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QCD axion window and false vacuum Higgs inflation

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ABSTRACT: The abundance of the QCD axion is known to be suppressed if the Hubble parameter during inflation, H_{inf} , is lower than the QCD scale, and if the inflation lasts sufficiently long. We show that the tight upper bound on the inflation scale can be significantly relaxed if the eternal old inflation is driven by the standard-model Higgs field trapped in a false vacuum at large field values. Specifically, H_{inf} can be larger than 100 GeV if the false vacuum is located above the intermediate scale. We also discuss the slow-roll inflation after the tunneling from the false vacuum to the electroweak vacuum.

KEYWORDS: Beyond Standard Model, Cosmology of Theories beyond the SM, Higgs Physics

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1 Introduction

The Peccei-Quinn (PQ) mechanism is the most plausible solution to the strong CP problem [1, 2]. It predicts the QCD axion, a pseudo Nambu-Goldstone boson, which arises as a consequence of the spontaneous breakdown of the global U(1) PQ symmetry [3, 4].

The QCD axion, a , is coupled to the standard model (SM) QCD as

$$\mathcal{L}_{\text{axion}} = \frac{g_3^2}{32\pi^2} \frac{a}{f_a} F_{\mu\nu}^\alpha \tilde{F}_\alpha^{\mu\nu}, \quad (1.1)$$

where f_a is the decay constant, g_3 the strong gauge coupling, $F_{\mu\nu}^\alpha$ the gluon field strength, and $\tilde{F}_\alpha^{\mu\nu}$ its dual. The global U(1) PQ symmetry is explicitly broken by non-perturbative effects of QCD through the above coupling. If there is no other explicit breaking, the axion acquires a mass

$$m_a(T)^2 = \frac{\chi(T)}{f_a^2}, \quad (1.2)$$

where $\chi(T)$ is the topological susceptibility. At $T \gg \Lambda_{\text{QCD}} \simeq \mathcal{O}(100) \text{ MeV}$, the topological susceptibility $\chi(T)$ is vanishingly small, and so, the axion is almost massless. $\chi(T)$ grows as the temperature decreases, and it approaches a constant value χ_0 at $T \lesssim \Lambda_{\text{QCD}}$. The detailed temperature dependence of $\chi(T)$ was studied by using the lattice QCD [5–10]. When the axion mass becomes comparable to the Hubble parameter, the axion starts

to oscillate around the potential minimum with an initial amplitude a_* . The oscillation amplitude decreases due to the subsequent cosmic expansion, and thus the effective strong CP phase is dynamically suppressed.

In the PQ mechanism some amount of coherent oscillations of the axion is necessarily produced by the misalignment mechanism [11–13], and those axions contribute to dark matter. If the initial misalignment angle, $\theta_* = a_*/f_a$, is of order unity, the observed dark matter abundance sets the upper bound of the so-called classical axion window,

$$10^8 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}, \quad (1.3)$$

where the lower bound is due to the neutrino burst duration of SN1987A [14–17]¹ or the cooling neutron star [19]. For instance, if the axion decay constant is of order the GUT scale or the string scale, i.e., $f_a = 10^{16-17} \text{ GeV}$, the axion abundance exceeds the observed dark matter abundance by many orders of magnitude. Therefore, for such large values of f_a , the initial misalignment angle θ_* must be fine-tuned to be of order 10^{-3} .

There are several ways to relax the upper bound of the classical axion window. The axion abundance can be suppressed by late-time entropy production [13, 20–23], the Witten effect [24–27], resonant conversion of the QCD axion into a lighter axion-like particle [28–31], tachyonic production of hidden photons [32, 33], dynamical relaxation [34] using the stronger QCD during inflation [35–38], and the anthropic selection of the small initial oscillation amplitude [39–41], etc.

Recently, it was proposed in refs. [42, 43] (including two of the present authors, FT and WY) that the axion overproduction problem can be ameliorated if the Hubble parameter during inflation, H_{inf} , is lower than the QCD scale, and if the inflation lasts long enough. The reason is as follows. First, the axion potential is already generated during inflation if the Gibbons-Hawking temperature, $T_{\text{inf}} = H_{\text{inf}}/2\pi$ [44], is below the QCD scale. Then, even though the axion mass is much smaller than the Hubble parameter, the axion field distribution reaches equilibrium after sufficiently long inflation when the quantum diffusion is balanced by the classical motion. The distribution is called the Bunch-Davies (BD) distribution [45]. If there are no light degrees of freedom which changes the strong CP phase (modulo 2π) during and after inflation [43, 46], the BD distribution is peaked at the CP conserving point [42, 43].² The variance of the BD distribution, $\sqrt{\langle a^2 \rangle}$, is determined by H_{inf} and the axion mass, and it can be much smaller than the decay constant f_a for sufficiently small H_{inf} . Thus, one can realize $|\theta_*| \ll 1$, which relaxes the upper bound of the classical axion window.³

So far we assume that the QCD scale during inflation is the same as in the present vacuum. However, this may not be the case. For instance, the Higgs field may take large

¹In ref. [18] it was pointed out that the accretion disk formed around the proto-neutron star (or black hole) may explain the late-time neutrino emission ($t \gtrsim 5 \text{ sec.}$) in which case the bound may be weakened.

²If the QCD axion has a mass mixing with a heavy axion whose dynamics induces a phase shift of π , one can dynamically realize a hilltop initial condition, $\theta_* \simeq \pi$ [47]. See also ref. [48] for realizing the hilltop initial condition in a supersymmetric set-up.

³The cosmological moduli problem of string axions can also be relaxed in a similar fashion [46]. See also refs. [49–53] for related topics.

field values in the early universe. It is well known that the SM Higgs potential allows another minimum at a large field value. It depends on the precise value of the top quark mass whether the other minimum is lower or higher than the electroweak vacuum [54]. It is also possible that the Higgs potential is uplifted by some new physics effects. If the Higgs field has a vacuum expectation value (VEV) much larger than the weak scale in the early universe, the renormalization group evolution of the strong coupling constant is modified due to the heavier quark masses, and the effective QCD scale becomes much larger than Λ_{QCD} .

In this paper, we revisit the axion with the BD distribution, focusing on a possibility that the effective QCD scale during inflation is higher than the present value due to a larger expectation value of the SM Higgs field. Our scenario is as follows. We assume that the Higgs is initially trapped in a false vacuum located at large field values where the eternal old inflation takes place. After exponentially long inflation, the Higgs tunnels into the electroweak vacuum through the bubble nucleation. We assume that the slow-roll inflation follows the quantum tunneling so that our observable universe is contained in the single bubble. What is interesting is that the axion acquires a relatively heavy mass due to the enhanced QCD scale and reaches the BD distribution during the old inflation. As we shall see later, the effective QCD scale can be as high as $\mathcal{O}(10^5)$ GeV in the minimal extension of the SM. Thus, the upper bound on the inflation scale as well as the required e-folding number can be greatly relaxed.

The possibility of a larger Higgs VEV was discussed in different contexts and set-ups. For instance, in a supersymmetric extension of the SM, there are flat directions containing the Higgs field, and some of which may take a large VEV during inflation [34–38, 55]. In this case, the axion field can be so heavy that it is stabilized at the minimum during the inflation, suppressing the axion isocurvature perturbation [38]. However, it is difficult to suppress the axion abundance because the CP phases of the SUSY soft parameters generically contribute to the effective strong CP phase [37] (see, however, ref. [48]). It was also pointed out that topological inflation takes place if the SM Higgs potential has two almost degenerate vacua at the electroweak and the Planck scales [56]. (See also refs. [57–60] for the Higgs inflation and the multiple point criticality.) We will come back to this possibility later in this paper. In the following we mainly consider the SM and its simple extension which does not involve any additional CP phases, and assume that the Higgs potential at the false vacuum drives the old inflation.

The rest of this paper is organized as follows. In section 2 we briefly review how the BD distribution suppresses the axion abundance. In section 3 we estimate the effective QCD scale when the Higgs field is trapped in a false vacuum at large field values, and derive the bound on the inflation scale H_{inf} for avoiding the axion overproduction. In section 4 we study the bubble formation and the subsequent evolution of the universe. The slow-roll inflation is discussed in section 5. The last section is devoted to discussion and conclusions.

2 The QCD axion and the Bunch-Davies distribution

Let us briefly review the QCD axion abundance and properties of the BD distribution. We refer an interested reader to the original references [42, 43] for more details.

The axion mass is temperature-dependent and it is parametrized by

$$m_a(T) \simeq \begin{cases} \frac{\sqrt{\chi_0}}{f_a} \left(\frac{T_{\text{QCD}}}{T} \right)^n & T \gtrsim T_{\text{QCD}} \\ 5.7 \times 10^{-6} \left(\frac{10^{12} \text{ GeV}}{f_a} \right) \text{ eV} & T \lesssim T_{\text{QCD}} \end{cases}, \quad (2.1)$$

where the exponent is given by $n \simeq 4.08$ [8], and we adopt $T_{\text{QCD}} \simeq 153 \text{ MeV}$ and $\chi_0 \simeq (75.6 \text{ MeV})^4$. At $T \gg T_{\text{QCD}}$, the axion is almost massless, while it acquires a nonzero mass as the temperature decreases down to T_{QCD} . Then, the axion starts to oscillate around the minimum when its mass becomes comparable to the Hubble parameter H . The axion abundance is given by [61]

$$\Omega_a h^2 \simeq 0.35 \left(\frac{\theta_*}{0.001} \right)^2 \times \begin{cases} \left(\frac{f_a}{3 \times 10^{17} \text{ GeV}} \right)^{1.17} & f_a \lesssim 3 \times 10^{17} \text{ GeV} \\ \left(\frac{f_a}{3 \times 10^{17} \text{ GeV}} \right)^{1.54} & f_a \gtrsim 3 \times 10^{17} \text{ GeV} \end{cases}, \quad (2.2)$$

where we assume that the axion starts to oscillate during the radiation-dominated era and there is no extra entropy production afterwards. One can see from the above equation that f_a cannot be larger than about 10^{12} GeV for $\theta_* \sim 1$, since otherwise the axion abundance would exceed the observed dark matter abundance, $\Omega_{\text{DM}} h^2 \simeq 0.12$ [62]. Thus, one needs $|\theta_*| \ll \mathcal{O}(1)$ for $f_a \gg 10^{12} \text{ GeV}$. If all values of $-\pi \leq \theta_* < \pi$ are equally likely, such an initial condition requires a fine-tuning. As we shall see below, however, this is not necessarily the case if the inflation scale is lower than the QCD scale.

During inflation one can define the Gibbons-Hawking temperature, $T_{\text{inf}} = H_{\text{inf}}/2\pi$ [44], associated with the horizon. Let us suppose that the temperature is close to or smaller than the QCD scale so that the QCD axion acquires a small but nonzero mass, $m_{a,\text{inf}}$. If the Higgs VEV is same as in the present vacuum, one can estimate it by $m_{a,\text{inf}} \simeq m_a(T_{\text{inf}})$.⁴ In the following we assume that $m_{a,\text{inf}}$ is much smaller than H_{inf} . As we are interested in the relatively large f_a and small θ_* , we can approximate the potential as the quadratic one,

$$V(a) \simeq \frac{1}{2} m_{a,\text{inf}}^2 a^2. \quad (2.3)$$

Then, after a sufficiently large number of e-folds, $N \gtrsim N_{\text{eq}} \equiv H_{\text{inf}}^2/m_{a,\text{inf}}^2$, the classical motion and the quantum diffusion are balanced, leading to the BD distribution peaked at the potential minimum. The typical value of the misalignment angle is then given by the variance of the BD distribution [42, 43],

$$|\theta_*| \sim \sqrt{\langle \theta_{\text{inf}}^2 \rangle} = \sqrt{\frac{3}{8\pi^2}} \frac{H_{\text{inf}}^2}{m_{a,\text{inf}} f_a}. \quad (2.4)$$

Thus, $|\theta_*|$ is naturally much smaller than unity if $H_{\text{inf}} \ll \sqrt{m_{a,\text{inf}} f_a}$. It is assumed here that the minimum of the axion potential does not change (modulo $2\pi f_a$) during and after

⁴It is possible that the axion mass during inflation for $T_{\text{inf}} \gtrsim T_{\text{QCD}}$ is slightly modified from eq. (2.1) due to the gravitational effects.

the inflation [43]. For instance, in the case that the Higgs VEV is at the weak scale, the axion window is open up to $f_a \sim 10^{16}$ GeV for $H_{\text{inf}} \sim 10$ MeV.

An even more interesting possibility is that the QCD scale is larger during inflation. In this case the axion mass at the very beginning of the universe was heavier than the current one, and the upper bound on H_{inf} to suppress the axion abundance is relaxed. As we shall see in the next section, this possibility can be realized if the Higgs field trapped in a false vacuum drives the eternal old inflation.

3 False vacuum Higgs inflation

In the present universe the SM Higgs develops a nonzero VEV of $v_{\text{EW}} \simeq 246$ GeV, which spontaneously breaks the electroweak symmetry. Let us express the Higgs doublet ϕ as

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h \end{pmatrix} \quad (3.1)$$

where h is the Higgs boson.

In the very early universe, the Higgs may be trapped in a false vacuum at $v_{\text{false}} \gg v_{\text{EW}}$. If its potential energy dominates the universe, it drives the eternal old inflation. The false vacuum subsequently decays into the electroweak vacuum through quantum tunneling, and an open bubble universe is nucleated. We assume that, inside the bubble, another scalar field drives the slow-roll inflation and decays into the SM particles for successful reheating. Then, our observable universe is well contained in the single bubble. We will provide such an inflation model later in this paper.

When the Higgs is trapped in the false vacuum, the SM quarks acquire heavier masses than in the electroweak vacuum, and so, the effective QCD scale is enhanced. If the old inflation lasts sufficiently long, the initial misalignment angle can be naturally suppressed by the BD distribution as we have seen before. In the following, we discuss the above scenario in detail. For simplicity, we assume that the QCD axion is a string axion or a KSVZ-type axion [63, 64], where the axion decay constant (or the PQ breaking scale) does not change during and after inflation.

3.1 A false vacuum in the Higgs potential

The quartic coupling of the Higgs field receives negative contributions from the top quark loops. For the pole mass of the top quark in the range of $171 \text{ GeV} \lesssim M_t \lesssim 176 \text{ GeV}$, the SM Higgs potential reaches a turning point at $10^8 \text{ GeV} \lesssim \Lambda_{\text{max}} \lesssim M_{\text{pl}}$ [65, 66], where $M_{\text{pl}} \simeq 2.4 \times 10^{18} \text{ GeV}$ is the reduced Planck mass. Above the turning point the potential starts to decrease and may become negative, implying that the electroweak vacuum is meta-stable. The lifetime of the electroweak vacuum is much longer than the present age of the universe [67, 68], but it is under discussion whether it is stable enough in the presence of small black holes [69–80] or compact objects without horizon [81].

Broadly speaking, there are two ways to make the electroweak vacuum absolutely stable. To this end, the vacuum at large field values must be uplifted. This can be realized

by either (i) the top quark mass of $M_t \lesssim 171$ GeV in the SM framework, or (ii) introducing new physics beyond the SM.

If the top quark pole mass is given by $M_t \simeq 171$ GeV, the potential has two vacua; one is at $\langle h \rangle = v_{EW}$, and the other at $\langle h \rangle \simeq M_{pl}$. For a certain value of the top quark mass, it is even possible to make the two vacua almost degenerate, which is known as the multiple point criticality [54]. This is the minimal possibility because no new physics is necessary, although the required top quark mass is not favored by the current measurements, $M_t = 172.9 \pm 0.4$ GeV [82].

The other possibility is that the Higgs potential is uplifted at scales above Λ_{max} by some new physics. As we shall see below, one can indeed stabilize the electroweak vacuum by introducing extra heavy particles coupled to the Higgs field. In either case, if the Higgs field is trapped in the false vacuum at large scales, the eternal old inflation occurs.

Here let us focus on the latter case, and show that one can stabilize the electroweak vacuum by introducing an extra real scalar field S . We consider the following potential for S ,

$$V_S = \frac{m_S^2}{2} S^2 + \frac{\lambda_S}{4!} S^4 + \frac{\lambda_P}{2} h^2 S^2 + \dots, \quad (3.2)$$

where we have imposed a Z_2 parity on S and the dots represent higher order terms suppressed by a large cutoff scale. For $m_S^2 > 0$, $\lambda_S > 0$, and $\lambda_P > 0$, S is stabilized at the origin. We assume that S is so heavy that the SM is reproduced in the low energy limit. The one-loop effective potential for the Higgs in the $\overline{MS}$ scheme is given by the general formula [83],

$$V_{eff}(h) = \sum_i \frac{(-1)^{2s_i}}{64\pi^2} (2s_i + 1) m_i^4(h) \left(\ln \frac{m_i^2(h)}{\mu_R^2} - c \right), \quad (3.3)$$

where i represents degrees of freedom coupled to the Higgs field, s_i is its spin, μ_R is the renormalization scale, $c = 3/2$ for $s_i = 0, 1/2$, and $c = 5/6$ for $s_i = 1$. At large fields values, the effective potential is conventionally expressed by

$$V_{eff}(h) \simeq \frac{\lambda_{eff}(h)}{4} h^4, \quad (3.4)$$

where $\lambda_{eff}(h)$ is the effective Higgs self-coupling, and it is approximately given by

$$\lambda_{eff}(h) \simeq \lambda(\mu_R) - \frac{3y_t^4}{16\pi^2} \left(\log \left(\frac{y_t^2 h^2}{2\mu_R^2} \right) - \frac{3}{2} \right) + \Delta^{NP} \lambda. \quad (3.5)$$

Here we have neglected the SM contributions other than the top Yukawa coupling. The first term is the quartic coupling at the renormalization scale μ_R , the second term is the dominant radiative correction from the top quark loop, and $\Delta^{NP} \lambda$ denotes the new physics contribution from the extra scalar S . Due to the minus sign of the second term, the top quark contribution tends to push the quartic coupling toward negative values. The scalar contribution can be estimated as

$$\Delta^{NP} \lambda \simeq \frac{1}{16\pi^2 h^4} \left((m_S^2 + \lambda_P h^2)^2 \left(\log \left(\frac{m_S^2 + \lambda_P h^2}{\max[\mu_R^2, m_S^2]} \right) - \frac{3}{2} \right) + \left(\frac{3}{2} m_S^4 + 2\lambda_P m_S^2 h^2 \right) \right), \quad (3.6)$$

where we have taken λ to be zero in the loop function as it is sub-dominant around the scale $\Lambda_{\max}$. (See e.g. ref. [84] for the derivation.) This is nothing but the Coleman-Weinberg potential [83] induced by S . In the logarithm we introduce the renormalization scale in such a way that the one-loop renormalization group equation of λ for both $\mu_R < m_S$ and $\mu_R > m_S$ can be derived by $\partial V_{\text{eff}}/\partial \mu_R = 0$. The last term in eq. (3.6) is the finite term contribution which guarantees the almost vanishing Higgs mass and cosmological constant terms at $v_{\text{EW}} \lesssim \mu_R \ll m_S$.

One can see from eqs. (3.5) and (3.6) that the contribution of S uplifts the potential if $\lambda_P^2 \gtrsim 3y_t^4$ at large field values. For a proper choice of m_S and λ_P , the potential can have a false vacuum with $V_{\text{eff}}(v_{\text{false}}) > V_{\text{eff}}(v_{\text{EW}})$ at $v_{\text{false}} \gg v_{\text{EW}}$. We show in figure 1 the effective potential of the Higgs field for $\lambda = 0, y_t = 0.6, \lambda_P = 1$, and $m_S = 6.3 \times 10^{10}$ GeV at $\mu_R = 10^{10}$ GeV. One can see that the potential turns around $\Lambda_{\max} = \mathcal{O}(10^{10})$ GeV, and the false vacuum is located at $v_{\text{false}} \approx 8.7 \times 10^{10}$ GeV.⁵ Thus, one can indeed uplift the Higgs potential at high scales so that there exists a false vacuum at $v_{\text{false}} \gtrsim \Lambda_{\max}$.

Here let us comment on the allowed range of m_S and λ_P for the false vacuum to exist. For $\lambda = 0, y_t = 0.6$, and $\lambda_P = 1$ at $\mu_R = 10^{10}$ GeV, we have found that the false vacuum exists for $m_S^2 \approx (39 - 42) \times (10^{10} \text{ GeV})^2$. If one further varies λ_P in the range of $\lambda_P = 0.7 - 1.6$, the false vacuum exists for $m_S \approx 2.9 \times 10^{10} - 1.4 \times 10^{11}$ GeV; if m_S is smaller, the EW vacuum will be the unique vacuum; if m_S is larger, the vacuum at large field values will be the true vacuum and the EW vacuum will be meta-stable. We have also checked that, for $\lambda_P \lesssim 0.6$, the potential cannot be sufficiently uplifted at $h \gg \Lambda_{\max}$. On the other hand, for $\lambda_P \gtrsim 1.6$, there appears a Landau pole at the scale $\mu_R \lesssim 10^{16}$ GeV.

We also note that m_S cannot be much larger than the location of the false vacuum $h = v_{\text{false}}$. This can be seen as follows. For $m_S \gg \mu_R \approx y_t h/\sqrt{2}$, one can expand the effective potential as

$$\frac{\lambda_{\text{eff}}}{4} h^4 \approx \frac{\lambda(\mu_R)}{4} h^4 + \frac{\lambda_P^3}{48\pi^2 m_S^2} h^6 + \mathcal{O}\left(\frac{(\lambda_P h^2)^4}{16\pi^2 m_S^4}\right) \quad \text{for } \mu_R \approx \frac{y_t h}{\sqrt{2}} \ll m_S. \quad (3.7)$$

The second term on the right-hand side is the effective dimension 6 operator induced by integrating out S . If the term becomes dominant at $h \gtrsim \Lambda_{\max}$, the potential can be up-lifted and a local minimum is formed at $v_{\text{false}} \sim \sqrt{y_t^4/\lambda_P^3 m_S}$. Here we have used $d(\lambda h^4)/dh \sim y_t^4 h^3/(16\pi^2)$ at $h \gtrsim \Lambda_{\max}$. Then, $m_S \gg y_t v_{\text{false}}/\sqrt{2}$ is equivalent to $\lambda_P \gg y_t^2$. This is also the condition that the dimension 8 and higher terms can be neglected compared with the dimension 6 term. However, since λ_P is bounded above to avoid the Landau pole below the GUT scale, the inequality is only marginally satisfied. Also, the effective theory description using the higher dimensional operators breaks down at $h \gg v_{\text{false}}$. Thus, one cannot have large hierarchy between m_S and v_{false} .

⁵The adopted values of y_t and λ correspond to the central values of the measured top and Higgs masses [82].

Figure 1. The effective potential for the Higgs field in the presence of the coupling with a singlet scalar S (red solid line). We take $\lambda = 0, y_t = 0.6, \lambda_P = 1$, and $m_S = 6.3 \times 10^{11}$ GeV at $\mu_R = 10^{10}$ GeV, for which the false vacuum is located at $v_{\text{false}} \approx 8.7 \times 10^{10}$ GeV. For comparison, the potential in the SM is also shown (gray dashed line).

If the Higgs field is trapped in the false vacuum, the eternal old inflation takes place with the Hubble parameter,

$$H_{\text{inf}} = \sqrt{\frac{V_{\text{eff}}(v_{\text{false}})}{3M_{\text{pl}}^2}}. \quad (3.8)$$

We can place an upper bound on H_{inf} as a function of v_{false} as follows. Since the Higgs potential reaches the turning point due to the top quark contributions, the value of the potential maximum is roughly given by

$$V_{\text{max}} \sim \frac{y_t^4}{16\pi^2} \Lambda_{\text{max}}^4 = \mathcal{O}(10^{-5} - 10^{-4}) v_{\text{false}}^4 \quad (3.9)$$

for $v_{\text{false}} \sim (1 - 3)\Lambda_{\text{max}}$. Since the false vacuum should have a lower energy than V_{max} , we obtain

$$H_{\text{inf}} \lesssim H_{\text{max}} \equiv \sqrt{\frac{V_{\text{max}}}{3M_{\text{pl}}^2}} \sim (10 - 100) \text{ GeV} \left(\frac{v_{\text{false}}}{10^{11} \text{ GeV}} \right)^2. \quad (3.10)$$

This gives an upper bound on the inflationary scale which can be realized using the Higgs false vacuum.

In the rest of this section we will see that the axion abundance also gives an upper limit on the inflationary scale. Specifically we will see that the QCD axion can explain dark matter even for f_a as large as the Planck scale when $v_{\text{false}} \gtrsim 10^{11}$ GeV. When $v_{\text{false}} \lesssim 10^{11}$ GeV, on the other hand, the bound (3.10) becomes stronger than that from the axion abundance with $f_a \lesssim M_{\text{pl}}$.⁶ This implies that the QCD axion can only contribute

⁶Note that the current best-fit values of the top mass and the strong coupling lead to $v_{\text{false}} \sim 10^{11}$ GeV, for which the QCD axion can be the dominant dark matter with $H_{\text{inf}} \sim H_{\text{max}}$ and $f_a \sim M_{\text{pl}}$.

to a fraction of the dark matter. Alternatively, one may introduce another sector that contributes to the Hubble parameter to saturate the upper limit and explain the QCD axion dark matter.

3.2 The effective QCD scale

The effective QCD scale during the eternal old inflation gets enhanced due to heavier quark masses. To see this we solve the one-loop renormalization group equation for the strong gauge coupling,

$$\frac{dg_3}{d \log(\mu_R)} = \frac{g_3^3}{16\pi^2} \left(-11 + \sum_i \frac{2}{3} \Theta \left(\mu_R - m_i \frac{v_{\text{false}}}{v_{\text{EW}}} \right) \right), \quad (3.11)$$

where m_i denotes the SM quark masses, i runs over the quark flavor, and Θ is the Heaviside step function. First we fix the initial value of the strong gauge coupling at some high-energy scales within the SM. To be concrete we adopt $g_3(M_{\text{pl}}) = g_3^{\text{pl}} \simeq 0.5$ at the Planck scale as the boundary condition. Then we solve the renormalization group equation toward lower scales, and define the effective QCD scale $\Lambda_{\text{QCD}}^{\text{eff}}(v_{\text{false}})$ by the renormalization scale μ_R where g_3^2 becomes equal to 4π .⁷ We show the numerical result of $\Lambda_{\text{QCD}}^{\text{eff}}(v_{\text{false}})$ in figure 2. For $v_{\text{false}} > 10^6 \text{ GeV}$, it can be fitted by

$$\Lambda_{\text{QCD}}^{\text{eff}}(v_{\text{false}}) \simeq 10^5 \text{ GeV} \left(\frac{v_{\text{false}}}{M_{\text{pl}}} \right)^{4/11}, \quad (3.12)$$

where all the SM quarks are already decoupled at the $\Lambda_{\text{QCD}}^{\text{eff}}(v_{\text{false}})$ for the parameters of our interest.

3.3 The QCD axion window opened wider

Here let us estimate how much the QCD axion window is relaxed if the eternal old inflation takes place in the Higgs false vacuum. We will come back to the issue of the decay rate of the false vacuum in the next section, and here we simply assume that the inflation lasts sufficiently long so that the probability distribution of the QCD axion reaches the BD distribution.

As we have seen before, the QCD axion acquires a small but nonzero mass if the inflation scale satisfies $T_{\text{inf}} < \Lambda_{\text{QCD}}^{\text{eff}}(v_{\text{false}})$. This is naturally realized if $v_{\text{false}} \lesssim 10^{11-12} \text{ GeV}$ due to eqs. (3.10) and (3.12). For $v_{\text{false}} \gg 10^{11} \text{ GeV}$, on the other hand, the two vacua must be almost degenerate in energy. Then, the axion mass during the eternal old inflation can be well approximated by

$$m_{a,\text{inf}} \simeq \frac{\Lambda_{\text{QCD}}^{\text{eff}}(v_{\text{false}})^2}{f_a}. \quad (3.13)$$

We substitute this mass into eq. (2.4) to evaluate the typical initial misalignment angle.

Let us recall that the axion window (1.3) was obtained by assuming the initial misalignment angle θ_* of order unity. Now θ_* is given by a function of H_{inf} and v_{false} (cf.

⁷Here we neglect contributions of the exotic quarks in the PQ sector, assuming that the PQ quarks are not coupled to the Higgs and they do not affect the Higgs potential through higher order corrections.

Figure 2. The effective QCD scale $\Lambda_{\text{QCD}}^{\text{eff}}$ as a function of the Higgs VEV v_{false} .

eqs. (2.4) and (3.13)), and in particular, it can be much smaller than unity for sufficiently small H_{inf} . Thus, the axion window can be expressed by the upper bound on H_{inf} for a given v_{false} and f_a . We show in figure 3 the upper bound on H_{inf} as a function of f_a for $v_{\text{false}} = 10^{18}$ GeV (red), 10^{11} GeV (green), 10^8 GeV (orange), and $\langle h \rangle = v_{\text{EW}}$ (blue) from top to bottom. The left vertical line at $f_a \sim 10^{12}$ GeV represents the classical axion window with $\theta_* = 1$. The red lines with $v_{\text{false}} = 10^{18}$ GeV correspond to the minimal scenario with $M_t \simeq 171$ GeV, in which case the false vacuum can be almost degenerate with the true vacuum in the framework of the SM. On the other hand, for $v_{\text{false}} < 10^{18}$ GeV (i.e. below the red line), one needs to introduce some new physics which uplifts the Higgs potential to make a false vacuum. For $v_{\text{false}} \sim 10^{11}$ GeV (i.e. the green line), the false vacuum is not necessarily degenerate with the electroweak one, because the upper bound on H_{inf} is comparable to H_{max} . For $v_{\text{false}} \lesssim 10^{11}$ GeV, the bound on H_{inf} from (3.10) is stronger than that from the axion abundance, and one needs another inflation sector to saturate the bound. The orange line approximately corresponds to the top quark mass ~ 176 GeV. The case of $v_{\text{false}} < 10^8$ GeV may also be possible in a more involved extension of the SM. One can see that, depending on v_{false} , the upper bound on H_{inf} is significantly relaxed compared to the SM shown by the blue line (the bottom one). Specifically, H_{inf} can be larger than ~ 10 GeV for $v_{\text{false}} \gtrsim 10^{11}$ GeV and 10^{12} GeV $\lesssim f_a \lesssim 10^{18}$ GeV.

4 False vacuum decay and slow-roll inflation

The false vacuum of the Higgs field is unstable and decays into the electroweak vacuum through the bubble nucleation. Let us denote by $\Gamma/\mathcal{V}$ the tunneling probability per unit time per unit volume. Then, the effective decay rate per the Hubble volume, $\mathcal{V}_{\mathcal{H}} \sim H_{\text{inf}}^{-3}$, is given by $\Gamma_{\text{eff,H}} = \Gamma \mathcal{V}_{\mathcal{H}}/\mathcal{V}$. If $\Gamma_{\text{eff,H}} \lesssim H_{\text{inf}}$, the inflation is eternal in a sense that in the whole universe there are always regions that continue to inflate [85–90] (see also [91–93]).

Figure 3. The upper bound on H_{inf} as a function of the decay constant for $v_{\text{false}} = 10^{18}$ GeV (red), 10^{11} GeV (green), 10^8 GeV (orange), and $\langle h \rangle = v_{\text{EW}}$ (blue), respectively, from top to bottom. The top three lines correspond to the currently allowed range of the top quark mass. The QCD axion explains dark matter on each line. Note that, for $v \lesssim 10^{11}$ GeV, the upper bound from (3.10) becomes stronger, and one needs to introduce another inflation sector to explain the QCD axion dark matter.

One can easily see this by noting that the physical volume of the inflating regions increases by a factor of

$$e^{-\Gamma_{\text{eff,H}} \Delta t} e^{3H_{\text{inf}} \Delta t} \quad (4.1)$$

over a time Δt . Therefore, in this case, the bubble formation is so rare that it cannot terminate the inflation as a whole; some fraction of the entire universe continues to inflate. It is important to note, however, that a *typical* e -folding number that a randomly picked observer experiences is not infinite, but finite (though exponentially large). Essentially, the universe at a later time is simply dominated by those regions where inflation continues, and in this sense, the eternity of eternal inflation relies on the volume measure [91, 94–96].⁸

Here we show that the typical e -folds the universe experiences before the tunneling, N_{dec} , is so large that the BD distribution of the QCD axion is reached. Specifically, we will show

$$N_{\text{dec}} \equiv \frac{H_{\text{inf}}}{\Gamma_{\text{eff,H}}} \gg N_{\text{eq}}, \quad (4.2)$$

where $N_{\text{eq}} = H_{\text{inf}}^2 / m_{a,\text{inf}}^2$ as defined below eq. (2.3). Using (3.12) and (3.13), we can rewrite the above condition as

$$\frac{\Gamma}{\mathcal{V}} \ll 10^{-6} \text{ GeV}^4 \left(\frac{v_{\text{false}}}{M_{\text{pl}}} \right)^{4/11} \left(\frac{H_{\text{inf}}}{10^3 \text{ GeV}} \right)^2 \left(\frac{f_a}{10^{16} \text{ GeV}} \right)^{-2}. \quad (4.3)$$

⁸It is possible to make the typical e -folds extremely large (e.g. $N \sim 10^{10^{10}}$) in a stochastic inflation model where the potential has a shallow local minimum around the hilltop [97].

If this condition is met, irrespective of whether the volume measure is adopted, our observable universe must have experienced a sufficiently long inflation in the past so that the initial axion field value obeys the BD distribution.

4.1 Thin-wall approximation ($10^{11} \text{ GeV} \ll v_{\text{false}} \ll M_{\text{pl}}$)

First, we consider the case of $10^{11} \text{ GeV} \ll v_{\text{false}} \ll M_{\text{pl}}$. In this case, the false vacuum must be almost degenerate with the true vacuum. This is because, as can be seen in figure 3, the Hubble parameter of the false vacuum Higgs inflation, H_{inf} , is bounded above by the QCD axion abundance, and it is much smaller than H_{max} when $v_{\text{false}} \gg 10^{11} \text{ GeV}$ (see eq. (3.10) for the definition of H_{max}). In the following we use a thin-wall approximation to estimate the vacuum decay rate. Later we will check the validity of the thin-wall approximation.

The false vacuum decay including gravitational effects was studied by Coleman and De Luccia [98] (CDL hereafter). We consider a thin-wall approximation for the CDL bubble nucleation assuming that the dominant bounce solution possesses an $O(4)$ symmetry. In the semiclassical approximation, the CDL tunneling rate per unit time per volume is given in the following form,

$$\frac{\Gamma}{V} \simeq A e^{-B}, \quad (4.4)$$

where higher order corrections are suppressed by the Planck constant. Here the prefactor A has mass dimension four, and B is the difference between the Euclidean actions for the bounce solution and the false vacuum one.

For precise determination of A , one needs to calculate the one-loop fluctuations around the bounce solution. The value of A for the standard-model Higgs potential was calculated in ref. [99]. In the presence of gravity, the calculations of A is hampered by e.g. gravitational fluctuations and renormalization of the graviton loops (see refs. [100–105] for the details). Here we simply assume that A is of order R_b^{-4} on dimensional grounds, where R_b is the physical size of the bubble at the nucleation.

In the absence of gravity, B is given by

$$B_0 = \int_0^\infty 2\pi^2 \rho^3 d\rho \left[\frac{1}{2} \left(\frac{d}{d\rho} h_{\text{sol}} \right)^2 + (V_{\text{eff}}(h_{\text{sol}}) - V_{\text{eff}}(v_{\text{false}})) \right], \quad (4.5)$$

where we have added a subscript of 0 to indicate that it does not include the effect of gravity, ρ is the radial coordinate in the Euclidean spacetime, and we have used the fact that the dominant bounce solution possesses an $O(4)$ symmetry. Here $h_{\text{sol}}(\rho)$ is the $O(4)$ symmetric bounce solution satisfying the Euclidean equation of motion,

$$\frac{d^2}{d\rho^2} h + \frac{3}{\rho} \frac{d}{d\rho} h = \frac{d}{dh} V_{\text{eff}}(h), \quad (4.6)$$

with the boundary conditions, $dh/d\rho|_{\rho \rightarrow 0} = 0$ and $h_{\rho \rightarrow \infty} = v_{\text{false}}$.

In the thin-wall approximation, one can estimate B_0 in a closed form [106],

$$B_0 = \frac{27\pi^2 \sigma^4}{2\epsilon^3}, \quad (4.7)$$

where σ is the tension of the bubble wall,

$$\sigma \equiv \int_{v_{\text{EW}}}^{v_{\text{false}}} dh \sqrt{2 [V_{\text{eff}}(h) - V_{\text{eff}}(v_{\text{false}})]}, \quad (4.8)$$

and ϵ is the energy difference between the two vacua,

$$\epsilon \equiv V_{\text{eff}}(v_{\text{false}}) - V_{\text{eff}}(v_{\text{EW}}) \quad (4.9)$$

In our case of the Higgs potential, they are given by

$$\sigma = \zeta v_{\text{false}}^3, \quad (4.10)$$

$$\epsilon \simeq 3H_{\text{inf}}^2 M_{\text{pl}}^2, \quad (4.11)$$

where ζ is a constant of $\mathcal{O}(10^{-2})$, and we have used the fact that the cosmological constant in the present vacuum is vanishingly small in eq. (4.11).

In the presence of gravity, the bounce action receives gravitational corrections [98]. Here we are interested in the case of the tunneling from the false vacuum dS to the true vacuum dS or Minkowski. In this case, the gravity effect suppresses B and makes the bubble nucleation more likely. This is because the cosmic expansion assists the bubble to expand after the nucleation. As the bounce solution deviates from the thin-wall approximation, the Gibbons-Hawking radiation induces upward fluctuations which also makes the bubble formation more likely [107]. Under the thin-wall approximation, a general form of the bounce action is given by [108]

$$B = B_0 r(x, y) \quad (4.12)$$

with

$$r(x, y) = 2 \frac{1 + xy - \sqrt{1 + 2xy + x^2}}{x^2(y^2 - 1)\sqrt{1 + 2xy + x^2}}, \quad (4.13)$$

$$x = \frac{\sigma^2}{\sigma_c^2} = \frac{3\sigma^2}{4\epsilon M_{\text{pl}}^2}, \quad (4.14)$$

$$y = \frac{V_{\text{eff}}(v_{\text{false}}) + V_{\text{eff}}(v_{\text{EW}})}{\epsilon}, \quad (4.15)$$

where the critical tension σ_c is defined by

$$\sigma_c = \sqrt{\frac{4\epsilon M_{\text{pl}}^2}{3}} \simeq 2H_{\text{inf}} M_{\text{pl}}^2. \quad (4.16)$$

In our scenario both x and y are positive, and in particular, $y > 1$. While x is always positive, y can be negative in a more general case.

The above bounce solution can be broadly classified into the two regimes, $x < 1$ or $x > 1$. The weak gravity limit is contained in the former case, while the gravitational effect is very important in the latter case. Note that the expression of B is the same for both cases. The bounce solutions corresponding to $x < 1$ and $x > 1$ are often called type A and type B bounces, respectively [107, 109].

In the limit of $V_{\text{eff}}(v_{\text{EW}}) \rightarrow 0$, i.e., $y \rightarrow 1$, the above expression of B is reduced to the original formula of CDL [98],

$$B = B_0 \left(1 + \frac{\sigma^2}{\sigma_c^2}\right)^{-2}. \quad (4.17)$$

In this limit, one can clearly see that the gravitational effect becomes relevant when $\sigma > \sigma_c$ (i.e. $x > 1$), while it is only a minor effect otherwise. In terms of our model parameters, we can rewrite the condition $x > 1$ as follows,

$$v_{\text{false}} \gtrsim v_c \simeq 2 \times 10^{13} \text{ GeV} \left(\frac{\zeta}{10^{-2}}\right)^{-\frac{1}{3}} \left(\frac{H_{\text{inf}}}{10 \text{ GeV}}\right)^{\frac{1}{3}}, \quad (4.18)$$

where v_c denotes the critical value of v_{false} . So, the gravitational effect becomes relevant as v_{false} increases.

Now we are ready to estimate B . Let us fix the inflation scale H_{inf} to some value (e.g. $H_{\text{inf}} = 10 \text{ GeV}$) for simplicity, and vary v_{false} .⁹ For $v_{\text{false}} \lesssim v_c$, the gravitational effect is negligible, and B increases in proportion to v_{false}^{12} . For $v_{\text{false}} \gtrsim v_c$, on the other hand, B becomes independent of v_{false} . Thus, B takes the smallest value at the smallest possible v_{false} . In other words, the vacuum decay rate is the largest at the smallest v_{false} in this regime where the thin-wall approximation is applicable.¹⁰ Using (4.12), (4.10), and (4.11), we obtain

$$B \simeq B_0 = \mathcal{O}(10^7) \left(\frac{\zeta}{10^{-2}}\right)^4 \left(\frac{v_{\text{false}}}{10^{12} \text{ GeV}}\right)^{12} \left(\frac{H_{\text{inf}}}{10^3 \text{ GeV}}\right)^{-6}. \quad (4.19)$$

Therefore, in the case of $10^{11} \text{ GeV} \ll v_{\text{false}} \ll M_{\text{pl}}$, B is so large that the vacuum decay rate is exponentially suppressed, and the condition (4.3) is trivially satisfied.

4.2 Non-degenerate vacua ($v_{\text{false}} \sim 10^{11} \text{ GeV}$)

In the case of $v_{\text{false}} \sim 10^{11} \text{ GeV}$, one can see from figure 3 that H_{inf} can be as large as $\mathcal{O}(10^2) \text{ GeV}$, which is comparable to H_{max} defined by the height of the potential barrier. If the upper bound on H_{inf} is saturated, the thin-wall approximation is not applicable, and a dedicated analysis is needed.

Let us make an order of magnitude estimate of B and the decay rate. The bubble size R_b can be roughly estimated as follows. The first term in (4.5) is of order v_{false}^2/R_b^2 , while the second term is of order V_{max} since we assume that the two vacua are not degenerate. By balancing the two terms, we get,

$$R_b \sim \frac{v_{\text{false}}}{V_{\text{max}}^{1/2}}. \quad (4.20)$$

Since the integral of (4.5) is essentially cut off around the size of the bounce solution, $\rho \sim R_b$, we obtain

$$B_0 \sim 2\pi^2 R_b^4 V_{\text{max}} = \mathcal{O}(10^{5-6}), \quad (4.21)$$

⁹This is justified because the upper bound on H_{inf} scales only as $v_{\text{false}}^{4/11}$. See eq. (3.12).

¹⁰The prefactor A may grow as v_{false} increases even when $v_{\text{false}} \gtrsim v_c$. However, the vacuum decay rate is mainly determined by B , and the increase of A does not change our conclusion.

Figure 4. Numerical result of the bounce solution by varying ρ . The potential is given in figure 1.

where we have used (3.9). Using $A \sim R_b^{-4}$, we arrive at

$$\frac{\Gamma}{\mathcal{V}} \lesssim 10^{-10^4} \left(\frac{v_{\text{false}}}{10^{11} \text{ GeV}} \right)^4 \text{ GeV}^4 \quad (4.22)$$

for which (4.3) is well satisfied.

We have numerically solved the equation of motion and obtained the bounce solution for the potential shown in figure 1. The bounce solution $v_{\text{false}} - h_{\text{sol}}$ is shown in figure 4 as a function of ρ . We have calculated B for this solution and obtained

$$B \approx 2.2 \times 10^5, \quad (4.23)$$

which agrees well the above naïve estimate, justifying our conclusion.

4.3 Thick-wall regime ($v_{\text{false}} \sim M_{\text{pl}}$)

When v_{false} approaches the Planck mass, the potential becomes relatively flat near the maximum. Then, the Higgs stays mostly around the potential maximum in the bounce solution, and the thin-wall approximation breaks down. Such a bounce solution can be interpreted as a combination of thermal fluctuations and the subsequent quantum tunneling [107]. In particular, when $v_{\text{false}} \sim M_{\text{pl}}$, the Hubble horizon in the wall, H_{dw}^{-1} , becomes comparable to the wall width, and the bounce is approximately given by the Hawking-Moss (HM) solution [110]. Once such a domain is formed, it will start to expand exponentially, and the topological Higgs inflation begins [56] (see refs. [111, 112] for the original topological inflation).

To see this, let us again fix H_{inf} and increase v_{false} , in other words, we increase x while y is fixed. In the limit of $x \gg 1$, the r -function in (4.13) approaches

$$r(x, y) \simeq \frac{2}{x^2(y+1)} \quad (x \gg 1). \quad (4.24)$$

When we also take $y \approx 1$, the corresponding bounce action reads,

$$B \approx \frac{24\pi^2 M_{\text{pl}}^4}{V_{\text{eff}}(v_{\text{false}})} = \frac{8\pi^2 M_{\text{pl}}^2}{H_{\text{inf}}^2}, \quad (4.25)$$

which coincides with the HM instanton [110]. We note that the thin-wall approximation actually breaks down in this limit. Nevertheless it is assuring that the large x limit of B obtained in the thin-wall approximation reproduces the HM instanton, which lends support to the above picture.

The topological Higgs inflation typically lasts only for a few e -folding number, because the potential curvature at the maximum is not very different from the Hubble parameter inside the domain.¹¹ Then, if the Higgs rolls down to the electroweak vacuum, the inflation ends and the cosmic expansion becomes decelerated. Since the initial configuration of the HM-like instanton does not necessarily possess the $O(4)$ symmetry, the universe will be inhomogeneous unless another inflation starts immediately. Thus, the energy scale of the second inflation is considered to be of order 10^{13} GeV.¹² Such high-scale inflation will generate quantum fluctuations of the QCD axion on top of the BD distribution. Thus, θ_* will be dominated by the quantum fluctuations, but it is still smaller than unity thanks to the initial BD distribution. In this case, the QCD axion cannot be the dominant component of dark matter because of its too large isocurvature perturbations. However, it may still be the subdominant component of dark matter, which gives rise to a non-negligible amount of non-Gaussianity of the isocurvature perturbations [113–116]. In the rest of this paper we are going to focus on the case of $v_{\text{false}} \ll M_{\text{pl}}$.

4.4 Open bubble universe and the dynamics

In the case of $v_{\text{false}} \ll M_{\text{pl}}$, the initial condition of the nucleated bubble is determined by the $O(4)$ symmetric CDL tunneling solution, $ds^2 = d\xi^2 + \rho^2(\xi)d\Omega_{S^3}^2$, where $d\Omega_{S^3}^2$ is the metric of S^3 and ξ is the radial coordinate. One can obtain the metric inside the bubble by an analytic continuation, the interior of the bubble looks like an infinite open universe for observers in the bubble [117, 118]. Infinitely many open bubble universes are created in the eternally inflating false vacuum. The open bubble universes are often discussed in the context of the string landscape [119–121]. Naively, such bubble universes would be almost empty with a small energy density, but it can be combined with the standard inflationary cosmology [122]. We will study the slow-roll inflation in the next section.

The created bubble universes might have some predictions such as distinct features of the primordial density perturbations [123–127] and the negative curvature $\mathcal{K} < 0$. The latest Planck data combined with the lensing and BAO gives $\Omega_{\mathcal{K}} = 0.0007 \pm 0.0019$ (68% CL) [62]. Although the observation does not give a statistically significant preference to a nonzero negative spatial curvature, a possibility of the open bubble universe is not excluded yet.

¹¹Here we do not adopt the volume measure.

¹²The inflation model discussed in section 5 can also be applied to this case.

5 Slow-roll inflation after the bubble nucleation

The slow-roll inflation is strongly supported by the observations of CMB and large-scale structure. The false vacuum Higgs inflation considered so far cannot explain the observed density perturbations, since the Higgs potential is not sufficiently flat to realize viable slow-roll inflation in our scenario. Thus, we need another sector that drives slow-roll inflation after the tunneling event.

Let us introduce an inflaton, φ , with the following potential,

$$V_{\text{inf}}(\varphi) = V_0 - \frac{m_0^2}{2}\varphi^2 - \frac{\lambda_\varphi}{4}\varphi^4 + \frac{\varphi^6}{6M^2}, \quad (5.1)$$

where λ_φ is a positive coupling constant, m_0 the mass parameter, M the cutoff scale, and V_0 the energy density during inflation at $\varphi \sim 0$. A supersymmetric version of the model was studied in refs. [128–130]. The inflaton φ may be identified with the B–L Higgs [51, 131–134] or an axion-like particle [47, 135, 136]. As we shall see shortly, λ_φ is much smaller than unity. For the moment we assume that m_0 is negligibly small in the following, but it is straightforward to take its effect on the inflaton dynamics. In fact, when we couple φ to the Higgs field, we will introduce a nonzero mass to realize the slow-roll inflation.

We briefly summarize here the known properties of the above inflation model. The minimum of $V_{\text{inf}}(\varphi)$ is located at

$$\varphi_{\text{min}} = \sqrt{\lambda_\varphi} M \ll M_{\text{pl}}. \quad (5.2)$$

The potential $V_{\text{inf}}(\varphi)$ is very flat around the origin, and so, if φ is initially around the origin, the slow-roll inflation takes place. The CMB normalization fixes the quartic coupling λ_φ , and the other parameters such as the inflation scale and the inflaton mass at the minimum are expressed in terms of φ_{min} as [131]

$$\lambda_\varphi \simeq 5 \times 10^{-13} \left(\frac{40}{N_*} \right)^3, \quad (5.3)$$

$$H_{\text{inf},\varphi} \simeq 10^{-7} \left(\frac{40}{N_*} \right)^{3/2} \left(\frac{\varphi_{\text{min}}^2}{M_{\text{pl}}} \right), \quad (5.4)$$

$$m_{\varphi,\text{min}} \simeq 10^{-6} \varphi_{\text{min}} \left(\frac{40}{N_*} \right)^{3/2}, \quad (5.5)$$

where N_* is the e-folding number at the horizon exit of the CMB scales. Since the inflation scale must be lower than that of the false vacuum Higgs inflation, N_* cannot exceed 50 or so. The spectral index is given by

$$n_s \simeq 1 - \frac{3}{N_*}, \quad (5.6)$$

which is too small to explain the observed value $n_s \simeq 0.965 \pm 0.004$ [62]. It is known however that the spectral index is extremely sensitive to the shape of the inflaton potential, and even a tiny deviation from the quartic potential can make it consistent with the observed value. For instance, one may introduce a Z_2 breaking linear term [137], a Coleman-Weinberg

potential, $\propto \varphi^2 \log[\varphi^2]$ [131], or a nonzero mass term [138]. While the mass term alone is not capable of increasing the spectral index sufficiently, a non-zero linear term is quite efficient. This is because the linear term shifts the inflaton field at the horizon exit of the CMB scale towards smaller values of the inflaton field, where the curvature is smaller, and thus n_s increases. Note that the linear term does not contribute to one of the slow-roll parameters, η , and the other slow-roll parameter ε is negligibly small in the low-scale inflation. The mass term does shift the inflaton field values at the horizon exit similarly, but it also gives a negative contribution to both η and n_s . As we shall shortly, the Z_2 breaking linear term will also be important for avoiding too long duration of the slow-roll inflation.

Here, the main question is if we can realize the hilltop initial condition of φ after the bubble nucleation from the false vacuum. In the thin-wall approximation, the universe inside the bubble is almost empty and dominated by the (negative) curvature term. In this case one cannot make use of fine-temperature effects to set the value of φ near the origin. On the other hand, if φ has a coupling to the Ricci curvature with a coupling of order unity, it acquires a mass of order the Hubble parameter during the false vacuum Higgs inflation, and can be stabilized the origin. Considering that $\varphi \approx 0$ is only the local minimum during the false vacuum Higgs inflation, however, it is not certain if $\varphi \approx 0$ is more likely than $\varphi \approx \varphi_{\min}$. One can still argue that $\varphi \approx 0$ is chosen based on the anthropic argument for the successful slow-roll inflation.

Now let us introduce the following renormalizable coupling to Higgs field to make the origin of φ the global minimum during the false vacuum Higgs inflation,

$$\delta V \simeq \frac{\lambda_{\varphi h}}{4}(\varphi^2 - \varphi_{\min}^2)h^2. \quad (5.7)$$

where $\lambda_{\varphi h}(> 0)$ is the coupling between φ and h . The second term is introduced to cancel the contribution on the Higgs boson mass at the EW vacuum. The basic picture is as follows. When the Higgs is in the false vacuum, φ acquires a large mass,

$$(m_{\varphi}^{\text{false}})^2 \simeq \frac{\lambda_{\varphi h} v_{\text{false}}^2}{4}, \quad (5.8)$$

which is larger than H_{inf}^2 unless $\lambda_{\varphi h}$ is extremely small. If the mass is large enough, it makes the origin the global minimum along the φ direction for the fixed $h = v_{\text{false}}$. After the tunneling, the Higgs field value becomes much smaller, and so does the mass of φ . Then, the slow-roll inflation starts along the φ direction with the potential given by (5.1). In particular, since φ remains heavy except when the Higgs field really approaches the “true” vacuum, the previous argument on the bubble nucleation is considered to remain valid.

To ensure the above-mentioned dynamics, a couple of conditions must be met. First, we do not want to modify the Higgs potential significantly by introducing the above coupling. The location of the potential barrier and the false vacuum remain almost intact if

$$V_{\text{max}} \gg \lambda_{\varphi h} \varphi_{\min}^2 v_{\text{false}}^2. \quad (5.9)$$

The potential energy at the false vacuum is still dominated by the Higgs contribution if

$$V_{\text{eff}}(v_{\text{false}}) \gg V_0. \quad (5.10)$$

In general, one needs to slightly shift the parameters to uplift the false vacuum to have the same value of $V_{\text{eff}}(v_{\text{false}})$, but this does not modify the previous argument on the bubble nucleation. In addition, the loop contribution of φ to the Higgs quartic coupling should be much smaller than that of the top quark. This amounts to $\lambda_{\varphi h} \ll y_t^2$. Secondly, the origin of φ is the global minimum in the φ -direction at $h = v_{\text{false}}$ if

$$\lambda_{\varphi h} \varphi_{\text{min}}^2 v_{\text{false}}^2 \gg \lambda_{\varphi} \varphi_{\text{min}}^4 \sim V_0. \quad (5.11)$$

Lastly, although we have required the locations of the potential maximum and the false vacuum along the Higgs direction remain almost unchanged, the location of the “true” vacuum is necessarily shifted from v_{EW} to $\tilde{v}_{\text{true}} \sim \sqrt{\lambda_{\varphi h} / \lambda_{\text{eff}}(\tilde{v}_{\text{true}})} \varphi_{\text{min}}$.¹³ This is because the Higgs acquires a negative mass from (5.7) if $\varphi = 0$. Thus, the mass of φ would remain positive even after the Higgs tunnels to $\tilde{v}_{\text{true}}$. (see eq. (5.7)). So, we introduce a nonzero mass m_0 in eq. (5.1) to cancel this contribution so that the effective mass is negative and its absolute magnitude is still smaller than $H_{\text{inf},\varphi} \sim \sqrt{V_0}/M_{\text{Pl}}$ at $h \approx \tilde{v}_{\text{true}}$. By integrating out the Higgs boson at around $\varphi \approx 0$, there is a tree-level contribution to the quartic potential $\delta\lambda_{\varphi} \sim -\lambda_{\varphi h}^2 / \lambda_{\text{eff}}$. If we require $|\delta\lambda_{\varphi}| < |\lambda_{\varphi}|$, we obtain

$$\lambda_{\varphi h} \lesssim 10^{-7} - 10^{-6}. \quad (5.12)$$

For larger $\lambda_{\varphi h}$, the CMB normalization condition would require cancellation between λ_{φ} and the contribution from integrating out the Higgs. Although this does not induce any technical problems, we assume that $\lambda_{\varphi h}$ satisfies the above inequality for simplicity.¹⁴ Note that the Higgs is so heavy around the origin of φ that integrating the Higgs does not lead to any logarithmic corrections to the inflaton potential. Also the inflaton should have only suppressed couplings to matter in a non-supersymmetric set-up, since otherwise the flat potential of the inflaton is spoiled by the Coleman-Weinberg corrections [83] (see refs. [131, 132, 140–142]).

The conditions (5.9), (5.10), and (5.11) are summarized as

$$\varphi_{\text{min}} \ll \text{Min} \left[\frac{10^{-2}}{\sqrt{\lambda_{\varphi h}}} v_{\text{false}}, 10^{13} \text{ GeV} \left(\frac{H_{\text{inf}}}{10^2 \text{ GeV}} \right)^{1/2}, 10^6 \sqrt{\lambda_{\varphi h}} v_{\text{false}} \right], \quad (5.13)$$

where we have used (3.9) and (5.3). Unless $\lambda_{\varphi h}$ is much smaller than unity, either the first or second term in the bracket gives the strongest condition on φ_{min} . Therefore, it is indeed possible to satisfy the constraints once the energy scale of the slow-roll inflation is taken to be sufficiently low. The energy density of the slow-roll inflation can be as large as $[10^{11-12} \text{ GeV}]^4$.

The slow-roll inflation model is of the hilltop type, and one may wonder that eternal inflation may take place, which would erase the BD distribution established during the false

¹³ $\tilde{v}_{\text{true}}$ should not be confused with v_{EW} .

¹⁴For larger $\lambda_{\varphi h}$, it is convenient to rewrite the relevant part of the potential as $V \approx (\lambda_{\text{eff}}/16) \times (\lambda_{\varphi h} / \lambda_{\text{eff}}(\varphi^2 - \varphi_{\text{min}}^2) + 2h^2)^2 + \text{const.}$, which contains a flat direction in the φ - h plane. By adding a certain quartic coupling of φ , one can have a successful hilltop inflation around $\varphi = 0$ and $h \sim \sqrt{\lambda_{\varphi h} / \lambda_{\text{eff}}(\tilde{v}_{\text{true}})} \varphi_{\text{min}}$. The inflaton dynamics is similar to the alchemical inflation model proposed in ref. [139], where the inflaton naturally turns into the Higgs after inflation. This implies that the SM sector is naturally reheated.

vacuum Higgs inflation. In fact, if one does not adopt the volume measure, the typical e -folding number is finite and is not so large, and the BD distribution established during the false vacuum Higgs inflation remains intact. We also note that a better fit to the observed spectral index is obtained if we add a tiny Z_2 breaking linear term in V_{inf} [137]. The linear term shifts the location of the potential maximum of φ which effectively reduces the total e -folding number. In any case, the BD distribution is not modified during the slow-roll inflation.

The reheating is considered to proceed through the perturbative decays

$$\varphi \rightarrow hh \quad (5.14)$$

or with the parametric resonance and dissipation effects via the quartic coupling (5.7). In particular, the tachyonic resonance transfer $\mathcal{O}(1)$ energy of φ into the Higgs field condensate [143, 144]. This happens when $\lambda_{\varphi h} \varphi_{\text{min}}^2 \gtrsim m_\varphi^2$ namely $\lambda_{\varphi h} \gtrsim |\lambda_\varphi|$. The Higgs fields soon form the SM thermal plasma and the reheating is successful. Depending on the size of the coupling, $\lambda_{\varphi h}$, the reheating can be instantaneous, in which case the reheating temperature can be as high as 10^{11-12} GeV. Then, thermal leptogenesis [145] or leptogenesis via active neutrino oscillation [146, 147] are possible. If φ is identified with the B–L Higgs boson, non-thermal leptogenesis may also take place as φ directly decays into right-handed neutrinos.

6 Discussion and conclusions

So far we have focused on the possibility that the Higgs field is trapped in the false vacuum and drives the eternal old inflation. After the bubble formation, another scalar field φ drives the slow-roll inflation. There is another possibility that the Higgs field value is set to be larger than the present one due to the inflaton field φ .¹⁵ In this case, φ drives both eternal (or extremely long) inflation and the subsequent slow-roll inflation that explains the observed primordial density perturbations (see e.g. ref. [97]). As we have seen before, the Higgs acquires a negative mass from the potential like (5.7) during inflation where $\varphi \simeq 0$. Thus, the field value of h can be largely displaced from v_{EW} during the inflation. Consequently, the QCD axion window can be similarly opened to large values of f_a , if the inflation scale is lower than the enhanced effective QCD scale and the inflation lasts long enough. Interestingly, the inflaton φ may be identified with the singlet S that uplifts the Higgs potential. In this case, the false vacuum of the Higgs potential will disappear due to the radiative corrections of φ because the inflaton is light. This is the main difference from the scenario discussed after (3.2).

The abundance of the QCD axion depends on the initial misalignment angle θ_* . For $f_a \gtrsim 10^{12}$ GeV, one usually assumes $\theta_* \ll 1$ to avoid the overproduction of the QCD axion. On the other hand, it was recently pointed out that θ_* follows the BD distribution and therefore can be suppressed, if the Hubble parameter during inflation is comparable to or lower than the QCD scale, and if the inflation lasts sufficiently long. To this end, one often

¹⁵The φ field does not have to be the inflaton, and it can be another scalar (such as curvaton) or fermion condensates. The only requirement is to keep the Higgs at large values during the eternal inflation.

needs to introduce another sector for the very long inflation. In this paper we have pointed out that the false vacuum Higgs inflation can do the job. Furthermore, the effective QCD scale is enhanced because of the Higgs field value larger than the present value, which significantly relaxes the upper bound on the inflation scale. We have found that the Hubble parameter during inflation can be larger than 100 GeV if the Higgs false vacuum is above the intermediate scale. We have also shown that the typical e -folding number that the universe experiences before the bubble nucleation is so large that the BD distribution of the QCD axion is realized. After the tunneling event, another slow-roll inflation must follow to generate the primordial density perturbation and to make the universe filled with radiation instead of the negative curvature. In a simple model based on the hilltop quartic inflation, the hilltop initial condition can be naturally realized if the inflaton has a quartic coupling with the Higgs field. Alternatively, through the coupling, the inflaton may be able to uplift the Higgs potential to erase the false vacuum, and it keeps the Higgs field at large values during inflation. The latter provides another attractive scenario, which warrants further investigation.

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