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AdS_2 solutions and their massive IIA origin

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Abstract: We consider warped $\mathrm{AdS}_2 \times \mathcal{M}_4$ backgrounds within $F(4)$ gauged supergravity in six dimensions. In particular, we are able to find supersymmetric solutions of the aforementioned type characterized by AdS_6 asymptotics and an $\mathcal{M}_4$ given by a three-sphere warped over a segment. Subsequently, we provide the 10D uplift of the solutions to massive type IIA supergravity, where the geometry is $\mathrm{AdS}_2 \times S^3 \times \tilde{S}^3$ warped over a strip. Finally we construct the brane intersection underlying one of these supergravity backgrounds. The explicit setup involves a D0-F1-D4 bound state intersecting a D4-D8 system.

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Contents

1 Introduction

Ever since the birth of the AdS/CFT correspondence [1, 2], the quest for supersymmetric AdS vacua in string theory has become a goal of utmost importance. All the research efforts in the last decades devoted to this task have delivered a wide range of results including partial or exhaustive classifications of AdS string vacua in diverse dimensions (see e.g. [3–19]). A further crucial element for providing a holographic interpretation of the corresponding AdS vacuum is to possess the underlying brane construction from which the solution emerges when taking the near-horizon limit (see [20, 21] for a non-exhaustive collection of examples).

While in higher dimensions the organizing pattern of the landscape of supersymmetric AdS solutions is well delineated, achieving such a goal in two and three dimensions turns out to be too hard of a task, at least in full generality. This is due to the vast and rich structure opening up when it comes to establishing the possible geometries and topologies of the internal space. However, some partial developments in this direction can be found in [22–28]. More recently in [29, 30] novel examples of AdS_2 & AdS_3 solutions were found in the context of massive type IIA string theory. In all of the examples there the ten dimensional background is given by the warped product of AdS times a sphere warped over a line, their striking general feature being the non-compactness of the would-be internal space.

Such a feature seems to emerge very naturally within the context of brane intersections in massive type IIA supergravity when these produce AdS_2 or AdS_3 geometries in their near-horizon limit. So far a similar thing seems to be happening in higher dimensions

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only when the corresponding AdS vacua are obtained by employing non-Abelian T duality (NATD) as a generating technique (see e.g. [31–34] for recent examples of this type). This issue makes the holographic interpretation of this type of supergravity constructions problematic. Nevertheless, within the context of [33], the proposed holographic picture is that of an infinite quiver theory arising from a possible deconstructed extra dimension.

Going back to the original goal of finding novel examples of supersymmetric lower dimensional AdS vacua, a very fruitful approach seems to be that of exploiting the existence of consistent truncations of string and M theory yielding lower dimensional gauged supergravities as effective descriptions. The reason why this can be so helpful is that one may restrict the search for solutions within a theory with a smaller amount of fields and excitations. Once in possession of new lower dimensional solutions, a ten (eleven) dimensional solution can be generated by using the needed uplift formula. In this context, new supersymmetric solutions were found in [35–37] and [38–44], by exploiting consistent truncations respectively down to $\mathcal{N} = (1, 1)$ supergravity in six dimensions and $\mathcal{N} = 1$ supergravity in seven dimensions.

The focus of this paper will be $F(4)$ supergravity in six dimensions as arising from a consistent truncation of massive type IIA supergravity on a squashed 4-sphere [45]. We will study supersymmetric warped AdS_2 solutions supported both by a non-trivial 2-form field and a non-trivial profile for the universal scalar field. We will show how such an *Ansatz* produces half-BPS solutions where the full six dimensional geometry is $\mathrm{AdS}_2 \times S^3$ warped over a segment. This, upon uplifting, will produce an AdS_2 solution in massive type IIA string theory where the ten dimensional background is given by $\mathrm{AdS}_2 \times S^3 \times \tilde{S}^3$ warped over a strip. Furthermore we will show how these ten dimensional backgrounds can be equally obtained by taking the near-horizon limit of a non-standard D0-F1-D4-D4′-D8 intersection specified by a certain brane charge distribution. Finally we will conclude by speculating on the physical interpretation of our construction.

2 $\mathrm{AdS}_2 \times \mathcal{M}_4$ solutions in $F(4)$ gauged supergravity

In this section we derive two supersymmetric $\mathrm{AdS}_2 \times \mathcal{M}_4$ warped backgrounds in $d = 6$ $F(4)$ gauged supergravity. The solutions preserve 8 real supercharges and, are characterized by an AdS_6 asymptotics and by a running profile for the 2-form gauge potential included in the supergravity multiplet. The 2-form wraps the internal directions of AdS_2 and supports the singular behaviors arising in the IR regime. As we will see this fact hints at the physical interpretation of these backgrounds in terms of branes intersecting the D4-D8 system giving rise to the AdS_6 vacuum.

We will firstly introduce our setup given by six dimensional $F(4)$ gauged supergravity in its minimal incarnation.[1] Then we will formulate a suitable *Ansatz* on the bosonic fields of the supergravity multiplet and on the corresponding Killing spinor. With this information at hand, we will derive the BPS equations and we will solve them analytically.

[1]We will restrict ourselves to the supergravity multiplet.

2.1 Minimal $\mathcal{N} = (1,1)$ gauged supergravity in $d = 6$

Minimal $\mathcal{N} = (1,1)$ supergravity in $d = 6$ [46, 47] is obtained by only retaining the pure supergravity multiplet and, as a consequence, the global isometry group breaks down to [47–49]

$$G_0 = \mathbb{R}^+ \times \mathrm{SO}(4)\,. \tag{2.1}$$

The R-symmetry group is realized as the diagonal $\mathrm{SU}(2)_R \subset \mathrm{SO}(4) \simeq \mathrm{SU}(2) \times \mathrm{SU}(2)$. The corresponding 16 supercharges of the theory are then organized in their irreducible chiral components. The fermionic field content of the supergravity multiplet is composed by two gravitini and two dilatini. Both the gravitini and the dilatini can be packed into pairs of Weyl spinors with opposite chiralities. Furthermore, in $d = 1 + 5$ spacetime dimensions, symplectic-Majorana-Weyl spinors[2] (SMW) may be introduced. The SMW formulation manifestly arranges the fermionic degrees of freedom of the theory into $\mathrm{SU}(2)_R$ doublets, which we respectively denote by ψ^a_μ and χ^a with $a = 1, 2$. It is worth mentioning that these objects have to respect a "pseudo-reality condtion" of the form in (B.5) in order for them to describe the correct number of propagating degrees of freedom.

The bosonic field content of the supergravity multiplet is given by the graviton e^m_μ, a positive real scalar X, a 2-form gauge potential $\mathcal{B}_{(2)}$, a non-Abelian SU(2) valued vector field A^i and an Abelian vector field A^0. The consistent deformations of the minimal theory consist in a gauging of the R-symmetry $\mathrm{SU}(2)_R \subset \mathrm{SO}(4)$, by making use of the vectors A^i, and a Stückelberg coupling inducing a mass term for the 2-form field $\mathcal{B}_{(2)}$. The strength of the former deformation is controlled by a coupling constant g, while the latter by a mass parameter m. The bosonic Lagrangian has the following form [45, 46, 50]

$$\begin{aligned}
\mathcal{L} = {} & R \star_6 1 - 4\, X^{-2} \star_6 dX \wedge dX - \frac{1}{2} X^4 \star_6 \mathcal{F}_{(3)} \wedge \mathcal{F}_{(3)} - V(X) \\
& - \frac{1}{2} X^{-2} \left(\star_6 \mathcal{F}^i_{(2)} \wedge \mathcal{F}^i_{(2)} + \star_6 \mathcal{H}_{(2)} \wedge \mathcal{H}_{(2)} \right) - \frac{1}{2} \mathcal{B}_{(2)} \wedge \mathcal{F}^0_{(2)} \wedge \mathcal{F}^0_{(2)} \\
& - \frac{1}{\sqrt{2}}\, m\, \mathcal{B}_{(2)} \wedge \mathcal{B}_{(2)} \wedge \mathcal{F}^0_{(2)} - \frac{1}{3} m^2\, \mathcal{B}_{(2)} \wedge \mathcal{B}_{(2)} \wedge \mathcal{B}_{(2)} - \frac{1}{2} \mathcal{B}_{(2)} \wedge \mathcal{F}^i_{(2)} \wedge \mathcal{F}^i_{(2)}\,,
\end{aligned} \tag{2.2}$$

where the field strengths are defined as

$$\begin{aligned}
\mathcal{F}_{(3)} &= d\mathcal{B}_{(2)}\,, \\
\mathcal{F}^0_{(2)} &= dA^0\,, \\
\mathcal{H}_{(2)} &= dA^0 + \sqrt{2}\, m\, \mathcal{B}_{(2)}\,, \\
\mathcal{F}^i_{(2)} &= dA^i + \frac{g}{2} \epsilon^{ijk} A^j \wedge A^k\,.
\end{aligned} \tag{2.3}$$

A combination of the gauging and the massive deformation induces the following scalar potential

$$V(X) = m^2 X^{-6} - 4\sqrt{2}\, gm\, X^{-2} - 2\, g^2 X^2\,, \tag{2.4}$$

which can be re-expressed in terms of a real "superpotential" $f(X)$ through

$$V(X) = 16\, X^2\, (D_X f)^2 - 80\, f(X)^2\,, \tag{2.5}$$

[2] For more details on Clifford algebras for $d = 1 + 5$ spacetime dimensions see appendix B.

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where $f(X)$ is given by

$$f(X) = \frac{1}{8}\left(m\,X^{-3} + \sqrt{2}\,g\,X\right) . \tag{2.6}$$

The supersymmetry variations of the fermionic fields are expressed in terms of a 6D Killing SMW spinor ζ^a as [46, 50]

$$\begin{aligned}
\delta_\zeta \psi^a_\mu &= \nabla_\mu\,\zeta^a + 4g\,(A_\mu)^a{}_b\,\zeta^b + \frac{X^2}{48}\,\Gamma_*\Gamma^{mnp}\,\mathcal{F}_{(3)\,mnp}\,\Gamma_\mu\zeta^a \\
&\quad + i\,\frac{X^{-1}}{16\sqrt{2}}\,\left(\Gamma_\mu{}^{mn} - 6\,e^m_\mu\,\Gamma^n\right)\,(\hat{\mathcal{H}}_{mn})^a{}_b\,\zeta^b - i f(X)\,\Gamma_\mu\Gamma_*\zeta^a\,, \\
\delta_\zeta \chi^a &= X^{-1}\Gamma^m\partial_m X\,\zeta^a + \frac{X^2}{24}\,\Gamma_*\Gamma^{mnp}\,\mathcal{F}_{(3)\,mnp}\,\zeta^a \\
&\quad - i\,\frac{X^{-1}}{8\sqrt{2}}\,\Gamma^{mn}(\hat{\mathcal{H}}_{mn})^a{}_b\,\zeta^b + 2i\,X D_X\,f(X)\,\Gamma_*\,\zeta^a\,,
\end{aligned} \tag{2.7}$$

with $\nabla_\mu\,\zeta^a = \partial_\mu\zeta^a + \frac{1}{4}\,\omega_\mu{}^{mn}\,\Gamma_{mn}\,\zeta^a$ and $(\hat{\mathcal{H}}_{mn})^a{}_b$ defined as

$$(\hat{\mathcal{H}}_{\mu\nu})^a{}_b = \mathcal{H}_{(2)\,\mu\nu}\,\delta^a{}_b - 4\,\Gamma_*\,(\mathcal{F}_{(2)\,\mu\nu})^a{}_b\,, \tag{2.8}$$

where we introduced the notation $A^a{}_b = \frac{1}{2}\,A^i(\sigma^i)^a{}_b$, σ^i being the Pauli matrices as given in (B.8). By varying the Lagrangian (2.2) with respect to all the bosonic fields one obtains the following equations of motion

$$\begin{aligned}
& R_{\mu\nu} - 4\,X^{-2}\,\partial_\mu X\,\partial_\nu\,X - \frac{1}{4}\,V(X)\,g_{\mu\nu} - \frac{1}{4}\,X^4\,\left(\mathcal{F}_{(3)\,\mu}{}^{\alpha\beta}\mathcal{F}_{(3)\,\nu\alpha\beta} - \frac{1}{6}\,\mathcal{F}^2_{(3)}\,g_{\mu\nu}\right) \\
&\quad - \frac{1}{2}\,X^{-2}\,\left(\mathcal{H}_{(2)\,\mu}{}^{\alpha}\,\mathcal{H}_{(2)\,\nu\alpha} - \frac{1}{8}\,\mathcal{H}^2_{(2)}g_{\mu\nu}\right) - \frac{1}{2}\,X^{-2}\,\left(\mathcal{F}^i_{(2)\,\mu}{}^{\alpha}\,\mathcal{F}^i_{(2)\,\nu\alpha} - \frac{1}{8}\mathcal{F}^{i\;2}_{(2)}\,g_{\mu\nu}\right) = 0\,, \\
& d\left(X^4\,\star_6\,\mathcal{F}_{(3)}\right) = -\frac{1}{2}\,\mathcal{H}_{(2)}\,\wedge\,\mathcal{H}_{(2)} - \frac{1}{2}\,\mathcal{F}^i_{(2)}\,\wedge\,\mathcal{F}^i_{(2)} - \sqrt{2}\,m\,X^{-2}\,\star_6\,\mathcal{H}_{(2)}\,, \\
& d\left(X^{-2}\,\star_6\,\mathcal{H}_{(2)}\right) = -\mathcal{H}_{(2)}\,\wedge\,\mathcal{F}_{(3)}\,, \\
& D\left(X^{-2}\,\star_6\,\mathcal{F}^i_{(2)}\right) = -\mathcal{F}^i_{(2)}\,\wedge\,\mathcal{F}_{(3)}\,, \\
& d\left(X^{-1}\star_6\,dX\right) + \frac{1}{8}\,X^{-2}\,\left(\star_6\,\mathcal{H}_{(2)}\,\wedge\,\mathcal{H}_{(2)} + \star_6\,\mathcal{F}^i_{(2)}\,\wedge\,\mathcal{F}^i_{(2)}\right) \\
&\quad - \frac{1}{4}\,X^4\,\star_6\,\mathcal{F}_{(3)}\,\wedge\,\mathcal{F}_{(3)} - \frac{1}{8}\,X\,D_X\,V(X)\star_6 1 = 0\,,
\end{aligned} \tag{2.9}$$

where D is the gauge covariant derivative defined as $D\omega^i = d\omega^i + g\,\epsilon_{ijk}\,A^j\wedge\omega^k$ for any ω^i transforming covariantly with respect to SU(2).

Finally we mention that the scalar potential (2.4) admits a critical point giving rise to an AdS_6 vacuum preserving 16 real supercharges. This vacuum is realized by the following value of vev for X

$$X = \frac{3^{1/4}\,m^{1/4}}{2^{1/8}\,g^{1/4}}\,, \tag{2.10}$$

while all the gauge potentials are zero.

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2.2 The general Ansatz

Let us consider a 6D metric of the general form

$$ds_6^2 = e^{2U(\alpha)}\, ds^2_{\mathrm{AdS}_2} + e^{2V(\alpha)}\, d\alpha^2 + e^{2W(\alpha)}\, ds^2_{S^3}\,, \tag{2.11}$$

associated to a warped backgrounds of the type $\mathrm{AdS}_2 \times \mathcal{M}_4$ where $\mathcal{M}_4$ is locally written as a fibration of a S^3 over an interval I_α. We point out that the warp factor V is non-dynamical and it has been introduced because its gauge-fixing will turn out to be crucial to analytically solve the obtained BPS equations.

As far as the 2-form gauge potential $\mathcal{B}_{(2)}$ is concerned, it will purely wrap AdS_2 as follows

$$\mathcal{B}_{(2)} = b(\alpha)\, \mathrm{vol}_{\mathrm{AdS}_2}\,. \tag{2.12}$$

We furthermore also assume a purely radial dependence for the scalar

$$X = X(\alpha)\,, \tag{2.13}$$

and, for simplicity, we will restrict ourselves to the case of vanishing vectors, i.e. $A^i = 0$ and $A^0 = 0$.

We need also a suitable *Ansatz* for the Killing spinor corresponding to the spacetime background given in (2.11) and (2.12). As we pointed out in [51], the action of the SUSY variations on the $\mathrm{SU}(2)_R$ indices of the Killing spinor ζ^a is trivial, so it is more natural to cast the components of a Killing spinor in a (1+5)-dimensional Dirac spinor ζ. Following the splitting of the Clifford algebra given in (B.9), the Killing spinors considered are of the form

$$\begin{aligned}
&\zeta(\alpha) = \zeta^+(\alpha) + \zeta^-(\alpha)\,,\\
&\zeta^+ = i\, Y(\alpha)\left(\cos\theta(r)\, \chi^+_{\mathrm{AdS}_2} \otimes \varepsilon_0 + \sin\theta(r)\, \chi^+_{\mathrm{AdS}_2} \otimes \sigma^3 \varepsilon_0\right) \otimes\ \eta_{S^3}\,,\\
&\zeta^- = Y(\alpha)\left(\sin\theta(r)\, \chi^-_{\mathrm{AdS}_2} \otimes \varepsilon_0 - \cos\theta(r)\, \chi^-_{\mathrm{AdS}_2} \otimes \sigma^3 \varepsilon_0\right) \otimes\ \eta_{S^3}\,.
\end{aligned} \tag{2.14}$$

The spinor η_{S^3} is a Dirac spinor, hence it has 4 real independent components and satisfies the following Killing equation

$$\nabla_{\theta^i}\, \eta_{S^3} = \frac{iR}{2}\, \gamma_{\theta^i}\, \eta_{S^3}\,, \tag{2.15}$$

where R^{-1} the radius of S^3 and γ_{θ^i} are the Dirac matrices introduced in (B.7) expressed in the curved basis $\{\theta^i\}$ on the 3-sphere.

Regarding the spinors $\chi^\pm_{\mathrm{AdS}_2}$, they are Majorana-Weyl Killing spinors on AdS_2 and only possess 1 real independent component each. They respectively solve the equations[3]

$$\nabla_{x^\alpha}\, \chi^\pm_{\mathrm{AdS}_2} = \pm\frac{iL}{2}\, \rho_{x^\alpha}\, \chi^\mp_{\mathrm{AdS}_2}\,, \tag{2.16}$$

[3]Since $\chi^\pm_{\mathrm{AdS}_2}$ are Weyl spinor, they respectively satisfy the conditions $\Pi(\pm\rho_*)\chi^\pm_{\mathrm{AdS}_2} = \pm\chi^\pm_{\mathrm{AdS}_2}$ with $\Pi = \frac{1}{2}(\mathbb{I} \pm \rho_*)$. It follows that they can organized in a Majorana doublet $\chi_{\mathrm{AdS}_2} = (\chi^+_{\mathrm{AdS}_2}, \chi^-_{\mathrm{AdS}_2})$ such that $\nabla_{x^\alpha}\, \chi_{\mathrm{AdS}_2} = \frac{iL}{2}\, \rho_*\, \rho_{x^\alpha}\, \chi_{\mathrm{AdS}_2}$.

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where L^{-1} is the radius of AdS_2 and ρ_{x^α} are the Dirac matrices introduced in (B.6) given in the curved basis $\{x^\alpha\}$ on AdS_2.

Finally ε_0 is a 2-dimensional real spinor encoding the two different chiral parts of ζ as

$$\Gamma_* \zeta = \pm \zeta \qquad \Longleftrightarrow \qquad \sigma^3 \varepsilon_0 = \pm \varepsilon_0 \,, \tag{2.17}$$

where we used the identity (B.10). Totally we have that ζ depends on 16 real independent supercharges that, as we will see, will be lowered by an algebraic projection condtion associated with the particular background considered.

2.3 $\mathrm{AdS}_2 \times S^3 \times I_\alpha$ warped solutions

Let us now derive two analytic warped solutions of the type $\mathrm{AdS}_2 \times S^3 \times I_\alpha$ associated with the general background 2.2. Both preserve 8 real supercharges (BPS/2), enjoy an AdS_6 asymptotics and a singular IR regime. The first solution is characterized by the following *Ansatz*,

$$\begin{aligned} ds_6^2 &= e^{2U(\alpha)}\, ds^2_{\mathrm{AdS}_2} + e^{2V(\alpha)}\, d\alpha^2 + e^{2W(\alpha)}\, ds^2_{S^3} \,, \\ \mathcal{B}_{(2)} &= b(\alpha)\, \mathrm{vol}_{\mathrm{AdS}_2} \,, \\ X &= X(\alpha) \,. \end{aligned} \tag{2.18}$$

If we now impose the algebraic condition

$$\sigma^2 \varepsilon_0 = \varepsilon_0 \,, \tag{2.19}$$

on the spinor ζ, written in (2.14), we can specify the SUSY variations of fermions (2.7) for the background (2.18). In this way we obtain the following set of BPS equations,

$$\begin{aligned} U' &= \frac{1}{4}\, e^V \, \cos(2\theta)^{-1} \Big((5 + 3\cos(4\theta))\, f + 6 \sin{(2\theta)}^2\, X\, D_X f + L\, e^{-U} \sin(2\theta) \Big) \,, \\ W' &= -\frac{1}{4}\, e^V \, \cos(2\theta)^{-1} \Big((-9 + \cos(4\theta))\, f + 2 \sin{(2\theta)}^2\, X\, D_X f - L\, e^{-U} \sin(2\theta) \Big) \,, \\ b' &= -\frac{e^{V+2U}}{X^2} \, \cos(2\theta)^{-1} \, \big(L\, e^{-U} + 2 \sin(2\theta) \, (f + 3\, X\, D_X f) \big) \,, \\ \theta' &= -e^V \, \sin(2\theta) \, (f - X\, D_X f) \,, \\ Y' &= \frac{Y}{8}\, e^V \, \cos(2\theta)^{-1} \Big((5 + 3\cos(4\theta))\, f + 6 \sin{(2\theta)}^2\, X\, D_X f + L\, e^{-U} \sin(2\theta) \Big) \,, \\ X' &= -\frac{1}{4}\, e^V\, X \, \cos(2\theta)^{-1} \, \big(L\, e^{-U} \sin(2\theta) + 2\, \sin(2\theta)^2\, f + (7 + \cos(4\theta))\, X\, D_X f \big) \,. \end{aligned} \tag{2.20}$$

In addition to the first-order equations, one has to impose the two additional constraints

$$\begin{aligned} b &\overset{!}{=} -\frac{e^{2U}}{m}\, X \, \big(L\, e^{-U} + 2\, \sin(2\theta) \, (f - \, X\, D_X f) \big) \,, \\ R &= \frac{1}{2}\, e^{-U+W}\, L \, \cos(2\theta)^{-1} + e^W \, \tan(2\theta) \, (3\, f + X\, D_X f) \,. \end{aligned} \tag{2.21}$$

If the superpotential f is given by (2.6), it is easy to see that the constraints (2.21) are satified. Let us now make the following gauge choice on the non-dynamical warp factor V

$$e^{V} = \left(\sin(2\theta)\,(X\,D_X f - f)\right)^{-1} . \tag{2.22}$$

Then, the equations (2.20) can be integrated analytically for $\alpha \in [0, \frac{\pi}{4}]$ and the corresponding solution is given by

$$\begin{aligned}
e^{2U} &= \frac{2^{1/2}}{3^{1/2}m^{1/2}}\left(2\sqrt{2}\,g + 3L\,(\cos(4\alpha)-1)\right)^{1/2}\sin(2\alpha)^{-1}\sin(4\alpha)^{-1}\,,\\
e^{2W} &= \left(2\sqrt{2}\,g - 6L\,\sin(2\alpha)^2\right)^{1/2}\tan(2\alpha)^{-1}\sin(2\alpha)^{-1}\,,\\
e^{2V} &= \frac{4\,3^{3/2}}{m^{1/2}}\cos(2\alpha)\tan(2\alpha)^{-2}\left(\sqrt{2}\,g - 3L\,\sin(2\alpha)^2\right)^{-3/2}\,,\\
b &= \frac{\sqrt{2}\,g - 3L}{3m}\sin(2\alpha)^{-1}\,\cos(2\alpha)^{-2}\,,\\
X &= 3^{1/4}\,m^{1/4}\,\cos(2\alpha)^{-1/2}\left(\sqrt{2}\,g - 3L\,\sin(2\alpha)^2\right)^{-1/4}\,,\\
Y &= \left(2\sqrt{2}\,g - 6L\,\sin(2\alpha)^2\right)^{1/8}\sin(\alpha)^{-1/2}\cos(2\alpha)^{-1/4}\,,\\
\theta &= -\alpha\,,
\end{aligned} \tag{2.23}$$

for $g > 0$ and $m > 0$. From the constraints in (2.21) we obtain the relation

$$R = -3^{1/4}2^{-1/4}\,g\,m^{1/4} + 3^{1/4}2^{-3/4}\,m^{1/4}\,L\,, \tag{2.24}$$

relating the radii R and L of the background to the gauging parameters g and m. The solution (2.23) endowed with the constraint (2.24) satisfies the equations of motion of $F(4)$ gauged supergravity written in (2.9). Finally, if we take the $\alpha \to 0$ limit, the solution (2.23) is locally described by the AdS$_6$ vacuum (2.10), while for $\alpha \to \frac{\pi}{4}$, the background becomes singular.

The second solution is simpler and it can be found by setting the two warp factors U and W of (2.18) equal. In this case, we produce a curved domain wall solution charged under the 2-form. The *Ansatz* in this case has the following form

$$\begin{aligned}
ds_6^2 &= e^{2U(\alpha)}\left(ds^2_{\mathrm{AdS}_2} + ds^2_{S^3}\right) + e^{2V(\alpha)}\,d\alpha^2\,,\\
\mathcal{B}_{(2)} &= b(\alpha)\,\mathrm{vol}_{\mathrm{AdS}_2}\,,\\
X &= X(\alpha)\,.
\end{aligned} \tag{2.25}$$

With this prescription the Killing spinor (2.14) boils down to

$$\zeta^{+} = Y(\alpha)\left(i\,\chi^{+}_{\mathrm{AdS}_2}\otimes\varepsilon_0 - \chi^{-}_{\mathrm{AdS}_2}\otimes\sigma^3\varepsilon_0\right)\otimes\ \eta_{S^3}\,. \tag{2.26}$$

Imposing again the algebraic condition (2.19) on (2.26), and plugging the *Ansatz* (2.25) into the SUSY variations of fermions (2.7), we obtain the following set of BPS equations

$$\begin{aligned}
U' &= -2\,e^{V} f\,, & Y' &= -Y\,e^{V} f\,,\\
b' &= -\frac{e^{U+V}\,L}{X^2}\,, & X' &= 2\,e^{V}\,X^2\,D_X f\,,
\end{aligned} \tag{2.27}$$

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that must be supplemented with the constraints

$$b \overset{!}{=} -\frac{e^U\, X\, L}{m}\,, \qquad \text{and} \qquad L = 2R\,. \tag{2.28}$$

Also in this case these constraints are satisfied if the superpotential has the form of (2.6). If we now make the gauge choice

$$e^V = \left(2\, X^2\, D_X f\right)^{-1}\,, \tag{2.29}$$

it is easy to see that the equations (2.27) are solved by the following expressions

$$\begin{aligned} e^{2U} &= \frac{1}{2^{1/3} g^{2/3}} \left(\frac{\alpha}{\alpha^4 - 1}\right)^{2/3}\,, \\ e^{2V} &= \frac{8}{g^2} \left(\frac{\alpha^2}{\alpha^4 - 1}\right)^{2}\,, \\ Y &= \frac{1}{2^{1/12} g^{1/6}} \left(\frac{\alpha}{\alpha^4 - 1}\right)^{1/6}\,, \\ b &= -\frac{3\, L}{2^{2/3} g^{4/3}} \frac{\alpha^{4/3}}{(\alpha^4 - 1)^{1/3}}\,, \\ X &= \alpha\,, \end{aligned} \tag{2.30}$$

with α running between 0 and 1 if we choose m and g such that $m = \frac{\sqrt{2}\, g}{3}$. The solution (2.30) solves the equations of motion (2.30) and, in the $\alpha \to 1$ limit, it locally reproduces the AdS_6 vacuum (2.10) with $m = \frac{\sqrt{2}\, g}{3}$, while, in $\alpha \to 0$, it manifests a singular behavior.

3 The massive IIA origin

We will now move to the 10D origin of these backgrounds in massive type IIA supergravity. We will start by discussing their uplifts by using the formula in [45]. Later, for the simpler case, we will also provide a brane solution which will allow us reinterpret the charged domain wall (2.30) as a particular background with polarized branes.

3.1 Uplifts and $\mathrm{AdS}_2 \times S^3 \times S^3 \times I_\alpha \times I_\xi$ backgrounds

In this section we present the consistent truncation of massive IIA supergravity around the $\mathrm{AdS}_6 \times S^4$ warped vacuum [45] and we discuss the uplifts of the $\mathrm{AdS}_2 \times \mathcal{M}_4$ solutions obtained in section 2.3. If one choose the 6D gauge parameters as it follows

$$m = \frac{\sqrt{2}\, g}{3}\,, \tag{3.1}$$

the 6D equations of motion (2.9) can be obtained from the following truncation *Ansatz* of the 10d background[4] [45]

$$ds^2_{10} = s^{-1/3}\, X^{-1/2}\, \Delta^{1/2}\, \left[ds^2_6 + 2g^{-2}\, X^2\, ds^2_4\right]\,, \tag{3.2}$$

[4] We use the string frame, while in [45] the truncation *Ansatz* is given in the Einstein frame. See appendix A.

where $\Delta = Xc^2 + X^{-3}s^2$ and ds_4^2 is the metric of a squashed 4-sphere locally written as a fibration of a 3-sphere $\tilde{S}^3$ over a segment,

$$ds_4^2 = d\xi^2 + \frac{1}{4}\,\Delta^{-1}\,X^{-3}\,c^2\,\sum_{i=1}^{3}\,\left(\theta^i - gA^i\right)^2\,, \tag{3.3}$$

with $c = \cos\xi$ and $s = \sin\xi$. The 3-sphere included in (3.3) is deformed and it is expressed as a SU(2) bundle with connections A^i and θ^i left-invariant 1-forms.[5] The fluxes and the dilaton are given by [45]

$$\begin{aligned}
F_{(4)} =& -\frac{\sqrt{2}}{6}\,g^{-3}\,s^{1/3}\,c^3\,\Delta^{-2}\,U\,d\xi\,\wedge\,\epsilon_{(3)} - \sqrt{2}\,g^{-3}\,s^{4/3}\,c^4\,\Delta^{-2}\,X^{-3}\,dX\,\wedge\,\epsilon_{(3)} \\
& - \sqrt{2}\,g^{-1}\,s^{1/3}\,c\,X^4\,\star_6\,\mathcal{F}_{(3)}\,\wedge\,d\xi - \frac{1}{\sqrt{2}}\,s^{4/3}\,X^{-2}\,\star_6\,\mathcal{H}_{(2)} \\
& + \frac{g^{-2}}{\sqrt{2}}\,s^{1/3}\,c\,\mathcal{F}^i_{(2)}\,h^i\,\wedge\,d\xi - \frac{g^{-2}}{4\sqrt{2}}\,s^{4/3}\,c^2\,\Delta^{-1}\,X^{-3}\,\epsilon_{ijk}\,\mathcal{F}^i_{(2)}\,\wedge h^j\,\wedge\,h^k\,, \\
F_{(2)} =& \frac{s^{2/3}}{\sqrt{2}}\,\mathcal{H}_{(2)}\,, \qquad\qquad H_{(3)} = s^{2/3}\,\mathcal{F}_{(3)} + g^{-1}\,s^{-1/3}\,c\,\mathcal{H}_{(2)}\,\wedge\,d\xi\,, \\
e^{\Phi} =& s^{-5/6}\,\Delta^{1/4}\,X^{-5/4}\,, \qquad F_{(0)} = m\,.
\end{aligned} \tag{3.4}$$

where $U = X^{-6}\,s^2 - 3X^2\,c^2 + 4\,X^{-2}\,c^2 - 6\,X^{-2}$ and $\epsilon_{(3)} = h^1\,\wedge\,h^2\,\wedge\,h^3$ with $h^i = \theta^i - gA^i$. The $\mathrm{AdS}_6 \times S^4$ warped vacuum of massive IIA is naturally obtained by uplifting the 6D vacuum (2.10). In particular, for $X = 1$ and vanishing gauge potentials, the manifold (3.3) becomes a round 4-sphere.[6] From (3.4) it follows that the $\mathrm{AdS}_6 \times S^4$ vacuum is supported by the 4-flux $F_{(4)}$ that, together with the dilaton, has the following form

$$F_{(4)} = \frac{5\sqrt{2}}{6}\,g^{-3}\,s^{1/3}\,c^3\,d\xi\,\wedge\,\epsilon_{(3)}\,, \qquad e^{\Phi} = s^{-5/6}\,. \tag{3.5}$$

These are exactly the flux and dilaton configurations corresponding to the near-horizon of the localized D4-D8 system of [45, 52].

The uplifts of the AdS_2 warped solutions obtained in section 2.3 can be easily derived by plugging the explicit form of the 6D backgrounds (2.23) and (2.30) into the truncation formulas (3.2) and (3.4). In both cases one obtains a 10D background $\mathrm{AdS}_2\times S^3\times\tilde{S}^3$ fibered over two intervals parametrized by the 6D coordinate α and by the internal coordinate ξ.

In particular we can write the corresponding 10D metric of the charged domain wall solution (2.30) as

$$ds_{10}^2 = s^{-1/3}\,X(\alpha)^{-1/2}\,\Delta^{1/2}\left[e^{2U(\alpha)}\left(ds^2_{\mathrm{AdS}_2} + ds^2_{S^3}\right) + e^{2V(\alpha)}\,d\alpha^2 + 2g^{-2}\,X(\alpha)^2\,ds_4^2\right], \tag{3.6}$$

where ds_4^2 is given by (3.3) in the particular case of vanishing vectors $A^i = 0$, i.e.

$$ds_4^2 = d\xi^2 + \frac{1}{4}\,\Delta^{-1}\,X(\alpha)^{-3}\,c^2\,ds^2_{\tilde{S}^3}\,. \tag{3.7}$$

The fluxes $F_{(4)}$, $F_{(2)}$ and $H_{(3)}$ can be easily derived from (3.4) by setting also the abelian 6D vector $A^0 = 0$, i.e. $\mathcal{H}_{(2)} = \sqrt{2}\,m\,\mathcal{B}_{(2)}$.

[5] They are defined by the relation $d\theta^i = -\frac{1}{2}\,\varepsilon_{ijk}\,d\theta^j\,\wedge\,d\theta^k$.

[6] As outlined in [52], this is only the upper hemisphere of a 4-sphere with a boundary appearing for $\xi \to 0$.

branes	t	ρ	φ^1	φ^2	φ^3	z	r	θ^1	θ^2	θ^3
D4	$\times$	$-$	$-$	$-$	$-$	$-$	$\times$	$\times$	$\times$	$\times$
D8	$\times$	$\times$	$\times$	$\times$	$\times$	$-$	$\times$	$\times$	$\times$	$\times$

Table 1. The brane picture underlying the 5d SCFT described by D4- and D8-branes. The above system is $\frac{1}{4}$-BPS.

3.2 D4-D8 system and AdS$_6$ vacua

In order to provide the explicit brane picture producing the 10D background (3.6) in its near-horizon limit, as a preliminary analysis, we review how the AdS$_6$ vacuum is obtained as the near-horizon limit of the D4-D8 intersection.

The complete brane system realizing this mechanism is sketched in table 1. The corresponding string frame supergravity background reads

$$\begin{aligned} ds^2_{10} &= H_{\mathrm{D4}}^{-1/2} H_{\mathrm{D8}}^{-1/2}\left(-dt^2\right) + H_{\mathrm{D4}}^{1/2} H_{\mathrm{D8}}^{1/2} dz^2 + H_{\mathrm{D4}}^{-1/2} H_{\mathrm{D8}}^{-1/2}\left(dr^2 + r^2 ds^2_{S^3}\right) + \\ &\quad + H_{\mathrm{D4}}^{1/2} H_{\mathrm{D8}}^{-1/2}\left(d\rho^2 + \rho^2 ds^2_{\tilde{S}^3}\right) , \end{aligned} \tag{3.8}$$

$$e^{\Phi} = H_{\mathrm{D4}}^{-1/4} H_{\mathrm{D8}}^{-5/4} , \qquad C_{(5)} = \left(H_{\mathrm{D4}}^{-1} - 1\right) dt \wedge \mathrm{vol}_{(4)} , \tag{3.9}$$

$$C_{(9)} = \left(H_{\mathrm{D8}}^{-1} - 1\right) dt \wedge \mathrm{vol}_{(4)} \wedge \tilde{\mathrm{vol}}_{(4)} , \tag{3.10}$$

where $\mathrm{vol}_{(4)}$ & $\tilde{\mathrm{vol}}_{(4)}$ represent the volume forms on the $\mathbb{R}^4$ factors respectively spanned by (r, θ^i) and (ρ, φ^i). The functions H_{D4} & H_{D8} specify a semilocalized D4-D8 intersection [52] and their explicit form is given by

$$H_{\mathrm{D8}} = Q_{\mathrm{D8}}\, z \ , \quad \text{and} \quad H_{\mathrm{D4}} = 1 + Q_{\mathrm{D4}}\left(\rho^2 + \frac{4}{9} Q_{\mathrm{D8}} z^3\right)^{-5/3} . \tag{3.11}$$

The above background yields a warped product of AdS$_6$ and a half S^4 in the limit where

$$z \to 0 \ , \quad \text{and} \quad \rho \to 0 \ , \quad \text{while} \quad \frac{z^3}{\rho^2} \sim \text{finite} . \tag{3.12}$$

In what follows we will consider the intersection of the D4-D8 system with a D0-F1-D4$'$ bound state. The presence of these new branes will break the isometry group of the AdS$_6 \times S^4$ vacuum producing the AdS$_2$ foliation.

3.3 The D0-F1-D4$'$-D4-D8 brane intersection

Given the above stringy picture, the complete brane system realizing the AdS$_2$ slicing of the 10D background is sketched in table 2. The corresponding supergravity background is that of a non-standard brane intersection in the spirit of [53], since there is no transverse direction which is common to all branes in the system. The explicit profile of the massive

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branes	t	ρ	φ^1	φ^2	φ^3	z	r	θ^1	θ^2	θ^3
D4	×	–	–	–	–	–	×	×	×	×
D8	×	×	×	×	×	–	×	×	×	×
D0	×	–	–	–	–	–	–	–	–	–
F1	×	–	–	–	–	×	–	–	–	–
D4′	×	×	×	×	×	–	–	–	–	–

Table 2. The brane picture underlying the 1d SCFT described by D0-F1-D4′ branes ending on a D4-D8 system. The above intersection is $\frac{1}{8}$-BPS.

IIA supergravity fields in the string frame reads

$$\begin{aligned} ds_{10}^2 &= H_{\rm D0}^{-1/2} H_{\rm F1}^{-1} H_{\rm D4}^{-1/2} H_{\rm D4'}^{-1/2} H_{\rm D8}^{-1/2} \left(-dt^2\right) + H_{\rm D0}^{1/2} H_{\rm F1}^{-1} H_{\rm D4}^{1/2} H_{\rm D4'}^{1/2} H_{\rm D8}^{1/2} dz^2 \\ &\quad + H_{\rm D0}^{1/2} H_{\rm D4}^{-1/2} H_{\rm D4'}^{1/2} H_{\rm D8}^{-1/2} \left(dr^2 + r^2 ds^2_{S^3}\right) + H_{\rm D0}^{1/2} H_{\rm D4}^{1/2} H_{\rm D4'}^{-1/2} H_{\rm D8}^{-1/2} \left(d\rho^2 + \rho^2 ds^2_{\tilde{S}^3}\right) , \end{aligned} \tag{3.13}$$

$$e^{\Phi} = H_{\rm D0}^{3/4} H_{\rm F1}^{-1/2} H_{\rm D4}^{-1/4} H_{\rm D4'}^{-1/4} H_{\rm D8}^{-5/4} , \qquad B_{(2)} = \left(H_{\rm F1}^{-1} - 1\right) dt \wedge dz \, , \tag{3.14}$$

$$C_{(5)} = H_{\rm D4'} \left(H_{\rm D4}^{-1} - 1\right) dt \wedge \text{vol}_{(4)} \; + \; H_{\rm D4} \left(H_{\rm D4'}^{-1} - 1\right) dt \wedge \tilde{\text{vol}}_{(4)} \, , \tag{3.15}$$

$$C_{(1)} = H_{\rm D8} \left(H_{\rm D0}^{-1} - 1\right) dt \, , \qquad C_{(9)} = \left(H_{\rm D8}^{-1} - 1\right) dt \wedge \text{vol}_{(4)} \wedge \tilde{\text{vol}}_{(4)} \, , \tag{3.16}$$

where the warp factors appearing in the above metric read

$$\begin{cases} H_{\rm D0} = H_{\rm F1} \; = \; 1 + \dfrac{Q_1}{\rho^2} + \dfrac{Q_2}{r^2} \, , \\ H_{\rm D4} = 1 + \dfrac{Q_1}{\rho^2} \, , \\ H_{\rm D4'} = 1 + \dfrac{Q_2}{r^2} \, , \\ H_{\rm D8} = 1 + Q_3 \, z \, . \end{cases} \tag{3.17}$$

If we now take the limit $\rho \to 0$ while keeping (z, r) finite, the metric becomes

$$ds_{10}^2 = H_{\rm D4'}^{-1/2} H_{\rm D8}^{-1/2} \left[Q_1 \left(ds^2_{\text{AdS}_2} + ds^2_{S^3}\right) + H_{\rm D4'} dr^2 + H_{\rm D4'} H_{\rm D8} dz^2 + r^2 H_{\rm D4'} ds^2_{\tilde{S}^3}\right] , \tag{3.18}$$

where $L_{\text{AdS}_2} = 1/2$, which is $\text{AdS}_2 \times S^3 \times \tilde{S}^3$ warped over the (z, r) coordinates. By comparing (3.17) with (3.2), one finds an explicit mapping between the (z, r) coordinates and the (α, ξ) coordinates appearing in the uplift formula. In particular, by comparing the warp factors in front of the $\text{AdS}_2 \times S^3$ block of the metric and the two expressions of the 10D dilaton, one gets the following two algebraic relations

$$\begin{cases} Q_1 \, H_{\rm D4'}^{-1/2} H_{\rm D8}^{-1/2} \overset{!}{=} s^{-1/3} \Delta^{1/2} X^{-1/2} e^{2U} \, , \\ \quad H_{\rm D4'}^{-1/4} H_{\rm D8}^{-5/4} \overset{!}{=} s^{-5/6} \Delta^{1/4} X^{-5/4} \, , \end{cases} \tag{3.19}$$

which, once combined with the matching condition for the $\tilde{S}^3$ block, give

$$\begin{cases} r = \dfrac{2}{\sqrt{g}} Q_1^{-3/4} e^{3U/2} X^{-1/2}\, c & , \\ z = Q_3^{-1}\left(Q_1^{-1/2} e^{U} X s^{2/3} - 1\right) & . \end{cases} \tag{3.20}$$

The complete forms of the two 10D backgrounds match through the coordinate change in (3.20), upon further identifying $Q_3 = m$, together with the condition (3.1) relating the couplings g & m.

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A Massive IIA supergravity

In this appendix we review the main features of massive IIA supergravity [54]. The theory is characterized by the bosonic fields g_{MN}, Φ, $B_{(2)}$, $C_{(1)}$ and $C_{(3)}$. The action has the following form

$$S_{\text{mIIA}} = \frac{1}{2\kappa_{10}^2}\left[\int \mathrm{d}^{10}x\, \sqrt{-g}\, e^{-2\Phi}\left(R + 4\,\partial_\mu\Phi\,\partial^\mu\Phi - \frac{1}{2}\,|H_{(3)}|^2\right) - \frac{1}{2}\sum_{p=0,2,4}|F_{(p)}|^2\right] + S_{\text{top}}\,, \tag{A.1}$$

where S_{top} is a topological term given by

$$\begin{aligned} S_{\text{top}} = -\frac{1}{2}\int \big(& B_{(2)}\wedge F_{(4)}\wedge F_{(4)} - \frac{1}{3}F_{(0)}\wedge B_{(2)}\wedge B_{(2)}\wedge B_{(2)}\wedge F_{(4)} \\ & + \frac{1}{20}F_{(0)}\wedge F_{(0)}\wedge B_{(2)}\wedge B_{(2)}\wedge B_{(2)}\wedge B_{(2)}\wedge B_{(2)}\big)\,, \end{aligned} \tag{A.2}$$

where $H_{(3)} = dB_{(2)}$, $F_{(2)} = dC_{(1)}$, $F_{(3)} = dC_{(3)}$ and the 0-form field strength $F_{(0)}$ is associated to the Romans' mass as $F_{(0)} = m$.

All the equations of motion can be derived[7] consistently from (A.1). They have the following form

$$\begin{aligned} R_{MN} - \frac{1}{2}T_{MN} &= 0\,, \\ \Box\Phi - |\partial\Phi|^2 + \frac{1}{4}R - \frac{1}{8}|H_{(3)}|^2 &= 0\,, \\ d\left(e^{-2\Phi}\star_{10} H_{(3)}\right) &= 0\,, \\ \left(d + H_{(3)}\wedge\right)\left(\star_{10}F_{(p)}\right) &= 0\,, \quad \text{with} \quad p = 2,4\,, \end{aligned} \tag{A.3}$$

[7] We set $\kappa_{10} = 8\pi G_{10} = 1$.

where $M, N, \cdots = 0, \ldots, 9$ and R and $\Box$ are respectively the 10D scalar curvature and the Laplacian. The stress-energy tensor is given by

$$\begin{aligned}T_{MN} = e^{2\Phi} \sum_p \left(\frac{p}{p!} F_{(p)\,MM_1\ldots M_{p-1}} F_{(p)\,N}{}^{M_1\ldots M_{p-1}} - \frac{p-1}{8} g_{MN} |F_{(p)}|^2 \right) \\ + \left(\frac{1}{2} H_{(3)\,MPQ} H_{(3)\,N}{}^{PQ} - \frac{1}{4} g_{MN} |H_{(3)}|^2 \right) - \left(4\nabla_M \nabla_N \Phi + \frac{1}{2} g_{MN} (\Box\Phi - 2|\partial\Phi|^2) \right),\end{aligned} \tag{A.4}$$

with ∇_M being associated with the Levi-Civita connection of the 10D background. The Bianchi identities take the form

$$\begin{aligned} dF_{(2)} &= F_{(0)} \wedge H_{(3)} \,, \\ dF_{(4)} &= -F_{(2)} \wedge H_{(3)} \,, \\ dH_{(3)} &= 0 \,, \\ dF_{(0)} &= 0 \,. \end{aligned} \tag{A.5}$$

As a consequence of (A.5), the following fluxes

$$F_{(0)} = m, \qquad H_{(3)} \qquad F_{(2)} - mB_{(2)} \,, \qquad F_{(4)} - B_{(2)} \wedge F_{(2)} + \frac{1}{2} mB_{(2)} \wedge B_{(2)} \,, \tag{A.6}$$

turn out to satisfy a Dirac quantization condition.

It may be worth mentioning that the truncation *Ansatz* of section 3.1 is obtained by casting massive IIA supergravity into the Einstein frame [45]. To convert the action (A.1), the equations of motions (A.3) and Bianchi identities (A.5) into the Einstein frame, one has to redefine the metric as $g_{MN} = e^{\Phi/2}\, g^{(\mathrm{E})}_{MN}$.

B Symplectic-Majorana-Weyl spinors in $d = 1 + 5$

In this appendix we collect the conventions and the fundamental relations involving irreducible spinors in $d = 1+5$. Subsequently, we construct an explicit representation of Dirac matrices. In $d = 1+5$ Dirac spinors enjoy 16 independent real components and they can be decomposed into irreducible Weyl spinors with opposite chirality and having 8 independent real components each. The 6D Clifford algebra is defined by the relation

$$\{\Gamma^m, \Gamma^n\} = 2\,\eta^{mn}\,\mathbb{I}_8 \,, \tag{B.1}$$

where $\{\Gamma^m\}_{m=0,\cdots 5}$ are the 8×8 Dirac matrices and $\eta = \mathrm{diag}(-1, +1, +1, +1, +1)$. The chirality operator Γ_* can be defined in the following way in terms of the above Dirac matrices

$$\Gamma_* = \Gamma^0\,\Gamma^1\,\Gamma^2\,\Gamma^3\,\Gamma^4\,\Gamma^5 \qquad \text{with} \qquad \Gamma_*\,\Gamma_* = \mathbb{I}_8 \,. \tag{B.2}$$

For $(1+5)$-dimensional backgrounds, we can choose the matrices A, B, C, respectively realizing Dirac, complex and charge conjugation, satisfying the following defining relations [55]

$$(\Gamma^m)^\dagger = -A\,\Gamma^m\,A^{-1} \,, \quad (\Gamma^m)^* = B\,\Gamma^m\,B^{-1} \,, \quad (\Gamma^m)^T = -C\,\Gamma^m\,C^{-1} \,, \tag{B.3}$$

with

$$B^T = C\,A^{-1}\,, \quad B^*\,B = -\mathbb{I}_8\,, \quad C^T = -C^{-1} = -C^\dagger = C\,. \tag{B.4}$$

The second identity in (B.4) implies that it is actually inconsistent to define a proper reality condition on Dirac (or Weyl) spinors. However, it is always possible to introduce $\mathrm{SU}(2)_R$ doublets ζ^a of Dirac spinors, called symplectic-Majorana (SM) spinors respecting a pseudo-reality condition [55] given by

$$\zeta_a \equiv (\zeta^a)^* \stackrel{!}{=} \epsilon_{ab}\,B\,\zeta^b\,, \tag{B.5}$$

where ϵ_{ab} is the SU(2) invariant Levi-Civita symbol. The condition (B.5) ensures us that the number of independent components of a SM spinor be the same of those of a Dirac spinor. Moreover, the above condition also turns out to be compatible with the projections onto the chiral components of a Dirac spinor. Hence it is possible to construct SM doublets of irreducible Weyl spinors that are called symplectic-Majorana-Weyl (SMW) spinors.

Let us now construct an explicit representation for the Dirac matrices satisfying (B.1). We firstly introduce the Dirac matrices $\{\rho^\alpha\}_{\alpha=0,1}$ for a $(1+1)$-dimensional background in the following representation

$$\rho^0 = i\sigma^2\,, \qquad \rho^1 = \sigma^1\,, \qquad \rho_* = \rho^0\rho^1 = \sigma^3\,, \tag{B.6}$$

and the Dirac matrices for a Euclidean 3-dimensional background $\{\gamma^i\}_{i=1,2,3}$ as

$$\gamma^1 = \sigma^1\,, \qquad \gamma^2 = \sigma^2\,, \qquad \gamma^3 = \sigma^3\,, \tag{B.7}$$

where

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,, \tag{B.8}$$

are the Pauli matrices. An explicit representation of the $(1+5)$-dimensional Dirac matriced satisying (B.1) can be defined in the following way

$$\begin{aligned} \Gamma^\alpha &= \rho^\alpha \otimes \sigma^1 \otimes \mathbb{I}_2\,, && \text{with} && \alpha = 0,\,1\,, \\ \Gamma^2 &= \rho_* \otimes \sigma^1 \otimes \mathbb{I}_2\,, \\ \Gamma^i &= \mathbb{I}_2 \otimes \sigma^2 \otimes \gamma^i\,, && \text{with} && i = 1,\,2\,,3\,. \end{aligned} \tag{B.9}$$

In this representation the chirality operator (B.2) takes the form

$$\Gamma_* = \mathbb{I}_2 \otimes \sigma^3 \otimes \mathbb{I}_2\,, \tag{B.10}$$

while the matrices A, B, C may be written as

$$\begin{aligned} A &= \Gamma^0 = i\,\sigma^2 \otimes \sigma^1 \otimes \mathbb{I}_2\,, \\ B &= -i\,\Gamma^4\,\Gamma^5 = \mathbb{I}_2 \otimes \mathbb{I}_2 \otimes \sigma^1\,, \\ C &= i\,\Gamma^0\,\Gamma^4\,\Gamma^5 = i\,\sigma^2 \otimes \sigma^1 \otimes \sigma^1\,. \end{aligned} \tag{B.11}$$

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