Published for SISSA by Springer

Received: April 13, 2017
Accepted: May 17, 2017
Published: May 29, 2017

More on microstate geometries of 4d black holes

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Abstract: We construct explicit examples of microstate geometries of four-dimensional black holes that lift to smooth horizon-free geometries in five dimensions. Solutions consist of half-BPS D-brane atoms distributed in $\mathbb{R}^3$. Charges and positions of the D-brane centers are constrained by the bubble equations and boundary conditions ensuring the regularity of the metric and the match with the black hole geometry. In the case of three centers, we find that the moduli spaces of solutions includes disjoint one-dimensional components of (generically) finite volume.

Keywords: Black Holes in String Theory, D-branes

ArXiv ePrint: 1701.05520

doi:10.1007/JHEP05(2017)147

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1 Introduction

Black holes are classical solutions of Einstein's equations with curvature singularities hidden behind event horizons. According to the "no-hair theorem", the solutions are unique in four dimensions, once the mass, charge and angular momentum are specified. At the classical level black holes are absolutely black and have zero statistical entropy $S = \log(1) = 0$. In a quantum theory however a black hole radiates as a black body with a finite temperature and an entropy given by one quarter of the area of its event horizon. To explain the microscopic origin of this entropy remains a primary task for any serious contender to a quantum theory of gravity.

In string theory, black holes can be realised in terms of D-branes intersecting in the internal space. The micro-states can be represented (and counted) in terms of excitations of the open strings connecting the building brane bits. Alternatively, one may think of the geometry generated by the excited brane state as the gravity representation of the micro-state. Since the micro-state geometry describes a pure state with zero entropy, it should have no horizon. This line of ideas motivates the "fuzzball" proposal that associates to every black hole micro-state a regular and horizon-free solution of classical gravity. The solutions, known as "fuzzballs" or "micro-state geometries", share with the would-be black hole the mass, charges and angular momentum but differ from it in the interior [1–9]. The black hole horizon and its entropy arise from a coarse graining superposition of the micro-state geometries.

In the last years, a large class of four and five dimensional black hole micro-state geometries have been produced [10–20]. The micro-state geometries are typically coded in smooth, horizon-less geometries with no closed time-like curves (CTC's) in five or six dimensions. This is the best one can achieve, since no-go theorems in four dimensions exclude the existence of non-singular asymptotically flat soliton solutions.[1] This is not the case in five or higher dimensions where the existence of Chern-Simons interactions and spatial sections with non-trivial topologies circumvent the no-go result [23]. From a four-dimensional perspective, the finiteness of the higher dimensional Riemann tensor (and its derivatives) results into a finite effective action with curvature divergences compensated by the singular behaviour of the scalars and gauge fields.

A black hole with finite area in four dimensions can be realised in several different frames. Popular choices include bound states of D1-D5-KK-p, of D0-D2-D4-D6 branes [24–30] or of intersecting D3-branes, wrapping three cycles in $T^2 \times T^2 \times T^2$ [31–33]. After reduction down to four dimensions, the solution can be viewed as a supersymmetric vacuum of an $\mathcal{N} = 2$ truncation of $\mathcal{N} = 8$ supergravity involving the gravity multiplet and three vector multiplets characterizing the complex structures of the internal torus. In a microscopic world-sheet description of the D3-brane system [32, 33], the harmonic functions characterising the gravity solution are sourced by disk diagrams with a boundary ending on a single, two or four different branes [32, 34, 35]. Higher multipole modes are generated by extra insertions of untwisted open string fields on the disk boundaries. The micro-state multiplicities can be computed by counting open strings in the D1-D5-p-KK system or vacua in the quantum mechanics associated to the D0-D2-D4-D6 realisation of the black hole [36, 37].

The aim of this paper is to construct explicit examples of micro-state geometries of four- dimensional BPS black holes. We follow the Bena-Warner ansatz [11, 23, 26] and look for regular five-dimensional geometries generated by distributions of half-BPS D-

[1] Regular solutions with AdS asymptotics have been recently found in [21, 22].

brane atoms in $\mathbb{R}^3$. The regularity of the five dimensional geometry is coded in the so called bubble equations that we generalise to account for the case of branes at angles. Boundary conditions at infinity further restrict the choices leading generically to a moduli space that consists of disjoint components. There are two classes of solutions. Scaling solutions are configurations that can be rigidly scaled (see [25, 38–40] for some results). The other class includes solutions where the distances between the centers are bounded by the charges.

We will consider micro-state geometries with both zero and non-zero angular momentum. The existence of supersymmetric solutions with non-zero angular momentum may look surprising, since single-center BPS black holes in four dimensions cannot carry angular momentum, because rotations of a black hole horizon are not compatible with supersymmetry. In our case, like for the multi center BPS black holes with non zero angular momentum considered in [41], the angular momentum is generated by the cross product of electric and magnetic fields of charges separated in $\mathbb{R}^3$. In the spirit of the fuzzball proposal, micro-states with non-zero angular momentum can be viewed as members of a canonical ensemble description of the black hole. The statistical average exposes zero angular momentum, even though each micro-state can carry some.[2]

The plan of the paper is as follows. In section 2 we present the BPS solutions describing systems of intersecting D3-branes on T^6 from the four dimensional perspective. We consider both cases of orthogonal and of intersecting D3-branes at angles. The Bena-Warner ansatz is introduced in section 3. The ansatz is generalised to accomodate for non-orthogonally intersecting D3-branes. In section 4 and 5 solutions to the bubble equations and to the boundary conditions are found for the 3-center case with orthogonal or D3-branes intersecting at angles. A preliminary discussion of the counting of the number of micro-states with fixed charges is presented in the Conclusions. In appendix A we derive the four dimensional solution from dimensional reduction of the intersecting D3-branes solution in ten dimensional type IIB supergravity.

2 Black holes from intersecting D3-branes

In this paper we consider a family of BPS solutions describing systems of intersecting D3-branes on T^6 from the four dimensional perspective. We refer the reader to the appendix for details on the ten dimensional solution and its reduction down to four dimensions. The four dimensional solution has been first explicitly derived in [26] in the D0-D2-D4-D6 type IIA frame and lifted to a system of M5 branes in M-theory. Here, for convenience, we consider the very symmetric formulation in terms of D3-branes, but the results can be easily translated into the type IIA and M-theory frame.

The four-dimensional geometries can be viewed as solutions of an $\mathcal{N} = 2$ truncation of $\mathcal{N} = 8$ supergravity involving the gravity multiplet and three vector multiplets. The scalars U_I in the vector multiplets, usually referred as STU, parametrise the complex structures of the three internal T^2's and span the moduli space $\mathcal{M}_{STU} = [\mathrm{SL}(2,R)/\mathrm{U}(1)]^3 \subset$

[2] We thank Iosif Bena for clarifications on this point.

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$E_{7(+7)}/\mathrm{SU}(8) = \mathcal{M}_{\mathcal{N}=8}$. Setting $16\pi G = 1$, the lagrangian can be written as

$$\mathcal{L} = \sqrt{g_4}\left(R_4 - \sum_{I=1}^{3}\frac{\partial_\mu U_I \partial^\mu \bar{U}_I}{2\left(\operatorname{Im} U_I\right)^2} - \frac{1}{4}F_a\mathcal{I}^{ab}F_b - \frac{1}{4}F_a\mathcal{R}^{ab}\widetilde{F}_b\right) \tag{2.1}$$

where $F_a = dA_a$, $\widetilde{F}_a = *_4 F_a$ with $a = 0, 1, 2, 3$, including the graviphoton, and

$$U_I = (\sigma + is, \tau + it, \nu + iu) \tag{2.2}$$

are the three complex scalars in the vector multiplets. In these variables the gauge kinetic functions read

$$\mathcal{I}^{ab} = stu\begin{pmatrix} 1+\frac{\sigma^2}{s^2}+\frac{\tau^2}{t^2}+\frac{\nu^2}{u^2} & -\frac{\sigma}{s^2} & -\frac{\tau}{t^2} & -\frac{\nu}{u^2} \\ -\frac{\sigma}{s^2} & \frac{1}{s^2} & 0 & 0 \\ -\frac{\tau}{t^2} & 0 & \frac{1}{t^2} & 0 \\ -\frac{\nu}{u^2} & 0 & 0 & \frac{1}{u^2} \end{pmatrix} \qquad \mathcal{R}^{ab} = \begin{pmatrix} 2\sigma\tau\nu & -\tau\nu & -\nu\sigma & -\sigma\tau \\ -\tau\nu & 0 & \nu & \tau \\ -\nu\sigma & \nu & 0 & \sigma \\ -\sigma\tau & \tau & \sigma & 0 \end{pmatrix} \tag{2.3}$$

The solutions will be written in terms of eight harmonic functions

$$\{V, L_I, K^I, M\} \tag{2.4}$$

on $\mathbb{R}^3$. It is convenient to introduce the combinations

$$\begin{aligned} Z_I &= L_I + \frac{|\varepsilon_{IJK}|}{2}\frac{K^J K^K}{V}, \\ \mu &= \frac{M}{2} + \frac{L_I K^I}{2V} + \frac{|\varepsilon_{IJK}|}{6}\frac{K^I K^J K^K}{V^2}. \end{aligned} \tag{2.5}$$

Here ϵ_{IJK} characterise the triple intersections among the three T_I^2 2-cycles in T^6.

The solutions can then be written as

$$\begin{aligned} ds^2 &= -e^{2U}\left(dt + w\right)^2 + e^{-2U} d|\mathbf{x}|^2 \\ A_a &= (A_0, A_I) = w_a + a_a\left(dt + w\right) \\ U_I &= -b^I + i\left(V e^{2U} Z_I\right)^{-1} \end{aligned} \tag{2.6}$$

with

$$\begin{aligned} b^I &= \frac{K^I}{V} - \frac{\mu}{Z_I} \qquad a_0 = -\mu V^2 e^{4U} \qquad a_I = V e^{4U}\left(-\frac{Z_1 Z_2 Z_3}{Z_I} + K^I \mu\right) \\ *_3 dw_a &= -dK_a \qquad K_a = \left(-V, K^I\right) \qquad *_3 dw = \frac{1}{2}\left(V dM - M dV + K^I dL_I - L_I dK^I\right) \end{aligned} \tag{2.7}$$

and

$$\begin{aligned} e^{-4U} &= \mathcal{I}_4(V, L_I, K^I, M) \equiv Z_1 Z_2 Z_3 V - \mu^2 V^2 \\ &= L_1\, L_2\, L_3\, V - K_1\, K_2\, K_3\, M + \sum_{I>J}^{3}\frac{K^I K^J L_I L_J}{2} - \frac{MV}{2}\sum_{I=1}^{3} K^I L_I \\ &\quad - \frac{M^2 V^2}{4} - \sum_{I=1}^{3}\frac{(K^I)^2 L_I^2}{4} \end{aligned} \tag{2.8}$$

has the same structure as the quartic U-duality invariant $\mathcal{I}_4$.

2.1 The asymptotic geometry

For general choices of the eight harmonic functions, the solution (2.6) is singular. Both naked and 'horizon-dressed' curvature singularities can be present. The generic solution is characterised by a mass $\mathfrak{M}$, associated to the Killing vector $\xi^{(t)M}\partial_M = \partial_t$, four electric charges Q^a and four magnetic charges P_a. Introducing the symplectic vector

$$\mathcal{F} = \begin{pmatrix} F_a \\ \frac{\delta\mathcal{L}}{\delta F_a} \end{pmatrix} = \begin{pmatrix} F_a \\ \star_4 \mathcal{I}^{ab}\, F_b - \mathcal{R}^{ab}\, F_b \end{pmatrix} \tag{2.9}$$

one finds for the charges

$$\begin{aligned} \mathfrak{M} &= -\frac{1}{8\pi G}\int_{S^2_\infty} \star_4\, d\xi^{(t)} \\ \begin{pmatrix} P_a \\ Q^a \end{pmatrix} &= -\frac{1}{4\pi}\int_{S^2_\infty} \mathcal{F} \end{aligned} \tag{2.10}$$

with $\xi^{(t)} = \xi^{(t)}_M\, dx^M$ and S^2_∞ the two-sphere at infinity. Solutions with extra symmetries arise for special choices of the harmonic functions. Axially symmetric solutions are characterised by the existence of an additional Killing vector $\xi^{(t)M}\partial_M = \partial_\phi$ associated to rotations around an axis in $\mathbb{R}^3$, and carry an extra quantum number, the angular momentum J given by[3]

$$J = -\frac{1}{16\pi G}\int_{S^2_\infty} \star_4\, d\xi^{(\phi)} \tag{2.11}$$

with $\xi^{(\phi)} = \xi^{(\phi)}_M\, dx^M$. Spherical symmetric solutions are invariant under rotations around the origin and are characterized by zero angular momentum.

In this paper we consider fuzzballs of spherically symmetric black holes. The harmonic functions specifying the general spherically symmetric solution can be written in the single-center form

$$V = v_0 + \frac{v}{r} \qquad L_I = \ell_{0I} + \frac{\ell_I}{r} \qquad K^I = k_0^I + \frac{k^I}{r} \qquad M = m_0 + \frac{m}{r} \tag{2.12}$$

and describe a general system of intersecting D3-branes wrapping three cycles on $T^2\times T^2\times T^2$ with one leg on each of the three T^2. The absence of Dirac-Misner strings requires that w vanishes at infinity or, equivalently, that $*_3 dw \sim r^{-3}$ at infinity leading to the constraint

$$v_0\, m - m_0\, v + k_0^I\, \ell_I - \ell_{0I}\, k^I = 0 \tag{2.13}$$

For simplicity we take $m_0 = m = 0$. For this choice one finds

$$e^{-4U} = V\, L_1\, L_2\, L_3 - \frac{1}{4}\left(\sum_{I=1}^{3} K^I L_I\right)^2 \tag{2.14}$$

[3] Being a surface integral, the expression for J holds true even when $\xi^{(\phi)}$ is only an asymptotic Killing vector. J is the z component of the vector $\mathbf{J}$, defined in (2.19).

Poles and zeros of this function are associated to horizons and curvature singularities respectively. If $e^{-4U}>0$ for all $r>0$ the solution describes a black hole with near horizon geometry $AdS_2\times S^2$ and entropy proportional to

$$\lim_{r\to 0} r^4\, e^{-4U} = \mathcal{I}_4(v,\ell_I,k^I,m=0) = v\,\ell_1\,\ell_2\,\ell_3 - \frac{1}{4}\left(\sum_{I=1}^{3} k^I \ell_I\right)^2 > 0 \tag{2.15}$$

If e^{-4U} has zeros for some positive r, the solution exposes a naked singularity.

The charges of the solution (or its fuzzball) are computed by the integrals (2.17) evaluated in the asymptotic geometries (2.12). Writing the three-dimensional metric in spherical coordinates

$$ds^2 = -e^{2U}(dt+w)^2 + e^{-2U}\left(dr^2 + r^2\, d\theta^2 + r^2\, \sin^2\theta\, d\phi^2\right) \tag{2.16}$$

and setting $G=(16\pi)^{-1}$ one finds for the charges[4]

$$\begin{aligned} \mathfrak{M} &= 8\pi\, r^2\, \partial_r\, e^{2U} \\ \begin{pmatrix} P_a \\ Q^a \end{pmatrix} &= \begin{pmatrix} (v,-k^I)^T \\ -r^2\,\partial_r\left(\mathcal{I}^{ab}\, a_b + \mathcal{R}^{ab}\, K_b\right) \end{pmatrix} \end{aligned} \tag{2.17}$$

where we used the fact that at infinity $w=0$ and $F_a = dw_a + da_a\, dt$. On the other hand the angular momentum of the fuzzball is computed by the integral (2.11). We notice that the evaluation of this integral requires a more detailed knowledge of the asymptotic geometry since angular momentum arises from the first dipole mode in the expansion of the harmonic function. Indeed, denoting by

$$H = h_0 + \frac{h_1}{r} + \frac{\mathbf{h}_2\cdot\mathbf{x}}{r^3} \tag{2.18}$$

one finds for the angular momentum

$$\mathbf{J} = 4\pi\,\left[m_0\,\mathbf{v}_2 - v_0\,\mathbf{m}_2 + \ell_{0I}\,\mathbf{k}_2^I - k_0^I\,\boldsymbol{\ell}_{2I}\right] \tag{2.19}$$

We anticipate here that apart from the scaling solutions, all fuzzball solutions we will find here carry a non-trivial angular momentum. We observe that for orthogonal branes angular momentum is carried by K-components (see appendix A.3 for details) corresponding to open string condensates on disks with boundary on two different D3-brane stacks. Indeed, an explicit microscopic description of the general supergravity solution exists if the harmonic functions satisfy the boundary conditions [32]

$$m_2 + \sum k_2^I = 0 \tag{2.20}$$

As we will see, only scaling solutions in the list of examples we find satisfy this restriction on the dipole modes.

[4] In our conventions $\star_4\, dr\wedge dt = e^{-2U}\, r^2\, \sin\theta\, d\theta\wedge d\phi$ and $\int \sin\theta d\theta\wedge d\phi = 4\pi$.

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2.1.1 Orthogonal branes

We first consider the supergravity solution characterised by the harmonic functions

$$V = 1 + \frac{v}{r} \qquad L_I = 1 + \frac{\ell_I}{r} \qquad K^I = M = 0 \tag{2.21}$$

describing a system of four stacks of D3-branes intersecting orthogonally on T^6. At large distances one finds

$$\begin{aligned} e^{-2U} &= \sqrt{V L_1 L_2 L_3} = 1 + \frac{(v + \ell_1 + \ell_2 + \ell_3)}{2\,r} + \dots \\ a_I &= -L_I^{-1} = -1 + \frac{\ell_I}{r} + \dots \qquad a_0 = 0 \\ U_I &= i\left(V e^{2U} L_I\right)^{-1} = i + \dots \end{aligned} \tag{2.22}$$

leading to

$$\mathfrak{M} = 4\pi\,(v + \ell_1 + \ell_2 + \ell_3) \tag{2.23}$$

$$\begin{pmatrix} P_a \\ Q^a \end{pmatrix} = \begin{pmatrix} (v, 0, 0, 0)^T \\ (0, \ell_1, \ell_2, \ell_3)^T \end{pmatrix} \tag{2.24}$$

The extremal Reissner Nordstrom solution corresponds to the choice $\ell_I = v = Q/2$, or equivalently

$$L_I = V = 1 + \frac{\ell}{r} \qquad M = K^I = 0 \tag{2.25}$$

after the identification $F_{\rm RN} = \frac{1}{2}\left(*F_0 + \sum_{I=1}^{3} F_I\right)$.

2.1.2 Branes at angles

We next consider the supergravity solution characterised by the harmonic functions

$$V = 1 + \frac{v}{r} \qquad L_I = 1 + \frac{\ell_I}{r} \qquad K^1 = g + \frac{k^1}{r} \qquad K^2 = g \qquad K^3 = M = 0 \tag{2.26}$$

The absence of Dirac-Misner strings (2.13) requires $k^1 = g(\ell_1 + \ell_2)$. The resulting solution is equivalent after a duality transformation to the solution found in [31] describing a system of D3 branes intersecting at a non-trivial angle at infinity, parametrised by g.

The asymptotic solution at large r becomes

$$\begin{aligned} & e^{2U} = 1 - \frac{v + \ell_1 + \ell_2 + \ell_3}{2r} + \dots \qquad U_I = \left(\mathrm{i}, \mathrm{i}, \frac{1}{g - \mathrm{i}}\right) + \dots \\ & w_a = (v\,\cos\theta\, d\phi, -k_1\,\cos\theta\, d\phi, 0, 0) + \dots \\ & a_a = \left(\frac{g\,\ell_3}{r}, \frac{(1+g^2)\ell_1}{r}, \frac{\ell_2 - g^2\ell_1}{r}, \frac{\ell_3}{r}\right) + \dots \\ & \mathcal{I}^{ab} = \begin{pmatrix} 1 & 0 & 0 & -g \\ 0 & \frac{1}{1+g^2} & 0 & 0 \\ 0 & 0 & \frac{1}{1+g^2} & 0 \\ -g & 0 & 0 & 1+g^2 \end{pmatrix} \qquad \mathcal{R}^{ab} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{g}{1+g^2} & 0 \\ 0 & \frac{g}{1+g^2} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned} \tag{2.27}$$

with dots denoting higher order terms in the expansion for large r. Plugging (2.27) into (2.17) one finds for the charges[5]

$$\mathfrak{M} = 4\pi\,(v + \ell_1 + \ell_2 + \ell_3) \tag{2.29}$$

$$\begin{pmatrix} P_a \\ Q^a \end{pmatrix} = \begin{pmatrix} (v, -g\,(\ell_1 + \ell_2), 0, 0)^T \\ (0, \ell_1, \ell_2, \ell_3)^T \end{pmatrix} \tag{2.30}$$

Due to different conventions and duality frames, it is not easy to compare the charges with the corresponding ones in [31].

3 Microstate geometries

In this section we review the Bena-Warner multi-Taub NUT ansatz for fuzzball geometries of four- and five-dimensional black holes. We slightly generalise the ansatz to accomodate for non-orthogonal brane intersections and derive the corresponding bubble equations. In the next section we present explicit horizon-free solutions with three centers.

3.1 The eleven dimensional lift

The four-dimensional solution (2.6) lifts to an eleven dimensional solution representing a systems of intersecting M5-branes with four electric and four magnetic charges. The eleven dimensional metric is given by [11]:

$$ds^2 = ds_5^2 + ds_{T^6}^2 \tag{3.1}$$

where

$$\begin{aligned} ds_5^2 &= -(Z_1\,Z_2\,Z_3)^{-\frac{2}{3}}\left[dt + \mu(d\Psi + w_0) + w\right]^2 + (Z_1\,Z_2\,Z_3)^{\frac{1}{3}}\left[V^{-1}(d\Psi + w_0)^2 + V d|\mathbf{x}|^2\right] \\ ds_{T^6} &= \sum_{I=1}^{3}\left(\frac{Z_1\,Z_2\,Z_3}{Z_I^3}\right)^{\frac{1}{3}}(dy_I^2 + d\tilde{y}_I^2) \end{aligned} \tag{3.2}$$

in which $\{t,\, \mathbf{x},\, \Psi,\, y_I,\, \tilde{y}_I\}$ with $I = 1,2,3$ are the coordinates of $\mathbb{R} \times \mathbb{R}^3 \times S^1 \times T^6$, respectively. Micro-states of the four dimensional black holes can be generically defined as smooth geometries with no horizons or curvature singularities in eleven dimensions carrying the same mass and charges as the corresponding black hole. Regular solutions can be constructed in terms of multi-center harmonic functions (V, L_I, K^I, M) with the positions of the centers and the charges chosen such that Z_I are finite and $\mu = 0$ near the centers.

[5] For the central charges $Z_I = e^{-U}\,(2\mathrm{Im}U_I)^{-1}\,r^2\,\partial_r U_I$, $Z_4 = \mathrm{i}\,e^{-U}\,r^2\,\partial_r U$ one finds at infinity

$$\begin{aligned} Z_1 &= 4\pi\left[2\,g\,\ell_1 + \mathrm{i}(v + \ell_1 - \ell_2 - \ell_3)\right] \qquad & Z_2 &= 4\pi\left[-2\,g\,\ell_1 + \mathrm{i}(v - \ell_1 + \ell_2 - \ell_3)\right] \\ Z_3 &= 4\pi\mathrm{i}(v - \ell_1 - \ell_2 + \ell_3) & Z_4 &= 4\pi\mathrm{i}(v + \ell_1 + \ell_2 + \ell_3) \end{aligned} \tag{2.28}$$

showing the saturation of the BPS bound $\mathfrak{M} = |Z_4| \geq |Z_i|$ for $i \neq 4$, when v and ℓ_I all have the same sign and g is sufficiently small.

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Under these assumptions one finds that the eleven dimensional metric (3.1) near the centers is $\mathbb{R} \times T^6 \times \mathbb{R}^{\not{4}}/\mathbb{Z}_{|q_i|}$. To avoid orbifold singularities we will henceforth take $|q_i| = 1$. Moreover, the absence of horizons and closed time-like curves requires that

$$Z_I V > 0 \qquad \text{and} \qquad e^{2U} > 0 \tag{3.3}$$

Let us remark that the condition $Z_I V > 0$ near the centers requires

$$Z_I\, V|_{r_i=0} = q_i \left(\ell_{0I} + \sum_{j\neq i} \frac{\ell_{I,j}}{r_{ij}} \right) + \ell_{I,i} \left(v_0 + \sum_{j\neq i} \frac{q_j}{r_{ij}} \right) + |\varepsilon_{IJK}| k_i^J \left(k_0^K + \sum_{j\neq i} \frac{k_j^K}{r_{ij}} \right) > 0 \tag{3.4}$$

It turns out that these necessary conditions are often sufficient to ensure the positivity of both $Z_I V$ and e^{2U} on the whole $\mathbb{R}^3$. In the next section we look for explicit solutions of these requirements satisfying the boundary conditions (2.12). We stress that the resulting solutions are regular everywhere in five dimensions and fall off at infinity to $\mathbb{R}^{1,3} \times S^1$. The four-dimensional fuzzball solution follows from reduction of this five-dimensional geometry down to four dimensions where the apparent singularity in the geometry is balanced by a blow up of the scalar fields.

3.2 The bubble equations

We consider N-center harmonic functions following the Bena-Warner ansatz [11, 23, 26]

$$\begin{aligned} V &= v_0 + \sum_{i=1}^N \frac{q_i}{r_i} \quad , \qquad & L_I &= \ell_{0I} + \sum_{i=1}^N \frac{\ell_{I,i}}{r_i} \\ K^I &= k_0^I + \sum_{i=1}^N \frac{k_i^I}{r_i} \quad , \qquad & M &= m_0 + \sum_{i=1}^N \frac{m_i}{r_i} \end{aligned} \tag{3.5}$$

with $r_i = |\mathbf{x}_i - \mathbf{x}|$ and $\mathbf{x}_i$ the position of the i^{th} center. We notice that (ℓ_{Ii}, m_i) and (q_i, k_i^I) describe the electric and magnetic fluxes of the four dimensional gauge fields through the sphere encircling the i^{th}- centers, so Dirac quantisation requires that they be quantised. Here we adopt units such that they are all integers. Alternatively, one can think of the eight charges as parametrising the number of D3-branes wrapping one of the eight three-cycles with a leg on each of the three tori T_I^2 in the factorisation $T^6 = T_1^2 \times T_2^2 \times T_3^2$. In other words, in our units each charge describes the number of D3-branes of a certain kind.

We look for regular five dimensional geometries behaving as $\mathbb{R} \times$ Taub$-$NUT near the centers. It is easy to see that w vanishes near the centers, so the Taub-NUT geometry factorises if Z_I are finite and μ vanishes near the centers, i.e.

$$Z_I\big|_{r_i\approx 0} \approx \zeta_i^I \qquad , \qquad \mu\big|_{r_i\approx 0} \approx 0 \tag{3.6}$$

with ζ_i^I some finite constants. The conditions that Z_I is finite near the centers can be solved by taking

$$\ell_{I,i} = -\frac{|\epsilon_{IJK}|}{2} \frac{k_i^J\, k_i^K}{q_i} \qquad , \qquad m_i = \frac{k_i^1\, k_i^2\, k_i^3}{q_i^2} \tag{3.7}$$

The vanishing of μ near the centers boils down to the so called *bubble equations*

$$\sum_{j=1}^{N} \frac{\Pi_{ij}}{r_{ij}} + v_0\, \frac{k_i^1\, k_i^2\, k_i^3}{q_i^2} - \ell_{0I}\, k_i^I - |\epsilon_{IJK}|\, \frac{k_0^I\, k_i^J\, k_i^K}{2\, q_i} - m_0 q_i = 0 \tag{3.8}$$

where $r_{ij} = |\mathbf{x}_i - \mathbf{x}_j|$ and

$$\Pi_{ij} = q_i\, q_j \prod_{I=1}^{3} \left(\frac{k_i^I}{q_i} - \frac{k_j^I}{q_j} \right) \tag{3.9}$$

represents the symplectic form $P_i^a Q_{aj} - P_j^a Q_{ai}$ that counts the number of D3-brane intersections. Indeed the conditions (3.7) ensure that both Z_I and μ are finite near the centers while the bubble equations follow from the requirement that μ vanishes near the center. The bubble equations ensure also the absence of Dirac-Misner strings. To see this, we notice that using the bubble equations, the w function defined by (2.7) can be written in the form

$$\begin{aligned} *_3 dw &= \sum_{i,j=1}^{N} (q_{[i}\, m_{j]} + k_{[i}^I\, \ell_{j],I}) \frac{1}{r_i} d\frac{1}{r_j} + \frac{1}{2} \sum_{i=1}^{N} \left(v_0\, m_i - m_0\, q_i - \ell_{0I}\, k_i^I + k_0^I\, \ell_{i,I} \right) d\frac{1}{r_i} \\ &= \frac{1}{2} \sum_{i,j=1}^{N} \Pi_{ij} \left(\frac{1}{r_j} - \frac{1}{r_{ij}} \right) d\frac{1}{r_i} \end{aligned} \tag{3.10}$$

where in the second line we used equations (3.7), (3.8) and $A_{[BC]}$ means $\frac{1}{2}(A_{BC} - A_{CB})$. The solution can be written in the form

$$w = \frac{1}{4} \sum_{i,j=1}^{N} \Pi_{ij}\, \omega_{ij} \tag{3.11}$$

in terms of the one forms ω_{ij} defined via the relation

$$*_3\, d\omega_{ij} = \left(\frac{1}{r_j} - \frac{1}{r_{ij}} \right) d\frac{1}{r_i} - \left(\frac{1}{r_i} - \frac{1}{r_{ij}} \right) d\frac{1}{r_j}, \tag{3.12}$$

that is

$$\omega_{ij} = \frac{(\mathbf{n}_i + \mathbf{n}_{ij}) \cdot (\mathbf{n}_j - \mathbf{n}_{ij})}{r_{ij}} d\phi_{ij} \tag{3.13}$$

with

$$\mathbf{n}_i = \frac{\mathbf{x} - \mathbf{x}_i}{r_i} \qquad \mathbf{n}_{ij} = \frac{\mathbf{x}_i - \mathbf{x}_j}{r_{ij}} \qquad d\phi_{ij} = \frac{\mathbf{n}_{ij} \times \mathbf{n}_i \cdot d\mathbf{x}}{r_i\, [1 - (\mathbf{n}_{ij} \cdot \mathbf{n}_i)^2]}.$$

It is easy to see that ω_{ij} is free of Dirac-Misner singularities. Indeed along the dangerous lines connecting any two centers the numerator of (3.13) always vanish so no string-like singularity arises. One can also see that near the centers w goes to a constant exact form.

Finally we notice that if the coefficients k_i^I satisfy the relation

$$v_0\, m_i - m_0 q_i - \ell_{0I}\, k_i^I + k_0^I\, \ell_{Ii} = 0 \tag{3.14}$$

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the system of equations is invariant under overall rescalings of the center positions $\mathbf{x}_i \to \lambda \mathbf{x}_i$. These solutions are known as "scaling solutions". Multiplying equation (3.14) by the positions of the centers $\mathbf{x}_i$ and summing one finds that the scaling solutions satisfy

$$m_0 \, \mathbf{v}_2 - v_0 \, \mathbf{m}_2 + \ell_{0I} \, \mathbf{k}_2^I - k_0^I \, \boldsymbol{\ell}_{2I} = 0 \tag{3.15}$$

and therefore according to (2.19) they carry zero angular momentum.

4 Fuzzballs of orthogonally intersecting branes

We look for regular geometries with the asymptotics (2.21), i.e.

$$\ell_{0I} = v_0 = 1 \qquad m_0 = m = k_0^I = k^I = 0 \tag{4.1}$$

For concreteness we take $q_i = 1$. The charges of the fuzzball solutions are then

$$P_0 = N \qquad , \qquad Q_I = -\sum_{i=1}^{N} \frac{|\epsilon_{IJK}| k_i^J \, k_i^K}{2} \tag{4.2}$$

The solution is specified by the positions $\mathbf{x}_i$ of the centers and the fluxes k_i^I. The positions of the centers are constrained by the bubble equations

$$\sum_{j \neq i}^{N} \frac{\Pi_{ij}}{r_{ij}} + \Lambda_i - \Gamma_i = 0 \tag{4.3}$$

with

$$\Pi_{ij} = \prod_{I=1}^{3} (k_i^I - k_j^I) \qquad\qquad \Gamma_i = \sum_{I=1}^{3} k^I{}_i \qquad\qquad \Lambda_i = k^1{}_i k^2{}_i k^3{}_i \tag{4.4}$$

while the boundary conditions require

$$\sum_{i=1}^{N} k_i^I = \sum_{i=1}^{N} k_i^1 \, k_i^2 \, k_i^3 = 0 \tag{4.5}$$

In addition, the absence of horizons and of closed time-like curves requires

$$Z_I \, V > 0 \qquad \text{and} \qquad e^{2U} > 0 \tag{4.6}$$

Configurations with one or two centers fail to meet the requirement $Q_I > 0$. We will consider solutions with three centers. We refer the reader to [43] for earlier discussions of three center solutions.[6]

[6] We thank Iosif Bena for pointing out this reference to us.

4.1 Three centers

The bubble equations (4.3) for three centers can be solved in general by taking

$$r_{12} = \frac{\Pi_{12}\, r_{23}}{\Pi_{23} - r_{23}\,(\Gamma_2 - \Lambda_2)} \qquad\qquad r_{13} = \frac{\Pi_{13}\, r_{23}}{-\Pi_{23} + r_{23}\,(\Gamma_1 + \Gamma_2 - \Lambda_1 - \Lambda_2)} \tag{4.7}$$

A solution given by (4.7) makes sense if the distances r_{ij} between the three centers are positive and they satisfy the triangle inequalities. This restricts significantly the choices for the k^I_i. A quick scan over the integers shows that boundary conditions are solved only if at least one of the fluxes k^I_i vanishes. Without loss of generality the general solution can then be parametrised in the form (up to permutations of rows and columns)

$$k^I{}_i = \begin{pmatrix} -\kappa_1\,\kappa_2 & -\kappa_1\,\kappa_3 & \kappa_1\,(\kappa_2+\kappa_3) \\ \kappa_3 & \kappa_2 & -\kappa_2-\kappa_3 \\ -\kappa_4 & \kappa_4 & 0 \end{pmatrix} \tag{4.8}$$

Consequently the harmonic functions takes the general form

$$\begin{aligned}
V &= 1 + \sum_{i=1}^{3} \frac{1}{r_i} & M &= \kappa_1\kappa_2\kappa_3\kappa_4 \left(\frac{1}{r_1} - \frac{1}{r_2}\right) \\
L_1 &= 1 + \kappa_4 \left(\frac{\kappa_3}{r_1} - \frac{\kappa_2}{r_2}\right) & L_2 &= 1 + \kappa_1\kappa_4 \left(-\frac{\kappa_2}{r_1} + \frac{\kappa_3}{r_2}\right) \\
L_3 &= 1 + \kappa_1 \left(\frac{\kappa_2\kappa_3}{r_1} + \frac{\kappa_2\kappa_3}{r_2} + \frac{(\kappa_2+\kappa_3)^2}{r_3}\right) & K_1 &= \kappa_1 \left(-\frac{\kappa_2}{r_1} - \frac{\kappa_3}{r_2} + \frac{\kappa_2+\kappa_3}{r_3}\right) \\
K_2 &= \frac{\kappa_3}{r_1} + \frac{\kappa_2}{r_2} - \frac{\kappa_2+\kappa_3}{r_3} & K_3 &= \kappa_4 \left(-\frac{1}{r_1} + \frac{1}{r_2}\right)
\end{aligned}$$

The charges and distances between the centers reduce to

$$\begin{aligned}
Q_1 &= \kappa_4(\kappa_3-\kappa_2) \qquad Q_2 = \kappa_1\kappa_4(\kappa_3-\kappa_2) \qquad Q_3 = \kappa_1(\kappa_2^2 + 4\kappa_2\kappa_3 + \kappa_3^2) \\
r_{12} &= \frac{2\kappa_1\kappa_4(\kappa_2-\kappa_3)^2 r_{23}}{\kappa_1\kappa_4(2\kappa_2^2 + 5\kappa_2\kappa_3 + 2\kappa_3^2) + (\kappa_2 + \kappa_4 - \kappa_1\kappa_3 + \kappa_1\kappa_2\kappa_3\kappa_4) r_{23}} \\
r_{13} &= \frac{\kappa_1\kappa_4(2\kappa_2+\kappa_3)(\kappa_2+2\kappa_3) r_{23}}{\kappa_1\kappa_4(2\kappa_2^2 + 5\kappa_2\kappa_3 + 2\kappa_3^2) - (\kappa_1 - 1)(\kappa_2+\kappa_3) r_{23}}.
\end{aligned} \tag{4.9}$$

4.1.1 Scaling solutions

The scaling solution corresponds to the choice

$$\kappa_2 = 0 \qquad \kappa_1 = 1 \qquad \kappa_3 = \kappa_4 = \kappa \tag{4.10}$$

One finds

$$\begin{aligned}
k^I{}_i &= \begin{pmatrix} 0 & -\kappa & \kappa \\ \kappa & 0 & -\kappa \\ -\kappa & \kappa & 0 \end{pmatrix} & r_{12} &= r_{23} = r_{13} = \ell \\
P_0 &= 3 & Q_1 &= Q_2 = Q_3 = \kappa^2
\end{aligned} \tag{4.11}$$

for any given ℓ. The regularity conditions become

$$
\begin{aligned}
e^{-4U} &= 1 + \frac{r_1r_2+r_1r_3+r_2r_3}{r_1r_2r_3} + \frac{\kappa^2\,(r_1r_2+r_1r_3+r_2r_3+3r_1+3r_2+3r_3)}{r_1r_2r_3} \\
&\quad + \frac{\kappa^4\,(r_1+r_2+r_3+9)}{r_1r_2r_3} + \frac{\kappa^6\,\left(2r_1r_2+2r_1r_3+2r_2r_3+r_1r_2r_3-r_1^2-r_2^2-r_3^2\right)}{r_1^2r_2^2r_3^2} > 0 \\
Z_1V &= 1 + \frac{r_1\,r_2+r_1\,r_3+r_2\,r_3+\kappa^2(2\,r_2+2r_3-r_1+r_2\,r_3)}{r_1r_2r_3} > 0
\end{aligned}
\tag{4.12}
$$

The conditions $Z_2V>0$ and $Z_3V>0$ follow from $Z_1V>0$ and the permutation symmetry of the system. The two conditions can be shown to be satisfied using the property

$$
r_1+r_2-r_3 \geq 0 \tag{4.13}
$$

satisfied by the distances r_i from any point $x\in\mathbb{R}^3$ to the three vertices of an equilateral triangle. This inequality can be proved using triangle inequalities [42].[7]

We conclude that the five-dimensional geometry defined by the multi-center solution is regular everywhere. We notice that the fluxes satisfy the scaling condition (3.14) and consequently a rigid rescaling of the positions of the centers generate a new solution. More precisely, the moduli space of solutions with this charge is spanned by a single continuous parameter ℓ and permutations of the rows or columns of the matrix (4.11). There are 12 inequivalent choices corresponding to the 3! permutations of the entries in the first line in (4.11) times the two choices for the position of the 0 in the second line. The remaining entries are determined by the conditions that the sum along rows and columns of the matrix k_i^I should vanish. The number 12 matches the number of apostles and the degeneracy of four-charge black hole micro-states with the minimal unit of charge $P_0=Q_I=1$ [36]! Moreover the solutions satisfy $\sum_{I=1}^3\mathbf{k}_2^I=\mathbf{m}_2=0$ and therefore according to (2.19) and (2.20) they carry zero angular momentum and admit a microscopic description in terms of orthogonal intersecting D3-branes along the lines of [32].

4.1.2 Non-scaling solutions

The analysis above can be repeated for more general choices of the fluxes but regularity conditions in general can only be verified numerically. Here we list some illustrative examples of the type of solutions one finds.

- $\kappa_2=0,\ \kappa_1=\kappa_3=1,\ \kappa_4=\kappa:$

$$
\begin{aligned}
&{k^I}_i = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -\kappa & \kappa & 0 \end{pmatrix} &\qquad& r_{13}=r_{23} &\qquad& r_{12}=\frac{2\,\kappa\,r_{23}}{2\,\kappa+(\kappa-1)\,r_{23}} \\
&P_0=3 && Q_1=Q_2=\kappa && Q_3=1
\end{aligned}
\tag{4.14}
$$

  Interestingly, triangle inequalities in this case do not constrain r_{23} that can take arbitrarily large value.

[7]We thank our mathematician colleagues Andrea Ianuzzi and Stefano Trapani for the proof of this property.

- $\kappa_2 = \kappa_4 = \kappa,\ \kappa_1 = 1,\ \kappa_3 = 2\,\kappa$

$$k^I{}_i = \begin{pmatrix} -\kappa & -2\,\kappa & 3\,\kappa \\ 2\kappa & \kappa & -3\,\kappa \\ -\kappa & \kappa & 0 \end{pmatrix} \qquad r_{12} = \frac{r_{23}}{10 + r_{23}} \qquad r_{13} = r_{23}.$$
$$P_0 = 3 \qquad Q_1 = Q_2 = \kappa^2 \qquad Q_3 = 13\,\kappa^2 \tag{4.15}$$

As before r_{23} can take arbitrarily large value.

- $\kappa_2 = 0,\ \kappa_1 = 3\,\kappa,\ \kappa_3 = 2\,\kappa,\ \kappa_4 = \kappa$

$$k^I{}_i = \begin{pmatrix} 0 & -3\,\kappa & 3\,\kappa \\ \kappa & 0 & -\kappa \\ -2\kappa & 2\kappa & 0 \end{pmatrix} \qquad r_{12} = \frac{12\,\kappa^2\, r_{23}}{12\,\kappa^2 - r_{23}} \qquad r_{13} = \frac{6\,\kappa^2\, r_{23}}{6\,\kappa^2 - r_{23}}$$
$$P_0 = 3 \qquad Q_1 = 2\,\kappa^2 \qquad Q_2 = 6\,\kappa^2 \qquad Q_3 = 3\,\kappa^2$$
$$r_{23} < 6\,(2 - \sqrt{2})\,\kappa^2 \tag{4.16}$$

We notice that triangle inequality in this case imposes an upper bound on r_{23} leading to a moduli space of finite volume.

5 Fuzzballs of branes at angles

We look for regular fuzzball geometries with the asymptotics (2.26), i.e.

$$\ell_{0I} = v_0 = 1 \qquad m_0 = m = k^3_0 = k^3 = k^2 = 0 \qquad k^1_0 = k^2_0 = g \qquad k^1 = g(\ell_1 + \ell_2) \tag{5.1}$$

This describes a fuzzball of a non-rotating black hole with charges

$$P_0 = N$$
$$Q_I = -\frac{|\epsilon_{IJK}|}{2} \sum_{i=1}^{N} \frac{k^J_i\, k^K_i}{q_i} \tag{5.2}$$

The fuzzball is specified by the parameters k^I_i describing the magnetic fluxes through the two-spheres encircling the centers and the positions $\mathbf{x}_i$. The positions of the centers are constrained by the bubble equations

$$\sum_{j \neq i}^{N} \frac{k^{(1)}_{ij}\, k^{(2)}_{ij}\, k^{(3)}_{ij}}{r_{ij}} + k^1_i\, k^2_i\, k^3_i - \sum_{I=1}^{3} k^I_i - g\, k^2_i\, k^3_i - g\, k^1_i\, k^3_i = 0 \tag{5.3}$$

while the boundary conditions require

$$\sum_{i=1}^{N} k^2_i = \sum_{i=1}^{N} k^3_i = \sum_{i=1}^{N} k^1_i\, k^2_i\, k^3_i = 0 \qquad \sum_{i=1}^{N} k^1_i = g\,(Q_1 + Q_2) \tag{5.4}$$

In addition, the absence of horizons and of closed time-like curves requires

$$Z_I\, V > 0 \qquad \text{and} \qquad e^{2U} > 0 \tag{5.5}$$

5.1 Three centers

The bubble equations (5.3) for three centers can be solved in general by taking

$$r_{12} = \frac{\Pi_{12}\, r_{23}}{\Pi_{23} - r_{23}\,(\Gamma_2 - \Lambda_2 + \Omega_2)} \qquad r_{13} = -\frac{\Pi_{13}\, r_{23}}{\Pi_{23} - r_{23}\,(\Gamma_1 + \Gamma_2 - \Lambda_1 - \Lambda_2 + \Omega_1 + \Omega_2)} \tag{5.6}$$

with

$$\Pi_{ij} = \prod_{I=1}^{3}(k_i^I - k_j^I) \qquad \Gamma_i = \sum_I k^I{}_i \qquad \Lambda_i = k^1{}_i k^2{}_i k^3{}_i \qquad \Omega_i = g\, k_i^2\, k_i^3 + g\, k_i^1\, k_i^3 \tag{5.7}$$

We consider the case

$$k^I{}_i = \begin{pmatrix} 0 & -\kappa_1 & \kappa_1 + g\kappa_3(\kappa_1+\kappa_2) \\ \kappa_2 & 0 & -\kappa_2 \\ -\kappa_3 & \kappa_3 & 0 \end{pmatrix} \tag{5.8}$$

with $\kappa_1, \kappa_2, \kappa_3$ three positive integers and g a rational number. One finds

$$\begin{aligned} Q_1 &= \kappa_2\,\kappa_3 \qquad Q_2 = \kappa_1\,\kappa_3 \qquad Q_3 = \kappa_1\,\kappa_2 + g\,\kappa_2\,\kappa_3(\kappa_1 + \kappa_2) \\ r_{12} &= \frac{2\,\kappa_1\,\kappa_2\,\kappa_3\, r_{23}}{2\,\kappa_1\,\kappa_2\,\kappa_3 + g\,\kappa_2\,\kappa_3^2\,(\kappa_1 + \kappa_2) - (\kappa_1 - \kappa_3 + g\,\kappa_1\,\kappa_3)\, r_{23}} \\ r_{13} &= \frac{2\,[\,\kappa_1\,\kappa_2\,\kappa_3 + g\,\kappa_2\,\kappa_3^2\,(\kappa_1 + \kappa_2)]\, r_{23}}{2\,\kappa_1\,\kappa_2\,\kappa_3 + g\,\kappa_2\,\kappa_3^2\,(\kappa_1 + \kappa_2) - [\kappa_1 - \kappa_2 + g\,\kappa_3\,(\kappa_1 + \kappa_2)]\, r_{23}} \end{aligned} \tag{5.9}$$

The harmonic functions become

$$V = 1 + \sum_{i=1}^{3} \frac{1}{r_i} \qquad L_I = 1 + \frac{Q_I}{r_I} \qquad M = 0 \tag{5.10}$$

$$K_1 = g - \left(\frac{\kappa_1}{r_2} - \frac{\kappa_1 + g\kappa_3(\kappa_1+\kappa_2)}{r_3}\right) \quad K_2 = g + \kappa_2\left(\frac{1}{r_1} - \frac{1}{r_3}\right) \quad K_3 = \kappa_3\left(-\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Some examples are

- $\kappa_1 = \kappa_2 = \kappa_3 = \kappa = (2g)^{-1}$

$$\begin{aligned} &k^I{}_i = \begin{pmatrix} 0 & -\kappa & 2\kappa \\ \kappa & 0 & -\kappa \\ -\kappa & \kappa & 0 \end{pmatrix} \qquad r_{12} = \frac{4\,\kappa^2\, r_{23}}{6\,\kappa^2 - r_{23}} \qquad r_{13} = \frac{4\,\kappa^2\, r_{23}}{3\,\kappa^2 - r_{23}} \\ &Q_0 = 3 \qquad Q_1 = Q_2 = \kappa^2 \qquad Q_3 = 2\,\kappa^2 \\ &r_{23} < \frac{9 - \sqrt{57}}{2}\kappa^2. \end{aligned} \tag{5.11}$$

- $\kappa_1 = \kappa_2 = \kappa$, $\kappa_3 = 2\,\kappa$, $g = (4\,\kappa)^{-1}$

$$\begin{aligned} &k^I{}_i = \begin{pmatrix} 0 & -\kappa & 2\,\kappa \\ \kappa & 0 & -\kappa \\ -2\,\kappa & 2\,\kappa & 0 \end{pmatrix} \qquad r_{12} = \frac{8\,\kappa^2\, r_{23}}{12\,\kappa^2 + r_{23}} \qquad r_{13} = \frac{8\,\kappa^2\, r_{23}}{6\,\kappa^2 - r_{23}}. \\ &Q_0 = 3 \qquad Q_1 = Q_2 = Q_3 = 2\,\kappa^2 \\ &r_{23} < \kappa^2\left(-11 + \sqrt{145}\right). \end{aligned} \tag{5.12}$$

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We notice that in the two examples considered here distances are bounded by triangle inequalities leading to moduli spaces of finite volume.

6 Conclusions

We have constructed explicit examples of micro-state geometries of four-dimensional black holes. Following Bena, Gibbons and Warner, we have considered solutions consisting of half-BPS D-brane atoms with centers in $\mathbb{R}^3$. Charges and positions of the centers are constrained by the bubble equations that ensure that the metric uplifts to a horizon-less and CTC-free metric in five dimensions and boundary conditions that grant the match of the fuzzball and black-hole geometries at infinity. As a result, divergences coming from curvature singularities in the four dimensional metric are compensated by the singular behaviours of the scalars and gauge fields, leading to a finite (higher-derivative) string effective action.

We have considered the case of three centers in some details and found that there are two broad classes of solutions: scaling solutions and non-scaling ones. The moduli space of scaling solutions is described by a disjoint union of 12 seven-dimensional components accounting for overall translations, rotations and scaling of the system.[8] These solutions carry zero angular momentum and admit a microscopic description in terms of intersecting D3-branes along the lines of [32]. Non-scaling solutions are described by disjoint unions of moduli space components with the system now confined on a finite volume region of space. These solutions generically carry non-zero angular momentum even though they should be thought of as micro-states of BPS and as such non-rotating BH's. The presence of several abelian vector fields and scalars may require an extension of the no-hair theorem that, to the best of our knowledge, has been formulated for the case of a single type of charge and no scalars.

Acknowledgments

We would like to thank A. Addazi, I. Bena, M. Bertolini, D. Consoli, G. Dall'Agata, S. Giusto, A, Iannuzzi, O. Lunin, S. Mathur, R. Russo, S. Trapani, M. Trigiante and G. Veneziano for very useful discussions and comments on the manuscript. We would like to thank the MIUR-PRIN contract 2015MP2CX4002 "Non-perturbative aspects of gauge theories and strings" and the University of Rome Tor Vergata 'Uncovering Excellence' grant "StaI - String Theory and Inflation" for partial support. L. P. would like to thank Queen Mary University of London for hospitality during completion of this work.

A The ten dimensional solution and its 4d reduction

In this appendix we collect some details on the dimensional reduction down to four dimensions of the eight harmonic family of BPS solutions describing a general system of intersecting D3-branes on T^6.

[8] Although in our case, $P_0 = 3$ the counting seems to match the results of the index for $P_0 = 1$. This may be interpreted as suggesting that our supergravity analysis only captures the 'untwisted' sector of the relevant supersymmetric quantum mechanics.

A.1 The ten dimensional solution

The eight harmonic family of BPS solutions associated to D3-branes intersecting on T^6 is characterised by a metric g_{MN} and a four form Ramond field C_4 of the form [32]

$$\begin{aligned} ds^2 &= g_{\mu\nu}\, dx^\mu\, dx^\nu + \sum_{I=1}^{3} h^I_{mn}\, dy_I^m\, dy_I^n \\ C_4 &= C_{\mu,mnp}\, dx^\mu \wedge dy_1^m \wedge dy_2^n \wedge dy_3^p, \end{aligned} \tag{A.1}$$

with $\mu = 0, \dots 3$, $m = 1, 2$. x^μ are the coordinates along the four-dimensional space time and $y_I^m = (y_I, \tilde{y}_I)$ span a $T^2 \times T^2 \times T^2$ torus with $I = 1, 2, 3$ labelling the three two-torus. More precisely we write[9]

$$\begin{aligned} ds^2 &= -e^{2U}\,(dt + w)^2 + e^{-2U} \sum_{i=1}^{3} dx_i^2 + \sum_{I=1}^{3} \frac{1}{\operatorname{Im} U_I}\, |dy_I + U_I d\tilde{y}_I|^2 \\ C_4 &= A_\Lambda \gamma^\Lambda = A_a \gamma^a + A_{\dot{a}}\, \gamma^{\dot{a}} \end{aligned} \tag{A.3}$$

with α^Λ one forms in four dimensions and γ_Λ three forms in the internal torus. $\Lambda = (mnp) = (a, \dot{a})$ is a collective index labelling the 8 different three cycles $[mnp]$ on T^6 entering in the solution

$$\begin{aligned} \gamma^a &= (dy_1 \wedge dy_2 \wedge dy_3, d\tilde{y}_1 \wedge dy_2 \wedge dy_3, dy_1 \wedge d\tilde{y}_2 \wedge dy_3, dy_1 \wedge dy_2 \wedge d\tilde{y}_3) \\ \gamma^{\dot{a}} &= (d\tilde{y}_1 \wedge d\tilde{y}_2 \wedge d\tilde{y}_3, dy_1 \wedge d\tilde{y}_2 \wedge d\tilde{y}_3, d\tilde{y}_1 \wedge dy_2 \wedge d\tilde{y}_3, d\tilde{y}_1 \wedge d\tilde{y}_2 \wedge dy_3) \end{aligned} \tag{A.4}$$

All functions entering in the metric and four form can be written in terms of eight harmonic functions

$$\{V, L_I, K^I, M\} \tag{A.5}$$

on $\mathbb{R}^3$ or equivalently in terms of the following combination

$$\begin{aligned} Z_I &= L_I + \frac{|\varepsilon_{IJK}|}{2} \frac{K^J K^K}{V}, \\ \mu &= \frac{M}{2} + \frac{L_I K^I}{2V} + \frac{|\varepsilon_{IJK}|}{6} \frac{K^I K^J K^K}{V^2}. \end{aligned} \tag{A.6}$$

with ϵ_{IJK} characterising the triple intersections between two cycles on T^6. One finds

$$\begin{aligned} e^{-4U} &= Z_1 Z_2 Z_3 V - \mu^2 V^2, \\ U_I &= \operatorname{Re} U_I + i \operatorname{Im} U_I = -b^I + i \left(V e^{2U} Z_I\right)^{-1} \qquad b^I = \frac{K^I}{V} - \frac{\mu}{Z_I}, \\ A_a &= (\alpha, \alpha^I - b^I \alpha) \\ A_{\dot{a}} &= \left(\beta - b^1 b^2 b^3 \alpha - \beta_I b^I + \frac{1}{2} |\varepsilon_{IJK}| \alpha^I b^J b^K,\ \beta^I + |\varepsilon_{IJK}| \left(\alpha b^J b^K - 2 b^J \alpha^K\right)\right) \end{aligned} \tag{A.7}$$

[9] In matrix form

$$h^I_{mn} = \frac{1}{\operatorname{Im} U_I} \begin{pmatrix} 1 & \operatorname{Re} U_I \\ \operatorname{Re} U_I & |U_I|^2 \end{pmatrix}, \qquad h_I^{mn} = \frac{1}{\operatorname{Im} U_I} \begin{pmatrix} |U_I|^2 & -\operatorname{Re} U_I \\ -\operatorname{Re} U_I & 1 \end{pmatrix}. \tag{A.2}$$

Finally the one-forms $\alpha, \alpha^I, \beta, \beta_I$ are defined in terms of the eight harmonic function via

$$\begin{aligned}
\alpha &= w_0 - \mu V^2 e^{4U}\left(dt+w\right), \\
\alpha^I &= -\frac{dt+w}{Z_I} + b^I w_0 + w^I, \\
\beta &= -v_0 + \frac{e^{-4U}}{V^2 Z_1 Z_2 Z_3}\left(dt+w\right) - b^I v_I + b^1 b^2 b^3 w_0 + \frac{|\varepsilon_{IJK}|}{2} b^I b^J w^K, \\
\beta_I &= -v_I + \frac{|\varepsilon_{IJK}|}{2}\left\{\frac{\mu\left(dt+w\right)}{Z_J Z_K} + b^J b^K w_0 + 2 b^J w^K\right\}
\end{aligned} \tag{A.8}$$

and

$$\begin{aligned}
*_3 dw_0 = dV, \quad *_3 dw^I = -dK^I, \quad *_3 dv_0 = dM, \quad *_3 dv_I = dL_I \\
*_3 dw = \frac{1}{2}\left(V dM - M dV + K^I dL_I - L_I dK^I\right)
\end{aligned} \tag{A.9}$$

A.2 The four dimensional model

After reduction to four dimensions, the ten dimensional solution can be viewed as a solution of a $\mathcal{N}=2$ truncation of maximal supersymmetric supergravity involving the gravity multiplet and three vector multiplets. The four dimensional model arises as a dimensional reduction of the ten dimensional lagrangian

$$\mathcal{L} = \sqrt{g_{10}}\left(R_{10} - \frac{1}{4{\cdot}5!} F_{M_1 \ldots M_5} F^{M_1 \ldots M_5}\right) \tag{A.10}$$

Plugging the ansatz (A.1) into (A.10) and taking all fields varying only along the four-dimensional spacetime one finds

$$\mathcal{L} = \sqrt{g_4}\left(R_4 - \sum_{I=1}^{3} \frac{\partial_\mu U_I \partial^\mu \bar{U}_I}{2\left(\operatorname{Im} U_I\right)^2} - \frac{1}{4{\cdot}2!} F_{\mu\nu,\Lambda} F^{\mu\nu,\Lambda}\right) \tag{A.11}$$

It is convenient to introduce a metric $\mathcal{H}^{\Lambda\Sigma}$ and its inverse to raise and lower the Λ indices. One writes

$$\mathcal{H}^{abc,def} = h_1^{ad}\, h_2^{be}\, h_3^{cf} \tag{A.12}$$

or in matrix form

$$\mathcal{H}^{\Lambda\Sigma} = \begin{pmatrix} \mathcal{H}_1 & \mathcal{H}_2 \\ \mathcal{H}_2^T & \mathcal{H}_3 \end{pmatrix}, \tag{A.13}$$

with $\mathcal{H}_1^{ab}$, $\mathcal{H}_2^{a\dot{a}}$, $\mathcal{H}_3^{\dot{a}\dot{b}}$ 4×4 matrices. The self-duality condition of the five form field in ten dimensions

$$F_{\mu\nu abc} = \frac{\sqrt{g_{10}}}{2} \varepsilon_{\mu\nu\rho\sigma abcdef} F^{\rho\sigma def}, \tag{A.14}$$

reduces to

$$F_{abc} = \varepsilon_{abcdef} \widetilde{F}^{def} \quad \Leftrightarrow \quad F_\Lambda = \varepsilon_{\Lambda\Sigma} \widetilde{F}^\Sigma \tag{A.15}$$

with $\widetilde{F} = *_4 F$ and $\varepsilon_{\Lambda\Sigma}$ an block off-diagonal antisymmetric matrix with the only non-trivial components

$$\varepsilon_{0\dot{0}} = -\varepsilon_{I\dot{I}} = -\varepsilon_{\dot{0}0} = \varepsilon_{\dot{I}I} = 1 \tag{A.16}$$

In components

$$F_a = \varepsilon_{a\dot{a}} \widetilde{F}^{\dot{a}} \qquad F_{\dot{a}} = \varepsilon_{\dot{a}a} \widetilde{F}^{a} \tag{A.17}$$

These self-duality relations can be used to express the components $F_{\dot{a}}$ in terms of the Poincare' duals of F_a. Indeed, inverting the first equation in (A.17) one finds[10]

$$F_{\dot{a}} = -(\mathcal{H}_3^{-1})_{\dot{a}\dot{b}} \left(\varepsilon^{\dot{b}c} \widetilde{F}_c + \mathcal{H}_2^{\dot{b}c} F_c \right) \tag{A.18}$$

with $\varepsilon^{\dot{a}a} = \mathrm{diag}(1,-1,-1,-1)$ the inverse of $\varepsilon_{a\dot{a}}$. Using these relations one can write

$$F_\Lambda F^\Lambda = \mathcal{L}_{stu} + \mathcal{L}_{\mathrm{top}} \tag{A.19}$$

with $\mathcal{L}_{\mathrm{top}} = -2\epsilon^{a\dot{a}} \widetilde{F}_{\dot{a}} F_a$ a total derivative,

$$\mathcal{L}_{stu} = 2(F_a \mathcal{I}^{ab} F_b + F_a \mathcal{R}^{ab} \widetilde{F}_b) \tag{A.20}$$

and[11]

$$\mathcal{I}^{ab} = stu \begin{pmatrix} 1+\frac{\sigma^2}{s^2}+\frac{\tau^2}{t^2}+\frac{\nu^2}{u^2} & -\frac{\sigma}{s^2} & -\frac{\tau}{t^2} & -\frac{\nu}{u^2} \\ -\frac{\sigma}{s^2} & \frac{1}{s^2} & 0 & 0 \\ -\frac{\tau}{t^2} & 0 & \frac{1}{t^2} & 0 \\ -\frac{\nu}{u^2} & 0 & 0 & \frac{1}{u^2} \end{pmatrix} \qquad \mathcal{R}^{ab} = \begin{pmatrix} 2\sigma\tau\nu & -\tau\nu & -\sigma\nu & -\sigma\tau \\ -\tau\nu & 0 & \nu & \tau \\ -\sigma\nu & \nu & 0 & \sigma \\ -\sigma\tau & \tau & \sigma & 0 \end{pmatrix} \tag{A.21}$$

where

$$U_I = (\sigma + is, \tau + it, \nu + iu) \tag{A.22}$$

Discarding the total derivative term, the four-dimensional Lagrangian can then be written as

$$\mathcal{L} = \sqrt{g_4} \left(R_4 - \sum_{I=1}^{3} \frac{\partial_\mu U_I \partial^\mu \bar{U}_I}{2\,(\mathrm{Im}\,U_I)^2} - \frac{1}{4} F_a \mathcal{I}^{ab} F_b - \frac{1}{4} F_a \mathcal{R}^{ab} \widetilde{F}_b \right) \tag{A.23}$$

The equations of motion read

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} &= \frac{1}{2\,(\mathrm{Im}\,U_I)^2} \left(\partial_\mu U_I \partial_\nu U_I - \frac{1}{2} g_{\mu\nu} (\partial U_I)^2 \right) \\ &\quad + \frac{1}{2} \mathcal{I}^{ab} \left(F_{a\mu\sigma} F^{\sigma}_{b\nu} - \frac{1}{4} g_{\mu\nu} F_a\, F_b \right) + \frac{1}{2} \mathcal{R}^{ab} \left(F_{a\mu\sigma} \tilde{F}^{\sigma}_{b\nu} - \frac{1}{4} g_{\mu\nu} F_a\, \tilde{F}_b \right) \\ \nabla_\mu \left\{ \mathcal{I}^{ab} F_b^{\mu\nu} + \mathcal{R}^{ab} \tilde{F}_b^{\mu\nu} \right\} &= 0 \\ -\nabla_\mu \frac{\nabla^\mu U_I}{(\mathrm{Im}\,U_I)^2} &= \mathrm{i} \frac{\partial_\mu U_I \partial^\mu \bar{U}_I}{(\mathrm{Im}\,U_I)^3} + \frac{1}{2} F_a \frac{\partial \mathcal{I}^{ab}}{\partial \bar{U}_I} F_b + \frac{1}{2} F_a \frac{\partial \mathcal{R}^{ab}}{\partial \bar{U}_I} \widetilde{F}_b \end{aligned} \tag{A.24}$$

[10] Here we use $*_4^2 = -1$.

[11] Equivalently

$$\begin{aligned} \mathcal{I}^{ab} &\equiv \mathcal{H}_1^{ab} + \varepsilon^{a\dot{b}} (\mathcal{H}_3^{-1})_{\dot{b}\dot{c}} \varepsilon^{\dot{c}b} - \mathcal{H}_2^{a\dot{b}} (\mathcal{H}_3^{-1})_{\dot{c}\dot{d}} \mathcal{H}_2^{\dot{d}b} \\ \mathcal{R}^{ab} &\equiv \varepsilon^{a\dot{b}} (\mathcal{H}_3^{-1})_{\dot{b}\dot{c}} \mathcal{H}_2^{\dot{c}b} + \mathcal{H}_2^{a\dot{b}} (\mathcal{H}_3^{-1})_{\dot{b}\dot{c}} \varepsilon^{\dot{c}b} \end{aligned}$$

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A.3 The basic solutions

A family of supersymmetric solutions to equations (A.24) is given in [32]. These solutions can be viewed as made of three different types of solutions, referred as K, L or M. In the following we display a representative of solution in each type.

A.3.1 L solutions

The L class of solutions can be represented by the choice

$$\begin{aligned} &V \equiv L\,(x)\,, \quad M = K^I = 0, \quad L_I = 1 \quad \Rightarrow \quad Z_I = 1, \quad \mu = 0 \\ &\mathcal{I} = \mathrm{diag}\left(L^{-3/2}, L^{-1/2}, L^{-1/2}, L^{-1/2}\right) \qquad \mathcal{R} = 0 \end{aligned} \tag{A.25}$$

The solution can be written as

$$\begin{aligned} ds^2 &= -L^{-\frac{1}{2}} dt^2 + L^{\frac{1}{2}} \sum_{i=1}^{3} dx_i^2 \\ A_0 &= w_0 \qquad *_3\, dw_0 = dL \\ U_1 &= U_2 = U_3 = \mathrm{i}\, L^{-\frac{1}{2}} \end{aligned} \tag{A.26}$$

A.3.2 K solutions

The K solutions correspond to the choice

$$K^3 = -M \equiv K\,(x)\,, \quad L_I = V = 1, \quad K^1 = K^2 = 0 \quad \Rightarrow \quad Z_I = 1, \quad \mu = 0$$

$$\mathcal{I} = \begin{pmatrix} 1+K^2 & 0 & 0 & K \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ K & 0 & 0 & 1 \end{pmatrix} \qquad \mathcal{R} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -K & 0 \\ 0 & -K & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{A.27}$$

The solution is given by

$$\begin{aligned} ds^2 &= -\,(dt+w)^2 + \sum_{i=1}^{3} dx_i^2, \\ U_1 &= U_2 = \mathrm{i} \qquad\qquad U_3 = -\,K + \mathrm{i} \\ A_0 &= A_3 = 0 \qquad\qquad A_1 = A_2 = -w \qquad *_3 dw = -\,dK \end{aligned} \tag{A.28}$$

A.3.3 M solutions

The M solutions correspond to the choice

$$K^2 = M \equiv M\,(x)\,, \qquad L_I = V = 1, \qquad K^1 = K^3 = 0 \quad \Rightarrow \quad \mu = M \quad Z_I = 1,$$

$$\mathcal{I} = a^{3/2} \begin{pmatrix} 1+2\frac{M^2}{a} & -\frac{M}{a} & 0 & -\frac{M}{a} \\ -\frac{M}{a} & a^{-1} & 0 & 0 \\ 0 & 0 & a^{-1} & 0 \\ -\frac{M}{a} & 0 & 0 & a^{-1} \end{pmatrix} \qquad \mathcal{R} = \begin{pmatrix} 0 & 0 & -M^2 & 0 \\ 0 & 0 & M & 0 \\ -M^2 & M & 0 & M \\ 0 & 0 & M & 0 \end{pmatrix} \tag{A.29}$$

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with $a = 1 - M^2$. The solution is given by

$$
\begin{aligned}
ds^2 &= -\frac{dt^2}{\sqrt{1-M^2}} + \sqrt{1-M^2}\sum_{i=1}^{3} dx_i^2, \\
U_1 &= U_3 = M + \mathrm{i}\sqrt{1-M^2} & U_2 &= \mathrm{i}\sqrt{1-M^2} \\
A_0 &= -\frac{M\,dt}{1-M^2} & A_1 &= A_3 = -\frac{dt}{1-M^2} \\
A_2 &= w_2 & *_3 dw_2 &= -\,dM
\end{aligned}
\tag{A.30}
$$

A.4 Sub-family of solutions

For completeness, we list some interesting sub-families of solutions included in the eight-harmonic class.

A.4.1 No scalars: IWP solution

Einstein-Maxwell theory can be embedded in four dimensional supergravity by restricting to geometries with a trivial internal square metric

$$
V e^{2U} Z = 1 \qquad b^I = 0 \tag{A.31}
$$

These equations can be solved in terms of two harmonic functions $\mathrm{Re}\,H$ and $\mathrm{Im}\,H$ via the identifications

$$
\begin{aligned}
V &= L_1 = L_2 = L_3 = \mathrm{Im}\,H \\
-M &= K^1 = K^2 = K^3 = \mathrm{Re}\,H
\end{aligned}
\tag{A.32}
$$

The general solution reduces to[12]

$$
\begin{aligned}
ds^2 &= -\,|H|^{-2}\,(dt+w)^2 + |H|^2\,d|\mathbf{x}|^2 \\
A_0 &= w_0 - \frac{\mathrm{Re}\,H}{|H|^2}\,(dt+w) \\
A_1 &= A_2 = A_3 = w^1 - \frac{\mathrm{Im}\,H}{|H|^2}\,(dt+w)
\end{aligned}
\tag{A.34}
$$

with H a complex harmonic function

$$
H = \mathrm{Re}\,H + \mathrm{i}\,\mathrm{Im}\,H \qquad\qquad \nabla^2 H = 0 \tag{A.35}
$$

and w and w_0, w^1 one forms defined as

$$
\begin{aligned}
dw &= \mathrm{i}\,*_3\left[H dH^* - H^* dH\right] \\
dw_0 &= *_3 d\,\mathrm{Im}\,H \qquad dw^1 = *_3 d\,\mathrm{Re}\,H
\end{aligned}
\tag{A.36}
$$

[12] In our conventions the Einstein-Maxwell lagrangian reads

$$
\mathcal{L} = \sqrt{g}\left[R - \frac{1}{4}F^2\right] \tag{A.33}
$$

We notice that the contribution to the stress energy tensor of gauge fields A_I exactly match that of A_0 , so we can replace the four gauge fields by a single one given by

$$A = 2w_0 - \frac{2\mathrm{Re}\, H}{|H|^2}\,(dt + w)$$

The resulting solution is known in the General Relativity literature as IWP (after Israel, Wilson and Perjes [44, 45]) and includes very well known examples of solutions of Maxwell-Einstein gravity:

- $AdS_2 \times S^2$. The harmonic function H reads[13]

$$H = \frac{1}{\sqrt{x_1^2 + x_2^2 + (x_3 - \mathrm{i}L)^2}} \tag{A.38}$$

  The geometry is regular everywhere. An infinite class of regular IWP geometries, obtained as bubbling of $AdS_2 \times S^2$ and parametrised by a string profile function has been recently constructed in [21].

- Kerr-Newman solution with $M = Q = q$, $P = 0$, $J = qL$. The harmonic function H reads

$$H = 1 + \frac{q}{\sqrt{x_1^2 + x_2^2 + (x_3 - \mathrm{i}L)^2}} \tag{A.39}$$

  The geometry has a naked curvature singularity at the zero of H.

- Reissner-Nordstrom with $M = Q = q$, $P = J = 0$. The harmonic function H reads

$$H = 1 + \frac{q}{\sqrt{x_1^2 + x_2^2 + x_3^2}} \tag{A.40}$$

  The geometry has a curvature singularities at the zero of H.

- Charged Taub-NUT with $M = Q = b_1$, $P = -b_2$, $J = 0$. The harmonic function H reads

$$H = 1 + \frac{b_1 + \mathrm{i}\, b_2}{\sqrt{x_1^2 + x_2^2 + x_3^2}} \tag{A.41}$$

  The geometry has no curvature singularities but it has a Dirac-Misner string-like singularity.

[13] Global coordinates are defined by

$$(x_1, x_2, x_3) = \left(\sqrt{(\rho^2 + L^2)(1 - \chi^2)}\, \cos\phi, \sqrt{(\rho^2 + L^2)(1 - \chi^2)}\, \sin\phi, \rho\,\chi\right) \tag{A.37}$$

with $\rho \in (-\infty, \infty)$, $\chi \in [-1, 1]$, $\phi \in [0, 2\pi]$. These coordinates cover twice the flat space with the points (ρ, χ) and $(-\rho, -\chi)$ identified.

A.4.2 One complex scalar: SWIP solutions

Next, we consider a solutions with single active scalar field, let us say U_1, with $U_2 = U_3 = \mathrm{i}$. These conditions can be solved in terms of two complex harmonic functions H_1, H_2 after the identification

$$L_1 = V = \mathrm{Re}\, H_2, \quad L_2 = L_3 \equiv \mathrm{Im}\, H_1 \quad K^1 = -M = \mathrm{Re}\, H_1 \quad K^2 = K^3 \equiv -\mathrm{Im}\, H_2 \tag{A.42}$$

For this choice the general solution reduces to

$$\begin{aligned}
ds^2 &= -\left[\mathrm{Im}\,\left(H_1\bar{H}_2\right)\right]^{-1}(dt+w)^2 + \mathrm{Im}\,\left(H_1\bar{H}_2\right)\sum_{i=1}^{3} dx_i^2,\\
U_1 &= \frac{H_1}{H_2}, \qquad\qquad U_2 = U_3 = \mathrm{i},\\
A_0 &= w_0 + \frac{\mathrm{Im}\, H_2}{\mathrm{Im}\,\left(H_1\bar{H}_2\right)}(dt+w)\,, \qquad A_1 = w^1 - \frac{\mathrm{Im}\, H_1}{\mathrm{Im}\,\left(H_1\bar{H}_2\right)}(dt+w)\\
A_2 &= A_3 = w^2 - \frac{\mathrm{Re}\, H_2}{\mathrm{Im}\,\left(H_1\bar{H}_2\right)}(dt+w)\\
*_3 dw &= -\mathrm{Re}\,\left(H_1 d\bar{H}_2 - \bar{H}_2 dH_1\right)\\
*_3 dw_0 &= \mathrm{Re}\, dH_2, \qquad *_3 dw^1 = -\mathrm{Re}\, dH_1, \qquad *_3 dw^2 = \mathrm{Im}\, dH_2.
\end{aligned} \tag{A.43}$$

The IWP class corresponds to the choice $H_1 = \mathrm{i}H_2 = H$. See for instance [46] for more information on the SWIP solution.

A.4.3 Two complex scalars

This solution corresponds to the choice

$$L_3 = L_2 \qquad K^3 = K^2 \tag{A.44}$$

leading to

$$\begin{aligned}
Z_1 &= L_1 + \frac{(K^2)^2}{V} \qquad Z_2 = Z_3 = L_2 + \frac{K^1K^2}{V}\\
\mu &= \frac{M}{2} + \frac{K^1\,(K^2)^2}{V^2} + \frac{K^1\,L_1}{2V} + \frac{K^2L_2}{V}
\end{aligned} \tag{A.45}$$

The solution reads

$$\begin{aligned}
ds^2 &= -e^{-2U}(dt+w)^2 + e^{2U}\sum_{i=1}^{3} dx_i^2, \qquad e^{-4U} = Z_1Z_2^2V - \mu^2V^2\\
U_1 &= -b^1 + \mathrm{i}(e^{2U}VZ_1)^{-1} \qquad U_2 = U_3 = -b^2 + \mathrm{i}(e^{2U}VZ_2)^{-1}\\
A_0 &= w_0 - \mu V^2 e^{4U}\,(dt+w) \qquad A_1 = w^1 + Ve^{4U}\,(dt+w)\left(Z_2^2 - K^1\,\mu\right)\\
A_2 &= A_3 = w^2 + Ve^{4U}\,(dt+w)\left(Z_1Z_2 - K^I\,\mu\right)
\end{aligned} \tag{A.46}$$

with

$$*_3 dw = \frac{1}{2}(VdM - MdV + K^1 dL_1 - L_1 dK^1 + 2K^2 dL_2 - 2L_2 dK^2)$$

$$*_3 dw_0 = dV, \qquad *_3 dw^1 = -dK^1 \qquad *_3\, dw^2 = -dK^2 \tag{A.47}$$

and

$$b^1 = \frac{K^1 L_1 - 2K^2 L_2 - MV}{2\left[(K^2)^2 + VL_1\right]} \qquad b^2 = b^3 = -\frac{K^1 L_1 + MV}{2(K^1 K^2 + VL_2)} \tag{A.48}$$

The SWIP solution is recovered for $L_1 = V$ and $K^1 = -M$ after the identifications (A.42).

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