Published for SISSA by Springer

Received: December 30, 2017
Revised: April 7, 2018
Accepted: May 4, 2018
Published: May 10, 2018

JHEP05(2018)072

Localization of U(1) gauge vector field on flat branes with five-dimension (asymptotic) AdS_5 spacetime

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Abstract: In order to localize U(1) gauge vector field on Randall-Sundrum-like braneworld model with infinite extra dimension, we propose a new kind of non-minimal coupling between the U(1) gauge field and the gravity. We propose three kinds of coupling methods and they all support the localization of zero mode. In addition, one of them can support the localization of massive modes. Moreover, the massive tachyonic modes can be excluded. And our method can be used not only in the thin braneword models but also in the thick ones.

Keywords: Field Theories in Higher Dimensions, Large Extra Dimensions

ArXiv ePrint: 1712.09843

https://doi.org/10.1007/JHEP05(2018)072

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1 Introduction

In the braneworld theory our universe can be taken as a brane embedding in the higher dimensional spacetime. So the localization of matter fields is an very important issue. Needless appending extra terms in the standard action the gravity can be localized on branes [1–3] as well as scalar fields [4]. The localization of fermions can be realized by introducing the usual Yukawa coupling [5] (or by another form of coupling [6]) with background scalar field in thick brane models [7–13]. The localization of U(1) gauge field is more complex than above two fields, especially, in Randall-Sundrum (RSII) like model [2]. Pomarol showed in ref. [14] that U(1) gauge field can not be localized in the RSII model with the following action

$$S = -\frac{1}{4}\int \mathrm{d}^5x\sqrt{-g}F_{MN}F^{MN}, \tag{1.1}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ is the field strength. But in some non-RSII models with non-flat brane U(1) fields can be localized, for examples, the thick de Sitter (dS) brane models [15–17], the thick Weyl brane model [18], and the thin wave brane models [19, 20].

The aim of this paper is to argue a new method to localize U(1) gauge field in 5D RSII-like braneworld models. For a RSII-like braneworld scenario which should hold the Z_2 symmetric along the infinite extr-dimension, 5D spacetime is an (asymptotic) anti-de Sitter (AdS_5) one, and the brane is flat. There are three common methods to realize the localization of U(1) gauge field.

The first one is to multiply $F_{MN}F^{MN}$ by a dilaton factor. For examples, in the work of ref. [21], dilaton factor is a function of scalar dilaton field and in the work of refs. [22, 23] the dilaton factor is a function of the background scalar field. It should be noted that the above two works only hold in thick brane models, because the scalar fields they needed only appears in thick brane models.

The second one is to append a mass term to above standard action (1.1). For examples, in the work of ref. [24] the authors appended a bulk constant mass term and a boundary mass term into the action, and in the work of ref. [25] the authors took the non-minimally coupling of vector and gravity into account, where the 5D scalar curvature is taken as the mass of 5D vector. What should be noted is the scalar curvature to be taken as the mass of 4D vector is first proposed in ref. [26] to be used in studying the inflation of our 4D universe. But adding a mass term into the 5D vector fields action will break its gauge invariance, so in ref. [27] the authors proposed a 5D Stueckelberg-like action to correct this problem.

The third one is to add a topology term to the 5D vector action. In ref. [28] Oda proposed a method by adding a 3-form potential and a topological term to the 5D vector field action to realized the localization of zero mode.

In this work we will follow the idea of the first approach [21, 22]. But in our method the dilaton factor is not a function of scalar fields but a function of scalar curvature. So our method can be used not only to thick brane models but also to the thin branes models.

This paper is constructed as follows. We review our method in section 2. The localization of zero and massive modes are discussed in section 3 and section 4, respectively. Applying our method to a concrete braneworld model is shown in section 5. Finally, we give our conclusions in section 6.

2 The method

The branes discussed here are flat and the line element of the five-dimensional space-time is assumed to be

$$\mathrm{d}s^2 = g_{MN}\mathrm{d}x^M dx^N = e^{2\alpha(y)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}y^2, \tag{2.1}$$

where $e^{2\alpha(y)}$ is the warp factor, $\alpha(y)$ is only the function of extra-dimensional coordinate y, and $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ is the metric on the branes. The braneworld which holds an (asymptotic) AdS_5 spacetime at infinity means that the scalar curvature R satisfies

$$\lim_{y\to\pm\infty} R(y) = -\mathrm{C}_R, \tag{2.2}$$

where $\mathrm{C}_R > 0$ is a constant. By using of the metric (2.1) one can get the expression of R:

$$R(y) = -20\alpha'(y)^2 - 8\alpha''(y). \tag{2.3}$$

To be consistent with the condition (2.2), $\alpha(y)$ should have the following asymptotic solution

$$\alpha(y \to \pm\infty) \to \mp\, k\, y + \mathrm{C}. \tag{2.4}$$

where parameter $k > 0$ and C is a constant, without loss of generality we set $\mathrm{C} = 0$ here and after. Substituting (2.4) into (2.3) one can get

$$\mathrm{C}_R = 20k^2. \tag{2.5}$$

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The action of the five-dimensional U(1) gauge field we proposed is

$$S = -\frac{1}{4}\int \mathrm{d}^5x\sqrt{-g}F(R)F_{MN}F^{MN}, \tag{2.6}$$

where $F(R)$ is a function of scalar curvature R, against the action in ref. [25], this action is gauge invariant.

How to confirm the form of $F(R)$? The rules we follow are:

1. when $R \to 0$ the action should return to the standard form (1.1);
2. $F(R)$ should satisfy the positivity condition

$$F(R) > 0 \tag{2.7}$$

and the finity condition at the same time

$$\int_{-\infty}^{+\infty} F(R)\,\mathrm{d}y < \infty \tag{2.8}$$

to preserve the canonical form of 4D action.

Models discussed here hold $\mathbb{Z}_2$ symmetrical along the extra-dimension, so $\alpha(y)$ is an even function. If $R(y)$ is monotonously decreasing as y varying form 0 to $\pm\infty$, $F(R)$ can has a simple form

$$F(R) = \chi(R) = 1 + \frac{R}{20k^2} \tag{2.9}$$

to satisfy the positivity condition (2.7). If $R(y)$ varies non-monotonic along the extra-dimension the form of $F(R)$ in (2.9) can not guarantee the positivity condition (2.7). But a form of

$$F(R) = \frac{1}{N}\sum_{n=1}^{N}(\chi(R)^2)^n, \tag{2.10}$$

where $N > 0$ and n is an integer, can hold the positivity condition (2.7), obviously.

It can be proven that the functions of $F(R)$ in equations (2.9) and (2.10) can only help us to obtain the localized zero mode. In order to obtain localized massive modes, we propose the following form:

$$F(R) = e^{\mathrm{C}_2\left(1-\left(\chi(R)^2\right)^{-\mathrm{C}_3/2}\right)}, \tag{2.11}$$

where C_2 and C_3 are under-determined positive parameters. We will show C_3 is related to the asymptotic behavior of the potential at infinity in eq. (4.5).

3 Localization of the zero mode

By means of the decomposition $A_\mu = \sum_n a_\mu(x)\rho_n(y)$ and the gauge condition $\partial_\mu A^\mu = 0$ and $A_4 = 0$, the above action (2.6) is reduced to

$$S = -\frac{1}{4}\int \mathrm{d}yF(R)\rho_n(y)^2 \int \mathrm{d}^4x(f_{\mu\nu}f^{\mu\nu} - 2m_n^2 a_\mu a^\mu), \tag{3.1}$$

where $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ is the four-dimensional gauge field strength tensor, and $\rho_n(y)$ satisfies the equation

$$\rho_n'' + \left(\frac{F'}{F} + 2\alpha'\right)\rho_n' = -m_n^2 \rho_n e^{-2\alpha}, \tag{3.2}$$

with the boundary conditions either the Neumann $\rho_n'(\pm\infty) = 0$ or the Dirichlet $\rho_n(\pm\infty) = 0$ [29], where the prime " $'$ " stands for the derivative with respect to y in this section. The localization of gauge field requires

$$I \equiv \int_{-\infty}^{+\infty} \mathrm{d}y F(R)\rho_n^2(y) = 1. \tag{3.3}$$

For the zero mode, $m_0 = 0$, eq. (3.2) reads

$$\rho_0'' + \left(\frac{F'}{F} + 2\alpha'\right)\rho_0' = 0. \tag{3.4}$$

By setting $\gamma' = 2\alpha' + F'/F$, the above equation (3.4) reads

$$\rho_0'' + \gamma'\rho_0' = 0. \tag{3.5}$$

The general solution of zero mode is

$$\rho_0 = \mathrm{c}_0 + \mathrm{c}_1 \int e^{-\gamma}\mathrm{d}y. \tag{3.6}$$

Here c_0 and c_1 are arbitrary constants. Models discussed here hold the $\mathbb{Z}_2$ symmetry along the extra-dimension, $F(R)$, $\alpha(y)$ and γ are even functions of y, so the second term in eq. (3.6) is odd. The Dirichlet boundary conditions $\rho_n(\pm\infty) = 0$ will lead to $\mathrm{c}_0 = 0$ and $\mathrm{c}_1 = 0$. But the Neumann boundary conditions $\rho_n'(\pm\infty) = 0$ only lead to $\mathrm{c}_1 = 0$, so the zero mode solution is

$$\rho_0 = \mathrm{c}_0. \tag{3.7}$$

The localization about zero mode can be realized when $F(R)$ satisfies the finity condition (2.8), namely

$$\int_{-\infty}^{+\infty} \mathrm{d}y F(R)\rho_0^2(y) = \mathrm{c}_0^2 \int_{-\infty}^{+\infty} F(R)\,\mathrm{d}y = 1. \tag{3.8}$$

Because $F(R)$ is continuous, the convergence of the above integration is determined by the asymptotic behavior of $F(R)$ at infinity along the extra dimension. The convergent condition is

$$F(R(y \to \pm\infty)) \propto y^{-p} \tag{3.9}$$

with $p > 1$. By using the $\mathbb{Z}_2$ symmetry, we may only need to discuss the case of $y \to +\infty$. The way of $\alpha(y)$ to its limit solution (2.4) is model dependent. In order to discuss the asymptotic behavior of χ at infinity, we add the first order infinitesimal term into $\alpha(y)$ (2.4)

$$\alpha(y \to +\infty) \sim -k\,y - \mathrm{C}y^{-q}. \tag{3.10}$$

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where $q > 0$. In order to satisfy positive condition C > 0 is need for $F(R)$ in eq. (2.9). Substituting (3.10) and (2.3) into (2.9) one get

$$\chi(R(y \to +\infty)) \sim \frac{2\mathrm{C}q\, y^{-(q+1)}}{k} \tag{3.11}$$

by neglecting higher order infinitesimal terms. It is easy to test that under the above asymptotic solution, the integration of $F(R)$, with the forms of (2.9), (2.10) and (2.11), are convergent. This leads to the conclusion that the zero mode can be localized.

4 Localization of massive modes

To discuss the localization of massive modes, it is more convenient to do the following coordinate transformation

$$\begin{cases} \mathrm{d}z &= e^{-\alpha(y)}\mathrm{d}y \\ z &= \int e^{-\alpha(y)}\mathrm{d}y \end{cases} \tag{4.1}$$

with the boundary condition $z(y = 0) = 0$. The line element (2.1) is recasted to the following conformally flat form namely,

$$\mathrm{d}s^2 = e^{2\alpha(z)}(\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}z^2). \tag{4.2}$$

Further, by using of the gauge choice $A_4 = 0$ and the decomposition $A_\mu = \sum_n a_\mu(x)\rho_n(z)$ the action of the five-dimensional gauge field (2.6) is reduced to

$$S = -\frac{1}{4}\sum_n \int \mathrm{d}z \tilde{\rho}_n^2(z) \int \mathrm{d}^4x (f_{\mu\nu}^{(n)} f^{(n)\mu\nu} - 2m_n^2 a_\mu^{(n)} a^{(n)\mu}), \tag{4.3}$$

where $\tilde{\rho}_n = \rho_n \sqrt{F(R)} e^{\alpha/2}$, and $\tilde{\rho}_n(z)$ satisfies the following Schrödinger-like equation

$$-\tilde{\rho}_n'' + V(z)\tilde{\rho}_n = m_n^2 \tilde{\rho}_n, \tag{4.4}$$

where the effective potential is given by

$$V(z) = \frac{1}{2}\alpha''(z) + \frac{1}{4}\alpha'^2(z) + \frac{\alpha'(z)F'(R)}{2F(R)} + \frac{F''(R)}{2F(R)} - \frac{F'^2(R)}{4F^2(R)}. \tag{4.5}$$

In this section the prime denotes the derivative with respect to the coordinate z. Equation (4.4) can be recast to

$$\left(\frac{\mathrm{d}}{\mathrm{d}z} + \Gamma'(z)\right)\left(-\frac{\mathrm{d}}{\mathrm{d}z} + \Gamma'(z)\right)\tilde{\rho}_n = m_n^2 \tilde{\rho}_n, \tag{4.6}$$

where

$$\Gamma'(z) = \frac{1}{2}\left(\frac{F'(R)}{F(R)} + \alpha'(z)\right), \tag{4.7}$$

and without loss of the generality we can take

$$\Gamma(z) = \frac{1}{2}\left(\ln F(R) + \alpha(z)\right). \tag{4.8}$$

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Equation (4.6) means that there is no tachyonic mode with $m_n^2 < 0$ in the spectrum of the KK modes [30]. It is worth to note that in order to get the effective action (4.3) of the four-dimensional vector field from the five-dimensional one (2.6), we have introduced the orthonormalization condition between different massive modes:

$$\int \mathrm{d}z \tilde{\rho}_m(z)\tilde{\rho}_n(z) = 0. \quad (m \neq n) \tag{4.9}$$

So the localization condition for $\tilde{\rho}_n(z)$ is

$$\int \mathrm{d}z \tilde{\rho}_n^2(z) < \infty. \tag{4.10}$$

The solution of zero mass mode at the z coordinate is easy to be found from (4.6),

$$\tilde{\rho}_0(z) = e^{\alpha(z)/2}\sqrt{F(R)}. \tag{4.11}$$

The existence of massive modes is determined by the potential $V(z)$. Based on the condition (2.4) we have shown the existence of zero mode, this means that at some finite area $V(z) < 0$ along the extra-dimension. So a necessary condition of the existence of localized massive modes is $V(\pm\infty) > 0$. $V(\pm\infty)$ is determined by asymptotic behavior of $\alpha(z)$ and $F(R(z))$. In the following we will discuss the limit of $V(z)$ with $z \to +\infty$ only, based on the $\mathbb{Z}_2$ symmetry $V(-\infty)$ is the same to $V(+\infty)$.

By using the limit solution $\alpha(y)$ (2.4) and the coordinate transformation (4.1), we obtain the limit solution of α with z coordinate at infinity,

$$\alpha(z) \to -\ln(k\,z). \tag{4.12}$$

For different models the ways to this limit solution are different. In order to discuss the asymptotic behavior of $V(z)$ at infinity, we add a first order infinitesimal term to $\alpha(z)$, namely,

$$\alpha(z) \to -\ln(k\,z) + \mathrm{C}_4 z^{-n}, \tag{4.13}$$

where C_4 and n are model dependent constants and $n > 0$. The scalar curvature R at the z coordinate reads

$$R(z) = -4e^{2\alpha(z)}(3\alpha'(z) + 2\alpha''(z)). \tag{4.14}$$

Substituting (4.13) into (4.14) one can get the asymptotic solution of R,

$$\begin{aligned} R(z \to +\infty) \to & -20k^2 e^{-2\mathrm{C}4z^{-n}} \\ & -(8\mathrm{C}_4k^2n^2 + 32\mathrm{C}_4k^2n)e^{-2\mathrm{C}_4z^{-n}}z^{-n} \\ & -12\mathrm{C}_4^2k^2n^2e^{-2\mathrm{C}_4z^{-n}}z^{-2n}. \end{aligned} \tag{4.15}$$

Because

$$\lim_{z\to+\infty} e^{-2\mathrm{C}_4z^{-n}} = 1, \tag{4.16}$$

and by ignoring the z^{-2n} term, we can further reduce $R(z \to +\infty)$ into

$$R(z \to +\infty) \to -20k^2 + \mathrm{C}_5\, z^{-n}, \tag{4.17}$$

where $\mathrm{C}_5 = -(8\mathrm{C}_4k^2n^2 + 32\mathrm{C}_4k^2n)$.

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Substituting eq. (4.17) into eq. (2.9), one can get the asymptotic solutions of $\chi(R)$, namely,

$$\chi(R(z\to+\infty))\to\frac{\mathrm{C}_5\, z^{-n}}{20k^2}. \tag{4.18}$$

For the first case of $F(R)$ in eq. (2.9), substituting eqs. (4.18) and (4.12) into eq. (4.5), we obtain the asymptotic solution of $V(z)$,

$$V_{\mathrm{I}}(z\to+\infty)\to\frac{1}{4}\mathrm{C}_4^2n^2z^{-2n-2}+\mathrm{C}_4n(n+1)z^{-n-2}+\frac{n^2+4n+3}{4}z^{-2}. \tag{4.19}$$

For the second case of $F(R)$ in eq. (2.10), the asymptotic solution of $V(z)$ is

$$V_{\mathrm{II}}(z\to+\infty)\to\left(n^2+2n+\frac{3}{4}\right)z^{-2}, \tag{4.20}$$

by ignoring the higher order infinitesimal terms. The limits of $V_{\mathrm{I}}(z)$ and $V_{\mathrm{II}}(z)$ are

$$\lim_{z\to+\infty}V_{\mathrm{I}}(z)=\lim_{z\to+\infty}V_{\mathrm{II}}(z)=0, \tag{4.21}$$

so there are no massive mode in that two cases.

For the third case of $F(R)$ in eq. (2.11), the asymptotic solution of $V(z)$ is

$$\begin{aligned}V_{\mathrm{III}}(z\to+\infty)\to\ &\mathrm{C}_6z^{2\mathrm{C}_3n-2}+\mathrm{C}_7z^{\mathrm{C}_3n-2}+\mathrm{C}_8z^{\mathrm{C}_3n-n-2}\\&+\frac{1}{4}\mathrm{C}_4^2n^2z^{-2(n+1)}+\frac{1}{2}\mathrm{C}_4n^2z^{-n-2}+\mathrm{C}_4nz^{-n-1}+\frac{3}{4}z^{-2},\end{aligned} \tag{4.22}$$

where

$$\begin{aligned}\mathrm{C}_6&=\mathrm{C}_2^2 4^{2\mathrm{C}_3-1}25^{\mathrm{C}_3}\mathrm{C}_3^2n^2|\mathrm{C}_5|^{-2\mathrm{C}_3}k^{4\mathrm{C}_3},\\ \mathrm{C}_7&=(2-\mathrm{C}_3n)\mathrm{C}_2 2^{2\mathrm{C}_3-1}\mathrm{C}_3n5^{\mathrm{C}_3}|\mathrm{C}_5|^{-\mathrm{C}_3}k^{2\mathrm{C}_3},\\ \mathrm{C}_8&=\mathrm{C}_2 2^{2\mathrm{C}_3-1}5^{\mathrm{C}_3}\mathrm{C}_3\mathrm{C}_4n^2|\mathrm{C}_5|^{-\mathrm{C}_3}k^{2\mathrm{C}_3}.\end{aligned}$$

From the above asymptotic solution, we find

$$V_{\mathrm{III}}(\infty)=\begin{cases}+\infty & \mathrm{C}_3>1/n & \text{infinitely deep potential}\\ \mathrm{C} & \mathrm{C}_3=1/n & \text{Pöschl-Teller potential}\\ 0 & 0<\mathrm{C}_3<1/n & \text{volcanic potential}\end{cases} \tag{4.23}$$

where C is a positive constant.

So the condition $\mathrm{C}_3=1/n$ means a finite number of localized massive modes, when $\mathrm{C}_3>1/n$, there will be infinite number of localized massive modes, and $0<\mathrm{C}_3<1/n$ corresponds to no massive mode can be localized on branes.

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5 Concrete braneworld model

In this section only the first case (2.9) and third case (2.11) of $F(R)$ are discussed, since the conclusion about second case (2.10) of $F(R)$ is the same with the first one (2.9). The model we used is proposed in ref. [31] without scalar field, where the five-dimensional action is

$$S = \frac{1}{4}\int \mathrm{d}^4x\mathrm{d}y\sqrt{-g}f(R), \tag{5.1}$$

and $f(R)$ reads

$$f(R) = \frac{4}{7}\left(6k^2+R\right)\cosh(\beta(w(R))) - \frac{2}{7}k^2\sqrt{480-\frac{36R}{k^2}-\frac{3R^2}{k^4}}\sinh(\beta(w(R))), \tag{5.2}$$

where $\beta(w)$ is defined as

$$\beta(w) \equiv 2\sqrt{3}\arctan\left(\tanh\left(\frac{w}{2}\right)\right), \tag{5.3}$$

with

$$w(R) = \pm\mathrm{arcsech}\left[\frac{\sqrt{20+R/k^2}}{2\sqrt{7}}\right]. \tag{5.4}$$

With above conditions the solution of $\alpha(y)$ is

$$\alpha(y) = -\ln(\cosh(k\,y)). \tag{5.5}$$

Under the z coordinate

$$\alpha(z) = -\frac{1}{2}\log\left(k^2z^2+1\right), \tag{5.6}$$

and

$$\begin{aligned} R(z) &= -4e^{-2\alpha(z)}\left(2\alpha''(z)+3\alpha'(z)^2\right) \\ &= \frac{8k^2-20k^4z^2}{k^2z^2+1}. \end{aligned} \tag{5.7}$$

With the above analytic solutions of $\alpha(z)$ (5.6) and $R(z)$ (5.7), one can easily get

$$V_{\mathrm{I}}(z) = \frac{3k^2(-2+5k^2z^2)}{4(1+k^2z^2)^2}, \tag{5.8}$$

and

$$\begin{aligned} V_{\mathrm{III}}(z) = \frac{1}{4}\left(\frac{25}{49}\right)^{\mathrm{C}_3}k^2\left(\frac{1}{k^2z^2+1}\right)^{2-2\mathrm{C}_3}\Bigg[&4\mathrm{C}_2^2\mathrm{C}_3^2k^2z^2+49^{\mathrm{C}_3}\left(3k^2z^2-2\right)\left(\frac{1}{5k^2z^2+5}\right)^{2\mathrm{C}_3} \\ &-4\mathrm{C}_27^{\mathrm{C}_3}\mathrm{C}_3\left(2(\mathrm{C}_3-1)k^2z^2+1\right)\left(\frac{1}{5k^2z^2+5}\right)^{\mathrm{C}_3}\Bigg]. \end{aligned} \tag{5.9}$$

The limit of $V_{\mathrm{I}}(z)$ is

$$\lim_{z\to+\infty}V_{\mathrm{I}}(z) = 0, \tag{5.10}$$

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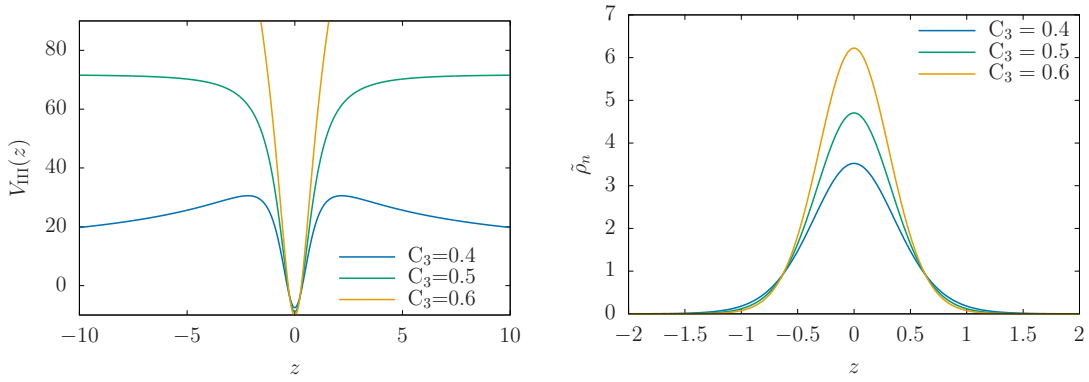

Figure 1. Plots of $V_{\rm III}(z)$ and $\tilde{\rho}_0$ with $C_2 = 20,\ k = 1$.

that is consistent with eq. (4.21). The asymptotic behavior of $R(z)$ is

$$R(z \to \infty) = -20k^2 + 8z^{-2}. \tag{5.11}$$

Compare eq. (5.11) with eq. (4.17), one can find that

$$n = 2,$$

which can be verified by the limit of $V_{\rm III}(z)$ (5.8), namely,

$$V_{\rm III}(\pm\infty) = \begin{cases} +\infty & C_3 > 1/2, \\ 5C_2^2k^2/28 & C_3 = 1/2, \\ 0 & 0 < C_3 < 1/2, \end{cases} \tag{5.12}$$

Equations (5.12) and (5.10) can be taken as a test of our conclusion made in section 4.

In the following we show some mode solutions with the potential $V_{\rm III}(z)$ (5.8). The zero mode solution (4.11) reads

$$\tilde{\rho}_0(z) = \sqrt{e^{C_2 - C_2\left(\frac{5}{7}\right)^{C_3}\left(\frac{1}{k^2z^2+1}\right)^{-C_3}}}\left(k^2z^2+1\right)^{-1/4}. \tag{5.13}$$

The plots of $V_{\rm III}(z)$ and $\tilde{\rho}_0(z)$ (non-normalized) with some specific values of parameters of C_2, k, and C_3 are shown in figure 1.

The localized massive mode solutions can be obtained by using of numerical method in the case of $C_3 \geq 1/2$. For example we show some massive solutions in figure 2 with parameters $C_3 = 0.5$, $C_2 = 20$, and $k = 1$. Another example is shown in figure 3 with parameters $C_3 = 0.6$, $C_2 = 20$, and $k = 1$.

6 Conclusions

In order to localize U(1) gauge vector fields on branes in 5D (asymptotic) AdS_5 spacetime with infinitely extra-dimension, we propose a 5D U(1) gauge vector fields action which include the non-minimal coupling with gravity and satisfy the gauge invariant. In this

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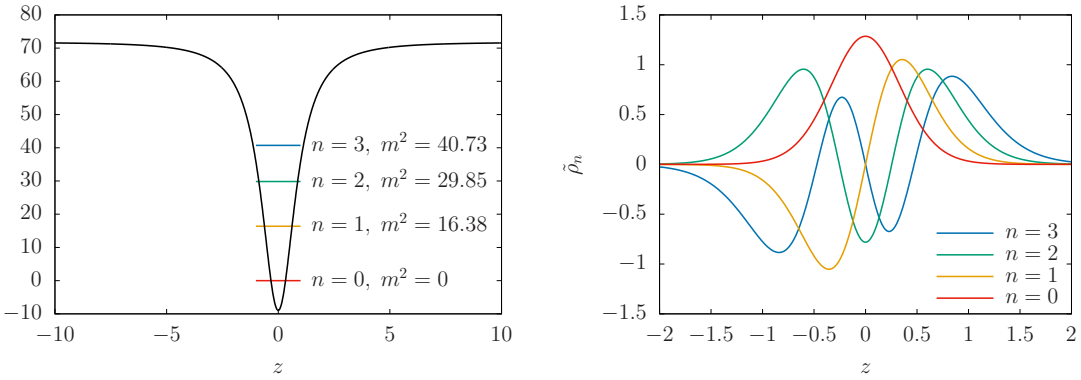

Figure 2. In the left figure, the black line represents the potential $V_{\rm III}(z)$, and colored lines represent the position of mass spectra in the potential. The right figure shows the corresponding solutions of $\tilde{\rho}_n$ (nomalized). Parameters $\mathrm{C}_3 = 0.5,\ \mathrm{C}_2 = 20$, and $k = 1$.

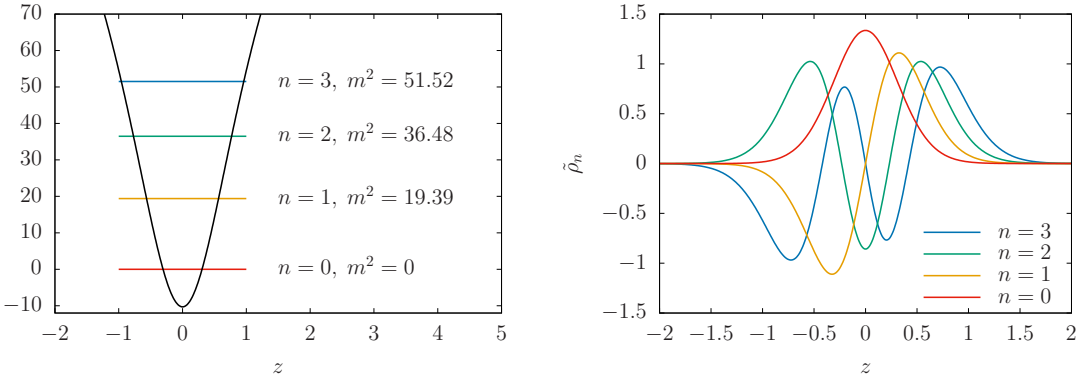

Figure 3. In the left figure, the black line represents the potential $V_{\rm III}(z)$, and colored lines represent the position of mass spectra in the potential. The right figure shows the corresponding solutions of $\tilde{\rho}_n$ (nomalized). Parameters $\mathrm{C}_3 = 0.6,\ \mathrm{C}_2 = 20$, and $k = 1$.

work we propose three kinds of coupling, they all support the localization of zero mode, but only the third one (2.11) support the localization of massive modes. With the third kind of coupling (2.11), there is a parameter C_3 which can control the localization of massive modes, namely, when

- $\mathrm{C}_3 = 1/n$, there are finite number of localized massive modes,
- $\mathrm{C}_3 > 1/n$, there are infinite number of localized massive modes,
- $0 < \mathrm{C}_3 < 1/n$, there is not localized massive mode,

where n is a positive number determined by the scalar curvature R (4.17). Moreover, our method can exclude the massive tachyonic modes and can be used not only in the thin braneword models but also in the thick ones.

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Acknowledgments

We thanks Prof. Yu-Xiao Liu for helpful discussion. We also thank the referee for helpful comments and suggestions. This work was supported by the National Natural Science Foundation of China (Grant Nos. 11305095, 11522541, 11375075, and 11705070), the Natural Science Foundation of Shandong Province, China (Grant No. ZR2013AQ016), and the Scientific Research Foundation of Shandong University of Science and Technology for Recruited Talents (Grant No. 2013RCJJ026).

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