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Measuring properties of a Heavy Higgs boson in the $H \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^-$ decay

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ABSTRACT: Suppose a heavy neutral Higgs or scalar boson H is discovered at the LHC, it is important to investigate its couplings to the standard model particles as much as possible. Here in this work we attempt to probe the CP-even and CP-odd couplings of the heavy Higgs boson to a pair of top quarks, through the decay $H \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^-$. We use the helicity-amplitude method to write down the most general form for the angular distributions of the final-state b quarks and W bosons. We figure out that there are 6 types of angular observables and, under CPT conservation, one-dimensional angular distributions can only reveal two of them. Nevertheless, the H couplings to the $t\bar{t}$ pair can be fully determined by exploiting the one-dimensional angular distributions. A Higgs-boson mass of 380 GeV not too far above the $t\bar{t}$ threshold is illustrated with full details. With a total of 10^4 events of $H \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^+$, one can determine the couplings up to 10–20% uncertainties.

KEYWORDS: Phenomenological Models

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1 Introduction

The scalar boson that was discovered at the LHC in 2012 [1, 2] turned out to be best described by the Standard Model (SM) Higgs boson [3], which is remarkable confirmation of the Higgs mechanism proposed in 1960s [4–6]. Among the Higgs boson couplings to the SM particles, the most constrained one is its coupling to the massive gauge bosons that is very close to the corresponding SM value with about 10% uncertainty [7]. Nevertheless, the couplings to fermions are much less constrained, especially for the first two generations. The coupling to the top-quark pair from global fits has about 20–30% uncertainty [7]. There was also direct measurements of the top Yukawa coupling in $pp \rightarrow t\bar{t}h$ production [8, 9], which still needs more data to have more precise measurements than the global fitting.

Even though the SM has achieved a great success in accounting for the interactions among the basic building blocks of matter, however extra particles and new interactions are required to explain the experimental observations of dark matter, non-vanishing neutrino mass, the baryon asymmetry of our Universe, inflation, etc. In most extensions beyond the SM, the Higgs sector is enlarged to include more than one Higgs doublet resulting in charged Higgs bosons and several neutral Higgs bosons in addition to the one discovered at the LHC. For example, the minimal supersymmetric extension of the SM, aka MSSM [10–12], requires two Higgs doublet fields, thus leading to a pair of charged Higgs bosons and 3 neutral ones. In the next-to-minimal supersymmetric standard model, there are two additional neutral Higgs bosons [13].

Suppose that in future experiments a neutral Higgs boson H heavier than the SM 125 GeV Higgs boson (denoted by h) is discovered. It is generally expected that the decays of the heavy Higgs boson H into gauge bosons would be suppressed as it becomes heavier

and heavier, because the measurement of the gauge-Higgs coupling of the SM-like 125 GeV boson allows only a little room for the H couplings to massive vector bosons. However, its couplings to fermions have no reasons to be small. Indeed, once it is above the $t\bar{t}$ threshold, the decay into $t\bar{t}$ pair could be dominant.

In this work, we assume that the heavy Higgs boson H is not too far above the $t\bar{t}$ threshold, say 380 GeV, and the dominant decay mode is $t\bar{t}$. Without requiring H to carry any definite CP-parity, we consider the possibility to probe its CP nature via the decay $H \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^-$. We employ the helicity-amplitude method [14] to calculate the decay amplitude taking into account all spin correlations in the decay chain. By measuring various angular distributions, in particular the angle between the decay planes of the top and anti-top quarks, one can discern the CP-even and CP-odd couplings of the Higgs boson. This is the main goal of this work. Other fermionic modes, in general, are either too small or suffer tremendously from SM backgrounds. The top quark also has the advantage that it decays before hadronization, in contrast to the bottom or charm quarks, and therefore the spin information is retained in the decay products of the top quark. Thus, the spin and CP properties of the parent Higgs boson can be determined by studying several kinematical distributions of the decay products of the top and anti-top quarks.

On the other hand, when the heavy Higgs boson H is below the $t\bar{t}$ threshold, its bosonic decay modes ZZ , hh , and hZ are useful for probing the CP nature of it. By taking account of the spin-0 nature of H , only the ZZ mode may lead to nontrivial angular correlations among the decay products of the Z bosons through the interferences among various helicity states of the two intermediate Z bosons before their decays [15]. This bosonic mode was suggested to determine the spin and parity of the Higgs boson [16] quite a number of years ago. After the 125 GeV Higgs-boson discovery, the method was practically applied to determine the spin and CP properties of the observed Higgs boson [17, 18]. We shall not concern with the bosonic modes in this work.

Under the current experimental status, in which active searches for heavy resonances decaying into a $t\bar{t}$ pair have been continually performed [19, 20], our study may show how well one can determine the properties of such a heavy scalar Higgs boson at the LHC and/or High Luminosity LHC (HL-LHC). We refer to, for example, ref. [21] for some previous studies at e^+e^- or $\mu^+\mu^-$ colliders.

The remainder of this article is organized as follows. In section 2, based on the helicity amplitude method [14], we present a formalism for the study of angular distributions in the decay $H \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^-$. We point out that there are 6 types of angular observables in general and we can classify them according to the CP and $C\tilde{P}$ parities of each observable. In section 3, we illustrate how well one can measure the couplings of the heavy Higgs boson by exploiting the angular observables introduced in section 2. Finally, section 4 is devoted to a brief summary, some prospects for future work and conclusions.

2 Formalism

Without loss of generality, the Lagrangian describing the interactions of the Higgs boson H with top quarks can be written as [22]

$$\mathcal{L}_{H\bar{t}t} = -g_t H \bar{t}(g^S + i\gamma_5 g^P)t = -g_t \sum_{A=L,R} (g^S + iAg^P) H \bar{t}P_A t \quad (2.1)$$

where $P_A = (1 + A\gamma_5)/2$ with $A = -(L), +(R)$. g_t is the overall strength of the $H-t-\bar{t}$ coupling and $g^{P(S)} = 0$ when H is the pure CP-even (odd) state. If H is a CP-mixed state, both g^S and g^P are non-vanishing. For the SM Higgs boson, $g_t = gm_t/(2M_W)$, $g^S = 1$, and $g^P = 0$. On the other hand, the Lagrangian describing the interactions of the top quarks with bottom quarks and W bosons is

$$\begin{aligned}\mathcal{L} &= -\frac{g}{\sqrt{2}} [W_\mu^- \bar{b}\gamma^\mu (f_L P_L + f_R P_R)t + W_\mu^+ \bar{t}\gamma^\mu (f_L^* P_L + f_R^* P_R)b] \\ &= -\frac{g}{\sqrt{2}} [(W_\mu^+)^* \bar{b}\gamma^\mu (f_L P_L + f_R P_R)t + (W_\mu^-)^* \bar{t}\gamma^\mu (f_L^* P_L + f_R^* P_R)b] .\end{aligned}\quad (2.2)$$

In the SM, $f_L = 1$ and $f_R = 0$.

2.1 Helicity amplitudes

We first present the helicity amplitude for the process

$$H \rightarrow t(p_t, \sigma_t) \bar{t}(\bar{p}_t, \bar{\sigma}_t) \rightarrow b(p_b, \sigma_b) W^+(k_1, \epsilon_1) \bar{b}(\bar{p}_b, \bar{\sigma}_b) W^-(k_2, \epsilon_2).$$

Here $p_{t,b}$ and $\bar{p}_{t,b}$ are four-momenta of the quarks (t, b) and antiquarks ($\bar{t}, \bar{b}$), respectively, and $\sigma_{t,b}$ and $\bar{\sigma}_{t,b}$ denote their helicities. The four-momenta of W^+ and W^- are denoted by k_1 and k_2 , respectively, with $p_t = p_b + k_1$ and $\bar{p}_t = \bar{p}_b + k_2$ and $\epsilon_1(\lambda_1)$ and $\epsilon_2(\lambda_2)$ are the polarization 4-vectors of W bosons. Depending on the helicities of the final-state particles, the amplitude can be cast into the form

$$\begin{aligned}i\mathcal{M}_{\sigma_b\lambda_1:\bar{\sigma}_b\lambda_2} &= \sum_{A,B,C=L,R} \left\{ \bar{u}_b(p_b, \sigma_b) \left[-i\frac{g}{\sqrt{2}} \not{\epsilon}_1^*(k_1) f_A P_A \right] \frac{i}{\not{p}_t - m_t} [-ig_t(g^S + iBg^P)P_B] \right. \\ &\quad \times \frac{i}{-\not{p}_t - m_t} \left[-i\frac{g}{\sqrt{2}} \not{\epsilon}_2^*(k_2) f_C^* P_C \right] v_b(\bar{p}_b, \bar{\sigma}_b) \Big\} \\ &= -i \frac{1}{p_t^2 - m_t^2 + im_t\Gamma_t} \frac{1}{\bar{p}_t^2 - m_t^2 + im_t\Gamma_t} \sum_{\sigma_t, \bar{\sigma}_t} \mathcal{M}_{\sigma_t:\sigma_b\lambda_1}^{t \rightarrow bW^+} \mathcal{M}_{\sigma_t\bar{\sigma}_t}^{H \rightarrow t\bar{t}} \mathcal{M}_{\bar{\sigma}_t:\bar{\sigma}_b\lambda_2}^{\bar{t} \rightarrow \bar{b}W^-}\end{aligned}\quad (2.3)$$

using

$$\sum_{\sigma_t} u(p_t, \sigma_t) \bar{u}(p_t, \sigma_t) = \not{p}_t + m_t, \quad \sum_{\bar{\sigma}_t} v(\bar{p}_t, \bar{\sigma}_t) + \bar{v}(\bar{p}_t, \bar{\sigma}_t) = \not{p}_t - m_t. \quad (2.4)$$

The helicity amplitude for the decay $H(q) \rightarrow t(p_t, \sigma_t) \bar{t}(\bar{p}_t, \bar{\sigma}_t)$ in the rest frame of H is given by

$$\mathcal{M}_{\sigma_t\bar{\sigma}_t}^{H \rightarrow t\bar{t}} = g_t \sqrt{s} \langle \sigma_t \rangle \delta_{\sigma_t\bar{\sigma}_t} e^{-i\sigma_t\phi_t} \quad (2.5)$$

where $s = q^2$, ϕ_t is the azimuthal angle of the t momentum, and

$$\langle + \rangle = X g^S - iY g^P, \quad \langle - \rangle = -X g^S - iY g^P. \quad (2.6)$$

The momentum-dependent X and Y are given by

$$\begin{aligned}X &= \frac{\left[(1 + \lambda_H^{1/2})^2 - (\alpha_t - \bar{\alpha}_t)^2 \right]^{1/2} - \left[(1 - \lambda_H^{1/2})^2 - (\alpha_t - \bar{\alpha}_t)^2 \right]^{1/2}}{2}, \\ Y &= \frac{\left[(1 + \lambda_H^{1/2})^2 - (\alpha_t - \bar{\alpha}_t)^2 \right]^{1/2} + \left[(1 - \lambda_H^{1/2})^2 - (\alpha_t - \bar{\alpha}_t)^2 \right]^{1/2}}{2},\end{aligned}\quad (2.7)$$

where $\lambda_H = (1 + \alpha_t^2 + \bar{\alpha}_t^2 - 2\alpha_t - 2\bar{\alpha}_t - 2\alpha_t\bar{\alpha}_t)$ with $\alpha_t = p_t^2/s$ and $\bar{\alpha}_t = \bar{p}_t^2/s$. When the top quarks are on-shell, $X = \beta_t = (1 - 4m_t^2/s)^{1/2}$ and $Y = 1$. One may take $\phi_t = 0$ without loss of generality.

The helicity amplitude for the decay $t(p_t, \sigma_t) \rightarrow b(p_b, \sigma_b)W^+(k_1, \epsilon_1)$ in the t rest frame is given by

$$\mathcal{M}_{\sigma_t: \sigma_b \lambda}^{t \rightarrow b W^+} = -\frac{g}{\sqrt{2}} \sqrt{2\sqrt{p_t^2} E_b} \langle \sigma_t : \sigma_b \lambda_1 \rangle_t, \quad (2.8)$$

where $2\sqrt{p_t^2} E_b = p_t^2 + m_b^2 - M_W^2$. The reduced helicity amplitudes $\langle \sigma_t : \sigma_b \lambda_1 \rangle_t$ are given by

$$\langle \sigma_t : \sigma_b \lambda_1 \rangle_t = \begin{cases} \sum_{A=L,R} \left[-A(\sqrt{2}f_A c_{\theta_1/2}) \frac{(1+A\sigma_t\beta_b)^{1/2}}{\sqrt{2}} \delta_{\sigma_t\lambda_1} \delta_{\sigma_t\sigma_b} \right. \\ \quad \left. + A\sigma_t(\sqrt{2}f_A s_{\theta_1/2} e^{+i\sigma_t\phi_1}) \frac{(1-A\sigma_t\beta_b)^{1/2}}{\sqrt{2}} \delta_{\sigma_t-\lambda_1} \delta_{\sigma_t-\sigma_b} \right] & \text{for } \lambda_1 = \pm \\ \sum_{A=L,R} \left[(f_A s_{\theta_1/2}) \frac{\beta_b E_b + A\sigma_t E_W}{M_W} \frac{(1+A\sigma_t\beta_b)^{1/2}}{\sqrt{2}} \delta_{\sigma_t\sigma_b} \right. \\ \quad \left. + \sigma_t(f_A c_{\theta_1/2} e^{+i\sigma_t\phi_1}) \frac{\beta_b E_b - A\sigma_t E_W}{M_W} \frac{(1-A\sigma_t\beta_b)^{1/2}}{\sqrt{2}} \delta_{\sigma_t-\sigma_b} \right] & \text{for } \lambda_1 = 0 \end{cases} \quad (2.9)$$

where θ_1 and ϕ_1 are the polar and azimuthal angles of the W^+ momentum in the t rest frame and $c_{\theta_1/2} = \cos(\theta_1/2)$ and $s_{\theta_1/2} = \sin(\theta_1/2)$. We note that $\sigma_b = \lambda_1$ when $\lambda_1 = \pm$ and the 4 amplitudes of $\langle \pm : + - \rangle_t$ and $\langle \pm : - + \rangle_t$ are identically vanishing. In the $m_b \rightarrow 0$ limit, the reduced amplitudes simplify and we have the following non-vanishing 8 amplitudes:¹

$$\begin{aligned} \langle - : -- \rangle_t &= +\sqrt{2}f_L c_{\theta_1/2}, & \langle - : -0 \rangle_t &= +\frac{\sqrt{p_t^2}}{M_W} f_L s_{\theta_1/2}, \\ \langle + : -- \rangle_t &= -\sqrt{2}f_L s_{\theta_1/2} e^{+i\phi_1}, & \langle + : -0 \rangle_t &= +\frac{\sqrt{p_t^2}}{M_W} f_L c_{\theta_1/2} e^{+i\phi_1}; \\ \langle + : ++ \rangle_t &= -\sqrt{2}f_R c_{\theta_1/2}, & \langle + : +0 \rangle_t &= +\frac{\sqrt{p_t^2}}{M_W} f_R s_{\theta_1/2}, \\ \langle - : ++ \rangle_t &= -\sqrt{2}f_R s_{\theta_1/2} e^{-i\phi_1}, & \langle - : +0 \rangle_t &= -\frac{\sqrt{p_t^2}}{M_W} f_R c_{\theta_1/2} e^{-i\phi_1}. \end{aligned} \quad (2.10)$$

We note that $A = \sigma_b$ in the $m_b = 0$ limit.

The helicity amplitude for the decay $\bar{t}(\bar{p}_t, \bar{\sigma}_t) \rightarrow \bar{b}(\bar{p}_b, \bar{\sigma}_b)W^-(k_2, \epsilon_2)$ in the $\bar{t}$ rest frame is similarly given by

$$\mathcal{M}_{\bar{\sigma}_t: \bar{\sigma}_b \lambda_2}^{\bar{t} \rightarrow \bar{b} W^-} \equiv -\frac{g}{\sqrt{2}} \sqrt{2\sqrt{\bar{p}_t^2} \bar{E}_b} \langle \bar{\sigma}_t : \bar{\sigma}_b \lambda_2 \rangle_{\bar{t}} \quad (2.11)$$

where $2\sqrt{\bar{p}_t^2} \bar{E}_b = \bar{p}_t^2 + m_b^2 - M_W^2$ and the reduced amplitudes $\langle \bar{\sigma}_t : \bar{\sigma}_b \lambda_2 \rangle_{\bar{t}}$ can be obtained by replacing f_A with f_{-A}^* in eq. (2.9) together with $\sigma_{t,b} \rightarrow \bar{\sigma}_{t,b}$, $\lambda_1 \rightarrow \lambda_2$, etc.² Further we

¹In ref. [23], the authors presented the helicity amplitudes in the $m_b \rightarrow 0$ limit. We find a minor discrepancy in four of the amplitudes with $\lambda_1 = \pm$ by an overall factor of $e^{-i\lambda_1\phi_1}$, which does not affect the full amplitude squared for the process $H \rightarrow t\bar{t} \rightarrow bW^+ \bar{b}W^-$.

²For details of the relation between the helicity amplitudes for the t and $\bar{t}$ decays, see appendix A.

note the relations

$$\begin{aligned}
 \langle - : -- \rangle_{\bar{t}} &= -\langle + : ++ \rangle_t^*, & \langle - : -0 \rangle_{\bar{t}} &= +\langle + : +0 \rangle_t^*, \\
 \langle + : -- \rangle_{\bar{t}} &= +\langle - : ++ \rangle_t^*, & \langle + : -0 \rangle_{\bar{t}} &= -\langle - : +0 \rangle_t^*, \\
 \langle + : ++ \rangle_{\bar{t}} &= -\langle - : -- \rangle_t^*, & \langle + : +0 \rangle_{\bar{t}} &= +\langle - : -0 \rangle_t^*, \\
 \langle - : ++ \rangle_{\bar{t}} &= +\langle + : -- \rangle_t^*, & \langle - : +0 \rangle_{\bar{t}} &= -\langle + : -0 \rangle_t^*.
 \end{aligned} \tag{2.12}$$

Collecting all the sub-amplitudes we obtain

$$\mathcal{M}_{\sigma_b \lambda_1 : \bar{\sigma}_b \lambda_2} \equiv -g^2 g_t \frac{\sqrt{s}}{2} \frac{\sqrt{p_t^2 + m_b^2 - M_W^2}}{p_t^2 - m_t^2 + im_t \Gamma_t} \frac{\sqrt{\bar{p}_t^2 + m_b^2 - M_W^2}}{\bar{p}_t^2 - m_t^2 + im_t \Gamma_t} \langle \sigma_b \lambda_1 : \bar{\sigma}_b \lambda_2 \rangle \tag{2.13}$$

where

$$\langle \sigma_b \lambda_1 : \bar{\sigma}_b \lambda_2 \rangle = \sum_{\sigma_t = \pm} \langle \sigma_t \rangle \langle \sigma_t : \sigma_b \lambda_1 \rangle_t \langle \sigma_t : \bar{\sigma}_b \lambda_2 \rangle_{\bar{t}}. \tag{2.14}$$

2.2 Angular coefficients and observables

The partial decay width of the process $H \rightarrow t\bar{t} \rightarrow bW^+ \bar{b}W^-$ is given by³

$$\begin{aligned}
 d\Gamma &= \frac{N_C}{2M_H} \left(\sum_{\sigma_b, \lambda_1, \bar{\sigma}_b, \lambda_2} |\mathcal{M}_{\sigma_b \lambda_1 : \bar{\sigma}_b \lambda_2}|^2 \right) d\Phi_4 \\
 &= \frac{N_C}{2^{13} \pi^6 M_H} \lambda_H^{1/2} \lambda_t^{1/2} \lambda_{\bar{t}}^{1/2} \sqrt{p_t^2} \sqrt{\bar{p}_t^2} \left(\sum |\mathcal{M}|^2 \right) d\sqrt{p_t^2} d\sqrt{\bar{p}_t^2} dc_{\theta_1} dc_{\theta_2} d\Phi
 \end{aligned} \tag{2.15}$$

where $N_C = 3$ and

$$\lambda_t = \left(1 - \frac{m_b^2}{p_t^2} - \frac{M_W^2}{p_t^2} \right)^2 - 4 \frac{m_b^2}{p_t^2} \frac{M_W^2}{p_t^2}, \quad \lambda_{\bar{t}} = \left(1 - \frac{m_b^2}{\bar{p}_t^2} - \frac{M_W^2}{\bar{p}_t^2} \right)^2 - 4 \frac{m_b^2}{\bar{p}_t^2} \frac{M_W^2}{\bar{p}_t^2}. \tag{2.16}$$

For any values of f_L and f_R , taking also account of finite m_b effects, the precise differential angular distribution $\frac{d\Gamma}{dc_{\theta_1} dc_{\theta_2} d\Phi}$ can be obtained numerically by integrating eq. (2.15) over $\sqrt{p_t^2}$ and $\sqrt{\bar{p}_t^2}$ and using eqs. (2.13), (2.14), (2.6), and (2.9).

On the other hand, in the $m_b \rightarrow 0$ limit, the amplitudes take much simpler forms and one can derive *analytic* expressions for the differential angular distributions in terms of physically meaningful angular coefficients and observables. When $f_L = 1$ and $f_R = 0$, there are only four non-vanishing amplitudes: $\langle -- : ++ \rangle$, $\langle -- : +0 \rangle$, $\langle -0 : ++ \rangle$, and $\langle -0 : +0 \rangle$. Explicitly, from table 1, we have

$$\begin{aligned}
 \langle -- : ++ \rangle &= 2 \left(\langle + \rangle s_{\theta_1/2} c_{\theta_2/2} e^{i\phi_1} - \langle - \rangle c_{\theta_1/2} s_{\theta_2/2} e^{-i\phi_2} \right) \\
 \langle -- : +0 \rangle &= -\sqrt{2} \frac{\sqrt{\bar{p}_t^2}}{M_W} \left(\langle + \rangle s_{\theta_1/2} s_{\theta_2/2} e^{i\phi_1} + \langle - \rangle c_{\theta_1/2} c_{\theta_2/2} e^{-i\phi_2} \right) \\
 \langle -0 : ++ \rangle &= -\sqrt{2} \frac{\sqrt{p_t^2}}{M_W} \left(\langle + \rangle c_{\theta_1/2} c_{\theta_2/2} e^{i\phi_1} + \langle - \rangle s_{\theta_1/2} s_{\theta_2/2} e^{-i\phi_2} \right) \\
 \langle -0 : +0 \rangle &= \frac{\sqrt{p_t^2} \sqrt{\bar{p}_t^2}}{M_W^2} \left(\langle + \rangle c_{\theta_1/2} s_{\theta_2/2} e^{i\phi_1} - \langle - \rangle s_{\theta_1/2} c_{\theta_2/2} e^{-i\phi_2} \right)
 \end{aligned} \tag{2.17}$$

³For the four-body phase space, see appendix B.

σ_t	σ_b	λ_1	$\langle \sigma_t : \sigma_b \lambda_1 \rangle_t$	$\bar{\sigma}_t$	$\bar{\sigma}_b$	λ_2	$\langle \bar{\sigma}_t : \bar{\sigma}_b \lambda_2 \rangle_{\bar{t}}$
+	−	−	$-\sqrt{2}s_{\theta_1/2}e^{i\phi_1}$	+	+	+	$-\sqrt{2}c_{\theta_2/2}$
+	−	0	$\frac{\sqrt{p_t^2}}{M_W}c_{\theta_1/2}e^{i\phi_1}$	+	+	0	$\frac{\sqrt{\bar{p}_t^2}}{M_W}s_{\theta_2/2}$
−	−	−	$\sqrt{2}c_{\theta_1/2}$	−	+	+	$-\sqrt{2}s_{\theta_2/2}e^{-i\phi_2}$
−	−	0	$\frac{\sqrt{p_t^2}}{M_W}s_{\theta_1/2}$	−	+	0	$-\frac{\sqrt{\bar{p}_t^2}}{M_W}c_{\theta_2/2}e^{-i\phi_2}$

Table 1. The non-vanishing amplitudes when $m_b = 0$ taking $f_L = 1$, and $f_R = 0$.

where $\theta_{1(2)}$ and $\phi_{1(2)}$ denote the direction of $W^{+(-)}$ in the $t(\bar{t})$ rest frame. And then, the sum of the amplitudes squared can be organized as

$$\begin{aligned}
 \sum_{\sigma_b, \lambda_1, \bar{\sigma}_1, \lambda_2} |\langle \sigma_b \lambda_1 : \bar{\sigma}_b \lambda_2 \rangle|^2 = & C_1 \left[\left(1 + \frac{p_t^2 \bar{p}_t^2}{4M_W^4} \right) (1 - c_{\theta_1} c_{\theta_2}) + \frac{p_t^2 + \bar{p}_t^2}{2M_W^2} (1 + c_{\theta_1} c_{\theta_2}) \right] \\
 & + C_2 \left[\left(-1 + \frac{p_t^2 \bar{p}_t^2}{4M_W^4} \right) (c_{\theta_1} - c_{\theta_2}) + \frac{p_t^2 - \bar{p}_t^2}{2M_W^2} (c_{\theta_1} + c_{\theta_2}) \right] \\
 & + C_3 \left(\frac{p_t^2}{2M_W^2} - 1 \right) \left(\frac{\bar{p}_t^2}{2M_W^2} - 1 \right) (-s_{\theta_1} s_{\theta_2} c_\Phi) \\
 & + C_4 \left(\frac{p_t^2}{2M_W^2} - 1 \right) \left(\frac{\bar{p}_t^2}{2M_W^2} - 1 \right) s_{\theta_1} s_{\theta_2} s_\Phi
 \end{aligned} \tag{2.18}$$

with $\Phi = \phi_1 + \phi_2$ denoting the angle between the two decay planes and the 4 angular coefficients are given by

$$\begin{aligned}
 C_1 &\equiv |\langle + \rangle|^2 + |\langle - \rangle|^2 = 2 \left[|X|^2 (g^S)^2 + |Y|^2 (g^P)^2 \right], \\
 C_2 &\equiv |\langle + \rangle|^2 - |\langle - \rangle|^2 = 4 \Im(X^* Y) g^S g^P, \\
 C_3 &\equiv 2 \Re[\langle + \rangle \langle - \rangle^*] = 2 \left[-|X|^2 (g^S)^2 + |Y|^2 (g^P)^2 \right], \\
 C_4 &\equiv 2 \Im[\langle + \rangle \langle - \rangle^*] = 4 \Re(X^* Y) g^S g^P.
 \end{aligned} \tag{2.19}$$

Under CP and CPT^4 transformations, the reduced H - $t\bar{t}$ helicity amplitudes transform as follows:

$$\langle \pm \rangle \xrightarrow{\text{CP}} \langle \mp \rangle, \quad \langle \pm \rangle \xrightarrow{\text{CPT}} \langle \mp \rangle^*. \tag{2.20}$$

We note that the CP parities of C_1 , C_2 , C_3 and C_4 are $+$, $-$, $+$, and $-$, respectively, implying that C_2 and C_4 are non-vanishing only when g^S and g^P exist simultaneously. Furthermore, the angular coefficient C_2 is CPT odd and it can be induced only in the presence of non-vanishing absorptive (or imaginary) parts of X and Y .

By integrating eq. (2.15) over $\sqrt{p_t^2}$ and $\sqrt{\bar{p}_t^2}$, we have

$$\frac{d\Gamma}{dc_{\theta_1} dc_{\theta_2} d\Phi} = N_C \frac{g^4 g_t^2 \sqrt{s}}{2^{15} \pi^6} \sum_i \mathcal{F}_i f_i(\theta_1, \theta_2, \Phi) \tag{2.21}$$

⁴ $\tilde{\text{T}}$ denotes the naive time-reversal transformation under which the matrix element gets complex conjugated.

where $\sqrt{s} = M_H$ and $i = 11, 12, 21, 22, 3, 4$. The angular functions are

$$\begin{aligned} f_{11}(\theta_1, \theta_2, \Phi) &= 1 - c_{\theta_1} c_{\theta_2}, & f_{12}(\theta_1, \theta_2, \Phi) &= 1 + c_{\theta_1} c_{\theta_2}, \\ f_{21}(\theta_1, \theta_2, \Phi) &= c_{\theta_1} - c_{\theta_2}, & f_{22}(\theta_1, \theta_2, \Phi) &= c_{\theta_1} + c_{\theta_2}, \\ f_3(\theta_1, \theta_2, \Phi) &= -s_{\theta_1} s_{\theta_2} c_{\Phi}, \\ f_4(\theta_1, \theta_2, \Phi) &= s_{\theta_1} s_{\theta_2} s_{\Phi}, \end{aligned} \quad (2.22)$$

and the numerical factors $\mathcal{F}_i$ are given by

$$\begin{aligned} \mathcal{F}_i &= \int \lambda_H^{1/2} \lambda_t^{1/2} \lambda_{\bar{t}}^{1/2} \sqrt{p_t^2} \sqrt{\bar{p}_t^2} \tilde{C}_i(p_t^2, \bar{p}_t^2) \\ &\quad \times \frac{p_t^2 + m_b^2 - M_W^2}{(p_t^2 - m_t^2)^2 + m_t^2 \Gamma_t^2} \frac{\bar{p}_t^2 + m_b^2 - M_W^2}{(\bar{p}_t^2 - m_t^2)^2 + m_t^2 \Gamma_t^2} d\sqrt{p_t^2} d\sqrt{\bar{p}_t^2} \end{aligned} \quad (2.23)$$

in which the tilded 6 angular coefficients $\tilde{C}_i$ are related to C_i as follows:

$$\begin{aligned} \tilde{C}_{11} &= C_1 \left(1 + \frac{p_t^2 \bar{p}_t^2}{4M_W^4} \right), & \tilde{C}_{12} &= C_1 \frac{p_t^2 + \bar{p}_t^2}{2M_W^2}, \\ \tilde{C}_{21} &= C_2 \left(-1 + \frac{p_t^2 \bar{p}_t^2}{4M_W^4} \right), & \tilde{C}_{22} &= C_2 \frac{p_t^2 - \bar{p}_t^2}{2M_W^2}, \\ \tilde{C}_3 &= C_3 \left(\frac{p_t^2}{2M_W^2} - 1 \right) \left(\frac{\bar{p}_t^2}{2M_W^2} - 1 \right), \\ \tilde{C}_4 &= C_4 \left(\frac{p_t^2}{2M_W^2} - 1 \right) \left(\frac{\bar{p}_t^2}{2M_W^2} - 1 \right). \end{aligned} \quad (2.24)$$

To proceed further, we have introduced weight factor w_i 's which are defined by

$$w_i \equiv \frac{\mathcal{F}_i}{\mathcal{F} \tilde{C}_i} \quad (2.25)$$

where

$$\mathcal{F} = \int \lambda_H^{1/2} \lambda_t^{1/2} \lambda_{\bar{t}}^{1/2} \sqrt{p_t^2} \sqrt{\bar{p}_t^2} \frac{p_t^2 + m_b^2 - M_W^2}{(p_t^2 - m_t^2)^2 + m_t^2 \Gamma_t^2} \frac{\bar{p}_t^2 + m_b^2 - M_W^2}{(\bar{p}_t^2 - m_t^2)^2 + m_t^2 \Gamma_t^2} d\sqrt{p_t^2} d\sqrt{\bar{p}_t^2} \quad (2.26)$$

and

$$\tilde{\tilde{C}}_i = \tilde{C}_i(p_t^2 = m_t^2, \bar{p}_t^2 = m_t^2) \quad (2.27)$$

are the constant tilded angular coefficients at the t pole. Explicitly,

$$\begin{aligned} \tilde{\tilde{C}}_{11} &= 2 \left(1 + \frac{m_t^4}{4M_W^4} \right) \left[\beta_t^2 (g_S)^2 + (g_P)^2 \right], & \tilde{\tilde{C}}_{12} &= 2 \left(\frac{m_t^2}{M_W^2} \right) \left[\beta_t^2 (g_S)^2 + (g_P)^2 \right], \\ \tilde{\tilde{C}}_{21,22} &= 0, \\ \tilde{\tilde{C}}_3 &= 2 \left(\frac{m_t^2}{2M_W^2} - 1 \right)^2 \left[-\beta_t^2 (g_S)^2 + (g_P)^2 \right], & \tilde{\tilde{C}}_4 &= 4 \left(\frac{m_t^2}{2M_W^2} - 1 \right)^2 \beta_t g^S g^P. \end{aligned} \quad (2.28)$$

We observe $\widetilde{C}_{21,22}$ are identically vanishing because $X = \beta_t$ and $Y = 1$ are real when $p_t^2 = \bar{p}_t^2 = m_t^2$. We also note that

$$\widetilde{C}_{11} + \widetilde{C}_{12} = 2 \left(1 + \frac{m_t^2}{2M_W^2} \right)^2 \left[\beta_t^2 (g_S)^2 + (g_P)^2 \right]. \quad (2.29)$$

Finally, we have obtained the normalized differential angular distribution

$$\frac{1}{\Gamma} \frac{d\Gamma}{dc_{\theta_1} dc_{\theta_2} d\Phi} = \sum_i \widetilde{R}_i \left(\frac{f_i(\theta_1, \theta_2, \Phi)}{8\pi} \right) \quad (2.30)$$

with the 6 angular observables defined by

$$\widetilde{R}_i \equiv \omega_i \widetilde{C}_i / (\omega_{11} \widetilde{C}_{11} + \omega_{12} \widetilde{C}_{12}). \quad (2.31)$$

After integrating over any two of the angles θ_1 , θ_2 , and Φ , one can obtain the following one-dimensional angular distributions in terms of the constant t -pole angular observables $\widetilde{R}_i$'s:

$$\begin{aligned} \frac{1}{\Gamma} \frac{d\Gamma}{dc_{\theta_1}} &= \frac{1}{2} + \frac{1}{2} (\widetilde{R}_{21} + \widetilde{R}_{22}) c_{\theta_1}, & \frac{1}{\Gamma} \frac{d\Gamma}{dc_{\theta_2}} &= \frac{1}{2} + \frac{1}{2} (-\widetilde{R}_{21} + \widetilde{R}_{22}) c_{\theta_2}, \\ \frac{1}{\Gamma} \frac{d\Gamma}{d\Phi} &= \frac{1}{2\pi} + \frac{\pi}{32} (-\widetilde{R}_3 c_\Phi + \widetilde{R}_4 s_\Phi) \end{aligned} \quad (2.32)$$

where the decay width is given by⁵

$$\Gamma = N_C \frac{g^4 g_t^2 \sqrt{s}}{2^{12} \pi^5} (\omega_{11} \widetilde{C}_{11} + \omega_{12} \widetilde{C}_{12}) \mathcal{F}. \quad (2.33)$$

When $M_H > 2m_t$ and the top quarks are on-shell, we have found that the weight factors do not deviate from unity by more than 1% and one may safely take $\widetilde{C}_i = \widetilde{C}_i$ in eq. (2.23) by adopting the narrow-width approximation (NWA) for the intermediate top quarks. Note that the only non-trivial one-dimensional angular distribution is $1/\Gamma d\Gamma/d\Phi$ since $\widetilde{R}_{21,22} = 0$. Again, we note that the analytic expressions for the normalized angular distributions, eq. (2.32), have been obtained in the $m_b = 0$ limit taking $f_L = 1$ and $f_R = 0$. We find that finite m_b effects are negligible as we shall show in figure 1 by comparing the distributions obtained by using the exact expression eq. (2.15) with finite m_b and the expression eq. (2.32) with $m_b = 0$. We then use eq. (2.32) to analyze angular distributions in our numerical analysis.

3 Numerical analysis

For our numerical analysis we are taking $M_H = 380 \text{ GeV}$. This choice of M_H ensures that the top and anti-top quarks are both on-shell and thus the decay channel $H \rightarrow t\bar{t}$ would be the dominant decay mode of the heavy Higgs boson H . Also, the production cross section of H would be substantially larger than the heavier Higgs boson with, for example,

⁵In appendix C, we show $\Gamma = \Gamma(H \rightarrow t\bar{t}) [B(t \rightarrow bW)]^2$ in the leading order (LO) by adopting the narrow width approximation for the intermediate top quarks.

	g^S	g^P	$(\widetilde{C}_{11} + \widetilde{C}_{12})[++]$	$\widetilde{R}_{11}[++]$	$\widetilde{R}_{12}[++]$	$\widetilde{R}_{21}[--]$	$\widetilde{R}_{22}[--]$	$\widetilde{R}_3[++]$	$\widetilde{R}_4[++]$
S1	1	0	3.76	0.580	0.420	0	0	-0.159	0
S2	0	1	22.1	0.580	0.420	0	0	0.159	0
S3	1	1	25.9	0.580	0.420	0	0	0.113	0.112
S4	1	-1	25.9	0.580	0.420	0	0	0.113	-0.112
S5	1	0.42	7.67	0.580	0.420	0	0	0.00295	0.159
S6	1	-0.42	7.67	0.580	0.420	0	0	0.00295	-0.159

Table 2. The values of $\widetilde{C}_{11} + \widetilde{C}_{12}$ and the 6 angular observables $\widetilde{R}_i = \widetilde{C}_i/(\widetilde{C}_1 + \widetilde{C}_3)$ with $i = 11, 12, 21, 22, 3, 4$ taking $w_i = 1$ for the 6 scenarios under consideration. The CP and CPT parities of each observable are shown in the square brackets.

$M_H \gtrsim 500$ GeV. For the t - b - W interaction, we assume the SM couplings: $f_L = 1$ and $f_R = 0$. These input values simplify our numerical analysis and there remain only 2 real input parameters of g^S and g^P to vary.⁶ In this case, we find that the total decay width is given by

$$\Gamma \simeq 22 g_t^2 \left[\beta_t^2 (g^S)^2 + (g^P)^2 \right] \text{ GeV} \quad (3.1)$$

with $\mathcal{F} \simeq 6100$. Note $\beta_t = 0.412$ for our choice of M_H close to the $2m_t$ threshold and $m_t = 173.1$ GeV and therefore the g^S contribution in the above equation is suppressed by the β_t^2 factor.

For the H couplings to top quarks, we consider the following 6 representative scenarios:

- **S1:** $(g^S, g^P) = (1, 0)$
- **S2:** $(g^S, g^P) = (0, 1)$
- **S3:** $(g^S, g^P) = (1, 1)$
- **S4:** $(g^S, g^P) = (1, -1)$
- **S5:** $(g^S, g^P) = (1, 0.42)$
- **S6:** $(g^S, g^P) = (1, -0.42)$

In the first two scenarios of **S1** and **S2**, only one of the couplings is non-vanishing and H is supposed to be a pure CP-even (odd) state in the **S1** (**S2**) scenario. In the other scenarios, CP is violated and the couplings g^S and g^P take on a relative phase. In the scenarios **S5** and **S6**, in particular, the relative sizes of the couplings are chosen such that $|g^P/g^S| \sim \beta_t$ in order that the two couplings contribute more or less equally to the amplitude squared.

In table 2, we show the values of $\widetilde{C}_{11} + \widetilde{C}_{12}$ and the 6 angular observables $\widetilde{R}_i = \widetilde{C}_i/(\widetilde{C}_1 + \widetilde{C}_3)$ for the 6 scenarios under consideration with $i = 11, 12, 21, 22, 3, 4$. We have taken $w_i = 1$. First of all, we observe that $\widetilde{R}_{11} = (1 + m_t^4/4M_W^4)/(1 + m_t^2/2M_W^2)^2$ and

⁶In passing, we note that the expressions given in the last section are very general and indeed can allow a more general analysis.

Figure 1. The normalized angular distributions (solid dots) generated according to eq. (2.15) integrating over $m_t - 5\Gamma_t \leq \sqrt{p_t^2} \leq m_t + 5\Gamma_t$ and $m_t - 5\Gamma_t \leq \sqrt{p_t^2} \leq m_t + 5\Gamma_t$. We have taken $f_L = 1$ and $f_R = 0$. In each frame, we are taking $(g^S, g^P) = (1, 0)$ (**S1**: upper-left), $(g^S, g^P) = (0, 1)$ (**S2**: upper-right), $(g^S, g^P) = (1, 1)$ (**S3**: middle-left), $(g^S, g^P) = (1, -1)$ (**S4**: middle-right), $(g^S, g^P) = (1, 0.42)$ (**S5**: lower-left), and $(g^S, g^P) = (1, -0.42)$ (**S6**: lower-right). The dashed lines are drawn using the expressions in eq. (2.32), which are obtained in the $m_b = 0$ limit, taking $w_i = 1$.

$\widetilde{\widetilde{R}}_{12} = (m_t^2/M_W^2)/(1 + m_t^2/2M_W^2)^2$ independent of the scenario and the CP $\widetilde{T}$ -odd $\widetilde{\widetilde{R}}_{21}$ and $\widetilde{\widetilde{R}}_{22}$ are identically vanishing in all the scenarios. This leaves only $\widetilde{\widetilde{R}}_3$ and $\widetilde{\widetilde{R}}_4$ as non-trivial angular observables which can be probed by studying the $d\Gamma/d\Phi$ distribution. The CP-odd $\widetilde{\widetilde{R}}_4$ observable is vanishing in the CP-conserving **S1** and **S2** scenarios and, if it is not vanishing, its sign is determined by the sign of the product of g^S and g^P . Further we note that $\widetilde{\widetilde{R}}_3$ is very suppressed in **S5** and **S6** because it is proportional to $-\beta_t^2 (g^S)^2 + (g^P)^2$.

In figure 1, we show the normalized angular distributions (solid dots) generated according to eq. (2.15) and compare them with those (dashed lines) obtained using the analytic

expressions eq. (2.32) obtained in the $m_b = 0$ limit taking $w_i = 1$. Without any noticeable differences between these two sets of distributions in all the scenarios,⁷ we conclude that the finite m_b effects are negligible and the NWA for the angular coefficients and observables works excellently. Thus, we conclude that the analytic expressions in eq. (2.32) provide a sufficient theoretical framework to analyze the angular distributions and to extract the g^S and g^P couplings when $M_H > 2m_t$. Incidentally, the behavior of the angular distribution in Φ can be easily understood as it varies according to $-\bar{R}_{3c\Phi} + \bar{R}_{3s\Phi}$: see eq. (2.32) and table 2.

Now we are going to illustrate how well one can measure the properties of the 380 GeV Higgs by taking the examples of scenarios **S4** and **S6**. For this purpose, we assume that one top quark decays hadronically and the other one leptonically with $\ell = e, \mu$. Then, the expected number of events would be

$$N_{\text{evt}} \sim (7 \times 10^3) \left[\frac{\sigma(H) \times B(H \rightarrow t\bar{t})}{10 \text{ pb}} \right] \left[\frac{B(t \rightarrow bW)}{1} \right]^2 \left(\frac{\epsilon_b}{0.7} \right)^2 \quad (3.2)$$

$$\times \left[\frac{B(W^+ \rightarrow \text{hadrons})}{0.6741} \right] \left[\frac{B(W^- \rightarrow \ell^- \bar{\nu})}{0.1071 + 0.1063} \right] \left(\frac{\epsilon_{\text{rec}}}{0.1} \right) \left(\frac{\mathcal{L}}{100 \text{ fb}^{-1}} \right) + (W^+ \leftrightarrow W^-)$$

where $\sigma(H)$ denotes the production cross section of the heavy Higgs boson H . When $M_H \lesssim 500 \text{ GeV}$, the experimental constraint on $\sigma(H)$ may come from the search for narrow scalar resonances in the b -tagged dijet mass spectrum: $\sigma(pp \rightarrow H) \cdot B(H \rightarrow b\bar{b}) \lesssim 10 \text{ pb}$ at 95% confidence level (CL) [24]. Taking $B(H \rightarrow b\bar{b}) \sim 0.1$, we have $\sigma(H) \lesssim 100 \text{ pb}$ at the LHC. Another 95% CL limit may be derived from $\sigma(gg \rightarrow H) \cdot B(H \rightarrow ZZ) \lesssim 0.1 \text{ pb}$ [25], which leads to $\sigma(H) \lesssim 10 \text{ pb}$ with $B(H \rightarrow ZZ) \sim 0.01$. In eq. (3.2), ϵ_b denotes the b -tagging efficiency and ϵ_{rec} stands for a collective efficiency of reconstructing the $H \rightarrow t\bar{t}$ system. Note that ϵ_{rec} includes the efficiency of fully reconstructing the four momenta of the t and $\bar{t}$ quarks which are necessary to measure the Φ distribution.⁸ We observe that ϵ_{rec} may also account for the dilutions due to interference with irreducible backgrounds and incorrect reconstruction of the neutrino momentum. One may achieve $\epsilon_{\text{rec}} \sim 1$ at the future e^+e^- colliders but the production cross sections would be much suppressed compared to pp colliders. In our analysis, we are taking $N_{\text{evt}} = 10^4$ as reference.

In figure 2, we show the normalized angular distributions for the **S4** (left) and **S6** (right) scenarios with $N_{\text{evt}} = 10^4$.⁹ The histograms represent the pseudo-data of a total of $N_{\text{event}} = 10^4$ generated according to eq. (2.15). The (red) solid lines present the results of fitting to the angular distributions using eq. (2.32). With $N_{\text{evt}} = 10^4$ and $\Delta\Phi = \pi/9$, we find that the absolute size of $1\text{-}\sigma$ errors of the output angular observables of $\bar{R}_{3,4}$ are about 0.02, see table 3. The input values are the same as in table 2. The output values together

⁷Numerically, we find that the absolute difference is smaller than 5×10^{-3} for the 6 scenarios under consideration.

⁸Using the pseudo-top algorithm, for example, the missing neutrino momentum can be reconstructed with a two-fold ambiguity at the LHC [26]. See also, for example, ref. [27] for more sophisticated algorithms for top-quark pairs.

⁹In practice, we have generated 10 pseudo datasets with each set having 10^4 events and take average of them to obtain the histograms.

Figure 2. **S4** (left), **S6** (right): the normalized angular distributions from the pseudo dataset generated with $\sqrt{p_t^2}, \sqrt{\bar{p}_t^2} = m_t \pm 5\Gamma_t$ and $\Delta\Phi = \pi/9$. The results of fitting to the angular distributions with eq. (2.32) are shown in the (red) solid lines.

S4	Input	Output	Best-fit	S6	Input	Output	Best-fit
$\chi^2_{\min} = 0.728$	value	value	value	$\chi^2_{\min} = 2.42$	value	value	value
$\bar{\bar{C}}_{11} + \bar{\bar{C}}_{12}$	25.9	25.9 ± 5.18	25.8	$\bar{\bar{C}}_{11} + \bar{\bar{C}}_{12}$	7.67	7.67 ± 1.53	7.73
$\bar{\bar{R}}_3$	+0.113	$+0.0985 \pm 0.0228$	+0.0877	$\bar{\bar{R}}_3$	+0.00295	-0.0102 ± 0.0228	-0.00842
$\bar{\bar{R}}_4$	-0.112	-0.149 ± 0.0229	-0.133	$\bar{\bar{R}}_4$	-0.159	-0.195 ± 0.0229	-0.159
g^S	+1	N/A	$+1.25 \pm 0.22$	g^S	+1	N/A	$+1.04 \pm 0.13$
g^P	-1	N/A	$-0.950^{+0.11}_{-0.10}$	g^P	-0.42	N/A	-0.405 ± 0.050

Table 3. Summary of the results obtained with $N_{\text{evt}} = 10^4$ and $\Delta\Phi = \pi/9$. The input values are the same as in table 2. The output values of $\bar{\bar{R}}_{3,4}$ have been obtained by fitting to the normalized angular distributions in figure 2. For $\bar{\bar{C}}_{11} + \bar{\bar{C}}_{12}$, we simply assume 20 % error. Implementing χ^2 analysis gives the best-fit values of g^S and g^P , see figures 3 and 4. Also shown are the best-fit values for $\bar{\bar{C}}_{11} + \bar{\bar{C}}_{12}$ and $\bar{\bar{R}}_{3,4}$ calculated using the best-fit values of g^S and g^P .

with parabolic errors have been obtained by fitting to the normalized angular distributions $1/\Gamma d\Gamma/d\Phi$ in figure 2.

Now we are ready to carry out a χ^2 analysis to achieve our ultimate goal of extracting the couplings g^S and g^P from the angular observables $\bar{\bar{R}}_3$ and $\bar{\bar{R}}_4$. To implement the analysis, we further need $\bar{\bar{C}}_{11} + \bar{\bar{C}}_{12}$. Using eqs. (2.29) and (3.1), we have

$$\bar{\bar{C}}_{11} + \bar{\bar{C}}_{12} \simeq \frac{\left(1 + \frac{m_t^2}{2M_W^2}\right)^2}{11g_t^2} B(H \rightarrow t\bar{t}) \frac{\Gamma_{\text{tot}}^H}{\text{GeV}}. \quad (3.3)$$

Assuming information on $B(H \rightarrow t\bar{t})$ and the coupling g_t can be eventually extracted from $\sigma \cdot B$ measurements by considering several H production and decay processes, together with an independent measurement of the total decay width, one may determine the combination

Figure 3. Upper-left: the confidence-level (CL) regions for scenario **S4**: $(g^S, g^P) = (1, -1)$ with $\Delta\chi^2 = 2.3$ (red), 5.99 (green), and 11.83 (blue) above the minimum, which correspond to confidence levels of 68.3%, 95%, and 99.7%, respectively. The vertical and horizontal lines show the best-fit values of (g^S, g^P) . The others: the scatter plots for $\Delta\chi^2$ versus g^S (upper-right), $\Delta\chi^2$ versus g^P (lower-left). The horizontal lines are for the 68.3% (red), 95% (green), and 99.7% (blue) CL regions.

of $\bar{\bar{C}}_{11} + \bar{\bar{C}}_{12}$. In our analysis, similar to the angular observables $\bar{\bar{R}}_{3,4}$, we simply assume 20 % error in $\bar{\bar{C}}_{11} + \bar{\bar{C}}_{12}$.

In the upper-left frame of figure 3, we show the confidence-level regions of the χ^2 analysis by varying g^S and g^P simultaneously for the scenario **S4**. We have found that $\chi^2_{\min}/d.o.f = 0.728/(3-2) = 0.728$ and the minimum occurs at

$$g^S = 1.25 \pm 0.22; \quad g^P = -0.950^{+0.11}_{-0.10}, \quad (3.4)$$

which are consistent with the input values $(g^S, g^P) = (+1, -1)$ within $\sim 1\text{-}\sigma$ range. Therefore, we conclude that the two couplings of H to the top-quark pair can be determined with about 10-20% errors when $N_{\text{evt}} = 10^4$ for the scenario **S4**. For the scenario **S6**, the

Figure 4. The same as in figure 3 but taking scenario **S6**: $(g^S, g^P) = (1, -0.42)$.

confidence-level regions are shown in figure 4. The minimum occurs at

$$g^S = 1.04 \pm 0.13; \quad g^P = -0.405 \pm 0.050, \quad (3.5)$$

with $\chi^2_{\min}/d.o.f = 2.42$. We again note that the fitted values are consistent with the input values $(g^S, g^P) = (+1, -0.42)$ safely within $1\text{-}\sigma$ range. Also, we conclude that the two couplings can be determined with about 13% errors in scenario **S6**. The results are also summarized in table 3.

4 Conclusions

We have performed a comprehensive study of the renormalizable CP-even and CP-odd couplings of a spin-0 heavy Higgs boson to a pair of top quarks, using the angular distributions in the decay $H \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^-$. Based on the helicity amplitude method, we figure out there are 6 types of angular observables $\bar{R}_i$ ($i = 11, 12, 21, 22, 3, 4$) according to their CP

and CPT parities. We found that $\bar{R}_{21,22}$ are identically zero unless CPT is violated through the presence of the absorptive part in the loop correction of the $Ht\bar{t}$ vertex. Furthermore, we find that among the 6 observables only the $\bar{R}_3$ and $\bar{R}_4$ observables can be probed by the one-dimensional angular distribution $d\Gamma/d\Phi$. This is our novel strategy for analyzing the decay $H \rightarrow t\bar{t} \rightarrow bW^+\bar{W}^-$ to measure the properties of a heavy Higgs boson H .

We have illustrated with 10^4 events for $H \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^-$ that the parameters g^S and g^P can be determined with about 10-20% uncertainties through the one-dimensional distribution $d\Gamma/d\Phi$. This is the major numerical result of this work.

We offer the further comments in our findings:

1. As long as the heavy Higgs boson is above the $t\bar{t}$ threshold and, at least, as long as the angular distributions are concerned, the narrow-width approximation is always a good one: the weight factors deviate from unity less than 1%.
2. The angular coefficient C_2 is CP odd and CPT odd which implies that it is only nonzero in presence of non-vanishing absorptive (or imaginary) parts of the $t\bar{t}H$ vertex *and* in the simultaneous existence of g^S and g^P .
3. The numerical analysis presented here is only limited to the left-handed decay of the top quark, i.e., $f_L = 1, f_R = 0$. Nevertheless, the formalism here is very general and can be applied to general studies.
4. In the current study, while respecting the present LHC upper limits on H production, we have used 10^4 events of $H \rightarrow t\bar{t} \rightarrow 2b2j\ell\nu$ with a luminosity of 100 fb^{-1} . The high-luminosity run of the LHC is supposed to collect 3000 fb^{-1} . The uncertainty in extracting the Yukawa couplings from the heavy Higgs boson decay would go down substantially.
5. When the heavy Higgs boson becomes heavier, of order 1 TeV, the decay products of the top and anti-top quarks become more boosted. In such a case, the angular analysis is more challenging and thus deserves a new study.

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A Relation between the t and $\bar{t}$ helicity amplitudes

The helicity amplitude for the decay $\bar{t}(\bar{p}_t, \bar{\sigma}_t) \rightarrow \bar{b}(\bar{p}_b, \bar{\sigma}_b)W^-(k_2, \epsilon_2)$ can be obtained from that for the decay $t(p_t, \sigma_t) \rightarrow b(p_b, \sigma_b)W^+(k_1, \epsilon_1)$ by replacing f_A with f_A^* together with $\sigma_{t,b} \rightarrow \bar{\sigma}_{t,b}$, $\lambda_1 \rightarrow \lambda_2$, etc. This can be easily understood through the relation

$$\bar{v}_t \gamma^\mu P_A v_b = \bar{u}_b \gamma^\mu P_{-A} u_t. \quad (\text{A.1})$$

The above relation can be shown by calculating both sides explicitly or by observing

$$C = i\gamma^2\gamma^0; \quad C = -C^{-1} = -C^T = -C^\dagger \quad (\text{A.2})$$

where C denotes charge conjugation and

$$u = C\bar{v}^T, \quad \bar{u} = v^T C = -v^T C^{-1} \quad (\text{or } v = C\bar{u}^T) \quad (\text{A.3})$$

together with

$$C(\gamma^\mu)^T C^{-1} = -\gamma^\mu, \quad C(\gamma^\mu\gamma_5)^T C^{-1} = +\gamma^\mu\gamma_5. \quad (\text{A.4})$$

Incidentally, $C\gamma_5^T C^{-1} = +\gamma_5$.

B The four-body phase space

For the $H \rightarrow t\bar{t} \rightarrow bW^+ \bar{b}W^-$ decay, the phase space can be factorized into

$$\begin{aligned} d\Phi_4(q \rightarrow p_t \bar{p}_t \rightarrow p_b k_1 \bar{p}_b k_2) &= \frac{dp_t^2}{2\pi} \frac{d\bar{p}_t^2}{2\pi} \frac{\lambda_H^{1/2}(1, p_t^2/s, \bar{p}_t^2/s)}{32\pi^2} d\cos\Theta^* d\Phi^* \\ &\times \frac{\lambda_t^{1/2}(1, m_b^2/p_t^2, M_W^2/p_t^2)}{32\pi^2} d\cos\theta_1 d\phi_1 \\ &\times \frac{\lambda_t^{1/2}(1, m_b^2/\bar{p}_t^2, M_W^2/\bar{p}_t^2)}{32\pi^2} d\cos\theta_2 d\phi_2 \end{aligned} \quad (\text{B.1})$$

where $s = q^2$ and $\lambda(1, a, b) = (1 - a - b)^2 - 4ab$. For our purpose, we may be able to take

$$d\Phi_4(q \rightarrow p_t \bar{p}_t \rightarrow p_b k_1 \bar{p}_b k_2) = \lambda_H^{1/2} \lambda_t^{1/2} \lambda_t^{1/2} \frac{dp_t^2}{2\pi} \frac{d\bar{p}_t^2}{2\pi} \frac{1}{8\pi} \frac{d\cos\theta_1 d\Phi}{32\pi^2} \frac{d\cos\theta_2}{16\pi}. \quad (\text{B.2})$$

C Narrow width approximation

Using

$$\delta(p^2 - m^2) = \lim_{\Gamma \rightarrow 0} \frac{m\Gamma}{\pi} \frac{1}{(p^2 - m^2)^2 + m^2\Gamma^2} \quad (\text{C.1})$$

and taking $\omega_i = 1$, we note that eq. (2.33) together with eq. (2.26) can be factorized into

$$\Gamma = \Gamma(H \rightarrow t\bar{t}) \left(\frac{\Gamma^{\text{LO}}(t \rightarrow bW)}{\Gamma_t} \right)^2 \quad (\text{C.2})$$

where, taking $m_b = 0$,

$$\begin{aligned} \Gamma(H \rightarrow t\bar{t}) &= N_C \frac{\beta_t g_t^2 M_H}{8\pi} \left[\beta_t^2 (g_S)^2 + (g_P)^2 \right], \\ \Gamma^{\text{LO}}(t \rightarrow bW) &= \frac{g^2 m_t}{2^6 \pi} \left(1 - \frac{M_W^2}{m_t^2} \right)^2 \left(2 + \frac{m_t^2}{M_W^2} \right) \\ &= \frac{G_F m_t^3}{8\pi\sqrt{2}} \left(1 - \frac{M_W^2}{m_t^2} \right)^2 \left(1 + 2 \frac{M_W^2}{m_t^2} \right) \end{aligned} \quad (\text{C.3})$$

with $G_F = \sqrt{2}g^2/8M_W^2$. In our analysis, we are taking $\Gamma^{\text{NLO}}(t \rightarrow bW)$ for Γ_t or $\Gamma_t = \Gamma^{\text{LO}}(t \rightarrow bW) \left[1 - \frac{2\alpha_s}{3\pi} \left(\frac{2\pi^2}{3} - \frac{5}{2} \right) \right]$ which is about 1.35 GeV.

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