Production and backreaction of spin-2 particles of SU(2) gauge field during inflation

A. Maleknejad[a] **and E. Komatsu**[a,b]

[a] *Max-Planck-Institute for Astrophysics,*
Karl-Schwarzschild-Str. 1, 85748 Garching, Germany

[b] *Kavli Institute for the Physics and Mathematics of the Universe (Kavli IPMU, WPI),*
UTIAS, The University of Tokyo,
5-1-5 Kashiwanoha, Kashiwa, Chiba 277-8583, Japan

E-mail: amalek@MPA-Garching.MPG.DE, komatsu@MPA-Garching.MPG.DE

ABSTRACT: Primordial SU(2) gauge fields with an isotropic background lead to the production of spin-2 particles during inflation. We provide a unified formalism to compute this effect in all of the inflation models with isotropic SU(2) gauge fields such as Gauge-flation and Chromo-Natural inflation with and without spectator axion fields or the mass of the gauge field from the Higgs mechanism. First, we calculate the number and energy densities of the spin-2 particles. We then obtain exact analytical formulae for their backreaction on the background equations of motion of SU(2) and axion fields in (quasi) de Sitter expansion, which were calculated only numerically for one particular model in the literature. We show that the backreaction is directly related to the number density of the spin-2 field. Second, we relate the number density of the spin-2 particles to the power spectrum and the energy density of the gravitational waves sourced by them. Finally, we use the size of the backreaction to constrain the parameter space of the models. We find that the tensor-to-scalar ratio of the sourced gravitational waves can at most be on the order of that of the vacuum contribution to avoid a large backreaction on slow-roll dynamics of the gauge and axion fields in quasi-de Sitter expansion.

Contents

1 Introduction

Inflation [1–4] with SU(2) gauge fields [5–8] has a rich phenomenology that is not shared by canonical single-scalar-field inflation models (see [9] for a review). As was first discovered by one of the authors (A.M.), when the conformal symmetry of Yang-Mills theory is broken by an effective $(F\tilde{F})^2$ term in the Lagrangian, non-Abelian gauge fields acquire an isotropic and homogeneous background (vacuum expectation value; VEV) solution during inflation [5, 6]. This VEV produces a copious amount of spin-2 particles which, in turn, linearly mix with tensor perturbations in the metric, i.e., gravitational waves. The same

phenomenology is obtained when the conformal symmetry is broken by a Chern-Simons interaction with an axion field $\varphi F\tilde{F}$ [7, 8].

In addition to the original models,[1] there are several more inflationary models with the SU(2) VEV which share the above features [11–16]. Despite differences in details of the models, their tensor sector can be presented in a unified manner. As the sourced tensor power spectrum is proportional to the density parameter of the gauge field during inflation, these models violate the Lyth bound [17, 18]. Depending on the details of the slow-roll dynamics of the gauge field VEV, e.g., a form of the axion potential, the tensor spectral index n_T can be negative or positive, and thus violates the conventional consistency relation of single-field slow-roll inflation, $n_T = -r/8$. Moreover, parity-violating interactions in linear perturbations make this spin-2 field chiral and hence generate observable circularly polarized gravitational waves as well as parity-odd correlations of cosmic microwave background (CMB) anisotropies, i.e., non-zero TB and EB [19]. Due to self-interactions of gauge fields, the gravitational waves can be highly non-Gaussian, yielding a large tensor bispectrum with approximately an equilateral shape [20–22]. Finally, chiral gravitational waves can generate baryon asymmetry via a gravitational anomaly [13, 23–25], and can serve as a natural leptogenesis mechanism during inflation to explain the observed baryon asymmetry in the Universe.

All of these signatures are robust consequences of having gauge fields during inflation and carry important information about the matter content of the early universe. The stochastic background of gravitational waves can be within reach of future CMB experiments [26–28], and that of $n_T > 0$ can be within reach of future gravitational wave interferometers [13, 19, 29]. As none of these features exists in canonical single-scalar-field inflation models, we can use them to distinguish the particle physics models of inflation. For example, these models could be embedded in supergravity [30] and string theory [31].

In this paper, we take a closer look at the phenomenology of the spin-2 particles generated from SU(2) gauge fields. Particularly significant is the backreaction of spin-2 particles on the background equations of motion of the gauge and axion fields, as it could spoil significant properties of inflation with SU(2) gauge fields. We also gain better insights into the power spectrum and energy density of primordial gravitational waves by relating them to the number density of the spin-2 particles.

This paper is organized as follows. In section 2, we briefly review the existing models of inflation with an SU(2) gauge field in a unified approach. We study the spin-2 particle production in section 3. In section 4, we compute the backreaction of this spin-2 field on the background field equations. We then relate the power spectrum and energy density of the sourced gravitational waves to the number density of the spin-2 particles in section 5. In section 6, we use the size of the backreaction to constrain the parameter space of the models. Finally, we conclude in 7. In appendix A, we discuss the symmetry structure of the background SU(2) gauge field. In appendix B, first, we discuss the action of the transverse-traceless perturbed SU(2) field around its VEV in a unified approach. In B.1, we prove that

---

[1]The original models of gauge-flation and chromo-natural inflation have been ruled out by the Planck data [8, 10].

a perturbed SU(2) field around its VEV has a spin-2 field. The details of our analytical study as well as some necessary mathematical tools are presented in appendices C–E.

2 Review of theory

Consider an inflationary model with a Friedmann-Lemaître-Robertson-Walker (FRLW) background

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \tag{2.1}$$

that can support slow-roll inflation with a slowly varying SU(2) gauge field VEV given by [5, 6]

$$\bar{A}_\mu(t) \equiv \bar{A}_\mu^a(t)T_a = \begin{cases} 0 & \mu = 0 \\ a\psi(t)\delta_i^a \ T_a & \mu = i, \end{cases} \tag{2.2}$$

where $\{T_a\}$ are the generators of the $su(2)$ algebra with $a = 1, 2, 3$

$$T_a T_b = \frac{1}{4}\delta_{ab}I_n + \frac{1}{2}i\epsilon^{abc}T_c, \tag{2.3}$$

where $I_n\delta_{ab}$ is the identity matrix and ϵ^{abc} is the totally antisymmetric matrix. In appendix A, we show that ansatz (2.2) is the general background solution for a gauge field with an isotropic and homogeneous energy-momentum tensor. We unify all the inflation models with an SU(2) field in the literature in the following Lagrangian

$$S = \int d^4x\sqrt{-g}\left[\mathcal{L}_A(A_\mu, \varphi) + \alpha_s\mathcal{L}_0(\chi) + \alpha_H\mathcal{L}_H(A_\mu, H)\right], \tag{2.4}$$

where $\mathcal{L}_A$ is the gauge field theory sector (with possibly an axion field φ), $\mathcal{L}_0$ is a (possible) scalar theory and $\mathcal{L}_H$ is a (possible) Higgs sector which makes the gauge field massive. The parameters α_s and α_H classify models as (see table 1)

$$\alpha_s = \begin{cases} 0 & \text{(axion)-SU(2) gauge field inflaton,} \\ 1 & \text{spectator (axion)-SU(2) gauge field,} \end{cases} \quad \text{and} \quad \alpha_H = \begin{cases} 0 & \text{massless models,} \\ 1 & \text{Higgsed models.} \end{cases}$$

In these models, the conformal symmetry of Yang-Mills theory is broken by either adding a $(F\tilde{F})^2$ effective term to the gauge theory, e.g. Gauge-flation [5, 6], or by coupling the gauge field sector to an axion field φ with slow-roll dynamics, e.g. chromo-natural [7, 8]. The former models are given by [5, 6]

$$\mathcal{L}_A \to \mathcal{L}_{Gf} \equiv -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{96}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2, \tag{2.5}$$

while the latter are given by[2] [7, 8]

$$\mathcal{L}_A \to \mathcal{L}_{Cn} \equiv -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda\varphi}{4f}F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi), \tag{2.6}$$

---

[2]A more general action including two dimension six operators, $\text{tr}(FFF)$ and the (PT violating) Weinberg operator $\text{tr}(FF\tilde{F})$ [32], has been considered in [23].

Model	α_s	α_H	$\mathcal{L}_A$	Original references
	0	0	$\mathcal{L}_{Gf}$	[5, 6]
			$\mathcal{L}_{Cn}$	[7, 8]
Spectator	1	0	$\mathcal{L}_{Gf}$	–
			$\mathcal{L}_{Cn}$	[15]
Higgsed	0	1	$\mathcal{L}_{Gf}$	[12, 16]
			$\mathcal{L}_{Cn}$	[14]
Spectator-Higgsed	1	1	$\mathcal{L}_{Gf}$	–
			$\mathcal{L}_{Cn}$	–

Table 1. Inflationary models involving an SU(2) gauge field in the literature and their relation to $\mathcal{L}_A$, α_s and α_H.

where $F_{\mu\nu} = T_a F^a_{\mu\nu}$ is the field strength tensor

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_A \epsilon^{abc} A^b_\mu A^c_\nu,$$

while $\tilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\lambda\sigma} F_{\lambda\sigma}$, φ is the axion and $V(\varphi)$ is the axion potential.[3] Moreover, in the Higgsed version of the models, the gauge field becomes massive by a Higgs field and we have an extra term for the dynamics of the Goldstone boson which in the Stueckelberg form is [33, 34][4]

$$\mathcal{L}_H = -g_A^2 Z_0^2 \mathrm{tr}\left(A_\mu - \frac{i}{g_A} U^{-1} \partial_\mu U \right)^2, \tag{2.8}$$

where $U = \exp(ig_A\pi)$ and π is the Goldstone mode corresponding to the Higgs fluctuations around its VEV, $\pi = \pi^a T_a$. We relate these models to the literature in table 1.

The $(F\tilde{F})^2$ term in (2.5) comes as an effective theory of (2.6) by integrating out the massive axion on energy scales below the mass of the axion, $M \simeq \mu^2/f$. In that case, the parameter κ is given as $\kappa = \frac{3\lambda^2}{\mu^4}$ [9, 35, 36]. Therefore, gauge-flation models are effectively equivalent to chromo-natural models in the limit that the axion is very massive, and they have the same tensor and vector perturbations [9]. (See also appendix B.)

These models can be specified in terms of three dimensionless parameters, ξ_A, ξ and ξ_{Z_0}, which are defined in the following. First, the almost constant gauge field configuration

---

[3]In the original chromo-natural model, the potential is the standard cosine potential, $V(\varphi) = \mu^4(1 + \cos(\frac{\varphi}{f}))$, with $f \ll M_{\mathrm{Pl}}$ and $\lambda \gtrsim 10^3 \frac{f}{M_{\mathrm{Pl}}}$.

[4]Here, the full Higgs field theory is

$$\mathcal{L}_Z = -\frac{1}{2} D_\mu Z D^\mu Z^\dagger - V(z), \tag{2.7}$$

with a VEV given as $\bar{Z}_A = Z_0(t)\delta^a_A T_a$ in which $A = 1, 2, 3$ is the field's internal index, and $D_\mu = \partial_\mu - ig_A A_\mu$ is the covariant derivative. However, we are interested in the limit that the Higgs mass is much greater the the Hubble scale. Therefore, the only relevant sector is the Goldstone boson part given in (2.8). Notice that $\mathcal{L}_H$ is gauge invariant and it can be written as $\mathcal{L}_H = -g_A^2 Z_0^2 \mathrm{tr}(U D_\mu U^{-1}/(-ig_A))^2$.

of the form (2.2) leads to a slowly-varying dimensionless parameter

$$\xi_{\mathrm{A}} \equiv \frac{g_{\mathrm{A}}\psi}{H}. \tag{2.9}$$

Validity of perturbation theory in the scalar sector of the SU(2) gauge field requires [8, 10, 17]

$$\xi_{\mathrm{A}} > \sqrt{2}. \tag{2.10}$$

A scalar mode in these models would have a negative frequency at $\frac{k}{a} \propto (2 - \xi_{\mathrm{A}}^2)$ which is unstable for $\xi_{\mathrm{A}} < \sqrt{2}$. In refs. [15, 20, 21], ξ_{A} has been called m_Q. The second dimensionless parameter is

$$\xi \equiv \frac{\lambda\dot{\varphi}}{2fH}. \tag{2.11}$$

In the Higgsed version of the models ($\alpha_H = 1$), we also have

$$\xi_{Z_0} \equiv \frac{g_{\mathrm{A}}Z_0}{H}. \tag{2.12}$$

Another important quantity in this setup is

$$\epsilon_A \equiv 2\, \frac{\bar{\rho}_{\mathrm{YM}}}{\bar{\rho}} = (1 + \xi_{\mathrm{A}}^2)\left(\frac{\psi}{M_{\mathrm{Pl}}}\right)^2, \tag{2.13}$$

which is the contribution of the gauge field to the total slow-roll parameter and equals twice the ratio of the energy density of the gauge field background $\bar{\rho}_{\mathrm{YM}}$ to the total energy density $\bar{\rho}$. In refs. [15, 20, 21], $\epsilon_B = \epsilon_A \xi_{\mathrm{A}}^2/(1 + \xi_{\mathrm{A}}^2)$ is used instead of ϵ_A.

The background field equation of the gauge field is given by the $\mu = i$ component of the following equation[5]

$$\bar{D}_\nu\left(\bar{F}^{\mu\nu} + \bar{\alpha}_A\epsilon^{\mu\nu\lambda\sigma}\bar{F}_{\lambda\sigma}\right) + \alpha_H g_{\mathrm{A}}^2 Z_0^2 \bar{A}^\mu = 0, \tag{2.15}$$

where a bar denotes a background quantity and D_ν is the covariant derivative

$$D_\mu \equiv \nabla_\mu - ig_{\mathrm{A}}A_\mu \quad (\bar{D}_\mu = \bar{\nabla}_\mu - ig_{\mathrm{A}}\bar{A}_\mu),$$

and $\bar{\alpha}_A$ is a function of the background fields which, depending on the form of $\mathcal{L}_A$, is given as[6]

$$\bar{\alpha}_A = \begin{cases} -\frac{2\kappa}{96}\overline{F\tilde{F}} & \text{for} \quad \mathcal{L}_A = \mathcal{L}_{Gf}, \\ \frac{\lambda}{2f}\bar{\varphi} & \text{for} \quad \mathcal{L}_A = \mathcal{L}_{Cn}. \end{cases} \tag{2.16}$$

Note that the zeroth component of (2.15) is a constraint equation which is equivalent to zero for our background ansatz.

---

[5]The explicit form of the background field equation of $\bar{A}_\mu$ is

$$\delta_i^a\left(\frac{(a\psi)\ddot{}}{a} + \frac{H(a\psi)\dot{}}{a} + 2g_{\mathrm{A}}^2\psi^3 + \alpha_H g_{\mathrm{A}}^2 Z_0^2\psi - 2\dot{\bar{\alpha}}_A g_{\mathrm{A}}\psi^2\right) = 0. \tag{2.14}$$

[6]In (2.16), we have $\overline{F\tilde{F}} = 12g_{\mathrm{A}}(H\psi + \dot{\psi})\psi^2$.

Assuming slow-roll dynamics in the background, equation (2.15) relates ξ_A, ξ_{Z_0}, and $\dot{\bar{\alpha}}_A/H$ as

$$\frac{\dot{\bar{\alpha}}_A}{H} \simeq \frac{(1 + \xi_A^2 + \frac{\alpha_H}{2}\xi_{Z_0}^2)}{\xi_A}. \tag{2.17}$$

In the $\mathcal{L}_{Cn}$ models, the background field equation of the axion is

$$\ddot{\varphi} + 3H\dot{\varphi} + V_\varphi + \frac{3\lambda g_A}{f}\psi^2(\dot{\psi} + H\psi) = 0. \tag{2.18}$$

2.1 Tensor perturbations

The existence of a spin-2 degree of freedom in the gauge field is a unique feature of the SU(2) inflation models. This is the primary focus of our work. Once we have a slow-roll background dynamics, the tensor perturbations in this family of models are entirely determined by the background quantities ξ, ξ_A, ξ_{Z_0}, and by the perturbed gauge field sector of the model, $\mathcal{L}_A + \alpha_H \mathcal{L}_H$. The vector and tensor perturbations in $\mathcal{L}_{Gf}$ and $\mathcal{L}_{Cn}$ are the same. Let us first briefly review the spin-2 part of the perturbed SU(2) gauge field. More details are presented in appendix B. See [5, 6] for the full decomposition of the field into the scalar, vector, and tensor perturbations.

Once we perturb the metric and the SU(2) gauge field around their homogeneous and isotropic solutions (2.2), we have the following spin-2 fluctuations

$$\delta_T g_{ij}(t, \vec{x}) = a^2 \gamma_{ij}(t, \vec{x}), \tag{2.19}$$

$$\delta_T A_i^a(t, \vec{x}) = M_{\text{Pl}}\, \delta^{aj} B_{ij}(t, \vec{x}), \tag{2.20}$$

where δ_T denotes the spin-2 subsector of the perturbed field. In appendix B.1, we prove that $\tilde{\gamma}_{ij} = B_{ij}/a$ is a (pseudo) spin-2 field. Nonetheless, throughout this paper, we shall call B_{ij} a spin-2 field. In Fourier space, the vacuum (free) γ_{ij} and B_{ij} can be expanded as

$$aM_{\text{Pl}}\gamma_{ij}(t, \vec{x}) = \sqrt{2} \sum_{\sigma=\pm 2} \int d^3k e^{i\vec{k}.\vec{x}} e_{ij}(\sigma, \hat{k}) \left[\hat{a}_\sigma(\vec{k})h_\sigma(\vec{k}) + \hat{a}_\sigma^\dagger(-\vec{k})h_\sigma^*(-\vec{k})\right], \tag{2.21}$$

$$M_{\text{Pl}}B_{ij}(t, \vec{x}) = \frac{1}{\sqrt{2}} \sum_{\sigma=\pm 2} \int d^3k e^{i\vec{k}.\vec{x}} e_{ij}(\sigma, \hat{k}) \left[\hat{b}_\sigma(\vec{k})B_\sigma(\vec{k}) + \hat{b}_\sigma^\dagger(-\vec{k})B_\sigma^*(-\vec{k})\right], \tag{2.22}$$

where h_σ and B_σ are the canonically normalized fields, $e_{ij}(\pm, \hat{k})$ are the polarization tensors associated with the ± 2 helicity states,[7] which are normalized as $e_{ij}(\sigma, \vec{k})e_{ij}^*(\sigma', \vec{k}) = 2\delta_{\sigma\sigma'}$, and $\hat{a}_\sigma(\vec{k})$ and $\hat{b}_\sigma(\vec{k})$ are the annihilation operators of the spin-2 modes of the metric and gauge field, respectively, satisfying

$$[\hat{a}_\sigma(\vec{k}), \hat{a}_{\sigma'}^\dagger(\vec{k}')] = [\hat{b}_\sigma(\vec{k}), \hat{b}_{\sigma'}^\dagger(\vec{k}')] = \delta^3(\vec{k} - \vec{k}')\delta_{\sigma\sigma'}.$$

---

[7]The polarization tensor of the spin-2 field in the direction $\hat{k} = -\hat{r}$ is given as

$$e_{ij}(\pm 2, \hat{k}) = \sqrt{2}e_i(\pm 1, \hat{k})e_j(\pm 1, \hat{k}) \quad \text{where} \quad \vec{e}(\pm 1, -\hat{r}) = \frac{1}{\sqrt{2}}(\hat{\theta} \mp i\hat{\phi}), \tag{2.23}$$

where $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$ are the local orthogonal unit vectors in the directions of increasing r, θ, and ϕ. Note that $e_{ij}(\sigma, \vec{k}) = e_{ij}^*(\sigma, -\vec{k})$, and $\vec{k} \times \vec{e}(\pm 1, \vec{k}) = \mp ik\vec{e}(\sigma, \vec{k})$.

	$\mathcal{L}_A$	references	$\frac{1}{2}\frac{m^2}{H^2}\left(\frac{\psi}{M_{\rm Pl}}\right)^{-2}$	$\beta_{\rm c}$	$\theta_{\rm c}$	$\delta_{\rm c}$	$\frac{\tilde{m}^2}{H^2}$
$\alpha_H = 0$	$\mathcal{L}_{Gf}$	[5, 6]	$(\xi_{\rm A}^2 - 1)$	$\xi_{\rm A}$	$\xi_{\rm A}^2$	$\frac{2(1+2\xi_{\rm A}^2)}{\xi_{\rm A}}$	$2(2+\xi_{\rm A}^2)$
	$\mathcal{L}_{Cn}$	[7, 8, 11, 13, 15]	$(\xi_{\rm A}^2 - 1)$	$\xi_{\rm A}$	$\xi_{\rm A}^2$	$2(\xi_{\rm A}+\xi)$	$2(1+\xi\xi_{\rm A})$
$\alpha_H = 1$	$\mathcal{L}_{Gf}$	[12, 16]	$(\xi_{\rm A}^2 - 1 + \xi_{Z_0}^2)$	$\xi_{\rm A}$	$\xi_{\rm A}^2 + \xi_{Z_0}^2$	$\frac{2(1+2\xi_{\rm A}^2)+\xi_{Z_0}^2}{\xi_{\rm A}}$	$2(2+\xi_{\rm A}^2+\xi_{Z_0}^2)$
	$\mathcal{L}_{Cn}$	[14]	$(\xi_{\rm A}^2 - 1 + \xi_{Z_0}^2)$	$\xi_{\rm A}$	$\xi_{\rm A}^2 + \xi_{Z_0}^2$	$2(\xi_{\rm A}+\xi)$	$2(1+\xi\xi_{\rm A})+\xi_{Z_0}^2$
Unified			$(\xi_{\rm A}^2 - 1 + \alpha_H\xi_{Z_0}^2)$	$\xi_{\rm A}$	$\xi_{\rm A}^2 + \alpha_H\xi_{Z_0}^2$	$2(\xi_{\rm A}+\frac{\dot\alpha_A}{H})$	$2(1+\xi_{\rm A}\frac{\dot\alpha_A}{H})+\alpha_H\xi_{Z_0}^2$

Table 2. Definition of parameters in the equations of tensor perturbations in (2.24)–(2.25) in terms of $\xi_{\rm A}$, ξ and ξ_{Z_0} for each type of models. The last row shows the parameters in the unified form for the generic action given in (2.4).

The tensor perturbations obey the following equations of motion

$$h_\sigma'' + \left[k^2 - \frac{a''}{a} + \frac{m^2}{H^2}\mathcal{H}^2\right]h_\sigma = \frac{2\psi}{M_{\rm Pl}}\mathcal{H}\left[(-\lambda_\sigma\beta_{\rm c}k + \theta_{\rm c}\mathcal{H})B_\sigma - B_\sigma'\right], \quad (2.24)$$

$$B_\sigma'' + \left[k^2 - \lambda_\sigma\delta_{\rm c}k\mathcal{H} - \frac{a''}{a} + \frac{\tilde{m}^2}{H^2}\mathcal{H}^2\right]B_\sigma = \mathcal{O}\left(\frac{\psi}{M_{\rm Pl}}h_\sigma\right), \quad (2.25)$$

where $\lambda_\pm = \pm 1$. See [5, 8, 14] for the expression on the right hand side of (2.25) which we ignore here.[8] The primes denote a derivative with respect to conformal time, τ, while $\frac{m^2}{H^2}$, $\frac{\tilde{m}^2}{H^2}$, $\beta_{\rm c}$, $\theta_{\rm c}$ and $\delta_{\rm c}$ are dimensionless slowly varying parameters defined in table 2 for each model. The field equation (2.25) can be written as a Whittaker equation as

$$\partial_z^2 B_\sigma + \left[-\frac{1}{4} + \frac{\kappa_\sigma}{z} + \frac{1}{z^2}\left(\frac{1}{4} - \mu^2\right)\right]B_\sigma = 0, \quad (2.26)$$

where $z = 2ik\tau$ and we used the slow-roll relation $aH \simeq -\frac{1}{\tau}$. The parameters κ_σ and μ are given as

$$\kappa_\sigma = -\frac{i\lambda_\sigma\delta_{\rm c}}{2} \quad \text{and} \quad \mu^2 = \frac{9}{4} - \frac{\tilde{m}^2}{H^2}. \quad (2.27)$$

Since $|\kappa_+| = |\kappa_-|$, we write

$$|\kappa| \equiv |\kappa_\sigma| = \frac{1}{2}\delta_c.$$

General solutions are given by linear combinations of the Whittaker functions $W_{\kappa_\lambda,\mu}(z)$, $M_{\kappa_\lambda,\mu}(z)$. Imposing the Bunch-Davies vacuum condition in the asymptotic past, we have

---

[8]The neglected term in r.h.s. of (2.25) is proportional to $\frac{\psi}{M_{\rm Pl}} \ll 1$ and therefore is subleading inside the horizon. However, after the horizon crossing when the homogeneous solution of B_σ decays due to its mass, this term acts like a small source term for B_σ. See for instance [18]. However, this effect makes a negligible correction to the sourced gravitational waves and the backreaction.

(see (C.12))

$$B_\sigma(\tau, \vec{k}) = \frac{e^{i\kappa_\sigma \pi/2}}{(2\pi)^{\frac{3}{2}} \sqrt{2k}} W_{\kappa_\sigma,\mu}(2ik\tau) \,. \tag{2.28}$$

Using the above in the field equation of h_σ (2.24), we find the sourced part of the gravitational waves.

Here, we summarize the main features of the spin-2 field with the field equation of (2.25) and the quadratic action of (B.6).

- $B_\pm$ evolves as a massive field in de Sitter space with a parity breaking linear derivative interaction term, $\mp \delta_c k \mathcal{H} B_\pm$, with δ_c given by table 2. In terms of $\bar\alpha_A$ (2.16), we can write it in a unified form, $\delta_c = 2(\xi_A + \frac{\dot{\bar\alpha}_A}{H})$.

- The first term in δ_c ($2\xi_A$) comes from the interaction of B_σ with the VEV of the gauge field through the covariant derivative D_μ, and is due to the self-interactions of gauge field in Yang-Mills theory.

- The second contribution in δ_c ($2\frac{\dot{\bar\alpha}_A}{H}$) is a time derivative of $\bar\alpha_A$. As shown in (2.16), for $\mathcal{L}_A = \mathcal{L}_{Gf}$, this parameter is due to the VEV of $F\tilde{F}$ while for $\mathcal{L}_A = \mathcal{L}_{Cn}$, it is due to the derivative interaction with the VEV of the axion field.

- The sound speeds of $B_\pm$ field and GWs are unity in all of the models in this family.[9]

- Due to the self-interactions of the gauge field, the B_σ is massive with a mass term $\frac{\tilde{m}^2}{H^2}$, given in table 2. The mass of the spin-2 field can be written in the unified form $\frac{\tilde{m}^2}{H^2} = 2(1 + \frac{\dot{\bar\alpha}_A}{H}\xi_A) + \alpha_H \xi_{Z_0}^2$.

- In a similar Abelian field case (see (B.5)), the first derivative interaction as well as the mass term are missing. Thus, the non-Abelian nature of the gauge field makes i) a more efficient particle production, while making ii) the transverse field massive and therefore decaying after horizon crossing.

- As we will see in section 3, these derivative interactions are responsible for production of the spin-2 particle by the background fields.

- In SU(2) gauge field setups, the right hand side of the field equation of the gravitational waves in (2.24) is non-zero and is given by an anisotropic inertia proportional to $\frac{\psi}{M_{\text{Pl}}}$. Therefore, the efficiency of the mixing between the spin-2 field and the gravitational waves is specified by the VEV of the SU(2) gauge field.

- This anisotropic inertia is parametrized in terms of θ_c, β_c, and a small mass term for the graviton, $\frac{m^2}{H^2}$, given in table 2.

- The term β_c is the coefficient of a linear derivative interaction which is equal to ξ_A regardless of the model. The other parameter can be written as $\theta_c = \xi_A^2 + \alpha_H \xi_{Z_0}^2$. The mass term is $\frac{m^2}{H^2} = 2(\psi/M_{\text{Pl}})^2 (\xi_A^2 - 1 + \alpha_H \xi_{Z_0}^2)$.

---

[9]This is also valid in the presence of dimension six operators, $\text{tr}(FFF)$ and the (PT violating) Weinberg operator $\text{tr}(FF\tilde{F})$ [23].

- All the interaction and parameters in the tensor perturbation sector are specified only by $\mathcal{L}_A$ and $\mathcal{L}_H$, and therefore independent of whether the gauge field sector is a spectator or not.

In this work, we assume (quasi) de Sitter expansion and keep terms up to first order in slow-roll. The slow-roll time evolution of ξ_A and ξ_{Z_0}, which is model dependent, contributes to the spectral tilt of the sourced gravitational waves. Depending on the details of the evolution of the gauge field VEV, the spectral tilt of the sourced gravitational waves can be positive or negative [37]. Since we are interested in the number density and the backreaction of the B_σ particle as well as in the amplitude of the sourced gravitational waves which are model independent, we neglect this effect in this paper.

3 Spin-2 Schwinger-type particle production

In this section, we study the spin-2 particle production due to their interactions with the VEV of the background fields. The background fields act as a classical source for the quantum fluctuations analogous to the well-known Schwinger effect [38]. However, unlike the standard Schwinger process in which the quantum field is sourced only by a background gauge field, here the spin-2 quantum field is sourced by both the backgrounds of axion and gauge fields. The derivation given in this section follows closely section 3.2 of [39].

To have a better qualitative understanding of the particle production process, let us write the field equation of Fourier modes using the (normalized) physical momentum

$$\tilde{\tau} = \frac{k}{aH},$$

as

$$\partial_{\tilde{\tau}}^2 B_\sigma(\tau, \vec{k}) + \omega_\sigma^2(\tilde{\tau}) B_\sigma(\tau, \vec{k}) = 0, \tag{3.1}$$

where $\omega_\sigma(\tilde{\tau})$ is the (time-dependent) effective frequency of the modes

$$\omega_\pm^2(\tilde{\tau}) = 1 \mp \frac{\delta_{\rm c}}{\tilde{\tau}} + \left(-2 + \frac{\tilde{m}^2}{H^2} \right) \frac{1}{\tilde{\tau}^2}. \tag{3.2}$$

In the limits that the effective frequency is slowly varying and

$$\left(\frac{\partial_{\tilde{\tau}} \omega_\sigma(\tilde{\tau})}{\omega_\sigma^2(\tilde{\tau})} \right)^2 \ll 1 \quad \text{and} \quad \left| \frac{\partial_{\tilde{\tau}}^2 \omega_\sigma(\tilde{\tau})}{\omega_\sigma^3(\tilde{\tau})} \right| \ll 1, \tag{3.3}$$

the particle production is zero and the modes are in an adiabatic vacuum state. Then the solution can be well-approximated by the WKB form,

$$B_{\sigma 1,2}^{\rm WKB}(\tau, \vec{k}) = \frac{1}{(2\pi)^{\frac{3}{2}} \sqrt{2k\omega_\sigma(\tilde{\tau})}} \exp\left(\pm i \int \omega_\sigma(\tilde{\tau}) d\tilde{\tau} \right), \tag{3.4}$$

in which $B_{\sigma 1}^{\rm WKB}$ and $B_{\sigma 2}^{\rm WKB}$ are the positive and negative frequency modes respectively. The WKB approximation is the exact solution of

$$\partial_{\tilde{\tau}}^2 B_\sigma + \omega_\sigma^2(\tilde{\tau}) \left(1 - \Omega_\sigma(\tilde{\tau}) \right) B_\sigma = 0, \tag{3.5}$$
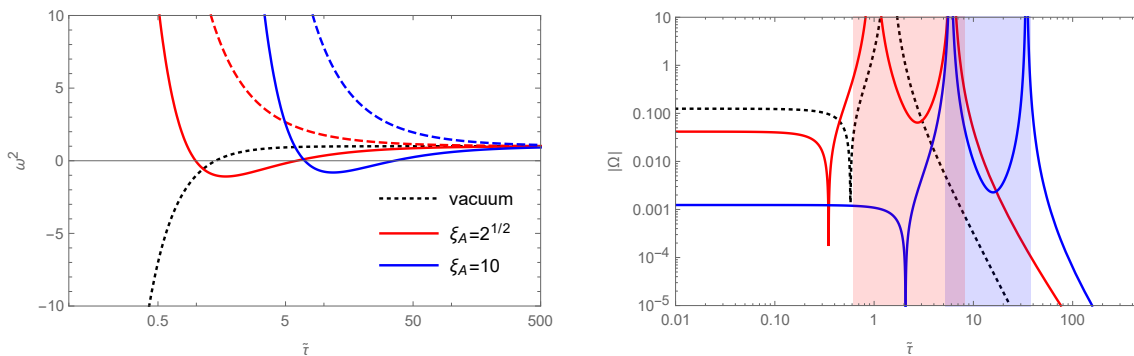
Figure 1. Effective frequency squared (ω^2) and deviation from adiabaticity ($|\Omega|$). The left panel shows ω^2 (3.2) as a function of $\tilde{\tau}$ while the right panel shows $|\Omega_\pm(\tilde{\tau})|$ (3.6) for massless systems ($\alpha_H = 0$) with $\xi_A = \sqrt{2}$ (red line) and 10 (blue line). For comparison, the dotted black line shows the vacuum gravitational waves. Here, the solid red and blue lines show the plus polarization modes while the dashed blue and red lines show the minus polarization. The shaded areas in the right panel show the particle production regime.
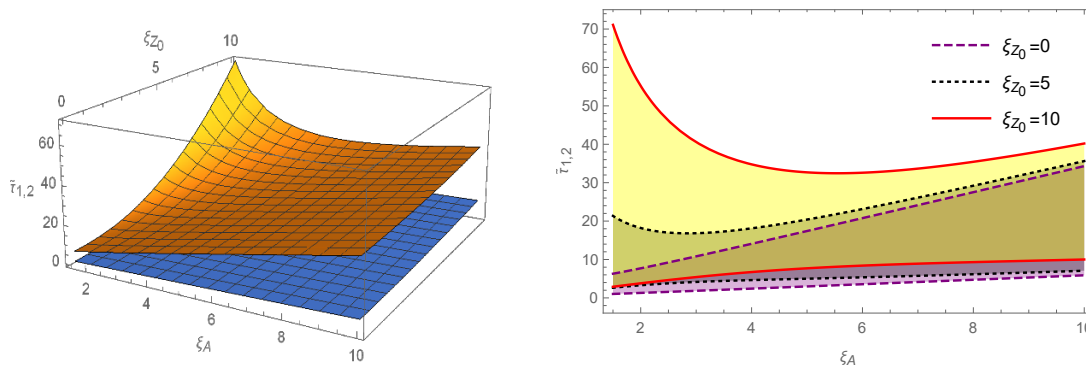
Figure 2. (left) $\tilde{\tau}_{1,2}$ interval for which B_σ has instability as a function of ξ_A and ξ_{Z_0}. (Right) three slices of the left panel. We show $\tilde{\tau}_{1,2}$ as a function of ξ_A for $\xi_{Z_0} = 0$ in (dashed) purple, $\xi_{Z_0} = 5$ in (dotted) black, and $\xi_{Z_0} = 10$ in (solid) red.

where $\Omega(\tilde{\tau})$ is defined as

$$\Omega_\sigma(\tilde{\tau}) \equiv \frac{3}{4}\left(\frac{\partial_{\tilde{\tau}}\omega_\sigma(\tilde{\tau})}{\omega_\sigma^2(\tilde{\tau})}\right)^2 - \frac{1}{2}\frac{\partial_{\tilde{\tau}}^2\omega_\sigma(\tilde{\tau})}{\omega_\sigma^3(\tilde{\tau})}, \tag{3.6}$$

which quantifies the deviation of our mode function from the exact adiabatic solution (see figure 1). We find that Ω_- is always very small and thus $B_-(\tau, \vec{k})$ remains adiabatic. However, Ω_+ becomes large around the roots of ω_+

$$\tilde{\tau}_{1,2} = \frac{1}{2}\left(\delta_c \pm \sqrt{8 + \delta_c^2 - 4\frac{\tilde{m}^2}{H^2}}\right) \simeq \left(|\kappa| \pm \sqrt{|\kappa|^2 - |\mu|^2}\right), \tag{3.7}$$

and the system experiences a large deviation from adiabaticity. The above roots are presented in figure 2.

When the adiabatic conditions hold, i.e., $|\Omega| \ll 1$, we have a well-defined adiabatic vacuum, and the field excitation about it describes particles. Deviations from adiabaticity in the asymptotic past and future are

$$\Omega_\pm(\tilde\tau) \to \begin{cases} \frac{1}{2\tilde\tau^3}\left(\mp \delta_c + \frac{3}{\tilde\tau}(\frac{7}{8}\delta_c^2 + 2 - \frac{\tilde m^2}{H^2}) \right) \simeq 0, & \text{if } \tilde\tau \to \infty \\ \frac{1}{8(1-\frac{\tilde m^2}{H^2})}, & \text{if } \tilde\tau \to 0. \end{cases} \tag{3.8}$$

In the asymptotic past, $\tilde\tau \to \infty$, the adiabaticity conditions are satisfied and the WKB solution (3.4) is the Bunch-Davies vacuum (2.28). At later times the adiabaticity conditions are violated and particles are produced. In the asymptotic future, recalling that $\xi_A > \sqrt{2}$ and using table 2, we have $\lim_{\tilde\tau\to 0}|\Omega_+(\tilde\tau)| \lesssim 10^{-2}$. Thus, the positive frequency modes (vacuum mode functions) in the asymptotic future, $v_\sigma(\tau, \vec k)$, are given by the WKB solution in (3.4), as

$$v_\sigma(\tau, \vec k) = \lim_{-k\tau \to 0} B_{\sigma 1}^{\text{WKB}}(\tau, \vec k) \simeq \frac{\sqrt{\tilde\tau}}{(2\pi)^{\frac{3}{2}}\sqrt{2k|\mu|}} e^{i|\mu|\ln\tilde\tau}, \tag{3.9}$$

in which we used $\mu = i|\mu|$ and $\lim_{\tilde\tau\to 0}\omega(\tilde\tau) \simeq |\mu|/\tilde\tau$. Using (C.12), the asymptotic future vacuum mode functions can be well approximated by the M-Whittaker functions as

$$v_\sigma(\tau, \vec k) = \frac{e^{i\mu\pi/2}}{2(2\pi)^{\frac{3}{2}}\sqrt{k|\mu|}} M_{\kappa_\sigma,\mu}(-2i\tilde\tau). \tag{3.10}$$

Using (C.14), the asymptotic past and future vacuum modes are related as

$$v_\sigma(\tau, \vec k) = e^{i(\kappa_\sigma-\mu)\frac{\pi}{2}}\frac{\sqrt{2|\mu|}\Gamma(2\mu)}{\Gamma(\frac{1}{2}+\mu+\kappa_\sigma)}B_\sigma(\tau, \vec k) + ie^{i(\kappa_\sigma+\mu)\frac{\pi}{2}}\frac{\sqrt{2|\mu|}\Gamma(2\mu)}{\Gamma(\frac{1}{2}+\mu-\kappa_\sigma)}B_\sigma^*(\tau, -\vec k). \tag{3.11}$$

Therefore, the spin-2 field can be either expanded in terms of the positive frequency modes in the asymptotic past (2.28) as in (2.22), or in terms of the positive frequency modes in the asymptotic future (3.10), as

$$M_{\text{Pl}}B_{ij}(\tau, \vec x) = \frac{1}{\sqrt{2}}\sum_{\sigma=\pm 2}\int d^3k\, e^{i\vec k.\vec x}e_{ij}(\sigma, \hat k)\left[\tilde b_\sigma(\vec k)v_\sigma(\vec k, \tau) + \tilde b_\sigma^\dagger(-\vec k)v_\sigma^*(-\vec k, \tau)\right], \tag{3.12}$$

where $\tilde b_\sigma(\vec k)$ and $\tilde b_\sigma^\dagger(\vec k)$ are the annihilation and creation operations of a particle with respect to the asymptotic future vacuum respectively. By definition, we have

$$b_\sigma(\vec k)|0_{\text{in}}\rangle = 0 \quad \text{and} \quad \tilde b_\sigma(\vec k)|0_{\text{out}}\rangle = 0, \tag{3.13}$$

where $|0_{\text{in}}\rangle$ and $|0_{\text{out}}\rangle$ are the vacuum states in the asymptotic past and future of the (quasi) de Sitter spacetime, respectively.

Using Bogoliubov transformation, we can write $\tilde b_\sigma(\vec k)$ in terms of $b_\sigma(\vec k)$ and $b_\sigma^\dagger(\vec k)$ as

$$\tilde b_\sigma(\vec k) = \alpha_{\sigma,\vec k}b_\sigma(\vec k) + \beta_{\sigma,\vec k}^*b_\sigma^\dagger(-\vec k), \tag{3.14}$$

where $\alpha_{\sigma,\vec k}$ and $\beta_{\sigma,\vec k}$ are Bogoliubov coefficients which satisfy the normalization condition

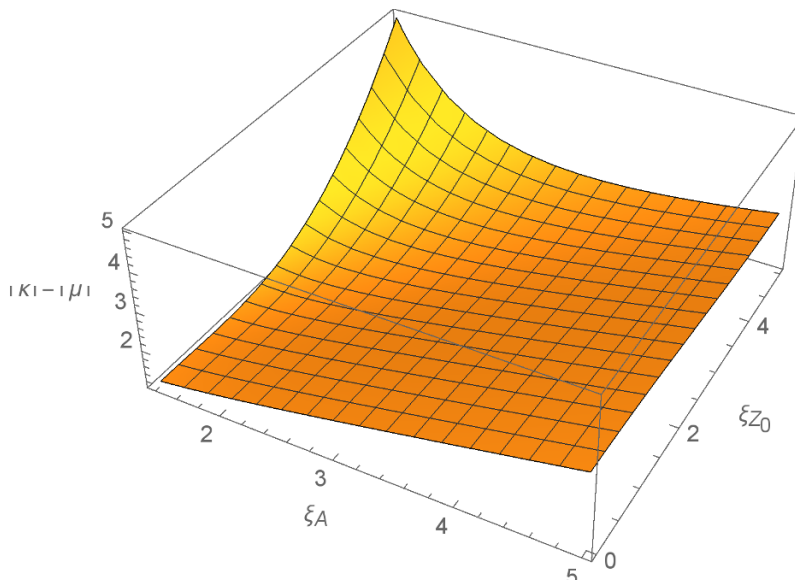$$|\alpha_{\sigma,\vec k}|^2 - |\beta_{\sigma,\vec k}|^2 = 1. \tag{3.15}$$

Figure 3. Efficiency of the particle production, $|\kappa| - |\mu|$, as a function of ξ_A and ξ_{Z_0}.

From the combination of eqs. (2.22), (3.11) and (3.12), we find

$$\alpha_{\sigma,\vec{k}} = \sqrt{2|\mu|}e^{(\lambda_\sigma|\kappa|+|\mu|)\pi/2}\frac{\Gamma(-2\mu)}{\Gamma(\frac{1}{2}-\mu-\kappa_\sigma)}, \tag{3.16}$$

$$\beta_{\sigma,\vec{k}} = -i\sqrt{2|\mu|}e^{(\lambda_\sigma|\kappa|-|\mu|)\pi/2}\frac{\Gamma(2\mu)}{\Gamma(\frac{1}{2}+\mu-\kappa_\sigma)}. \tag{3.17}$$

Having the $\beta_{\sigma,\vec{k}}$ coefficients, we are ready to determine the particle number density as well as the vacuum-vacuum transition amplitude. The efficiency of the particle production for $\sigma = +2$ is given by the exponent $|\kappa| - |\mu|$, which, recalling (2.10) and using (2.27), can be approximated as

$$|\kappa| - |\mu| \approx \frac{\xi_A}{2}\left[1 + \left(\sqrt{1 + \alpha_H(\frac{\xi_{Z_0}}{\xi_A})^2} - \sqrt{2}\right)^2\right] > \frac{\xi_A}{2}. \tag{3.18}$$

This quantity is presented in figure 3 as a function of ξ_A and ξ_{Z_0} and has the following asymptotic forms

$$|\kappa| - |\mu| \approx \begin{cases} (2 - \sqrt{2})\xi_A & \text{if } \frac{\xi_{Z_0}}{\xi_A} \ll 1 \\ \frac{1}{2}\alpha_H\frac{\xi_{Z_0}^2}{\xi_A} & \text{if } \frac{\xi_{Z_0}}{\xi_A} \gg 1. \end{cases} \tag{3.19}$$

The number density of the created particles with a given comoving momentum, $\vec{k}$, in the asymptotic future is

$$n_\sigma(\vec{k}) = \langle 0_{\text{in}}|\tilde{b}_{\sigma,\vec{k}}^\dagger\tilde{b}_{\sigma,\vec{k}}|0_{\text{in}}\rangle = |\beta_\sigma|^2 = \frac{e^{2\lambda_\sigma|\kappa|\pi} + e^{-2|\mu|\pi}}{2\sinh(2|\mu|\pi)}, \tag{3.20}$$

which has a k-independent spectrum for each polarization state. As we see, there is a large pair production in the plus polarization while it is almost zero for the minus state

$$n_+(\vec{k}) \gtrsim e^{\xi_A\pi} \quad \text{and} \quad n_-(\vec{k}) \lesssim e^{-6\xi_A\pi}. \tag{3.21}$$

The total particle creation from the asymptotic past to the asymptotic future, therefore, is

$$N_\sigma = \frac{1}{(2\pi)^3} |\beta_\sigma|^2 \int d^3k = \frac{1}{(2\pi)^2} \frac{e^{2\lambda_\sigma |\kappa|\pi} + e^{-2|\mu|\pi}}{\sinh(2|\mu|\pi)} \int_0^\infty k^2 dk, \tag{3.22}$$

which is divergent since it expresses the number of pairs created for all times. The physically meaningful quantity, however, is the pair production rate, i.e., the number of pairs produced per unit time per unit physical volume

$$\Gamma^\sigma_{\text{pairs}} = \frac{1}{a(\tau)^4} \frac{dN_\sigma}{d\tau}. \tag{3.23}$$

To calculate the derivative we need to convert the wavenumber integral into a time integral. It has been shown in the right panel of figure 1 that the system has two sharp deviations from adiabaticity around the roots of ω_+. Therefore, pairs of particles with $+2$ helicity state of a given comoving momentum, k, are produced mostly around (3.7)

$$\tau_{1,2}(k) \simeq -(|\kappa| \pm \sqrt{|\kappa|^2 - |\mu|^2})/k. \tag{3.24}$$

Note that $\tilde\tau = k/(aH) \simeq -k\tau$. As a result, the total particle creation at $\tau_1(k)$ and $\tau_2(k)$ are given respectively by

$$N_1 \approx (|\kappa| + \sqrt{|\kappa|^2 - |\mu|^2})^3 \frac{e^{2|\kappa|\pi} + e^{-2|\mu|\pi}}{(2\pi)^2 \sinh(2|\mu|\pi)} \int_{-\infty}^0 d\tau \, (a(\tau)H)^4, \tag{3.25}$$

$$N_2 \approx (|\kappa| - \sqrt{|\kappa|^2 - |\mu|^2})^3 \frac{e^{2|\kappa|\pi} + e^{-2|\mu|\pi}}{(2\pi)^2 \sinh(2|\mu|\pi)} \int_{-\infty}^0 d\tau \, (a(\tau)H)^4. \tag{3.26}$$

The corresponding production rates are

$$\Gamma^1_{\text{pairs}} \approx (|\kappa| + \sqrt{|\kappa|^2 - |\mu|^2})^3 \frac{e^{2|\kappa|\pi} + e^{-2|\mu|\pi}}{(2\pi)^2 \sinh(2|\mu|\pi)} H^4, \tag{3.27}$$

$$\Gamma^2_{\text{pairs}} \approx (|\kappa| - \sqrt{|\kappa|^2 - |\mu|^2})^3 \frac{e^{2|\kappa|\pi} + e^{-2|\mu|\pi}}{(2\pi)^2 \sinh(2|\mu|\pi)} H^4. \tag{3.28}$$

We find that the particle production during τ_1 is much more efficient than during τ_2, i.e.

$$\frac{\Gamma^2_{\text{pairs}}}{\Gamma^1_{\text{pairs}}} \simeq 10^{-2}. \tag{3.29}$$

Therefore, we can neglect the burst of particles created at τ_2. After integrating eq. (3.23), we find that the physical number densities of pairs created up to time τ are also time independent

$$n_{\text{pairs}} = \frac{1}{a(\tau)^3} \int_{-\infty}^{\tau'} d\tau \, a(\eta)^4 \Gamma^1_{\text{pairs}} \approx \frac{\Gamma^1_{\text{pairs}}}{3H}, \tag{3.30}$$

i.e., gravitational and Schwinger-type particle production are exactly balanced by the gravitational redshifting. We can approximate the above as

$$n_{\text{pairs}} \approx (|\kappa| + \sqrt{|\kappa|^2 - |\mu|^2})^3 \frac{H^3}{6\pi^2} e^{2(|\kappa| - |\mu|)\pi}. \tag{3.31}$$
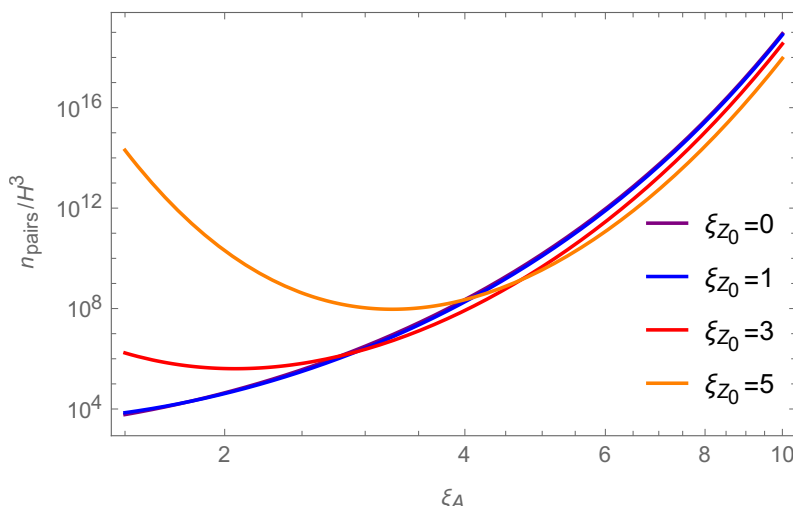
Figure 4. The number density of the created pairs, n_{pairs}/H^3, as a function of ξ_A and ξ_{Z_0}.

The particle production increases exponentially with $(|\kappa| - |\mu|)$ and has the following asymptotic forms

$$\frac{n_{\text{pairs}}}{H^3} \to \begin{cases} \frac{(2+\sqrt{2})^3}{6\pi^2} \xi_A^3 e^{2(2-\sqrt{2})\xi_A \pi} & \text{if } \frac{\xi_{Z_0}}{\xi_A} \ll 1 \\ \frac{1}{6\pi^2} \left(\frac{\xi_{Z_0}}{\xi_A}\right)^3 e^{\frac{\xi_{Z_0}^2}{\xi_A}\pi} & \text{if } \frac{\xi_{Z_0}}{\xi_A} \gg 1 \quad (\alpha_H = 1) , \end{cases} \tag{3.32}$$

which may cause a large backreaction on the background VEV fields. This is the next subject of our study, given in section 4. The number density of the created particles is presented in figure 4.

Another interesting quantity to compute is the vacuum-vacuum transition amplitude defined by

$$|\langle 0_{\text{out}}|0_{\text{in}}\rangle|^2 \equiv e^{\left(-\iint d^3 x \, d\tau a^4 \Upsilon_{\text{vac}}\right)} = \exp\left[-\frac{1}{(2\pi)^3} \int d^3 x \int dk^3 \ln(1 + |\beta_{+,\vec{k}}|^2) \right], \tag{3.33}$$

where Υ_{vac} is the vacuum decay rate. Using eq. (3.20) and (3.24), we obtain

$$\Upsilon_{\text{vac}} = -2 \frac{H^4}{(2\pi)^2} (|\kappa| + \sqrt{|\kappa|^2 - |\mu|^2})^3 \ln\left[1 + \left(\frac{e^{2|\kappa|\pi} + e^{-2|\mu|\pi}}{2\sinh(2|\mu|\pi)}\right)\right], \tag{3.34}$$

which is well approximated by

$$\Upsilon_{\text{vac}} \approx -\frac{H^4}{\pi} (|\kappa| + \sqrt{|\kappa|^2 - |\mu|^2})^3 (|\kappa| - |\mu|) , \tag{3.35}$$

implying a sizable vacuum decay rate. Finally, in the Minkowski limit with $H \to 0$, this setup with an isotropic SU(2) gauge field does not experience Schwinger-type particle production [39].

4 Backreaction

In this section, we compute the induced current and backreaction of the spin-2 field on the inflationary background.

4.1 Induced current and backreaction

The continuous global SU(2) symmetry (A.5) leads to the conserved Noether current as

$$J_A^\mu = -T^a \frac{\delta(\delta_B \mathcal{L})}{\delta \bar{A}_\mu^a} = \delta_B \left[D_\nu \left(\bar{g}^{\mu\lambda} \bar{g}^{\nu\sigma} + \bar{\alpha}_A \epsilon^{\mu\nu\lambda\sigma} \right) F_{\lambda\sigma} \right], \tag{4.1}$$

where δ_B denotes quadratic order action with respect to B_{ij} field, a bar denotes background quantities, and $D_\mu \equiv \partial_\mu - ig_A A_\mu$ is the covariant derivative. The current affects the background equations of motion as (see (2.15))

$$(\nabla_\nu - ig_A \bar{A}_\nu)\left(\bar{F}^{\mu\nu} + \bar{\alpha}_A \epsilon^{\mu\nu\lambda\sigma} \bar{F}_{\lambda\sigma} \right) + \alpha_H g_A^2 Z_0^2 \bar{A}^\mu = -\langle J_A^\mu \rangle, \tag{4.2}$$

where $\bar{\alpha}_A$ is a function of background fields given in (2.16). The expectation value of the zero-component of J_A^μ vanishes

$$\langle J_A^0 \rangle = 0. \tag{4.3}$$

Thus, the background field equation of ψ in (2.14) is sourced by $\mathcal{J}_A$ defined by

$$\mathcal{J}_A \equiv \frac{a}{3} \delta_i^a J_{Aa}^i = \frac{g_A M_{\mathrm{Pl}}^2}{3a^2}\left(\frac{1}{a}\epsilon_{qp}^i B_{jq}\partial_i B_{jp} + \dot{\bar{\alpha}}_A B_{ij}^2 \right)$$

$$= \frac{g_A}{3a^3}\sum_\sigma \int d^3k \left(-\lambda_\sigma k + \mathcal{H}\frac{\dot{\bar{\alpha}}_A}{H} \right)|B_\sigma(\vec{k})|^2. \tag{4.4}$$

In the $\mathcal{L}_A = \mathcal{L}_{Cn}$ models with axion-gauge field coupling, B_{ij} induces another backreaction term of the form $\nabla_\mu P_\varphi^\mu$ where

$$P_\varphi^\mu = \frac{\lambda M_{\mathrm{Pl}}^2}{2a^3 f}\delta_0^\mu \left(\epsilon_{qp}^i B_{qj}\partial_i B_{pj} + g_A a\psi B_{ij}B^{ij} \right), \tag{4.5}$$

with $P_\varphi^i = 0$. The divergence of the zero-competent then backreacts on the background axion field as

$$\ddot{\varphi} + 3H\dot{\varphi} + V_\varphi + \frac{3\lambda g_A}{f}\psi^2(\dot{\psi} + H\psi) = \langle \mathcal{P}_\varphi \rangle, \tag{4.6}$$

where

$$\langle \mathcal{P}_\varphi \rangle \equiv \langle \nabla_\mu P_\varphi^\mu \rangle = \frac{\lambda}{2a^3 f}\sum_\sigma \frac{d}{dt}\left[\int d^3k \left(-\lambda_\sigma k + \mathcal{H}\xi_A \right)|B_\sigma(\vec{k})|^2 \right]. \tag{4.7}$$

Note that P_φ^μ is *not* a Noether current. In figure 5, we plotted $\mathcal{J}_A$ and $\mathcal{P}_\varphi$ with respect to ξ_A.

Both $\langle \mathcal{J}_A \rangle$ and $\langle \mathcal{P}_\varphi \rangle$ can be written in terms of the following momentum integral

$$\mathcal{K}[X] \equiv \sum_\sigma (2\pi)^2 \int \frac{d^3k}{2\mathcal{H}^3}\left(-\lambda_\sigma k + \mathcal{H}X \right)|B_\sigma(\vec{k})|^2. \tag{4.8}$$

We find

$$\langle \mathcal{J}_A \rangle = \frac{2g_A H^3}{3(2\pi)^2}\mathcal{K}[\dot{\bar{\alpha}}_A/H], \tag{4.9}$$

$$\langle \mathcal{P}_\varphi \rangle = \frac{\lambda}{(2\pi)^2 a^3 f}\frac{d}{dt}\left(a^3 H^3 \mathcal{K}[\xi_A] \right) \simeq \frac{3\lambda H^4}{(2\pi)^2 f}\mathcal{K}[\xi_A], \tag{4.10}$$
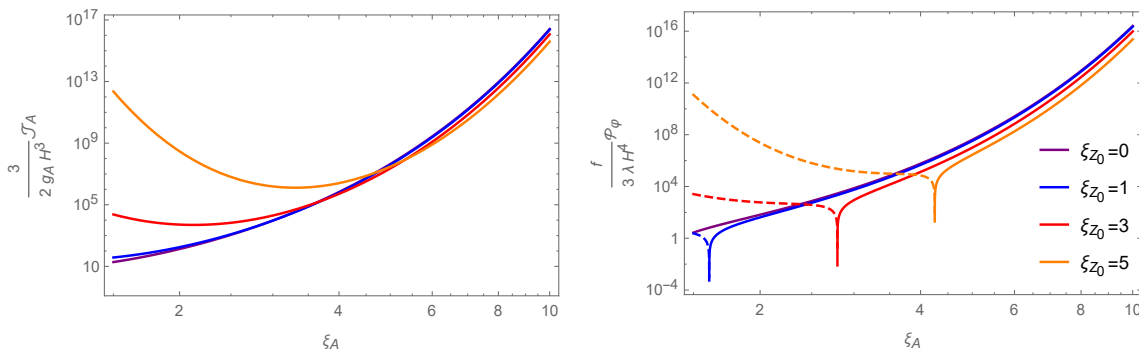
Figure 5. Backreaction terms on the gauge ($\mathcal{J}_A$) and axion ($\mathcal{P}_\varphi$) fields as a function of ξ_A and ξ_{Z_0}. The dashed lines show negative values. We show $\frac{3}{2g_A H^3}\mathcal{J}_A$ and $\frac{f}{3\lambda H^4}\mathcal{P}_\varphi$.

where we have used $\frac{\dot{\bar{\alpha}}_A}{H} \simeq \frac{(1+\xi_A^2+\frac{\alpha_H}{2}\xi_{Z_0}^2)}{\xi_A}$ from (2.17). We work out the integral of $\mathcal{K}[X]$ and its renormalization (using the adiabatic subtraction technique) in appendix D. Here we present its final regularized form

$$\mathcal{K}_{\rm reg}[X] = \frac{1}{6}e^{2(|\kappa|-|\mu|)\pi}\Bigg(|\mu|(-4|\mu|^2+15|\kappa|^2-4-9|\kappa|X)+\frac{\mathcal{C}(X)}{4}{\rm Re}\Bigg[\psi^{(0)}\left(\frac{1}{2}+i|\kappa|-i|\mu|\right)$$
$$-\psi^{(0)}\left(\frac{1}{2}+i|\kappa|+i|\mu|\right)\Bigg]\Bigg), \quad (4.11)$$

where $\psi^{(0)}(z) \equiv \frac{d}{dz}\ln\Gamma(z)$ is the digamma function, and $\mathcal{C}(X)$ is

$$\mathcal{C}(X) \equiv 3X\left(\frac{1}{2}+6\kappa^2-2\mu^2\right)-\frac{3}{2}(7+20\kappa^2-12\mu^2)|\kappa|. \quad (4.12)$$

We find that $\mathcal{K}_{\rm reg}[X]$ is proportional to $e^{2(|\kappa|-|\mu|)\pi} \gg 1$ (see figure 3). In the limit that $|\kappa|-|\mu| \gg 1$, we can further simply $\mathcal{K}_{\rm reg}[X]$ to

$$\mathcal{K}_{\rm reg}[X] = \frac{1}{6}e^{2(|\kappa|-|\mu|)\pi}\left[|\mu|(-4|\mu|^2+15|\kappa|^2-4-9|\kappa|X)+\frac{\mathcal{C}(X)}{4}\ln\left(\frac{|\kappa|-|\mu|}{|\kappa|+|\mu|}\right)\right], \quad (4.13)$$

in which we used (C.8) to expand $\psi^{(0)}(z)$. This completes our derivation of the analytical formulae for the backreaction terms.[10] Using (3.31), we can relate $\mathcal{K}_{\rm reg}[X]$ to the number density of the spin-2 field as

$$\mathcal{K}_{\rm reg}[X] \simeq \frac{\pi^2}{10}\mathcal{I}_{BR}[X]\left(\frac{n_{\rm pairs}}{H^3}\right), \quad (4.15)$$

---

[10]As we see in figure 3, the generated pair particle number is very large and thus we are in the classical regime. Therefore, we can estimate the induced gauge field current, J_A^μ, by semi-classical approximations. Specifically, we can approximate it as $\mathcal{J}_A \sim 2g_A n_{\rm pairs}v$. Assuming that the particles travel with the speed of light $v \sim 1$, we can approximate the induced gauge field current as

$$\mathcal{J}_A \sim \frac{2g_A H^3}{6\pi^2}\big(|\kappa|+\sqrt{|\kappa|^2-|\mu|^2}\big)^3 e^{2(|\kappa|-|\mu|)\pi}, \quad (4.14)$$

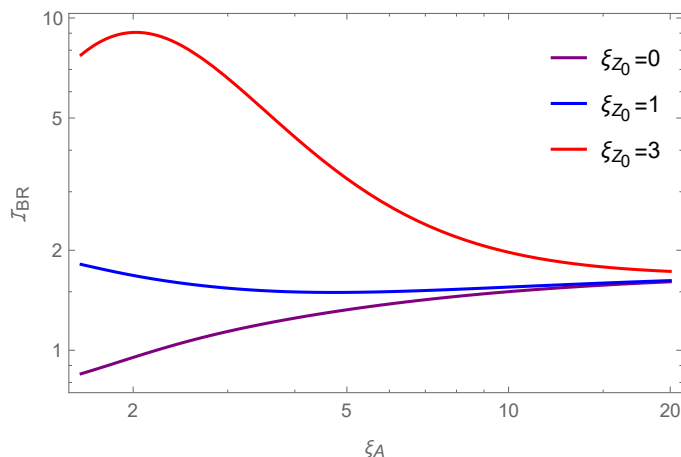which is in agreement with the result of our exact solution in (4.9).

Figure 6. The prefactor $\mathcal{I}_{BR}[(1 + \xi_{\mathrm{A}}^2 + \frac{\alpha_H}{2}\xi_{Z_0}^2)/\xi_{\mathrm{A}}]$ as a function of ξ_{A} and for different values of ξ_{Z_0}. Note that for massive cases with $\xi_{Z_0} \neq 0$, we set $\alpha_H = 1$.

where $\mathcal{I}_{BR}[X]$ is of order unity and given as

$$\mathcal{I}_{BR}[X] \equiv 10 \left[\frac{|\mu|(-4|\mu|^2 + 15|\kappa|^2 - 4 - 9|\kappa|X) + \frac{\mathcal{C}(X)}{4}\ln\left(\frac{|\kappa|-|\mu|}{|\kappa|+|\mu|}\right)}{(|\kappa| + \sqrt{|\kappa|^2 - |\mu|^2})^3} \right]. \tag{4.16}$$

The backreaction is also directly related to n_{pairs}. In particular, the backreaction to the field equation of the gauge field is

$$\mathcal{J}_A \simeq \frac{g_{\mathrm{A}}\mathcal{I}_{BR}[(1 + \xi_{\mathrm{A}}^2 + \frac{\alpha_H}{2}\xi_{Z_0}^2)/\xi_{\mathrm{A}}]}{60} \, n_{\mathrm{pairs}}, \tag{4.17}$$

where $\mathcal{I}_{BR}[(1 + \xi_{\mathrm{A}}^2 + \frac{\alpha_H}{2}\xi_{Z_0}^2)/\xi_{\mathrm{A}}]$ is of order unity as presented in figure 6. Similar to $\mathcal{J}_A$, the backreaction to the axion field equation is also directly given by the number density of the spin-2 field, i.e. $\mathcal{P}_\varphi \sim \frac{\lambda H}{f}n_{\mathrm{pairs}}$. This relation is valid for all the models considered in this paper.

Equations (4.9)–(4.11) and (4.17) are the first main results of this paper. Equations (4.9)–(4.11) have been estimated only numerically for one model in this family previously in [40] which are in agreement with our formulae over the region where the comparison is possible. The relation between the backreaction and the number density of the spin-2 field in (4.17) is derived here for the first time.

4.2 Energy density of spin-2 fields

The extra spin-2 field has a sizable energy-momentum density. The expectation value of its energy density adds to the total energy density in the background, $\bar{\rho}$, as

$$\bar{\rho}(t) = \sum_I \bar{\rho}_I(t) + \langle \delta_{\mathrm{B}}\rho(t)\rangle_{\vec{k}=0}. \tag{4.18}$$

Perturbing the energy momentum tensor and considering only terms quadratic in B_{ij}, we have

$$\delta_{\mathrm{B}}T_{00} = -\bar{g}_{00} \, \delta_{\mathrm{B}}\rho \quad \text{and} \quad \delta_{\mathrm{B}}T_{ij} = \bar{g}_{ij} \, \delta_{\mathrm{B}}P. \tag{4.19}$$
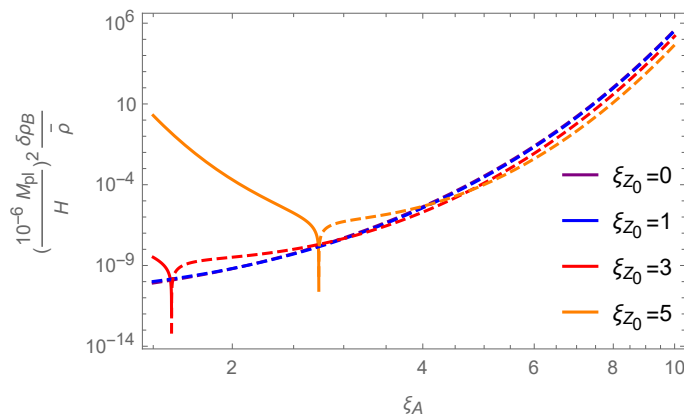
Figure 7. The energy density of the spin-2 field, $\delta_{\rm B}\rho$, as a function of $\xi_{\rm A}$ and for different values of ξ_{Z_0}. The dashed lines show negative values.

The contribution of B_{ij} to the energy density, $\delta_{\rm B}\rho$, is

$$\langle\delta_{\rm B}\rho\rangle = M_{\rm Pl}^2\langle\frac{1}{2a^2}(\partial_0 B_{ij})^2 + \frac{1}{2a^4}(\partial_k B_{ij})^2 + \frac{g_{\rm A}\psi}{a^3}\epsilon^{qpi}B_{jq}\partial_i B_{jp} + \frac{1}{2}\alpha_H g_{\rm A}^2 Z_0^2 B_{ij}^2\rangle. \quad (4.20)$$

The contribution to the isotropic pressure is

$$\langle\delta_{\rm B}P\rangle = \langle\frac{1}{3}\delta_{\rm B}\rho\rangle|_{\alpha_H=0} - \frac{1}{6}\alpha_H g_{\rm A}^2 Z_0^2 M_{\rm Pl}^2\langle B_{ij}^2\rangle. \quad (4.21)$$

In the absence of interaction with the Higgs field, $\alpha_H = 0$, B_{ij} field has the equation of state of radiation. On the other hand, for a massive gauge field with $\alpha_H = 1$, the B_{ij} field gets closer to dust.

We compute the normalized energy density in appendix D.2, and only show the result here. Using (D.41), we find the energy density fraction in the spin-2 field B_σ as

$$\frac{\langle\delta_{\rm B}\rho\rangle_{\rm reg}}{\bar\rho} \approx \left(\frac{H}{M_{\rm Pl}}\right)^2 \frac{(\delta_c + 2\xi_{\rm A})}{3(2\pi)^2}\mathcal{K}_{\rm reg}\left[\frac{3\xi_{\rm A}(\delta_c - 3\xi_{\rm A}) - 1}{\delta_c + 2\xi_{\rm A}}\right], \quad (4.22)$$

where $\frac{3\xi_{\rm A}(\delta_c - 3\xi_{\rm A}) - 1}{\delta_c + 2\xi_{\rm A}} \gtrsim 1$. We show the energy density as a function of $\xi_{\rm A}$ and ξ_{Z_0} in figure 7. Let us summarize the main features of the energy density:

- The energy density of B_- is always positive while the energy density of B_+ can be negative.

- The total energy density in B_{ij} is negative for $\xi_{Z_0} \lesssim \frac{9}{5}\xi_{\rm A}$.

- The energy density fraction is of order $\frac{\langle\delta_{\rm B}\rho\rangle}{\bar\rho} \sim \left(\frac{H}{M_{\rm Pl}}\right)^2\delta_c|\mu|^3 e^{2(|\kappa|-|\mu|)\pi}$. Therefore, reducing the energy scale of inflation decreases the energy fraction in B_{ij} as $\left(\frac{H}{M_{\rm Pl}}\right)^2$.

- Validity of perturbation theory requires $\frac{\langle\delta_{\rm B}\rho\rangle}{\bar\rho} \lesssim 10^{-5}$, which constrains the parameter space of the models as a function of the energy scale of inflation.

Although $\langle\delta_{\rm B}\rho\rangle_{\rm reg}$ is negative in most of the parameter space, it is always a small part of the total energy density of the setup. The total energy-momentum tensor satisfies null and weak energy conditions, while the B_+ field violates both. The reason underlying the negative energy of the plus polarization is the existence of a short phase of instability for each $\vec{k}$-mode of B_+ around horizon crossing. However, this phase ends as soon as the mode exists the horizon; thus, the existence of the cosmic horizon evades (dangerous) infinite energy extraction of negative energy systems, unlike in flat space.

5   Gravitational waves

Each polarization state of the spin-2 field B_{ij} mixes with the corresponding polarization of the gravitational waves, γ_{ij} (see (2.24)). In particular, in the presence of the gauge field, we have

$$\gamma_\sigma(t,\vec{x}) = \gamma_\sigma^{\rm vac}(t,\vec{x}) + \gamma_\sigma^{\rm s}(t,\vec{x}), \tag{5.1}$$

where $\gamma_\sigma^{\rm vac}(t,\vec{x})$ is the vacuum gravitational waves (i.e. by quantum fluctuations of the spacetime [41, 42]) with helicity σ while $\gamma_\sigma^{\rm s}(t,\vec{x})$ is the part sourced by the spin-2 field. Here, we compute the power spectrum and the energy density of $\gamma_\sigma^{\rm s}$ in terms of $\xi_{\rm A}$ and ξ_{Z_0} and relate them with the number density of B_σ, $n_{\rm pairs}$. We work out the exact form of the sourced gravitational waves in (quasi) de Sitter in appendix E. The result in the super horizon limit is

$$h_\sigma^{\rm s}(\tau,\vec{k}) = \frac{e^{i\kappa_\sigma\pi/2}}{(2\pi)^{\frac{3}{2}}}\left(\frac{\psi}{M_{\rm Pl}}\right)\left(\frac{aH}{\sqrt{2}k^{\frac{3}{2}}}\right)\mathcal{G}_\sigma(\xi_{\rm A},\xi_{Z_0}), \tag{5.2}$$

where the explicit form of $\mathcal{G}_\sigma(\xi_{\rm A},\xi_{Z_0})$ is given in (E.6) and shown in figure 8. It can be well approximated as

$$\mathcal{G}_\sigma(\xi_{\rm A},\xi_{Z_0}) = \frac{\pi}{\cos(\pi\mu)\Gamma(-\kappa_\sigma)}\left[\frac{(i+\lambda_\sigma\beta_c)}{\kappa_\sigma} + \frac{(i-\lambda_\sigma\beta_c)\Gamma^2(-\kappa_\sigma)}{\Gamma(\frac{1}{2}-\kappa_\sigma-\mu)\Gamma(\frac{1}{2}-\kappa_\sigma+\mu)}\right]. \tag{5.3}$$

Taking the (classical) limit, $|\mu|\gg 1$, and using (E.8), we have

$$\gamma_+^{\rm s}(\tau,\vec{k}) \simeq \frac{\mathcal{A}_+}{(2\pi)^{\frac{3}{2}}}e^{(|\kappa|-|\mu|)\pi}\left(\frac{\psi}{M_{\rm Pl}}\right)\left(\frac{H}{k^{\frac{3}{2}}}\right)\sqrt{\pi|\kappa|}, \tag{5.4}$$

where $\mathcal{A}_+$ is (see (E.9))

$$\mathcal{A}_+ \simeq \left[\frac{(i-\xi_{\rm A})\Gamma^2\left(i\left(2\xi_{\rm A}+\frac{\xi_{Z_0}^2+2}{2\xi_{\rm A}}\right)\right)}{\Gamma\left(\frac{1}{2}+i[2\xi_{\rm A}+\frac{\xi_{Z_0}^2+2}{2\xi_{\rm A}}-\sqrt{2}(1+\xi_{\rm A}^2+\xi_{Z_0}^2)^{\frac{1}{2}}]\right)\Gamma\left(\frac{1}{2}+i[2\xi_{\rm A}+\frac{\xi_{Z_0}^2+2}{2\xi_{\rm A}}+\sqrt{2}(1+\xi_{\rm A}^2+\xi_{Z_0}^2)^{\frac{1}{2}}]\right)}\right.$$
$$\left.+\frac{i\xi_{\rm A}-1}{2\xi_{\rm A}+\frac{\xi_{Z_0}^2+2}{2\xi_{\rm A}}}\right]. \tag{5.5}$$

Figure 8 shows $|\mathcal{A}_+|^2$ as a function of $\xi_{\rm A}$ and ξ_{Z_0}. We find that $|\mathcal{A}_+|^2$ oscillates between zero and unity as a function of $\xi_{\rm A}$.
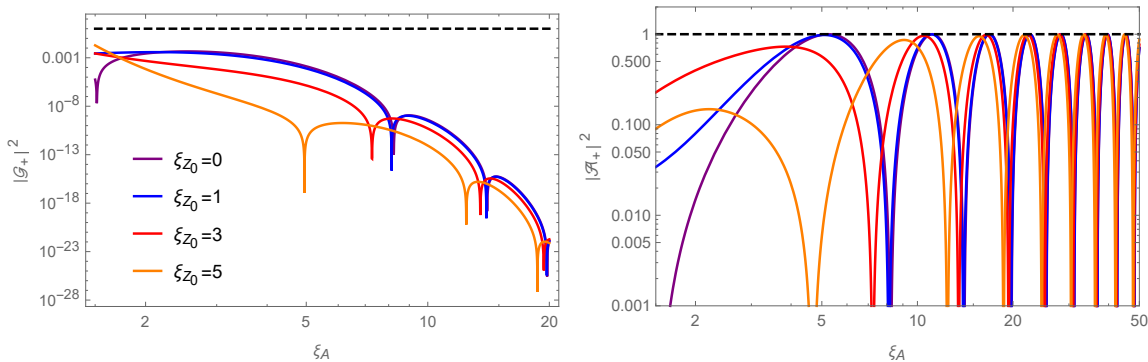
Figure 8. The prefactors $|\mathcal{G}_+|^2$ (left) and $|\mathcal{A}_+|^2$ (right) as functions of ξ_A and ξ_{Z_0}. The (black) dashed line shows unity.
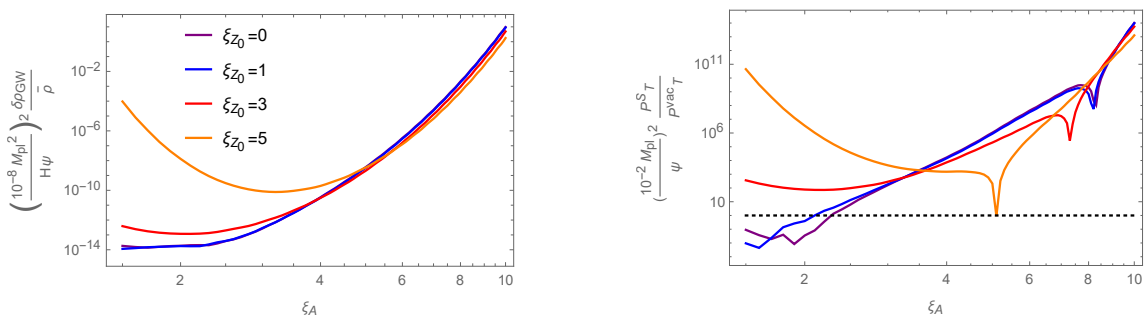

Figure 9. The energy density (left) and power spectrum (right) of gravitational waves sourced by the gauge field as a function of ξ_A and ξ_{Z_0}. We show $\left(10^{-8}M_{\rm Pl}^2/H\psi\right)^2 \delta\rho_{\rm s}^{\rm GW}/\bar\rho$ (left) and $\left(10^{-2}M_{\rm Pl}/\psi\right)^2 P_T^{\rm s}/P_T^{\rm vac}$ (right). The dashed line in the right panel shows unity.

Finally, we can write the ratio of the power spectra of sourced and vacuum gravitational waves in term of the spin-2 fields number density as

$$\frac{P_T^{\rm s}}{P_T^{\rm vac}} = \frac{\langle \gamma_+^{\rm s}\gamma_+^{\rm s}\rangle}{\langle \gamma_+^{\rm vac}\gamma_+^{\rm vac}\rangle}\bigg|_{-k\tau \ll 1} \simeq \left(\frac{\psi}{M_{\rm Pl}}\right)^2\left(\frac{n_{\rm pairs}}{H^3}\right)\frac{6\pi^3|\mathcal{A}_+|^2|\kappa_+|}{(|\kappa_+| + \sqrt{|\kappa_+|^2 - |\mu|^2})^3}, \tag{5.6}$$

where $P_T^{\rm x}(k)\delta^{(3)}(\vec{k}+\vec{k}') \equiv 8\pi k^3\langle\gamma_+^{\rm x}(\vec{k})\gamma_+^{\rm x}(\vec{k}')\rangle$ is the power spectrum of $\gamma_+^{\rm x}$ where x = (s, vac). Moreover, $\langle\gamma_+^{\rm vac}(\vec{k})\gamma_+^{\rm vac}(\vec{k}')\rangle = \frac{H^2}{(2\pi)^3k^3}$ where $a\gamma_+(\vec{k}')$ is a canonically normalized field. In the right panel of figure 9, we show $\left(10^{-2}M_{\rm Pl}/\psi\right)^2 P_T^{\rm s}/P_T^{\rm vac}$. This is the second main result of this paper.

The next interesting quantity is the energy density of the gravitational waves. The energy-momentum tensor of the gravitational wave is

$$t_{\mu\nu}^{\rm GW} = \frac{M_{\rm Pl}^2}{4}\langle\partial_\mu\gamma_{ij}\partial_\nu\gamma_{ij}\rangle, \tag{5.7}$$

which gives the energy density in the sourced part of the gravitational waves, $\delta\rho_{\rm s}^{\rm GW} = t_{00}^{\rm GW}$, as

$$\delta\rho_{\rm s}^{\rm GW} = \frac{M_{\rm Pl}^2}{4a^2}\langle\gamma_{ij}^{s\prime}\gamma_{ij}^{s\prime}\rangle. \tag{5.8}$$

In terms of the physical momentum, it can be written as

$$\delta\rho_{\rm s}^{\rm GW} = \frac{2H^4}{(2\pi)^2}\sum_{\sigma=\pm}\int \tilde{\tau}^3 d\tilde{\tau}k\langle\left(\partial_{\tilde{\tau}}h_\sigma^{s*} + \frac{\mathcal{H}}{k}h_\sigma^{s*}\right)\left(\partial_{\tilde{\tau}}h_\sigma^{s} + \frac{\mathcal{H}}{k}h_\sigma^{s}\right)\rangle. \tag{5.9}$$

In the left panel of figure 9, we show $\left(10^{-8}M_{\rm Pl}^2/H\psi\right)^2\delta\rho_{\rm s}^{\rm GW}/\bar{\rho}$. The energy density can be written as

$$\delta\rho_{\rm s}^{\rm GW} = \frac{4H^4}{3\pi^2}\left(\frac{\psi}{M_{\rm Pl}}\right)^2\mathcal{I}_{\rm GW}(\xi_{\rm A},\xi_{Z_0})e^{2(|\kappa|-|\mu|)\pi}, \tag{5.10}$$

where $\mathcal{I}_{\rm GW}(\xi_{\rm A},\xi_{Z_0})$ is roughly

$$\mathcal{I}_{\rm GW}(\xi_{\rm A},\xi_{Z_0}) \sim (|\kappa| + \sqrt{|\kappa|^2 - |\mu|^2})^2. \tag{5.11}$$

Comparing to the number density of the (gauge field's) spin-2 particles in (3.31), we find

$$\delta\rho_{\rm s}^{\rm GW} \sim \frac{8\pi}{(|\kappa_+| + \sqrt{|\kappa_+|^2 - |\mu|^2})}\left(\frac{\psi}{M_{\rm Pl}}\right)^2 Hn_{\rm pairs}. \tag{5.12}$$

The above relation is correct up to order unity coefficients. The energy density in the sourced gravitational wave is proportional to $\left(\frac{\psi}{M_{\rm Pl}}\right)^2$, the number density of B_+, $Hn_{\rm pairs}$, and the inverse of $\tilde{\tau}_1$ in (3.7).

6 Constraints on the parameter space

In this section, we use the size of the backreaction to constrain the parameter space of the models. Validity of perturbation theory requires that the backreaction terms be much smaller than the other terms in the field equations. To do this, we normalize $\mathcal{J}_A$ and $\mathcal{P}_\phi$ by $H^2\psi$ and $\lambda Hg_{\rm A}\psi^3/f$ respectively to construct two dimensionless backreaction terms

$$\mathcal{B}_A \equiv \frac{\mathcal{J}_A}{H^3}\frac{H}{M_{\rm Pl}}\frac{M_{\rm Pl}}{\psi} = \left(\frac{10^{-2}M_{\rm Pl}}{\psi}\right)^2\left(\frac{H}{10^{-6}M_{\rm Pl}}\right)^2\frac{2\times 10^{-8}}{3(2\pi)^2}\xi_{\rm A}\mathcal{K}[\frac{\dot{\bar{\alpha}}_A}{H}], \tag{6.1}$$

$$\mathcal{B}_\varphi \equiv \frac{\mathcal{P}_\varphi}{\lambda Hg_{\rm A}\psi^3/f} = \left(\frac{10^{-2}M_{\rm Pl}}{\psi}\right)^2\left(\frac{H}{10^{-6}M_{\rm Pl}}\right)^2\frac{3\times 10^{-8}}{(2\pi)^2}\frac{1}{\xi_{\rm A}}\mathcal{K}[\xi_{\rm A}], \tag{6.2}$$

where the explicit forms of $\mathcal{K}[X]$ for $X = \dot{\bar{\alpha}}_A/H \simeq (1 + \xi_{\rm A}^2 + \frac{\alpha_H}{2}\xi_{Z_0}^2)/\xi_{\rm A}$ and $X = \xi_{\rm A}$ are given in (4.11) and a good approximation is given in (4.13). Another backreaction is the contribution of the spin-2 field, B_σ particle, to the total energy density which is presented in figure 7 and is subleading comparing to the above quantities. As we see, both $\mathcal{B}_A$ and $\mathcal{B}_\varphi$ are proportional to the scale of inflation as $(H/M_{\rm Pl})^2$ and hence decrease by reducing the scale of inflation. These dimensionless backreaction terms are shown in figure 10. We require both $\mathcal{B}_A$ and $\mathcal{B}_\varphi$ to be lower than 0.01 so that they are smaller than the slow-roll suppressed terms in the background field equation. Figure 11 shows the available parameter space corresponding to each scale of inflation.

Among the three backreactions, $\mathcal{B}_A$, $\mathcal{B}_\varphi$, and $\frac{\delta_B\rho}{\bar{\rho}}$, the first one is the largest. $\mathcal{B}_A$ is an exponential function of $\xi_{\rm A}$ and ξ_{Z_0} while it depends on the scale of inflation as H^2. For a
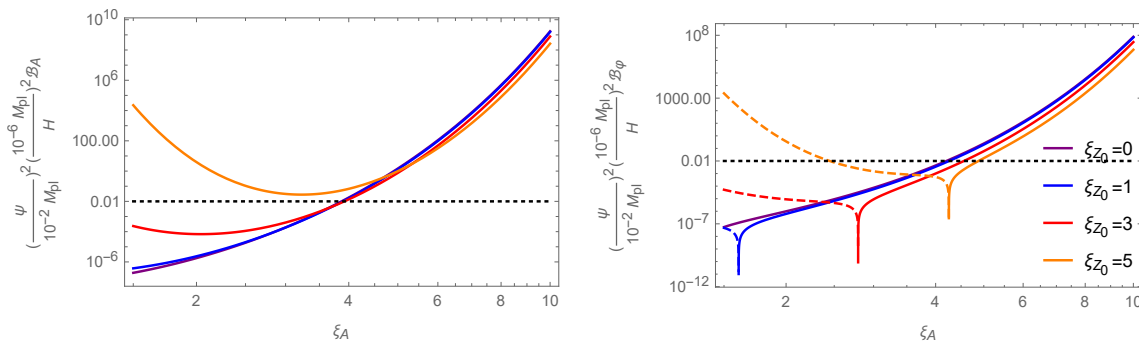
Figure 10. The dimensionless backreaction terms to the gauge and axion field equations as a function of ξ_A. The dotted black line shows 0.01. We show $\left(10^2\psi/M_{\rm Pl}\right)^2\left(10^{-6}M_{\rm Pl}/H\right)^2\mathcal{B}_A$ and $\left(10^2\psi/M_{\rm Pl}\right)^2\left(10^{-6}M_{\rm Pl}/H\right)^2\mathcal{B}_\varphi$. The dashed lines show negative values.
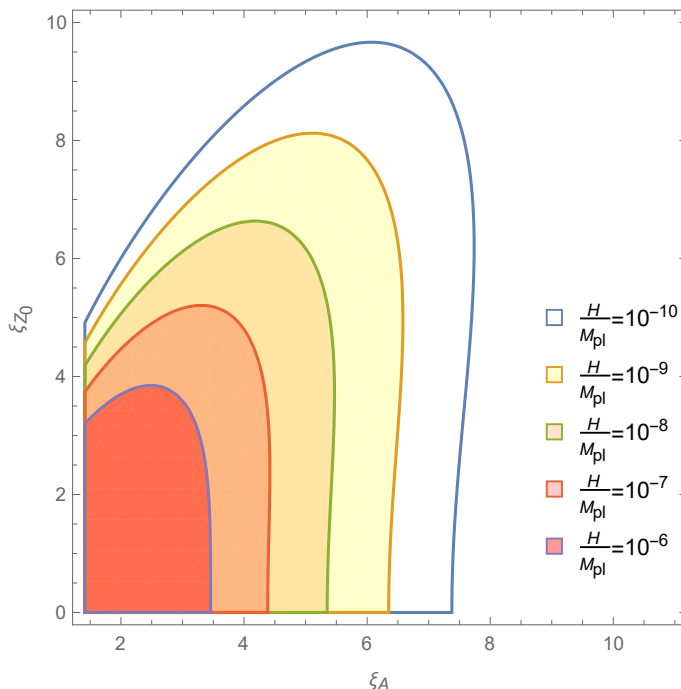
Figure 11. The parameter space $(\xi_A, \xi_{Z_0}, \frac{H}{M_{\rm Pl}}, \epsilon_A = 2 \times 10^{-4})$ with dimensionless backreactions less than 10^{-2}.

given ψ and a bound on $\mathcal{B}_A$, lowering H by a factor of 10 enlarges the acceptable domain of ξ_A and ξ_{Z_0} by one unit (see figure 11). For instance, for a $\frac{\psi}{M_{\rm Pl}} \sim 10^{-2}$ and a GUT scale inflation with $H \sim 10^{-6}M_{\rm Pl}$, we find that $0 \le \xi_{Z_0} < 4$ and $\sqrt{2} < \xi_A \lesssim 3.5$ (lower bound comes from stability of the scalar sector) are allowed. However, for a lower scale of inflation with $H \sim 10^{-7}M_{\rm Pl}$, we find that $0 \le \xi_{Z_0} < 5$ and $\sqrt{2} < \xi_A \lesssim 4.5$ are allowed. In this region, the energy density fraction of the spin-2 field is $\frac{\delta_B\rho}{\bar{\rho}} \lesssim 10^{-7}$ and decreases linearly with the decrease of H.

6.1 Parameter space of massless models

Up to now, our formulae have been presented in the unified form and are valid for any models assuming slow-roll dynamics of the VEV of the gauge field and quasi-de Sitter expansion. In this section, we use our backreaction formulae to further constrain the massless models, i.e., $\xi_{Z_0} = 0$ [11, 15, 40].

The total slow-roll parameter, ϵ, is given by

$$\epsilon = \epsilon_\varphi + \alpha_s \epsilon_\chi + \epsilon_A, \tag{6.3}$$

where ϵ_φ and ϵ_A are respectively the contributions of the axion and the gauge fields to the slow-roll parameter, whereas ϵ_χ is the contribution of the inflaton field in the spectator case (with $\alpha_s = 1$).

In massless models with $\xi_{Z_0} = 0$ and in the regime $\xi_A \gtrsim 2.5$, the extra scalar fields have little effect on the scalar power spectrum and scalar tilt, and thus we have the standard result given by $P_\zeta \simeq \frac{1}{2\epsilon}\left(\frac{H}{2\pi M_{\rm Pl}}\right)^2$ and $n_s - 1 \simeq -2(3\epsilon - \eta)$. On the other hand, adding the sourced tensor power spectrum given in (5.6) to the standard vacuum part, we have the total tensor power spectrum given by

$$P_T \simeq 2\left[1 + \left(\frac{\psi}{M_{\rm Pl}}\right)^2 \frac{e^{|\kappa_+|\pi}}{2}|\mathcal{G}_+(\xi_{\rm A})|^2\right]\left(\frac{H}{\pi M_{\rm Pl}}\right)^2. \tag{6.4}$$

The tensor-to-scalar ratio, $r \equiv P_T/P_\zeta$, is

$$r = r_{\rm vac} + r_{\rm source} \simeq 16\epsilon\left[1 + \left(\frac{\psi}{M_{\rm Pl}}\right)^2 \frac{e^{|\kappa_+|\pi}}{2}|\mathcal{G}_+(\xi_{\rm A})|^2\right], \tag{6.5}$$

where $r_{\rm vac} = 16\epsilon$ is the standard vacuum value, whereas $r_{\rm source}$ is the contribution from the gauge field. Using $P_\zeta = 2.2 \times 10^{-9}$ and ϵ_A in (2.13), we write r and $\mathcal{B}_A$ in terms of $r_{\rm vac}$, ϵ_A/ϵ and $\xi_{\rm A}$ as

$$r \simeq r_{\rm vac}\left[1 + \frac{\epsilon_A}{2}\frac{e^{\frac{(1+2\xi_{\rm A}^2)}{\xi_{\rm A}}\pi}}{(1+\xi_{\rm A}^2)}|\mathcal{G}_+(\xi_{\rm A})|^2\right] = r_{\rm vac}\left[1 + \frac{r_{\rm vac}}{32}\frac{\epsilon_A}{\epsilon}\frac{e^{\frac{(1+2\xi_{\rm A}^2)}{\xi_{\rm A}}\pi}}{(1+\xi_{\rm A}^2)}|\mathcal{G}_+(\xi_{\rm A})|^2\right], \tag{6.6}$$

and

$$\mathcal{B}_A \simeq \frac{8.8}{3 \times 10^9}\frac{\epsilon}{\epsilon_A}\xi_{\rm A}(1+\xi_{\rm A}^2)\,\mathcal{K}\left[\frac{1+\xi_{\rm A}^2}{\xi_{\rm A}}\right]. \tag{6.7}$$

The exact form of $\mathcal{K}[\frac{1+\xi_{\rm A}^2}{\xi_{\rm A}}]$ is given in (4.11). It is approximately given by $\mathcal{K}[\frac{1+\xi_{\rm A}^2}{\xi_{\rm A}}] \sim \xi_{\rm A}^3 e^{2\left((2-\sqrt{2})\xi_{\rm A}+\frac{2}{\xi_{\rm A}}\right)\pi}$.

In this section we constrain the parameter space of models in ϵ_B-$\xi_{\rm A}$ plane, following refs. [21, 39]. Here, $\epsilon_B \equiv g^2\psi^4/(H^2 M_{\rm Pl}^2)$ is related to ϵ_A as $\epsilon_B = \epsilon_A\xi_{\rm A}^2/(1+\xi_{\rm A}^2)$. We can relate this to the backreaction term as

$$\frac{\epsilon_B}{\epsilon} \simeq \frac{8.8}{3 \times 10^9}\frac{\xi_{\rm A}^3}{\mathcal{B}_A}\mathcal{K}\left[\frac{1+\xi_{\rm A}^2}{\xi_{\rm A}}\right], \tag{6.8}$$

which puts a lower bound on $\frac{\epsilon_B}{\epsilon}$ from backreaction.

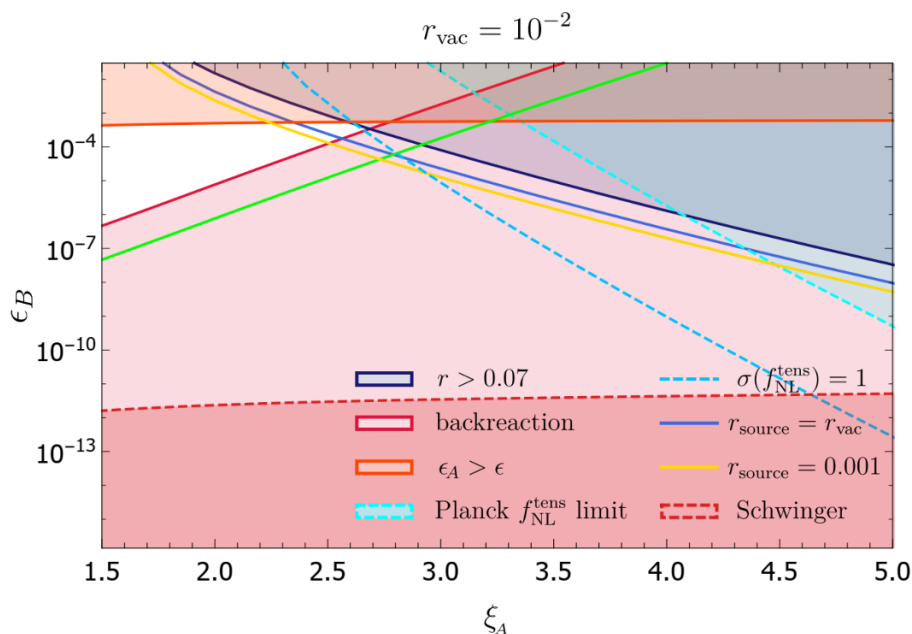We restrict the parameter space by imposing the following constraints:

Figure 12. Excluded parameter space of the massless models with $r_{\rm vac} = 10^{-2}$. The blue shaded area is excluded by the tensor-to-scalar ratio, the light red area by the large backreaction ($\mathcal{B}_A > 10^{-2}$), the orange area by the inconsistent slow-roll condition ($\epsilon_A > \epsilon$), the cyan area by the tensor non-Gaussianity, and the dark red area by the Schwinger pair-creation of scalar fields. The blue and yellow lines show $r_{\rm source} = r_{\rm vac}$ and $r_{\rm source} = 10^{-3}$, respectively, while the dashed cyan line shows $f_{\rm NL}^{\rm tens} = 1$. The green line corresponds to the amount of backreaction as large as $\mathcal{B}_A = 0.1$.

- The BICEP2/Keck and Planck (BKP) upper bound on the tensor-to-scalar ratio, $r < 0.07$ (95% C.L) [43];

- The Planck upper bound on the tensor non-Gaussianity parameter, $f_{\rm NL}^{\rm tens}$ [21, 44];

- Small backreaction given by $\mathcal{B}_A < 10^{-2}$ or 10^{-1};

- Small Schwinger pair-creation of scalar fields [39];

- Consistency of the slow-roll parameter, $\epsilon_A < \epsilon$;

We show the constraints in figure 12 and 13 for $r_{\rm vac} = 10^{-2}$, 10^{-3}, and 10^{-4}. We find that large parameter space is excluded already. In particular, we find that $r_{\rm source}$ cannot be much greater than $r_{\rm vac}$.

To see this in more detail, in figure 14 we show the ratio $r_{\rm source}/r_{\rm vac} = (r - r_{\rm vac})/r_{\rm vac}$ as a function of ξ_A for given values of ϵ_A/ϵ and $r_{\rm vac}$. Imposing the bound on the size of the backreaction, $\mathcal{B}_A < 10^{-2}$, as well as on $\epsilon_A/\epsilon < 0.9$, we find that the maximum possible value of $(r - r_{\rm vac})/r_{\rm vac}$ can be at most 5 for $r = 3 \times 10^{-3}$, and smaller for smaller $r_{\rm vac}$ because of $r_{\rm source}/r_{\rm vac} \propto r_{\rm vac}$ for a given ϵ_A/ϵ; see (6.6). The constraints weaken when we impose a weaker bound on the backreaction, $\mathcal{B}_A < 10^{-1}$. For larger values of $r_{\rm vac} = 3 \times 10^{-2}$, the strongest constraint comes from the BKP upper bound on $r < 0.07$ rather than from backreaction, yielding $(r - r_{\rm vac})/r_{\rm vac} < 1.3$.
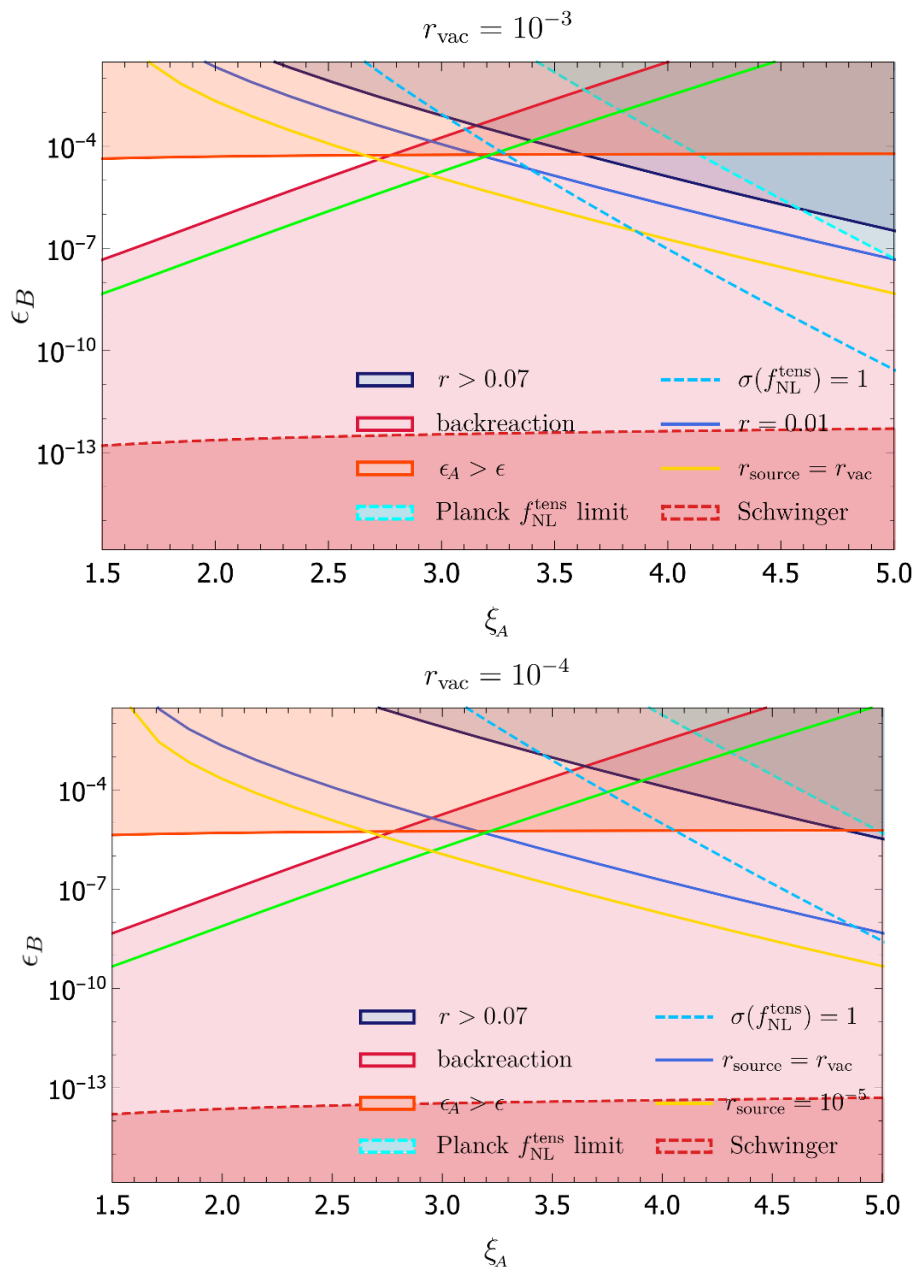
Figure 13. Same as figure 12 but for $r_{\mathrm{vac}} = 10^{-3}$ (top) and 10^{-4} (bottom). In the top panel the blue and yellow lines show $r = 10^{-2}$ and $r_{\mathrm{source}} = r_{\mathrm{vac}}$, respectively, whereas in the bottom panel they show $r_{\mathrm{source}} = r_{\mathrm{vac}}$ and $r_{\mathrm{source}} = 10^{-5}$, respectively.
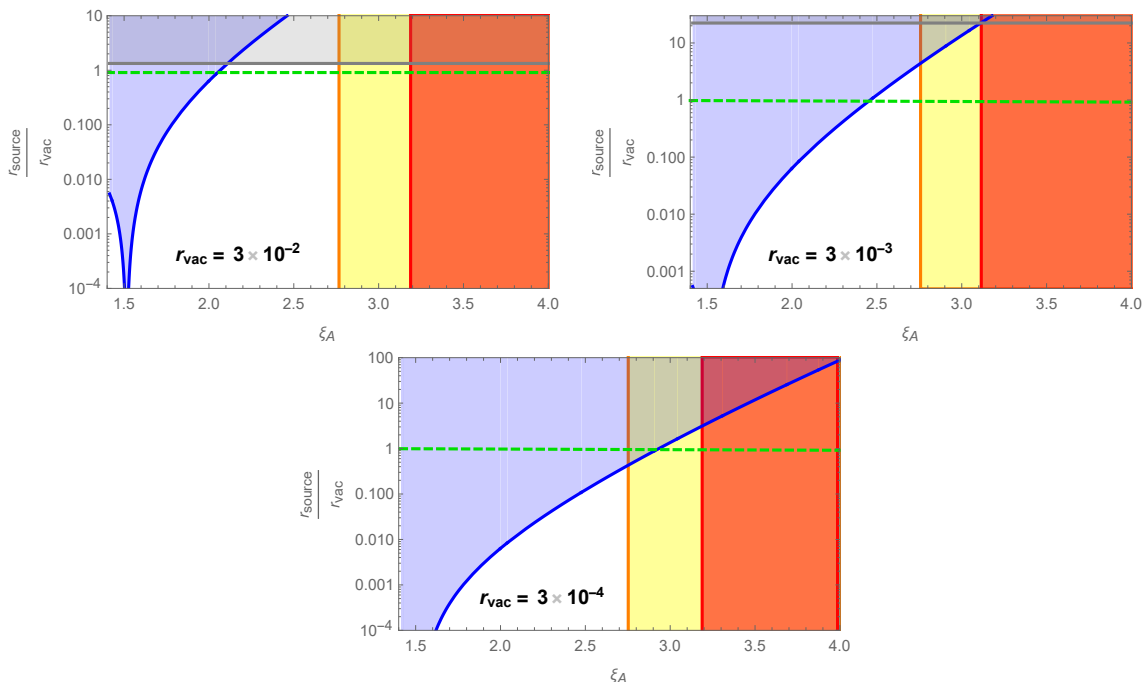
Figure 14. Constraints on the fractional contribution of the sourced gravitational waves relative to the vacuum one, $r_{\text{source}}/r_{\text{vac}} = (r - r_{\text{vac}})/r_{\text{vac}}$, as a function of ξ_A for three different values of $r_{\text{vac}} = 3 \times 10^{-2}$ (top left), 3×10^{-3} (top right), and 3×10^{-4} (bottom). The blue shaded area is excluded by the inconsistency of the slow-roll condition ($\epsilon_A/\epsilon > 0.9$), the yellow and orange areas by the large backreaction $\mathcal{B}_A > 10^{-2}$ and 10^{-1}, respectively, and the gray area by the tensor-to-scalar ratio $r > 0.07$. The dashed green line marks $r_{\text{source}}/r_{\text{vac}} = 1$.

The allowed parameter space we find in this section is much more constrained than that found in the literature for the spectator axion-SU(2) model [15, 40]. The reason is two folds. First, the upper limit of $\epsilon_B \simeq 10^{-2}$ adopted by the previous study is too conservative to satisfy the consistency of the slow-roll parameters given by $\epsilon_B/\epsilon = 16\epsilon_B/r_{\text{vac}} < 1$ for a given value of $r_{\text{vac}} < 0.07$. Second, the region of the strong backreaction was defined as $\mathcal{J}_A/(g_A\lambda\psi^2\dot{\varphi}/f) < 1$, which is too conservative to satisfy the slow-roll dynamics of the gauge field. The field equation of ψ for the massless case is given in (2.14)

$$\frac{(a\ddot{\psi})}{a} + \frac{H(a\dot{\psi})}{a} + 2g_A^2\psi^3 - \frac{g_A\lambda\dot{\varphi}}{f}\psi^2 = \mathcal{J}_A. \tag{6.9}$$

Assuming slow-roll dynamics of the gauge field, i.e., $\frac{\ddot{\psi}}{H^2\psi} \ll \frac{\dot{\psi}}{H\psi} \ll 1$, we can write it as

$$3H\dot{\psi} + \dot{H}\psi + V_{\text{eff},\psi}(\psi) \simeq 0, \tag{6.10}$$

where the field derivative of the effective potential of ψ is

$$V_{\text{eff},\psi}(\psi) \simeq 2H^2\psi(1 + \xi_A^2) - \frac{g_A\lambda\dot{\varphi}}{f}\psi^2. \tag{6.11}$$

Slow-roll demands $V_{\text{eff},\psi}(\psi) \ll 1$, while each of the terms in the right hand side can be much larger, e.g., $\frac{g_A \lambda \dot\varphi}{f} \psi^2 / V_{\text{eff},\psi} \gg 1$. On the other hand, $\mathcal{J}_A$ should be at most on the order of the slow-roll suppressed terms, i.e., $\frac{\mathcal{J}_A}{H^2 \psi} \equiv \mathcal{B}_A \ll 1$, which is more restrictive.

Before we leave this section, let us comment on the Higgsed models, e.g., Higgsed gauge-flation and Higgsed chromo-natural models [12, 14, 16]. Both scalar and tensor perturbations are amplified by the Higgs VEV in this set up, with the scalar ones being more strongly amplified; thus, the Higgs VEV, quantified by ξ_{Z_0}, reduces the tensor-to-scalar ratio [14, 16]. On the other hand, in section 4, we find that the size of the backreaction is much stronger in the Higgsed models with $\xi_{Z_0} \neq 0$. Hence, requiring slow-roll dynamics in the gauge field sector during (quasi)-de Sitter expansion, the Higgsed models cannot evade $r/r_{\text{vac}} < \mathcal{O}(1)$ bound either.

7 Conclusion

A background of axion and SU(2) gauge fields produces a copious amount of spin-2 particles during inflation. In this paper, we have calculated the number [eq. (3.31)] and energy densities [eq. (4.22)] of the spin-2 particles as well as their backreaction on the equations of motion of the axion and gauge field backgrounds [eqs. (4.9) and (4.10)]. We provided analytical formulae which are valid for all the inflation models with SU(2) gauge fields studied in the literature (see eq.s (2.24) and (2.25) for the definition of the model parameters and table 2 for their correspondence to the literature). The former results are new. The latter results were presented only numerically for a single model in this family but in [40]. With that exception, this is the first time that the backreaction constrains this family of models on the equations of motion. The analytical formulae derived in this paper allow us to easily estimate the importance of the backreaction for any parameters and constrain the parameter space that is consistent with perturbation theory. Moreover, it enables us to relate the backreaction in the gauge field and axion background equations to the number density of the spin-2 field as $\mathcal{J}_A \sim g_A n_{\text{pairs}}$ and $\mathcal{P}_\varphi \sim \frac{\lambda H}{f} n_{\text{pairs}}$ respectively.

These spin-2 particles mix with gravitational waves. We related the number density of the spin-2 particles to the power spectrum [eq. (5.6)] and energy density [eq. (5.12)] of primordial gravitational waves from inflation as well as to the size of the backreaction [eq. (4.17)]. The relation to the energy density $\delta\rho_s^{\text{GW}}$ is intuitive: $\delta\rho_s^{\text{GW}}$ is given by the number density of spin-2 particles times particle's physical momentum at horizon crossing (i.e., $k/a \sim H$), times the coupling strength squared, i.e., $\delta\rho_s^{\text{GW}} \sim (\psi/M_{\text{Pl}})^2 H n_{\text{pairs}}$. Moreover, the ratio of the power spectra of sourced and vacuum gravitational waves is also proportional to the number density of the spin-2 field and the VEV of the SU(2) gauge field, i.e., $P_T^s / P_T^{\text{vac}} \sim (\psi/M_{\text{Pl}})^2 (n_{\text{pairs}}/H^3)$. That gives us a physical insight into how the strength of gravitational waves from SU(2) gauge fields is determined.

Finally, we constrained the parameter space of the massless models in this class of inflationary scenarios. We find that the backreaction and the consistency of the slow-roll condition exclude most of the parameter space. In particular, the tensor-to-scalar ratio of the gravitational waves sourced by the gauge field can at most be on the order of that of

the standard vacuum contribution. Going beyond the massless models, we argue that the Higgsed models cannot evade this conclusion either.

The analytical study presented in this paper is based on quasi-de Sitter expansion and slow-roll dynamics of the background gauge field. For more general situations, full numerical analysis is required. At second order in perturbation, the spin-2 field couples to the scalar sector and contributes to the scalar power spectrum and non-Gaussianity. This non-linear effect may be important [45]. We expect that the loop contribution to the scalar power spectrum is related to the number density of the spin-2 field, n_{pairs}^2/H^6, that we computed in this paper.

Acknowledgments

A Symmetry of the VEV SU(2) field

In section 2, we present the metric and the VEV of the gauge field in a specific coordinate system in which the spatial metric is $a^2\delta_{ij}$. Here we present the general solution of the VEV of the gauge field which generates a homogeneous and isotropic energy-momentum tensor in general spatial coordinates.

Fixing the time-function, t, under a global rotation, we have

$$x^{'\mu} \mapsto x^\mu = \Lambda^\mu_{\ \nu} x^{'\nu}, \tag{A.1}$$

where $\Lambda^0_{\ \nu} = 0$ and the tetrad fields ($\eta^{\alpha\beta} = \mathbf{e}^\alpha_\mu \mathbf{e}^{\beta\mu}$) transform as

$$\mathbf{e}^{'\alpha}_\mu(t) \mapsto \mathbf{e}^\alpha_\mu(t) = \Lambda^\nu_{\ \mu} \mathbf{e}^{'\alpha}_\nu(t). \tag{A.2}$$

Since we choose to use the tetrad system with $\mathbf{e}^{'a}_0(t) = 0$ and $\mathbf{e}^{'a}_i(t) = a\delta^a_i$, under the action of (A.1), we have $\mathbf{e}^a_0(t) = 0$. The general form of the gauge field's VEV in the temporal gauge that can generate a homogeneous and isotropic energy-momentum tensor (hence respect the symmetries of the FLRW background) is given by

$$\bar{A}^a_\mu(t, \vec{x}) = \psi(t)\mathbf{e}^a_\mu(t). \tag{A.3}$$

More precisely, using (A.2) and the above solution, the field strength tensor is

$$\bar{F}^a_{0i} = \frac{1}{a}\partial_0\big(a\psi\big)\mathbf{e}^a_i(t) \quad \text{and} \quad \bar{F}^a_{ij} = g_{\text{A}}\psi^2\epsilon^a_{bc}\mathbf{e}^b_i(t)\mathbf{e}^c_j(t), \tag{A.4}$$

which leads to a homogeneous and isotropic energy-momentum tensor for the gauge field sector. In addition to the above global rotational symmetry, there is a residual gauge symmetry, a continuous global SU(2) symmetry, as well

$$U(\alpha) = \exp(i\alpha^a T_a), \tag{A.5}$$

which respects the temporal gauge and the form of the VEV in (A.3)

$$\bar{A}_\mu \mapsto \frac{1}{-ig_A} U \bar{D}_\mu U^{-1} = \psi(t) \mathbf{e}_\mu^a(t) U T_a U^{-1} = \psi(t) \mathbf{e}_\mu^a(t) T_a'. \tag{A.6}$$

It is straightforward to see that the VEV gauge field in (A.3) also satisfies

$$\nabla_\mu \bar{A}^\mu(t) = 0. \tag{A.7}$$

Finally, action has a Z_2 symmetry, Parity, as

$$\psi \mapsto -\psi \quad (\text{and for} \quad \mathcal{L}_A = \mathcal{L}_{Cn} \quad \varphi \mapsto -\varphi),$$

which can be spontaneously broken by the VEV while it is still the symmetry of the *background* energy-momentum tensor.

B Transverse-traceless field

In this part, we expand the gauge field's action around the VEV as[11]

$$A_\mu(x^\nu) = \bar{A}_\mu(t) + A_\mu^{\mathrm{SV}}(x^\nu) + B_\mu(x^\nu), \tag{B.1}$$

where $A_\mu^{\mathrm{SV}}(x^\nu)$ parameterizes the scalar and vector modes in A_μ^a (longitudinal modes),[12] while $B_\mu(x^\nu)$ is the transverse part of the gauge field

$$\nabla_\mu B^\mu(x^\nu) = 0. \tag{B.3}$$

Notice that under the action of the continuous global symmetry (A.5), B_μ transforms similar to (A.6). We have

$$\mathrm{tr}(B_\mu \bar{A}^\mu) = 0. \tag{B.4}$$

Perturbing action (2.5) and (2.6) and using (B.4), the quadratic action of $B_\mu(x^\nu)$ is[13]

$$\delta_B \mathcal{L}_A[\varphi, A] = -\frac{1}{2} [\tilde{D}_\mu B_\nu]^a [\tilde{D}^\mu B^\nu]^a + \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \bar{\alpha}_A \; B_\nu^a \partial_\lambda B_\sigma^a - \frac{1}{2} \alpha_H \; g_A^2 Z_0^2 B_\mu^a B_a^\mu$$
$$-\frac{1}{2} [M_{\mu\nu}^2]^a [B^\mu B^\nu]^a - \frac{1}{2} [M_\mu B_\nu]^a [M^\nu B^\mu]^a, \tag{B.6}$$

---

[11]Interestingly, the linear Einstein equations combine the B_μ with the GWs. However, it is independent of the other parts of the perturbed gauge field. In other words, the linearized Einstein equations do not combine B_i and A_i^{SV} which makes the decomposition (B.1) physically meaningful and possible.

[12]More precisely, after fixing the gauge, the perturbed gauge fields have $3 \times 4 - 3 = 9$ degrees of freedom which can be decomposed in terms of 3 scalars, 2 vectors and one tensor fluctuation. In particular, in the temporal gauge, we have

$$\delta A_i^a = \delta_i^a Q + \delta^{ak} \partial_{ik} \tilde{Z} + g_A \psi a \epsilon^a_{\;i}{}^k \partial_k (Z - \tilde{Z}) + \delta_a^j \partial_i v_j + \epsilon^a_{\;i}{}^j w_j + a \delta^{aj} \tilde{\gamma}_{ij}, \tag{B.2}$$

where Q, Z, and $\tilde{z}$ are scales, v_i, w_i and $\tilde{\gamma}_{ij}$ are transverse fields and $\tilde{\gamma}_{ij}$ is symmetric. However, $\tilde{\gamma}_{ij}$ is the only field that contributes to the transverse part of the A_i^a.

[13]For an Abelian field with action (2.6), the quadratic action of B_μ is

$$\delta_B \mathcal{L}_A[\varphi, A] = -\frac{1}{2} \nabla_\mu B_\nu \nabla^\mu B^\nu + \frac{\lambda}{2f} \epsilon^{\mu\nu\lambda\sigma} \partial_\mu \varphi B_\nu \partial_\lambda B_\sigma - \frac{\alpha_H}{2} g_A^2 Z_0^2 B_\mu B^\mu. \tag{B.5}$$

where δ_{B} denotes terms quadratic order in B_μ, $\bar{\alpha}_A$ is defined in (2.16), while the explicit form of the covariant derivative, $\tilde{D}_\mu$, and the mass terms are[14]

$$\tilde{D}_\mu \equiv \nabla_\mu - 2ig_{\mathrm{A}}\bar{A}_\mu, \tag{B.7}$$

$$M^2_{\mu\nu} \equiv 4ig_{\mathrm{A}}\epsilon_{\mu\nu}{}^{\lambda\sigma}\partial_\lambda\bar{\alpha}_A \ \bar{A}^a_\sigma T_a, \tag{B.8}$$

$$M_\mu \equiv 2g_{\mathrm{A}}\bar{A}^a_\mu T_a. \tag{B.9}$$

Note that $(M_\mu B_\nu)^2$ cancels the term proportional to $\bar{A}_\mu\bar{A}_\nu$ in the $(\tilde{D}_\mu B_\nu)^2$. Therefore, the only actual masses are the one proportional to $\bar{\alpha}_A$ and α_H.

One can determine $\dot{\bar{\alpha}}$ as a function of $\bar{A}_\mu$ by the gauge field background equation in (2.15). Notice that a given value of $\bar{A}_\mu$ and Z_0 gives the same value for $\bar{\alpha}_A$ regardless of whether $\mathcal{L}_A$ is $\mathcal{L}_{Gf}$ or $\mathcal{L}_{Cn}$. That indicates that their tensor sectors are the same at the linear order.

We emphasize that unlike the U(1) field in which its transverse part is a spin-1 field, the transverse part of A^a_μ (in the temporal gauge) can be written as

$$B^a_i = \delta^a_j B_{ij} = a\delta^a_j\tilde{\gamma}_{ij}. \tag{B.10}$$

In the following, we prove that $\tilde{\gamma}_{ij}$ is a (pseudo) spin-2 field.

B.1 (pseudo) spin-2 in perturbed SU(2) field?

At first, it may come as a surprise that there is a (pseudo) spin-2 degree of freedom in a spin-1 SU(2) gauge field as in (2.20). In this appendix, we show that once the gauge field is perturbed around its isotropic and homogeneous solution (2.2), there is a sector in the perturbed field that transforms as a spin-2 field under rotations and is an odd eigenstate of parity.[15]

It is convenient to write the fields in the complex spherical coordinates $(r, z, \bar{z})$ which are related to the standard spherical coordinates (r, θ, ϕ) as

$$z = e^{i\phi}\tan\frac{\theta}{2} \quad \text{and} \quad \bar{z} = e^{-i\phi}\tan\frac{\theta}{2}. \tag{B.11}$$

The FLRW background geometry in this coordinate system is

$$ds^2 = -dt^2 + a^2(t)\left(dr^2 + 2r^2\eta_{z\bar{z}}dzd\bar{z}\right), \tag{B.12}$$

where

$$\eta_{z\bar{z}} = \frac{2}{(1 + z\bar{z})^2}. \tag{B.13}$$

Moreover, consider a choice of the spatial triads $(\mathbf{e}^\alpha_i\mathbf{e}_{\alpha j} = g_{ij})$ as

$$\mathbf{e}^\alpha_r = a(t)\delta^\alpha_r \quad \text{and} \quad \mathbf{e}^\alpha_z = a(t)\delta^\alpha_z, \tag{B.14}$$

---

[14]Note that comparing to the background covariant derivative $\bar{D}_\mu = \nabla_\mu - ig_{\mathrm{A}}\bar{A}_\mu$, the gauge field has a factor of 2 in (B.7). That comes from the fact that $[X_\mu Y_\nu]^a = \frac{1}{2}[X_\mu, Y_\nu]^a$ and therefore $([\bar{D}_\mu, B_\nu])^a = (\tilde{D}_\mu B_\nu)^a$.

[15]Another way to see the pseudo-tensor nature of the spin-2 degree of freedom is the fact that it always couples with the tensor metric perturbation with a factor of ψ which is a pseudo-scalar.

where α is the index of the SO(3) algebra and runs from 1 to 3. One can write B_i^a in (B.10) as

$$B_i^a = \tilde{\gamma}_{ij}(t,\vec{x})\mathbf{e}^{\alpha j}\delta_\alpha^a. \tag{B.15}$$

Then in the $(r, z, \bar{z})$ coordinates and for a wave propagating in the direction $\hat{k} = -\hat{r}$, we have $B_r^a = 0$ while B_z^a and $B_{\bar{z}}^a$ are non-zero. Moreover, given the fact that $\mathrm{tr}(\tilde{\gamma}_{ij}) = 0$, the only non-zero components of $\tilde{\gamma}_{ij}$ are $\tilde{\gamma}_{zz}$ and $\tilde{\gamma}_{\bar{z}\bar{z}}$. Under the action of a rotation around the x^3-direction, we have

$$z \xmapsto{R} w = e^{i\delta}z \quad \text{and} \quad \bar{z} \xmapsto{R} \bar{w} = e^{-i\delta}\bar{z}, \tag{B.16}$$

and

$$\mathbf{e}^{\alpha z} \xmapsto{R} \mathbf{e}^{\alpha w} = e^{i\delta}\mathbf{e}^{\alpha z} \tag{B.17}$$

$$\delta_T A_z^a \xmapsto{R} \delta_T A_w^a = e^{-i\delta}\delta_T A_z^a. \tag{B.18}$$

Finally, from the combination of (B.17) and (B.18) we arrive at the desired result

$$\tilde{\gamma}_{zz} \xmapsto{R} \tilde{\gamma}_{ww} = e^{-2i\delta}\tilde{\gamma}_{zz}, \tag{B.19}$$

which shows that $\tilde{\gamma}_{ij}$ transforms as a spin-2 field under the action of rotations. Moreover, under the action of parity, we have

$$\delta A_i^a \xmapsto{P} -\delta A_i^a, \tag{B.20}$$

which leads to

$$\tilde{\gamma}_{ij} \xmapsto{P} -\tilde{\gamma}_{ij}. \tag{B.21}$$

Thus, $\tilde{\gamma}_{ij}$ is a pseudo-tensor. This completes the proof that $\tilde{\gamma}_{ij}$ is a (pseudo) spin-2 degree of freedom.

C Mathematical supplement

Here, we present some mathematical formulae and relations which we need throughout this work including some properties of Gamma and Whittaker functions as well as the asymptotic form of the Meijer G-functions in the large argument limit.

The Gamma function has simple poles for non-positive integers

$$\mathrm{Res}(\Gamma, -n) = \frac{(-1)^n}{n!} \quad (n \in \mathbb{N}). \tag{C.1}$$

Moreover, for any complex z, it satisfies

$$\Gamma(z+1) = z\Gamma(z), \tag{C.2}$$

which for non-integer values of z, gives

$$\Gamma(z)\Gamma(-z) = -\frac{\pi}{z\sin(\pi z)} \quad (z \notin \mathbb{Z}). \tag{C.3}$$

The derivative of the Γ-function can be written as a polygamma function

$$\psi^{(d-1)}(z) = \frac{d^d}{dz^d} \ln \Gamma(z), \tag{C.4}$$

which has the following series representation

$$\psi^{(d-1)}(z) = (-1)^d (d-1)! \sum_{j=0}^{\infty} \frac{1}{(z+j)^d}, \tag{C.5}$$

which holds for $d > 1$ and any complex z not equal to a negative integer. Therefore, the harmonic series can be written as

$$\sum_{q=1}^{n} \frac{1}{q} = -\psi^{(0)}(1) + \psi^{(0)}(n+1). \tag{C.6}$$

Furthermore, the asymptotic series

$$\lim_{|z| \to \infty} \Gamma(z) \simeq \sqrt{2\pi} z^{z-\frac{1}{2}} \exp(-z) \left(1 + \frac{1}{12z} + \frac{1}{288z^2} - \frac{139}{51840z^3} + \dots \right), \tag{C.7}$$

which is valid in the sector $|\arg(z)| < \pi$, leads to the following asymptotic expansion of the digamma function

$$\psi^{(0)}(z) = \ln(z) - \frac{1}{2z} - \frac{1}{12z^2} + \frac{1}{120z^4} + \dots , \quad \text{where} \quad |z| \to \infty. \tag{C.8}$$

For complex values of z in which $z = x + iy$ with finite real x and $y \to \infty$, we have [46]

$$\Gamma(x+iy) \simeq \sqrt{2\pi} |y|^{x-\frac{1}{2}} e^{-\pi y/2} e^{-i(y+\frac{\pi}{2}(\frac{1}{2}-x))}. \tag{C.9}$$

Finally, in the limit that z goes to zero, Gamma functions satisfy

$$\lim_{z \to 0} z\Gamma(-z-n) = -\frac{(-1)^n}{n!} \quad \text{and} \quad \lim_{z \to 0} \frac{d}{dz}(z\Gamma(-z-n)) = \frac{(-1)^n}{n!} \psi^{(0)}(n+1), \tag{C.10}$$

where $n \in \mathbb{N}$. For later convenience, we recall that in the complex analysis, if $f(z)$ has a pole of order k at $z = z_0$ then the residues are given as

$$\text{Res}(f, z_0) = \frac{1}{(k-1)!} \frac{d^{k-1}}{dz^{k-1}} \left((z-z_0)^k f(z) \right) \Big|_{z=z_0}. \tag{C.11}$$

The W and M Whittaker functions, which are the solutions of (2.26), have the asymptotic expansions

$$W_{\kappa,\mu} \sim e^{-z/2} z^\kappa (1 + \mathcal{O}(\tfrac{1}{z})) \qquad \text{for} \quad |z| \to \infty, \tag{C.12}$$

$$M_{\kappa,\mu} \sim z^{\mu+\frac{1}{2}} (1 + \mathcal{O}(z)) \qquad \text{for} \quad |z| \to 0, \tag{C.13}$$

implying that W/M functions correspond to positive frequency modes in the asymptotic past/future limits of de Sitter, respectively.

The W function satisfies the following relation

$$W_{\kappa,-\mu}(z) = W_{\kappa,\mu}(z).$$

In our setup in which z, μ and κ are all imaginary quantities, we have $W^*_{\kappa,\mu}(z) = W_{-\kappa,\mu}(-z)$. The W and M functions are related as

$$M_{\kappa,\mu}(z) = \frac{\Gamma(2\mu+1)}{\Gamma(\frac{1}{2}+\mu+\kappa)}e^{-i(\frac{1}{2}+\mu-\kappa)\pi}W_{\kappa,\mu}(z) + \frac{\Gamma(2\mu+1)}{\Gamma(\frac{1}{2}+\mu-\kappa)}e^{i\kappa\pi}W_{-\kappa,\mu}(e^{i\pi}z), \quad \text{(C.14)}$$

which holds when 2μ is not an integer and $-\frac{3\pi}{2} < |\text{argz}| < \frac{\pi}{2}$ [47]. The W-Whittaker functions have the Mellin-Barnes integral representation [46]

$$W_{\kappa,\mu}(z) = \frac{e^{-\frac{z}{2}}}{2i\pi}\int_{-i\infty}^{i\infty}\frac{\Gamma(\frac{1}{2}+\mu+s)\Gamma(\frac{1}{2}-\mu+s)\Gamma(-\kappa-s)}{\Gamma(\frac{1}{2}+\mu-\kappa)\Gamma(\frac{1}{2}-\mu-\kappa)}z^{-s}ds \quad \text{where} \quad |\arg(z)| < \frac{3}{2}\pi, \tag{C.15}$$

which holds when $\frac{1}{2}\pm\mu-\kappa \neq 0,-1,-2,\ldots$, and the contour of the integration separates the poles of $\Gamma(\frac{1}{2}+\mu+s)\Gamma(\frac{1}{2}-\mu+s)$ from poles of $\Gamma(-\kappa-s)$.

The Whittaker functions satisfy the following integral identities

$$\int x^n e^{-ix}W_{\kappa,\mu}(-2ix)dx = \frac{x^{n+1}\text{G}^{2,2}_{2,3}\left(-2ix\middle|\begin{matrix}-n, & 1+\kappa\\ \frac{1}{2}-\mu, & \mu+\frac{1}{2}, & -n-1\end{matrix}\right)}{\Gamma(\frac{1}{2}-\kappa-\mu)\Gamma(\frac{1}{2}-\kappa+\mu)}, \quad \text{(C.16)}$$

$$\int x^n e^{ix}W_{\kappa,\mu}(-2ix)dx = x^{n+1}\text{G}^{2,1}_{2,3}\left(-2ix\middle|\begin{matrix}-n, & 1-\kappa\\ \frac{1}{2}-\mu, & \mu+\frac{1}{2}, & -n-1\end{matrix}\right). \quad \text{(C.17)}$$

The Meijer G-functions with $Re(p) > 0$, $Re(q) > 0$ and $p - q \neq 0$, has the following asymptotic form for $x \gg 1$

$$\frac{1}{x^{p-1}}\text{G}^{2,1}_{2,3}\left(-2ix\middle|\begin{matrix}p, & q\\ \frac{1}{2}-\mu, & \frac{1}{2}+\mu, & p-1\end{matrix}\right) \simeq \frac{i(-2i)^p}{2}\Gamma\left(\frac{3}{2}-p-\mu\right)\Gamma\left(\frac{3}{2}-p+\mu\right)\Gamma(p-q),$$

$$+ \frac{i(-2i)^q}{2(q-p)}\Gamma\left(\frac{3}{2}-q-\mu\right)\Gamma\left(\frac{3}{2}-q+\mu\right)x^{q-p}. \text{ (C.18)}$$

$$\frac{1}{x^{p-1}}\text{G}^{2,2}_{2,3}\left(-2ix\middle|\begin{matrix}p, & q\\ \frac{1}{2}-\mu, & \frac{1}{2}+\mu, & p-1\end{matrix}\right) \simeq \frac{i(-2i)^p}{2}\frac{\Gamma(\frac{3}{2}-p-\mu)\Gamma(\frac{3}{2}-p+\mu)}{\Gamma(1-p+q)}. \quad \text{(C.19)}$$

D Computation of the induced currents

In this appendix, we compute the momentum integral $\mathcal{K}[X]$ in (4.8) which is necessary for the induced currents and energy density.[16] First, we work out the total integral which is divergent. In appendix D.1, we regularize this current by using adiabatic subtraction. Finally, in appendix D.2, we compute the energy density of the spin-2 fluctuations.

---

[16]The scalar induced current in the same setup has been worked out in [39]. The scalar induced current due to a U(1) case has been worked out in [48].

It is convenient to decompose $\mathcal{K}[X]$ in terms of polarizations as

$$\mathcal{K}[X] = \sum_{\sigma = \pm 2} \mathcal{K}_\sigma[X].$$

Using (2.28) and (4.8), we can write $\mathcal{K}_\sigma[X]$ as

$$\mathcal{K}_\sigma[X] = \lim_{\Lambda \to \infty} e^{i\kappa_\sigma \pi} \int_0^\Lambda \frac{\tilde{\tau} d\tilde{\tau}}{2} \left(-\lambda_\sigma \tilde{\tau} + X \right) |W_{\kappa_\sigma, \mu}(-2i\tilde{\tau})|^2, \tag{D.1}$$

where $\tilde{\tau}$ is a rescaled physical momentum and Λ is the physical UV cutoff which in the end will be sent to infinity

$$\tilde{\tau} \equiv \frac{k}{aH} \quad \text{and} \quad \Lambda = \frac{k_{\mathrm{UV}}}{aH}.$$

Note that in our setup, both κ and μ are pure imaginary. In the following, for notational convenience, we drop the argument X of $\mathcal{K}_\sigma[X]$ and σ subscript in κ_σ and λ_σ, unless otherwise stated. Upon using the integral representation of Whittaker functions in (C.15), we find

$$\mathcal{K}_\sigma = \lim_{\Lambda \to \infty} \frac{1}{8(2\pi)^2} e^{i\kappa\pi} \left[\Gamma\left(\frac{1}{2}+\mu-\kappa\right) \Gamma\left(\frac{1}{2}-\mu-\kappa\right) \Gamma\left(\frac{1}{2}+\mu^*+\kappa\right) \Gamma\left(\frac{1}{2}-\mu^*+\kappa\right) \right]^{-1}$$

$$\times \int_{-i\infty}^{i\infty} ds \int_{-i\infty}^{i\infty} ds' e^{i(s-s')\frac{\pi}{2}} (2\Lambda)^{2-s-s'} \left(-\frac{\lambda\Lambda}{3-s-s'} + \frac{X}{2-s-s'} \right) \Gamma\left(\frac{1}{2}+\mu+s\right) \Gamma\left(\frac{1}{2}-\mu+s\right)$$

$$\times \Gamma(-\kappa-s)\Gamma\left(\frac{1}{2}+\mu^*+s'\right) \Gamma\left(\frac{1}{2}-\mu^*+s'\right) \Gamma(\kappa-s'). \tag{D.2}$$

The integrand has singularities at $s' = -\frac{1}{2} \pm \mu - n$, $\kappa + n$, $3 - s$, and $2 - s$. Moreover, it is proportional to $\Lambda^{2-s-s'}$ which vanishes for $\mathrm{Re}(s') > 3 - \mathrm{Re}(s)$ in the limit $\Lambda \to \infty$. Upon choosing the contour of s such that $\mathrm{Re}(s) > -1$ and closing the s'-contour in the right-half plane without passing through the poles,[17] we are left with the following six poles

$$s_1' = \kappa, \ \ s_2' = \kappa + 1, \ \ s_3' = \kappa + 2, \ \ s_3' = \kappa + 3, \ \ s_5' = 2 - s, \quad \text{and} \quad s_6' = 3 - s. \tag{D.3}$$

Doing the s'-integral, we obtain

$$\mathcal{K}_\sigma = \frac{e^{i\kappa\pi}}{24(2\pi)^2} \left[\Gamma\left(\frac{1}{2}+\mu-\kappa\right) \Gamma\left(\frac{1}{2}-\mu-\kappa\right) \Gamma\left(\frac{1}{2}+\mu^*+\kappa\right) \Gamma\left(\frac{1}{2}-\mu^*+\kappa\right) \right]^{-1}$$

$$\times \lim_{\Lambda \to \infty} \int_{-i\infty}^{i\infty} ds\, \Gamma\left(\frac{1}{2}+\mu+s\right) \Gamma\left(\frac{1}{2}-\mu+s\right) \Gamma(-\kappa-s) K_\sigma(s, \Lambda). \tag{D.4}$$

---

[17]Note that the integral of (D.2) over a finite path along the real axis vanishes at $\lim \mathrm{Im}(s') \to \pm\infty$.

where $K_\sigma(s, \Lambda)$ is

$$
\begin{aligned}
K_\sigma(s, \Lambda) = (2i\pi)e^{-i\kappa\pi}e^{i(s+\kappa)\frac{\pi}{2}} \Bigg[& 3X\Gamma\left(\frac{5}{2}+\mu^*-s\right)\Gamma\left(\frac{5}{2}-\mu^*-s\right)\Gamma(\kappa-2+s) \\
& +\frac{3}{2}i\lambda\Gamma\left(\frac{7}{2}+\mu^*-s\right)\Gamma\left(\frac{7}{2}-\mu^*-s\right)\Gamma(\kappa-3+s) \\
& + (2\Lambda)^{2-s-\kappa}\left(\frac{3\lambda\Lambda}{s+\kappa-3}-\frac{3X}{s+\kappa-2}\right)\Gamma\left(\frac{1}{2}+\mu^*+\kappa\right)\Gamma\left(\frac{1}{2}-\mu^*+\kappa\right) \\
& + i(2\Lambda)^{1-s-\kappa}\left(\frac{3\lambda\Lambda}{s+\kappa-2}-\frac{3X}{s+\kappa-1}\right)\Gamma\left(\frac{3}{2}+\mu^*+\kappa\right)\Gamma\left(\frac{3}{2}-\mu^*+\kappa\right) \\
& - \frac{1}{2!}(2\Lambda)^{-s-\kappa}\left(\frac{3\lambda\Lambda}{s+\kappa-1}-\frac{3X}{s+\kappa}\right)\Gamma\left(\frac{5}{2}+\mu^*+\kappa\right)\Gamma\left(\frac{5}{2}-\mu^*+\kappa\right) \\
& - \frac{i}{4}(2\Lambda)^{-s-\kappa}\frac{\lambda}{s+\kappa}\Gamma\left(\frac{7}{2}+\mu^*+\kappa\right)\Gamma\left(\frac{7}{2}-\mu^*+\kappa\right) \Bigg].
\end{aligned} \tag{D.5}
$$

This integral is divergent and includes terms proportional to Λ^3, Λ^2 and Λ.

Let us first compute the (first two) finite terms in (D.5). Using (C.3), we write them as

$$
\mathcal{K}_{\sigma,1} = \frac{1}{24}\left[\Gamma\left(\frac{1}{2}+\mu-\kappa\right)\Gamma\left(\frac{1}{2}-\mu-\kappa\right)\Gamma\left(\frac{1}{2}+\mu^*+\kappa\right)\Gamma\left(\frac{1}{2}-\mu^*+\kappa\right)\right]^{-1}\mathcal{I}_\sigma, \tag{D.6}
$$

where

$$
\begin{aligned}
\mathcal{I}_\sigma = \frac{3\pi}{(2i\pi)}\int_{-i\infty}^{+i\infty}ds\, & \frac{e^{i(s+\kappa)\pi}}{\sin(\pi(s+\kappa))}\Gamma\left(\frac{1}{2}+\mu^*-s\right)\Gamma\left(\frac{1}{2}-\mu^*-s\right)\Gamma\left(\frac{1}{2}+\mu+s\right)\Gamma\left(\frac{1}{2}-\mu+s\right) \\
& \times\frac{\left(\left(\frac{3}{2}-s\right)^2-\mu^2\right)\left(\left(\frac{1}{2}-s\right)^2-\mu^2\right)}{(\kappa+s)(\kappa+s-1)(\kappa+s-2)}\left[X+i\lambda\frac{\left(\frac{5}{2}-s\right)^2-\mu^2}{2(\kappa+s-3)}\right].
\end{aligned} \tag{D.7}
$$

It can be further simplified as

$$
\begin{aligned}
\mathcal{I}_\sigma = \frac{\pi}{(2i\pi)}\int_{-i\infty}^{+i\infty}ds\, & \Gamma\left(\frac{1}{2}+\mu^*-s\right)\Gamma\left(\frac{1}{2}-\mu^*-s\right)\Gamma\left(\frac{1}{2}+\mu+s\right)\Gamma\left(\frac{1}{2}-\mu+s\right)\frac{e^{i(s+\kappa)\pi}}{\sin(\pi(\kappa+s))} \\
& \times\left[\tilde{\mathcal{C}}(s)-\tilde{\mathcal{C}}(s-1)+\frac{\mathcal{C}(X)}{(\kappa+s)}\right],
\end{aligned} \tag{D.8}
$$

in which $\mathcal{C}(X)$ and $\tilde{\mathcal{C}}(s)$ are

$$
\mathcal{C}(X) \equiv 3X\left(\frac{1}{2}+6\kappa^2-2\mu^2\right)-\frac{3}{2}(7+20\kappa^2-12\mu^2)|\kappa|, \tag{D.9}
$$

$$
\begin{aligned}
\tilde{\mathcal{C}}(s) \equiv\ & \frac{c_1(s+3)-3c_1(s+2)+3c_1(s+1)-c_2(s+2)+2c_2(s+1)}{\kappa+s} \\
& +\frac{c_1(s+2)-3c_1(s+1)-c_2(s+1)}{\kappa+s-1}+\frac{c_1(s+1)}{\kappa+s-2}+9Xs(s+1-2\kappa) \\
& +\frac{3}{2}i\lambda(7+20\kappa^2-12\mu^2)s+5i\lambda s(s+1)(1+2s-3\kappa).
\end{aligned} \tag{D.10}
$$

Here, $c_1(s)$ and $c_2(s)$ are the following functions of s

$$c_1(s) = -\frac{i\lambda}{4}\left[\left(\frac{5}{2} - s\right)^2 - \mu^2\right]\left[\left(\frac{3}{2} - s\right)^2 - \mu^2\right]\left[\left(\frac{1}{2} - s\right)^2 - \mu^2\right], \qquad \text{(D.11)}$$

$$c_2(s) = \frac{3X}{2}\left[\left(\frac{3}{2} - s\right)^2 - \mu^2\right]\left[\left(\frac{1}{2} - s\right)^2 - \mu^2\right]. \qquad \text{(D.12)}$$

Recalling that μ is pure imaginary, the first two terms in integral (D.8) can be written as

$$\mathcal{I}_{\sigma,1} = \frac{\pi}{(2i\pi)}\left(\int_{-i\infty}^{+i\infty} ds - \int_{-i\infty-1}^{+i\infty-1} ds\right)\Gamma\left(\frac{1}{2} + \mu^* - s\right)\Gamma\left(\frac{1}{2} - \mu^* - s\right)$$
$$\times \Gamma\left(\frac{1}{2} + \mu + s\right)\Gamma\left(\frac{1}{2} - \mu + s\right)\frac{e^{i(s+\kappa)\pi}}{\sin(\pi(\kappa + s))}\tilde{\mathcal{C}}(s). \qquad \text{(D.13)}$$

which has poles at

$$s_{n,\pm} = -\frac{1}{2} \pm \mu - n, \qquad\qquad s_{n,0} = -\kappa + n, \qquad\qquad\qquad \text{(D.14)}$$

$$\tilde{s}_{n,\pm} = \frac{1}{2} \pm \mu + n, \qquad\qquad \tilde{s}_{n,0} = -\kappa - n, \qquad (n \in \mathbb{N}). \qquad \text{(D.15)}$$

Notice that $s_{n,0}$ poles with $n = 0, 1, 2$ are 2nd rank while the rest are simple poles. Therefore, it is more convenient to close the contour path of s on the left half-plane which only includes simple poles below

$$s_1 = -1 - \kappa, \quad s_{0,\pm} = -\frac{1}{2} \pm \mu.$$

Doing the complex integral, we have

$$\mathcal{I}_{\sigma,1} = \Gamma\left(\frac{1}{2} + \mu^* + \kappa\right)\Gamma\left(\frac{1}{2} - \mu^* + \kappa\right)\Gamma\left(\frac{1}{2} + \mu - \kappa\right)\Gamma\left(\frac{1}{2} - \mu - \kappa\right)\left\{\tilde{\mathcal{C}}(-1 - \kappa)\right.$$
$$\left. + \frac{i}{2}\left[\frac{e^{2i\pi\kappa} + e^{2i\pi\mu}}{\sin(2\pi\mu)}\tilde{\mathcal{C}}\left(-\frac{1}{2} + \mu\right) - \frac{e^{2i\pi\kappa} + e^{-2i\pi\mu}}{\sin(2\pi\mu)}\tilde{\mathcal{C}}\left(-\frac{1}{2} - \mu\right)\right]\right\}, \qquad \text{(D.16)}$$

in which the explicit forms of $\tilde{\mathcal{C}}(-\frac{1}{2} + \mu)$ and $\tilde{\mathcal{C}}(-\frac{1}{2} - \mu)$ are

$$\frac{1}{2}\left[\tilde{\mathcal{C}}\left(-\frac{1}{2} + \mu\right) + \tilde{\mathcal{C}}\left(-\frac{1}{2} - \mu\right)\right] = \frac{9}{4}X(-1 + 4\kappa + 4\mu^2) - \frac{3i\lambda}{4}(7 + 20\kappa^2 - 12\mu^2 + 5\kappa(-1 + 4\mu^2)),$$

$$\frac{1}{2}\left[\tilde{\mathcal{C}}\left(-\frac{1}{2} + \mu\right) - \tilde{\mathcal{C}}\left(-\frac{1}{2} - \mu\right)\right] = 2\lambda|\mu|\left[-9X|\kappa| - 4 - 15\kappa^2 + 4\mu^2\right]. \qquad \text{(D.17)}$$

Now we turn to the last term in (D.8) which is

$$\mathcal{I}_{\sigma,2} = \frac{\mathcal{C}(X)}{(2i\pi)}\pi \int_{-i\infty}^{+i\infty} ds\, \Gamma\left(\frac{1}{2} + \mu^* - s\right)\Gamma\left(\frac{1}{2} - \mu^* - s\right)\Gamma\left(\frac{1}{2} + \mu + s\right)\Gamma\left(\frac{1}{2} - \mu + s\right)$$
$$\times \frac{e^{i(s+\kappa)\pi}}{(\kappa + s)\sin(\pi(\kappa + s))}. \qquad \text{(D.18)}$$

Closing the contour on the left half-plane with an infinite radius semicircle,[18] we have the poles $s_{n,\pm}$ and $\tilde{s}_{n+1,0}$, which are an infinite number of simple poles. Summing up the contribution of s_n poles and using (C.5), we have

$$\mathcal{I}_{\sigma,2}\big|_{\tilde{s}_{n+1,0}-poles} = \mathcal{C}(X)\,\psi(1)\,\Gamma\left(\frac{1}{2}+\mu^*+\kappa\right)\Gamma\left(\frac{1}{2}-\mu^*+\kappa\right)\Gamma\left(\frac{1}{2}+\mu-\kappa\right)\Gamma\left(\frac{1}{2}-\mu-\kappa\right). \tag{D.19}$$

Moreover, using (C.5), we find the contribution of $s_{n,\pm}$ poles as

$$\mathcal{I}_{\sigma,2}\big|_{s_{n,\pm}-poles} = \mathcal{C}(X)\,\Gamma\left(\frac{1}{2}+\mu^*+\kappa\right)\Gamma\left(\frac{1}{2}-\mu^*+\kappa\right)\Gamma\left(\frac{1}{2}+\mu-\kappa\right)\Gamma\left(\frac{1}{2}-\mu-\kappa\right)$$
$$\times\frac{i}{2\sin(2\mu\pi)}\left((e^{2i\kappa\pi}+e^{2i\mu\pi})\psi^{(0)}\left(\frac{1}{2}-\kappa-\mu\right)-(e^{2i\kappa\pi}+e^{-2i\mu\pi})\psi^{(0)}\left(\frac{1}{2}-\kappa+\mu\right)\right), \tag{D.20}$$

where $\psi^{(0)}(z) = \frac{d}{dz}\ln\Gamma(z)$ is the digamma function. Finally, adding (D.16), (D.19) and (D.20), we obtain

$$\mathcal{K}_{\sigma,1} = \frac{1}{24}\left[\tilde{\mathcal{C}}(-1-\kappa)+2i\lambda|\mu|(4\mu^2-15\kappa^2-4-9|\kappa|X)\left[\frac{\cos(2\pi\mu)+e^{2i\pi\kappa}}{\sin(2\pi\mu)}\right]-\frac{9}{4}(4\mu^2-1)X\right.$$
$$+\mathcal{C}(X)\left(\psi^{(0)}(1)+\frac{i}{2}\frac{(e^{2i\kappa\pi}+e^{2i\mu\pi})}{\sin(2\mu\pi)}\psi^{(0)}\left(\frac{1}{2}-\kappa-\mu\right)-\frac{i}{2}\frac{(e^{2i\kappa\pi}+e^{-2i\mu\pi})}{\sin(2\mu\pi)}\psi^{(0)}\left(\frac{1}{2}-\kappa+\mu\right)\right)$$
$$\left.+\frac{3i\lambda}{4}(7+20\kappa^2-12\mu^2+12X|\kappa|)-\frac{15}{4}|\kappa|(1-4\mu^2)\right]. \tag{D.21}$$

Now we turn to the remaining 4 (Λ-dependent) lines in (D.5). Here, we close the contour in the right-half plane which encloses the following poles at the $\Lambda\to\infty$ limit

$$s_1 = -\kappa,\ s_2 = 1-\kappa,\ s_3 = 2-\kappa,\quad\text{and}\quad s_4 = 3-\kappa.$$

Some of the above poles are second rank and therefore the integral includes a derivative of the Gamma function, i.e. $\psi^{(0)}(z)$. Doing the second complex integral in (D.4) and using (C.10) and (C.11), we find

$$\mathcal{K}_{\sigma,2} = \frac{1}{24}\lim_{\Lambda\to\infty}\left[-4\lambda\Lambda^3+6(X-|\kappa|)\Lambda^2+3\lambda\Lambda\left(4|\kappa|X+\frac{1}{2}(1+12\kappa^2-4\mu^2)\right)-\mathcal{C}(X)\ln(2\Lambda)\right.$$
$$+\mathcal{C}(X)\left(\frac{i\pi}{2}-\psi^{(0)}(1)+\psi^{(0)}\left(\frac{1}{2}+\mu-\kappa\right)+\psi^{(0)}\left(\frac{1}{2}-\mu-\kappa\right)\right)-\tilde{\mathcal{C}}(-1-\kappa)$$
$$\left.-\frac{3i\lambda}{4}\left(7+20\kappa^2-12\mu^2+12X|\kappa|\right)-|\kappa|(20+37\kappa^2)+X\left(\frac{3}{2}+21\kappa^2+6\mu^2\right)\right]. \tag{D.22}$$

---

[18]More precisely, we use $(\kappa+s)^d$ $(d>1)$ in the denominator of (D.18) and then compute the $d\to 1$ limit solution. As a result, the added infinite radius semicircle integral vanishes.

Adding (D.21) and (D.22), we finally find the desired $\mathcal{K}_\sigma$ as

$$\mathcal{K}_\sigma = \frac{1}{24}\lim_{\Lambda\to\infty}\left[-4\lambda_\sigma\Lambda^3 + 6(X-|\kappa|)\Lambda^2 + 3\lambda_\sigma\Lambda\left(4|\kappa|X + \frac{1}{2}(1+12\kappa^2-4\mu^2)\right)\right.$$
$$-\mathcal{C}(X)\ln(2\Lambda) - |\kappa|\left(\frac{95}{4}+37\kappa^2-15\mu^2\right) + 2i\lambda_\sigma|\mu|(4\mu^2-15\kappa^2-4-9|\kappa|X)\left[\frac{\cos(2\pi\mu)+e^{2i\pi\kappa_\sigma}}{\sin(2\pi\mu)}\right]$$
$$+\frac{i}{2}\mathcal{C}(X)\left(\pi + \frac{(e^{2i\kappa_\sigma\pi}+e^{-2i\mu\pi})}{\sin(2\mu\pi)}\psi^{(0)}\left(\frac{1}{2}-\kappa_\sigma-\mu\right) - \frac{(e^{2i\kappa_\sigma\pi}+e^{2i\mu\pi})}{\sin(2\mu\pi)}\psi^{(0)}\left(\frac{1}{2}-\kappa_\sigma+\mu\right)\right)$$
$$\left.+3X\left(\frac{5}{4}+7\kappa^2-\mu^2\right)\right]. \tag{D.23}$$

It has divergent terms of the order 3, 2, 1 and log of Λ. Summing over the polarization states, we find

$$\mathcal{K} = \frac{1}{12}\lim_{\Lambda\to\infty}\left\{6(X-|\kappa|)\Lambda^2 - \mathcal{C}(X)\ln(2\Lambda) - |\kappa|\left(\frac{95}{4}-37|\kappa|^2+15|\mu|^2\right)+3X\left(\frac{5}{4}-7|\kappa|^2+|\mu|^2\right)\right.$$
$$+2|\mu|(-4|\mu|^2+15|\kappa|^2-4-9|\kappa|X)\frac{\sinh(2\pi|\kappa|)}{\sinh(2\pi|\mu|)}$$
$$+\frac{\mathcal{C}(X)}{4}\mathrm{Re}\left[\frac{(e^{2|\kappa|\pi}+e^{2|\mu|\pi})}{\sinh(2|\mu|\pi)}\psi^{(0)}\left(\frac{1}{2}+i|\kappa|-i|\mu|\right)\right.$$
$$-\frac{(e^{2|\kappa|\pi}+e^{-2|\mu|\pi})}{\sinh(2|\mu|\pi)}\psi^{(0)}\left(\frac{1}{2}+i|\kappa|+i|\mu|\right)+\frac{(e^{-2|\kappa|\pi}+e^{2|\mu|\pi})}{\sinh(2|\mu|\pi)}\psi^{(0)}\left(\frac{1}{2}-i|\kappa|-i|\mu|\right)$$
$$\left.\left.-\frac{(e^{-2|\kappa|\pi}+e^{-2|\mu|\pi})}{\sinh(2|\mu|\pi)}\psi^{(0)}\left(\frac{1}{2}-i|\kappa|+i|\mu|\right)\right]\right\}. \tag{D.24}$$

We find that $\mathcal{K}$ is real and has divergent terms of the order 2 and log of Λ.

Before renormalizing $\mathcal{K}$ and removing Λ terms, let us take a closer look at the finite terms in (D.24). Recalling (2.10) and (3.18) ($|\kappa| > 3.5$), we realize that the dominant finite terms are proportional to $e^{2(|\kappa|-|\mu|)\pi} > e^{\pi\xi_A} \gg 1$. Thus, we can approximate $\mathcal{K}[X]$ as

$$\mathcal{K}[X] = \frac{1}{6}e^{2(|\kappa|-|\mu|)\pi}\left\{|\mu|(-4|\mu|^2+15|\kappa|^2-4-9|\kappa|X)+\frac{\mathcal{C}(X)}{4}\mathrm{Re}\left[\psi^{(0)}\left(\frac{1}{2}+i|\kappa|-i|\mu|\right)\right.\right.$$
$$\left.\left.-\psi^{(0)}\left(\frac{1}{2}+i|\kappa|+i|\mu|\right)\right]\right\}+\lim_{\Lambda\to\infty}\left[\frac{1}{2}(X-|\kappa|)\Lambda^2 - \frac{\mathcal{C}(X)}{12}\ln(2\Lambda)\right]+\mathcal{O}(\xi_A^{3-m}\xi_{Z_0}^m), \tag{D.25}$$

where $\mathcal{O}(\xi_A^{3-m}\xi_{Z_0}^m)$ with $m=0,1,2,3$ are the next leading terms and hence negligible. The quantity $(|\kappa|-|\mu|)$ is presented in figure 3. In the next section, we renormalize the above and find the physical quantity, $\mathcal{K}_{\mathrm{reg}}[X]$, as

$$\mathcal{K}_{\mathrm{reg}}[X] \equiv \mathcal{K}[X] - \mathcal{K}^{c.t.}[X], \tag{D.26}$$

where $\mathcal{K}^{c.t.}[X]$ is the counter-term.

D.1 Regularized current

In this section, we use the adiabatic subtraction technique in curved QFT to remove the divergent terms in the current. The mode function $B_{\sigma,\vec{k}}(\tau)$ has the following WKB form

$$B_{\sigma,\vec{k}}^{WKB}(\tau) = \frac{1}{(2\pi)^{\frac{3}{2}}\sqrt{2W_{\sigma,\vec{k}}(\tau)}}e^{-i\int_{-\infty}^{\tau}d\tilde{\tau}W_{\sigma,\vec{k}}(\tilde{\tau})}, \tag{D.27}$$

where $W_{\sigma,\vec{k}}^2$ can be written in terms of the instantaneous frequency, $\omega_{\sigma,\vec{k}}^2$, defined by

$$\omega_{\sigma,\vec{k}}^2 = k^2 - \lambda_\sigma \delta_{\rm c} k\mathcal{H} + \frac{\tilde{m}^2}{H^2}\mathcal{H}^2, \tag{D.28}$$

as

$$W_{\sigma,\vec{k}}^2 = \omega_{\sigma,\vec{k}}^2 - \frac{a''}{a} + \frac{3}{4}\left(\frac{W_{\sigma,\vec{k}}'}{W_{\sigma,\vec{k}}}\right)^2 - \frac{1}{2}\frac{W_{\sigma,\vec{k}}''}{W_{\sigma,\vec{k}}}. \tag{D.29}$$

If $W_{\sigma,\vec{k}}$ is real and positive, then $B_{\sigma,\vec{k}}^{WKB}(\tau)$ corresponds to canonically normalized positive frequency modes in the asymptotic past. For regularization in 4 dimension, we need to expand $W_{\sigma,\vec{k}}$ up to the second order of time derivatives with respect to a as

$$W_{\sigma,\vec{k}}^2 = \omega_{\sigma,\vec{k}}^2 - \frac{a''}{a} + \frac{3}{4}\left(\frac{\omega_{\sigma,\vec{k}}'}{\omega_{\sigma,\vec{k}}}\right)^2 - \frac{1}{2}\frac{\omega_{\sigma,\vec{k}}''}{\omega_{\sigma,\vec{k}}}. \tag{D.30}$$

Using (D.30) in (4.8), we obtain $\mathcal{K}_\sigma^{c.t.}[X]$ as[19]

$$\mathcal{K}_\sigma^{c.t.} = \lim_{\Lambda\to\infty}\int^\Lambda \tilde{\tau}'^2 d\tilde{\tau}' \frac{\mathcal{H}}{2\omega_{\sigma,\vec{k}}}\big(-\lambda_\sigma\tilde{\tau}' + X\big)\left(1 + \frac{\mathcal{H}^2}{\omega_{\sigma,\vec{k}}^2} + \frac{\mathcal{H}^4}{4\omega_{\sigma,\vec{k}}^4}\Big(-2\lambda_\sigma|\kappa|\tilde{\tau}' + \frac{27}{4} - 3\mu^2\Big)\right.$$
$$\left. - \frac{5}{8}\frac{\mathcal{H}^6}{\omega_{\sigma,\vec{k}}^6}\left(-\lambda_\sigma|\kappa|\tilde{\tau}' + \frac{9}{4} - \mu^2\right)^2\right). \tag{D.31}$$

Doing the integral, and summing over the polarization states, we obtain

$$\mathcal{K}_\sigma^{c.t.} = \frac{1}{12}\lim_{\Lambda\to\infty}\left[6(X - |\kappa|)\Lambda^2 - \mathcal{C}(X)\ln(2\Lambda)\right] + \mathcal{O}(\xi_{\rm A}^{3-m}\xi_{Z_0}^m). \tag{D.32}$$

The above counter term has divergences of the order 2 and log of Λ which cancel with the divergences of the total $\mathcal{K}$ in (D.25). Moreover, it has finite terms of order $\mathcal{O}(\xi_{\rm A}^{3-m}\xi_{Z_0}^m)$ which are subleading compared to the dominate terms in $\mathcal{K}$ and we neglect them here. Finally, subtracting (D.32) from (D.25), we have the desired regularized $\mathcal{K}[X]$ as

$$\mathcal{K}_{\rm reg}[X] = \frac{1}{6}e^{2(|\kappa|-|\mu|)\pi}\left\{|\mu|(-4|\mu|^2 + 15|\kappa|^2 - 4 - 9|\kappa|X) + \frac{\mathcal{C}(X)}{4}\mathrm{Re}\left[\psi^{(0)}\left(\frac{1}{2} + i|\kappa| - i|\mu|\right)\right.\right.$$
$$\left.\left. - \psi^{(0)}\left(\frac{1}{2} + i|\kappa| + i|\mu|\right)\right]\right\} + \mathcal{O}(\xi_{\rm A}^{3-m}\xi_{Z_0}^m), \tag{D.33}$$

where $\mathcal{O}(\xi_{\rm A}^{3-m}\xi_{Z_0}^m)$ with $m = 0, 1, 2, 3$ are the next leading terms and hence negligible.

---

[19]Note that the instantaneous frequency squared is negative in the interval $\tilde{\tau} \in (\tilde{\tau}_2, \tilde{\tau}_1)$ in (3.7). Therefore, technically, we have to consider an IR cut-off for the momentum integral, $\Lambda_{IR} \gtrsim \tilde{\tau}_1$. In principle it can be a problem since Λ_{IR} explicitly appears in the finite terms. However, this effect and the other finite terms in $\mathcal{K}^{c.t.}$ are at most of the order $\mathcal{O}(\xi_{\rm A}^{3-m}\xi_{Z_0}^m)$ with $m = 0, 1, 2, 3$ and are subleading comparing with the dominant finite terms of $\mathcal{K}$ in (D.25). As a result, a more careful regularization process would not improve our results.

D.2 Energy density

Here we compute the energy density in the spin-2 fluctuations of the gauge field which is presented in (4.20). Going to Fourier space, we can write $\langle \delta_{\rm B} \rho \rangle$ as

$$\langle \delta_{\rm B} \rho \rangle = \frac{1}{2a^4} \sum_\sigma \int_0^\Lambda dk^3 \left[|B'_\sigma|^2 + \left(k^2 - 2\lambda_\sigma \xi_{\rm A} k \mathcal{H} + \alpha_H \xi_{Z_0}^2 \mathcal{H}^2 \right) |B_\sigma|^2 \right], \qquad \text{(D.34)}$$

where in the end we send Λ to infinity. From (2.28), we can write B_σ as

$$B_\sigma(\tau, \vec{k}) = \frac{1}{\sqrt{k}} \bar{B}_\sigma(\tilde{\tau}).$$

We then write the energy density as

$$\langle \delta_{\rm B} \rho \rangle = (2\pi) H^4 \sum_\sigma \int_0^\Lambda \tilde{\tau} d\tilde{\tau} \left[\tilde{\tau}^2 \left(|\partial_{\tilde{\tau}} \bar{B}_\sigma|^2 + |\bar{B}_\sigma|^2 \right) + \left(-2\lambda_\sigma \xi_{\rm A} \tilde{\tau} + \alpha_H \xi_{Z_0}^2 \right) |\bar{B}_\sigma|^2 \right]. \quad \text{(D.35)}$$

The first term in the integral can be written as

$$\int_0^\Lambda \tilde{\tau}^3 d\tilde{\tau} |\partial_{\tilde{\tau}} \bar{B}_\sigma|^2 = \int_0^\Lambda d\tilde{\tau} \left[\partial_{\tilde{\tau}} \left(\tilde{\tau}^3 \bar{B}_\sigma \partial_{\tilde{\tau}} \bar{B}_\sigma - \frac{3}{2} \tilde{\tau}^2 \bar{B}_\sigma^2 \right) + 3\tilde{\tau} \bar{B}_\sigma^2 - \tilde{\tau}^3 \bar{B}_\sigma \partial_{\tilde{\tau}}^2 \bar{B}_\sigma \right]. \quad \text{(D.36)}$$

Using the field equation of $\tilde{B}_\sigma$ in (2.25), we arrive at

$$\int_0^\Lambda \tilde{\tau}^3 d\tilde{\tau} |\partial_{\tilde{\tau}} \bar{B}_\sigma|^2 = \int_0^\Lambda \tilde{\tau} d\tilde{\tau} \left[1 + \tilde{\tau}^2 - \lambda_\sigma \delta_c \tilde{\tau} + \frac{\tilde{m}^2}{H^2} \right] |\bar{B}_\sigma|^2 \right] + \tilde{\mathcal{C}}_{c.t.}, \qquad \text{(D.37)}$$

where $\tilde{\mathcal{C}}_{c.t.}$ comes from integrating the total derivative term

$$\tilde{\mathcal{C}}_{c.t.} = \lim_{\Lambda \to \infty} \left(\tilde{\tau}^3 \bar{B}_\sigma \partial_{\tilde{\tau}} \bar{B}_\sigma - \frac{3}{2} \tilde{\tau}^2 \bar{B}_\sigma^2 \right) \bigg|_{\tilde{\tau} = \Lambda}. \qquad \text{(D.38)}$$

Using (D.36), we can write the bare energy density as

$$\langle \delta_{\rm B} \rho \rangle = (2\pi) H^4 \left\{ \sum_\sigma \int_0^\Lambda \tilde{\tau} d\tilde{\tau} \left[(\delta_c + 2\xi_{\rm A}) \left(-\lambda_\sigma \tilde{\tau} + \frac{1 + \frac{\tilde{m}^2}{H^2} + \alpha_H \xi_{Z_0}^2}{\delta_c + 2\xi_{\rm A}} \right) + 2\tilde{\tau}^2 \right] |\bar{B}_\sigma|^2 + \mathcal{C}_{c.t.} \right\}. \tag{D.39}$$

From (4.8), the first two terms can be written in terms of $\mathcal{K} \left[\frac{1 + \frac{\tilde{m}^2}{H^2} + \alpha_H \xi_{Z_0}^2}{\delta_c + 2\xi_{\rm A}} \right]$ and we have its renormalized form in (D.33). The *regularized* part of the last term inside the brackets satisfies the following inequality (see figure 15)

$$\sum_\sigma \int d\tilde{\tau} \tilde{\tau}^3 |\bar{B}_\sigma|^2 \lesssim 0.1 \times (\delta_c + 2\xi_{\rm A}) \sum_\sigma \int |\bar{B}_\sigma|^2 \tilde{\tau}^2 d\tilde{\tau}, \qquad \text{(D.40)}$$

which implies that it is negligible comparing to the first term in (D.39). Thus, the *regularized* energy density can be well approximated as

$$\langle \delta_{\rm B} \rho \rangle_{\rm reg} \approx \frac{H^4}{(2\pi)^2} (\delta_c + 2\xi_{\rm A}) \mathcal{K}_{\rm reg} \left[\frac{1 + \frac{\tilde{m}^2}{H^2} + \alpha_H \xi_{Z_0}^2}{\delta_c + 2\xi_{\rm A}} \right], \qquad \text{(D.41)}$$

which, as we see, is given in terms of the regularized $\mathcal{K}_{\rm reg}[X]$ in (D.33) with $X = \frac{1 + \frac{\tilde{m}^2}{H^2} + \alpha_H \xi_{Z_0}^2}{\delta_c + 2\xi_{\rm A}}$.
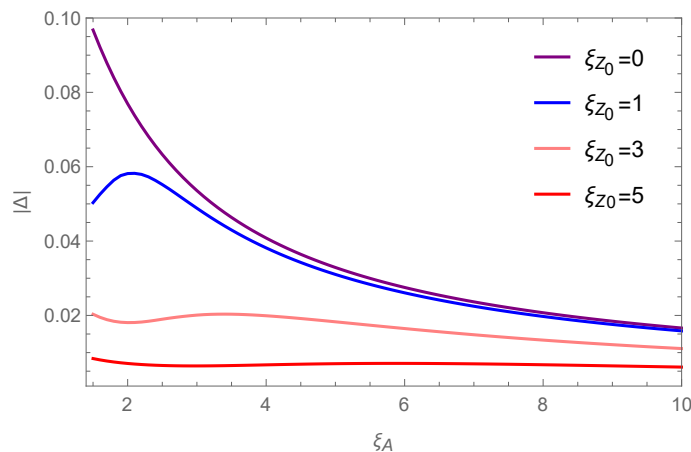
Figure 15. The ratio $\Delta \equiv \left(\frac{\int_0^{\tilde{\tau}_{\mathrm{IR}}} d\tilde{\tau} \tilde{\tau}^3 |\bar{B}_+|^2}{(\delta_c + 2\xi_{\mathrm{A}}) \int_0^{\tilde{\tau}_{\mathrm{IR}}} d\tilde{\tau} \tilde{\tau}^2 |\bar{B}_+|^2} \right)$ as a function of ξ_{A} and ξ_{Z_0}. Here $\tilde{\tau}_{\mathrm{IR}}$ is the physical momentum in which the deviation from adiabaticity is $\Omega_+(\tilde{\tau}_{\mathrm{IR}}) = 0.1$. The ratio Δ for the minus polarization is smaller with the same order of magnitude.

E Sourced graviational waves

In this appendix, we work out the analytical form of the gravitational waves sourced by the gauge field for the general action (2.4). The derivation given here follows closely [11].

The inhomogeneous solution of (2.24) sourced by B_σ is given as

$$ h_\sigma^s(\tau, \vec{k}) = \int_{\tilde{\tau}}^\Lambda G(\tilde{\tau}, \tilde{\tau}') S_\sigma^T(\tilde{\tau}') d\tilde{\tau}', \tag{E.1} $$

where the source term, $S_\sigma^T(\tilde{\tau}')$, and the retarded Green's function, $G(\tilde{\tau}, \tilde{\tau}')$, are given by

$$ S_\sigma^T(\tilde{\tau}') = \frac{2}{\tilde{\tau}'} \left(\frac{\psi}{M_{\mathrm{Pl}}} \right) \left[\left(-\lambda_\sigma \beta_c + \frac{\theta_c}{\tilde{\tau}'} \right) B_\sigma(\tilde{\tau}', \vec{k}) + \partial_{\tilde{\tau}'} B_\sigma(\tilde{\tau}', \vec{k}) \right], \tag{E.2} $$

$$ G(\tilde{\tau}, \tilde{\tau}') = \left(\frac{\tilde{\tau}' - \tilde{\tau}}{\tilde{\tau}' \tilde{\tau}} \cos(\tilde{\tau}' - \tilde{\tau}) - \left(1 + \frac{1}{\tilde{\tau} \tilde{\tau}'} \right) \sin(\tilde{\tau}' - \tilde{\tau}) \right) \Theta(\tilde{\tau}' - \tilde{\tau}), \tag{E.3} $$

respectively. Here $\Theta(\tilde{\tau}' - \tilde{\tau})$ is the Heaviside step function. Using the integral relations (C.16) and (C.17) and doing the integral (E.1) for $-k\tau = \tilde{\tau} \ll 1$, we obtain

$$
\begin{aligned}
h_\sigma^s(\tau, \vec{k}) \simeq \frac{e^{i\kappa_\sigma \pi/2}}{(2\pi)^{\frac{3}{2}}} \Bigg[&-(i + \lambda_\sigma \beta_c) \Bigg\{ \frac{\mathrm{G}_{2,3}^{2,1}\left(-2i\tilde{\tau}_\Lambda \left|\begin{array}{c} 1, \quad 1 + \kappa_\sigma \\ \frac{1}{2} - \mu, \ \frac{1}{2} + \mu, \ 0 \end{array} \right. \right)}{\Gamma(\frac{1}{2} - \kappa_\sigma - \mu)\Gamma(\frac{1}{2} - \kappa_\sigma + \mu)} \\
&+ \mathrm{G}_{2,3}^{2,2}\left(-2i\tilde{\tau}_\Lambda \left|\begin{array}{c} 1, \quad 1 - \kappa_\sigma \\ \frac{1}{2} - \mu, \ \frac{1}{2} + \mu, \ 0 \end{array} \right. \right) \Bigg\} \\
&- \frac{1}{\tilde{\tau}_\Lambda} \Bigg\{ \mathrm{G}_{2,3}^{2,2}\left(-2i\tilde{\tau}_\Lambda \left|\begin{array}{c} 2, \quad -\kappa_\sigma \\ \frac{1}{2} - \mu, \ \frac{1}{2} + \mu, \ 1 \end{array} \right. \right) - (1 - \kappa_\sigma - i\lambda_\sigma \beta_c + \theta_c) \mathrm{G}_{2,3}^{2,2}\left(-2i\tilde{\tau}_\Lambda \left|\begin{array}{c} 2, \quad 1 - \kappa_\sigma \\ \frac{1}{2} - \mu, \ \frac{1}{2} + \mu, \ 1 \end{array} \right. \right)
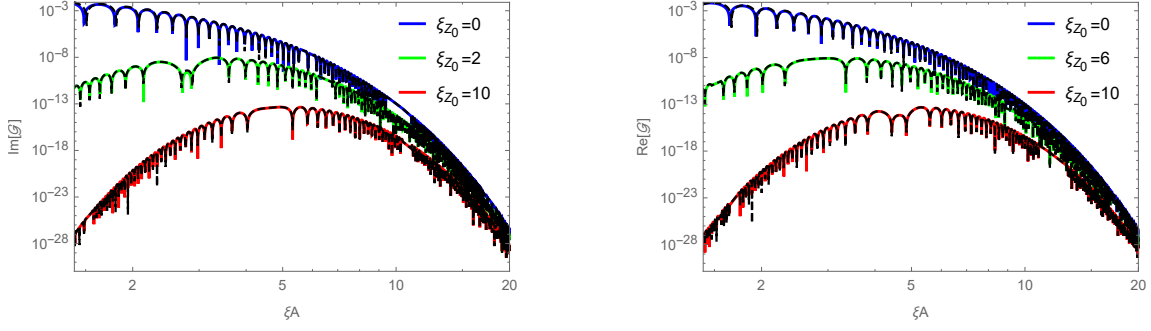\end{aligned}
$$

Figure 16. The exact form of $\mathcal{G}_\sigma(\xi_A, \xi_{Z_0})$ in (E.6) compared to the approximated from in (E.7) with respect to ξ_A for different values of ξ_{Z_0}. The solid (black) lines are the exact forms while the dashed line on the top of each curve is its approximated form.

$$+\frac{\mathrm{G}_{2,3}^{2,1}\left(-2i\tilde{\tau}_\Lambda\,\middle|\,\begin{matrix}2, & 2+\kappa_\sigma\\\frac{1}{2}-\mu, & \frac{1}{2}+\mu, & 1\end{matrix}\right)}{\Gamma(-\frac{1}{2}-\kappa_\sigma-\mu)\Gamma(-\frac{1}{2}-\kappa_\sigma+\mu)}+(1+\kappa_\sigma-i\lambda_\sigma\beta_c-\theta_c)\frac{\mathrm{G}_{2,3}^{2,1}\left(-2i\tilde{\tau}_\Lambda\,\middle|\,\begin{matrix}2, & 1+\kappa_\sigma\\\frac{1}{2}-\mu, & \frac{1}{2}+\mu, & 1\end{matrix}\right)}{\Gamma(\frac{1}{2}-\kappa_\sigma-\mu)\Gamma(\frac{1}{2}-\kappa_\sigma+\mu)}\Bigg\}$$

$$+\frac{1}{\tilde{\tau}_\Lambda^2}\Bigg\{i(\theta_c-\kappa_\sigma)\left[\mathrm{G}_{2,3}^{2,2}\left(-2i\tilde{\tau}_\Lambda\,\middle|\,\begin{matrix}3, & 1-\kappa_\sigma\\\frac{1}{2}-\mu, & \frac{1}{2}+\mu, & 2\end{matrix}\right)-\frac{\mathrm{G}_{2,3}^{2,1}\left(-2i\tilde{\tau}_\Lambda\,\middle|\,\begin{matrix}3, & 1+\kappa_\sigma\\\frac{1}{2}-\mu, & \frac{1}{2}+\mu, & 2\end{matrix}\right)}{\Gamma(\frac{1}{2}-\kappa_\sigma-\mu)\Gamma(\frac{1}{2}-\kappa_\sigma+\mu)}\right] \qquad (E.4)$$

$$-i\mathrm{G}_{2,3}^{2,2}\left(-2i\tilde{\tau}_\Lambda\,\middle|\,\begin{matrix}3, & -\kappa_\sigma\\\frac{1}{2}-\mu, & \frac{1}{2}+\mu, & 2\end{matrix}\right)+i\frac{\mathrm{G}_{2,3}^{2,1}\left(-2i\tilde{\tau}_\Lambda\,\middle|\,\begin{matrix}3, & 2+\kappa_\sigma\\\frac{1}{2}-\mu, & \frac{1}{2}+\mu, & 2\end{matrix}\right)}{\Gamma(-\frac{1}{2}-\kappa_\sigma-\mu)\Gamma(-\frac{1}{2}-\kappa_\sigma+\mu)}\Bigg\}\Bigg]\times\left(\frac{\psi}{M_{\mathrm{Pl}}}\right)\left(\frac{aH}{\sqrt{2}k^{\frac{3}{2}}}\right).$$

which implies that the sourced part of the gravitational wave can be written as

$$h_\sigma^s(\tau,\vec{k}) = \frac{e^{i\kappa_\sigma\pi/2}}{(2\pi)^{\frac{3}{2}}}\left(\frac{\psi}{M_{\mathrm{Pl}}}\right)\left(\frac{aH}{\sqrt{2}k^{\frac{3}{2}}}\right)\mathcal{G}_\sigma(\xi_A,\xi_{Z_0}). \qquad (E.5)$$

Using the asymptotic form of Meijer-G functions at $\tilde{\tau}_\Lambda \gg 1$ in (C.18)–(C.19), we can simplify $\mathcal{G}_\sigma$ as

$$\mathcal{G}_\sigma(\xi_A,\xi_{Z_0}) = \frac{\Gamma(\frac{1}{2}-\mu)\Gamma(\frac{1}{2}+\mu)}{\Gamma(-\kappa_\sigma)}\Bigg\{\left[\frac{(i+\lambda_\sigma\beta_c)}{\kappa_\sigma}-\frac{2(\lambda_\sigma\beta_c+i(2+\theta_c))}{(\frac{1}{4}-\mu^2)}+\frac{4i(2+\theta_c)(1+\kappa_\sigma)}{(\frac{9}{4}-\mu^2)(\frac{1}{4}-\mu^2)}\right]$$

$$+\left[i-\lambda_\sigma\beta_c+\frac{2i\kappa_\sigma(i\lambda_\sigma\beta_c+\theta_c)}{(\frac{1}{4}-\mu^2)}+\frac{4i\kappa_\sigma\left(\frac{1}{4}-\mu^2+2\kappa_\sigma+\theta(\kappa_\sigma-1)\right)}{(\frac{9}{4}-\mu^2)(\frac{1}{4}-\mu^2)}\right]$$

$$\times\frac{\Gamma^2(-\kappa_\sigma)}{\Gamma(\frac{1}{2}-\kappa_\sigma-\mu)\Gamma(\frac{1}{2}-\kappa_\sigma+\mu)}\Bigg\}. \qquad (E.6)$$

We show $|\mathcal{G}_+(\xi_A,\xi_{Z_0})|^2$ in figure 8. As we see, this function decreases with the increase of ξ_A and ξ_{Z_0} and due to the Gamma function has infinite number of roots on the ξ_A axis. We can well approximate the above as (see figure 16)

$$\mathcal{G}_\sigma(\xi_A,\xi_{Z_0}) \simeq \frac{\pi}{\cos(\pi\mu)\Gamma(-\kappa_\sigma)}\left[\frac{(i+\lambda_\sigma\beta_c)}{\kappa_\sigma}+\frac{(i-\lambda_\sigma\beta_c)\Gamma^2(-\kappa_\sigma)}{\Gamma(\frac{1}{2}-\kappa_\sigma-\mu)\Gamma(\frac{1}{2}-\kappa_\sigma+\mu)}\right]. \qquad (E.7)$$

Finally, after using (C.9) in the limit that $|\mu| \gg 1$, and up to a phase factor, we have

$$\mathcal{G}_\sigma(\xi_{\mathrm{A}}, \xi_{Z_0}) \simeq \mathcal{A}_\sigma \sqrt{2\pi|\kappa|} e^{\pi(\frac{1}{2}\lambda_\sigma|\kappa| - |\mu|)}, \tag{E.8}$$

where $\mathcal{A}_\sigma$ is the following quantity

$$\mathcal{A}_\sigma \equiv \left(\frac{(i + \lambda_\sigma \beta_c)}{\kappa_\sigma} + \frac{(i - \lambda_\sigma \beta_c)\Gamma^2(-\kappa_\sigma)}{\Gamma(\frac{1}{2} - \kappa_\sigma - \mu)\Gamma(\frac{1}{2} - \kappa_\sigma + \mu)} \right). \tag{E.9}$$

In figure 8, we present $|\mathcal{A}_+|^2$ as a function of ξ_{A} and ξ_{Z_0}.

References

[1] A.H. Guth, *The inflationary universe: a possible solution to the horizon and flatness problems*, *Phys. Rev.* **D 23** (1981) 347 [INSPIRE].

[2] K. Sato, *First order phase transition of a vacuum and expansion of the universe*, *Mon. Not. Roy. Astron. Soc.* **195** (1981) 467 [INSPIRE].

[3] A.D. Linde, *A new inflationary universe scenario: a possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems*, *Phys. Lett.* **B 108** (1982) 389 [INSPIRE].

[4] A. Albrecht and P.J. Steinhardt, *Cosmology for grand unified theories with radiatively induced symmetry breaking*, *Phys. Rev. Lett.* **48** (1982) 1220 [INSPIRE].

[5] A. Maleknejad and M.M. Sheikh-Jabbari, *Non-Abelian gauge field inflation*, *Phys. Rev.* **D 84** (2011) 043515 [arXiv:1102.1932] [INSPIRE].

[6] A. Maleknejad and M.M. Sheikh-Jabbari, *Gauge-flation: inflation from non-Abelian gauge fields*, *Phys. Lett.* **B 723** (2013) 224 [arXiv:1102.1513] [INSPIRE].

[7] P. Adshead and M. Wyman, *Chromo-natural inflation: natural inflation on a steep potential with classical non-Abelian gauge fields*, *Phys. Rev. Lett.* **108** (2012) 261302 [arXiv:1202.2366] [INSPIRE].

[8] P. Adshead, E. Martinec and M. Wyman, *Perturbations in chromo-natural inflation*, *JHEP* **09** (2013) 087 [arXiv:1305.2930] [INSPIRE].

[9] A. Maleknejad, M.M. Sheikh-Jabbari and J. Soda, *Gauge fields and inflation*, *Phys. Rept.* **528** (2013) 161 [arXiv:1212.2921] [INSPIRE].

[10] R. Namba, E. Dimastrogiovanni and M. Peloso, *Gauge-flation confronted with Planck*, *JCAP* **11** (2013) 045 [arXiv:1308.1366] [INSPIRE].

[11] A. Maleknejad, *Axion inflation with an* SU(2) *gauge field: detectable chiral gravity waves*, *JHEP* **07** (2016) 104 [arXiv:1604.03327] [INSPIRE].

[12] C.M. Nieto and Y. Rodriguez, *Massive gauge-flation*, *Mod. Phys. Lett.* **A 31** (2016) 1640005 [arXiv:1602.07197] [INSPIRE].

[13] R.R. Caldwell and C. Devulder, *Axion gauge field inflation and gravitational leptogenesis: a lower bound on B modes from the matter-antimatter asymmetry of the universe*, *Phys. Rev.* **D 97** (2018) 023532 [arXiv:1706.03765] [INSPIRE].

[14] P. Adshead, E. Martinec, E.I. Sfakianakis and M. Wyman, *Higgsed chromo-natural inflation*, *JHEP* **12** (2016) 137 [arXiv:1609.04025] [INSPIRE].

[15] E. Dimastrogiovanni, M. Fasiello and T. Fujita, *Primordial gravitational waves from axion-gauge fields dynamics*, *JCAP* **01** (2017) 019 [arXiv:1608.04216] [INSPIRE].

[16] P. Adshead and E.I. Sfakianakis, *Higgsed gauge-flation*, *JHEP* **08** (2017) 130 [arXiv:1705.03024] [INSPIRE].

[17] E. Dimastrogiovanni and M. Peloso, *Stability analysis of chromo-natural inflation and possible evasion of Lyth's bound*, *Phys. Rev.* **D 87** (2013) 103501 [arXiv:1212.5184] [INSPIRE].

[18] P. Adshead, E. Martinec and M. Wyman, *Gauge fields and inflation: chiral gravitational waves, fluctuations and the Lyth bound*, *Phys. Rev.* **D 88** (2013) 021302 [arXiv:1301.2598] [INSPIRE].

[19] B. Thorne, T. Fujita, M. Hazumi, N. Katayama, E. Komatsu and M. Shiraishi, *Finding the chiral gravitational wave background of an axion-SU(2) inflationary model using CMB observations and laser interferometers*, *Phys. Rev.* **D 97** (2018) 043506 [arXiv:1707.03240] [INSPIRE].

[20] A. Agrawal, T. Fujita and E. Komatsu, *Large tensor non-Gaussianity from axion-gauge field dynamics*, *Phys. Rev.* **D 97** (2018) 103526 [arXiv:1707.03023] [INSPIRE].

[21] A. Agrawal, T. Fujita and E. Komatsu, *Tensor non-Gaussianity from axion-gauge-fields dynamics: parameter search*, *JCAP* **06** (2018) 027 [arXiv:1802.09284] [INSPIRE].

[22] E. Dimastrogiovanni, M. Fasiello, R.J. Hardwick, H. Assadullahi, K. Koyama and D. Wands, *Non-Gaussianity from axion-gauge fields interactions during inflation*, *JCAP* **11** (2018) 029 [arXiv:1806.05474] [INSPIRE].

[23] A. Maleknejad, *Chiral gravity waves and leptogenesis in inflationary models with non-Abelian gauge fields*, *Phys. Rev.* **D 90** (2014) 023542 [arXiv:1401.7628] [INSPIRE].

[24] A. Maleknejad, *Gravitational leptogenesis in axion inflation with SU(2) gauge field*, *JCAP* **12** (2016) 027 [arXiv:1604.06520] [INSPIRE].

[25] P. Adshead, A.J. Long and E.I. Sfakianakis, *Gravitational leptogenesis, reheating and models of neutrino mass*, *Phys. Rev.* **D 97** (2018) 043511 [arXiv:1711.04800] [INSPIRE].

[26] T. Matsumura et al., *Mission design of LiteBIRD*, arXiv:1311.2847 [INSPIRE].

[27] SIMONS OBSERVATORY collaboration, *The Simons observatory: science goals and forecasts*, *JCAP* **02** (2019) 056 [arXiv:1808.07445] [INSPIRE].

[28] CMB-S4 collaboration, *CMB-S4 science book, first edition*, arXiv:1610.02743 [INSPIRE].

[29] V. Domcke, B. Mares, F. Muia and M. Pieroni, *Emerging chromo-natural inflation*, *JCAP* **04** (2019) 034 [arXiv:1807.03358] [INSPIRE].

[30] G. Dall'Agata, *Chromo-natural inflation in supergravity*, *Phys. Lett.* **B 782** (2018) 139 [arXiv:1804.03104] [INSPIRE].

[31] E. McDonough and S. Alexander, *Observable chiral gravitational waves from inflation in string theory*, *JCAP* **11** (2018) 030 [arXiv:1806.05684] [INSPIRE].

[32] S. Weinberg, *Larger Higgs exchange terms in the neutron electric dipole moment*, *Phys. Rev. Lett.* **63** (1989) 2333 [INSPIRE].

[33] H. Ruegg and M. Ruiz-Altaba, *The Stueckelberg field*, *Int. J. Mod. Phys.* **A 19** (2004) 3265 [hep-th/0304245] [INSPIRE].

[34] T. Kunimasa and T. Goto, *Generalization of the Stueckelberg formalism to the massive Yang-Mills field*, *Prog. Theor. Phys.* **37** (1967) 452 [INSPIRE].

[35] M.M. Sheikh-Jabbari, *Gauge-flation vs chromo-natural inflation*, *Phys. Lett.* **B 717** (2012) 6 [arXiv:1203.2265] [INSPIRE].

[36] P. Adshead and M. Wyman, *Gauge-flation trajectories in chromo-natural inflation*, *Phys. Rev.* **D 86** (2012) 043530 [arXiv:1203.2264] [INSPIRE].

[37] T. Fujita, E.I. Sfakianakis and M. Shiraishi, *Tensor spectra templates for axion-gauge fields dynamics during inflation*, arXiv:1812.03667 [INSPIRE].

[38] J.S. Schwinger, *On gauge invariance and vacuum polarization*, *Phys. Rev.* **82** (1951) 664 [INSPIRE].

[39] K.D. Lozanov, A. Maleknejad and E. Komatsu, *Schwinger effect by an* SU(2) *gauge field during inflation*, *JHEP* **02** (2019) 041 [arXiv:1805.09318] [INSPIRE].

[40] T. Fujita, R. Namba and Y. Tada, *Does the detection of primordial gravitational waves exclude low energy inflation?*, *Phys. Lett.* **B 778** (2018) 17 [arXiv:1705.01533] [INSPIRE].

[41] A.A. Starobinsky, *Spectrum of relict gravitational radiation and the early state of the universe*, *JETP Lett.* **30** (1979) 682 [*Pisma Zh. Eksp. Teor. Fiz.* **30** (1979) 719] [INSPIRE].

[42] L.P. Grishchuk, *Statistics of the microwave background anisotropies caused by the squeezed cosmological perturbations*, *Phys. Rev.* **D 53** (1996) 6784 [gr-qc/9504045] [INSPIRE].

[43] BICEP2 and KECK ARRAY collaborations, *Improved constraints on cosmology and foregrounds from BICEP2 and Keck Array cosmic microwave background data with inclusion of 95 GHz band*, *Phys. Rev. Lett.* **116** (2016) 031302 [arXiv:1510.09217] [INSPIRE].

[44] PLANCK collaboration, *Planck* 2015 *results. XVII. Constraints on primordial non-Gaussianity*, *Astron. Astrophys.* **594** (2016) A17 [arXiv:1502.01592] [INSPIRE].

[45] A. Papageorgiou, M. Peloso and C. Unal, *Nonlinear perturbations from the coupling of the inflaton to a non-Abelian gauge field, with a focus on chromo-natural inflation*, *JCAP* **09** (2018) 030 [arXiv:1806.08313] [INSPIRE].

[46] F.W. Olver, D.W. Lozier, R.F. Boisvert and C.W. Clark, *NIST handbook of mathematical functions*, 1st ed., Cambridge University Press, New York, NY, U.S.A. (2010).

[47] Z. Wang and D. Guo., *Special functions*, World Scientific, Singapore (1989).

[48] T. Kobayashi and N. Afshordi, *Schwinger effect in* 4D *de Sitter space and constraints on magnetogenesis in the early universe*, *JHEP* **10** (2014) 166 [arXiv:1408.4141] [INSPIRE].