Erratum: Initial conditions and degrees of freedom of non-local gravity

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The authors did not realize that some of the initial conditions indicated to solve the Cauchy problem in nonlocal systems are not independent, so that the number of initial conditions is reduced from 4 to 2 in the case of the scalar field. Consequently, and correcting also a couple of minor typos, the text should be changed as follows.

- Abstract: “the non-local scalar and gravitational theories with exponential operators are both characterized by four initial conditions” should read “the non-local scalar and gravitational theories with exponential operators are characterized by, respectively, two and four initial conditions”
- Section 1, page 4: “The counting of initial conditions and degrees of freedom is carried out in section 2.3, where we find that this number is, respectively, 4 and 1 for the real non-local scalar” should be “The counting of initial conditions and degrees of freedom is carried out in section 2.3, where we find that this number is, respectively, 2 and 1 for the real non-local scalar”.

- Section 2, page 11, item 3: “In particular, these conditions are valid at $r = \beta r_*$, so that also the Cauchy problem of the non-local system (2.1) is solved by four initial conditions, corresponding (via (2.16)) to $\phi(t_i, \mathbf{x})$ and its first three time derivatives. We will find a similar result also in non-local gravity, with four initial conditions for the metric. In general, given a non-local action with exponential non-locality for tensorial field $\phi_{\mu\nu\dots}$ representing n physical degrees of freedom, the diffusion-equation method relies on a second-order localized system for a field $\Phi_{\mu\nu\dots}$ and an auxiliary field $\chi^{\mu\nu\dots}$ with the same symmetry properties as ϕ , thus leading to $2n$ initial conditions.” should be “In particular, these conditions are valid at $r = \beta r_*$, where, however, χ is fully determined once ϕ is known. Therefore, the Cauchy problem of the non-local system (2.1) is solved by two initial conditions, corresponding (via (2.16)) to $\phi(t_i, \mathbf{x})$ and its first time derivative.”
- Section 2, page 16, the paragraph “The novelty brought in by the diffusion-equation method is that it allows one to go beyond the free theory and count the extra number of initial conditions. Surprisingly, in the scalar-field case this number is two, which sums to the two initial conditions of the free theory for a total of four. This result goes against the belief, implicitly endorsed in some literature, that the information from the free theory is complete. In particular, when one says that the number of initial conditions for solving the Cauchy problem of the theory (2.1) is two, one should specify that this is true only for the free, perturbative case.” should read “The novelty brought in by the diffusion-equation method is that it allows one to go beyond the free theory and count the extra number of initial conditions. Surprisingly, in the scalar-field case this number is zero and there are no extra initial conditions with respect to the free theory.”
- Section 2, page 16, claim (2.34): “As a consequence, the Cauchy problem of the non-local non-perturbative real scalar field theory (2.1) in D dimensions is specified by four initial conditions $\phi(t_i, \mathbf{x})$, $\dot{\phi}(t_i, \mathbf{x})$, $\ddot{\phi}(t_i, \mathbf{x})$, $\dddot{\phi}(t_i, \mathbf{x})$.” should be “As a consequence, the Cauchy problem of the non-local non-perturbative real scalar field theory (2.1) in D dimensions is specified by two initial conditions $\phi(t_i, \mathbf{x})$ and $\dot{\phi}(t_i, \mathbf{x})$.”
- Section 2, page 19, next-to-last bullet: “The specific prediction of the diffusion method is (2.33), for any non-quadratic potential.” should read “The specific prediction of the diffusion method is (2.33) and (2.34), for any non-quadratic potential.”
- Section 3, page 22 above (3.12) and page 23 above (3.16): “ $\gamma_{r_*}(\square) = -r_* + (r_*^2/2)\square + O(\square^3)$ ” should be “ $\gamma_{r_*}(\square) = -r_* + (r_*^2/2)\square + O(\square^2)$ ”.
- Section 4, page 28, first paragraph: the sentence “This is a striking resemblance with the scalar-field case, where we had to specify the same number of initial conditions once the auxiliary field was removed” should be removed.

The authors apologize to the reader.

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