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SO(32) heterotic line bundle models

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ABSTRACT: We search for the three-generation standard-like and/or Pati-Salam models from the SO(32) heterotic string theory on smooth, quotient complete intersection Calabi-Yau threefolds with multiple line bundles, each with structure group U(1). These models are S- and T-dual to intersecting D-brane models in type IIA string theory. We find that the stable line bundles and Wilson lines lead to the standard model gauge group with an extra $U(1)_{B-L}$ via a Pati-Salam-like symmetry and the obtained spectrum consists of three chiral generations of quarks and leptons, and vector-like particles. Green-Schwarz anomalous U(1) symmetries control not only the Yukawa couplings of the quarks and leptons but also the higher-dimensional operators causing the proton decay.

KEYWORDS: Superstrings and Heterotic Strings, Superstring Vacua

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1 Introduction

String theory is a most successful candidate of a unified theory including both the gauge and gravitational interactions. Among the perturbative superstring theories, the heterotic string theory [1, 2] is an attractive one, since the gauge group and matter representations are uniquely determined by the ten-dimensional gauge and gravitational anomaly cancellation conditions. In the low-energy effective action of the heterotic string, the background gauge field strength is related to the curvature of internal manifold through the Bianchi identity. The simplest approach to solve the Bianchi identity is the “standard embedding”, where the gauge bundle with $SU(3)$ structure group is identified with the holomorphic tangent bundle of the Calabi-Yau (CY) manifolds [3]. (See, e.g. refs. [4, 5] for the detailed analysis of three-generation models.) On the other hand, it is possible to construct consistent heterotic models on smooth CY threefolds, even if the gauge bundles are not directly related to the tangent bundle [6]. In this approach, holomorphic gauge bundles satisfying the Hermitian Yang-Mills equations lead to semi-realistic standard-like models as discussed in several CY threefolds with stable non-abelian vector bundles [7–12] and line bundles [13, 14].

So far, $E_8 \times E_8$ heterotic string models have been well discussed in contrast to $SO(32)$ heterotic ones. This is because the adjoint representation of E_8 gauge group naturally involves the spectrum of E_6 , $SO(10)$ and $SU(5)$ grand unified theories (GUTs). On the

other hand, the adjoint representation of $SO(32)$ does not contain the spinor representation of $SO(10)$, but it contains the spectrum of the standard model. Indeed, as pioneered in ref. [15], several standard-like models from the $SO(32)$ heterotic string theory are constructed on toroidal orbifold [16–19], torus with magnetic fluxes [20, 21], elliptically fibered CY manifolds with stable vector bundles [22] given by the spectral cover construction [23, 24] and smooth CY threefolds with line bundles [25]. However, the $SO(32)$ heterotic line bundle models on smooth CY threefolds have not been fully explored.

S- and T-dualities tell us that the $SO(32)$ heterotic line bundle models correspond to intersecting D6-brane models in type IIA string theory, where several stacks of branes directly lead to the minimal supersymmetric standard model (MSSM) and/or Pati-Salam model [26–28].¹ It therefore motivates us to search for standard-like models from the $SO(32)$ heterotic string theory on CY threefolds without an intermediate GUT.² To obtain realistic three-generation models, we consider multiple line bundles in the Cartan directions of $SO(32)$ rather than non-abelian bundles. These line bundles allow us to decompose the $SO(32)$ gauge group and compute the net chiral asymmetries of quarks and leptons, taking into account that the choice of line bundles is constrained by the consistency conditions such as anomaly cancellation conditions, masslessness conditions of the hypercharge gauge boson, supersymmetric conditions, and so-called K-theory condition. It turns out that these theoretical and phenomenological requirements indicate the enhancement of the standard model gauge group to the Pati-Salam one. To break the Pati-Salam gauge symmetry, we concentrate on quotient complete intersection Calabi-Yau manifolds (CICYs) [30, 31] by a similar argument as in $E_8 \times E_8$ heterotic line bundle models [13, 14], where one can introduce Wilson lines into the internal components of $U(1)$ s because of the existence of a freely-acting discrete symmetry group. We systematically search for standard-like models on several CICYs with multiple line bundles, where the number of Kähler moduli is restricted to $1 \leq h^{1,1} \leq 5$, with $h^{1,1}$ being the hodge number of CICYs. We find that such restrictive line bundles and certain Wilson lines lead to the standard-like models where the gauge group consists of $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ via the Pati-Salam-like symmetry. The spectrum in the visible sector contains the three generations of quarks and leptons without chiral exotics, and Higgs doublets appear as the vector-like particles with respect to the standard model gauge group with an extra $U(1)_{B-L}$. The gauge symmetries of the low-energy effective action allow for the perturbative Yukawa couplings of quarks and leptons.

The remainder of this paper is organized as follows. We first briefly review several consistency conditions on the basis of the low-energy effective action of the heterotic string theory in section 2.1. In section 2.2, we present the group decomposition of $SO(32)$ gauge symmetry employing multiple line bundles along the line of ref. [20]. Even if line bundles satisfy the consistency conditions, they are further constrained by the masslessness conditions of the $U(1)_Y$ gauge boson, originating from the couplings between the $U(1)_Y$ gauge boson and closed string axions as discussed in section 2.3. A search for concrete three-generation standard-like models on several quotient CICYs indicates in section 3

¹For more details, see e.g. ref. [29] and references therein.

²For the model building realizing the $SU(5)$ -like spectrum, we refer to ref. [25].

that the visible gauge group consists of $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ as a result of multiple line bundles and certain Wilson lines. Other $U(1)$ gauge bosons become massive through the Green-Schwarz mechanism. The obtained spectrum contains the three generations of quarks and leptons, vector-like Higgs and extra vector-like particles with respect to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$. Extra $U(1)$ gauge symmetries including $U(1)_{B-L}$ control the Yukawa couplings among the elementary particles and higher-dimensional operators causing the proton decay. Finally, section 4 is devoted to the conclusion.

2 Setup

2.1 Consistency conditions in the low-energy effective action

We briefly review the low-energy effective action of the $SO(32)$ heterotic string theory on CY manifolds with multiple line bundles. (For more details, we refer to refs. [10, 11, 32].) At the order of α' , the bosonic part of the low-energy effective action is given by

$$S_{\text{bos}} = \frac{1}{2\kappa_{10}^2} \int_{M^{(10)}} e^{-2\phi_{10}} \left[R + 4d\phi_{10} \wedge *d\phi_{10} - \frac{1}{2} H \wedge *H \right] - \frac{1}{2g_{10}^2} \int_{M^{(10)}} e^{-2\phi_{10}} \text{tr}(F \wedge *F) - \sum_s N_s T_5 \int_{M^{(10)}} B^{(6)} \wedge \delta(\gamma_s), \quad (2.1)$$

where ϕ_{10} denotes the dilaton. Trace of F and R , “tr”, is taken in the fundamental representations of $SO(32)$ and $SO(1,9)$, κ_{10} and g_{10} are the gravitational and gauge couplings normalized as $2\kappa_{10}^2 = (2\pi)^7(\alpha')^4$ and $g_{10}^2 = 2(2\pi)^7(\alpha')^3$. Here, we include the Wess-Zumino term in the presence of heterotic five-branes with tension $T_5 = ((2\pi)^5(\alpha')^3)^{-1}$. Heterotic (anti) five-branes wrapping the holomorphic two-cycles γ_s correspond to the positive (negative) N_s and the Poincaré dual four-form of γ_s is represented as $\delta(\gamma_s)$. $H = dB^{(2)} - \frac{\alpha'}{4}(w_{\text{YM}} - w_L)$ involves the gauge and gravitational Chern-Simons three-forms, w_{YM} and w_L . Note that Kalb-Ramond two-form $B^{(2)}$ is related to $B^{(6)}$ under the ten-dimensional Hodge duality, namely $*dB^{(2)} = e^{2\phi_{10}} dB^{(6)}$.

Throughout this paper, we consider the following internal gauge bundle

$$W = \bigoplus_{a=1}^M L_a, \quad (2.2)$$

where L_a are the multiple line bundles, each with structure group $U(1)$. The concrete embedding of $U(1)$ into $SO(32)$ is discussed later. The inclusion of such line bundles breaks $SO(32)$ into the four-dimensional gauge group G and the adjoint representation of $SO(32)$ is decomposed as

$$496 \rightarrow \bigoplus_p (R_p, C_p), \quad (2.3)$$

where R_p and C_p stand for certain representations of G and W . Given these line bundles, we can construct standard-like models only if internal gauge bundles satisfy several consistency conditions which are enumerated as follows.

First of all, when we denote the internal background field strengths as $\bar{F}$ and $\bar{R}$, the Bianchi identity of the Kalb-Ramond field $B^{(6)}$

$$d(e^{2\phi_{10}} * dB^{(6)}) = -\frac{\alpha'}{4} \left(\text{tr} \bar{F}^2 - \text{tr} \bar{R}^2 - 4(2\pi)^2 \sum_s N_s \delta(\gamma_s) \right) \quad (2.4)$$

constrains the abelian bundles as

$$\text{ch}_2(W) + c_2(T\mathcal{M}) = \sum_s N_s \delta(\gamma_s), \quad (2.5)$$

where $\text{ch}_2(W)$ and $c_2(T\mathcal{M})$ are the second Chern character and second Chern class of W and the tangent bundle of the CY manifold $\mathcal{M}$ respectively. To keep the stability of the system, we require the vanishing anti heterotic five-branes, namely

$$\text{ch}_2(W) + c_2(T\mathcal{M}) \geq [0], \quad (2.6)$$

in cohomology.

Next, since the spinorial representation appears in the first excited mode in the heterotic string [1, 2], we require that the first Chern class of the total gauge bundle lies in second even integral cohomology basis of the CY manifold:

$$c_1(W) = \sum_a n_a c_1(L_a) \in H^2(\mathcal{M}, 2\mathbb{Z}), \quad (2.7)$$

where n_a are the integers depending on the embedding of line bundles into $\text{SO}(32)$. Such a condition is also related to the K-theory condition [33, 34] in the S-dual type I superstring theory.

Finally, the nonvanishing $\text{U}(1)$ field strengths on the CY manifold are constrained by the supersymmetric condition. From the supersymmetric transformations of the gauginos, the internal field strengths have to obey

$$\bar{F}_{ij} = \bar{F}_{\bar{i}\bar{j}} = 0, \quad g^{i\bar{j}} \bar{F}_{i\bar{j}} = 0, \quad (2.8)$$

implying that the vector bundles should be holomorphic. It is known that the constraint equation for the $(1,1)$ form of F is called the Hermitian Yang-Mills equation. It can be solved when holomorphic bundles satisfy the following condition

$$\mu(L_a) = \frac{1}{2l_s^4} \int_{\mathcal{M}} J \wedge J \wedge c_1(L_a) + \frac{\mathcal{V}}{6\pi s} \int_{\mathcal{M}} \left[c_1^3(L_a) + \frac{1}{4} c_1(L_a) \wedge c_2(T\mathcal{M}) \right] = 0, \quad (2.9)$$

for all a . $J = l_s^2 \sum_{i=1}^{h^{1,1}} t^i w_i$ is the Kähler form expressed in terms of a basis w_i of $H^{1,1}(\mathcal{M}, \mathbb{R})$, where t^i denote the Kähler moduli normalized by the string length $l_s = 2\pi\sqrt{\alpha'}$. Now, we include the string loop corrections [10, 11] characterized by the volume of CY $\mathcal{V} = \frac{1}{6} d_{ijk} t^i t^j t^k$ and the dilaton $s = \mathcal{V}/(2\pi g_s^2)$. d_{ijk} stand for the triple intersection numbers of CY manifolds, namely $d_{ijk} = \int_{\mathcal{M}} w_i \wedge w_j \wedge w_k$. Hence, the first Chern class of each line bundle should be properly chosen such that the moduli spaces of Kähler moduli reside in the supergravity reliable domain $t^i > 1$ in string units. Otherwise, the nonvanishing

D -terms appear in the four-dimensional supergravity action. In this paper, we restrict ourselves to line bundles satisfying all the consistency conditions presented so far.

In this approach, we can calculate the net-number of chiral massless fermions. When we denote the internal bundle $\mathcal{C}_p$ to each C_p , the left-handed and the right-handed fermionic zero-modes in R_p are counted by the Dolbeault cohomology $H^1(\mathcal{M}, \mathcal{C}_p)$ and $H^2(\mathcal{M}, \mathcal{C}_p)$ respectively. Since the μ -stable bundles ($\mu(L_a) = 0$) give the vanishing zeroth and third cohomology $H^0(\mathcal{M}, \mathcal{C}_p) = H^3(\mathcal{M}, \mathcal{C}_p) = 0$ ³ and $H^2(\mathcal{M}, \mathcal{C}_p) \simeq H^1(\mathcal{M}, \mathcal{C}_p^*)$ by Serre duality, Hirzebruch-Riemann-Roch theorem tells us that the net-number of chiral massless fermions (chiral supermultiplets) is counted by the corresponding Euler number

$$\begin{aligned}\chi(\mathcal{M}, \mathcal{C}_p) &= \sum_{i=0}^3 (-1)^i \dim(H^i(\mathcal{M}, \mathcal{C}_p)) = -\dim(H^1(\mathcal{M}, \mathcal{C}_p)) + \dim(H^1(\mathcal{M}, \mathcal{C}_p^*)) \\ &= \int_{\mathcal{M}} \left[\text{ch}_3(\mathcal{C}_p) + \frac{1}{12} c_2(T\mathcal{M}) c_1(\mathcal{C}_p) \right].\end{aligned}\quad (2.10)$$

Note that we now consider the complex representation $\mathcal{C}_p$. When some bundles in eq. (2.2) are trivial bundles $\mathcal{O}_{\mathcal{M}}$, the $U(1)$ gauge symmetries are enhanced to non-abelian ones as shown in section 3 and corresponding cohomology becomes $\dim(H^0(\mathcal{M}, \mathcal{O}_{\mathcal{M}})) = \dim(H^3(\mathcal{M}, \mathcal{O}_{\mathcal{M}})) = 1$ and $\dim(H^1(\mathcal{M}, \mathcal{O}_{\mathcal{M}})) = \dim(H^2(\mathcal{M}, \mathcal{O}_{\mathcal{M}})) = 0$. This is because the zero-modes of the Dirac operator are the $(0, 0)$ and $(0, 3)$ forms under the Dolbeault operator $\bar{\partial}$ on manifolds of $SU(3)$ holonomy. For more details we refer to ref. [35].

In this way, proper internal line bundles have the potential to yield three generations of chiral fermions for a large class of CY manifolds, although, in the standard embedding scenario, three-generation models are restricted to specific CY manifolds with small hodge numbers.

2.2 Matter content and group decomposition

In the following, we proceed to extract standard-like models from the $SO(32)$ heterotic string theory. Along the line of ref. [20], we first decompose the $SO(32)$ gauge group as

$$\begin{aligned}SO(32) &\rightarrow SO(12) \times SO(20), \\ 496 &\rightarrow (1, 190) + (12_v, 20_v) + (66, 1),\end{aligned}\quad (2.11)$$

where $SO(12)$ and $SO(20)$ are further decomposed by the insertion of line bundles,

$$\begin{aligned}SO(12) &\rightarrow SO(8) \times SU(2)_L \times U(1)_1 \rightarrow SU(4)_C \times U(1)_2 \times SU(2)_L \times U(1)_1 \\ &\rightarrow SU(3)_C \times U(1)_3 \times U(1)_2 \times SU(2)_L \times U(1)_1, \\ SO(20) &\rightarrow U(1)_4 \times \cdots \times U(1)_{13}.\end{aligned}\quad (2.12)$$

³As commented in ref. [14], non-trivial line bundles L_a have $\dim(H^0(\mathcal{M}, L_a)) = 0$ if $\mu(L_a) < 0$ and also $\dim(H^3(\mathcal{M}, L_a)) = 0$ if $\mu(L_a) > 0$. Such positive and negative slopes $\mu(L_a)$ exist in the Kähler moduli space, since we focus on the case with $\mu(L_a) = 0$. Hence, we obtain $\dim(H^0(\mathcal{M}, L_a)) = \dim(H^3(\mathcal{M}, L_a)) = 0$ according to the theorem in ref. [36].

SO(12)	$(\text{SU}(4)_C \times \text{SU}(2)_L)_{\text{U}(1)_1, \text{U}(1)_2}$	$(\text{SU}(3)_C \times \text{SU}(2)_L)_{\text{U}(1)_1, \text{U}(1)_2, \text{U}(1)_3}$	$\text{U}(1)_Y$	Matter
66	$(15, 1)_{0,0}$	$(3, 1)_{0,0,4}$	$2/3$	$\bar{u}_{R_1}^c$
		$(\bar{3}, 1)_{0,0,-4}$	$-2/3$	$u_{R_1}^c$
	$(6, 1)_{0,2}$	$(\bar{3}, 1)_{0,2,2}$	$1/3$	$d_{R_1}^c$
		$(3, 1)_{0,2,-2}$	$-1/3$	$\bar{d}_{R_2}^c$
	$(6, 1)_{0,-2}$	$(\bar{3}, 1)_{0,-2,2}$	$1/3$	$d_{R_2}^c$
		$(3, 1)_{0,-2,-2}$	$-1/3$	$\bar{d}_{R_1}^c$
	$(4, 2)_{1,1}$	$(3, 2)_{1,1,1}$	$1/6$	Q_1
		$(1, 2)_{1,1,-3}$	$-1/2$	L_1
	$(\bar{4}, 2)_{1,-1}$	$(\bar{3}, 2)_{1,-1,-1}$	$-1/6$	$\bar{Q}_2$
		$(1, 2)_{1,-1,3}$	$1/2$	$\bar{L}_2$
	$(4, 2)_{-1,1}$	$(3, 2)_{-1,1,1}$	$1/6$	Q_2
		$(1, 2)_{-1,1,-3}$	$-1/2$	L_2
	$(\bar{4}, 2)_{-1,-1}$	$(\bar{3}, 2)_{-1,-1,-1}$	$-1/6$	$\bar{Q}_1$
		$(1, 2)_{-1,-1,3}$	$1/2$	$\bar{L}_1$
	$(1, 1)_{2,0}$	$(1, 1)_{2,0,0}$	0	n_1
	$(1, 1)_{-2,0}$	$(1, 1)_{-2,0,0}$	0	$\bar{n}_1$

Table 1. Matter content from the adjoint representation of SO(12), where the hypercharge U(1)_Y is identified as U(1)₃/6. Here, the subscript indices label the U(1) charges.

Under the above decomposition, we take the Cartan directions of SO(32), H_a ($a = 1, 2, \dots, 16$), as $H_1 - H_2$ and $H_1 + H_2 - 2H_3$ for SU(3)_C and $H_5 - H_6$ for SU(2)_L. Other U(1) directions are chosen as

$$\begin{aligned}
 \text{U}(1)_1 &: (0, 0, 0, 0, 1, 1; 0, \dots, 0), \\
 \text{U}(1)_2 &: (1, 1, 1, 1, 0, 0; 0, \dots, 0), \\
 \text{U}(1)_3 &: (1, 1, 1, -3, 0, 0; 0, \dots, 0), \\
 \text{U}(1)_4 &: (0, 0, 0, 0, 0, 0; 1, 0, \dots, 0), \\
 \text{U}(1)_5 &: (0, 0, 0, 0, 0, 0; 0, 1, 0, \dots, 0), \\
 &\vdots \\
 \text{U}(1)_{13} &: (0, 0, 0, 0, 0, 0; 0, \dots, 0, 1),
 \end{aligned} \tag{2.13}$$

in the basis H_a and SO(32) roots are $(\pm 1, \pm 1, 0, \dots, 0)$ under H_a , ($a = 1, \dots, 16$). The underline represents all the possible permutations.

Under this Cartan basis, the adjoint representation of SO(12), 66, involves all the candidates of the standard model particles, except for the right-handed leptons as summarized in table 1. Hence, it motivates us to further decompose SO(20) into multiple U(1)s. When SO(20) is decomposed into all the U(1)s, the candidates of right-handed leptons appear

from the vector representation and the singlet of $\text{SO}(12)$, 12_v and 1,

$$\begin{aligned}
 (12_v, 20_v) &\rightarrow \begin{cases} L_3^a = (1, 2)_{1,0,0;-1(a)} \\ L_4^a = (1, 2)_{-1,0,0;-1(a)} \\ u_{R_2}^c = (\bar{3}, 1)_{0,-1,-1;-1(a)} \\ d_{R_3}^c = (\bar{3}, 1)_{0,-1,-1;1(a)} \\ e_{R_1}^c = (1, 1)_{0,-1,3;1(a)} \\ n_2^c = (1, 1)_{0,-1,3;-1(a)} \end{cases}, \quad (a = 4, 5, \dots, 13), \\
 (1, 190) &\rightarrow \begin{cases} e_{R_2}^{c\,ab} = (1, 1)_{0,0,0;1(a),1(b)} \\ n_3^{c\,ab} = (1, 1)_{0,0,0;1(a),-1(b)} \end{cases}, \quad (a, b = 4, 5, \dots, 13, a < b), \quad (2.14)
 \end{aligned}$$

where the subscript indices label the $\text{U}(1)_{1,2,3}$ and non-zero $\text{U}(1)_{a,b}$ charges. This decomposition results in the candidates of right-handed quarks, charged-leptons and/or Higgs.

To realize the correct hypercharge, we redefine the hypercharge as

$$\text{U}(1)_Y = \frac{1}{6} \left(\text{U}(1)_3 + 3 \sum_{c=4}^{13} \text{U}(1)_c \right), \quad (2.15)$$

and the matter content and associated cohomology are then summarized in table 2. It is notable that we just decompose $\text{SO}(20)$ into tenth $\text{U}(1)$ s by inserting line bundles into all the Cartan directions. If the first Chern numbers of certain $\text{U}(1)$ s are vanishing or correlated with each other, these $\text{U}(1)$ gauge symmetries are enhanced to non-abelian ones as demonstrated in section 2.4.

2.3 $\text{U}(1)_Y$ masslessness conditions

Before searching for three generations of quarks and leptons, let us discuss the couplings between string axions and the $\text{U}(1)_Y$ gauge boson. Such axionic couplings will cause the mass term of the $\text{U}(1)_Y$ gauge boson, even when gauge bundles satisfy the consistency conditions presented in section 2.1. The authors of refs. [10, 11] pointed out that background gauge fluxes induce the couplings between string axions and $\text{U}(1)$ gauge bosons through the Green-Schwarz term [37, 38],

$$S_{\text{GS}} = \frac{1}{24(2\pi)^5 \alpha'} \int B^{(2)} \wedge X_8, \quad (2.16)$$

where

$$X_8 = \frac{1}{24} \text{Tr} F^4 - \frac{1}{7200} (\text{Tr} F^2)^2 - \frac{1}{240} (\text{Tr} F^2) (\text{tr} R^2) + \frac{1}{8} \text{tr} R^4 + \frac{1}{32} (\text{tr} R^2)^2. \quad (2.17)$$

Now “Tr” is taken in the adjoint representation of $\text{SO}(32)$.

Under the expansion of the Kalb-Ramond field in a basis of $H^2(\mathcal{M}, \mathbb{Z})$,

$$\begin{aligned}
 B^{(2)} &= b_0^{(2)} + l_s^2 \sum_{i=1}^{h^{1,1}} b_i^{(0)} w_i, \\
 B^{(6)} &= l_s^6 b_0^{(0)} \text{vol}_6 + l_s^4 \sum_{i=1}^{h^{1,1}} b_i^{(2)} \hat{w}^i, \quad (2.18)
 \end{aligned}$$

Matter	Repr.	Cohomology
Q_1	$(3, 2)_{1,1,1}$	$H^*(\mathcal{M}, L_1 \otimes L_2 \otimes L_3)$
Q_2	$(3, 2)_{-1,1,1}$	$H^*(\mathcal{M}, L_1^{-1} \otimes L_2 \otimes L_3)$
L_1	$(1, 2)_{1,1,-3}$	$H^*(\mathcal{M}, L_1 \otimes L_2 \otimes L_3^{-3})$
L_2	$(1, 2)_{-1,1,-3}$	$H^*(\mathcal{M}, L_1^{-1} \otimes L_2 \otimes L_3^{-3})$
L_3^a	$(1, 2)_{1,0,0;-1_{(a)}}$	$H^*(\mathcal{M}, L_1 \otimes L_a^{-1})$
L_4^a	$(1, 2)_{-1,0,0;-1_{(a)}}$	$H^*(\mathcal{M}, L_1^{-1} \otimes L_a^{-1})$
$u_{R_1}^c$	$(\bar{3}, 1)_{0,0,-4}$	$H^*(\mathcal{M}, L_3^{-4})$
$u_{R_2}^{c\,a}$	$(\bar{3}, 1)_{0,-1,-1;-1_{(a)}}$	$H^*(\mathcal{M}, L_2^{-1} \otimes L_3^{-1} \otimes L_a^{-1})$
$d_{R_1}^c$	$(\bar{3}, 1)_{0,2,2}$	$H^*(\mathcal{M}, L_2^2 \otimes L_3^2)$
$d_{R_2}^c$	$(\bar{3}, 1)_{0,-2,2}$	$H^*(\mathcal{M}, L_2^{-2} \otimes L_3^2)$
$d_{R_3}^{c\,a}$	$(\bar{3}, 1)_{0,-1,-1;1_{(a)}}$	$H^*(\mathcal{M}, L_2^{-1} \otimes L_3^{-1} \otimes L_a)$
$e_{R_1}^{c\,a}$	$(1, 1)_{0,-1,3;1_{(a)}}$	$H^*(\mathcal{M}, L_2^{-1} \otimes L_3^3 \otimes L_a)$
$e_{R_2}^{c\,ab}$	$(1, 1)_{0,0,0;1_{(a)},1_{(b)}}$	$H^*(\mathcal{M}, L_a \otimes L_b)$
n_1	$(1, 1)_{2,0,0}$	$H^*(\mathcal{M}, L_1^2)$
$n_2^{c\,a}$	$(1, 1)_{0,-1,3;-1_{(a)}}$	$H^*(\mathcal{M}, L_2^{-1} \otimes L_3^3 \otimes L_a^{-1})$
$n_3^{c\,ab}$	$(1, 1)_{0,0,0;1_{(a)},-1_{(b)}}$	$H^*(\mathcal{M}, L_a \otimes L_b^{-1})$

Table 2. Massless spectrum and corresponding cohomology. In the second column, the subscript indices label the $U(1)_{1,2,3}$ and non-zero $U(1)_{a,b}$ charges.

where $\hat{w}^i$ are the Hodge dual four-forms of the Kähler forms w_i and vol_6 denotes the volume form of the CY manifold, $b_i^{(0)}$ and $b_0^{(0)}$ represent the model-dependent and -independent axions respectively. Those are connected with two-forms, $b_i^{(2)}$ and $b_0^{(2)}$ under the ten-dimensional hodge duality $*dB^{(2)} = e^{2\phi_{10}}dB^{(6)}$.

On the line bundle background, the $U(1)_a$ gauge field strengths are decomposed as

$$F_a = f_a + \bar{f}_a, \quad (2.19)$$

where f_a are four-dimensional parts and

$$\bar{f}_a = 2\pi \sum_{i=1}^{h^{1,1}} m_a^i w_i \quad (2.20)$$

denote the background gauge fluxes in a basis of $H^2(\mathcal{M}, \mathbb{Z})$. Here, m_a^i are the integers subject to the Dirac quantization condition and we assume that the first Chern numbers of all the $U(1)$ s are independent, otherwise these $U(1)$ gauge symmetries are enhanced to non-abelian ones. Such a gauge background gives rise to the axionic couplings through the

Green-Schwarz term,

$$\frac{1}{3(2\pi)^3 l_s^2} \int b_0^{(2)} \wedge \left[\text{tr} T_1^4 \bar{f}_1^3 f_1 + (\text{tr} T_2^4 \bar{f}_2^3 + 3(\text{tr} T_2^2 T_3^2) \bar{f}_2 \bar{f}_3^2 + (\text{tr} T_2 T_3^3) \bar{f}_3^3) f_2 \right. \\ \left. + (\text{tr} T_3^4 \bar{f}_3^3 + 3(\text{tr} T_2 T_3^3) \bar{f}_2 \bar{f}_3^2 + 3(\text{tr} T_2^2 T_3^2) \bar{f}_2^2 \bar{f}_3) f_3 + \sum_{c=4}^{13} \text{tr} T_c^4 \bar{f}_c^3 f_c \right], \quad (2.21)$$

for the model-independent axion⁴ and

$$\frac{1}{l_s^2} \int b_i^{(2)} \wedge \left(\sum_{a=1}^{13} \text{tr}(T_a^2) m_a^i \right) f_a, \quad (2.22)$$

for the model-dependent axions.⁵ Here, T_a denote the $U(1)_a$ generators whose directions are chosen as in eq. (2.13). These Stueckelberg couplings yield the mass terms of the $U(1)_a$ gauge bosons as mentioned before.

Recalling the fact that the hypercharge $U(1)_Y$ is defined as $U(1)_Y = \frac{1}{6}(U(1)_3 + 3 \sum_{c=4}^{13} U(1)_c)$, the masslessness of the $U(1)_Y$ gauge field is ensured when

$$\text{tr}(T_3^4) d_{ijk} m_3^i m_3^j m_3^k + 3 \text{tr}(T_2 T_3^3) d_{ijk} m_2^i m_3^j m_3^k + 3 \text{tr}(T_2^2 T_3^2) d_{ijk} m_2^i m_2^j m_3^k \\ + 3 \sum_{c=4}^{13} \text{tr}(T_c^4) d_{ijk} m_c^i m_c^j m_c^k = 0 \quad (2.23)$$

and

$$\text{tr}(T_3^2) m_3^i + 3 \sum_{c=4}^{13} \text{tr}(T_c^2) m_c^i = 0 \quad (2.24)$$

are satisfied simultaneously.

2.4 Pati-Salam-like models

The simplest way to satisfy all the requirements is to set

$$m_5^i = -m_4^i, \quad m_3^i = m_d^i = 0, \quad (2.25)$$

for $d = 6, \dots, 13$ and $i = 1, 2, \dots, h^{1,1}$ and non-zero values for other line bundles. Under the above ansatz, one can easily satisfy the K-theory condition

$$c_1(W) = 2c_1(L_1) + 4c_1(L_2) + \sum_{c=4}^{13} c_1(L_c) = 0 \pmod{2}, \quad (2.26)$$

and the $U(1)_Y$ masslessness conditions in eqs. (2.23) and (2.24). Here and in what follows, the traces of $U(1)$ generators are taken as $\text{tr}(T_1) = 2$, $\text{tr}(T_2) = 4$, $\text{tr}(T_c) = 1$, where $U(1)_1$ and $U(1)_2$ are embedded into $U(2)_L$ and $U(4)_C$ gauge groups respectively. Note

⁴Note that the curvature terms in eq. (2.17) are irrelevant to the masslessness conditions of $U(1)_Y$.

⁵For the detailed derivation, see ref. [20].

that the gauge symmetries $SU(2)_L$ and $SU(4)_C$ of the Pati-Salam model are embedded as $SU(2)_L \subset U(2)_L$ and $SU(4)_C \subset U(4)_C$. These first Chern classes are of the form

$$c_1(L_a) = \sum_{i=1}^{h^{1,1}} m_a^i w_i, \quad (2.27)$$

where the quantized fluxes m_a^i are the integers as mentioned before. However, the above choice of line bundles results in the Pati-Salam-like gauge group plus hidden gauge group,

$$SO(32) \rightarrow SU(4)_C \times SU(2)_L \times SU(2)_R \times \Pi_{c=1}^2 U(1)_c \times U(1)' \times SO(16), \quad (2.28)$$

where $SU(3)_C$ is enhanced to $SU(4)_C$ due to the vanishing $c_1(L_3)$. The correlated first Chern classes $m_5^i = -m_4^i$ also lead to the gauge enhancement from $U(1)_{4,5}$ to $SU(2)_R \otimes U(1)'$ whose Cartan directions are taken as $H_4 + H_5$ for $SU(2)_R$ and $H_4 - H_5$ for $U(1)' \simeq S(U(1)_4 \times U(1)_5)$. For our purpose, we search for the three-generation models using $S(U(1)_4 \times U(1)_5)$ rather than $U(1)'$ in the latter analysis, where two $U(1)_{4,5}$ charge vectors $\mathbf{q} = (q_4, q_5)$ and $\mathbf{q}' = (q'_4, q'_5)$ are identified if $\mathbf{q} - \mathbf{q}' \in \mathbb{Z} \times (1, 1)$ because of $L_5 \simeq L_4^{-1}$. Other $U(1)$ s, especially $SO(2)$ s, are enhanced to hidden $SO(16)$ due to the vanishing first Chern classes. As a result, the remaining gauge symmetry on $\mathcal{M}$ is similar to the Pati-Salam model. It is remarkable that the above discussion presented so far is irrelevant to the underlying CY geometries thanks to the constraints on the line bundles (2.25). Although it is possible to directly derive the standard model gauge group with hypercharge flux breaking, we require a concrete model-by-model search. We therefore leave the detailed analysis for a future work.

When the vacuum expectation values of singlet fields under the Pati-Salam-like gauge group are taken zero, the number of massive $U(1)$ s is given by the rank of the following mass matrix in units of string length $l_s = 2\pi\sqrt{\alpha'}$ [10, 11],

$$M_{ai} = \begin{cases} 2\pi \text{tr}(T_a) m_a^i & \text{for } i = 1, 2, \dots, h^{1,1} \\ d_{jkl} \text{tr}(T_a) \left[\frac{1}{6} m_a^j m_a^k m_a^l + \frac{1}{24} m_a^j c_2^{kl}(T\mathcal{M}) \right] & \text{for } i = 0 \end{cases}. \quad (2.29)$$

The first line is derived from the Stueckelberg couplings between the model-dependent axions and $U(1)_a$ gauge bosons, whereas the second line is originating from the model-independent axion associated with the dilaton field. It turns out that when $h^{1,1} = 1$, the maximum rank of mass matrix (2.29) is 2 in which at least one of the three $U(1)$ s remains massless due to the correlated Chern numbers (2.25). When $h^{1,1} \geq 2$, it is possible to consider three massive $U(1)$ gauge bosons at the compactification scale. Note that the Cartan direction of $U(1)_Y = \frac{1}{6}(U(1)_3 + 3U(1)_4 + 3U(1)_5)$ remains massless and the remaining $U(1)_3$ could be identified with $U(1)_{B-L}$ after the Pati-Salam symmetry breaking.

In addition to the constraints on the line bundles (2.25), we restrict ourselves to line bundles satisfying the D -term conditions (2.9) and the stability condition (2.6) simplified as

$$2\text{ch}_2(L_1) + 4\text{ch}_2(L_2) + \text{ch}_2(L_4) + \text{ch}_2(L_5) + c_2(T\mathcal{M}) \geq [0], \quad (2.30)$$

in cohomology, where $\text{ch}_2(L_a) = \frac{1}{2}c_1^2(L_a) = \frac{1}{2}d_{ijk}m_a^j m_a^k \tilde{w}^i$ with $c_1^i(L_a) = m_a^i$. Note that we use the relations $\text{tr}(T_1^2) = 2$, $\text{tr}(T_2^2) = 4$ and $\text{tr}(T_{4,5}^2) = 1$. In the light of D -term

conditions (2.9), it is difficult to realize the vanishing D -terms for $h^{1,1} = 1$. Hence, we demand $h^{1,1} \geq 2$, yielding three massive $U(1)$ gauge bosons except for $U(1)_Y$ and $U(1)_{B-L}$.

One option to break the Pati-Salam symmetry is its spontaneous symmetry breaking in the presence of vector-like Higgs $(\bar{4}, 1, 2, 1) + \text{c.c.}$. In this paper, we take into account another option, namely to introduce Wilson lines into the internal components of $SU(4)_C$ and $SU(2)_R$ on the quotient CY manifolds.

3 Three-generation models on quotient complete intersection CY manifolds

We are now ready to searching for the three-generation models on CY manifolds with multiple line bundles satisfying all the consistency conditions. In particular, we concentrate on smooth, quotient CICYs $\tilde{\mathcal{M}} = \mathcal{M}/\Gamma$ [30, 31], where $\mathcal{M}$ is a simply connected CICY with a freely-acting discrete symmetry group Γ . CICYs defined in the ambient space $\mathbb{P}^{n_1} \times \dots \times \mathbb{P}^{n_m}$ are characterized by the following $m \times R$ configuration matrix,

$$\begin{array}{c} \mathbb{P}^{n_1} \\ \mathbb{P}^{n_2} \\ \vdots \\ \mathbb{P}^{n_m} \end{array} \begin{bmatrix} q_1^1 & q_2^1 & \cdots & q_R^1 \\ q_1^2 & q_2^2 & \cdots & q_R^2 \\ \vdots & \vdots & \ddots & \vdots \\ q_1^m & q_2^m & \cdots & q_R^m \end{bmatrix}_{\chi}^{h^{1,1}, h^{1,2}}, \quad (3.1)$$

where the subscript and superscripts denote the Euler number and Hodge numbers of CICYs respectively. When the homogeneous coordinates of $\mathbb{P}^{n_l}$ are represented as x_{α}^l with $l = 1, \dots, m$ and $\alpha = 0, \dots, n_l$, the positive integers q_r^l ($r = 1, \dots, R$) specify the multi-degree of R homogeneous polynomials on $\mathbb{P}^{n_1} \times \dots \times \mathbb{P}^{n_m}$. Then, the common zero locus of such R polynomials corresponds to the defining equations of CICYs. The CY condition, namely a vanishing first Chern class of the tangent bundle, is realized by setting $\sum_{r=1}^R q_r^l = n_l + 1$, for all l and the dimension of CICYs becomes 3 under $\sum_{l=1}^m n_l - R = 3$. Through out this paper, we focus on the favorable CICYs where the second cohomology of CY descends from that of the ambient space.

Among them, some of CICYs admit a freely-acting discrete symmetry group Γ classified in ref. [39]. Those quotient CICYs are of particular importance, since one can turn on Wilson lines because of the existence of the discrete symmetry group. Furthermore, Γ reduces the number of complex structure moduli.⁶ As discussed in section 2.4, some of the line bundles satisfying all the conditions in section 2 should be correlated with one another. The gauge symmetry of the standard model in turn is enhanced to the Pati-Salam-like symmetry. We therefore introduce Wilson lines into the internal components of $U(1)$ s on the quotient CICYs such that the Pati-Salam group is broken to the standard model one. It turns out that one cannot find realistic models on CICYs with $h^{1,1} = 1, 2, 3, 5$, since the underlying CY geometries and consistency conditions constrain the choice of background

⁶In general, the number of Kähler moduli on $\mathcal{M}$ and $\tilde{\mathcal{M}} = \mathcal{M}/\Gamma$ is different from each other, but in this paper we focus on CICYs with $h^{1,1}(\mathcal{M}) = h^{1,1}(\tilde{\mathcal{M}})$.

line bundles. In the next but one subsection 3.2, we show the standard-like models on a concrete quotient CICY with $h^{1,1}(\tilde{\mathcal{M}}) = 4$.

3.1 Standard-like models via the Pati-Salam-like symmetry

To break the Pati-Salam-like symmetry into the standard model one, we consider the quotient CICYs $\tilde{\mathcal{M}} = \mathcal{M}/\Gamma$, where one can introduce Wilson lines into the internal components of $U(1)_{3,4,5}$ such that $SU(4)_C$ and $SU(2)_R \times U(1)'$ are broken to $SU(3)_C \times U(1)_3$ and $U(1)_{4,5}$ respectively. (For more details, we refer to ref. [35].) When the discrete symmetry group Γ consists of only Abelian discrete symmetries $\mathbb{Z}_n$, namely $\Gamma = \otimes_i \mathbb{Z}_{n_i}$, the representations of $\mathbb{Z}_{n_i}$ are represented as $e^{2\pi i p_i / n_i}$ for $p_i \in \{0, 1, \dots, n_i - 1\}$. If the Wilson lines are in the center of $SU(4)_C$ and $SU(2)_R$, the Pati-Salam symmetry is unbroken. Hence, one of the Abelian symmetries should be different from $\mathbb{Z}_4$ and $\mathbb{Z}_2$. In such a case, the low-energy gauge group on the quotient CICYs $\tilde{\mathcal{M}}$ becomes

$$SO(32) \rightarrow SU(3)_C \times SU(2)_L \times \Pi_{c=1}^5 U(1)_c \times SO(16), \quad (3.2)$$

where three $U(1)$ s except for $U(1)_Y$ and $U(1)_{B-L}$ become massive through the axionic couplings.

On quotient CICYs $\tilde{\mathcal{M}}$, the net-number of massless chiral supermultiplets is subject to the discrete symmetry group Γ . Indeed, the gauge bundles on $\mathcal{M}$ reduce to those on $\tilde{\mathcal{M}}$ if such bundles are equivariant bundles, and the cohomology associated with the matter field on $\tilde{\mathcal{M}}$ is described by the subspace of $H^1(\mathcal{M}, V)$. Note that all line bundles have an equivariant structure for a single $\mathbb{Z}_n$, namely $\Gamma = \mathbb{Z}_n$. (For more details, we refer to ref. [14].) The number of generation on $\mathcal{M}$ is divided by the group order, $|\Gamma|$:

$$\chi(\tilde{\mathcal{M}}, \mathcal{C}_p \otimes \mathcal{S}) = \frac{\chi(\mathcal{M}, \mathcal{C}_p)}{|\Gamma|}, \quad (3.3)$$

where $\mathcal{S}$ is a certain representation of Γ . Note that the chiral asymmetry is independent of Wilson lines.

Under these circumstances, we search for three generations of quarks and leptons such that the indices in table 3 become

$$\begin{aligned} m_{(4,2,1,1)_{1,1,0}} + m_{(4,2,1,1)_{-1,1,0}} &= \chi(\mathcal{M}, L_1 \otimes L_2) + \chi(\mathcal{M}, L_1^{-1} \otimes L_2) = -3|\Gamma|, \\ m_{(\bar{4},1,2,1)_{0,-1,-1}} + m_{(\bar{4},1,2,1)_{0,-1,1}} &= \chi(\mathcal{M}, L_2^{-1} \otimes L_4^{-1}) + \chi(\mathcal{M}, L_2^{-1} \otimes L_4) = -3|\Gamma|, \end{aligned} \quad (3.4)$$

within $-3|\Gamma| \leq m_{(4,2,1,1)_{\pm 1,1,0}}, m_{(\bar{4},1,2,1)_{0,-1,\pm 1}} \leq 0$ and indices of other massless states in table 3 are integrally quantized on $\mathcal{M}$ and $\tilde{\mathcal{M}}$. Now, we have used $c_1(L_3) = 0$ and each index is calculated using the formula in appendix A. Note that we have not taken into account the generation of Higgs and Higgsino fields which could be identified with $L_{3,4}^{4,5}$ and/or their conjugates in the latter analysis. We further require that the exotic modes such as $(4, 1, 1, 16)_{0,-1,0}$, $(1, 2, 1, 16)_{1,0,0}$ and $(1, 1, 2, 16)_{0,0,1}$ in table 3 should not be chiral, namely

$$\chi(L_1) = 0, \quad \chi(L_2) = 0, \quad \chi(L_4) = -\chi(L_5) = 0. \quad (3.5)$$

When the heterotic five-branes present in the system, there exist chiral fermions under the fundamental representations of $\text{SO}(32) \times \text{Sp}(2N_s)$, namely $(32, 2N_s)$. Here, N_s denotes the number of heterotic five-branes which have the symplectic gauge degrees of freedom as confirmed in the S-dual type I superstring theory [40]. Since these chiral fermions have $(4, 1, 1, 16, 2N_s)_{0,-1,0}$, $(1, 2, 1, 16, 2N_s)_{1,0,0}$ and $(1, 1, 2, 16, 2N_s)_{0,0,1}$ representations of $\text{SU}(4)_C \times \text{SU}(2)_L \times \text{SU}(2)_R \times \prod_{c=1}^2 \text{U}(1)_c \times \text{U}(1)' \times \text{SO}(16) \times \text{Sp}(2N_s)$, the condition (3.5) also ensures the absence of these chiral fermions. Here, the subscript indices label the $\text{U}(1)_1$, $\text{U}(1)_2$ and $\text{U}(1)'$ charges.

To count the number of vector-like pairs for each representation, we have to calculate the dimensions of its corresponding cohomology $H^*(\mathcal{M}, \mathcal{C}_p)$ where $\mathcal{C}_p$ denotes the internal bundle. For our purpose, it is sufficient to consider the number of chiral modes to derive the particles in the standard-like models. Recall that the Higgs and higgsino fields will be identified with $L_{3,4}^{4,5}$ and/or their conjugates in the latter analysis. Although it is possible to break the Pati-Salam-like symmetry by the vector-like Higgs $(\bar{4}, 1, 2, 1) + \text{c.c.}$, in this paper, we concentrate on Wilson lines to break the Pati-Salam-like symmetry, namely $\text{SU}(4)_C \rightarrow \text{SU}(3)_C \times \text{U}(1)_3$ and $\text{SU}(2)_R \times \text{U}(1)' \rightarrow \text{U}(1)_4 \times \text{U}(1)_5$. It can be achieved by the introduction of discrete Wilson lines on the quotient CICYs $\tilde{\mathcal{M}} = \mathcal{M}/\Gamma$, where Γ should be different from $\mathbb{Z}_4$ and $\mathbb{Z}_2$. However, in such a scenario, $\text{U}(1)_{B-L}$ gauge symmetry is still unbroken on the quotient CICYs. One of the possibilities to break $\text{U}(1)_{B-L}$ gauge symmetry is the non-zero vacuum expectation value of at least one of the right-handed sneutrino which carries the $\text{U}(1)_{B-L}$ charge not the $\text{U}(1)_Y$ charge. When the soft mass of the right-handed sneutrino is tachyonic at the TeV scale through renormalization group effects, the sneutrino develops a vacuum expectation value. Such a spontaneous $\text{U}(1)_{B-L}$ breaking is demonstrated in the B-L MSSM derived from the $E_8 \times E_8$ heterotic string theory [41, 42]. This $\text{U}(1)_{B-L}$ breaking scenario requires a deeper understanding of the mechanism of supersymmetry breaking and the stabilization of moduli fields. It will be the subject of future work.

3.2 Explicit three-generation models

Let us search for the models passing all the requirements on the quotient CICYs within $2 \leq h^{1,1} \leq 5$. For the 6 CICYs with $h^{1,1}(\mathcal{M}) = 2$ and 12 CICYs with $h^{1,1}(\mathcal{M}) = 3$, there are totally four quotient CICYs where one of the discrete symmetry is different from $\mathbb{Z}_2$ and $\mathbb{Z}_4$ [39]. They are characterized by the configuration matrices listed in appendix B. However, one cannot find realistic models within the range $|m_a^i| \leq 8$.

In a similar way, we next search for the models on $\mathcal{M}$ with $h^{1,1}(\mathcal{M}) = 4$. Although there exist two possible CICYs different from $\mathbb{Z}_2$ and $\mathbb{Z}_4$ for the 19 CICYs with $h^{1,1}(\mathcal{M}) = 4$, the following CICY,

$$\begin{array}{c} \mathbb{P}^2 \\ \mathbb{P}^2 \\ \mathbb{P}^2 \\ \mathbb{P}^2 \end{array} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{array}{c} 4,40 \\ \\ \\ -72 \end{array}, \quad (3.6)$$

leads to realistic models. It admits the $\mathbb{Z}_3$ and $\mathbb{Z}_3 \times \mathbb{Z}_3$ freely-acting discrete symmetries. The nonvanishing intersection numbers and second Chern number of the tangent bundle

$(\text{SU}(4)_C \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SO}(16))_{\text{U}(1)_1, \text{U}(1)_2, \text{U}(1)_4, \text{U}(1)_5}$	$(\text{SU}(3)_C \times \text{SU}(2)_L \times \text{SO}(16))_{\text{U}(1)_1, \text{U}(1)_2, \text{U}(1)_3, \text{U}(1)_4, \text{U}(1)_5}$	Matter	Index
$(4, 2, 1, 1)_{1,1,0,0}$	$(3, 2, 1)_{1,1,1,0,0}$ $(1, 2, 1)_{1,1,-3,0,0}$	Q_1 L_1	$\chi(\mathcal{M}, L_1 \otimes L_2)/ \Gamma $
$(4, 2, 1, 1)_{-1,1,0,0}$	$(3, 2, 1)_{-1,1,1,0,0}$ $(1, 2, 1)_{-1,1,-3,0,0}$	Q_2 L_2	$\chi(\mathcal{M}, L_1^{-1} \otimes L_2)/ \Gamma $
$(15, 1, 1, 1)_{0,0,0,0}$	$(\bar{3}, 1, 1)_{0,0,-4,0,0}$	$u_{R_1}^c$	$\chi(\mathcal{M}, \mathcal{O}_{\mathcal{M}})/ \Gamma $
$(6, 1, 1, 1)_{0,2,0,0}$	$(\bar{3}, 1, 1)_{0,2,2,0,0}$ $(3, 1, 1)_{0,2,-2,0,0}$	$d_{R_1}^c$ $\bar{d}_{R_2}^c$	$\chi(\mathcal{M}, L_2^2)/ \Gamma $
$(1, 1, 1, 1)_{2,0,0,0}$	$(1, 1, 1)_{2,0,0,0,0}$	n_1	$\chi(\mathcal{M}, L_1^2)/ \Gamma $
$(\bar{4}, 1, 2, 1)_{0,-1,-1,0}$	$(\bar{3}, 1, 1)_{0,-1,-1,-1,0}$ $(1, 1, 1)_{0,-1,3,0,1}$ $(1, 1, 1)_{0,-1,3,-1,0}$ $(\bar{3}, 1, 1)_{0,-1,-1,0,1}$	$u_{R_2}^{c4}$ $e_{R_1}^{c5}$ n_2^{c4} $d_{R_3}^{c5}$	$\chi(\mathcal{M}, L_2^{-1} \otimes L_4^{-1})/ \Gamma $
$(\bar{4}, 1, 2, 1)_{0,-1,1,0}$	$(\bar{3}, 1, 1)_{0,-1,-1,1,0}$ $(1, 1, 1)_{0,-1,3,1,0}$ $(1, 1, 1)_{0,-1,3,0,-1}$ $(\bar{3}, 1, 1)_{0,-1,-1,0,-1}$	$d_{R_3}^{c4}$ $e_{R_1}^{c4}$ n_2^{c5} $u_{R_2}^{c5}$	$\chi(\mathcal{M}, L_2^{-1} \otimes L_4)/ \Gamma $
$(1, 2, 2, 1)_{1,0,-1,0}$	$(1, 2, 1)_{1,0,0,-1,0}$ $(1, 2, 1)_{1,0,0,0,1}$	L_3^4 $\bar{L}_4^5$	$\chi(\mathcal{M}, L_1 \otimes L_4^{-1})/ \Gamma $
$(1, 2, 2, 1)_{1,0,1,0}$	$(1, 2, 1)_{1,0,0,0,-1}$ $(1, 2, 1)_{1,0,0,1,0}$	L_3^5 $\bar{L}_4^4$	$\chi(\mathcal{M}, L_1 \otimes L_4)/ \Gamma $
$(1, 1, 3, 1)_{0,0,0,0}$ $(1, 1, 1, 1)_{0,0,2,0}$	$(1, 1, 1)_{0,0,0,1,1}$ $(1, 1, 1)_{0,0,0,1,-1}$	$e_{R_2}^{c45}$ n_3^{c45}	$\chi(\mathcal{M}, \mathcal{O}_{\mathcal{M}})/ \Gamma $ $\chi(\mathcal{M}, L_4^2)/ \Gamma $
$(\bar{4}, 1, 1, 16)_{0,-1,0,0}$	$(\bar{3}, 1, 16)_{0,-1,-1,0,0}$ $(1, 1, 16)_{0,-1,3,0}$	— —	$\chi(\mathcal{M}, L_2^{-1})/ \Gamma $
$(1, 2, 1, 16)_{1,0,0,0}$	$(1, 2, 16)_{1,0,0,0,0}$	—	$\chi(\mathcal{M}, L_1)/ \Gamma $
$(1, 1, 2, 16)_{0,0,1,0}$	$(1, 1, 16)_{0,0,0,1,0}$ $(1, 1, 16)_{0,0,0,0,-1}$	— —	$\chi(\mathcal{M}, L_4)/ \Gamma $
$(1, 1, 1, 120)_{0,0,0,0}$	$(1, 1, 120)_{0,0,0,0,0}$	—	$\chi(\mathcal{M}, \mathcal{O}_{\mathcal{M}})/ \Gamma $

Table 3. Massless spectrum and corresponding index. In the first and second columns, the entries refer to the representations under $(\text{SU}(4)_C \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{SO}(16))_{\text{U}(1)_1, \text{U}(1)_2, \text{U}(1)_4, \text{U}(1)_5}$ and $(\text{SU}(3)_C \times \text{SU}(2)_L \times \text{SO}(16))_{\text{U}(1)_1, \text{U}(1)_2, \text{U}(1)_3, \text{U}(1)_4, \text{U}(1)_5}$ and the subscript indices label the U(1) charges. Note that in the first column, two $\text{U}(1)_{4,5}$ charge vectors $\mathbf{q} = (q_4, q_5)$ and $\mathbf{q}' = (q'_4, q'_5)$ are identified if $\mathbf{q} - \mathbf{q}' \in \mathbb{Z} \times (1, 1)$ because of $L_5 \simeq L_4^{-1}$.

expressed in terms of a basis $\tilde{w}^i$ on $\mathcal{M}$ are given by

$$\begin{aligned} d_{123} &= 6, & d_{124} &= d_{134} = d_{234} = 5, & d_{112} &= d_{113} = d_{122} = d_{133} = d_{223} = d_{233} = 3, \\ d_{114} &= d_{144} = d_{224} = d_{244} = d_{334} = d_{344} = 2, \end{aligned} \quad (3.7)$$

and

$$c_2(T\mathcal{M}) = (36, 36, 36, 36), \quad (3.8)$$

respectively. Since one cannot find realistic models satisfying all the consistency conditions within the range $|m_a^i| \leq 4$ for the $\mathbb{Z}_3 \times \mathbb{Z}_3$ case, we focus on the $\mathbb{Z}_3$ case where the number of complex structure moduli as well as the Euler number on the quotient CY manifold $\tilde{\mathcal{M}} = \mathcal{M}/\mathbb{Z}_3$ reduce to be $h^{2,1} = 16$ and $\chi = -24$.

When we search for the line bundles within the range $|m_a^i| \leq 4$ to realize eqs. (3.4) and (3.5), namely the three generations of quarks and leptons and no chiral exotics on the quotient CICY $\tilde{\mathcal{M}}$, it turns out that the lists of the first Chern numbers in tables 4 and 5 yield realistic models. In both cases with tables 4 and 5, the rank of the mass matrix of U(1)s in eq. (2.29) is 3 and the remaining two U(1) gauge symmetries are U(1)_Y and U(1)_{B-L} = U(1)₃/3. The distinction between the spectrum of tables 4 and 5 are the existence of Higgs fields which could be identified with $L_{3,4}^{4,5}$ and/or their conjugates. The indices of Higgs fields $L_{3,4}^{4,5}$ vanish for the choice of line bundles in table 4, whereas Higgs fields appear in the case of table 5. As commented before, the exotic modes such as d_{R_1} and $d_{R_2}^c$ are the vector-like particles with respect to the standard model gauge group with an extra U(1)_{B-L} = U(1)₃/3 on the quotient CICY. Indeed, the net-number of chiral modes is the same because of $\chi(L_2^2) = -\chi(L_2^{-2})$. We expect that such modes could become massive due to the loop-effects for the anomalous U(1)₂ symmetry. Since the obtained spectrum illustrated in table 6 consists of the three generations of quarks and leptons, vector-like Higgs and extra vector-like particles with respect to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, this models are free of gauge and gravitational anomalies. For other examples in table 5, the particle spectrum is similar to that in table 6, but the flavor structure of Yukawa couplings could be different from one another. Indeed, for some of the line bundles listed in table 5, the number of chiral generation is determined by only $Q_1(L_1)$ or $Q_2(L_2)$ in contrast to the case of table 6.

Let us take a closer look at the possible Yukawa couplings among the standard model particles with an emphasis on the case of table 6. In this model, we find that the gauge symmetries of the low-energy effective action allow for the following Yukawa couplings of quarks and leptons,

$$\begin{aligned} (Q_1, \bar{L}_3^5, u_{R_2}^{c\ 5}), & \quad (Q_2, \bar{L}_4^5, u_{R_2}^{c\ 5}), & (Q_1, L_4^4, d_{R_3}^{c\ 4}), & \quad (Q_2, L_3^4, d_{R_3}^{c\ 4}), \\ (L_1, L_4^4, e_{R_1}^{c\ 4}), & \quad (L_2, L_3^4, e_{R_1}^{c\ 4}), & (L_1, \bar{L}_3^5, n_2^{c\ 5}), & \quad (L_2, \bar{L}_4^5, n_2^{c\ 5}), \end{aligned} \quad (3.9)$$

by identifying the up-and down-type Higgs doublets as $\bar{L}_{3,4}^5$ and $L_{3,4}^4$. Here, $\bar{L}_4^5$ stands for the conjugate representation of L_4^5 and those Higgs fields are vector-like particles under the standard model gauge group with an extra U(1)_{B-L}. Three right-handed neutrinos are identified with n_2^5 . Although those couplings are allowed by the gauge symmetries, there

$(m_1^1, m_1^2, m_1^3, m_1^4)$	$(m_2^1, m_2^2, m_2^3, m_2^4)$	$(m_4^1, m_4^2, m_4^3, m_4^4)$	# of Five-branes
$(\mp 2, 0, \pm 1, 0)$	$(0, 1, -2, 0)$	$(\mp 1, \pm 1, 0, 0)$	$(6, 24, 12, 0)$

Table 4. Number of heterotic five-branes in the basis (w_1, w_2, w_3, w_4) and the first Chern numbers $c_1^i(L_a) = m_a^i$ in agreement with all the requirements, where other line bundles are constrained as $c_1(L_5) = -c_1(L_4)$ and $c_1(L_3) = 0$. We omit the other sign flipping and possible permutations among the m_a^i . The list of this table gives rise to the vanishing indices of Higgs fields $L_{3,4}^{4,5}$.

$(m_1^1, m_1^2, m_1^3, m_1^4)$	$(m_2^1, m_2^2, m_2^3, m_2^4)$	$(m_4^1, m_4^2, m_4^3, m_4^4)$	# of Five-branes
$(\mp 3, 0, 0, \pm 3)$	$(1, 0, -1, 2)$	$(0, \mp 1, \pm 1, \pm 4)$	$(30, 48, 30, 0)$
$(\mp 1, 0, \pm 1, \mp 3)$	$(0, 1, -1, -2)$	$(\mp 1, \pm 1, 0, \pm 1)$	$(42, 75, 51, 12)$
$(\mp 1, 0, \pm 1, \mp 3)$	$(1, -1, 0, -1)$	$(0, \mp 1, \pm 1, \mp 1)$	$(39, 27, 66, 12)$
$(\mp 1, 0, \pm 1, \mp 2)$	$(0, 1, -1, -1)$	$(\mp 1, \pm 1, 0, 0)$	$(18, 45, 33, 12)$

Table 5. Number of heterotic five-branes in the basis (w_1, w_2, w_3, w_4) and the first Chern numbers $c_1^i(L_a) = m_a^i$ in agreement with all the requirements, where other line bundles are constrained as $c_1(L_5) = -c_1(L_4)$ and $c_1(L_3) = 0$. We omit the other sign flipping and possible permutations among the m_a^i . One of the chiral spectrum is illustrated in table 6.

exist topological selection rules for their Yukawa couplings as discussed in refs. [43–45]. We leave the detailed study of the Yukawa couplings for a future work. With the help of $U(1)_{B-L}$ and other anomalous $U(1)$ symmetries, R-parity violating and the dimension-four and -five proton decay operators such as $QQQL$ and $u^c u^c d^c e^c$ are forbidden. As stated already, we will also postpone the detailed study of $U(1)_{B-L}$ breaking for a future analysis.

For the 23 CICYs with $h^{1,1} = 5$, we have searched for the three-generation models and among them, there are four possible candidates where one of the discrete symmetry group is different from $\mathbb{Z}_2$ and $\mathbb{Z}_4$ [39]. However, it turns out that the line bundle background does not pass all the requirements within the range $|m_a^i| \leq 4$.

4 Conclusion

We have searched for the three-generation standard-like models from the $SO(32)$ heterotic string theory on quotient complete intersection Calabi-Yau manifolds with line bundles. The line bundles with $U(1)$ structure groups, in contrast to non-abelian bundles, are promising tool to realize standard-like models not only in the $E_8 \times E_8$ heterotic string, but also the $SO(32)$ one. Such stable line bundles lead to the three chiral generations of quarks and leptons.

In this paper, we have presented that the Wilson lines and line bundles satisfying the supersymmetric conditions, the $U(1)_Y$ masslessness conditions and K-theory condition yield the standard-like model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ via a Pati-Salam-like symmetry. Other $U(1)$ gauge bosons become massive through the Green-Schwarz mechanism. The obtained spectrum contains the three generations of quarks and leptons, vector-like Higgs and extra vector-like particles with respect to $SU(3)_C \times SU(2)_L \times$

Matter	Repr.	Index	Multiplicities	Total
Q_1	$(3, 2, 1)_{1,1,1,0,0}$	$\chi(\mathcal{M}, L_1 \otimes L_2)/ \Gamma $	-1	-3
Q_2	$(3, 2, 1)_{-1,1,1,0,0}$	$\chi(\mathcal{M}, L_1^{-1} \otimes L_2)/ \Gamma $	-2	
L_1	$(1, 2, 1)_{1,1,-3,0,0}$	$\chi(\mathcal{M}, L_1 \otimes L_2)/ \Gamma $	-1	-3
L_2	$(1, 2, 1)_{-1,1,-3,0,0}$	$\chi(\mathcal{M}, L_1^{-1} \otimes L_2)/ \Gamma $	-2	
L_3^4	$(1, 2, 1)_{1,0,0,-1,0}$	$\chi(\mathcal{M}, L_1 \otimes L_4^{-1})/ \Gamma $	-4	
L_3^5	$(1, 2, 1)_{1,0,0,0,-1}$	$\chi(\mathcal{M}, L_1 \otimes L_4)/ \Gamma $	-28	
L_4^4	$(1, 2, 1)_{-1,0,0,-1,0}$	$\chi(\mathcal{M}, L_1^{-1} \otimes L_4^{-1})/ \Gamma $	28	
L_4^5	$(1, 2, 1)_{-1,0,0,0,-1}$	$\chi(\mathcal{M}, L_1^{-1} \otimes L_4)/ \Gamma $	4	
$u_{R_1}^c$	$(\bar{3}, 1, 1)_{0,0,-4,0,0}$	$\chi(\mathcal{M}, \mathcal{O}_{\mathcal{M}})/ \Gamma $	0	-3
$u_{R_2}^{c\ 4}$	$(\bar{3}, 1, 1)_{0,-1,-1,-1,0}$	$\chi(\mathcal{M}, L_2^{-1} \otimes L_4^{-1})/ \Gamma $	0	
$u_{R_2}^{c\ 5}$	$(\bar{3}, 1, 1)_{0,-1,-1,0,-1}$	$\chi(\mathcal{M}, L_2^{-1} \otimes L_4)/ \Gamma $	-3	
$d_{R_1}^c$	$(\bar{3}, 1, 1)_{0,2,2,0,0}$	$\chi(\mathcal{M}, L_2^2)/ \Gamma $	-12	-3
$d_{R_2}^c$	$(\bar{3}, 1, 1)_{0,-2,2,0,0}$	$\chi(\mathcal{M}, L_2^{-2})/ \Gamma $	12	
$d_{R_3}^{c\ 4}$	$(\bar{3}, 1, 1)_{0,-1,-1,1,0}$	$\chi(\mathcal{M}, L_2^{-1} \otimes L_4)/ \Gamma $	-3	
$d_{R_3}^{c\ 5}$	$(\bar{3}, 1, 1)_{0,-1,-1,0,1}$	$\chi(\mathcal{M}, L_2^{-1} \otimes L_4^{-1})/ \Gamma $	0	
$e_{R_1}^{c\ 4}$	$(1, 1, 1)_{0,-1,3,1,0}$	$\chi(\mathcal{M}, L_2^{-1} \otimes L_4)/ \Gamma $	-3	-3
$e_{R_1}^{c\ 5}$	$(1, 1, 1)_{0,-1,3,0,1}$	$\chi(\mathcal{M}, L_2^{-1} \otimes L_4^{-1})/ \Gamma $	0	
$e_{R_2}^{c\ 45}$	$(1, 1, 1)_{0,0,0,1,1}$	$\chi(\mathcal{M}, \mathcal{O}_{\mathcal{M}})/ \Gamma $	0	
n_1	$(1, 1, 1)_{2,0,0,0,0}$	$\chi(\mathcal{M}, L_1^2)/ \Gamma $	0	0
$n_2^{c\ 4}$	$(1, 1, 1)_{0,-1,3,-1,0}$	$\chi(\mathcal{M}, L_2^{-1} \otimes L_4^{-1})/ \Gamma $	0	0
$n_2^{c\ 5}$	$(1, 1, 1)_{0,-1,3,0,-1}$	$\chi(\mathcal{M}, L_2^{-1} \otimes L_4)/ \Gamma $	-3	-3
$n_3^{c\ 45}$	$(1, 1, 1)_{0,0,0,1,-1}$	$\chi(\mathcal{M}, L_4^2)/ \Gamma $	-24	-24
—	$(1, 2, 16)_{1,0,0,0,0}$	$\chi(\mathcal{M}, L_1)/ \Gamma $	0	0
—	$(\bar{3}, 1, 16)_{0,-1,-1,0,0}$	$\chi(\mathcal{M}, L_2^{-1})/ \Gamma $	0	0
—	$(1, 1, 16)_{0,-1,3,0,0}$	$\chi(\mathcal{M}, L_2^{-1})/ \Gamma $	0	0
—	$(1, 1, 16)_{0,0,0,1,0}$	$\chi(\mathcal{M}, L_4)/ \Gamma $	0	0
—	$(1, 1, 16)_{0,0,0,0,-1}$	$\chi(\mathcal{M}, L_4)/ \Gamma $	0	0
—	$(1, 1, 120)_{0,0,0,0,0}$	$\chi(\mathcal{M}, \mathcal{O}_{\mathcal{M}})/ \Gamma $	0	0

Table 6. Supersymmetric spectrum in visible and hidden sectors for the line bundles $(m_1^1, m_1^2, m_1^3, m_1^4) = (-3, 0, 0, 3)$, $(m_2^1, m_2^2, m_2^3, m_2^4) = (1, 0, -1, 2)$ and $(m_4^1, m_4^2, m_4^3, m_4^4) = (0, -1, 1, 4)$ in table 5. In the second column, the entries refer to the representations under $SU(3)_C$, $SU(2)_L$ and $SO(16)$ and the subscript indices label the $U(1)$ charges.

$U(1)_Y \times U(1)_{B-L}$. Although the Yukawa couplings are allowed by the gauge symmetries of the low-energy effective action, we leave the derivation of selection rules for the holomorphic Yukawa couplings on CICYs [44–46] for a future work.

Motivated by S- and T-dual models of intersecting D-branes in type IIA string theory, we have focused on the Pati-Salam-like gauge group to derive the standard model one. It would be interesting to check whether $SO(32)$ gauge group is directly broken to the standard model gauge group by a hypercharge flux, although there exists a no-go theorem on the existence of a standard model spectrum consistent with the hypercharge flux breaking and gauge coupling unification in the $E_8 \times E_8$ heterotic string [47]. The gauge couplings of non-abelian gauge groups embedded in the $SO(32)$ heterotic string are non-universal in contrast to those in the $E_8 \times E_8$ heterotic string [10, 11]. Such non-universal gauge couplings have the potential to explain the differences between the gauge couplings of $SU(3)_C$ and $SU(2)_L$ at the string scale as demonstrated in the toroidal background [48].

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A Net-number of chiral fields

In this appendix, we present the index formula to calculate the net-number of chiral fields in table 3,

$$\begin{aligned}\chi(L_a^p) &= \int_{\mathcal{M}} \left[\text{ch}_3(L_a^p) + \frac{c_2(TM)c_1(L_a^p)}{12} \right] = d_{ijk} \left[\frac{p^3}{6} m_a^i m_a^j m_a^k + \frac{p}{12} m_a^i c_2^{jk}(TM) \right], \\ \chi(L_a^p \otimes L_b^q) &= \chi(L_a^p) + \chi(L_b^q) + \int_{\mathcal{M}} \left[c_1(L_a^p) \text{ch}_2(L_b^q) + \text{ch}_2(L_a^p) c_1(L_b^q) \right] \\ &= \chi(L_a^p) + \chi(L_b^q) + \frac{1}{2} d_{ijk} \left[pq^2 m_a^i m_b^j m_b^k + p^2 q m_a^i m_a^j m_b^k \right],\end{aligned}\tag{A.1}$$

where $c_1^i(L_a) = m_a^i$ and p, q, r are the integers.

B List of CICYs to be applicable for the Pati-Salam models

To break the Pati-Salam-like symmetry by the inclusion of Wilson lines, the freely-acting discrete symmetry groups of CICYs should be different from $\mathbb{Z}_4$ and $\mathbb{Z}_2$. We list such topologically distinguishable CICYs for $2 \leq h^{1,1} \leq 5$ in table 7 [39], where $c_2(TM)$ denotes the second Chern number in the basis $\hat{w}^i$.

$(h^{1,1}, h^{2,1})$	Configuration matrix	$c_2(T\mathcal{M})$	Γ
(2, 52)	$\mathbb{P}^4 \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$	(24,24)	Z_5
(2, 56)	$\mathbb{P}^2 \begin{bmatrix} 0 & 1 & 1 & 1 \\ 3 & 1 & 1 & 1 \end{bmatrix}$	(24,24,24,24,24)	$Z_3 \times Z_3$
(2, 83)	$\mathbb{P}^2 \begin{bmatrix} 3 \\ 3 \end{bmatrix}$	(24,24,24,24,24)	$Z_3 \times Z_3$
(3, 39)	$\mathbb{P}^2 \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$	(36, 36, 54)	$Z_3, Z_3 \times Z_3$
(3, 48)	$\mathbb{P}^2 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$	(36, 36, 36)	$Z_3, Z_3 \times Z_3$
(4, 68)	$\mathbb{P}^1 \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}$	(24,24,24,24)	Z_8^a
(4, 40)	$\mathbb{P}^2 \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$	(36, 36, 36, 36)	$Z_3, Z_3 \times Z_3$
(5, 32)	$\mathbb{P}^2 \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$	(36,36,36,36,36)	Z_3
(5, 37)	$\mathbb{P}^1 \begin{bmatrix} 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 2 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$	(24,24,24,24,64)	Z_8
(5, 45)	$\mathbb{P}^1 \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$	(24,24,24,24,24)	Z_5, Z_{10}

Table 7. List of favorable CICYs to be applicable for the Pati-Salam models.

^aAlthough this tetra-quadric has other discrete symmetries $Z_4, Z_2 \times Z_2, Z_4 \times Z_2, Q_8, Z_4 \times Z_4, Z_4 \rtimes Z_4, Z_8 \times Z_2, Z_8 \rtimes Z_2, Z_2 \times Q_8$, we focus on the Z_8 case.

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