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What can we learn about the lepton CP phase in the next 10 years?

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ABSTRACT: We discuss how the lepton CP phase can be constrained by accelerator and reactor measurements in an era without dedicated experiments for CP violation search. To characterize globally the sensitivity to the CP phase δ_{CP} , we use the CP exclusion fraction, which quantifies what fraction of the δ_{CP} space can be excluded at given input values of θ_{23} and δ_{CP} . Using the measure we study the CP sensitivity which may be possessed by the accelerator experiments T2K and NO ν A. We show that, if the mass hierarchy is known, T2K and NO ν A alone may exclude, respectively, about 50%–60% and 40%–50% of the δ_{CP} space at 90% CL by 10 years running, provided that a considerable fraction of beam time is devoted to the antineutrino run. The synergy between T2K and NO ν A is remarkable, leading to the determination of the mass hierarchy through CP sensitivity at the same CL.

KEYWORDS: Neutrino Physics, CP violation

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1 Introduction

After accumulating hints and indications, the elusive lepton mixing angle θ_{13} was finally discovered to be non-zero and measured with high precision [1–9]. Thus, we are left with the CP violating phase δ_{CP} , the unique unknown parameter in the lepton flavor mixing matrix [10], which could remain a mystery for sometime together with the problem of determining the neutrino mass hierarchy. Lepton CP violation due to δ_{CP} , in association with the one by the possible Majorana phases, may hide the secret behind the baryon number asymmetry in our universe [11]. However, because of the smallness of the effects of δ_{CP} , being suppressed by the small ratio of two Δm^2 and products of mixing angles, its measurement will require dedicated facilities such as Hyper-Kamiokande [12] and LBNE [13] as well as intense neutrino beams.

Here, the potential problem is that it will take a long time, ~ 10 years, to construct and operate such facilities. Therefore, it may be worthwhile to ask the question, "What can be done in the next 10 years toward the observation of lepton CP violation?". To sharpen up our concern we may ask a more scrutinizing question: "How can an experiment that is not actually capable of observing CP violation induced by δ_{CP} help us to pave the way to the final discovery?". It is the purpose of this paper to give a partial answer to these questions. We argue that one of the most important goals related to lepton CP violation that may be reached by the ongoing and the upcoming experiments is to exclude a significant fraction of the δ_{CP} space. It is interesting and timely to discuss the following questions: "What is the impact of running T2K also in the antineutrino mode on the determination of δ_{CP} ? What would be the optimal time sharing between neutrino and antineutrino beams in order that T2K can say something meaningful on δ_{CP} ? How T2K and NO ν A compare with each other in δ_{CP} sensitivity? Can the combination of equal-time running of T2K and NO ν A say more on δ_{CP} than each one of these experiments with doubled running time? Or, rephrasing, is there a synergy between them?" In what follows we will address all these questions based on the CP exclusion fraction which quantify how much the CP phase can be constrained for a given input parameters and experimental set up.

2 CP exclusion fraction; a measure of CP sensitivity for non-conclusive experiments

In this paper, to quantify the experimental sensitivity of T2K and NO ν A to δ_{CP} in a global way, we use the "CP exclusion fraction" $\equiv f_{CPX}$ where f_{CPX} is defined as the fraction of δ_{CP} values which can be disfavored at a given confidence level for a given set of input parameters. It is closely related to "CP coverage" [14, 15] as $f_{CPX} = 1 - (CP \text{ coverage}/360^\circ)$. In this work, we use the standard parameterization of the neutrino mixing matrix [16].

Here we make some clarifying remarks on the relation between f_{CPX} and the widely used "CP violation (CPV) fraction" (see, e.g., refs. [12, 17]). The CPV fraction gives us the fraction of δ_{CP} values for which CP violation can be established, conveying a clear cut message by focusing on "yes or no" to CP violation. But, due to the definition, it suffers from the "bias" of choosing special reference points ($\delta_{\text{CP}} = 0 \text{ or } \pi$) to discuss the sensitivity to CP phase. For example, a CPV fraction plot neither tells us whether the experiment is able to exclude, $\delta_{\text{CP}} = \pi/2$ or $-\pi/2$, nor allows us to extract the precision on δ_{CP} determination at these values of δ_{CP} . Notice that one of them, depending upon the mass hierarchy, is likely to be the initial footprint of the near future experiments. Since f_{CPX} does not have any bias issue it is particularly useful in dealing with "non-conclusive experiment," and it provides a better ground for a fruitful discussion of synergies.

Furthermore, $f_{\rm CPX}$ is intimately related to the uncertainty on the determination of $\delta_{\rm CP}$, at a certain CL (see e.g., ref. [18]). Since $1 - f_{\rm CPX}$ is equal to the fraction of the allowed range of $\delta_{\rm CP}$, one could naively expect $(1 - f_{\rm CPX})/2$ to be the uncertainty associated with $\delta_{\rm CP}/2\pi$. This interpretation fails if the allowed range of $\delta_{\rm CP}$ is disconnected or if there are multiple fake solutions and non-Gaussianities in the χ^2 [15], as it is likely the case for T2K and NO ν A with ~ 10 years running perspective. On the other hand, in the precision measurement era of $\delta_{\rm CP}$, the χ^2 will become locally Gaussian and $(1 - f_{\rm CPX})/2$ will turn smoothly to be the uncertainty on $\delta_{\rm CP}/2\pi$.

3 Sensitivity to CP phase expected by T2K

In this and the following sections we discuss the results of our analyses, the sensitivities to CP phase determination or exclusion to be expected by the T2K and NO ν A experiments,

respectively, assuming accurate measurement of θ_{13} by the reactor experiments. Details of our analysis method are described in appendix A. An intuitive explanation of some of the salient features of the analysis results by using bi-probability plot will be offered in appendix B.

Considering the nature of the experiments as the initial stage of CP phase measurement we will use, throughout this work, the CP exclusion fraction in $\delta_{\rm CP} - \sin^2 \theta_{23}$ space defined at 90% CL to display the sensitivity to CP phase. We note that while 90% CL may not guarantee high enough confidence for exclusion, the criterion is often used to place useful constraints on physics parameters in the literatures, for example, in the reports from Bugey [19], Chooz [20], and T2K [1] experiments. While we show only the results corresponding to 90% CL in this paper, we have also performed the computations to obtain the contours at 95% CL ($\simeq 2\sigma$ CL). Very roughly speaking, the change of CP exclusion fraction when we use 95% CL is that the contours of equal $f_{\rm CPX}$ at 90% CL are to be interpreted as $f_{\rm CPX} - (0.1 - 0.15)$ at 95% CL, the precise values of $f_{\rm CPX}$ reduction depend on $\delta_{\rm CP}$ and $\sin^2 \theta_{23}$.

We focus our discussion primarily on the possibility of a total of 10 years of data taking. The reason being, as we will see shortly, that after a total of 5 running years T2K will only be able to exclude 50% of δ_{CP} values in a very limited parameter space in the $\delta_{CP} - \sin^2 \theta_{23}$ plane, even if we assume that the mass hierarchy is known. We would like to explore the possibility of increasing the CP sensitivity of the experiment in a longer time span. As we mentioned in section 1, most probably, the construction of a dedicated CP explorer needs longer than 10 years from now, so that it is not an unrealistic scenario to examine.

The inverted mass hierarchy has been favored by some experimental analyses [21, 22], however feebly. Hence, the choice of the hierarchy to be displayed in our figures is basically arbitrary, and we opt for the inverted one for this section. Our treatment will not be completely equal for T2K and NO ν A, because our analysis of NO ν A can not be as mature as that of T2K for which we can profit from the informations of the experiment in operation.

3.1 Total of 5 running years $(5 \times 10^{21} \text{ POT})$

In figure 1, the contours of equal CP exclusion fraction for T2K experiment are plotted in the space spanned by the true values of $\delta_{\rm CP}$ and $\sin^2 \theta_{23}$. A total running time of 5 years is assumed with the nominal design luminosity, and the results for the $\nu + \bar{\nu}$ beam time sharing of 5 + 0, 3 + 2, and 2 + 3 years are shown (panels from left to right). Intermediate runnings, like 4+1 years, lie between the results shown. In the upper panels (lower panels) of figure 1 the inverted (normal) hierarchy is assumed as the input true mass hierarchy. It is quite likely that the mass hierarchy will not be determined with high confidence level when T2K completes its running period of 5 years. Therefore, we present here only the case where we fit for an unknown mass hierarchy, obtained by marginalizing over both cases.

The numbers on the isolines correspond to the CP exclusion fraction that can be achieved at 90% CL. By comparing the CP exclusion fractions of the three cases of $\nu + \bar{\nu}$ running periods of 5 + 0, 3 + 2, and 2 + 3 years in figure 1, it is evident that running in antineutrino mode helps to improve the CP sensitivity. It is notable that the performance of 3 + 2 and 2 + 3 years of runnings are roughly comparable to each other.

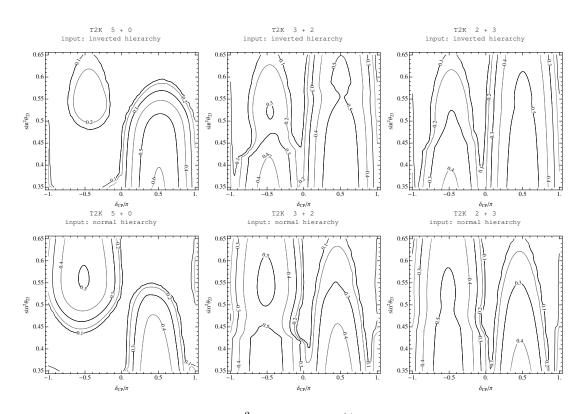


Figure 1. f_{CPX} isolines on the $\delta_{\text{CP}} - \sin^2 \theta_{23}$ plane at 90 % CL, for T2K running in $\nu + \bar{\nu}$ mode for n + n' years.

We note some characteristic features of the exclusion fraction iso-contour lines: (1) overall, the regions of higher sensitivity to CP are centered around $\delta_{\rm CP} \simeq \pm \pi/2$, and (2) in the 5 + 0 years running option the CP sensitive region is restricted mostly to two regions centered at ($\delta_{\rm CP} \simeq \pi/2$, low $\sin^2 \theta_{23}$) and ($\delta_{\rm CP} \simeq -\pi/2$, high $\sin^2 \theta_{23}$), whereas in 3 + 2 and 2 + 3 years running options (center and right panels) the dependence on $\sin^2 \theta_{23}$ is weakened, particularly at around $\delta_{\rm CP} \simeq \pi/2$ and $\delta_{\rm CP} \simeq -\pi/2$ for the inverted and the normal hierarchies, respectively.

From the probability point of view, one naively expects that the highest sensitivity to CP would be at $\delta_{\rm CP} \simeq \pm \pi/2$, in agreement with the first feature mentioned above. However, as statistics increases these most favorable values become less favorable than $\delta_{\rm CP} = 0$, depending on θ_{23} and our knowledge on the mass hierarchy, as will be shown in figures 2–4. For a qualitative explanation, see appendix B.

3.2 Total of 10 running years (10^{22} POT)

In figure 2 we present similar contours of equal CP exclusion fraction for a total of 10 running years with $\nu + \bar{\nu}$ beam time sharing of 10 + 0, 7 + 3, and 5 + 5 years (panels from left to right), assuming the nominal design luminosity for T2K but extended to 10 years. The results for 3 + 7 running years (not shown) are similar to the latter two cases, which represent the best sensitivities among the studied cases of a total of 10 running years. The results presented in the top panels were obtained by marginalizing over the mass hierarchies

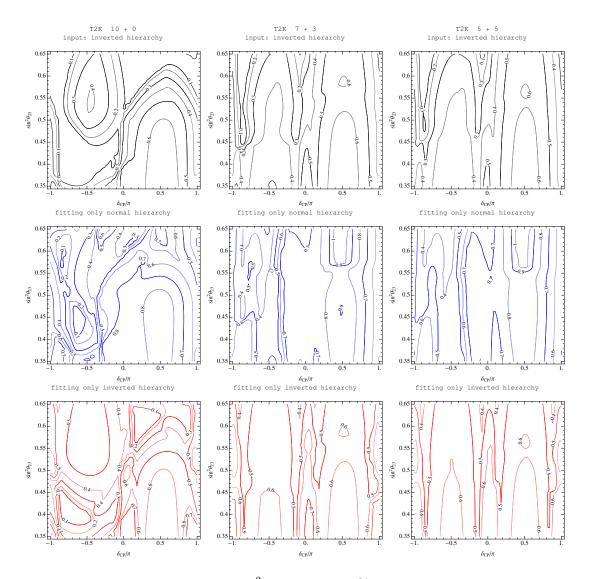


Figure 2. f_{CPX} isolines on the $\delta_{\text{CP}} - \sin^2 \theta_{23}$ plane at 90 % CL, for T2K running in $\nu + \bar{\nu}$ mode for n + n' years. From top to bottom, we marginalize over, fit only normal, and only inverted hierarchies.

(black contours). The middle and bottom panels are for cases of a fit assuming the normal (blue contours) and the inverted (red contours) mass hierarchies, respectively. In figure 2, only the case for inverted mass hierarchy as input is shown.

The main features of the CP exclusion fraction contours for the normal mass hierarchy as input may be obtained, in the zeroth order approximation, by doing the reparameterization $\delta_{\text{CP}} \rightarrow \pi - \delta_{\text{CP}}$ in figure 2. This approximation is valid because of the small matter effect in the T2K setting. The particular case of T2K 5 + 5 running years with the normal hierarchy as input is shown in the next section, see figure 3.

It should be emphasized first that as in the case of 5 years of data taking, the inclusion of antineutrino running time significantly improves the sensitivity to CP phase. Some of the distinctive features of running T2K for 10 years, shown in figure 2, compared to the results in 5 years running shown in figure 1, are: (1) with marginalization over the mass hierarchies the null sensitivity regions become significantly smaller, in particular, if we compare the last two top panels of each figure; (2) the 7+3 and 5+5 years running results, when fitted assuming the correct hierarchy, can exclude at least 50% of the values of $\delta_{\rm CP}$ in almost the entire $\delta_{\rm CP} - \sin^2 \theta_{23}$ plane allowed by the current oscillation data. This can be seen in the bottom center and right panels; and (3) the 7+3 and 5+5 years running results, when fitted using the normal mass hierarchy, can exclude a fraction of $\delta_{\rm CP}$ values up to 80%-90% for $\delta_{\rm CP} > 0$. The higher exclusion power is due to the assumption of the wrong mass hierarchy. But for $\delta_{\rm CP} < 0$, specially when θ_{23} is in the second octant, the exclusion fraction tends to be much less than the one for the right hierarchy.

What is the meaning of doing a fit assuming the wrong mass hierarchy? We argue that it is an alternative and useful way of probing the mass hierarchy sensitivity in terms of the CP exclusion fraction. Since this point will become clearer in the discussion of $NO\nu A$ results we will come back to it in the next section.

4 Sensitivity to CP phase expected by $NO\nu A$ and by its combination with T2K

4.1 10 running years: NO ν A (6 × 10²¹ POT)

In figure 3 the f_{CPX} contours are plotted for a total of 10 running years of the NO ν A experiment with $\nu + \bar{\nu}$ beam time sharing of 5 + 5 years. The left and middle panels are for the case of inverted and normal mass hierarchies, respectively. The results for 7 + 3 running years (not shown) are similar to the ones in figure 3. To make the comparison with T2K easier, we also show T2K 5 + 5 years running with normal hierarchy as input. As in figure 2 the upper panels are for cases marginalized over the mass hierarchies (black contours). The middle and bottom panels are for cases of a fit assuming the normal (blue contours) and the inverted (red contours) mass hierarchies, respectively.

We notice the following two significant features of NO ν A's CP sensitivity in comparison to that of T2K: (1) the sensitivity of NO ν A to CP phase is worse than that of T2K when marginalized over the mass hierarchies (top panels), almost losing the sensitivity in the negative (positive) half plane of δ_{CP} for the input inverted (normal) mass hierarchy; and (2) similarly, T2K is slightly better than NO ν A in the CP sensitivity assuming the right mass hierarchy (middle panels of the second and third columns), having 60% contours of CP exclusion in both half planes of δ_{CP} . On the other hand, in the wrong mass hierarchy fit the NO ν A CP sensitivity is overwhelming, making almost a complete exclusion at 90% CL of one of the half planes possible.

It appears that the relatively low NO ν A CP sensitivity compared to that of T2K comes partly from the relatively low statistics, as T2K generally accumulates 20–30% more statistics than NO ν A. In addition, the fact that the major axis of the CP ellipse for NO ν A is shorter than that for T2K (see figure 7 in appendix B) makes the CP sensitivity of NO ν A worse than that of T2K even with similar statistics, this is confirmed by explicit computation.

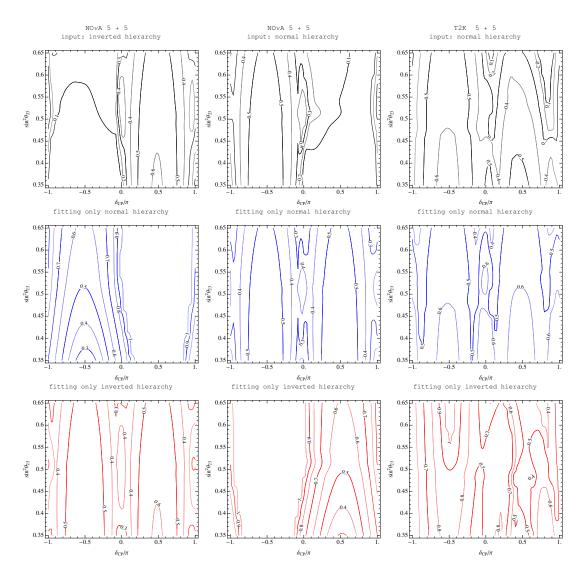


Figure 3. Similar plots as in figure 2 but for NO ν A and T2K running in $\nu + \bar{\nu}$ modes for 5+5 years.

On the other hand, the powerfulness of excluding almost half the space (positive δ_{CP} region for the inverted, and negative δ_{CP} region for the normal mass hierarchies) in the wrong hierarchy fit is due to the larger matter effect thanks to the longer baseline of NO ν A. Using this property the CP exclusion fraction may be used as a powerful indicator of the mass hierarchy though in a particular region of δ_{CP} . Therefore, it appears to us that these two experiments complement each other quite nicely.

4.2 Combination of $NO\nu A$ with T2K and the synergy

One of the most intriguing questions would be how high is the sensitivity to the CP phase when T2K and NO ν A are combined, and to what extent a synergy can be expected. To answer these questions, we present in figure 4 the contours of CP exclusion fraction obtained by combining 5+5 years running of T2K and NO ν A (a total of 10 years each) for the inverted (left panels) and normal (middle panels) input mass hierarchies. To extract

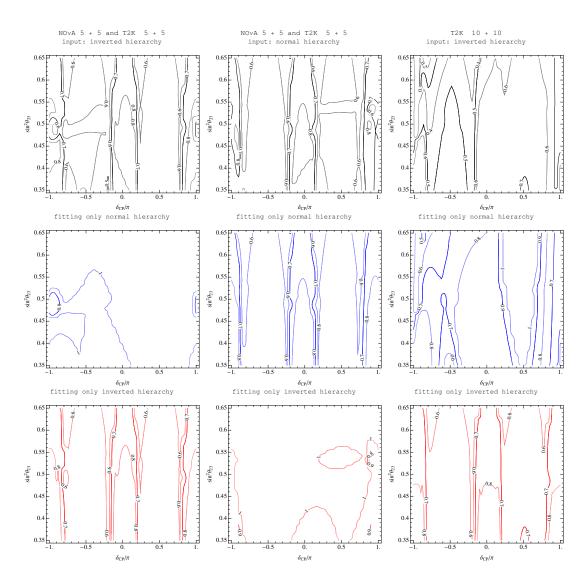


Figure 4. Similar plots as in figure 2 but for NO ν A and T2K running in $\nu + \bar{\nu}$ modes for 5+5 years (each) combined (left and middle panels) as well as T2K alone for 10 + 10 years (right panels).

the effect of the synergy we place in the right panel of figure 4 the contours obtained by a hypothetical 10 + 10 years running of T2K (a total 20 years). Although we do not consider it a realistic option, we show it for the sake of revealing the synergy.

The distinctive features of figure 4 are as follows: (1) when both experiments are combined, the wrong mass hierarchy is excluded at 90% CL in almost the entire allowed region of $\delta_{\rm CP} - \sin^2 \theta_{23}$ space; (2) also for the combination, the entire $\delta_{\rm CP} - \sin^2 \theta_{23}$ space is covered by 60% or higher exclusion fraction region, even marginalizing over the mass hierarchies; (3) for all cases assuming known hierarchy, the region of the highest sensitivity tends to exist at $\delta_{\rm CP} \sim 0$ or $\pm \pi$, which is different from the cases of lower statistics where the highest sensitivity is likely to occur at $\delta_{\rm CP} \sim \pm \pi/2$ (see appendix B for a qualitative discussion); and (4) finally, the effect of the synergy is evident when T2K and NO ν A combination, 5 + 5 years running each, is compared to T2K running for 20 years, particularly when marginalizing over the hierarchy.

5 The interplay between $\delta_{\rm CP}$ and θ_{23} octant for the experimental strategy

Until now, we have focused on the sensitivity to CP phase and discussed some strategy to optimize it. Actually, T2K and NO ν A can endeavor to measure another very important unknown: the octant of θ_{23} . Then, we raise the straightforward question (see also [23]) "How the strategies for determining δ_{CP} and the θ_{23} octant are related?" See refs. [24, 25] which also discussed the octant determination by combining T2K and NO ν A.

To answer this question, let us first recollect some relevant features of the θ_{23} octant measurement. Due to high statistics of the disappearance channels $\nu_{\mu} \rightarrow \nu_{\mu}$ and $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{\mu}$, $\sin^2 2\theta_{23}$ can be measured with high precision, but they are insensitive to the θ_{23} octant. On the other hand, because of its relatively low statistics, the appearance channels $\nu_{\mu} \rightarrow \nu_{e}$ and $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ have the capability of breaking the octant degeneracy only if the determinations of $\sin^2 2\theta_{13}$ and $\sin^2 2\theta_{23}$ are precise enough. For concreteness, let us focus on T2K. After 10 years of running, we expect that the determination of θ_{23} by the disappearance channels is dominated by systematic errors. Hence its sensitivity to $\sin^2 2\theta_{23}$ would be approximately independent of the running configuration.

Now, if T2K runs solely in the neutrino mode, the octant degeneracy becomes virtually unsolvable. From figure 7 found in appendix B, we can see that by only using the neutrino mode, even if we know the true mass hierarchy and the precise value of the oscillation probability, $P(\nu_{\mu} \rightarrow \nu_{e})$, θ_{23} different octants can be confused. This is in general true apart from the case where θ_{23} lies in the 1st (2nd) octant and δ_{CP} is close to $\pi/2$ ($-\pi/2$).

The impact of the spectral information is rather poor, as can be seen by analysing figure 5, where we show the appearance probabilities for neutrino (left panel) and antineutrino (right panel) as a function of the neutrino energy for the case where $0.95 < \sin^2 2\theta_{23} < 0.97$ and for various different values of $\delta_{\rm CP}$. We can see from the left panel that the two cases of $\delta_{\rm CP} = -\pi/2$ with θ_{23} in the 1st octant and $\delta_{\rm CP} = 0$ with θ_{23} in the 2nd octant are easily confused even if we take into account the energy spectrum. However, these two cases give very different probabilities in the antineutrino modes. The importance of the antineutrino run in resolving the octant degeneracy was inherent in the analysis in ref. [26], and some of the related points are discussed recently in refs. [24, 25].

When the antineutrino running is incorporated in T2K, the comparison between the event rates as well as the energy spectra of the $\nu + \bar{\nu}$ modes challenges the degeneracy toward its resolution in a more robust way. To understand how well the mechanism works, we present in figure 6 the regions of resolution of the octant degeneracy in $\delta_{\rm CP} - \sin^2 \theta_{23}$ space, calculated by imposing a Gaussian uncertainty on $\sin^2 2\theta_{23}$ of 0.02 at 68% CL. The regions colored in blue, green and red represent the region on the plane of the true values of $\delta_{\rm CP}$ and $\sin^2 \theta_{23}$ in which the octant of θ_{23} can be distinguished at 1σ , 2σ , and 3σ CL, respectively. Around maximal θ_{23} , no identification of the preferred θ_{23} octant exists. Shown in the panels from left to right in figure 6 are 10 + 0, 7 + 3, and 5 + 5 years running cases. It is also worth mentioning that the sensitivity for 3 + 7 years running is similar to the former two.

As can be seen, the inclusion of the antineutrino run significantly improves the sensitivity to the octant determination of θ_{23} . We also notice that for a fraction of time allocated

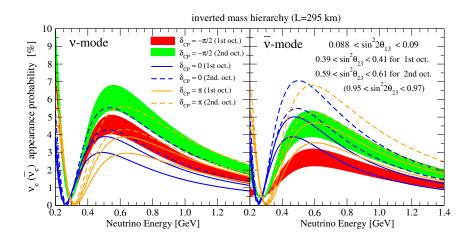


Figure 5. Appearance probabilities $P(\nu_{\mu} \rightarrow \nu_{e})$ for neutrino (left panel) and $P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e})$ for antineutrino (right panel) as a function of the neutrino energy, for $\delta_{\rm CP} = 0, \pm \pi/2$ for the case where $0.95 < \sin^2 2\theta_{23} < 0.97$.

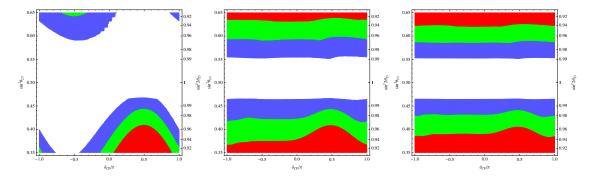


Figure 6. Regions in which the θ_{23} octant degeneracy is resolved on the $\delta - \sin^2 \theta_{23}$ plane. From left to right: 10 + 0, 7 + 3, and 5 + 5 years of $\nu + \bar{\nu}$ running of T2K, assuming inverted hierarchy as input.

for antineutrino running within 30%-70% of the total running time, the sensitivity to the octant of θ_{23} is remarkably stable. The CP sensitivity relies more strongly on the optimal proportion of antineutrino to neutrino time.

6 Conclusion

In the near future, 5 to 10 years from now, we do not expect to be able to measure the lepton CP phase, since we will not yet dispose of neutrino experiments designed to discover CP violation due to non-zero $\sin \delta_{\rm CP}$. However, the accelerator based neutrino oscillation experiments, T2K and NO ν A, after the precise measurement of $\sin^2 \theta_{13}$ by the reactor experiments, will have some sensitivity to $\delta_{\rm CP}$. This sensitivity will depend on the true values of $\delta_{\rm CP}$, $\sin^2 \theta_{23}$, the neutrino mass hierarchy as well as the amount of data taking in neutrino and antineutrino modes.

To study the maximal sensitivity to δ_{CP} attainable by a single or a set of experiments we employed the CP exclusion fraction, which quantifies the range of δ_{CP} that can be excluded,

at a certain confidence level (we adopted 90% in this paper), by a set of experimental observables. We expect that the CP exclusion fraction is particularly useful to examine the potential of exploring CP phase possessed by the near future experiments which may be the unique sources of information on the CP phase in an era without CP violation dedicated apparatus.

By using the CP exclusion fraction we have analyzed the CP sensitivity of T2K and NO ν A experiments. We have shown that it is important to run T2K in the antineutrino mode in order to significantly enhance the CP sensitivity of this experiment. The optimal situation seems to be to share the time equally between neutrino and antineutrino beams. For both hierarchies, if one could run T2K for 10 years one would be able to exclude 50% or more of the $\delta_{\rm CP}$ values in almost an entire half plane and $\sin^2 \theta_{23} \in [0.35, 0.65]$. If the neutrino mass hierarchy is known by that time, one could extend this result for almost the entire $\delta_{\rm CP} - \sin^2 \theta_{23}$ plane.

We have shown that NO ν A is less powerful than T2K for the CP sensitivity as measured with the CP exclusion fraction. By combining both experiments, we come across a synergy, excluding 60% or more of δ_{CP} values, as well as the wrong mass hierarchy at 90% CL in almost the entire $\delta_{CP} - \sin^2 \theta_{23}$ space.

We have also examined T2K sensitivity to the θ_{23} octant, showing that adding antineutrino run also helps the experimental sensitivity to $\sin^2 \theta_{23}$. The determination of this parameter will further help constraining δ_{CP} , as it will exclude part of the currently allowed region of δ_{CP} and $\sin^2 \theta_{23}$.

We emphasize that the 10% uncertainty we adopt in our analyses, for both experiments, may be a very conservative choice, in particular for the analysis of 10 years running. This is because T2K already achieved the uncertainty of $\simeq 10\%$ for running in neutrino mode, and it is conceivable that this will be improved in the future. A caution is, however, that so far little experimental information is accumulated in the antineutrino mode.

The results of our analysis in this paper underlines the necessity of dedicated experiments specially designed to access the lepton CP violating phase δ_{CP} . Examples for such apparatus include Hyper-Kamiokande or LBNE. Nonetheless, we emphasize the importance of getting as much information as we can on δ_{CP} before the day of dedicated machines arrives. It will certainly help us to lay the foundations for winning perhaps the long-term hardest job of hunting for the lepton CP phase, the marathon in neutrino physics.

A Analysis method

We follow the conventional χ^2 method to calculate the likelihood, at a given confidence level, of rejecting points in the parameter space $(\sin^2 \theta_{23}, \delta_{\rm CP})$ for a given input value of the parameters $(\sin^2 \theta_{23}^{\rm in}, \delta_{\rm CP}^{\rm in})$. Toward the goal, we compute the expected number of events T_i in the *i*-th energy bin as a function of the input parameters, $T_i(\theta_{13}^{\rm in}, \theta_{23}^{\rm in}, \delta_{\rm CP}^{\rm in}, h^{\rm in})$, where $h^{\rm in}$ is the input neutrino mass hierarchy. We also compute the expected number of events F_i in the *i*-th energy bin for a given set of fit and nuisance parameters $\{\alpha\}, F_i(\theta_{13}^{\rm fit}, \theta_{23}^{\rm fit}, \delta_{\rm CP}^{\rm fit}, \{\alpha\})$. These numbers include neutrino and antineutrino events,

	$0.55{ m GeV}$		$0.75{ m GeV}$	
	width (MeV)	shift (MeV)	width (MeV)	shift (MeV)
ν QE	85	-10	98	-15
ν nQE	70	-325	110	-390
$\overline{\nu}$ QE	57	-20	60	-20
$\overline{\nu}$ nQE	100	-270	120	-310

Table 1. T2K energy reconstruction parameters used in this paper.

according to the assumed exposure. With these we can build the likelihood function

$$-2\ln\mathcal{L}\left(\theta_{23}^{\mathrm{in}},\delta_{\mathrm{CP}}^{\mathrm{in}},h^{\mathrm{in}},\delta_{\mathrm{CP}}^{\mathrm{fit}}\right) = \min_{\left\{\theta_{13}^{\mathrm{fit}},\theta_{23}^{\mathrm{fit}},h^{\mathrm{fit}},\left\{\alpha\right\}\right\}} \left\{\sum_{i=1}^{\mathrm{nb}} 2\left(F_i - T_i + T_i\ln\frac{T_i}{F_i}\right) + \sum_j \left(\frac{\alpha_j}{\sigma_j}\right)^2 + \left(\frac{\sin^2 2\theta_{13}^{\mathrm{in}} - \sin^2 2\theta_{13}^{\mathrm{fit}}}{\sigma_{13}}\right)^2\right\},\tag{A.1}$$

where we set $\sin^2 2\theta_{13}^{\text{in}} = 0.089$ [8]. The expected number of events includes the contribution from signal and background so that schematically $F_i = \alpha_j F_i^{\text{signal}} + \alpha_{j+1} F_i^{\text{bck}}$. The likelihood (A.1) will be used to calculate, at a given confidence level, the fraction of values of δ_{CP} that are not compatible with the assumed input values. For T2K, we use 23 energy bins of 50 MeV and for NO ν A 20 bins of 150 MeV. In both cases we assume $\sigma_{13} = 0.005$, and all $\sigma_j = 0.1$.

In order to simulate T2K $\nu_{\mu} \rightarrow \nu_{e}$ and $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}$ events, we used a similar machinery as the one developed in ref. [27]. We took the fluxes for the neutrino and antineutrino modes, as well as the backgrounds, from the Hyper-Kamiokande letter of intent [12], normalizing the numbers to the T2K experimental parameters. We used the cross sections from ref. [26]. The migration of events were taken into account as below, in a similar way as done in [27]. We considered four systematic uncertainties, that is, the signal and background absolute normalizations for both neutrino and antineutrino modes. We took all of them to be 10%. In view of the fact that T2K comes already very close to 10% level systematic errors, it is a conservative choice for the neutrino mode, but may be a reasonable choice for the antineutrino mode.

To mimic the T2K neutrino energy reconstruction, we built migration matrices for quasi-elastic (QE) and non-quasi-elastic (nQE) events for both ν and $\bar{\nu}$ modes. For each migration matrix, we set Gaussian energy distributions at two values of the true neutrino energy, 0.55 GeV and 0.75 GeV, with a width and a parameter to shift the centre of the Gaussian, and we inter/extrapolated the form to all energies of interest. The precise values we used are shown in table 1. The efficiencies were taken to be almost constant for QE events, around 80%, and slightly decreasing for nQE, around 25% and 45% for the neutrino and the antineutrino channels, respectively. We simulate T2K disappearance modes according to ref. [26], obtaining a sensitivity to $\sin^2 2\theta_{23}$ around 0.02 (0.013) at 90% CL for a 5 (10) years running only in the neutrino mode.

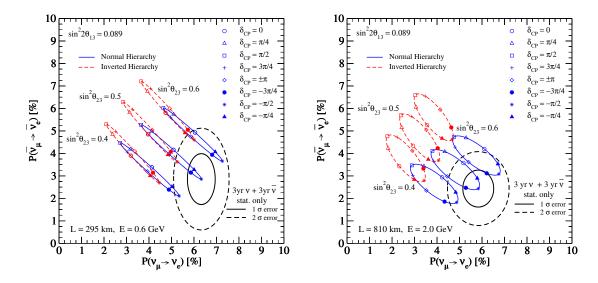


Figure 7. Bi-probability plots for T2K (left panel) and NO ν A (right panel) setups. The rough expected statistical uncertainties for 3+3 running years are indicated by the solid (1 σ) and dashed (2 σ) black curves for the normal mass hierarchy, $\delta_{\rm CP} = -\pi/2$ and $\sin^2 \theta_{23} = 0.5$.

Due to the small impact on the appearance channel, we set $|\Delta m_{31}^2|$ to $2.47 \times 10^{-3} \text{ eV}^2$ (2.43 × 10⁻³ eV²) for the normal (inverted) hierarchy [28], $\sin^2 \theta_{12} = 0.31$ and $\Delta m_{21}^2 = 7.54 \times 10^{-5} \text{ eV}^2$. Implementing the precisely measured value of θ_{13} is an indispensable ingredient in our method of detecting CP violation by ongoing and near future accelerator experiments [29]. To incorporate the precision reactor measurement of θ_{13} , we assume the final sensitivity to match Daya Bay's current systematic uncertainty of $\sin^2 2\theta_{13}$, that is, $\delta(\sin^2 2\theta_{13}) = 0.005$ [7, 8].

Regarding NO ν A simulation, we have based it on the simulation done in [17, 30], considering both the appearance and disappearance channels for the neutrino and antineutrino modes, using the latest experimental configuration [31, 32]. We used the fluxes available from [33] and take the cross sections from refs. [34, 35].

We assume that 1 year running of T2K and NO ν A corresponds, respectively, to delivery of 10^{21} and 6×10^{20} protons on target (POT). The fiducial mass of Super-Kamiokande is taken as 22.5 kt and NO ν A detector as 14 kt.

B Qualitative discussions using the bi-probability plot

Here we present a simple way to understand some of the notable features in the analysis results presented in sections 3 and 4 by using the bi-probability plots [36]. In figure 7 ellipses are drawn in $P(\nu_{\mu} \rightarrow \nu_{e}) - P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e})$ space by varying $\delta_{\rm CP}$ from $-\pi$ to π , keeping the other oscillation parameters fixed. We show in the left and right panels, the bi-probability plots which correspond roughly to the T2K (L = 295 km and E = 0.6 GeV) and the NO ν A (L = 810 km and E = 2.0 GeV) setups, respectively. We also placed in figure 7 a roughly estimated statistical error for 3 + 3 running years for the case where the mass hierarchy is normal, $\delta_{\rm CP} = -\pi/2$ and $\sin^2 \theta_{23} = 0.5$. Some general remarks on figure 7: (1) the CP ellipses for T2K are thinner and their major axes, which are proportional to $\sin \delta_{CP}$, are longer than those of NO ν A. The property follows because the peak neutrino energy taken for T2K is closer to the first oscillation maximum, $|\Delta m_{32}^2|L/(4E) = \pi/2$. (2) For a given set of oscillation parameters, the CP ellipses for different hierarchies are more separated for NO ν A than for T2K due to a stronger matter effect in the former setup. Hence, T2K should prevail in sensitivity to δ_{CP} , while NO ν A has higher sensitivity to the mass hierarchy. We also notice that, by comparing the error ellipses to the CP ones in figure 7, it is clear that CP violation cannot be established at 3σ after a 3 + 3 years, even when combining both experiments.

The importance of exploiting both neutrino and the antineutrino modes, can be accessed by the bi-probability plot for the T2K experiment. For the normal mass hierarchy and $\sin^2 2\theta_{23} = 0.96$, that is $\sin^2 \theta_{23} = 0.4$ or 0.6, suppose that only the neutrino mode is observed with $P(\nu_{\mu} \rightarrow \nu_{e}) = 5\%$. Then, we can not distinguish the cases between $\sin^2 \theta_{23} = 0.4$ with $-3\pi/4 \leq \delta_{\rm CP} \leq -\pi/4$, and $\sin^2 \theta_{23} = 0.6$ with $\pi/4 \leq \delta_{\rm CP} \leq 3\pi/4$. However, a distinction can be made by including the antineutrino mode, since $P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}) \sim 2\%$ for $\sin^2 \theta_{23} = 0.4$ while $P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}) \sim 6\%$ for $\sin^2 \theta_{23} = 0.6$.

What would be the values of $\delta_{\rm CP}$ which give larger or smaller CP exclusion fractions? Let us simplify the discussion by not considering the hierarchy and octant degeneracies. We can mentally translate the error ellipses to $\delta_{\rm CP} = 0$ and compare with the case displayed in figure 7. If the error is large (low statistics), the edges of the ellipses ($\delta_{\rm CP} = \pm \pi/2$) would correspond to a higher sensitivity. As to error shrinks, if $\delta_{\rm CP} = 0$, the extremes starts to be excluded, and this case becomes comparable to the former due to a Jacobian effect. If the statistics increases further, the exclusion fraction for $\delta_{\rm CP} = 0$ is expected to be larger than that for $\delta_{\rm CP} = \pm \pi/2$. This behaviour is confirmed by our results shown in sections 3 and 4, by comparing, for instance, T2K 2 + 3 years running in figure 1 with T2K 10 + 10 years running in figure 4.

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