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Notes on the ambitwistor pure spinor string

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Abstract: In this work, some aspects of the ambitwistor pure spinor string are investigated. The b ghost is presented and its main properties are derived in a simple way, very similar to the usual pure spinor b ghost construction. The heterotic case is also addressed with a new proposal for the BRST charge. The BRST cohomology is shown to correctly describe the heterotic supergravity spectrum and a semi-composite b ghost is constructed.

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JHEP05(2016)116

Contents

1 Introduction

The so-called ambitwistor string was proposed in [1] and corresponds to a chiral infinite tension limit ($\alpha' \to 0$) of the string, therefore containing only the massless spectrum. Quantization of this model remarkably leads to the Cachazo-He-Yuan (CHY) tree level amplitudes [2].

Soon after Mason and Skinner's work, Berkovits came up with the pure spinor version of the ambitwistor superstring [3], successfully describing the CHY formulas in an explicitly supersymmetric way, as characteristic of the pure spinor superstring.

The coupling of the RNS ambitwistor string to (NS-NS) curved backgrounds was developed in [4], where quantum consistency naturally imposed the non-linear $D = 10$ supergravity equations of motion.

Following an analogous idea, Chandia and Vallilo [5, 6] analyzed the type II supergravity background coupled to the pure spinor string in the $\alpha' \to 0$ limit and found that Berkovits' original proposal had an extra nilpotent symmetry in the action. As it turned out, a consistent redefinition of the pure spinor BRST charge enabled a more natural coupling of the action to the Kalb-Ramond field and superpartner, leading to the expected type II supergravity constraints of [7].

It is interesting to point out that the ambitwistor string of Mason and Skinner have a pair of ghost fields (b,$\tilde{b}$) satisfying

$$\{Q, b\} = T,\ \{Q, \tilde{b}\} = H, \tag{1.1}$$

where Q is the BRST charge, T is the energy-momentum tensor and $H = \frac{1}{2}P^2$ is the particle-like Hamiltonian. Berkovits' pure spinor version does not seem to have a BRST-trivial energy-momentum tensor. On the other hand, as will be shown here, the results

JHEP05(2016)116

of [5, 6] can be interpreted as a splitting in the holomorphic theory which is responsible for a very simple construction of the b ghost and a generalization of the operator $\tilde{b}$ of (1.1). In simple terms, one can define for each "sector" a new field, b_+ and b_-, satisfying $\{Q, b_\pm\} = T_\pm$, with

$$T_+ + T_- = T, \tag{1.2a}$$

$$T_+ - T_- = \frac{1}{2}P^2 + \ldots. \tag{1.2b}$$

The dots in (1.2b) are extra terms required to make the right hand side BRST-closed. The operators b_+ and b_- are very similar to the composite pure spinor b ghost but their geometrical interpretation is not clear yet. Unlike in Berkovits' proposal, a concrete form for the integrated vertex operator is still lacking in Chandia and Vallilo's modification and a better understanding on the newly introduced b_+ and b_- might help to solve this issue.

Concerning the heterotic case, also proposed in [3], the energy-momentum tensor is clearly BRST-trivial but there does not seem to exist a $\tilde{b}$ operator trivializing the particle-like Hamiltonian. Maybe a bit more worrying is the fact that the supergravity states do not have a satisfactory vertex operator description.

Motivated by the holomorphic sectorization of the type II case, the heterotic BRST charge will be modified to

$$Q = \oint \{\lambda^\alpha d_\alpha + \bar{c}T_+ - \bar{b}\bar{c}\partial\bar{c}\}, \tag{1.3}$$

where λ^α is the pure spinor ghost, d_α is the improved operator proposed in [5, 6], ($\bar{b}$,$\bar{c}$) are the reparametrization ghosts and T_+ is a *fake* energy-momentum satisfying

$$T_+(x)\, T_+(y) \sim \frac{2T_+}{(z-y)^2} + \frac{\partial T_+}{(z-y)}, \tag{1.4a}$$

$$T_+(x)\, \lambda^\alpha d_\alpha(y) \sim \text{regular}. \tag{1.4b}$$

Besides having the (b,$\tilde{b}$) structure mentioned above, the BRST charge of (1.3) will be shown to correctly describe the massless heterotic spectrum (super Yang-Mills and supergravity). In terms of the redefined supersymmetric invariants, the heterotic action will be rewritten such that the coupling with the Kalb-Ramond field is manifest, exactly like in the type II case.

This work is organized as follows. Section 2 discusses the type II case of the infinite tension limit of the pure spinor string. After a review of Berkovits' proposal and the modification proposed by Chandia and Vallilo, the holomorphic sectorization is studied and the construction of the composite b ghost is presented in detail, together with several properties. Section 3 starts with a review of the heterotic case, explaining why the natural choice for the supergravity vertex is incomplete. With the new proposal for the BRST charge, this flaw is corrected and a semi-composite b ghost is introduced. Section 4 discusses the results and possible directions to follow.

JHEP05(2016)116

2 The type II ambitwistor pure spinor string

The $\alpha' \to 0$ limit of the pure spinor superstring was first discussed in [3]. For the type II case, the proposed action is simply

$$S = \int d^2z\{P_m \overline{\partial} X^m + p_\alpha \overline{\partial}\theta^\alpha + w_\alpha \overline{\partial}\lambda^\alpha + \hat{p}_{\hat{\alpha}} \overline{\partial}\hat{\theta}^{\hat{\alpha}} + \hat{w}_{\hat{\alpha}} \overline{\partial}\hat{\lambda}^{\hat{\alpha}}\}, \tag{2.1}$$

where $\{P_m,\, p_\alpha,\, \hat{p}_{\hat{\alpha}}\}$ denote the conjugate momenta to the $\mathcal{N}=2$ superspace coordinates $\{X^m,\, \theta^\alpha,\, \hat{\theta}^{\hat{\alpha}}\}$, and $(w_\alpha, \lambda^\alpha)$ and $(\hat{w}_{\hat{\alpha}}, \hat{\lambda}^{\hat{\alpha}})$ are the usual pure spinor ghost conjugate pairs. For convenience, the same chirality is being considered for the superspace coordinates θ and $\hat{\theta}$ (type IIB) but the results are easily generalized to the type IIA case.

The first order action S is supersymmetric with respect to the charges

$$q_\alpha = \oint \left\{ p_\alpha + \frac{1}{2} P_m (\gamma^m \theta)_\alpha \right\}, \tag{2.2a}$$

$$\hat{q}_{\hat{\alpha}} = \oint \left\{ \hat{p}_{\hat{\alpha}} + \frac{1}{2} P_m (\gamma^m \hat{\theta})_{\hat{\alpha}} \right\}, \tag{2.2b}$$

which define the invariants P_m and

$$\Pi^m = \partial X^m + \frac{1}{2}(\theta\gamma^m\partial\theta) + \frac{1}{2}(\hat{\theta}\gamma^m\partial\hat{\theta}), \tag{2.3a}$$

$$d_\alpha = p_\alpha - \frac{1}{2} P_m (\gamma^m \theta)_\alpha, \tag{2.3b}$$

$$\hat{d}_{\hat{\alpha}} = \hat{p}_{\hat{\alpha}} - \frac{1}{2} P_m (\gamma^m \hat{\theta})_{\hat{\alpha}}. \tag{2.3c}$$

As usual, S has to be provided with the BRST charge

$$Q = \oint \{\lambda^\alpha d_\alpha + \hat{\lambda}^{\hat{\alpha}} \hat{d}_{\hat{\alpha}}\}, \tag{2.4}$$

Nilpotency of Q follows from the pure spinor constraints $(\lambda\gamma^m\lambda) = (\hat{\lambda}\gamma^m\hat{\lambda}) = 0$.

As expected, the type II supergravity spectrum is in the cohomology of (2.4). BRST-closedness of the vertex

$$U_{SG} = \lambda^\alpha \hat{\lambda}^{\hat{\alpha}} A_{\alpha\hat{\alpha}}(\theta, \hat{\theta}) e^{ik_m X^m}, \tag{2.5}$$

imply the linearized supergravity equations of motion for the superfield $A_{\alpha\hat{\alpha}}$:

$$\gamma^{\alpha\beta}_{mnpqr} D_\beta A_{\alpha\hat{\alpha}} = 0, \quad \gamma^{\hat{\alpha}\hat{\beta}}_{mnpqr} \hat{D}_{\hat{\beta}} A_{\alpha\hat{\alpha}} = 0. \tag{2.6}$$

Here, $D_\alpha \equiv \partial_\alpha + \frac{i}{2}(\gamma^m\theta)_\alpha k_m$ and $\hat{D}_{\hat{\alpha}} = \partial_{\hat{\alpha}} + \frac{i}{2}(\gamma^m\hat{\theta})_{\hat{\alpha}} k_m$ are the supersymmetric derivatives for momentum eigenstates. The gauge transformations come from the BRST-exact states of the form $\Lambda = \lambda^\alpha \Lambda_\alpha + \hat{\lambda}^{\hat{\alpha}} \hat{\Lambda}_{\hat{\alpha}}$, implying the gauge transformation $\delta A_{\alpha\hat{\alpha}} = D_\alpha \hat{\Lambda}_{\hat{\alpha}} + \hat{D}_{\hat{\alpha}} \Lambda_\alpha$, as long as the superfield parameters satisfy $D\gamma^{mnpqr}\Lambda = \hat{D}\gamma^{mnpqr}\hat{\Lambda} = 0$.

For convenience, $A_{\alpha\hat{\alpha}}$ in (2.5) can be cast as $A_{\alpha\hat{\alpha}} = A_\alpha(\theta)\hat{A}_{\hat{\alpha}}(\hat{\theta})$, such that one can introduce the usual auxiliary fields satisfying

$$A_m = \frac{1}{8}(D_\alpha \gamma_m^{\alpha\beta} A_\alpha), \tag{2.7a}$$

$$W^\alpha = \frac{1}{10}\gamma_m^{\alpha\beta}[D_\beta A_m - ik^m \gamma^{\alpha\beta}_m A_\beta], \tag{2.7b}$$

$$F_{mn} = \frac{i}{2}\left(k_m A_n - k_n A_m\right), \tag{2.7c}$$

JHEP05(2016)116

and similar equations for $\hat{A}_m$, $\hat{W}^{\hat{\alpha}}$ and $\hat{F}_{mn}$ in terms of $\hat{A}_{\hat{\alpha}}$. These auxiliary fields are the basic ingredients of the integrated vertex V presented in [3], given by:

$$V = \int d^2z\, \overline{\delta}(k^m P_m)[P_m A^m + d_\alpha W^\alpha + N^{mn} F_{mn}][P_m \hat{A}^m + \hat{d}_{\hat{\alpha}} \hat{W}^{\hat{\alpha}} + \hat{N}^{mn} \hat{F}_{mn}] e^{ik_m X^m}. \quad (2.8)$$

N_{mn} and $\hat{N}_{mn}$ are the ghost Lorentz currents, defined as

$$N^{mn} \equiv \tfrac{1}{2}(\lambda\gamma^{mn}w), \quad \hat{N}^{mn} \equiv \tfrac{1}{2}(\hat{\lambda}\gamma^{mn}\hat{w}), \quad (2.9)$$

and the operator $\overline{\delta}(k^m P_m)$ is detailedly described in [1], having the right conformal dimensions necessary to make V a worldsheet scalar. Observe that BRST-closedness and gauge transformations of V ($\delta A_m = k_m \Lambda$ and $\delta \hat{A}_m = k_m \hat{\Lambda}$) can be shown to be proportional to $\overline{\delta}(k \cdot P)\, k^m P_m$.

The pure spinor tree level amplitudes computed using the massless vertices described above have explicit spacetime supersymmetry and were shown to agree with the RNS computations [3, 8].

In spite of the interesting outcomes, Berkovits' proposal has yet to be better understood. The BRST cohomology of (2.4) is not clear enough and a consistent coupling with curved backgrounds seems to require a slight modification of the flat space limit just presented [5, 6]. These features will be discussed, reviewed and extended in the rest of the section.

2.1 Extra elements in the BRST cohomology

The simple form of the BRST charge (2.4) hides a fundamental feature of the closed string spectrum that is the decoupling of the left-moving and right-moving sectors. Of course, the *chiral* action (2.1) is not able to encode this information and this has an interesting consequence, as there might be extra states in the BRST cohomology.

Most of the cohomology analysis for the $\alpha' \to 0$ limit reviewed above can be parallelized with the $\mathcal{N} = 2$ pure spinor superparticle. In [9] there is a thorough discussion on the physical spectrum coming from the quantization of the superparticle, in particular that of the zero-momentum states. Of course, to talk about *physical* spectrum one has to define the *physical* state conditions. This will be discussed in section 4 because it is fundamentally related to the developments to be presented in the next subsections.

For now, it will be pointed out that at zero-momentum there are also non-vanishing conformal weight states in the BRST cohomology. Consider, for example, the operator

$$(\lambda\gamma^m \partial\theta),$$

which is BRST-closed and have conformal weight one. In the full superstring (finite α'), it would correspond to the BRST transformation of the operator Π^m. However here,

$$[Q, \Pi^m] = (\lambda\gamma^m \partial\theta) + (\hat{\lambda}\gamma^m \partial\hat{\theta}). \quad (2.10)$$

In fact, there does not seem to exist an operator O^m such that $[Q, O^m] = (\lambda\gamma^m \partial\theta)$. The same holds for $(\hat{\lambda}\gamma^m \partial\hat{\theta})$.

JHEP05(2016)116

Usually, BRST-closed states with nonvanishing conformal weight can be argued to be BRST-exact. This follows from the fact that the energy-momentum tensor itself is BRST-trivial, i.e. this argument relies on the existence of a b ghost satisfying $\{Q, b\} = T$. For the action (2.1), the energy momentum-tensor is given by

$$T = -P_m \partial X^m - p_\alpha \partial \theta^\alpha - \hat{p}_{\hat{\alpha}} \partial \hat{\theta}^{\hat{\alpha}} - w_\alpha \partial \lambda^\alpha - \hat{w}_{\hat{\alpha}} \partial \hat{\lambda}^{\hat{\alpha}}, \tag{2.11}$$

and the known procedure to build the composite pure spinor b ghost [10, 11] does not work here. This will be clarified soon, but technically it is related to the mixing of the variables that would describe the left and right-moving sector of the finite tension superstring.

The above observation raises the question about massive states, which are usually built out of non-vanishing conformal weight fields. Since the operators of the form $\exp(ik_m X^m)$ are worldsheet scalars in the $\alpha' \to 0$ limit, in a BRST trivial energy-momentum scenario this would mean that the cohomology consists of massless states only . On the other hand, the action S has further symmetries. One of them, in particular, is generated by the particle-like Hamiltonian

$$\mathcal{H}_B = -\frac{1}{2} P_m P^m, \tag{2.12}$$

which can be interpreted as the mass operator and commutes with the BRST charge. If one requires the physical states to be annihilated by $\mathcal{H}_B$, that would automatically project out any possible massive BRST-closed state.

As it turns out, $\mathcal{H}_B$ is BRST-exact [12]. To show that, consider first the following:

$$\begin{aligned} g^\alpha &\equiv \frac{1}{4}(\gamma^m d)^\alpha P_m, \quad \{Q, g^\alpha\} = \frac{1}{2}\lambda^\alpha \mathcal{H}_B, \\ \hat{g}^{\hat{\alpha}} &\equiv \frac{1}{4}(\gamma^m \hat{d})^{\hat{\alpha}} P_m, \quad \{Q, \hat{g}^{\hat{\alpha}}\} = \frac{1}{2}\hat{\lambda}^{\hat{\alpha}} \mathcal{H}_B. \end{aligned} \tag{2.13}$$

Next, defining

$$B^+ \equiv \frac{C \cdot g}{C \cdot \lambda} + \frac{\hat{C} \cdot \hat{g}}{\hat{C} \cdot \hat{\lambda}}, \tag{2.14a}$$

$$B^- \equiv \frac{C \cdot g}{C \cdot \lambda} - \frac{\hat{C} \cdot \hat{g}}{\hat{C} \cdot \hat{\lambda}}, \tag{2.14b}$$

for any nonvanishing $(C \cdot \lambda)$ and $(\hat{C} \cdot \hat{\lambda})$, with C_α and $\hat{C}_{\hat{\alpha}}$ constant spinors, it can be demonstrated that

$$\{Q, B^+\} = \mathcal{H}_B, \tag{2.15a}$$

$$\{Q, B^-\} = 0. \tag{2.15b}$$

In particular, it implies that any BRST-closed eigenstate of $\mathcal{H}_B$ with nonzero eigenvalue is BRST-exact. The operator B^- is BRST-closed and the absence of a b ghost makes it hard to tell whether it is BRST-exact, although unlikely. The covariant versions of these operators would require the introduction of the nonminimal sector [11] and have been defined also in [13], similarly to what is done in subsection 2.3.

Altogether, these observations indicate that the original proposal of [3] might be incomplete, since the BRST cohomology is enhanced when compared to the zero-momentum spectrum of the superstring and it is not clear whether this is relevant for a well-defined worldsheet theory for supergravity.

In fact, Chandia and Vallilo [5, 6] already considered this possibility from another perspective. In an attempt to obtain the supergravity constraints from a consistent coupling of the type II background to the free action (2.1), they noticed another symmetry which led to a modification of the flat space limit and a redefinition of the supersymmetry charge and consequently the operators d_α and $\hat{d}_{\hat{\alpha}}$. This will be reviewed next.

2.2 Review of the improved BRST-charge

The key observation in [5, 6] is that the action S is also invariant under another nilpotent symmetry generated by

$$\mathcal{K} \equiv \oint \left\{ (\lambda\gamma_m\theta)\left[\partial X^m + \frac{1}{2}(\theta\gamma^m\partial\theta)\right] - (\hat{\lambda}\gamma_m\hat{\theta})\left[\partial X^m + \frac{1}{2}(\hat{\theta}\gamma^m\partial\hat{\theta})\right]\right\}. \tag{2.16}$$

Although the two terms (hatted and unhatted) above are independent symmetries of the action, only this particular combination is BRST-closed. Concerning supersymmetry, it is easy to show that $\mathcal{K}$ is supersymmetric up to BRST-exact terms:

$$\{q_\alpha, \mathcal{K}\} = \{Q, \oint (\gamma_m\theta)_\alpha \left[\partial X^m + \frac{1}{2}(\theta\gamma^m\partial\theta)\right]\}, \tag{2.17a}$$

$$\{\hat{q}_{\hat{\alpha}}, \mathcal{K}\} = -\{Q, \oint (\gamma_m\hat{\theta})_{\hat{\alpha}} \left[\partial X^m + \frac{1}{2}(\hat{\theta}\gamma^m\partial\hat{\theta})\right]\}. \tag{2.17b}$$

Based on Berkovits' suggestion that $Q+\mathcal{K}$ should be the BRST charge instead, Chandia and Vallilo made a consistent redefinition of the supersymmetry charges and supersymmetric invariants.[1] The operators d_α and $\hat{d}_{\hat{\alpha}}$ were redefined as

$$d_\alpha \equiv p_\alpha - \frac{1}{2}(P_m - \partial X_m)(\gamma^m\theta)_\alpha + \frac{1}{4}(\theta\gamma_m\partial\theta)(\gamma^m\theta)_\alpha, \tag{2.18a}$$

$$\hat{d}_{\hat{\alpha}} \equiv \hat{p}_{\hat{\alpha}} - \frac{1}{2}(P_m + \partial X_m)(\gamma^m\hat{\theta})_{\hat{\alpha}} - \frac{1}{4}(\hat{\theta}\gamma_m\partial\hat{\theta})(\gamma^m\hat{\theta})_{\hat{\alpha}}, \tag{2.18b}$$

together with the supersymmetry charges

$$q_\alpha \equiv \oint \left\{ p_\alpha + \frac{1}{2}(P_m - \partial X_m)(\gamma^m\theta)_\alpha - \frac{1}{12}(\theta\gamma_m\partial\theta)(\gamma^m\theta)_\alpha \right\}, \tag{2.19a}$$

$$\hat{q}_{\hat{\alpha}} \equiv \oint \left\{ \hat{p}_{\hat{\alpha}} + \frac{1}{2}(P_m + \partial X_m)(\gamma^m\hat{\theta})_{\hat{\alpha}} + \frac{1}{12}(\hat{\theta}\gamma_m\partial\hat{\theta})(\gamma^m\hat{\theta})_{\hat{\alpha}} \right\}. \tag{2.19b}$$

P_m is no longer invariant under supersymmetry, only the combination

$$P_m - \frac{1}{2}(\theta\gamma_m\partial\theta) + \frac{1}{2}(\hat{\theta}\gamma_m\partial\hat{\theta}).$$

[1]In fact, the action (2.1) has two other nilpotent symmetries, generated by $\mathcal{K}_1 = \oint (\lambda\gamma_m\theta)(\hat{\theta}\gamma^m\partial\hat{\theta})$ and $\mathcal{K}_2 = \oint (\hat{\lambda}\gamma_m\hat{\theta})(\theta\gamma^m\partial\theta)$, but there does not seem to be any operator redefinition consistent with $\mathcal{N} = 2$ supersymmetry that would incorporate them, as they mix the spinor chiralities.

JHEP05(2016)116

It is convenient, however, to write it in a linear combination with Π^m defined in (2.3a), introducing two other supersymmetric invariants that will appear naturally in the OPE algebra:

$$P_m^- \equiv P_m - \partial X_m - (\theta\gamma_m\partial\theta), \tag{2.20a}$$
$$P_m^+ \equiv P_m + \partial X_m + (\hat\theta\gamma_m\partial\hat\theta). \tag{2.20b}$$

The action S in (2.1) can be rewritten in terms of the newly defined operators as

$$\begin{aligned} S =& \int d^2z\Big\{\frac{1}{2}(P_m^+ + P_m^-)\overline{\Pi}^m + d_\alpha\overline{\partial}\theta^\alpha + w_\alpha\overline{\partial}\lambda^\alpha + \hat d_{\hat\alpha}\overline{\partial}\hat\theta^{\hat\alpha} + \hat w_{\hat\alpha}\overline{\partial}\hat\lambda^{\hat\alpha}\Big\} \\ &-\frac{1}{2}\int d^2z\{\Pi_m[(\theta\gamma^m\overline{\partial}\theta) - (\hat\theta\gamma^m\overline{\partial}\hat\theta)] - [(\theta\gamma^m\partial\theta) - (\hat\theta\gamma^m\partial\hat\theta)]\overline{\Pi}_m\} \\ &-\frac{1}{4}\int d^2z\{(\theta\gamma_m\partial\theta)(\hat\theta\gamma^m\overline{\partial}\hat\theta) - (\hat\theta\gamma_m\partial\hat\theta)(\theta\gamma^m\overline{\partial}\theta)\}, \end{aligned} \tag{2.21}$$

where $\overline{\Pi}^m$ is just the antiholomorphic version of Π^m. The BRST charge Q has the same form (2.4), but now with the modified d_α and $\hat d_{\hat\alpha}$ of (2.18). It is worth to point out the the integrated vertex displayed in (2.8) is no longer BRST-closed with respect to the modified charge and this is so far an unsolved issue.

The relevant OPE's for the improved set of operators can be summarized as

$$\begin{aligned} d_\alpha(z)d_\beta(y) &\sim -\frac{P_m^-\gamma^m_{\alpha\beta}}{(z-y)}, & \hat d_{\hat\alpha}(z)\hat d_{\hat\beta}(y) &\sim -\frac{P_m^+\gamma^m_{\hat\alpha\hat\beta}}{(z-y)}, \\ d_\alpha(z)P_m^-(y) &\sim -2\frac{(\gamma_m\partial\theta)_\alpha}{(z-y)}, & \hat d_{\hat\alpha}(z)P_m^+(y) &\sim 2\frac{(\gamma_m\partial\hat\theta)_{\hat\alpha}}{(z-y)}, \\ P_m^-(z)P_n^-(y) &\sim 2\frac{\eta_{mn}}{(z-y)^2}, & P_m^+(z)P_n^+(y) &\sim -2\frac{\eta_{mn}}{(z-y)^2}, \\ d_\alpha(z)\Pi^m(y) &\sim \frac{(\gamma_m\partial\theta)_\alpha}{(z-y)}, & \hat d_{\hat\alpha}(z)\Pi^m(y) &\sim \frac{(\gamma_m\partial\hat\theta)_{\hat\alpha}}{(z-y)}, \\ P_m^-(z)\Pi^n(y) &\sim -\frac{\delta_m^n}{(z-y)^2}, & P_m^+(z)\Pi^n(y) &\sim -\frac{\delta_m^n}{(z-y)^2}. \end{aligned} \tag{2.22}$$

Notice that there is a clear splitting and the two sectors $\{P_m - \partial X_m, p_\alpha, \theta^\alpha, w_\alpha, \lambda^\alpha\}$ and $\{P_m + \partial X_m, \hat p_{\hat\alpha}, \hat\theta^{\hat\alpha}, \hat w_{\hat\alpha}, \hat\lambda^{\hat\alpha}\}$ are "decoupled". Next subsection will extend this idea and introduce the pure spinor b ghost for the type II ambitwistor string.

2.3 Holomorphic sectorization and the b ghost

The proposal of [5, 6] splits the chiral action S in two sectors which emulate the would-be left and right-moving sectors of the superstring. It can be shown that this feature easily solves the cohomology issues discussed in subsection 2.1. In fact it enables a very simple construction for the composite b ghost. To do that, the two sectors have to be better understood.

It is interesting to observe, for example, that the energy-momentum tensor of (2.11) can be written in a way that makes this splitting explicit. Using the operators defined in (2.18) and (2.20), T is written as

$$T = T_+ + T_-, \tag{2.23}$$

where

$$T_- \equiv \frac{1}{4}\eta^{mn} P^-_m P^-_n - d_\alpha \partial\theta^\alpha - w_\alpha \partial\lambda^\alpha, \tag{2.24a}$$

$$T_+ \equiv -\frac{1}{4}\eta^{mn} P^+_m P^+_n - \hat{d}_{\hat\alpha} \partial\hat\theta^{\hat\alpha} - \hat{w}_{\hat\alpha} \partial\hat\lambda^{\hat\alpha}. \tag{2.24b}$$

Both T_- and T_+ are BRST-closed and can be viewed as *fake* anomaly-free energy-momentum tensors for each sector:[2]

$$T_-(x)T_-(y) \sim \frac{2T_-}{(z-y)^2} + \frac{\partial T_-}{(z-y)}, \tag{2.25a}$$

$$T_+(x)T_+(y) \sim \frac{2T_+}{(z-y)^2} + \frac{\partial T_+}{(z-y)}, \tag{2.25b}$$

$$T_-(x)T_+(y) \sim \text{regular}. \tag{2.25c}$$

Note that $\mathcal{H}_B$ is not BRST-closed with respect to the new BRST-charge, which comes from the fact that $[\mathcal{K}, \mathcal{H}_B] \neq 0$ in subsection 2.2. However, one can define

$$\mathcal{H}_{CV} \equiv T_+ - T_-, \tag{2.26}$$

which is interpreted as a generalization of $\mathcal{H}_B$ in (2.12) [5, 6]. A natural question is whether $\mathcal{H}_{CV}$ is BRST-exact or not. If so, given the sectorization so far observed, it is likely that both T_+ and T_- are BRST-exact, leading to a trivialization of the energy-momentum tensor.

Motivated by the original proposal for the pure spinor b ghost [10], one can define the operators

$$G^\alpha \equiv -\frac{1}{4}\eta^{mn}\gamma^{\alpha\beta}_m (d_\beta, P^-_m) - \frac{1}{4}N_{mn}(\gamma^{mn}\partial\theta)^\alpha - \frac{1}{4}J\partial\theta^\alpha - \partial^2\theta^\alpha, \tag{2.27a}$$

$$\hat{G}^{\hat\alpha} \equiv \frac{1}{4}\eta^{mn}\gamma^{\hat\alpha\hat\beta}_m (\hat{d}_{\hat\beta}, P^+_m) - \frac{1}{4}\hat{N}_{mn}(\gamma^{mn}\partial\hat\theta)^{\hat\alpha} - \frac{1}{4}\hat{J}\partial\hat\theta^{\hat\alpha} - \partial^2\hat\theta^{\hat\alpha}. \tag{2.27b}$$

N_{mn} and $\hat{N}_{mn}$ are ghost Lorentz currents displayed in (2.9), and J and $\hat{J}$ are the ghost number currents:

$$J \equiv -w\cdot\lambda, \quad \hat{J} \equiv -\hat{w}\cdot\hat\lambda. \tag{2.28}$$

Observe that one has to take into account quantum effects of non-commuting operators and the ordering prescription that will be adopted here is

$$(A, B)(y) \equiv \frac{1}{2\pi i}\oint \frac{dz}{z-y} A(z)B(y). \tag{2.29}$$

[2] One has to be careful with this interpretation because only the linear combination in (2.23) has the expected properties of a energy-momentum tensor when acting on X^m or P_m, e.g.

$$T_+(x)\,X^m(y) \sim \tfrac{1}{2}\tfrac{(\partial X^m + P^m)}{(z-y)}, \quad T_-(x)\,X^m(y) \sim \tfrac{1}{2}\tfrac{(\partial X^m - P^m)}{(z-y)}.$$

JHEP05(2016)116

It is straightforward to show that the operators in (2.27) satisfy the following properties,

$$\begin{aligned}\{Q, G^{\alpha}\} &= (\lambda^{\alpha}, T_{-}),\\ \{Q, \hat{G}^{\hat{\alpha}}\} &= (\hat{\lambda}^{\hat{\alpha}}, T_{+}),\end{aligned}$$

resembling the usual holomorphic construction.

In order to present a covariant version of the b ghost, the known chain of operators introduced in [10, 11] will be mirrored here. In fact there is little to change, only some overall factors. These operators are defined as

$$H^{\alpha\beta} \equiv -\frac{1}{768}\gamma^{\alpha\beta}_{mnp}(d\gamma^{mnp}d + 24N^{mn}\eta^{pq}P^{-}_{q}), \tag{2.31a}$$

$$\hat{H}^{\hat{\alpha}\hat{\beta}} \equiv \frac{1}{768}\gamma^{\hat{\alpha}\hat{\beta}}_{mnp}(\hat{d}\gamma^{mnp}\hat{d} + 24\hat{N}^{mn}\eta^{pq}P^{+}_{q}), \tag{2.31b}$$

$$K^{\alpha\beta\gamma} \equiv \frac{1}{192}N_{mn}\gamma^{[\alpha\beta}_{mnp}(\gamma^{p}d)^{\gamma]}, \tag{2.31c}$$

$$\hat{K}^{\hat{\alpha}\hat{\beta}\hat{\gamma}} \equiv -\frac{1}{192}\hat{N}_{mn}\gamma^{[\hat{\alpha}\hat{\beta}}_{mnp}(\gamma^{p}\hat{d})^{\hat{\gamma}]}, \tag{2.31d}$$

$$L^{\alpha\beta\gamma\lambda} \equiv \frac{1}{6144}(N^{mn}, N^{rs})\eta^{pq}\gamma^{[\alpha\beta}_{mnp}\gamma^{\gamma]\lambda}_{qrs}, \tag{2.31e}$$

$$\hat{L}^{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\lambda}} \equiv -\frac{1}{6144}(\hat{N}^{mn}, \hat{N}^{rs})\eta^{pq}\gamma^{[\hat{\alpha}\hat{\beta}}_{mnp}\gamma^{\hat{\gamma}]\hat{\lambda}}_{qrs}, \tag{2.31f}$$

and satisfy

$$\begin{aligned}[]
[Q, H^{\alpha\beta}] &= (\lambda^{[\alpha}, G^{\beta]}), & [Q, \hat{H}^{\hat{\alpha}\hat{\beta}}] &= (\hat{\lambda}^{[\hat{\alpha}}, G^{\hat{\beta}]}),\\
\{Q, K^{\alpha\beta\gamma}\} &= (\lambda^{[\alpha}, H^{\beta\gamma]}), & \{Q, \hat{K}^{\hat{\alpha}\hat{\beta}\hat{\gamma}}\} &= (\hat{\lambda}^{[\hat{\alpha}}, \hat{H}^{\hat{\beta}\hat{\gamma}]}),\\
[Q, L^{\alpha\beta\gamma\lambda}] &= (\lambda^{[\alpha}, K^{\beta\gamma\lambda]}), & [Q, \hat{L}^{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\lambda}}] &= (\hat{\lambda}^{[\hat{\alpha}}, \hat{K}^{\hat{\beta}\hat{\gamma}\hat{\lambda}]}),\\
(\lambda^{[\alpha}, L^{\beta\gamma\lambda\sigma]}) &= 0, & (\hat{\lambda}^{[\hat{\alpha}}, \hat{L}^{\hat{\beta}\hat{\gamma}\hat{\lambda}\hat{\sigma}]}) &= 0.
\end{aligned} \tag{2.32}$$

The square brackets denote indices antisymmetrization and it can be read as

$$[\alpha_1 \ldots \alpha_n] = \frac{1}{n!}\left(\alpha_1 \ldots \alpha_n + \text{all antisymmetric permutations}\right). \tag{2.33}$$

The next step is to introduce the non-minimal variables of [11], which enter the action as

$$S_{nm} = \int d^2z\{\overline{w}^{\alpha}\overline{\partial}\overline{\lambda}_{\alpha} + s^{\alpha}\overline{\partial}r_{\alpha} + \hat{\overline{w}}^{\hat{\alpha}}\overline{\partial}\hat{\overline{\lambda}}_{\hat{\alpha}} + \hat{s}^{\hat{\alpha}}\overline{\partial}\hat{r}_{\hat{\alpha}}\}, \tag{2.34}$$

with energy-momentum tensor

$$T_{nm} = -\overline{w}^{\alpha}\partial\overline{\lambda}_{\alpha} - s^{\alpha}\partial r_{\alpha} - \hat{\overline{w}}^{\hat{\alpha}}\partial\hat{\overline{\lambda}}_{\hat{\alpha}} - \hat{s}^{\hat{\alpha}}\partial\hat{r}_{\hat{\alpha}}. \tag{2.35}$$

The variables $\overline{\lambda}_{\alpha}$ and $\hat{\overline{\lambda}}_{\hat{\alpha}}$ are also pure spinors while r_{α} and $\hat{r}_{\hat{\alpha}}$ are anticommuting spinors satisfying the constraints $(\overline{\lambda}\gamma^{m}r) = 0$ and $(\hat{\overline{\lambda}}\gamma^{m}\hat{r}) = 0$. The BRST charge is modified accordingly,

$$J_{\text{BRST}} \equiv \lambda^{\alpha}d_{\alpha} + \hat{\lambda}^{\hat{\alpha}}\hat{d}_{\hat{\alpha}} + \overline{w}^{\alpha}r_{\alpha} + \hat{\overline{w}}^{\hat{\alpha}}\hat{r}_{\hat{\alpha}}, \tag{2.36a}$$

$$Q \equiv \oint J_{\text{BRST}}, \tag{2.36b}$$

JHEP05(2016)116

but this does not affect the previous cohomology because any dependence on the non-minimal variables can be gauged away (quartet argument).

The final step is the definition of b_- and b_+ as

$$\begin{aligned} b_- =& \left(\frac{\bar\lambda_\alpha}{(\bar\lambda\cdot\lambda)}, G^\alpha\right) - 2!\left(\frac{\bar\lambda_\alpha r_\beta}{(\bar\lambda\cdot\lambda)^2}, H^{\alpha\beta}\right) - 3!\left(\frac{\bar\lambda_\alpha r_\beta r_\gamma}{(\bar\lambda\cdot\lambda)^3}, K^{\alpha\beta\gamma}\right) \\ &+4!\left(\frac{\bar\lambda_\alpha r_\beta r_\gamma r_\lambda}{(\bar\lambda\cdot\lambda)^4}, L^{\alpha\beta\gamma\lambda}\right) - s^\alpha\partial\bar\lambda_\alpha - \partial\left(\frac{\bar\lambda_\alpha\bar\lambda_\beta}{(\bar\lambda\cdot\lambda)^2}\right)\lambda^\alpha\partial\theta^\beta, \end{aligned} \tag{2.37}$$

and

$$\begin{aligned} b_+ =& \left(\frac{\hat{\bar\lambda}_{\hat\alpha}}{(\hat{\bar\lambda}\cdot\hat\lambda)}, \hat G^{\hat\alpha}\right) - 2!\left(\frac{\hat{\bar\lambda}_\alpha \hat r_\beta}{(\hat{\bar\lambda}\cdot\hat\lambda)^2}, \hat H^{\hat\alpha\hat\beta}\right) - 3!\left(\frac{\hat{\bar\lambda}_{\hat\alpha}\hat r_{\hat\beta}\hat r_{\hat\gamma}}{(\hat{\bar\lambda}\cdot\hat\lambda)^3}, \hat K^{\hat\alpha\hat\beta\hat\gamma}\right) \\ &+4!\left(\frac{\hat{\bar\lambda}_{\hat\alpha}\hat r_{\hat\beta}\hat r_{\hat\gamma}\hat r_{\hat\lambda}}{(\hat{\bar\lambda}\cdot\hat\lambda)^4}, \hat L^{\hat\alpha\hat\beta\hat\gamma\hat\lambda}\right) - \hat s^{\hat\alpha}\partial\hat{\bar\lambda}_{\hat\alpha} - \partial\left(\frac{\hat{\bar\lambda}_{\hat\alpha}\hat{\bar\lambda}_{\hat\beta}}{(\hat{\bar\lambda}\cdot\hat\lambda)^2}\right)\hat\lambda^{\hat\alpha}\partial\hat\theta^{\hat\beta}. \end{aligned} \tag{2.38}$$

The last terms in b_- and b_+ are quantum ordering contributions.

The operators b_- and b_+ anticommute with the BRST charge Q to give the non-minimal version of T_- and T_+:

$$\begin{aligned} \{Q, b_-\} &= T_- - \bar w^\alpha\partial\bar\lambda_\alpha - s^\alpha\partial r_\alpha, \\ &\equiv \mathcal{T}_- \end{aligned} \tag{2.39a}$$

$$\begin{aligned} \{Q, b_+\} &= T_+ - \hat{\bar w}^{\hat\alpha}\partial\hat{\bar\lambda}_{\hat\alpha} - \hat s^{\hat\alpha}\partial\hat r_{\hat\alpha}. \\ &\equiv \mathcal{T}_+ \end{aligned} \tag{2.39b}$$

The demonstration of (2.39) is a bit lengthy because of the reordering operations. Using the operators chain of (2.27) and (2.31), the b ghost defined by

$$b \equiv b_- + b_+, \tag{2.40}$$

can be shown to satisfy

$$\{Q, b\} = \mathcal{T}_+ + \mathcal{T}_-, \tag{2.41}$$

which is equal to the energy momentum tensor of the action $S + S_{nm}$,

$$\begin{aligned} \mathcal{T} =& -P_m\partial X^m - p_\alpha\partial\theta^\alpha - \hat p_{\hat\alpha}\partial\hat\theta^{\hat\alpha} - w_\alpha\partial\lambda^\alpha - \hat w_{\hat\alpha}\partial\hat\lambda^{\hat\alpha} \\ &-\bar w^\alpha\partial\bar\lambda_\alpha - s^\alpha\partial r_\alpha - \hat{\bar w}^{\hat\alpha}\partial\hat{\bar\lambda}_{\hat\alpha} - \hat s^{\hat\alpha}\partial\hat r_{\hat\alpha}. \end{aligned} \tag{2.42}$$

The existence of the b ghost (2.40) ensures that the BRST cohomology is composed of worldsheet scalars only, excluding the extra states described in subsection 2.1. Therefore, BRST-closed massive states are unequivocally BRST-exact.

Observe that the operator $\mathcal{H}_{CV}$ defined in (2.26) can be rewritten as

$$\mathcal{H}_{CV} = \{Q, (b_+ - b_- + \hat s^{\hat\alpha}\partial\hat{\bar\lambda}_{\hat\alpha} - s^\alpha\partial\bar\lambda_\alpha)\}, \tag{2.43}$$

JHEP05(2016)116

but once the non-minimal sector is included, it makes sense to define

$$\mathcal{H} \equiv \mathcal{T}_+ - \mathcal{T}_-, \tag{2.44}$$

which is also BRST-exact.

The properties of b_- and b_+ are now easy to determine because they have the same structure of the the composite b ghost of [11]. Nilpotency, for example, follows from the same arguments of [14, 15] and it can be shown that

$$b_\pm(z) b_\pm(y) \sim 0.$$

Clearly, the OPE $b_+(z)b_-(y)$ is also regular, but this follows from the sector splitting. With respect to the BRST current, the OPE's with $b_\pm$ are computed to be

$$J_{\text{BRST}}(z) b_\pm(y) \sim \frac{3}{(z-y)^3} + \frac{J_\pm}{(z-y)^2} + \frac{\mathcal{T}_\pm}{(z-y)},$$

where J_- and J_+ are interpreted as the ghost number currents for each sector, defined as

$$J_- \equiv J + r_\alpha s^\alpha - 2\frac{(\bar\lambda \cdot \partial\lambda)}{(\bar\lambda\cdot\lambda)} + 2\frac{(r\cdot\partial\theta)}{(\bar\lambda\cdot\lambda)} - 2\frac{(r\cdot\lambda)(\bar\lambda\cdot\partial\theta)}{(\bar\lambda\cdot\lambda)^2} \tag{2.45a}$$

$$J_+ \equiv \hat{J} + \hat{r}_{\hat\alpha} \hat{s}^{\hat\alpha} - 2\frac{(\hat{\bar\lambda} \cdot \partial\hat\lambda)}{(\hat{\bar\lambda}\cdot\hat\lambda)} + 2\frac{(\hat{r}\cdot\partial\hat\theta)}{(\hat{\bar\lambda}\cdot\hat\lambda)} - 2\frac{(\hat{r}\cdot\hat\lambda)(\hat{\bar\lambda}\cdot\partial\hat\theta)}{(\hat{\bar\lambda}\cdot\hat\lambda)^2}. \tag{2.45b}$$

The unusual terms in $J_\pm$ are BRST-exact [11] and can in fact be eliminated by a BRST transformation of the b ghost [16]. The ghost number currents have the following OPE's:

$$\mathcal{T}_\pm(z) J_\pm(y) \sim -\frac{3}{(z-y)^3} + \frac{J_\pm}{(z-y)^2} + \frac{\partial J_\pm}{(z-y)}, \tag{2.46}$$

$$J_\pm(z) J_\pm(y) \sim \frac{3}{(z-y)^2}. \tag{2.47}$$

Altogether, these results can be summarized as

$$b(z)b(y) \sim 0, \tag{2.48a}$$

$$J_{\text{BRST}}(z) b(y) \sim \frac{6}{(z-y)^3} + \frac{J_g}{(z-y)^2} + \frac{\mathcal{T}}{(z-y)}, \tag{2.48b}$$

$$J_g(z) J_g(y) \sim \frac{6}{(z-y)^2}, \tag{2.48c}$$

$$\mathcal{T}(z) J_g(y) \sim -\frac{6}{(z-y)^3} + \frac{J_g}{(z-y)^2} + \frac{\partial J_g}{(z-y)}, \tag{2.48d}$$

with

$$J_g \equiv J_- + J_+ \tag{2.49}$$

defined as the *total* ghost number current.

The equations displayed in (2.48) resemble the $\mathcal{N}=2$ topological algebra of [11] but now with $\hat{c}=6$ and no antiholomorphic currents.

In the next section the heterotic ambitwistor string will be discussed.

JHEP05(2016)116

3 The heterotic ambitwistor pure spinor string

In [3], Berkovits also introduced the infinite tension limit of the heterotic pure spinor superstring. The chiral action is given by

$$S = \int d^2z \{P_m \overline{\partial} X^m + p_\alpha \overline{\partial}\theta^\alpha + w_\alpha \overline{\partial}\lambda^\alpha + \overline{b}\overline{\partial}\overline{c} + \mathcal{L}_C\}, \tag{3.1}$$

where $(\overline{b}, \overline{c})$ is the known Virasoro ghost pair for the heterotic string. $\mathcal{L}_C$ accounts for the Lagrangian of the SO(32) or $E(8) \times E(8)$ current algebra with central charge 16 and (holomorphic) generators J^I, with I denoting the adjoint representation of the gauge group. The action S is invariant under the $\mathcal{N} = 1$ supersymmetry transformations generated by the charge

$$q_\alpha = \oint \left\{ p_\alpha + \frac{1}{2} P_m (\gamma^m \theta)_\alpha \right\}. \tag{3.2}$$

The heterotic pure spinor BRST charge was proposed to be

$$Q = \oint \{\lambda^\alpha d_\alpha + \overline{c}(-P_m \Pi^m - d_\alpha \partial\theta^\alpha - w_\alpha \partial\lambda^\alpha - \overline{b}\partial\overline{c} + T_C)\}, \tag{3.3}$$

where

$$\Pi^m = \partial X^m + \frac{1}{2}(\theta\gamma^m\partial\theta), \tag{3.4}$$

d_α is the same of (2.3b) and T_C is the energy-momentum tensor associated to $\mathcal{L}_C$. The full energy-momentum tensor is given by

$$T = -P_m \partial X^m - p_\alpha \partial\theta^\alpha - w_\alpha \partial\lambda^\alpha - \overline{b}\partial\overline{c} - \partial(\overline{b}\overline{c}) + T_C. \tag{3.5}$$

The massless spectrum of the heterotic string includes the non-abelian super Yang-Mills fields and $\mathcal{N} = 1$ supergravity. The former can be encoded by the vertex operator

$$U_{SYM} = \lambda^\alpha \overline{c} A^I_\alpha(\theta) J^I e^{ik\cdot X}, \tag{3.6}$$

where $A^I_\alpha(\theta)$ satisfies

$$D_\alpha \gamma^{\alpha\beta}_{mnpqr} A^I_\beta = 0. \tag{3.7}$$

The gauge transformations of U_{SYM} are described by the BRST-exact operator

$$\begin{aligned} \delta U_{SYM} &\equiv \{Q, \overline{c}\Lambda^I(\theta) J^I e^{ik\cdot X}\}, \\ &= \lambda^\alpha \overline{c}(D_\alpha \Lambda^I) J^I e^{ik\cdot X}. \end{aligned} \tag{3.8}$$

As for the supergravity states, the natural guess for the vertex operator would be

$$U_{SG} = \lambda^\alpha \overline{c} A^m_\alpha(\theta) P_m e^{ik\cdot X}. \tag{3.9}$$

BRST-closedness of U_{SG} implies

$$D_\alpha \gamma^{\alpha\beta}_{mnpqr} A^s_\beta = 0, \tag{3.10a}$$
$$k_m A^m_\alpha = 0, \tag{3.10b}$$

which are the usual equations for the supergravity field A^m_α. However, the expected gauge transformation $\delta A^m_\alpha = D_\alpha \Lambda^m + k^m \Lambda_\alpha$ does not come from a BRST-exact state:

$$\begin{aligned}\delta U_{SG} &\equiv \lambda^\alpha \bar{c}(D_\alpha \Lambda^m + k^m \Lambda_\alpha) P_m e^{ik\cdot X},\\ &\neq \{Q, \text{something}\}.\end{aligned} \tag{3.11}$$

Therefore, the vertex (3.9) does not seem to properly describe the heterotic supergravity spectrum [3].

Next, motivated by the work of Chandia and Vallilo and the analysis of the previous section, a new BRST charge for the action (3.1) will be presented. The BRST cohomology will be shown to correctly describe the massless spectrum of the heterotic string and the correspondent b ghost will be constructed.

3.1 New proposal for the BRST charge

The action (3.1) also has a nilpotent symmetry that commutes with the BRST charge (3.3), generated by

$$\mathcal{K} = \oint (\lambda\gamma_m\theta)\left[\partial X^m + \frac{1}{2}(\theta\gamma^m\partial\theta)\right]. \tag{3.12}$$

Therefore, there should be an analogous procedure to absorb this symmetry and redefine the BRST charge consistently.

First, the supersymmetry charge will be redefined as

$$q_\alpha = \oint \left\{ p_\alpha + \frac{1}{2}(P_m - \partial X_m)(\gamma^m\theta)_\alpha - \frac{1}{12}(\theta\gamma_m\partial\theta)(\gamma^m\theta)_\alpha \right\}, \tag{3.13}$$

exactly like in subsection 2.2, which provides the supersymmetric invariants:

$$d_\alpha = p_\alpha - \frac{1}{2}P_m(\gamma^m\theta)_\alpha + \frac{1}{2}\Pi^m(\gamma_m\theta)_\alpha, \tag{3.14a}$$

$$P^-_m = P_m - \partial X_m - (\theta\gamma_m\partial\theta), \tag{3.14b}$$

$$P^+_m = P_m + \partial X_m. \tag{3.14c}$$

Note that $P^\pm_m$ and Π^m are not all independent as $P^+_m = P^-_m + 2\Pi_m$, cf. equation (3.4). In terms of the new invariants, the action can be rewritten as

$$\begin{aligned}S = &\int d^2z \left\{ \frac{1}{2}(P^+_m + P^-_m)\overline{\Pi}^m + d_\alpha\bar\partial\theta^\alpha + \omega_\alpha\bar\partial\lambda^\alpha + \bar{b}\partial\bar{c} + \mathcal{L}_C \right\}\\ &-\frac{1}{2}\int d^2z \{\Pi^m(\theta\gamma_m\bar\partial\theta) - \overline{\Pi}^m(\theta\gamma_m\partial\theta)\},\end{aligned} \tag{3.15}$$

and the relevant non-regular OPE's are simply

$$\begin{aligned}
d_\alpha(z)d_\beta(y) &\sim -\frac{P^-_m\gamma^m_{\alpha\beta}}{(z-y)}, & P^\pm_m(z)P^\pm_n(y) &\sim \mp 2\frac{\eta_{mn}}{(z-y)^2},\\
d_\alpha(z)P^-_m(y) &\sim -2\frac{(\gamma_m\partial\theta)_\alpha}{(z-y)}, & d_\alpha(z)\Pi^m(y) &\sim \frac{(\gamma_m\partial\theta)_\alpha}{(z-y)},\\
P^-_m(z)\Pi^n(y) &\sim -\frac{\delta^n_m}{(z-y)^2}, & P^+_m(z)\Pi^n(y) &\sim -\frac{\delta^n_m}{(z-y)^2}.
\end{aligned} \tag{3.16}$$

The analogy with the type II case can be pushed further and a similar sectorization can be shown to hold in the heterotic case. The energy-momentum of (3.5) can be cast as

$$T = T_+ + T_-, \tag{3.17}$$

where

$$T_- \equiv \frac{1}{4}\eta^{mn}P^-_m P^-_n - d_\alpha\partial\theta^\alpha - w_\alpha\partial\lambda^\alpha, \tag{3.18a}$$
$$T_+ \equiv -\frac{1}{4}\eta^{mn}P^+_m P^+_n + T_C - \bar{b}\partial\bar{c} - \partial(\bar{b}\bar{c}), \tag{3.18b}$$

satisfying the same set of OPE's of (2.25).

As before, the new BRST current should naturally incorporate this splitting and will be defined as

$$J_{\text{BRST}} \equiv \lambda^\alpha d_\alpha + \bar{c}\Big(-\frac{1}{4}\eta^{mn}P^+_m P^+_n + T_C - \bar{b}\partial\bar{c}\Big) + \frac{3}{2}\partial^2\bar{c}, \tag{3.19}$$

cf. (3.14). The last term is introduced to make J_{BRST} a conformal primary operator, but disappears in the BRST charge $Q = \oint J_{\text{BRST}}$, such that:

$$Q = \oint \{\lambda^\alpha d_\alpha + \bar{c}T_+ - \bar{b}\bar{c}\partial\bar{c}\}. \tag{3.20}$$

It is straightforward to show that the action is invariant under the BRST transformations generated by (3.20).

3.2 BRST cohomology and the semi-composite b ghost

Concerning the cohomology of the BRST charge of (3.20), only a minor modification is required. The super Yang-Mills states are still described by the vertex U_{SYM} and gauge transformation δU_{SYM} displayed in (3.6) and (3.8) respectively. On the other hand, the $\mathcal{N}=1$ supergravity vertex will be corrected to

$$U_{SG} = \lambda^\alpha\bar{c}A^m_\alpha(\theta)P^+_m e^{ik\cdot X}. \tag{3.21}$$

BRST-closedness will again provide the equations displayed in (3.10). The gauge transformations of U_{SG} are given in terms of BRST-exact states of the form

$$[Q,\Lambda] = \lambda^\alpha\bar{c}P^+_m(D_\alpha\Lambda^m + ik^m\Lambda_\alpha)e^{ik\cdot X}, \tag{3.22}$$

where $\Lambda = 2\lambda^\alpha\Lambda_\alpha - \bar{c}P^+_m\Lambda^m$ and $\lambda^\alpha\lambda^\beta D_\alpha\Lambda_\beta = k_m\Lambda^m = 0$.

Defining $\mathbb{A}^m_\alpha(X,\theta) \equiv A^m_\alpha e^{ik\cdot X}$, the superfield equations of motion of U_{SG} can be cast as

$$\gamma^{\alpha\beta}_{mnpqr}D_\beta\mathbb{A}^m_\alpha = 0, \tag{3.23a}$$
$$\partial^n\partial_n\mathbb{A}^m_\alpha - \partial^m\partial_n\mathbb{A}^n_\alpha = 0, \tag{3.23b}$$

with gauge transformations given by

$$\delta\mathbb{A}^m_\alpha = D_\alpha\Lambda^m + \partial^m\Lambda_\alpha. \tag{3.24}$$

JHEP05(2016)116

As long as the gauge parameters satisfy

$$D\gamma_{mnpqr}\Lambda = 0, \tag{3.25a}$$

$$\partial^n\partial_n\Lambda^m - \partial^m\partial_n\Lambda^n = 0, \tag{3.25b}$$

(3.24) can be seen as a BRST transformation of U_{SG}, as opposed to (3.11) in the original formulation. Note that even with this improvement with respect to Berkovits' proposal, it is not clear whether the supergravity theory can be recovered through these vertices. In the RNS ambitwistor string of [1], the tree level amplitudes in the heterotic theory could not be interpreted in terms of standard space-time gravity. This has yet to be clarified here and will be left for a future work.

The absence of massive states in the cohomology is ensured by the existence of a semi-composite b ghost. While in [3] the fundamental $\bar{b}$ fits the role of such operator, the modifications introduced in the BRST charge (3.20) imply the sectorization of the new b ghost as well. Note that $\{Q, \bar{b}\} = T_+$, i.e. only part of the energy-momentum tensor (3.5). Defining

$$b \equiv \bar{b} + b_-, \tag{3.26}$$

where b_- has the same form of (2.37) in terms of the non-minimal variables, it is direct to show that $\{Q, b\} = \mathcal{T}$, with

$$\begin{aligned}\mathcal{T} = &-P_m\partial X^m - p_\alpha\partial\theta^\alpha - w_\alpha\partial\lambda^\alpha + T_C \\ &-\bar{b}\partial\bar{c} - \partial(\bar{b}\bar{c}) - \overline{w}^\alpha\partial\overline{\lambda}_\alpha - s^\alpha\partial r_\alpha.\end{aligned} \tag{3.27}$$

The heterotic b ghost consists of a fundamental part $\bar{b}$ and the usual (pure spinor) composite one, b_-.

For completeness, the heterotic case can be shown to have a similar OPE set as the one displayed in (2.48),

$$b(z)b(y) \sim 0, \tag{3.28a}$$

$$J_{\text{BRST}}(z)b(y) \sim \frac{6}{(z-y)^3} + \frac{J_g}{(z-y)^2} + \frac{\mathcal{T}}{(z-y)}, \tag{3.28b}$$

$$J_g(z)J_g(y) \sim \frac{4}{(z-y)^2}, \tag{3.28c}$$

$$\mathcal{T}(z)J_g(y) \sim -\frac{6}{(z-y)^3} + \frac{J_g}{(z-y)^2} + \frac{\partial J_g}{(z-y)}, \tag{3.28d}$$

with

$$J_g \equiv J_- + cb, \tag{3.29}$$

where J_- is defined in (2.45a).

4 Final remarks

The results presented in [5, 6] and developed here have to be further explored, but it is interesting to see that the sectorization of the holomorphic $\alpha' \to 0$ limit of the pure spinor

JHEP05(2016)116

superstring describes the expected massless spectrum in a very simple way and enables a natural definition for the composite b ghost.

The geometrical interpretation of (b_+,b_-) for type II and $(\overline{b},b_-)$ for heterotic theories is not clear yet. The ideas of [17] might shed some light in the pure spinor construction, since there the 1-loop scattering equations of the ambitwistor formulation for the RNS string were studied in detail. To make it more precise, notice that in [1], the operators b and $\tilde{b}$ satisfy

$$\{Q,b\} = T, \tag{4.1a}$$

$$\{Q,\tilde{b}\} = \frac{1}{2}P_m P^m, \tag{4.1b}$$

while in the pure spinor case discussed here one has

$$\{Q,b\} = T, \tag{4.2a}$$

$$\begin{aligned}\{Q,\tilde{b}\} &= T_+ - T_-,\\ &= \frac{1}{2}P_m P^m + \dots .\end{aligned} \tag{4.2b}$$

The operator $\tilde{b}$ is defined as

$$\tilde{b}_{\mathrm{II}} \equiv b_+ - b_-, \tag{4.3a}$$

$$\tilde{b}_{\mathrm{het}} \equiv \overline{b} - b_-, \tag{4.3b}$$

according to the results of subsections 2.3 and 3.2. Since the parallel is clear, a natural step now would be to investigate the consistency (e.g. modular invariance) of the 1-loop amplitude prescription in the same line of [17] with the adequate identifications. In [13], an amplitude prescription was presented following Berkovits' proposal [3]. There, because of the absence of a true b ghost satisfying $\{Q,b\} = T$, BRST-invariance of the amplitude does not have the usual surface terms in the *moduli* space integration but is achieved through the $\overline{\delta}(P^2)$ insertions proposed in [17], much like BRST invariance of Berkovits' integrated vertex depends on the $\overline{\delta}(k \cdot P)$ operator (see equation (2.8)). It would be interesting to have an alternative prescription using the sectorized construction and to compare both approaches.

From another perspective, the operators $\tilde{b}_{\mathrm{II}}$ and $\tilde{b}_{\mathrm{het}}$ of (4.3) seem to provide the analogous of the physical state condition in the closed string,

$$(b_L)_0 - (b_R)_0 \,|\psi\rangle = 0. \tag{4.4}$$

The index 0 denotes the zero mode of the left and right-moving b ghost of the closed string. Physical states here will then be defined as elements of the BRST cohomology satisfying

$$(\tilde{b}_{\mathrm{II}})_0 \,|\psi\rangle \approx 0, \tag{4.5a}$$

$$(\tilde{b}_{\mathrm{het}})_0 \,|\psi\rangle \approx 0. \tag{4.5b}$$

The symbol $\approx$ means equal up to BRST-exact terms.

The integrated form of the vertex operators is still lacking, but the sectorized b ghost operators ($b_\pm$ and $\bar{b}$) might play an important role. In [17] there is a direct relation between the integrated and the unintegrated vertices through b and $\tilde{b}$ insertions. It is very likely that a similar construction can be found here to build the integrated vertices associated to (2.5), (3.6) and (3.21). This idea has to be further developed and certainly deserves more attention for a precise formulation of the ambitwistor string in the pure spinor formalism.

Last, concerning the heterotic case, there is a very straightforward test for the new BRST charge proposed in (3.20). The heterotic action

$$\begin{aligned} S_{\text{het}} = & \int d^2z \Big\{ \frac{1}{2}(P_m^+ + P_m^-)\overline{\Pi}^m + d_\alpha \bar{\partial}\theta^\alpha + \omega_\alpha \bar{\partial}\lambda^\alpha + b\bar{\partial}c + \mathcal{L}_C \Big\} \\ & -\frac{1}{2}\int d^2z \{ \Pi^m (\theta\gamma_m \bar{\partial}\theta) - \overline{\Pi}^m (\theta\gamma_m\partial\theta) \}, \end{aligned} \tag{4.6}$$

given in terms of the redefined supersymmetric invariants of (3.14), naturally presents the coupling with the Kalb-Ramond field in the zero-momentum limit (second line), analogous to Chandia and Vallilo's proposal for the type II case [5, 6]. Therefore, the curved background embedding of S_{het} should provide the heterotic supergravity constraints of [7] through a sensible curved space generalization of $T_\pm$ [18]. As mentioned in section 3, the RNS ambitwistor string does not provide the expected supergravity amplitudes. A similar analysis will have to be performed here. It seems, however, that one might expect similar results and possible inconsistencies could show up in determining the supergravity constraints.

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JHEP05(2016)116