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Two-loop master integrals for the mixed QCD-electroweak corrections for $H \rightarrow b\bar{b}$ through a $Ht\bar{t}$ -coupling

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ABSTRACT: We present the two-loop master integrals relevant to the $\mathcal{O}(\alpha\alpha_s)$ -corrections to the decay $H \rightarrow b\bar{b}$ through a $Ht\bar{t}$ -coupling. We keep the full dependence on the heavy particle masses, but neglect the b -quark mass. The occurring square roots can be rationalised and the result is expressed in terms of multiple polylogarithms.

KEYWORDS: Higgs Physics, Perturbative QCD

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1 Introduction

The precise determination of the properties of the Higgs boson is now a central pillar of the experimental programs at the LHC. As the Higgs boson decays predominately to $b\bar{b}$, the partial width for this decay is of central importance [1]. This raises immediately the question, how precise can we predict this partial width within the Standard Model from theory? This requires the computation of quantum corrections.

The state-of-the-art for the partial decay width for $H \rightarrow b\bar{b}$ is as follows: in massless QCD the corrections are known to $\mathcal{O}(\alpha_s^4)$ [2]. Keeping the mass dependence, QCD corrections are known to $\mathcal{O}(\alpha_s^3)$ [3–6]. The QCD calculation of order $\mathcal{O}(\alpha_s^2)$ for the decay $H \rightarrow b\bar{b}$ through a $Ht\bar{t}$ -coupling has become available quite recently [7]. Two-loop QED corrections and mixed QED-QCD corrections have been considered in [8]. For the mixed electroweak-QCD corrections to the decay $H \rightarrow b\bar{b}$ of order $\mathcal{O}(\alpha\alpha_s)$ the leading term in an expansion in m_H^2/m_t^2 has been obtained in [9, 10]. This has been improved by including systematically more terms in [11]. As far as the two-loop electroweak corrections of order $\mathcal{O}(\alpha^2)$ are concerned only the leading term in an expansion in m_H^2/m_t^2 is known [12, 13]. In recent years, there has been a substantial progress in our abilities to compute Feynman integrals, and Feynman integrals which previously could only be calculated approximately come into reach. In this article we present the two-loop master integrals relevant to the $\mathcal{O}(\alpha\alpha_s)$ -corrections to the decay $H \rightarrow b\bar{b}$ through a $Ht\bar{t}$ -coupling. We keep the full dependence on the heavy particle masses (m_t , m_H and m_W), but neglect the b -quark

Figure 1. Examples of Feynman diagrams contributing to the mixed $\mathcal{O}(\alpha\alpha_s)$ -corrections to the decay $H \rightarrow b\bar{b}$ through a $Ht\bar{t}$ -coupling. The Higgs boson is denoted by a dashed line, a top quark by a green line, a bottom quark with a black line and a gluon by a curly line. Particles with mass m_W are drawn with a wavy line.

mass. In Higgs and top physics there are two-loop Feynman integrals related to elliptic curves [14–18]. One might fear that we are in a similar situation here. To some relief this is not the case. All master integrals can be expressed entirely in terms of multiple polylogarithm. The difficulty of these integrals is entirely due to the required simultaneous rationalisation of two square roots.

Large parts of the calculation are based on by now standard techniques: we first derive a system of differential equations for the master integrals [19–29], using integration-by-parts identities [30, 31] and the Laporta algorithm [32]. We then bring the system of differential equations into an ε -form [25]. This will introduce two square roots. It turns out that the square roots can be rationalised simultaneously with the methods of [33]. This is the essential new ingredient of our calculation. The resulting system of differential equations involves only dlog-forms and can be solved in terms of multiple polylogarithms. We are able to express all master integrals in terms of multiple polylogarithms with an alphabet consisting of 13 letters.

This paper is organised as follows: in the next section we introduce our notation. In particular we define the kinematic variables which will rationalise the square roots. In section 3 we define the master integrals, for which the system of differential equations is in ε -form. The system of differential equations is presented in section 4. The analytic results for the master integrals are given in section 5. In addition, section 6 gives numerical results for the most interesting case $p^2 = m_H^2$. Finally, our conclusions are contained in section 7. The appendix shows for all master integrals the corresponding Feynman diagrams and describes the content of the supplementary electronic file attached to this article.

2 Notation

We are interested in the mixed $\mathcal{O}(\alpha\alpha_s)$ -corrections to the decay $H \rightarrow b\bar{b}$ through a $Ht\bar{t}$ -coupling. Examples of Feynman diagrams are shown in figure 1. Not shown are diagrams whose master integrals are related to the master integrals of the diagrams of figure 1 by symmetry. We will neglect the b -quark mass. However, we will treat the dependence on the top quark mass m_t , the W -boson mass m_W and the momentum p of the Higgs boson exactly. For an on-shell Higgs-boson we have $p^2 = m_H^2$. For the two-loop contributions to the Higgs decay we have two independent external momenta p_1 and p_2 , which label

Figure 2. Auxiliary diagrams for the two planar vertex corrections, the non-planar vertex correction and topology D . The internal masses of the propagators are encoded by the colour of the propagators: massless (black), m_t (green), m_W (red).

the momenta of b -quark and $\bar{b}$ -quark, respectively. With two independent loop momenta we thus have seven linearly independent scalar products. For each of the four Feynman diagrams G_A - G_D we introduce an auxiliary topology with seven propagators. These are shown in figure 2. The master integrals related to diagram G_E are a subset of the master integrals related to diagram G_A (and likewise a subset of the master integrals related to diagram G_D). We consider the integrals

$$I_{\nu_1\nu_2\nu_3\nu_4\nu_5\nu_6\nu_7}^X = e^{2\gamma_E\epsilon} (\mu^2)^{\nu-D} \int \frac{d^D k_1}{i\pi^{\frac{D}{2}}} \frac{d^D k_2}{i\pi^{\frac{D}{2}}} \prod_{j=1}^7 \frac{1}{(P_j^X)^{\nu_j}}, \quad X \in \{A, B, C, D\}, \quad (2.1)$$

where $D = 4 - 2\epsilon$ denotes the number of space-time dimensions, γ_E denotes the Euler-Mascheroni constant, μ is an arbitrary scale introduced to render the Feynman integral dimensionless, and the quantity ν is defined by

$$\nu = \sum_{j=1}^7 \nu_j. \quad (2.2)$$

The inverse propagators P_j^X are defined as follows:

Topology A:

$$\begin{aligned} P_1^A &= -k_1^2 + m_t^2, & P_2^A &= -(k_1 - p_1 - p_2)^2 + m_t^2, & P_3^A &= -(k_1 + k_2)^2, \\ P_4^A &= -k_2^2 + m_t^2, & P_5^A &= -(k_2 + p_1)^2 + m_W^2, & P_6^A &= -(k_2 + p_1 + p_2)^2 + m_t^2, \\ P_7^A &= -(k_1 - p_1)^2 + m_t^2. \end{aligned} \quad (2.3)$$

Topology B:

$$\begin{aligned} P_1^B &= -k_1^2 + m_t^2, & P_2^B &= -(k_1 - p_1 - p_2)^2 + m_t^2, & P_3^B &= -(k_1 + k_2)^2 + m_W^2, \\ P_4^B &= -k_2^2, & P_5^B &= -(k_2 + p_1)^2, & P_6^B &= -(k_2 + p_1 + p_2)^2, \\ P_7^B &= -(k_1 - p_1)^2 + m_t^2. \end{aligned} \quad (2.4)$$

Topology C:

$$\begin{aligned} P_1^C &= -k_1^2 + m_t^2, & P_2^C &= -(k_1 - p_1 - p_2)^2 + m_t^2, & P_3^C &= -(k_1 + k_2)^2, \\ P_4^C &= -(k_1 + k_2 - p_1)^2, & P_5^C &= -k_2^2 + m_W^2, & P_6^C &= -(k_2 + p_2)^2 + m_t^2, \\ P_7^C &= -(k_1 - p_1)^2 + m_t^2. \end{aligned} \quad (2.5)$$

Topology D:

$$\begin{aligned} P_1^D &= -k_1^2 + m_t^2, & P_2^D &= -(k_1 - p_1 - p_2)^2 + m_t^2, & P_3^D &= -(k_1 - p_1)^2 + m_W^2, \\ P_4^D &= -(k_1 + k_2)^2 + m_t^2, & P_5^D &= -k_2^2, & P_6^D &= -(k_2 + p_1)^2, \\ P_7^D &= -(k_2 + p_1 + p_2)^2. \end{aligned} \quad (2.6)$$

For all topologies our conventions are such that we are interested in the integrals with $\nu_7 \leq 0$.

The Feynman parameter representations for the four topologies are given by

$$I_{\nu_1 \nu_2 \nu_3 \nu_4 \nu_5 \nu_6 \nu_7}^X = e^{2\gamma_E \varepsilon} \frac{\Gamma(\nu - D)}{\prod_{j=1}^7 \Gamma(\nu_j)} \int_{x_i \geq 0} d^7 x \, \delta \left(1 - \sum_{j=1}^7 x_j \right) \left(\prod_{j=1}^7 x_j^{\nu_j - 1} \right) \frac{\mathcal{U}_X^{\nu - \frac{3}{2}D}}{\mathcal{F}_X^{\nu - D}}. \quad (2.7)$$

The graph polynomials are given by

$$\begin{aligned}
 \mathcal{U}_A &= (x_1 + x_2 + x_7)(x_4 + x_5 + x_6) + x_3(x_1 + x_2 + x_4 + x_5 + x_6 + x_7), \\
 \mathcal{F}_A &= [x_1 x_2 (x_3 + x_4 + x_5 + x_6) + x_4 x_6 (x_1 + x_2 + x_3 + x_7) + x_3 (x_1 x_6 + x_2 x_4)] \left(\frac{-p^2}{\mu^2} \right) \\
 &\quad + \mathcal{U}_A \left[(x_1 + x_2 + x_4 + x_6 + x_7) \frac{m_t^2}{\mu^2} + x_5 \frac{m_W^2}{\mu^2} \right], \\
 \mathcal{U}_B &= (x_1 + x_2 + x_7)(x_4 + x_5 + x_6) + x_3(x_1 + x_2 + x_4 + x_5 + x_6 + x_7), \\
 \mathcal{F}_B &= [x_1 x_2 (x_3 + x_4 + x_5 + x_6) + x_4 x_6 (x_1 + x_2 + x_3 + x_7) + x_3 (x_1 x_6 + x_2 x_4)] \left(\frac{-p^2}{\mu^2} \right) \\
 &\quad + \mathcal{U}_B \left[(x_1 + x_2 + x_7) \frac{m_t^2}{\mu^2} + x_3 \frac{m_W^2}{\mu^2} \right], \\
 \mathcal{U}_C &= (x_1 + x_2 + x_7)(x_5 + x_6) + (x_3 + x_4)(x_1 + x_2 + x_5 + x_6 + x_7), \\
 \mathcal{F}_C &= [x_1 x_2 (x_3 + x_4 + x_5 + x_6) + x_1 x_4 x_6 + x_2 x_3 x_5 - x_3 x_6 x_7] \left(\frac{-p^2}{\mu^2} \right) \\
 &\quad + \mathcal{U}_C \left[(x_1 + x_2 + x_6 + x_7) \frac{m_t^2}{\mu^2} + x_5 \frac{m_W^2}{\mu^2} \right], \\
 \mathcal{U}_D &= (x_1 + x_2 + x_3)(x_5 + x_6 + x_7) + x_4(x_1 + x_2 + x_3 + x_5 + x_6 + x_7), \\
 \mathcal{F}_D &= [x_1 x_2 (x_4 + x_5 + x_6 + x_7) + x_5 x_7 (x_1 + x_2 + x_3 + x_4) + x_4 (x_1 x_7 + x_2 x_5)] \left(\frac{-p^2}{\mu^2} \right) \\
 &\quad + \mathcal{U}_D \left[(x_1 + x_2 + x_4) \frac{m_t^2}{\mu^2} + x_3 \frac{m_W^2}{\mu^2} \right]. \tag{2.8}
 \end{aligned}$$

Let us introduce an operator $\mathbf{i}^+$, which raises the power of the propagator i by one, e.g.

$$\mathbf{1}^+ I_{\nu_1 \nu_2 \nu_3 \nu_4 \nu_5 \nu_6 \nu_7}^X = I_{(\nu_1+1) \nu_2 \nu_3 \nu_4 \nu_5 \nu_6 \nu_7}^X. \tag{2.9}$$

In addition we define two operators $\mathbf{D}^\pm$, which shift the dimension of space-time by two through

$$\mathbf{D}^\pm I_{\nu_1 \nu_2 \nu_3 \nu_4 \nu_5 \nu_6 \nu_7}^X(D) = I_{\nu_1 \nu_2 \nu_3 \nu_4 \nu_5 \nu_6 \nu_7}^X(D \pm 2). \tag{2.10}$$

The dimensional shift relations read [34, 35]

$$\mathbf{D}^- I_{\nu_1 \nu_2 \nu_3 \nu_4 \nu_5 \nu_6 \nu_7}^X(D) = \mathcal{U}_X(\nu_1 \mathbf{1}^+, \nu_2 \mathbf{2}^+, \nu_3 \mathbf{3}^+, \nu_4 \mathbf{4}^+, \nu_5 \mathbf{5}^+, \nu_6 \mathbf{6}^+, \nu_7 \mathbf{7}^+) I_{\nu_1 \nu_2 \nu_3 \nu_4 \nu_5 \nu_6 \nu_7}^X(D). \tag{2.11}$$

In the following we will set

$$\mu^2 = m_t^2. \tag{2.12}$$

After setting $\mu^2 = m_t^2$ the master integrals depend kinematically on two dimensionless quantities. A naive choice is

$$v = \frac{p^2}{m_t^2}, \quad w = \frac{m_W^2}{m_t^2}, \tag{2.13}$$

Topology	Number of master integrals
A	18
B	15
C	31
D	14

Table 1. The number of master integrals for a given topology.

with $p = p_1 + p_2$. However, with this choice we will encounter square roots. In particular, the square roots

$$\sqrt{-v(4-v)} \quad \text{and} \quad \sqrt{\lambda(v, w, 1)} \quad (2.14)$$

will occur. The Källén function is defined by

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx. \quad (2.15)$$

In order to rationalise the square roots we introduce dimensionless quantities x and y through

$$\frac{p^2}{m_t^2} = v = -\frac{(1-x)^2}{x}, \quad \frac{m_W^2}{m_t^2} = w = \frac{(1-y+2xy)(x-2y+xy)}{x(1-y^2)}. \quad (2.16)$$

The first transformation is standard and has occurred in many places before, the second one is easily obtained with the methods of ref. [33]. The Feynman integrals are then functions of x, y and the dimensional regularisation parameter ε . The inverse transformations are given by

$$x = \frac{1}{2} \left(2 - v - \sqrt{-v(4-v)} \right), \quad y = \frac{\sqrt{\lambda(v, w, 1)} - \sqrt{-v(4-v)}}{1 - w + 2v}, \quad (2.17)$$

such that $x = 0$ corresponds to $v = \infty$ and $y = 0$ corresponds to $w = 1$. Let us also note that the point $(v, w) = (0, 1)$ is blown up in (x, y) -space to the hypersurface $x = 1$. This motivates our final change of coordinates and we introduce

$$x' = 1 - x. \quad (2.18)$$

3 Master integrals

For the reduction to master integrals we use the programs **Reduze** [36], **Kira** [37] or **Fire** [38] combined with **LiteRed** [39, 40]. Each topology involves a certain number of master integrals. This number is shown in table 1 and corresponds to the number of master integrals if we just consider one topology in isolation. Of course, the various topologies share some master integrals and the final number of master integrals which we have to compute is

lower. We have the following relations

$$\begin{aligned}
 I_{\mu 00\nu 000}^A &= I_{\mu 0000\nu 0}^C = I_{\mu 00\nu 000}^D, \\
 I_{\mu 000\nu 00}^A &= I_{\mu 0\nu 0000}^B = I_{\mu 000\nu 00}^C = I_{00\nu\mu 000}^D, \\
 I_{\mu\nu 0\rho 000}^A &= I_{\mu\nu 000\rho 0}^C = I_{\mu\nu 0\rho 000}^D, \\
 I_{\mu\nu 00\rho 00}^A &= I_{\mu\nu\rho 0000}^B = I_{\mu\nu 00\rho 00}^C, \\
 I_{0\mu\nu\rho 000}^A &= I_{\mu 00\nu 0\rho 0}^C = I_{0\mu 0\rho\nu 00}^D, \\
 I_{0\mu\nu\rho 000}^B &= I_{0\mu\rho 0\nu 00}^C, \\
 I_{\mu 0\nu 0\rho 00}^A &= I_{\mu 0\rho\nu 000}^B = I_{\mu 0\nu 0\rho 00}^C = I_{00\rho\mu\nu 00}^D, \\
 I_{\mu 00\nu\rho\sigma 0}^A &= I_{\nu\sigma\rho\mu 000}^D, \\
 I_{\mu\nu\rho\sigma 000}^B &= I_{\mu\nu\sigma 0\rho 00}^C, \\
 I_{\mu\nu\rho 00\sigma 0}^C &= I_{\mu\nu 0\sigma 0\rho 0}^D, \\
 I_{\mu\nu\rho 0\sigma 00}^A &= I_{\mu\nu\sigma 0\rho 00}^B = I_{\mu\nu 0\rho\sigma 00}^C, \\
 I_{0\mu\nu\rho\sigma 00}^A &= I_{\mu 00\nu\sigma\rho 0}^C = I_{0\rho\sigma\mu\nu 00}^D, \\
 I_{\mu\nu\rho\sigma\kappa 00}^A &= I_{\mu\nu 0\rho\kappa\sigma 0}^C, \\
 I_{\mu\nu\rho\sigma\kappa 00}^B &= I_{\mu\nu\sigma\kappa\rho 00}^C, \\
 I_{\mu 0\nu\rho\sigma\kappa 0}^C &= I_{0\kappa\sigma\mu\nu\rho 0}^D.
 \end{aligned} \tag{3.1}$$

In total we have to consider 39 master integrals, which are grouped into 25 blocks such that one block corresponds to one sub-topology. Some of the master integrals are taken as integrals in $D - 2 = 2 - 2\varepsilon$ space-time dimensions. Of course, with the help of the dimensional shift relations in eq. (2.11) they are easily expressed as (longer) linear combinations of master integrals in $D = 4 - 2\varepsilon$ space-time dimensions. A system of master integrals is given by

$$\begin{aligned}
 J_1 &= \varepsilon^2 \mathbf{D}^- I_{1001000}^A, \\
 J_2 &= \varepsilon^2 \mathbf{D}^- I_{1000100}^A, \\
 J_3 &= \varepsilon^2 \frac{(1-x^2)}{2x} \mathbf{D}^- I_{1101000}^A, \\
 J_4 &= \varepsilon^2 \frac{(1-x^2)}{2x} \mathbf{D}^- I_{1100100}^A, \\
 J_5 &= \frac{1}{2} \varepsilon^2 v \mathbf{D}^- I_{1001010}^B, \\
 J_6 &= \frac{1}{2} \varepsilon^2 v \mathbf{D}^- I_{0011010}^B, \\
 J_7 &= \varepsilon^2 \frac{(1-x^2)}{2x} \mathbf{D}^- I_{0111000}^A, \\
 J_8 &= \varepsilon^2 \mathbf{D}^- I_{(-1)111000}^A, \\
 J_9 &= \varepsilon^2 \frac{(1-x)(1+x-2y+y^2+2xy+xy^2)}{x(1-y^2)} \mathbf{D}^- I_{0111000}^B, \\
 J_{10} &= \varepsilon^2 \left[\mathbf{D}^- I_{(-1)111000}^B - (1-w) \mathbf{D}^- I_{0111000}^B \right],
 \end{aligned}$$

$$\begin{aligned}
 J_{11} &= 2\varepsilon^2 \mathbf{D}^- I_{0111(-1)00}^B - 2\varepsilon^2 \mathbf{D}^- I_{0110000}^B, \\
 J_{12} &= \varepsilon^2 (1-w) \mathbf{D}^- I_{1010100}^A, \\
 J_{13} &= \varepsilon^2 \frac{(1-x^2)^2}{4x^2} \mathbf{D}^- I_{1101010}^A, \\
 J_{14} &= \varepsilon^2 \frac{(1-x)^3(1+x)}{4x^2} \mathbf{D}^- I_{1101010}^B, \\
 J_{15} &= \varepsilon^3 (1-\varepsilon) v I_{1001110}^A, \\
 J_{16} &= \varepsilon^2 \frac{(1-x^2)}{x} \left[(1-w) \mathbf{D}^- I_{1111000}^B - \mathbf{D}^- I_{0111000}^B \right], \\
 J_{17} &= \varepsilon^3 v I_{1120010}^C, \\
 J_{18} &= \varepsilon^3 v I_{1110020}^C, \\
 J_{19} &= \varepsilon^2 \frac{(1-x^2)}{x} \left[(1-2\varepsilon) I_{2110010}^C + \varepsilon I_{1110020}^C \right], \\
 J_{20} &= \varepsilon^3 v I_{1120100}^A, \\
 J_{21} &= \varepsilon^3 v I_{1110200}^A, \\
 J_{22} &= 2\varepsilon^2 \frac{(1-x)}{x(1+w)} \left[(1-2\varepsilon)(1+x) I_{2110100}^A - \varepsilon(x-w) I_{1120100}^A + \varepsilon(1-x+2w) I_{1110200}^A \right. \\
 &\quad \left. - \frac{1}{2}(1+x) \mathbf{D}^- I_{1100100}^A \right], \\
 J_{23} &= 2\varepsilon^3 v I_{0211100}^A, \\
 J_{24} &= 2\varepsilon^3 v I_{0121100}^A, \\
 J_{25} &= 2\varepsilon^3 v I_{0210110}^C, \\
 J_{26} &= 2\varepsilon^3 v I_{0120110}^C, \\
 J_{27} &= \varepsilon^3 \frac{(1-x)^2}{x(1+x)} \left[(1-2\varepsilon)(1-x) I_{1101110}^A + 2(1-\varepsilon) I_{1001110}^A \right], \\
 J_{28} &= 2\varepsilon^3 (1-2\varepsilon) v I_{1111010}^B, \\
 J_{29} &= 2\varepsilon^3 v (1-w) I_{1111200}^A, \\
 J_{30} &= 2\varepsilon^3 v \left(I_{1112100}^A + I_{1111200}^A \right), \\
 J_{31} &= 4\varepsilon^3 \frac{(1-x)^2}{x(1+x^2)} \left[(1-x^2) I_{2111100}^A - (1+w) I_{1111200}^A - 2I_{1112100}^A + 2I_{1120100}^A + I_{1110200}^A \right. \\
 &\quad \left. - I_{0211100}^A - 2I_{0121100}^A \right], \\
 J_{32} &= 4\varepsilon^4 v I_{1111100}^B, \\
 J_{33} &= 2\varepsilon^3 v (1-w) I_{1110120}^C, \\
 J_{34} &= 2\varepsilon^3 v w \left(I_{1110210}^C + I_{1110120}^C \right), \\
 J_{35} &= 2\varepsilon^3 v (1-w) I_{1120110}^C, \\
 J_{36} &= 4\varepsilon^3 \frac{(1-x)^2}{x(1+x^2)} \left[(1-x^2) I_{2110110}^C - (1+w) I_{1110120}^C - 2w I_{1110210}^C + 2(1-w) I_{1120110}^C \right. \\
 &\quad \left. + 2I_{1120010}^C + I_{1110020}^C - I_{0210110}^C - 2I_{0120110}^C \right],
 \end{aligned}$$

$$\begin{aligned}
J_{37} &= 2\varepsilon^4 v I_{1011110}^C, \\
J_{38} &= \varepsilon^3 vw I_{1112010}^D, \\
J_{39} &= 4\varepsilon^4 \frac{(1-x)^2 (1-x+x^2-xw)}{x^2} I_{1111110}^C.
\end{aligned} \tag{3.2}$$

4 The system of differential equations

Let us set $\vec{J} = (J_1, \dots, J_{39})^T$ for the vector of master integrals. In this basis the system of differential equations is in ε -form [25]. We have

$$d\vec{J} = \varepsilon A \vec{J}, \tag{4.1}$$

where the matrix A is independent of ε . In order to construct the basis of master integrals leading to an ε -form, we could have followed the same procedure as in [17, 18]. In practice, the results of [17, 18] allow us to guess the master integrals or at least indicate an ansatz for the master integrals. In each topology, our ansatz involves only a few parameters, which are fixed by requiring that terms of order ε^0 in the matrix A vanish.

Let us describe the singularities of the system of differential equations. The singularities are on hypersurfaces and each hypersurface is defined by a polynomial in x and y . There are sixteen polynomials, which are given by

$$\begin{aligned}
p_1 &= x, \\
p_2 &= x - 1, \\
p_3 &= x + 1, \\
p_4 &= y, \\
p_5 &= y - 1, \\
p_6 &= y + 1, \\
p_7 &= xy + x - y + 1, \\
p_8 &= xy + x - 2y, \\
p_9 &= 2xy - y + 1, \\
p_{10} &= xy^2 + 2xy - 2y^2 + x + 2y, \\
p_{11} &= xy^2 + 2xy + y^2 + x - 2y + 1, \\
p_{12} &= xy^2 + 2xy - y^2 + x + 2y - 1, \\
p_{13} &= 2xy^2 + 2xy - y^2 + 2y - 1, \\
p_{14} &= 2xy^2 + 2xy - 3y^2 + 2y + 1, \\
p_{15} &= 3xy^2 + 2xy - 2y^2 - x + 2y, \\
p_{16} &= 3xy^2 + 2xy - 3y^2 - x + 2y + 1.
\end{aligned} \tag{4.2}$$

We note that the polynomials p_k are maximally of degree 3. The highest degree in the variable y is two, the highest degree in the variable x is one. The entries of the matrix A

are $\mathbb{Q}$ -linear combinations of dlog-forms of these polynomials:

$$A_{ij} = \sum_{k=1}^{16} \tilde{c}_{ijk} d \ln(p_k(x, y)), \quad \tilde{c}_{ijk} \in \mathbb{Q}. \quad (4.3)$$

We find that the matrix A contains only fifteen $\mathbb{Q}$ -independent linear combinations of dlog-forms. A basis for these is given by

$$\begin{aligned} \omega_1 &= \frac{ds}{s} = 2d \ln p_2 - d \ln p_1, \\ \omega_2 &= \frac{ds}{s - 4m_t^2} = 2d \ln p_3 - d \ln p_1, \\ \omega_3 &= \frac{ds}{\sqrt{-s(4m_t^2 - s)}} = d \ln p_1, \\ \omega_4 &= \frac{dm_W^2}{m_W^2} = d \ln p_8 + d \ln p_9 - d \ln p_5 - d \ln p_6 - d \ln p_1, \\ \omega_5 &= \frac{dm_W^2}{m_W^2 - m_t^2} = d \ln p_7 - d \ln p_5 - d \ln p_6 + d \ln p_4 + d \ln p_2 - d \ln p_1, \\ \omega_6 &= d \ln(s + m_W^2 - m_t^2) = d \ln p_{16} - d \ln p_5 - d \ln p_6 + d \ln p_2 - d \ln p_1, \\ \omega_7 &= d \ln(s - m_W^2 + m_t^2) = d \ln p_{12} - d \ln p_5 - d \ln p_6 + d \ln p_2 - d \ln p_1, \\ \omega_8 &= \frac{1}{2} d \ln(sm_W^2 + (m_t^2 - m_W^2)^2) \\ &= \frac{1}{2} d \ln p_{15} + \frac{1}{2} d \ln p_{14} - d \ln p_5 - d \ln p_6 + d \ln p_2 - d \ln p_1, \\ \omega_9 &= \frac{1}{2} d \ln((2m_t^2 - m_W^2)s + (m_t^2 - m_W^2)^2) \\ &= \frac{1}{2} d \ln p_{13} + \frac{1}{2} d \ln p_{10} - d \ln p_5 - d \ln p_6 + d \ln p_2 - d \ln p_1, \\ \omega_{10} &= \frac{1}{2} d \ln(\lambda(s, m_W^2, m_t^2)) \\ &= d \ln p_{11} - d \ln p_5 - d \ln p_6 + d \ln p_2 - d \ln p_1, \\ \omega_{11} &= \frac{1}{2} d \ln p_{15} - \frac{1}{2} d \ln p_{14}, \\ \omega_{12} &= \frac{1}{2} d \ln p_{13} - \frac{1}{2} d \ln p_{10}, \\ \omega_{13} &= \frac{1}{2} d \ln p_9 - \frac{1}{2} d \ln p_8, \\ \omega_{14} &= \frac{1}{2} d \ln p_6 - \frac{1}{2} d \ln p_5, \\ \omega_{15} &= d \ln p_4 - \frac{1}{2} d \ln p_6 - \frac{1}{2} d \ln p_5. \end{aligned} \quad (4.4)$$

The entries of A are therefore of the form

$$A_{ij} = \sum_{k=1}^{15} c_{ijk} \omega_k, \quad c_{ijk} \in \mathbb{Q}. \quad (4.5)$$

By a rescaling of the master integrals with constant factors we may actually achieve

$$c_{ijk} \in \mathbb{Z}. \quad (4.6)$$

For our choice of basis of master integrals $\vec{J}$ we have $c_{ijk} \in \mathbb{Z}$. Equivalently, we may express the matrix A as

$$A = \sum_{k=1}^{15} C_k \omega_k, \quad (4.7)$$

where the entries of the 39×39 -matrices C_k are integer numbers. The matrix A is given in the supplementary electronic file attached to this article.

On specific hypersurfaces the differential forms simplify considerably. On the hypersurface $y=0$ (i.e. for the case $m_W = m_t$) the differential forms reduce to a linear combination of

$$\frac{dx}{x}, \quad \frac{dx}{x-1}, \quad \frac{dx}{x+1}. \quad (4.8)$$

On the hypersurface $x=0$ (i.e. for the case $p^2 \rightarrow \infty$) the differential forms reduce to a linear combination of

$$\frac{dy}{y}, \quad \frac{dy}{y-1}, \quad \frac{dy}{y+1}, \quad \frac{dy}{y+\frac{1}{3}}. \quad (4.9)$$

On the hypersurface $x=1$ (i.e. for the case $p^2=0$ and $m_W^2 = m_t^2$) the differential forms reduce to a linear combination of

$$\frac{dy}{y}, \quad \frac{dy}{y-1}, \quad \frac{dy}{y+1}, \quad \frac{2ydy}{y^2+1}, \quad \frac{2(y-2)dy}{y^2-4y-1}, \quad \frac{2(y+2)dy}{y^2+4y-1}. \quad (4.10)$$

The derivative of the master integrals is given by the product of the matrix A with the vector $\vec{J}$:

$$d\vec{J} = \varepsilon A \vec{J}. \quad (4.11)$$

The right-hand side vanishes if $\vec{J}$ is in the kernel of A . This is what happens on the hypersurface $x=1$: although $A \neq 0$ we have

$$\left. \frac{\partial \vec{J}}{\partial y} \right|_{x=1} = 0. \quad (4.12)$$

It follows that the master integrals are constant on the hypersurface $x=1$. This is a significant simplification for solving the differential equations.

5 Analytical results

The analytic result for the master integrals at a point (x, y) is obtained from the value of the master integrals at a boundary point (x_i, y_i) by integrating the system of differential equations along a path from (x_i, y_i) to (x, y) . The result does not depend on the chosen path, but only on the homotopy class of the path. For the case at hand we have several significant simplification:

1. All master integrals are constant on the hypersurface $x = 1$ (corresponding to $p^2 = 0$ and $m_W^2 = m_t^2$) and we may take the values of the master integrals on this hypersurface as boundary values. We therefore have a boundary line.
2. It is then sufficient to integrate the differential equation along a straight line from $(1, y)$ to (x, y) with $y = \text{const}$. The polynomials p_1 - p_3 and p_7 - p_{16} are all linear in x (the polynomials p_4 - p_6 don't contribute along $y = \text{const}$), avoiding the need to factorise higher-order polynomials in x .
3. The boundary values on the hypersurface $x = 1$ are particularly simple: 35 out of the 39 master integrals vanish on this hypersurface, the four non-vanishing master integrals are products of one-loop integrals.

Since all integration kernels are dlog-forms, the result can be expressed in terms of multiple polylogarithms. Multiple polylogarithms are defined for $l_k \neq 0$ by [41–44]

$$G(l_1, l_2, \dots, l_k; z) = \int_0^z \frac{dz_1}{z_1 - l_1} \int_0^{z_1} \frac{dz_2}{z_2 - l_2} \cdots \int_0^{z_{k-1}} \frac{dz_k}{z_k - l_k}. \quad (5.1)$$

We have

$$G(l_1, l_2, \dots, l_k; z) = \int_0^z \frac{dz_1}{z_1 - l_1} G(l_2, \dots, l_k; z_1). \quad (5.2)$$

We define $G(0, \dots, 0; z)$ with k zeros for l_1 to l_k to be

$$G(0, \dots, 0; z) = \frac{1}{k!} (\ln z)^k. \quad (5.3)$$

This permits us to allow trailing zeros in the sequence $(l_1, \dots, l_k)$ by defining the function G with trailing zeros via eq. (5.2) and eq. (5.3). For $l_k \neq 0$ we have the scaling relation

$$G(l_1, \dots, l_k; z) = G(\lambda l_1, \dots, \lambda l_k; \lambda z). \quad (5.4)$$

Let us introduce the short-hand notation

$$G_{m_1, \dots, m_k}(l_1, \dots, l_k; z) = G(\underbrace{0, \dots, 0}_{m_1-1}, l_1, \dots, l_{k-1}, \underbrace{0, \dots, 0}_{m_k-1}, l_k; z), \quad (5.5)$$

where all l_j for $j = 1, \dots, k$ are assumed to be non-zero. The sum representation of multiple polylogarithms is defined by

$$\text{Li}_{m_1, \dots, m_k}(x_1, \dots, x_k) = \sum_{n_1 > n_2 > \dots > n_k > 0} \frac{x_1^{n_1}}{n_1^{m_1}} \cdots \frac{x_k^{n_k}}{n_k^{m_k}}, \quad (5.6)$$

and related to the integral representation by

$$\begin{aligned} \text{Li}_{m_1, \dots, m_k}(x_1, \dots, x_k) &= (-1)^k G_{m_1, \dots, m_k} \left(\frac{1}{x_1}, \frac{1}{x_1 x_2}, \dots, \frac{1}{x_1 \dots x_k}; 1 \right), \\ G_{m_1, \dots, m_k}(l_1, \dots, l_k; z) &= (-1)^k \text{Li}_{m_1, \dots, m_k} \left(\frac{z}{l_1}, \frac{l_1}{l_2}, \dots, \frac{l_{k-1}}{l_k} \right). \end{aligned} \quad (5.7)$$

The master integrals J_1 , J_2 , J_5 and J_6 are products of one-loop integrals and rather simple. They are given by

$$\begin{aligned} J_1 &= e^{2\gamma_E \varepsilon} (\Gamma(1 + \varepsilon))^2, \\ J_2 &= e^{2\gamma_E \varepsilon} (\Gamma(1 + \varepsilon))^2 w^{-\varepsilon}, \\ J_5 &= e^{2\gamma_E \varepsilon} \frac{(\Gamma(1 + \varepsilon))^2 (\Gamma(1 - \varepsilon))^2}{\Gamma(1 - 2\varepsilon)} (-v)^{-\varepsilon}, \\ J_6 &= e^{2\gamma_E \varepsilon} \frac{(\Gamma(1 + \varepsilon))^2 (\Gamma(1 - \varepsilon))^2}{\Gamma(1 - 2\varepsilon)} (-v)^{-\varepsilon} w^{-\varepsilon}. \end{aligned} \quad (5.8)$$

These are the only master integrals, which do not vanish on the hypersurface $x = 1$. All other master integrals vanish on the hypersurface $x = 1$. This fully specifies the boundary conditions on the line $(x, y) = (1, y)$. Note that the integrals J_5 and J_6 have logarithmic singularities at $x = 1$. This is most easily seen from eq. (2.16) and the comments made there. The hypersurface $x = 1$ corresponds to the point $(v, w) = (0, 1)$ and J_5 and J_6 have logarithmic singularities at $v = 0$.

For the integration in (x, y) -space from $(1, y)$ to (x, y) it is better to change variables from x to $x' = 1 - x$. We therefore integrate in (x', y) -space from $(0, y)$ to (x', y) , where y is treated as a parameter. This integration gives multiple polylogarithms of the form

$$G(l'_1, \dots, l'_k; x'), \quad (5.9)$$

where the letters $l'_1, \dots, l'_k$ are from the alphabet

$$\mathcal{A} = \{0, 1, 2, x'_7, x'_8, x'_9, x'_{10}, x'_{11}, x'_{12}, x'_{13}, x'_{14}, x'_{15}, x'_{16}\}. \quad (5.10)$$

The non-trivial letters x'_7 - x'_{16} are given by

$$\begin{aligned} x'_7 &= \frac{2}{1+y}, & x'_8 &= \frac{1-y}{1+y}, & x'_9 &= \frac{1+y}{2y}, \\ x'_{10} &= \frac{1+4y-y^2}{(1+y)^2}, & x'_{11} &= \frac{2(1+y^2)}{(1+y)^2}, & x'_{12} &= \frac{4y}{(1+y)^2}, \\ x'_{13} &= -\frac{1-4y-y^2}{2y(1+y)}, & x'_{14} &= \frac{1+4y-y^2}{2y(1+y)}, & x'_{15} &= \frac{1-4y-y^2}{(1+y)(1-3y)}, \\ x'_{16} &= -\frac{4y}{(1+y)(1-3y)}. \end{aligned} \quad (5.11)$$

For all basis integrals we write

$$J_k = \sum_{j=0}^{\infty} \varepsilon^j J_k^{(j)}. \quad (5.12)$$

The quantities $J_k^{(j)}$ are given for $1 \leq k \leq 39$ and $0 \leq j \leq 4$ in the supplementary electronic file attached to this article. To give an example, let us show the first non-vanishing term

of the most complicated integral, the non-planar vertex correction J_{39} . We find

$$\begin{aligned}
 J_{39}^{(3)} = & 8G(0, 1, 1; x') - 4G(0, 1, x'_8; x') - 4G(0, x'_8, 1; x') + 4G(0, x'_8, x'_9; x') + 4G(0, x'_9, x'_8; x') \\
 & - 8G(1, 2, 1; x') + 8G(1, x'_{15}, 1; x') - 4G(1, x'_{15}, x'_8; x') - 4G(1, x'_{15}, x'_9; x') \\
 & - 16G(1, x'_7, 1; x') + 8G(1, x'_7, x'_8; x') + 12G(1, x'_7, x'_9; x') + 4G(1, x'_8, 1; x') \\
 & - 4G(1, x'_8, x'_9; x') - 4G(1, x'_9, x'_8; x') + 8G(x'_{14}, 2, 1; x') - 8G(x'_{14}, x'_{15}, 1; x') \\
 & + 4G(x'_{14}, x'_{15}, x'_8; x') + 4G(x'_{14}, x'_{15}, x'_9; x') + 16G(x'_{14}, x'_7, 1; x') - 8G(x'_{14}, x'_7, x'_8; x') \\
 & - 12G(x'_{14}, x'_7, x'_9; x') - 4G(x'_{14}, x'_8, 1; x') + 4G(x'_{14}, x'_8, x'_9; x') + 4G(x'_{14}, x'_9, x'_8; x') \\
 & + 8G(x'_{15}, 1, 1; x') + 4G(x'_{15}, 1, x'_8; x') - 8G(x'_{15}, 2, 1; x') + 4G(x'_{15}, x'_{14}, x'_8; x') \\
 & + 4G(x'_{15}, x'_{14}, x'_9; x') + 4G(x'_{15}, x'_7, 1; x') - 12G(x'_{15}, x'_7, x'_8; x') - 8G(x'_{15}, x'_7, x'_9; x') \\
 & - 4G(x'_{15}, x'_8, 1; x') + 4G(x'_{15}, x'_8, x'_9; x') + 4G(x'_{15}, x'_9, x'_8; x') - 8G(x'_7, 1, 1; x') \\
 & + 4G(x'_7, 1, x'_8; x') + 4G(x'_7, x'_8, 1; x') - 4G(x'_7, x'_8, x'_9; x') - 4G(x'_7, x'_9, x'_8; x'). \quad (5.13)
 \end{aligned}$$

6 Numerical results

Of particular interest are numerical results for

$$p^2 = m_H^2. \quad (6.1)$$

Since $p^2 > 0$, we are not in the Euclidean region. Feynman's $i0$ -prescription instructs us to take a small imaginary part into account: $p^2 \rightarrow p^2 + i0$. This selects the correct branches for the two square roots $\sqrt{-v(4-v)}$ and $\sqrt{\lambda(v, w, 1)}$. With

$$m_W = 80.38 \text{ GeV}, \quad m_H = 125.2 \text{ GeV}, \quad m_t = 173.1 \text{ GeV} \quad (6.2)$$

we obtain for the variables x and y

$$x = 0.7384 + 0.6743i, \quad y = 0.3987i. \quad (6.3)$$

The values of the master integrals at this point are given to 8 digits in table 2. They are easily computed to arbitrary precision by evaluating the multiple polylogarithms with the help of GiNaC [45, 46]. In addition we verified the first few digits at various kinematic points with the help of the programs `sector_decomposition` [47] and `pySecDec` [48, 49].

7 Conclusions

In this paper we presented the two-loop master integrals relevant to the $\mathcal{O}(\alpha\alpha_s)$ -corrections to the decay $H \rightarrow b\bar{b}$ through a $Ht\bar{t}$ -coupling. We kept the exact dependence of the masses of the heavy particles (m_W and m_t) and the momentum p^2 of the Higgs boson, but neglected the b -quark mass. All master integrals are expressed in terms of multiple polylogarithms with an alphabet of 13 letters. They can be evaluated to arbitrary precision with the help of the GiNaC-library. For the special case $p^2 = m_H^2$ we presented the numerical values.

	ε^0	ε^1	ε^2	ε^3	ε^4
J_1	1	0	1.6449341	-0.80137127	1.8940657
J_2	1	1.5342081	2.8218314	2.3241685	2.8313617
J_3	0	-0.74005414 <i>i</i>	-0.069477641 <i>i</i>	-1.2212434 <i>i</i>	0.47861551 <i>i</i>
J_4	0	-0.74005414 <i>i</i>	-1.2048747 <i>i</i>	-2.1988043 <i>i</i>	-1.9222094 <i>i</i>
J_5	1	0.64791401 + 3.1415927 <i>i</i>	-4.7249059 + 2.0354819 <i>i</i>	-6.357481 - 4.5083042 <i>i</i>	-2.2935901 - 13.276148 <i>i</i>
J_6	1	2.1821222 + 3.1415927 <i>i</i>	-2.5539737 + 6.8553389 <i>i</i>	-12.242073 + 2.3118807 <i>i</i>	-16.987211 - 15.906447 <i>i</i>
J_7	0	0.74005414 <i>i</i>	-1.2490335 <i>i</i>	4.0114542 <i>i</i>	-7.931414 <i>i</i>
J_8	0	0	-0.54768013	0.47549335	-2.5261306
J_9	0	1.4585842 <i>i</i>	0.050868134 <i>i</i>	6.4537259 <i>i</i>	-4.1565512 <i>i</i>
J_{10}	0	1.5342081	-1.7502763	5.7981619	-12.790768
J_{11}	0	0	2.2406312	1.5008239	10.399293
J_{12}	0	-1.5342081	0.91070302	-5.9968338	8.8611857
J_{13}	0	0	-0.54768013	-0.10283443	-0.91150188
J_{14}	0	0.74005414 <i>i</i>	-2.3249487 + 0.54896908 <i>i</i>	-1.7246372 - 3.4477675 <i>i</i>	3.1827067 - 5.0304701 <i>i</i>
J_{15}	0	0	-0.40473314	-0.19458947	-0.73218618
J_{16}	0	-1.4801083 <i>i</i>	2.8153204 <i>i</i>	-7.8872093 <i>i</i>	15.843952 <i>i</i>
J_{17}	0	0	-0.27384007	0.25481947	-1.2423838
J_{18}	0	0	0	0.54349879	-0.505106
J_{19}	0	-0.74005414 <i>i</i>	1.3879888 <i>i</i>	-3.8717735 <i>i</i>	8.3164903 <i>i</i>
J_{20}	0	0	-0.40473314	-0.012836717	-1.766483
J_{21}	0	0	0	1.0679293	0.50513135
J_{22}	0	0	0.40473314 + 2.8771124 <i>i</i>	-2.1230218 - 1.197403 <i>i</i>	0.75622025 + 11.086074 <i>i</i>
J_{23}	0	0	0	1.4266372	-0.44333884
J_{24}	0	0	-0.80946628	0.078302369	-3.4351805
J_{25}	0	0	0	1.447072	-0.3662879
J_{26}	0	0	-0.83957331	-0.038098053	-3.7104375
J_{27}	0	0	0.40473314	0.19458947 - 0.29952444 <i>i</i>	0.73218618 - 0.17212665 <i>i</i>
J_{28}	0	0	0	2.4399925 + 2.5430133 <i>i</i>	-6.9287969 + 2.8702957 <i>i</i>
J_{29}	0	0	0	0.69594214	1.4888349
J_{30}	0	0	0	1.3639124	2.6684304
J_{31}	0	0	0	1.530613 - 0.59904887 <i>i</i>	2.5899951 - 1.0224563 <i>i</i>
J_{32}	0	0	0	-1.6890882	-0.27250855
J_{33}	0	0	0	0.372767	0.60475274
J_{34}	0	0	0	0.30226015	0.61200492
J_{35}	0	0	-0.29189318	-0.65744728	-1.3431597
J_{36}	0	0	0	1.5567823 - 0.61339819 <i>i</i>	2.7076661 - 1.0872344 <i>i</i>
J_{37}	0	0	0	-0.78991058	0.12604664
J_{38}	0	0	0	0.050162573	0.077279399
J_{39}	0	0	0	0.18876826	0.41154739

Table 2. Numerical results for the first five terms of the ε -expansion of the master integrals J_1 - J_{39} at the kinematic point $s = m_H^2$.

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A Master topologies

In this appendix we show in tables 3–6 the diagrams of all master topologies.

J_1

J_2

Figure 3. Master topologies (part 1).

J_3

J_4

J_5

J_6

$J_7 - J_8$

$J_9 - J_{11}$

J_{12}

J_{13}

Figure 4. Master topologies (part 2).

Figure 5. Master topologies (part 3).

J_{28}

$J_{29} - J_{31}$

J_{32}

$J_{33} - J_{36}$

J_{37}

J_{38}

J_{39}

Figure 6. Master topologies (part 4).

B Supplementary material

Attached to this article is a supplementary electronic file in ASCII format with Maple syntax, defining the quantities

$$A, \quad J.$$

The matrix A appears in the differential equation

$$d\vec{J} = \varepsilon A \vec{J}. \quad (\text{B.1})$$

The entries of the matrix A are $\mathbb{Z}$ -linear combinations of $\omega_1, \dots, \omega_{15}$, defined in eq. (4.4). These differential forms are denoted by

$$\text{omega_1}, \dots, \text{omega_15}.$$

The vector J contains the results for the master integrals up to order ε^4 in terms of multiple polylogarithms. The variable ε is denoted by `eps`, $\zeta_2, \zeta_3, \zeta_4$ by

$$\text{zeta_2}, \text{zeta_3}, \text{zeta_4},$$

respectively. For the notation of multiple polylogarithms we give an example: $G(x'_7, x'_8, 1; x')$ is denoted by

$$\text{Glog}([\text{xp7}, \text{xp8}, 1], \text{xp}).$$

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