RECEIVED: February 17, 2020

REVISED: April 10, 2020

ACCEPTED: April 23, 2020

PUBLISHED: May 14, 2020

HQ collisional energy loss in a magnetized medium

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ABSTRACT: We study the effect of the magnetic field on the collisional energy loss of heavy quark (HQ) moving in a magnetized thermal partonic medium. This is investigated in the strong field approximation where the lowest Landau level (LLL) becomes relevant. We work in the limit $g\sqrt{eB} \ll T \ll \sqrt{eB}$ which is relevant for heavy ion collisions. Effects of the magnetic field are incorporated through the resummed gluon propagator in which the dominant contribution arises from the quark loop. We also take the approximation $\sqrt{eB} \ll M$, M being the HQ mass, so that the HQ is not Landau quantized. It turns out that there are only two types of scatterings that contribute to the energy loss of HQ; the Coulomb scattering of HQ with light quarks/anti-quarks and the t-channel Compton scattering. It is observed that for a given magnetic field, the dominant contribution to the collisional energy loss arises from Compton scattering process i.e., $Qg \rightarrow Qg$. On the other hand, of the two processes, the Coulomb scattering i.e., $Qq \rightarrow Qq$ is more sensitive to the magnetic field. The net collisional energy loss is seen to increase with increase in the magnetic field. For a reasonable strength of the magnetic field, the field dependent contribution to the collisional energy loss is of the same order as to the case without magnetic field which can be important for the jet quenching phenomena in the heavy ion collision experiments.

KEYWORDS: Heavy Ion Phenomenology, QCD Phenomenology

ARXIV EPRINT: [2002.04922](https://arxiv.org/abs/2002.04922)

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1 Introduction

Experimental observations suggest that the Heavy Ion Collisions (HICs) create a novel state of matter consisting of deconfined light quarks and gluons, called the Quark Gluon Plasma (QGP). There are a plethora of theoretical studies investigating various properties of QGP governed by Quantum Chromo Dynamics(QCD) at high temperature(T). More recent developments indicate production of a strong magnetic field, with an initial strength few times the pion mass square, i.e., $eB \sim m_\pi^2$ at RHIC and few tens of pion mass square i.e., $\sim 15m_\pi^2$ at LHC in a non-central HIC [1–6]. The existence of such a magnetic field opens up new directions towards the theoretical studies of properties of QGP leading to diverse experimental consequences. The imprint, the magnetic field lays on QGP brings in some of the most important theoretical studies including Chiral Magnetic Effect(CME) [1, 7], Chiral Magnetic Wave leading to a charge-dependent elliptic flow [8–11], transport properties of QGP in magnetic field [12–14], quarkonia suppression [15, 16], dilepton and photon production [17, 18], HQ drag and diffusion coefficients [19, 20], jet quenching [21] etc. To what extent the magnetic field embosses its influence on the deconfined medium depends on several salient properties of the HICs at the early stage. Despite high initial strength, the magnetic field eventually decays at a significant rate.

The initial dynamics of the system of deconfined matter plays a pivotal role in deciding the longevity of the magnetic field at the later stages. It can be hypothesized that the magnetic field can induce an amount of electrical conductivity which becomes adequate enough for the decaying magnetic field to persist [18, 25–27]. This in turn can induce a current which opposes the rate of decrease of the magnetic field as per Lenz’s Law [22–24]. Therefore, it is imperative that the external magnetic field persists long enough so that it can impart crucial effects on various properties of the medium. It is instructive to investigate the effect of this field on the space-time evolution of the medium. Some studies are trying to see this effect by reconstructing Hydrodynamic evolution in presence of magnetic field, i.e. Magneto hydrodynamics [28–30].

In the case of QGP, a more realistic approach of estimation of the magnetic field and its relaxation time must include the medium effects i.e., electrical conductivity (σ) of the medium. The phenomenological models that are used to describe QGP evolution show that the strongly interaction system in HICs is thermalised just after the collision ($\tau \sim 0.5\text{fm}$) where magnetic field is near its maximum value [25, 31]. In a conducting medium, magnetic field satisfies a diffusion equation with diffusion coefficient $(\sigma\mu)^{-1}$, where μ is the magnetic permeability. With this one finds that the time scale over which magnetic field remain reasonably strong over a length scale L is $\tau = L^2\sigma/4$ [32]. For the electrical conductivity $\sigma \simeq 0.04\text{T}$ from ref. [33] at $T = 200\text{MeV}$, the electrical conductivity $\sigma = 8\text{MeV}$. This leads to the relaxation time $\tau \sim 1\text{fm}$ for a system size of the order of 10fm . This also suggest that the magnetic field is a slowly varying function of time and can remain reasonably strong for a longer period of time compared to the case of without a medium. For higher temperatures σ will be higher [34] increasing the value of τ . Further, it is shown in ref. [25] in an expanding medium the magnetic field remain somewhat constant for a longer time.

Consequently, it is of utmost importance to explore to what extent the magnetic field inside QGP affects different observables of the deconfined matter. To this end, surveying the in-medium properties of heavy quarks [35–37] and quarkonia have become quite relevant in the context of magnetic field [15, 16, 38–48]. However, since the HQs are moving in real time inside the QGP, understanding and estimating the dynamical properties of HQs are also necessary. In this context, transport coefficients like drag and diffusion of HQ have been estimated in presence of a strong external magnetic field in some of the recent literatures [19, 20]. AdS/CFT has also been employed to have an estimation of the drag force of HQ [49]. Most of the calculations with strong magnetic field have been performed using perturbative QCD (pQCD) techniques in Leading Order (LO) of the strong coupling α_s in the limit $M \gg \sqrt{eB}$ so that the motion of the HQ is not directly affected by the external magnetic field. Nonetheless, the light quarks/anti-quarks are affected by the magnetic field with the gluons remaining unaffected. The thermally equilibrated light quarks are Landau quantized. The magnetic field also affects the gluon self energy through the quark loop. Further they also affect the HQ light thermal parton scattering cross-sections.

It is well known that a high energy particle created in the initial stages of the heavy ion collision loses its energy in the medium by interacting with the medium partons. This leads to the phenomena of jet quenching, which as anticipated many years ago, is one of the prominent probes of QGP [50–53]; for a recent review see ref. [54]. Generally, there are two

types of processes that contribute to the energy loss namely; radiative process [55–57] and collisional process [58–60]. Experimental results for quenching of heavy flavors [61] were suggestive of including both radiative as well as collisional energy loss has been discussed in refs. [59, 63]. In the present investigation, we focus our attention to the collisional energy loss of HQ in the background of a constant magnetic field which may be relevant for the HICs as discussed in literature [63–67].

The present investigation intends to estimate the HQ collisional energy loss ($-dE/dx$) in the low coupling regime and strong magnetic field. Specifically we will consider the hierarchy in the scales i.e., $\sqrt{\alpha_s eB} \ll T \ll \sqrt{eB}$ and $\sqrt{eB} \ll M$. To do so, we first calculate the resummed gluon propagator in the strong magnetic field background. Let us note here that only quark loop contributes to the resummed gluon propagator in the limit $eB \gg T^2$. This resummed propagator is used to estimate the collisional energy loss. In this hierarchy of scales, two types of processes contribute to the scatterings of HQ with the light thermal partons affecting the HQ energy loss. As we shall see, the collisional energy loss increases with the magnetic field and for a given magnetic field, the field dependent contribution to the collisional energy loss could be similar in magnitude to the collisional energy loss in the absence of magnetic field. This can be important for the jet quenching in the heavy ion collisions.

The paper is organized as follows. In section 2, we standardize the mathematical notations, followed by a brief description of the real-time formalism. Further, in the real-time formalism of thermal field theory, we also discuss the fermion propagator in section 3.1 and resummed gluon propagator in section 3.2 in LLL approximation. In section 4, we discuss the formalism to calculate HQ energy loss i.e., $-dE/dx$ with descriptions of both the cases; (a) when HQ is interacting with light quarks (section 4.1) and (b) HQ scattering with the thermal gluons (section 4.2). Our findings are presented in section 5 with the relevant plots and the possible explanation. In section 6, we summarize the present findings and discuss the possible outlook. In the appendices, we present the detailed calculations of the gluon self energy in a magnetized thermal medium and scattering amplitudes for the process $Qg \rightarrow Qg$.

2 Set-up

For the present investigation, we assume here that the magnetic field is constant and is along the $\hat{z}$ direction i.e., $\vec{B} = B\hat{z}$. In the subsequent subsection, we shall discuss the quark propagator in the real-time formalism of thermal field theory and in the presence of such a magnetic field. For this purpose we use the following notation. The notations $\parallel$ and $\perp$ represents the components parallel and perpendicular to the magnetic field of the corresponding quantities. For the metric tensor, we use

$$g_{\mu\nu}^{\parallel} = (1, 0, 0, -1) \quad g_{\mu\nu}^{\perp} = (0, -1, -1, 0). \quad (2.1)$$

The parallel (i.e., $a_{\mu}^{\parallel} = g_{\mu\nu}^{\parallel} a^{\nu}$) and perpendicular (i.e., $a_{\mu}^{\perp} = g_{\mu\nu}^{\perp} a^{\nu}$) components of a four-vector a_{μ} are represented as

$$a_{\mu}^{\parallel} = (a_0, 0, 0, -a_3) \quad a_{\mu}^{\perp} = (0, -a_1, -a_2, 0). \quad (2.2)$$

The four-vector product ($a^\mu b_\mu = a \cdot b$) can be written as

$$a \cdot b = a_{\parallel} \cdot b_{\parallel} - a_{\perp} \cdot b_{\perp}. \quad (2.3)$$

Similarly, both the components of square of a four-vector is

$$a_{\parallel}^2 = a_0^2 - a_3^2 \quad a_{\perp}^2 = a_1^2 + a_2^2. \quad (2.4)$$

3 Real-time formalism

In this section, we first summarize the basic formulation for real time thermal field theory in the presence of magnetic field in a self contained manner that will be used to obtain the results on energy loss. In thermal field theories (TFT), due to the Kubo-Martin-Schwinger conditions, the time argument for the fields varies from 0 to $-i\beta$, β being the inverse of the temperature. In the real time formulation of TFT, the time contour can be deformed to go from $t = 0$ to $t = \infty$ infinitesimally above the real axis and then back to $t = -i\beta$ below the real axis leading to a 2×2 matrix structure for the propagator. Corresponding to different structure for the propagation along the contour, one can define the following four functions e.g. for fermionic fields $\psi_\alpha(x)$, ($\alpha = 1, 2, 3, 4$) as

$$S^>(X, Y)_{\alpha\beta} = -i\langle\psi_\alpha(X)\bar{\psi}_\beta(Y)\rangle \quad (3.1)$$

$$S^<(X, Y)_{\alpha\beta} = i\langle\bar{\psi}_\beta(Y)\psi_\alpha(X)\rangle \quad (3.2)$$

$$S(X, Y)_{\alpha\beta} = -i\langle\hat{T}[\psi(X)_\alpha\bar{\psi}_\beta(Y)]\rangle \quad (3.3)$$

$$S^*(X, Y)_{\alpha\beta} = -i\langle\hat{T}^*[\psi_\alpha(X)\bar{\psi}_\beta(Y)]\rangle \quad (3.4)$$

Similarly, for bosonic fields i.e., $A^\mu(x)$ ($\mu = 0, 1, 2, 3$), one can define the following propagators

$$D_{\mu\nu}^>(X, Y) = -i\langle A_\mu(X)A_\nu(Y)\rangle \quad (3.5)$$

$$D_{\mu\nu}^<(X, Y) = -i\langle A_\nu(Y)A_\mu(X)\rangle \quad (3.6)$$

$$D_{\mu\nu}(X, Y) = -i\langle\hat{T}[A_\mu(X)A_\nu(Y)]\rangle \quad (3.7)$$

$$D_{\mu\nu}^*(X, Y) = -i\langle\hat{T}^*[A_\mu(X)A_\nu(Y)]\rangle. \quad (3.8)$$

In the above, $\hat{T}$ and $\hat{T}^*$ are the time ordering and anti-time ordering operators respectively which are defined as

$$\hat{T}[A(X)B(Y)] = \theta(x_0 - y_0)A(X)B(Y) \pm \theta(y_0 - x_0)B(Y)A(X). \quad (3.9)$$

$$\hat{T}^*[A(X)B(Y)] = \theta(y_0 - x_0)A(X)B(Y) \pm \theta(x_0 - y_0)B(Y)A(X). \quad (3.10)$$

In terms of these four functions, the 2×2 propagator is given by, e.g. for fermion fields

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \equiv \begin{pmatrix} S & S^< \\ S^> & S^* \end{pmatrix}. \quad (3.11)$$

Here, spinor indices in the second matrix are suppressed. The 11 component corresponds to the conventional time ordered propagator while the 22 component corresponds to anti time

ordering as the contour ordering along the contour below the real axis is conversely ordered in time. The off diagonal components correspond to the Wightman propagators. From the definitions of the functions, it is clear that all the four functions are not independent and are related by

$$S_{11} + S_{22} = S_{12} + S_{21}. \quad (3.12)$$

In the Keldysh representation, a linear combination of above causal propagators is used to define the retarded (S_R), advanced (S_A) and Feynman (S_F) propagators as

$$S_R = S_{11} - S_{12}, \quad S_A = S_{11} - S_{21}, \quad S_F = S_{11} + S_{22}. \quad (3.13)$$

Feynman propagator can also be obtained from the advanced and the retarded propagators as

$$S_F(K) = \left(\frac{1}{2} - \tilde{f}(k_0) \right) [S_R(K) - S_A(K)] \quad (3.14)$$

where $\tilde{f}(k_0)$ is the distribution function of the fermions. One can invert the relations of eq. (3.13) to re-write the propagators in the Keldysh basis in terms of those in the RA basis as

$$S_{11}(K) = \frac{1}{2} \left(S_F(K) + S_A(K) + S_R(K) \right), \quad (3.15)$$

$$S_{12}(K) = \frac{1}{2} \left(S_F(K) + S_A(K) - S_R(K) \right), \quad (3.16)$$

$$S_{21}(K) = \frac{1}{2} \left(S_F(K) - S_A(K) + S_R(K) \right), \quad (3.17)$$

$$S_{22}(K) = \frac{1}{2} \left(S_F(K) - S_A(K) - S_R(K) \right). \quad (3.18)$$

It may be mentioned that these formulations can also be applied to nearly equilibrium systems. For thermal equilibrium systems $\tilde{f}(k_0)$ becomes the Fermi-Dirac distribution function. In the next section we discuss the quark propagator in the background of magnetic field.

3.1 Fermion propagator in LLL

The retarded and advanced propagators (S_R, S_A) of a free quark of electric charge q_f and mass m in the presence of magnetic field B can be given as [68]

$$S_{R/A}(K) = \sum_{n=0}^{\infty} \left[\frac{i\Xi_n(K)}{K^2 - m^2} \right]_{k_0 \rightarrow k_0 \pm i\epsilon} \quad (3.19)$$

where the retarded (R)/advanced (A) corresponds to $+i\epsilon / -i\epsilon$. The sum is over all the Landau levels (LLs) that is represented by n . Four momentum squared $K^2 = k_0^2 - k_z^2 - 2n|q_f B|$. All LLs except the lowest ($n = 0$) are doubly degenerate. The numerator of eq. (3.19) has the Dirac structure as [68]

$$\Xi_n(K) = (\not{k}_{\parallel} + m)[\mathcal{P}_+ \Theta_n(\zeta) + \mathcal{P}_- \Theta_{n-1}(\zeta)] + \not{k}_{\perp} \Phi_{n-1}(\zeta) \quad (3.20)$$

where $\zeta = 2 \frac{k_{\perp}^2}{|q_f B|}$ and

$$\Theta_l(\zeta) = 2e^{-\frac{\zeta}{2}}(-1)^l L_l(\zeta) \quad (3.21)$$

$$\Phi_l(\zeta) = 4e^{-\frac{\zeta}{2}}(-1)^{l-1} L_{l-1}^1(\zeta). \quad (3.22)$$

$L_l(\zeta)$ and $L_l^1(\zeta)$ are associated Laguerre Polynomials. In eq. (3.19), the projection operator ($\mathcal{P}_{\pm}$) that projects the spin in the direction of magnetic field is defined as $\mathcal{P}_{\pm} = (1 \pm \text{sgn}(q_f B) i \gamma^1 \gamma^2)/2$. Note, here that the projection operator depends on the electric charge of the quark as the spin magnetic moment depends on the charge of the quark. As mentioned earlier, we take the strong magnetic field limit so that only LLL is relevant. We shall further assume that, at finite temperature $\sqrt{eB} \gg T, m$ so that the dynamics of light quark is governed by the magnetic field. In the LLL approximation the associated Laguerre Polynomials $L_{-1}(\zeta) = 0$ and $L_0(\zeta) = 1$, so that eq. (3.19) reduces to

$$S_{R/A}(K) = i \exp\left(-\frac{k_{\perp}^2}{|q_f B|}\right) \frac{2(k_{\parallel} + m)\mathcal{P}_{+}}{k_{\parallel}^2 - m^2 \pm i\epsilon k_0}, \quad (3.23)$$

and the Feynman propagator can be obtained from

$$\begin{aligned} S_F(K) &= \left(\frac{1}{2} - \tilde{f}(k_0)\right) \left[S_R(K) - S_A(K)\right] \\ &\equiv \left(\frac{1}{2} - \tilde{f}(k_0)\right) \rho_F(K), \end{aligned} \quad (3.24)$$

where $\tilde{f}(k_0)$ is Fermi-Dirac distribution function and $\rho_F(K)$ is quark spectral density. From eq. (3.23), it is clear that in the limit $k_{\perp}^2 \ll q_f B$, the motion of a quark is restricted in the transverse directions and allowed only in the direction parallel to the magnetic field. It can also be observed that for the infrared limit i.e., $m^2, T^2, k_{\perp}^2, k_0^2 \ll eB$, the dimensional reduction from 3+1-dimension to 1+1-dimension takes place. This dimensional reduction in the LLL approximation suggests that the pairing dynamics of quarks occurs in 1+1 dimension and spontaneous chiral symmetry breaking occurs even at weak interaction between quarks in 3+1 dimension [69].

3.2 Resummed gluon propagator in LLL

Next we consider the resummed retarded/advanced gluon propagator in the presence of magnetic field within the LLL approximation. The resummed propagator is obtained by inserting the self energy corrections in the bare propagator and can be written as

$$D_{\mu\nu}^{R/A}(K) = [(D_{\mu\nu}^{R/A}(K))_0^{-1} + \Pi^{R/A}(K)_{\mu\nu}]^{-1} \quad (3.25)$$

where the bare gluon propagator $(D_{\mu\nu}^{R/A}(K))_0$ in covariant gauge is given as

$$(D_{\mu\nu}^{R/A}(K))_0 = -\frac{P_{\mu\nu}(K)}{(k_0 \pm i\epsilon)^2 - k^2} + \xi \frac{K_{\mu} K_{\nu}}{((k_0 \pm i\epsilon)^2 - k^2)^2}. \quad (3.26)$$

In eq. (3.26), the projection operator ($P_{\mu\nu}(K)$) is defined as

$$P_{\mu\nu}(K) = -g_{\mu\nu} + \frac{K_\mu K_\nu}{(k_0 \pm i\epsilon)^2 - k^2} \quad (3.27)$$

and ξ is the gauge parameter. The retarded/advanced resummed gluon propagator depends on the retarded/advanced gluon self energy that, in general, can get contribution from both the gluon loop and the quark loop. The magnetic field does not affect the contribution from the gluon loop. However, the magnetic field modifies the quark loop contribution. The leading contribution from the gluon loop to the self energy at finite temperature T is proportional to $g^2 T^2$ while we shall see that the leading contribution from the quark loop at finite T, B is proportional to $g^2 |q_f B|$ as given in eq. (A.12). Since we work in the limit $eB \gg T^2, m^2$, we shall drop the gluon loop contribution and keep quark loop contribution in the retarded/advanced gluon self energies. Taking the magnetic field in the $\hat{z}$ direction, the most general form of gluon self energy at finite T and B can be written in terms of seven independent tensors as

$$\Pi_{\mu\nu}^{R/A}(K) = \sum_{j=\parallel, \perp, T, L} \Pi^{R/A}(K)_j P_{\mu\nu}^j(K) + \Pi_P \frac{K_\mu K_\nu}{K^2} + \Pi_n n_\mu n_\nu + \Pi_b b_\mu b_\nu \quad (3.28)$$

where $L, T, \parallel, \perp$ respectively are for longitudinal, transverse, parallel and perpendicular components of the gluon self energy. The four-vectors $n^\mu = (1, \mathbf{0})$ and $b^\mu = (0, 0, 0, -1)$, break Lorentz and rotational symmetry due to thermal medium and the magnetic field respectively. Further, the projection operators are transverse to the momentum i.e., $P_\mu P_j^{\mu\nu} = 0$. It turns out that only the four projection tensors contribute to the retarded/advanced self energies in the leading order. Explicitly these projection operators are given as

$$P_{\mu\nu}^T(K) = -g_{\mu\nu} + \frac{k_0}{k^2} \left[K_\mu n_\nu + n_\mu K_\nu \right] - \frac{1}{k^2} \left[K_\mu K_\nu + K^2 n_\mu n_\nu \right], \quad (3.29)$$

$$P_{\mu\nu}^L(K) = -\frac{k_0}{k^2} \left[K_\mu n_\nu + n_\mu K_\nu \right] + \frac{1}{k^2} \left[\frac{k_0^2}{K^2} K_\mu K_\nu + K^2 n_\mu n_\nu \right], \quad (3.30)$$

$$P_{\mu\nu}^\parallel(k) = -g_{\mu\nu}^\parallel + \frac{k_\mu^\parallel k_\nu^\parallel}{k_\parallel^2}, \quad (3.31)$$

$$P_{\mu\nu}^\perp(k) = -g_{\mu\nu}^\perp + \frac{k_\mu^\perp k_\nu^\perp}{k_\perp^2}. \quad (3.32)$$

The parallel and perpendicular components of self energy comes from the quark loop while the longitudinal and transverse components are from the gluon loop. Taking contribution from both quark and gluon loop the resummed retarded gluon propagator is given as [70]

$$D_{\mu\nu}^R(K) = -\frac{1}{\Delta(K)} \left[(K^2 - \Pi_R^\parallel(K) - \Pi_R^L(K)) P_{\mu\nu}^T(K) + (K^2 - \Pi_R^\parallel(K) - \Pi_R^L(K)) P_{\mu\nu}^L(K) \right. \\ \left. + \Pi_R^\parallel(K) P_{\mu\nu}^\parallel(K) + D_\perp(K) P_{\mu\nu}^\perp(K) \right] + \xi \frac{K_\mu K_\nu}{(K^2)^2} \quad (3.33)$$

where

$$\begin{aligned} \Delta(K) = (K^2 - \Pi_R^T(K))(K^2 - \Pi_R^L(K)) - \Pi_R^\parallel(K) \left[K^2 - a\Pi_R^T(K) \frac{K^2}{k_\parallel^2} \right. \\ \left. - \Pi_R^L(K)(1-a) \frac{k_0^2}{k_\parallel^2} \right], \end{aligned} \quad (3.34)$$

and

$$\begin{aligned} D_\perp(K) = \frac{1}{K^2 - \Pi_R^T(K) - \Pi_R^L(K)} \left[\Pi_\parallel(K)(\Pi_R^L(K) - \Pi_R^T(K))(1-a) \frac{k_0^2}{k_\parallel^2} \right. \\ \left. + \Pi_R^\perp(K)(K^2 - \Pi_R^L(K) - \Pi_R^\parallel(K)) \right] \end{aligned} \quad (3.35)$$

and $a = k_3^2/k^2$. In the LLL approximation for the propagator as in eq. (3.23) lead to $\Pi_\perp = 0$. This is due to the fact that there is no current in the transverse direction in LLL approximation. All the expressions for the gluon self energies (Π_L , Π_T and $\Pi_\parallel$) are explicitly given in the appendix A. As may be noted, the gluon loop contribution to the gluon self energy is of the order of $\alpha_s T^2$ which can also be dropped with respect to the quark loop contribution which is of the order of $\alpha_s eB$. In this approximation, the resummed retarded gluon propagator becomes

$$D_{\mu\nu}^R(K) = -\frac{1}{\Delta} [(K^2 - \Pi_R^\parallel)(P_{\mu\nu}^T + P_{\mu\nu}^L) + \Pi_R^\parallel P_{\mu\nu}^\parallel + D_\perp(K)P_{\mu\nu}^\perp] + \xi \frac{K_\mu K_\nu}{(K^2)^2} \quad (3.36)$$

where

$$P_{\mu\nu}^L + P_{\mu\nu}^T = -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}. \quad (3.37)$$

Since $D_\perp(K) \sim \alpha_s^2 eBT^2 \ll \Pi_\parallel \sim \alpha_s eB$, the third term in eq. (3.36) may be dropped. Further as may be observed from eq. (3.34) $\Delta \approx K^2(K^2 - \Pi_\parallel(K))$ with similar argument. Thus eq. (3.36), taking appropriate $i\epsilon$ factors into account, can be approximated as

$$D_{\mu\nu}^R(K) = -\frac{P_{\mu\nu}(K)}{(k_0 + i\epsilon)^2 - k^2} - \frac{\Pi_R^\parallel(k_0 + i\epsilon, \mathbf{k})P_{\mu\nu}^\parallel(K)}{K^2(K^2 - \Pi_R^\parallel(k_0 + i\epsilon, \mathbf{k}))} + \xi \frac{K_\mu K_\nu}{((k_0 + i\epsilon)^2 - k^2)^2}. \quad (3.38)$$

Similarly, the resummed advanced and Feynman gluon propagators can be written as

$$D_{\mu\nu}^A(K) = -\frac{P_{\mu\nu}(K)}{(k_0 - i\epsilon)^2 - k^2} - \frac{\Pi_A^\parallel(k_0 - i\epsilon, \mathbf{k})P_{\mu\nu}^\parallel(K)}{K^2(K^2 - \Pi_A^\parallel(k_0 - i\epsilon, \mathbf{k}))} + \xi \frac{K_\mu K_\nu}{((k_0 - i\epsilon)^2 - k^2)^2}, \quad (3.39)$$

$$D_{\mu\nu}^F(K) = (1 + 2f(k_0)) \left[D_{\mu\nu}^R(K) - D_{\mu\nu}^A(K) \right]. \quad (3.40)$$

We shall use eqs. (3.38), (3.39) and (3.40) to estimate the HQ energy loss.

Figure 1. Feynman diagram for heavy quark self energy. The bold solid line refers to HQ while the curly line corresponds to gluon. The black blob here shows the gluon resummed propagator in the presence of a magnetic field background. In the LLL, the contribution to the resummed gluon propagator comes from the quark loop only.

4 Formalism

In this section, we shall calculate the energy loss of HQ of mass M , moving in a magnetized thermal medium of light quarks/anti-quarks and gluons, with momentum $\mathbf{p}$ and energy $E = \sqrt{\mathbf{p}^2 + M^2}$. The rate of energy loss of a heavy fermion moving with velocity $\mathbf{v}$ in a thermal medium has been considered earlier in refs. [58, 59] for QED plasma and for QGP in ref. [60]. In general the energy loss of the heavy fermion is given by

$$-\frac{dE}{dx} = \frac{1}{v} \int_M^\infty dE' (E - E') \frac{d\Gamma}{dE'} \quad (4.1)$$

where Γ is the interaction rate of heavy fermion with the medium particles. For the collisional energy loss, we confine our attention to $2 \rightarrow 2$ processes only as has been considered earlier [60]. For large momentum of HQ, however, the energy loss by radiative processes will also be important. In general, for the case of HQ moving in a QGP medium of light partons there can be two types of scatterings namely the Coulomb scattering i.e., $Qq \rightarrow Qq$ and Compton scattering i.e., $Qg \rightarrow Qg$. As mentioned, we shall consider the limit $M \gg \sqrt{eB} \gg T$ so that HQ is not directly affected by the magnetic field and the light quarks are populated only in LLL. In the following sections we shall calculate the interaction rate and subsequently estimate the energy loss of the HQ.

4.1 Energy loss due to scattering with light quark: $Qq \rightarrow Qq$

The interaction rate Γ of the HQ is related to imaginary part of its retarded self energy in the medium [71]

$$\Gamma(E) = -\frac{1}{2E} (1 - \tilde{f}(E)) \text{Tr} \left[(\not{P} + M) \Im \Sigma_R(P) \right] \quad (4.2)$$

where $\tilde{f}(E)$ is Fermi-Dirac distribution function and $\Sigma_R(P)$ is the retarded self energy of HQ as shown in figure 1.

In the RTF, similar to the retarded propagator eq. (3.13), the retarded self energy can be written in terms of 11 and 12 components of self energy as

$$\Sigma_R(P) = \Sigma_{11}(P) - \Sigma_{12}(P). \quad (4.3)$$

Using the propagators for quark and gluon in the Keldysh basis the retarded self energy of HQ can be written as

$$\Sigma_R(P) = ig^2(t_a t_b) \int \frac{d^4 K}{(2\pi)^4} \left[D_{11}^{\mu\nu}(K) \gamma_\nu S_{11}(P') \gamma_\mu - D_{12}^{\mu\nu}(K) \gamma_\nu S_{12}(P') \gamma_\mu \right]. \quad (4.4)$$

The HQ propagators are $S_{11}(K) = (\not{K} + M) \Delta_{11}(K)$ and $S_{12}(K) = (\not{K} + M) \Delta_{12}(K)$ where $\Delta_{11}(K)$ and $\Delta_{12}(K)$ are given as

$$\Delta_{11}(K) = \frac{1}{K^2 - M^2 + i\epsilon} + 2\pi i \tilde{f}(k_0) \delta(K^2 - M^2) \quad (4.5)$$

and

$$\Delta_{12}(K) = 2\pi i \tilde{f}(k_0) \delta(K^2 - M^2). \quad (4.6)$$

Here, $\tilde{f}(k_0)$ is the fermion distribution function of the HQ. With these simplifications, eq. (4.4) becomes

$$\begin{aligned} \Sigma_R(P) = ig^2(t_a t_b) \int \frac{d^4 K}{(2\pi)^4} \left[D_{11}^{\mu\nu}(K) \gamma_\nu (\not{P}' + M) \gamma_\mu \Delta_{11}(P') \right. \\ \left. - D_{12}^{\mu\nu}(K) \gamma_\nu (\not{P}' + M) \gamma_\mu \Delta_{12}(P') \gamma_\mu \right]. \end{aligned} \quad (4.7)$$

It is easier for the evaluation of the retarded self energy to convert the Keldysh propagators to the RA basis using the relations as given in eqs. (3.15) and (3.16) for the propagators $D_{ab}^{\mu\nu}$ and Δ_{ab} . Let us note that in eq. (4.7), the propagators $D_{11}^{\mu\nu}$ and $D_{12}^{\mu\nu}$ are resummed gluon propagators and S_{11}, S_{12} for the HQ and hence the corresponding Δ_{11}, Δ_{12} are bare propagators. Eq. (4.7) for the retarded self energy then can be rewritten as

$$\begin{aligned} \Sigma_R(P) = \frac{ig^2(t_a t_b)}{8} \int \frac{d^4 K}{(2\pi)^4} \left[D_F^{\mu\nu}(K) \lambda_{\mu\nu} \Delta_R(P') + D_A^{\mu\nu}(K) \lambda_{\mu\nu} \Delta_R(P') \right. \\ \left. + D_R^{\mu\nu}(K) \lambda_{\mu\nu} \Delta_F(P') + D_R^{\mu\nu}(K) \lambda_{\mu\nu} \Delta_A(P') \right], \end{aligned} \quad (4.8)$$

where $\lambda_{\mu\nu} = \gamma_\mu (\not{P}' + M) \gamma_\nu$. The leading contribution to the self energy comes from the gluon propagator arising in the soft momentum transfer limit i.e., $|\mathbf{k}| \sim g\sqrt{eB}$. In the limit $E \gg \sqrt{eB} \gg T \gg |\mathbf{k}|$, out of the four terms in eq. (4.8) only the first term becomes dominant. This is because $D_F^{\mu\nu}$ involves gluonic distribution function. Thus eq. (4.8) reduces to

$$\Sigma_R(P) = \frac{ig^2(t_a t_b)}{8} \int \frac{d^4 K}{(2\pi)^4} \left[D_F^{00} \gamma_0 (\not{P}' + M) \gamma_0 + 2D_F^{0i} \gamma_0 (\not{P}' + M) \gamma_i + D_F^{ij} \gamma_i (\not{P}' + M) \gamma_j \right] \Delta_R(P'). \quad (4.9)$$

Using eqs. (3.38), (3.39) and (3.40) in the eqs. (4.2) and (4.8), the interaction rate of HQ can be given as

$$\begin{aligned} \Gamma(E) = \frac{g^2}{8E} (1 - \tilde{f}(E)) \int \frac{d^4 K}{(2\pi)^4} (1 + 2\tilde{f}(k_0)) \text{Tr} \left[(\not{P} + M) \gamma_0 (\not{P}' + M) \gamma_0 P_{\parallel}^{00} \right. \\ \left. + 2(\not{P} + M) \gamma_0 (\not{P}' + M) \gamma_i P_{\parallel}^{0i} + (\not{P} + M) \gamma_i (\not{P}' + M) \gamma_j P_{\parallel}^{ij} \right] \rho_L \Im(\Delta_R(P')) \end{aligned} \quad (4.10)$$

where

$$\rho_L = \frac{2\Im\Pi_R^\parallel(K)}{(K^2 - \Re\Pi_R^\parallel(K))^2 + (\Im\Pi_R^\parallel(K))^2}. \quad (4.11)$$

In the above equation, $\Im\Pi_R^\parallel(K)$ and $\Re\Pi_R^\parallel(K)$ respectively are the imaginary and the real parts of retarded gluon self energy as defined in eqs. (A.12) and (A.13). Further, let us note that the imaginary part in the retarded self energy of HQ comes from the propagator $\Delta_R(P')$ which can be written as

$$\Im(\Delta(P')) = -\frac{\pi}{2E'} \left(\delta(E - k_0 - E') + \delta(E - k_0 + E') \right). \quad (4.12)$$

The second delta function in eq. (4.12) does not contribution due to kinematic reasons ($k_0 \ll M$). Therefore, we drop this term and continue with the first delta function. Thus, the interaction rate becomes

$$\begin{aligned} \Gamma(E) = & -\frac{g^2}{8E} (1 - \tilde{f}(E)) \int \frac{d^4K}{(2\pi)^4} (1 + 2f(k_0)) \left[(4EE' + 4p^2 - 4\mathbf{p} \cdot \mathbf{k} + 4M^2) k_z^2 + 8(E(\mathbf{p}' \cdot \mathbf{k}) \right. \\ & \left. + E'(\mathbf{p} \cdot \mathbf{k}')) k_0 + 4(\mathbf{p} \cdot (\mathbf{p} - \mathbf{k}) + EE' - M^2) k_0^2 \right] \frac{\rho_L \pi}{2E' k_\parallel^2} \delta(E - k_0 - E'). \end{aligned} \quad (4.13)$$

Since we are working in the limit where $|\mathbf{k}| \sim g\sqrt{eB} \ll T$ so in this limit $E \gg |\mathbf{k}|$ and one can write $E' = \sqrt{(\mathbf{p}^2 + m^2)} \equiv E - \mathbf{v} \cdot \mathbf{k}$. The delta function in the above equation can be simplified as $\delta(E - k_0 + E') \sim \delta(k_0 - \mathbf{v} \cdot \mathbf{k})$. Further, we assume that HQ moves parallel to the magnetic field. The integration over azimuthal angle in eq. (4.13) can be done trivially and the integration over the polar angle can be done by using the energy delta function. The energy loss is thus obtained as

$$\begin{aligned} -\frac{dE}{dx} \Big|_{Qq \rightarrow Qq} = & -\frac{g^2}{2E} (1 - \tilde{f}(E)) \int_0^\infty \frac{k dk}{(2\pi)^2} \int_{-kv}^{kv} \frac{k_0 dk_0}{2\pi} (1 + 2f(k_0)) \left[4(2E^2 - 2Ek_0) k_0^2 \right. \\ & \left. - 4v^2(2E^2 k_0 - Ek^2) k_0 + 4v^2(2E^2 - 2M^2 - 2Ek_0) k_0^2 \right] \frac{\rho_L \pi}{2Ek_0^2(v^2 - 1)}. \end{aligned} \quad (4.14)$$

In the above equation we have used $1/E \approx 1/E'$ as the momentum of the gluon is soft. The magnetic field dependence in the energy loss for $Qq \rightarrow Qq$ scattering comes through ρ_L as defined in eq. (4.11) which depends on the real and the imaginary parts of retarded self energy of gluon.

4.2 HQ gluon scattering: $Qg \rightarrow Qg$

The other contribution to the HQ energy loss comes from the scattering of the gluons off the HQ i.e., $Q(P) + g(K) \rightarrow Q(P') + g(K')$ as shown in figure 2. We shall discuss this process in some detail. The interaction rate is given as

$$\begin{aligned} \Gamma(E) = & \frac{1}{2E} \int \frac{d\mathbf{p}'}{(2\pi)^3 2E'} \int \frac{d\mathbf{k}}{(2\pi)^3 2k} f(k) \int \frac{d\mathbf{k}'}{(2\pi)^3 2k'} (1 + f(k')) \\ & \times (2\pi)^4 \delta^4(P + K - P' - K') |\bar{\mathcal{M}}|^2 \end{aligned} \quad (4.15)$$

Figure 2. Feynman diagram for heavy quark and gluon scattering. In the strong magnetic field limit the contribution to the resummed gluon propagator comes from the quark loop only.

where $|\bar{\mathcal{M}}|^2$ is the matrix element squared averaged over initial spin and color degrees of freedom and summed over final spin and color for the $Qg \rightarrow Qg$ scattering and $f(k)$ is the gluonic thermal distribution function. Generally, there are three types of processes i.e., through s, t and u channels that can contribute to the total scattering amplitude for this process. However, we are considering the strong magnetic field limit (LLL approximation) i.e., $M \gg \sqrt{eB} \gg T$ and assume that the effect of the magnetic field on HQ is suppressed due to its large mass so that the s and the u channels, where HQ propagator arises, does not give any additional contribution arising from the magnetic field. Therefore, the only contribution to eq. (4.15) comes from the t -channel scattering as shown in figure 2. Let us note that as mentioned earlier the resummed gluon propagator has thermal contribution ($\sim \alpha_s T^2$) from gluon loop and the magnetic field contribution ($\sim \alpha_s eB$) from quark loop in the gluon self energy. In LLL approximation, the thermal contributions are negligible as compared with that of the magnetic field. Consequently, we shall use the resummed gluon propagator as given in eq. (3.38). The t -channel scattering amplitude can be written as

$$\mathcal{M} = -ig^2 f_{acb} t_{ji}^a [\bar{u}_j(P') \gamma^\alpha u_i(P)] D_{\delta\alpha}^R(Q) C^{\delta\mu\nu}(Q, K, -K') \epsilon_\mu(K) \epsilon_\nu^*(K') \quad (4.16)$$

where $Q = P - P' = K' - K$ is four-momentum vector for the exchanged gluon and we have kept explicitly the color indices as in figure 2. The resulting matrix element squared can be written after performing the color and spin sum over the final state and averaging over the initial state as

$$\begin{aligned} |\bar{\mathcal{M}}|^2 &= \frac{1}{4} g^4 C_F \text{Tr}[(\not{P}' + M) \gamma^\alpha (\not{P} + M) \gamma^{\alpha'}] D_{\delta\alpha}^R(Q) D_{\delta'\alpha'}^R(Q) C^{\delta\mu\nu}(Q, K, K') \\ &\quad \times C^{\delta'\mu'\nu'}(Q, K, K') \sum \epsilon_\mu(K) \epsilon_{\mu'}(K) \sum \epsilon_\nu(K') \epsilon_{\nu'}(K') \\ &= \frac{1}{4} g^4 C_F \text{Tr}[(\not{P}' + M) \gamma^\alpha (\not{P} + M) \gamma^{\alpha'}] D_{\delta\alpha}^R(Q) D_{\delta'\alpha'}^R(Q) \\ &\quad \times C^{\delta\mu\nu}(Q, K, -K') C_{\mu\nu}^{\delta'}(Q, K, -K') \end{aligned} \quad (4.17)$$

where $C_F = 1/2$ is the color factor and $C^{\mu\nu\alpha}(P, Q, R) = (P - Q)^\alpha g^{\mu\nu} + (Q - R)^\mu g^{\nu\alpha} + (R - P)^\nu g^{\mu\alpha}$. To further simplify eq. (4.17), we will use the transversality condition

$\epsilon(K) \cdot K = \epsilon(K') \cdot K' = 0$ for the gluons to obtain

$$\begin{aligned} C^{\delta\mu\nu}(Q, K, -K') &= -2g^{\delta\mu}K^\nu + g^{\mu\nu}(K + K')^\delta - 2g^{\nu\delta}K'^\mu \\ C_{\mu\nu}^{\delta'}(Q, K, -K') &= -2g_{\mu}^{\delta'}K_\nu + g_{\mu\nu}(K + K')^{\delta'} - 2g_{\nu}^{\delta'}K'_\mu \end{aligned} \quad (4.18)$$

so that,

$$C^{\delta\mu\nu}(Q, K, -K')C_{\mu\nu}^{\delta'}(Q, K, -K') = 4(K^\delta K'^{\delta'} + K'^\delta K^{\delta'}). \quad (4.19)$$

The product of propagators that appear in eq. (4.17) can be simplified to

$$\begin{aligned} D_{\delta\alpha}^R(Q)D_{\delta'\alpha'}^R(Q) &= \frac{g_{\delta\alpha}g_{\delta'\alpha'}}{Q^4} - \frac{\Pi_R^\parallel(Q)g_{\delta\alpha}P_{\delta'\alpha'}^\parallel + \Pi_R^\parallel(Q)g_{\delta'\alpha'}P_{\delta\alpha}^\parallel}{Q^4(Q^2 - \Pi_R^\parallel(Q))} \\ &\quad + \frac{(\Pi_R^\parallel(Q))^2 P_{\delta\alpha}^\parallel P_{\delta'\alpha'}^\parallel}{Q^4(Q^2 - \Pi_R^\parallel(Q))^2}. \end{aligned} \quad (4.20)$$

The first term in eq. (4.20) corresponds to the vacuum contribution. Since we are interested in the medium contribution (i.e., T and B), we will not consider this term. The medium dependent term that appears in eq. (4.17) can be written in compact manner as

$$(D_{\delta\alpha}^R(q)D_{\delta'\alpha'}^R(q))(C^{\delta\mu\nu}(K, K')C_{\mu\nu}^{\delta'}(K, K')) = \mathcal{A}_{\alpha\alpha'} + \mathcal{B}_{\alpha\alpha'} + \mathcal{C}_{\alpha\alpha'} \quad (4.21)$$

where

$$\mathcal{A}_{\alpha\alpha'} = -\frac{4\Pi_R^\parallel(q)}{q^4(q^2 - \Pi_R^\parallel(q))}(K^\delta K'^{\delta'} + K'^\delta K^{\delta'})g_{\delta\alpha}P_{\delta'\alpha'}^\parallel \quad (4.22)$$

$$\mathcal{B}_{\alpha\alpha'} = -\frac{4\Pi_R^\parallel(q)}{q^4(q^2 - \Pi_R^\parallel(q))}(K^\delta K'^{\delta'} + K'^\delta K^{\delta'})g_{\delta'\alpha'}P_{\delta\alpha}^\parallel \quad (4.23)$$

$$\mathcal{C}_{\alpha\alpha'} = \frac{4(\Pi_R^\parallel(q))^2}{q^4(q^2 - \Pi_R^\parallel(q))^2}(K^\delta K'^{\delta'} + K'^\delta K^{\delta'})P_{\delta\alpha}^\parallel P_{\delta'\alpha'}^\parallel \quad (4.24)$$

So eq. (4.17), can be written as

$$|\bar{\mathcal{M}}|^2 = \frac{1}{4} \times \frac{1}{2} g^4 (\mathcal{T}_1^{\alpha\alpha'} + \mathcal{T}_2^{\alpha\alpha'}) (\mathcal{A}_{\alpha\alpha'} + \mathcal{B}_{\alpha\alpha'} + \mathcal{C}_{\alpha\alpha'}) \quad (4.25)$$

where $\mathcal{T}_1^{\alpha\alpha'} = Tr[\not{P}'\gamma^\alpha \not{P}\gamma^{\alpha'}]$ and $\mathcal{T}_2^{\alpha\alpha'} = M^2 Tr[\gamma^\alpha \gamma^{\alpha'}]$ are traces over Dirac space. Six tensor contracted terms of eq. (4.25) are simplified in appendix B. Further simplification leads to the final form of scattering amplitude as

$$\begin{aligned} |\bar{\mathcal{M}}|^2 &= 4g^4 \frac{(\Pi_R^\parallel(Q))^2}{Q^4(Q^2 - \Pi_R^\parallel(Q))^2} [(P.P_\parallel.K)(P'.P_\parallel.K') + (P.P_\parallel.K')(K.P_\parallel.P') \\ &\quad + (P.P')(K.P_\parallel.K')] - \frac{4g^4 \Pi_R^\parallel(Q)}{Q^4(Q^2 - \Pi_R^\parallel(Q))} [(P.K)(P'.P_\parallel.K') + (P.K')(K.P_\parallel.P') \\ &\quad + (K.P')(P.P_\parallel.K') + (P'.K')(P.P_\parallel.K) - 2(P.P')(K.P_\parallel.K')] \\ &\quad - 4g^4 M^2 \left[\frac{2\Pi_R^\parallel(Q)}{Q^4(Q^2 - \Pi_R^\parallel(Q))} (K.P_\parallel.K') + \frac{(\Pi_R^\parallel(Q))^2}{Q^4(Q^2 - \Pi_R^\parallel(Q))^2} (K.P_\parallel.K') \right]. \end{aligned} \quad (4.26)$$

where the tensor product is

$$P.P_{\parallel}.K = P_{\mu}P_{\parallel}^{\mu\nu}K_{\nu} = \frac{(P.q_{\parallel})(K.q_{\parallel})}{q_{\parallel}^2} - (P.k_{\parallel}) \quad (4.27)$$

The expression for the retarded self energy $\Pi_R^{\parallel}$ is given explicitly in appendix A. This completely defines the matrix element squared.

Energy loss of HQ due to thermal gluons. The contribution of t -channel Compton scattering i.e, $Qg \rightarrow Qg$, to the HQ energy loss can be obtained by using eq. (4.1) and the interaction rate (Γ) as given in eq. (4.15). Hence, one can write [58]

$$\begin{aligned} -\frac{dE}{dx}\Big|_{Qg \rightarrow Qg} &= \frac{1}{2vE} \int \frac{d\mathbf{p}'}{(2\pi)^3 2E'} \int \frac{d\mathbf{k}}{(2\pi)^3 2k} f(k_0) \int \frac{d\mathbf{k}'}{(2\pi)^3 2k'} (1 + f(k'_0)) \\ &\quad \times (2\pi)^4 \delta^4(P + K - P' - K')(E - E') |\bar{\mathcal{M}}|^2 \end{aligned} \quad (4.28)$$

The energy and momentum transfer in each scattering are, $\omega = E - E' = |\mathbf{k}'| - |\mathbf{k}|$ and $\mathbf{q} = \mathbf{p} - \mathbf{p}' = \mathbf{k}' - \mathbf{k}$. A comment regarding the use of resummed gluon propagator may be relevant here. The hard contributions to the energy loss ($|\mathbf{q}| \sim T$) can be obtained by using the bare gluon propagator for the $Qg \rightarrow Qg$ scatterings since the self energy corrections are negligible at leading order (LO). We confine our attention here for the soft momentum transfer in the range $g\sqrt{eB} \leq |\mathbf{q}| \ll T \ll \sqrt{eB}$ [19]. This requires the resummation of the gluon propagator in the LLL approximation as we have used here. The $|\mathbf{p}'|$ integration in eq. (4.28) can be performed with the help of momentum delta function. In the soft momentum transfer limit the energy $E' \approx E - \mathbf{v} \cdot \mathbf{q}$ so that the energy delta function reduces to $\delta(\omega - \mathbf{v} \cdot \mathbf{q})$. With these simplifications, eq. (4.28) becomes

$$-\frac{dE}{dx}\Big|_{Qg \rightarrow Qg} = \frac{(2\pi)}{16vE^2} \int \frac{d\mathbf{k}}{(2\pi)^3 k} f(k) \int \frac{d\mathbf{k}'}{(2\pi)^3 k'} (1 + f(k')) \delta(\omega - \mathbf{v} \cdot \mathbf{q}) \omega |\bar{\mathcal{M}}|^2. \quad (4.29)$$

Now, the simplification of the above 6-dimensional integration requires a proper choice of the co-ordinate system which should also be compatible with the terms appearing in the matrix element squared. For this purpose, we choose the direction of the momentum of the incoming HQ along the $\hat{z}$ -axis which is also the direction of the magnetic field. We denote the angle made by $\mathbf{k}$ and $\mathbf{k}'$ with the $\hat{z}$ -axis as θ_k and $\theta_{k'}$ respectively. Further, the azimuthal angles made by the two momenta of the incoming and outgoing gluon are ϕ_k and $\phi_{k'}$ respectively. Therefore, the energy loss can be written as

$$\begin{aligned} -\frac{dE}{dx}\Big|_{Qg \rightarrow Qg} &= \frac{1}{(2\pi)^5 16vE^2} \int k dk f(k) \int k' dk' (1 + f(k')) \int_{-1}^1 dx \int_{-1}^1 dy \int_0^{2\pi} d\phi_k \\ &\quad \times \int_0^{2\pi} d\phi_{k'} |\bar{\mathcal{M}}|^2 \omega \delta(\omega - vk'y + vkx) \end{aligned} \quad (4.30)$$

where, $x = \cos \theta_k$ and $y = \cos \theta_{k'}$. Now, we introduce a new integration variable ω which will take care of one of the angular integrations x or y .

$$\int_0^{\infty} d\omega \delta[\omega - k(1 - vx)] = 1 \quad (4.31)$$

Using these two δ -functions, we perform x and y angular integrations by writing the δ -functions as $\delta[\omega - k(1 - vx)] = \frac{1}{vk} \delta(x - \frac{k-\omega}{vk})$ and $\delta(\omega - vk'y + vkx) = \frac{1}{vk'} \delta(y - \frac{\omega+vkx}{vk'})$. So, the final expression of the energy loss becomes:

$$-\frac{dE}{dx}\Big|_{Qg \rightarrow Qg} = \frac{1}{(2\pi)^5 16vE^2} \int_0^\infty \omega d\omega \int_{\frac{\omega}{1+v}}^{\frac{\omega}{1-v}} f(k) dk \int (1 + f(k')) dk' \int_0^{2\pi} d\phi_k \times \int_0^{2\pi} d\phi_{k'} |\bar{\mathcal{M}}|^2 \quad (4.32)$$

The explicit form of the four-vector product and the tensor contractions in the matrix element squared are elaborately given in the appendix C.

5 Results and discussion

The two scatterings that contribute to the HQ energy loss are $Qq \rightarrow Qq$ evaluated in eq. (4.14) and t channel $Qg \rightarrow Qg$ evaluated in eq. (4.32). as mentioned earlier, we have confined our attention to the case where the typical momentum transfer from light partons to HQ is soft i.e., $g\sqrt{eB} \leq |\mathbf{k}| \ll T \ll M$. Therefore, we have used the resummed gluon propagator in the evaluation of the matrix element squared for these two processes. To simplify our calculations for energy loss we take HQ momentum $\vec{p} = (0, 0, p_z)$ and magnetic field $\vec{B} = B\hat{z}$. For the numerical purpose we take HQ as the charm quark with mass $M = 1.2 \text{ GeV}$, temperature $T = 0.25 \text{ GeV}$ and magnetic field $eB = 0.1 \text{ GeV}^2$ ($\sim 5m_\pi^2$) so that the condition $eB \gg T^2$ is satisfied. Since $eB \gg T^2$, only LLL will be populated by the light quarks. We also take finite mass (m_f) of light quarks in the gluon self energy diagram. The energy loss contributions using eqs. (4.32) and (4.14) has been plotted in figure 3. In the left side of figure 3, we have shown the variation of the energy loss taking contributions from both the scattering processes as a function of velocity of HQ. We have taken here $m_f = 10 \text{ MeV}$ for the light quark mass and number of flavors $n_f = 2$. At low velocity the collisional energy loss is very small $\sim 10^{-6} \text{ GeV}^2$, however with increase in HQ velocity the energy loss increases. While both the scattering processes contribute to the energy loss, it is observed numerically that the $Qg \rightarrow Qg$ gives the dominant contribution to the energy loss for a given value of velocity of the HQ essentially due to larger color factor compared to the Coulomb scattering.

This behavior is similar to the case of vanishing magnetic field. It ought to be mentioned here that in the presence of magnetic field the energy loss is of similar order as compared to the case of vanishing magnetic field as estimated in ref. [60]. Indeed, the asymptotic value for the energy loss ($v \rightarrow 1$) of ref. [60] is given by

$$\frac{dE}{dx} = \frac{4\pi}{33 - 2n_f} m_D^2 \quad (5.1)$$

with $m_D^2 = 4\pi \left(1 + \frac{n_f}{6}\right) \alpha_s T^2$ which with the parameters $\alpha_s = 0.3$, $n_f = 2$ and $T = 0.25 \text{ GeV}$ turns out to be 0.15 GeV^2 . This may be compared with the magnetic field contribution given by the red curve in the left panel of figure 3 which reaches to the value 0.1 GeV^2 in

Figure 3. *Left panel:* magnetic field dependent contribution to the collisional energy loss as function of HQ velocity. Here we have taken the light quark mass $m_f = 10$ MeV, flavor $n_f = 2$, magnetic field $eB = 0.1 \text{ GeV}^2 (\sim 5m_\pi^2)$ and temperature $T = 0.25$ GeV. The blue, red and black curves corresponds to the coupling constant $\alpha_s = 0.2$, $\alpha_s = 0.3$ and $\alpha_s = 0.4$ respectively. *Right panel:* variation of the collisional energy loss scaled with T^2 as a function of scaled variable eB/m_π^2 for light quark mass $m_f = 10$ MeV, temperature $T = 0.25$ GeV and coupling constant $\alpha_s = 0.2$.

the same limit. Further, it may be relevant to note that in the limit of vanishing light quark mass i.e., $m_f = 0$, the magnetic field contribution to the gluon self energy vanishes as $\Pi_{\parallel}^R$ is proportional to m_f^2 . This will lead to vanishing of the magnetic field dependent contribution to the energy loss. This is similar to the vanishing of magnetic field contribution for the dilepton production [17] in the same limit.

In the right panel of figure 3, we display the energy loss scaled with T^2 as a function of magnetic field scaled by pion mass square i.e., eB/m_π^2 , where m_π is pion mass. We have taken here $T = 0.25$ GeV and the magnitude of the heavy quark velocity $v = 0.6$. As may be observed in the figure the energy loss increases with the increase in the magnetic field. Numerically, it is seen that $Qg \rightarrow Qg$ contribution is not affected too much with the magnetic field. For this process, the magnetic field dependence arises from the resummed gluon propagator with the field dependent contribution of the quark loop. This quark loop contribution increases with the magnetic field leading to a mild decrease of the energy loss due to this process as quark loop contribution lies in the retarded propagator. On the other hand, for the Coulomb scattering process i.e., $Qq \rightarrow Qq$ the contribution increase with increase in the magnetic field. This can be understood as follows; the energy loss is proportional to ρ_L that depends on real and imaginary parts of retarded self energies and related to the spectral function. The spectral function increases with increase in the magnetic field. This increase of ρ_L with the magnetic field was also observed in ref. [17]. It turns out that this increase is significant and the contribution becomes similar order as the $Qg \rightarrow Qg$ for larger magnetic field leading to increase of the total energy loss with magnetic field as observed in figure 3.

6 Summary and conclusion

In the present investigation, we have studied the effect of the magnetic field on the HQ collisional energy loss in a thermalized QGP medium. We have done the analysis for the case of strong field limit i.e., $\sqrt{eB} \gg T$ so that the light quarks are populated only in the LLL. On the other hand, the heavy quark mass M is much larger than the strength of the magnetic field (i.e., $M \gg \sqrt{eB}$), so that heavy quark is not Landau quantized. The effect of the magnetic field manifests through resummed retarded/advanced gluon propagator through quarks loop. Since the gluon loop contribution to the gluon self energy is proportional to $\alpha_s T^2$ and the quark loop contribution is proportional to $\alpha_s eB$, in the limit $\sqrt{eB} \gg T$ the gluon loop contribution in the gluon resummed propagator is not taken into account. For the scattering of HQ with the thermalized light partons we have considered the soft momentum transfer limit i.e., $g\sqrt{eB} \leq |\mathbf{k}| \ll T \ll \sqrt{eB}$. With $M \gg \sqrt{eB}$ and the in the LLL approximation the relevant scattering processes are Coulomb scattering i.e., $Qq \rightarrow Qq$ and t -channel Compton scattering $Qg \rightarrow Qg$. The u and the s channels of the Compton scatterings are not affected by the magnetic field. For a given magnitude of the heavy quark velocity, the Compton scattering process is dominant over the Coulomb scattering process for the range of the magnetic field considered here.

It is observed that of the two processes, the Coulomb scattering process is more sensitive to the magnetic field as its contribution to the energy loss is proportional to the spectral function which increases with increase in the magnetic field. This leads to a net increase in the energy loss with increase in the magnetic field. It turns out that in this strong field limit, the magnetic field dependent collisional energy loss (for $eB = 5m_\pi^2$) is comparable to the same in the vanishing field limit and therefore could be important for the jet quenching phenomena in HICs. However, In a realistic situation in HIC up to what extent the magnetic field can affect the collisional energy loss will also depend on the medium response to the magnetic field and require further investigations. In the vacuum, the magnetic field decreases very rapidly however, in a system with finite electrical conductivity, magnetic field satisfies diffusion equation and the relaxation time of the external magnetic field depends on the electrical conductivity of the system. In a system with larger conductivity magnetic field can sustain and reasonably be strong for a longer period of time. However, it ought to be mentioned that this requires a proper estimation of the electrical conductivity of the medium as well as solutions of magneto hydrodynamic equations which needs further investigations. Furthermore, for smaller values of magnetic field one must include the effect of higher Landau levels.

We would like to mention here that the present investigation is a first attempt to include the effect of magnetic field in the collisional energy loss. However, for the large momentum of heavy quark, the radiative contribution to the energy loss may also be relevant and could be affected by the magnetic field. This apart, for a moderate value of magnetic field $eB \sim T^2$ the contributions arising from the higher Landau levels could also be important for the energy loss and hence jet quenching. In this case, one must take the contribution of gluon loops in the gluon self energy for the resummed gluon propagator as well as the contribution from the s and u channels of Compton scattering. Some of these problems

are relegated to future investigations. We also emphasize on the fact that our results are reliable only in the strong magnetic field limit where the LLL approximation is reasonable.

A Gluon self energy

In the RTF, the retarded self energy of gluon in the Keldysh basis can be written as

$$\Pi_R^{\mu\nu}(p_0, \mathbf{p}) = \Pi_{11}^{\mu\nu}(p_0, \mathbf{p}) + \Pi_{12}^{\mu\nu}(p_0, \mathbf{p}) \quad (\text{A.1})$$

where $\Pi_{11}^{\mu\nu}$ and $\Pi_{12}^{\mu\nu}$ are 11 and 12 component of the gluon self energy. Both 11 and 22 components of self energy can be obtained by using the quark propagators to acquire

$$\Pi_R^{\mu\nu}(p_0, \mathbf{p}) = -ig^2 t^a t^b \Omega \int \frac{d^2 k_{\parallel}}{(2\pi)^2} \left(\text{Tr}[\gamma^\mu S_{11}(Q) \gamma^\nu S_{11}(K) - \gamma^\mu S_{21}(Q) \gamma^\nu S_{12}(Q)] \right), \quad (\text{A.2})$$

where

$$\Omega = \frac{|q_f B|}{8\pi} \exp\left(-\frac{p_{\perp}^2}{|2q_f B|}\right). \quad (\text{A.3})$$

In eq. (A.2), the B dependent term (i.e., Ω) comes from the transverse part in the quark propagator. In the LLL, the dynamics in the transverse direction is restricted due to dimensional reduction from (3+1)-dimension to (1+1)-dimension. Therefore, the gauge invariant form of the gluon self energy in a magnetic field background can be written as

$$\Pi_R^{\mu\nu}(p_0, \mathbf{p}) = \Pi_R^{\parallel}(P) \left(g_{\parallel}^{\mu\nu} - \frac{p_{\parallel}^{\mu} p_{\parallel}^{\nu}}{p_{\parallel}^2} \right). \quad (\text{A.4})$$

To estimate the energy loss, we need $\Pi_R^{\parallel}(P)$ which can be obtained from the relation $\Pi_R^{\parallel}(P) = -(p_{\parallel}^2/p_z^2) \Pi_R^{00}(p_0, \mathbf{p})$. Therefore, we focus only on the time-like component (Π_R^{00}) of retarded self energy. Taking the trace over Dirac matrices in eq. (A.2), we get

$$\Pi_R^{00}(p_0, \mathbf{p}) = -i8g^2 t^a t^b \Omega \int \frac{d^2 k_{\parallel}}{(2\pi)^2} (\mathcal{K} \cdot \mathcal{Q}) \left(\Delta_{11}(Q) \Delta_{11}(K) - \Delta_{21}(Q) \Delta_{12}(K) \right) \quad (\text{A.5})$$

where Δ_{11} and Δ_{12} are defined in eqs. (4.5) and (4.6) with $\mathcal{K} \cdot \mathcal{Q} = k_0 q_0 + k_z q_z + m^2$. Similar to Δ_{11} , Δ_{21} can also be obtained from S_{21} . Eq. (A.5) can further be simplified by writing the propagators Δ_{ij} in terms of the retarded, advanced and Feynman propagators similar to eqs. (3.15), (3.16) and (3.17) to acquire

$$\Pi_R^{00}(p_0, \mathbf{p}) = -i4g^2 t^a t^b \Omega \int \frac{d^2 k_{\parallel}}{(2\pi)^2} (\mathcal{K} \cdot \mathcal{Q}) \left(\Delta_F(Q) \Delta_R(K) + \Delta_A(Q) \Delta_F(K) \right). \quad (\text{A.6})$$

Replacing $K \rightarrow -Q$ and using the relation $\Delta_R(-Q) = \Delta_A(Q)$, eq. (A.6) becomes

$$\Pi_R^{00}(p_0, \mathbf{p}) = 2\pi g^2 t^a t^b \Omega \int \frac{d^2 k_{\parallel}}{(2\pi)^2} (\mathcal{K} \cdot \mathcal{Q}) (1 - 2\tilde{f}(k_0)) \delta(k_{\parallel}^2 - m_f^2) \frac{1}{q_{\parallel}^2 - m_f^2 - i\epsilon q_0} \quad (\text{A.7})$$

The first term of eq. (A.7) which is independent of the Fermi Dirac distribution function, is the vacuum contribution to the time-like component of the gluon self energy. The time-like

component of the gluon self energy can be separated into the vacuum and the thermal parts to get

$$\Pi_R^{00}(p_0, \mathbf{p}) = \Pi_R^{00}(p_0, \mathbf{p})|_{vac} + \Pi_R^{00}(p_0, \mathbf{p})|_{th} \quad (\text{A.8})$$

where the vacuum term is given as [19]

$$\Pi_R^{00}(p_0, \mathbf{p})|_{vac} = \frac{2g^2 t^a t^b \Omega p_z^2}{p_{\parallel}^2 + i\epsilon p_0} \left[1 - \frac{4m_f^2}{\sqrt{p_{\parallel}^2(4m_f^2 - p_{\parallel}^2)}} \arctan \left(\frac{p_{\parallel}^2}{\sqrt{p_{\parallel}^2(4m_f^2 - p_{\parallel}^2)}} \right) \right]. \quad (\text{A.9})$$

For the thermal contribution to the time-like component of the gluon self energy, the energy integral in eq. (A.7) can be done by using the energy delta function to obtain

$$\Pi_R^{00}(p_0, \mathbf{p})|_{th} = -\frac{2\pi g^2 t^a t^b \Omega m_f^2 p_z^2}{p_{\parallel}^2} \left(\mathcal{J}_0(P) + \frac{2p_z}{p_{\parallel}^2} \mathcal{J}_1(P) \right), \quad (\text{A.10})$$

with

$$\mathcal{J}_a = \int_{-\infty}^{\infty} \frac{dk_z}{2\pi E} \tilde{f}(E) \frac{k_z^a}{(k_z - p_z/2)^2 - p_0^2/4 + p_0^2 m_f^2/p_{\parallel}^2 - i p_0 \epsilon} \quad (\text{A.11})$$

where $a = 0, 1$. Using eq. (A.10) and the relation $\Pi_R^{\parallel}(P) = -(p_{\parallel}^2/p_z^2)\Pi_R^{00}(p_0, \mathbf{p})$, the parallel component of the medium dependent gluon self energy can be written as

$$\Pi_R^{\parallel}(P) = 2\pi g^2 t^a t^b \Omega m_f^2 \left[\mathcal{J}_0(P) + \frac{2p_z}{p_{\parallel}^2} \mathcal{J}_1(P) \right] \quad (\text{A.12})$$

Imaginary part of the gluon self energy. The imaginary part of the retarded self energy of the gluon can be obtained from the imaginary part of the 11 component of gluon self energy by using the relation [72]

$$\Im \Pi_R^{\mu\nu}(p_0, \mathbf{p}) = (1 - \tilde{f}(p_0)) \Im \Pi_{11}^{\mu\nu}(p_0, \mathbf{p}) \quad (\text{A.13})$$

where

$$\Pi_{11}^{\mu\nu}(p_0, \mathbf{p}) = i g^2 t^a t^b \int \frac{d^2 k_{\perp}}{(2\pi)^2} e^{-\frac{k_{\perp}^2 + q_{\perp}^2}{|q_f B|}} \int \frac{d^2 k_{\parallel}}{(2\pi)^2} \text{Tr}[\gamma^{\mu} S_{11}(K) \gamma^{\nu} S_{11}(Q)]. \quad (\text{A.14})$$

$S_{11}(P)$ in eq. (A.14), is the 11 component of the quark propagator in RTF that can be obtained from the retarded, advanced and Feynman propagators by using the relation as given in eq. (3.15). Using the eqs. (3.23) and (3.24), the imaginary part of $\Pi_{11}^{\mu\nu}(P)$ can be written as

$$\begin{aligned} \Im \Pi_{11}^{\mu\nu}(p_0, \mathbf{p}) = & \pi g^2 t^a t^b \Omega \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \frac{1}{4E_k E_q} \left[\left(1 - \tilde{f}(E_k) - \tilde{f}(E_q) + 2\tilde{f}(E_k)\tilde{f}(E_q) \right) \right. \\ & \times \left(\mathcal{N}^{\mu\nu}(k_0 = E_k) \delta(p_0 - E_k - E_q) + \mathcal{N}^{\mu\nu}(k_0 = -E_k) \delta(p_0 + E_k + E_q) \right) \\ & + \left(-\tilde{f}(E_k) - \tilde{f}(E_q) + 2\tilde{f}(E_k)\tilde{f}(E_q) \right) \left(\mathcal{N}^{\mu\nu}(k_0 = -E_k) \delta(p_0 - E_k + E_q) \right. \\ & \left. \left. + \mathcal{N}^{\mu\nu}(k_0 = E_k) \delta(p_0 + E_k - E_q) \right) \right] \quad (\text{A.15}) \end{aligned}$$

where

$$\mathcal{N}^{\mu\nu} = \text{Tr}[\gamma^\mu \mathcal{S}_0(K) \gamma^\nu \mathcal{S}_0(Q)] \quad (\text{A.16})$$

with

$$\mathcal{S}_0(K) = (\not{k}_\parallel + m_f)(1 + i\gamma^1\gamma^2). \quad (\text{A.17})$$

The imaginary part of the retarded self energy i.e., $\Im\Pi_R^\parallel$ can be obtained from $\Im\Pi_R^{00}$ by using the general structure of the retarded self energy as given in eq. (A.4). So only the time-like component of $\Im\Pi_{11}^{\mu\nu}$ i.e., $\Im\Pi_{11}^{00}$ is relevant which can be obtained from eq. (A.15). The momentum integration in eq. (A.15) can be done by using the energy delta functions. Let us first simplify the energy delta functions and re-write those in terms of k_z . By using the relation

$$\delta(f(x)) = \sum_n \frac{\delta(x - x_n)}{\left| \frac{\partial f(x)}{\partial x} \right|_{x=x_n}}, \quad (\text{A.18})$$

one can write

$$\delta(p_0 - E_k - E_q) = \frac{\delta(k_z - k_z^0) E_{k_z^0} E_{q_z^0}}{k_z^0 (E_{k_z^0} + E_{q_z^0})} + \frac{\delta(k_z - k_z^1) E_{k_z^1} E_{q_z^1}}{k_z^1 (E_{k_z^1} + E_{q_z^1})}, \quad (\text{A.19})$$

$$\delta(p_0 + E_k - E_q) = \frac{\delta(k_z - k_z^0) E_{k_z^0} E_{q_z^0}}{k_z^0 (E_{k_z^0} - E_{q_z^0})} + \frac{\delta(k_z - k_z^1) E_{k_z^1} E_{q_z^1}}{k_z^1 (E_{k_z^1} - E_{q_z^1})}, \quad (\text{A.20})$$

where

$$k_z^0 = -\frac{p_z}{2} + \frac{1}{2|p_\parallel|} \sqrt{p_z^2 p_\parallel^2 - 4p_0^2 m_f^2 + p_\parallel^4}, \quad (\text{A.21})$$

and

$$k_z^1 = -\frac{p_z}{2} - \frac{1}{2|p_\parallel|} \sqrt{p_z^2 p_\parallel^2 - 4p_0^2 m_f^2 + p_\parallel^4}. \quad (\text{A.22})$$

With further simplification, the imaginary part of Π_{11}^{00} can be written as

$$\begin{aligned} \Im\Pi_{11}^{00}(p_0, \mathbf{p}) = & g^2 t^a t^b \Omega \pi 2m_f^2 \left[\left(\frac{1}{k_z^0 (E_{k_z^0} + E_{q_z^0})} + \frac{1}{k_z^1 (E_{k_z^1} + E_{q_z^1})} \right) - (\tilde{f}(k_z^0) + \tilde{f}(q_z^0)) \right. \\ & - 2\tilde{f}(k_z^0) \tilde{f}(q_z^0) \left(\frac{2E_{k_z^0}}{k_z^0 p_z (2k_z^0 + p_z)} \right) - (\tilde{f}(k_z^1) + \tilde{f}(q_z^1) - 2\tilde{f}(k_z^1) \tilde{f}(q_z^1)) \\ & \left. \times \left(\frac{2E_{k_z^1}}{k_z^1 p_z (2k_z^1 + p_z)} \right) \right]. \end{aligned} \quad (\text{A.23})$$

As mentioned earlier, eq. (A.23) can be used to obtain $\Im\Pi_R^\parallel$. In the HTL limit, the thermal contribution to the transverse and longitudinal components of the gluon self energy that comes from the gluon loop is given as

$$\Pi_R^L(P) = \frac{g^2 T^2 N_c}{3} [-1 + Q(x)] \quad (\text{A.24})$$

and

$$\Pi_R^T(P) = \frac{g^2 T^2 N_c}{6} [x^2 + (1 - x^2)Q(x)] \quad (\text{A.25})$$

where $x = \frac{p_0}{|\mathbf{p}|}$. The function $Q(x)$ is given as

$$Q(x) = \frac{x}{2} \left(\ln \left| \frac{1+x}{1-x} \right| - i\pi\theta(1-x^2) \right). \quad (\text{A.26})$$

B $Qg \rightarrow Qg$ scattering

The contracted terms of eq. (4.25) are

Term-1:

$$\begin{aligned}\mathcal{T}_1^{\mu\nu}\mathcal{A}_{\mu\nu} &= -\frac{16\Pi_R^{\parallel}(q)}{q^4(q^2 - \Pi_R^{\parallel}(q))} [P^\mu P'^\nu + P'^\mu P^\nu - (P.P')g^{\mu\nu}] \left[K_\mu K'^\delta P_{\delta\nu}^{\parallel} + K'_\mu K^\delta P_{\delta\nu}^{\parallel} \right] \\ &= -\frac{16\Pi_R^{\parallel}(q)}{q^4(q^2 - \Pi_R^{\parallel}(q))} \left[(P.K)(K'.P_{\parallel}.P') + (P.K')(K.P_{\parallel}.P') \right. \\ &\quad \left. + (K.P')(K'.P_{\parallel}.P) + (P'.K')(K.P_{\parallel}.P) - 2(P.P')(K'.P_{\parallel}.K) \right].\end{aligned}\quad (\text{B.1})$$

Term-2:

$$\begin{aligned}\mathcal{T}_1^{\mu\nu}\mathcal{B}_{\mu\nu} &= -\frac{16\Pi_R^{\parallel}(q)}{q^4(q^2 - \Pi_R^{\parallel}(q))} [P^\mu P'^\nu + P'^\mu P^\nu - (P.P')g^{\mu\nu}] \left[K^\delta P_{\delta\mu}^{\parallel} K'_\nu + K'^\delta P_{\delta\mu}^{\parallel} K_\nu \right] \\ &= -\frac{16\Pi_R^{\parallel}(q)}{q^4(q^2 - \Pi_R^{\parallel}(q))} \left[(P'.K')(K.P_{\parallel}.P) + (K.P')(K'.P_{\parallel}.P) \right. \\ &\quad \left. + (P.K')(K.P_{\parallel}.P') + (P.K)(K'.P_{\parallel}.P') - 2(P.P')(K.P_{\parallel}.K') \right].\end{aligned}\quad (\text{B.2})$$

Term3:

$$\begin{aligned}\mathcal{T}_1^{\mu\nu}\mathcal{C}_{\mu\nu} &= \frac{16(\Pi_R^{\parallel}(q))^2}{q^4(q^2 - \Pi_R^{\parallel}(q))^2} [P^\mu P'^\nu + P'^\mu P^\nu - (P.P')g^{\mu\nu}] \left[K^\delta P_{\delta\mu}^{\parallel} K'^{\delta'} P_{\delta'\nu}^{\parallel} + K'^\delta P_{\delta\mu}^{\parallel} K^{\delta'} P_{\delta'\nu}^{\parallel} \right] \\ &= \frac{16(\Pi_R^{\parallel}(q))^2}{q^4(q^2 - \Pi_R^{\parallel}(q))^2} \left[(K.P_{\parallel}.P)(K'.P_{\parallel}.P') + (K'.P_{\parallel}.P)(K.P_{\parallel}.P') \right. \\ &\quad \left. + (K.P_{\parallel}.P')(K'.P_{\parallel}.P) + (K'.P_{\parallel}.P')(K.P_{\parallel}.P) + 2(P.P')(K.P_{\parallel}.K') \right].\end{aligned}\quad (\text{B.3})$$

Here, we make use of the identity:

$$g^{\mu\nu} P_{\delta\mu}^{\parallel} P_{\delta'\nu}^{\parallel} = -P_{\delta\delta'}^{\parallel}. \quad (\text{B.4})$$

Term-4:

$$\begin{aligned}\mathcal{T}_2^{\mu\nu}\mathcal{A}_{\mu\nu} &= -\frac{16M^2\Pi_R^{\parallel}(q)}{q^4(q^2 - \Pi_R^{\parallel}(q))} g^{\mu\nu} \left[K_\mu K'^\delta P_{\delta\nu}^{\parallel} + K'_\mu K^\delta P_{\delta\nu}^{\parallel} \right] \\ &= -\frac{32M^2\Pi_R^{\parallel}(q)}{q^4(q^2 - \Pi_R^{\parallel}(q))} (K.P_{\parallel}.K').\end{aligned}\quad (\text{B.5})$$

Term-5:

$$\begin{aligned}\mathcal{T}_2^{\mu\nu}\mathcal{B}_{\mu\nu} &= -\frac{16M^2\Pi_R^{\parallel}(q)}{q^4(q^2 - \Pi_R^{\parallel}(q))} g^{\mu\nu} \left[K^\delta P_{\delta\mu}^{\parallel} K'_\nu + K'^\delta P_{\delta\mu}^{\parallel} K_\nu \right] \\ &= -\frac{32M^2\Pi_R^{\parallel}(q)}{q^4(q^2 - \Pi_R^{\parallel}(q))} (K.P_{\parallel}.K').\end{aligned}\quad (\text{B.6})$$

Term-6:

$$\begin{aligned}\mathcal{T}_2^{\mu\nu}\mathcal{C}_{\mu\nu} &= \frac{16M^2(\Pi_R^{\parallel}(q))^2}{q^4(q^2 - \Pi_R^{\parallel}(q))^2} g^{\mu\nu} \left[K^\delta P_{\delta\mu}^{\parallel} K'^{\delta'} P_{\delta'\nu}^{\parallel} + K'^\delta P_{\delta\mu}^{\parallel} K^{\delta'} P_{\delta'\nu}^{\parallel} \right] \\ &= -\frac{32M^2(\Pi_R^{\parallel}(q))^2}{q^4(q^2 - \Pi_R^{\parallel}(q))^2} (K.P_{\parallel}.K').\end{aligned}\quad (\text{B.7})$$

C Four vector product and tensor contractions

With the assumption that the HQ quark moves in the direction of the magnetic field, the four-vector products in the matrix element squared i.e., $|\bar{\mathcal{M}}|^2$ as given in eq. (4.26) can be given as

$$\begin{aligned}
 P.K &= Ek_0 - \mathbf{p} \cdot \mathbf{k} = Ek_0 - pk \cos \theta_k = Ek_0 - pkx \\
 P.K' &= Ek'_0 - \mathbf{p} \cdot \mathbf{k}' = Ek'_0 - pk'y \\
 P'.K &= E'k_0 - (\mathbf{p} + \mathbf{k} - \mathbf{k}') \cdot \mathbf{k} = Ek_0 - vkk'y + vk^2x - pkx - k^2 + \mathbf{k} \cdot \mathbf{k}' \\
 P'.K' &= E'k'_0 - (\mathbf{p} + \mathbf{k} - \mathbf{k}') \cdot \mathbf{k}' = Ek'_0 - vk'^2y + vkk'x - pk'y + k'^2 - \mathbf{k} \cdot \mathbf{k}' \\
 P.P' &= EE' - \mathbf{p} \cdot (\mathbf{p} - \mathbf{q}) = M^2.
 \end{aligned} \tag{C.1}$$

Here $\mathbf{k} \cdot \mathbf{k}' = kk'[\sqrt{(1-x^2)(1-y^2)}(\cos \phi_k \cos \phi_{k'} + \sin \phi_k \sin \phi_{k'}) + xy]$ with $x = \cos \theta_k$, $y = \cos \theta_{k'}$ and with the magnitude of momenta, in general, $k = |\mathbf{k}|$. The tensor contractions can be splitted into the four-vector dot products as

$$\begin{aligned}
 P.P_{\parallel}.K &= \frac{(P.q_{\parallel})(K.q_{\parallel})}{q_{\parallel}^2} - P.K_{\parallel} \\
 P.P_{\parallel}.K' &= \frac{(P.q_{\parallel})(K'.q_{\parallel})}{q_{\parallel}^2} - P.K'_{\parallel} \\
 P'.P_{\parallel}.K &= \frac{(P'.q_{\parallel})(K.q_{\parallel})}{q_{\parallel}^2} - P'.K_{\parallel} \\
 P'.P_{\parallel}.K' &= \frac{(P'.q_{\parallel})(K'.q_{\parallel})}{q_{\parallel}^2} - P'.K'_{\parallel} \\
 K.P_{\parallel}.K' &= \frac{(K.q_{\parallel})(K'.q_{\parallel})}{q_{\parallel}^2} - K.K'_{\parallel}
 \end{aligned} \tag{C.2}$$

The dot products in eq. (C.2) can also be simplified by taking the same assumption for the heavy quark motion as

$$\begin{aligned}
 P.q_{\parallel} &= E\omega - pq_z = E\omega - p(k'_z - k_z) = E\omega - pk'y + pkx \\
 K.q_{\parallel} &= \omega k_0 - k_z q_z = \omega k_0 - k_z(k'_z - k_z) = \omega k_0 - kk'xy + k^2x^2 \\
 K'.q_{\parallel} &= \omega k'_0 - k'_z(k'_z - k_z) = \omega k'_0 - k'^2y^2 + kk'xy \\
 P'.q_{\parallel} &= E'\omega - (p + k_z - k'_z)(k'_z - k_z) \\
 &= E\omega - v\omega k'y + v\omega kx - pk'y + pkx + k^2x^2 + k'^2y^2 - 2kk'xy \\
 P.K_{\parallel} &= Ek_0 - pk_z = Ek_0 - pkx \\
 P.K'_{\parallel} &= Ek'_0 - pk'_y \\
 P'.K_{\parallel} &= E'k_0 - (p_z + k_z - k'_z)k_z = Ek_0 - vkk'y + vk^2x - pkx - k^2x^2 + kk'xy \\
 P'.K'_{\parallel} &= E'k'_0 - (p + k_z - k'_z)k'_z = Ek'_0 - vk'^2y + vkk'x - pk'y - kk'xy + k'^2y^2 \\
 K.K'_{\parallel} &= k_0k'_0 - k_zk'_z
 \end{aligned} \tag{C.3}$$

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