chi_{[2]}^b + \chi_{[4]}^b + 2\chi_{[4]}^u + \chi_{[1,1]}^y + \chi_{[2]}^a \chi_{[3,0]}^y + \chi_{[2]}^b \chi_{[3,0]}^y + \chi_{[3,0]}^y \\ & \quad + \chi_{[2]}^u (\chi_{[2]}^a + \chi_{[2]}^b - \chi_{[1,1]}^y + \chi_{[3,0]}^y - 1) + 2) q^8 \\ & + (\chi_{[1]}^a \chi_{[1]}^b \chi_{[3]}^u \chi_{[2,0]}^y - \chi_{[1]}^a \chi_{[1]}^b \chi_{[1]}^u \chi_{[0,1]}^y) q^{\frac{26}{3}} \end{aligned}$$

$$\begin{aligned}
 & + (2\chi_{[2]}^a \chi_{[1,0]}^y + \chi_{[4]}^a \chi_{[1,0]}^y + 2\chi_{[2]}^a \chi_{[2]}^b \chi_{[1,0]}^y + 2\chi_{[2]}^b \chi_{[1,0]}^y + \chi_{[4]}^b \chi_{[1,0]}^y + 2\chi_{[4]}^u \chi_{[1,0]}^y \\
 & + 2\chi_{[1,0]}^y + \chi_{[2,1]}^y + \chi_{[2]}^a \chi_{[4,0]}^y + \chi_{[2]}^b \chi_{[4,0]}^y + \chi_{[4,0]}^y \\
 & + \chi_{[2]}^u (2\chi_{[2]}^a \chi_{[1,0]}^y + 2\chi_{[2]}^b \chi_{[1,0]}^y + 2\chi_{[1,0]}^y - \chi_{[2,1]}^y + \chi_{[4,0]}^y)) q^{\frac{28}{3}} \\
 & + (\chi_{[1]}^a \chi_{[1]}^b \chi_5^u - \chi_{[1]}^a \chi_{[1]}^b \chi_{[1]}^u \chi_{[1,1]}^y + \chi_{[3]}^u (2\chi_{[1]}^a \chi_{[1]}^b + \chi_{[3]}^a \chi_{[1]}^b + \chi_{[1]}^a \chi_{[3,0]}^y \chi_{[1]}^b + \chi_{[1]}^a \chi_{[3]}^b)) q^{10} \\
 & + (3\chi_{[2]}^a \chi_{[0,1]}^y + \chi_{[2]}^a \chi_{[2]}^b \chi_{[0,1]}^y + 3\chi_{[2]}^b \chi_{[0,1]}^y - \chi_{[0,1]}^y + 3\chi_{[2]}^a \chi_{[2,0]}^y + 2\chi_{[4]}^a \chi_{[2,0]}^y \\
 & + 3\chi_{[2]}^a \chi_{[2]}^b \chi_{[2,0]}^y + 3\chi_{[2]}^b \chi_{[2,0]}^y + 2\chi_{[4]}^b \chi_{[2,0]}^y + 6\chi_{[2,0]}^y + \chi_{[4]}^u (3\chi_{[2,0]}^y - 2\chi_{[0,1]}^y) \\
 & + \chi_{[3,1]}^y + \chi_{[2]}^a \chi_{[5,0]}^y + \chi_{[2]}^b \chi_{[5,0]}^y + \chi_{[5,0]}^y + \chi_{[2]}^u (3\chi_{[0,1]}^y + 3\chi_{[2]}^a \chi_{[2,0]}^y + 3\chi_{[2]}^b \chi_{[2,0]}^y \\
 & + 3\chi_{[2,0]}^y - \chi_{[3,1]}^y + \chi_{[5,0]}^y)) q^{\frac{32}{3}} \\
 & + (2\chi_{[1]}^a \chi_{[1]}^b \chi_{[5]}^u \chi_{[1,0]}^y + \chi_{[1]}^u (2\chi_{[1]}^a \chi_{[1]}^b \chi_{[1,0]}^y - \chi_{[1]}^a \chi_{[1]}^b \chi_{[2,1]}^y) + \chi_{[3]}^u (6\chi_{[1]}^a \chi_{[1]}^b \chi_{[1,0]}^y \\
 & + 2\chi_{[3]}^a \chi_{[1]}^b \chi_{[1,0]}^y + 2\chi_{[1]}^a \chi_{[3]}^b \chi_{[1,0]}^y + \chi_{[1]}^a \chi_{[1]}^b \chi_{[4,0]}^y)) q^{\frac{34}{3}} + \mathcal{O}(q^{12}). \tag{4.9}
 \end{aligned}$$

4.3.2 $k = 3$

The global symmetry for $N = 3$ and $k = 3$ is $G_{\text{flavor}} = \text{SU}(3)_a \times \text{SU}(3)_b \times \text{U}(1)_F$. The formula (3.3) gives

$$\begin{aligned}
 \mathcal{I}_{N=3,k=3}^{\text{grav}} = & 1 + (\chi_{[1,1]}^a + \chi_{[1,1]}^b + 1)q^4 + (\chi_{[1,1]}^a \chi_{[1,0]}^y + \chi_{[1,1]}^b \chi_{[1,0]}^y + 2\chi_{[1,0]}^y)q^{\frac{16}{3}} \\
 & + (u^3 \chi_{[1,0]}^a \chi_{[0,1]}^b + u^3 + u^{-3} \chi_{[0,1]}^a \chi_{[1,0]}^b + u^{-3})q^6 + (\chi_{[1,1]}^a \chi_{[2,0]}^y + \chi_{[1,1]}^b \chi_{[2,0]}^y + 2\chi_{[2,0]}^y)q^{\frac{20}{3}} \\
 & + (u^3 \chi_{[1,0]}^a \chi_{[0,1]}^b \chi_{[1,0]}^y + u^3 \chi_{[1,0]}^y + u^{-3} \chi_{[0,1]}^a \chi_{[1,0]}^b \chi_{[1,0]}^y + u^{-3} \chi_{[1,0]}^y)q^{\frac{22}{3}} \\
 & + (\chi_{[1,1]}^a \chi_{[1,1]}^b + \chi_{[1,1]}^a \chi_{[3,0]}^y + 2\chi_{[1,1]}^a + \chi_{[2,2]}^a + \chi_{[1,0]}^a \chi_{[0,1]}^b \\
 & + \chi_{[0,1]}^a \chi_{[1,0]}^b + 2\chi_{[1,1]}^b + \chi_{[2,2]}^b + \chi_{[1,1]}^b \chi_{[3,0]}^y + 2\chi_{[3,0]}^y + 2)q^8 \\
 & + (-u^3 \chi_{[0,1]}^y + u^3 \chi_{[1,0]}^a \chi_{[0,1]}^b \chi_{[2,0]}^y + u^3 \chi_{[2,0]}^y - u^{-3} \chi_{[0,1]}^y + u^{-3} \chi_{[0,1]}^a \chi_{[1,0]}^b \chi_{[2,0]}^y + u^{-3} \chi_{[2,0]}^y)q^{\frac{26}{3}} \\
 & + (\chi_{[0,3]}^a \chi_{[1,0]}^y + 5\chi_{[1,1]}^a \chi_{[1,0]}^y + \chi_{[2,2]}^a \chi_{[1,0]}^y + \chi_{[3,0]}^a \chi_{[1,0]}^y + \chi_{[1,0]}^a \chi_{[0,1]}^b \chi_{[1,0]}^y + \chi_{[0,3]}^b \chi_{[1,0]}^y \\
 & + \chi_{[0,1]}^a \chi_{[1,0]}^b \chi_{[1,0]}^y + 2\chi_{[1,1]}^a \chi_{[1,1]}^b \chi_{[1,0]}^y + 5\chi_{[1,1]}^b \chi_{[1,0]}^y + \chi_{[2,2]}^b \chi_{[1,0]}^y + \chi_{[3,0]}^b \chi_{[1,0]}^y \\
 & + 4\chi_{[1,0]}^y + \chi_{[1,1]}^a \chi_{[4,0]}^y + \chi_{[1,1]}^b \chi_{[4,0]}^y + 2\chi_{[4,0]}^y)q^{\frac{28}{3}} \\
 & + (u^3 \chi_{[1,1]}^a + u^3 \chi_{[0,2]}^a \chi_{[0,1]}^b + 2u^3 \chi_{[1,0]}^a \chi_{[0,1]}^b + u^3 \chi_{[2,1]}^a \chi_{[0,1]}^b + u^3 \chi_{[0,1]}^a \chi_{[1,0]}^b + u^3 \chi_{[1,1]}^b \\
 & + u^3 \chi_{[1,0]}^a \chi_{[1,2]}^b + u^3 \chi_{[1,0]}^a \chi_{[2,0]}^b - u^3 \chi_{[1,1]}^y + u^3 \chi_{[1,0]}^a \chi_{[0,1]}^b \chi_{[3,0]}^y + u^3 \chi_{[3,0]}^y + u^3 \\
 & + u^{-3} \chi_{[1,1]}^a + u^{-3} \chi_{[1,0]}^a \chi_{[0,1]}^b + u^{-3} \chi_{[0,1]}^a \chi_{[0,2]}^b + 2u^{-3} \chi_{[0,1]}^a \chi_{[1,0]}^b + u^{-3} \chi_{[1,2]}^a \chi_{[1,0]}^b \\
 & + u^{-3} \chi_{[2,0]}^a \chi_{[1,0]}^b + u^{-3} \chi_{[1,1]}^b + u^{-3} \chi_{[0,1]}^a \chi_{[2,1]}^b - u^{-3} \chi_{[1,1]}^y + u^{-3} \chi_{[0,1]}^a \chi_{[1,0]}^b \chi_{[3,0]}^y \\
 & + u^{-3} \chi_{[3,0]}^y + u^{-3})q^{10} \\
 & + (\chi_{[0,3]}^a \chi_{[0,1]}^y + 3\chi_{[1,1]}^a \chi_{[0,1]}^y + \chi_{[3,0]}^a \chi_{[0,1]}^y - \chi_{[1,0]}^a \chi_{[0,1]}^b \chi_{[0,1]}^y + \chi_{[0,3]}^b \chi_{[0,1]}^y - \chi_{[0,1]}^a \chi_{[1,0]}^b \chi_{[0,1]}^y \\
 & + \chi_{[1,1]}^a \chi_{[1,1]}^b \chi_{[0,1]}^y + 3\chi_{[1,1]}^b \chi_{[0,1]}^y + \chi_{[3,0]}^b \chi_{[0,1]}^y + \chi_{[0,1]}^y + \chi_{[0,3]}^a \chi_{[2,0]}^y + 8\chi_{[1,1]}^a \chi_{[2,0]}^y \\
 & + 2\chi_{[2,2]}^a \chi_{[2,0]}^y + \chi_{[3,0]}^a \chi_{[2,0]}^y + \chi_{[1,0]}^a \chi_{[0,1]}^b \chi_{[2,0]}^y + \chi_{[0,3]}^b \chi_{[2,0]}^y + \chi_{[0,1]}^a \chi_{[1,0]}^b \chi_{[2,0]}^y \\
 & + 3\chi_{[1,1]}^a \chi_{[1,1]}^b \chi_{[2,0]}^y + 8\chi_{[1,1]}^b \chi_{[2,0]}^y + 2\chi_{[2,2]}^b \chi_{[2,0]}^y + \chi_{[3,0]}^b \chi_{[2,0]}^y + 9\chi_{[2,0]}^y + \chi_{[1,1]}^a \chi_{[5,0]}^y \\
 & + \chi_{[1,1]}^b \chi_{[5,0]}^y + 2\chi_{[5,0]}^y)q^{\frac{32}{3}}
 \end{aligned}$$

$$\begin{aligned}
 & + (2u^3 \chi_{[1,1]}^a \chi_{[1,0]}^y + 2u^3 \chi_{[0,2]}^a \chi_{[0,1]}^b \chi_{[1,0]}^y + 6u^3 \chi_{[1,0]}^a \chi_{[0,1]}^b \chi_{[1,0]}^y + 2u^3 \chi_{[2,1]}^a \chi_{[0,1]}^b \chi_{[1,0]}^y \\
 & + u^3 \chi_{[0,1]}^a \chi_{[1,0]}^b \chi_{[1,0]}^y + 2u^3 \chi_{[1,1]}^b \chi_{[1,0]}^y + 2u^3 \chi_{[1,0]}^a \chi_{[1,2]}^b \chi_{[1,0]}^y + 2u^3 \chi_{[1,0]}^a \chi_{[2,0]}^b \chi_{[1,0]}^y \\
 & + 4u^3 \chi_{[1,0]}^y - u^3 \chi_{[2,1]}^y + u^3 \chi_{[1,0]}^a \chi_{[0,1]}^b \chi_{[4,0]}^y + u^3 \chi_{[4,0]}^y + 2u^{-3} \chi_{[1,1]}^a \chi_{[1,0]}^y \\
 & + u^{-3} \chi_{[1,0]}^a \chi_{[0,1]}^b \chi_{[1,0]}^y + 2u^{-3} \chi_{[0,1]}^a \chi_{[0,2]}^b \chi_{[1,0]}^y + 6u^{-3} \chi_{[0,1]}^a \chi_{[1,0]}^b \chi_{[1,0]}^y + 2u^{-3} \chi_{[1,2]}^a \chi_{[1,0]}^b \chi_{[1,0]}^y \\
 & + 2u^{-3} \chi_{[2,0]}^a \chi_{[1,0]}^b \chi_{[1,0]}^y + 2u^{-3} \chi_{[1,1]}^b \chi_{[1,0]}^y + 2u^{-3} \chi_{[0,1]}^a \chi_{[2,1]}^b \chi_{[1,0]}^y + 4u^{-3} \chi_{[1,0]}^y \\
 & - u^{-3} \chi_{[2,1]}^y + u^{-3} \chi_{[0,1]}^a \chi_{[1,0]}^b \chi_{[4,0]}^y + u^{-3} \chi_{[4,0]}^y) q^{\frac{34}{3}} + \mathcal{O}(q^{12}). \tag{4.10}
 \end{aligned}$$

5 Conclusions and discussions

In this paper we calculated the superconformal index of $\mathcal{N} = (1, 0)$ theories realized on N M5-branes at the $\mathbb{C}^2/\mathbb{Z}_k$ singularity. We used the holographic description of the theories, M-theory on $AdS_7 \times S^4/\mathbb{Z}_k$. For small k and N we confirmed that the indices are consistent with the expected flavor symmetries in table 1. Namely, each of the index is expanded in terms of characters of the flavor symmetry. To see the symmetry enhancement at $N = 2$ it is crucial to include the finite N corrections due to wrapped M2-branes. We use the formula (3.4) including the single-wrapping contributions.

In fact for the $N = 1$ case the 6d index was reproduced from 5d abelian quiver theories in [22]. It is nice if one can generalize their results and compare directly with our results for $N \geq 2$.

There are many ways of extension. We consider only the simple class of $\mathcal{N} = (1, 0)$ theories realized on a D6-NS5 system. It is known that this is generalized by introducing D8-branes [2–4]. Because the corresponding supergravity solutions are known [23–25] it would be possible to apply our method to these theories. Inclusion of orientifold planes and M9-planes [1, 26, 27] may also be interesting.

In this paper we focus only on the single-wrapping contributions. This is because we have not yet understood how to determine the integration contours in the gauge integrals that we need to perform to calculate the contribution of multiple branes. In the case of $(2, 0)$ theory we can define Schur-like index and it was found in [12] that by adopting an appropriate pole selection rule we can reproduce the known Schur-like index. Although we cannot define the Schur-like index for $\mathcal{N} = (1, 0)$ theory, unfortunately, this strongly suggests that we can reproduce the all order index by including multiple-brane contributions.

Acknowledgments

We would like to thank R. Arai and D. Yokoyama for wonderful discussions and useful comments.

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