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Deformations of large $N = (4,4)$ 2D SCFT from 3D gauged supergravity

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Abstract: Supersymmetric and non-supersymmetric deformations of large $N = (4,4)$ SCFT with superconformal symmetry $D^1(2,1;\alpha) \times D^1(2,1;\alpha)$ are explored in the gravity dual described by a Chern-Simons $N = 8$, $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$ gauged supergravity in three dimensions. For $\alpha > 0$, the gauged supergravity describes an effective theory of the maximal supergravity in nine dimensions on $AdS_3 \times S^3 \times S^3$ with the parameter α being the ratio of the two S^3 radii. We consider the scalar manifold of the supergravity theory of the form $\mathrm{SO}(8,8)/\mathrm{SO}(8) \times \mathrm{SO}(8)$ and find a number of stable non-supersymmetric AdS_3 critical points for some values of α. These correspond to non-supersymmetric IR fixed points of the UV $N = (4,4)$ SCFT dual to the maximally supersymmetric critical point. We study the associated RG flow solutions interpolating between these fixed points and the UV $N = (4,4)$ SCFT. Possible supersymmetric flows to non-conformal field theories and half-supersymmetric domain walls within this gauged supergravity are also investigated.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence, Supergravity Models, dS vacua in string theory

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Contents

1 Introduction

$\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence is interesting in various aspects. Unlike in higher dimensional cases, much more insight to the AdS/CFT correspondence [1] is expected since both gravity and field theory sides are well under control. It is also useful in the study of black hole entropy, see for example [2] and [3]. Until now, various gravity backgrounds implementing $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence have been proposed. Some of them are obtained from Kaluza-Klein dimensional reductions of higher dimensional supergravities on spheres or other internal manifolds. The other are constructed directly within the three dimensional framework of Chern-Simons gauged supergravity, but, in some cases particularly for compact and non-compact gauge groups, higher dimensional origins are still mysterious.

One of the most interesting backgrounds for $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence is string theory on $AdS_3 \times S^3 \times S^3 \times S^1$. The background is half-supersymmetric and dual to large $N = (4, 4)$ SCFT in two dimensions, see [4] for a classification of $N = 4$ SCFT in two dimensions. In string theory, this arises as a near horizon limit of the double D1-D5 brane system [5–7]. The Kaluza-Klein spectrum for small S^1 radius has been computed in [8]. Apart from the non-propagating supergravity multiplet in three dimensions, the spectrum contains massive multiplets of various spins. The full symmetry of $AdS_3 \times S^3 \times S^3$ is $D^1(2,1;\alpha)\times D^1(2,1;\alpha)$ whose bosonic subgroup is $\mathrm{SO}(2,2)\times \mathrm{SO}(4)\times \mathrm{SO}(4)$ corresponding to the isometry of $AdS_3 \times S^3 \times S^3$, respectively. Additionally, the holography of large $N = 4$ SCFT has recently been studied in the context of higher spin AdS_3 dual [9].

Like in higher dimensions, it would be useful to have an effective theory in three dimensions that describes the above $S^3 \times S^3$ dimensional reduction. The $AdS_3 \times S^3 \times S^3$ background will become an AdS_3 vacuum preserving sixteen supercharges and $\mathrm{SO}(4) \times \mathrm{SO}(4)$ gauge symmetry, which is the isometry of $S^3 \times S^3$. This can be achieved by a gauged matter-coupled supergravity in three dimensions [10–12]. The gauge group should contain the $\mathrm{SO}(4)\times\mathrm{SO}(4)$ factor. The natural construction should be the $N = 8$ gauged supergravity since the number of supersymmetry is exactly the same as that of the $AdS_3 \times S^3 \times S^3$ background. A theory describing supergravity coupled to massive spin-$\frac{1}{2}$ multiplets has been studied in [13] in which some critical points and a holographic RG flow have been discussed. The resulting theory is in the form of $N = 8$ gauged supergravity with compact $\mathrm{SO}(4)\times\mathrm{SO}(4)$ gauge group and $\mathrm{SO}(8,n)/\mathrm{SO}(8)\times\mathrm{SO}(n)$ scalar manifold.

When coupled to massive spin-1 multiplets, the theory needs to accompany for massive vector fields. For a theory coupled to two spin-1 multiplets, the corresponding gauge group is a non-semisimple group $(\mathrm{SO}(4)\times\mathrm{SO}(4))\ltimes \mathbf{T}^{12}$. It has been argued that the effective theory is the $N = 8$ gauged supergravity with $\mathrm{SO}(8,8)/\mathrm{SO}(8)\times\mathrm{SO}(8)$ scalar manifold [14]. The gauging is a straightforward extension of the $\mathrm{SO}(4)\ltimes\mathbf{T}^6$ gauging of [15] in which the effective theory of six-dimensional supergravity reduced on $AdS_3 \times S^3$ has been given. Some supersymmetric vacua of the $(\mathrm{SO}(4)\times\mathrm{SO}(4))\ltimes\mathbf{T}^{12}$ gauged theory have already been identified in [16]. All of these vacua are related to the maximally supersymmetric vacuum by marginal deformations. The theory with only the $\mathrm{SO}(4)\times\mathrm{SO}(4)$ semisimple part of the gauge group being gauged has been study in [17], and the solution corresponding to a marginal deformation from $N = (4,4)$ to $N = (3,3)$ SCFT, describing a D5-brane reconnection, has been explicitly given.

In this paper, we will reexamine the full $(\mathrm{SO}(4)\times\mathrm{SO}(4))\ltimes\mathbf{T}^{12}$ gauging and look for other deformations apart from the marginal ones. This could be relevant for $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence and black hole physics. The holographic study of the conformal symmetry $D^1(2,1;\alpha)$ is not only useful in the context of $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence but also in $\mathrm{AdS}_2/\mathrm{CFT}_1$ correspondence. This is because the symmetry $D^1(2,1;\alpha)$ also arises in superconformal quantum mechanics [18–20]. The isometry of AdS_2 is $\mathrm{SO}(2,1)$ which is a subgroup of the AdS_3 isometry $\mathrm{SO}(2,2)\sim\mathrm{SO}(2,1)\times\mathrm{SO}(2,1)$. Accordingly, the superconformal symmetry in one dimension contains only a single $D^1(2,1;\alpha)$. The holographic study of $\mathrm{AdS}_2/\mathrm{CFT}_1$ correspondence directly from two dimensional gauged supergravity has not been performed extensively. This is in part due to the lack of gauged supergravities in two dimensions. Until now, only the maximal gauged supergravity and its truncation have appeared [21, 22]. Since AdS_2 can be obtained by dimensional reduction of AdS_3 on S^1 via a very-near-horizon limit [23, 24], the results obtained here might be useful in the study of deformations in $D^1(2,1;\alpha)$ superconformal mechanics.

The paper is organized as follow. In section 2, we will give a brief review of $N = 8$, $(\mathrm{SO}(4)\times\mathrm{SO}(4))\ltimes\mathbf{T}^{12}$ gauged supergravity along with some relations to the $N = (4,4)$

SCFT. Section 3 deals with a description of new critical points, and the stability condition for some of them is verified. In section 4, we study possible supersymmetric flows to non-conformal field theories and $\frac{1}{2}$-BPS domain walls. We also comment on some numerical RG flow solutions describing deformations of the $N = (4, 4)$ SCFT to other CFTs in the IR. We end the paper by giving some conclusions and discussions in section 5. The appendices summarize necessary ingredients needed in the construction of $N = 8$ theory and relevant formulae including the explicit form of some scalar potentials.

2 $N = 8$, $(\mathrm{SO}(4)\times\mathrm{SO}(4))\ltimes\mathbf{T}^{12}$ gauged supergravity in three dimensions

We now review the construction of $N = 8$ gauged supergravity with $(\mathrm{SO}(4)\times\mathrm{SO}(4))\ltimes\mathbf{T}^{12}$ gauge group. The theory has partially been studied before in [16]. We will explore the scalar potential of this theory in more details. Rather than follow the parametrization of $\mathrm{SO}(8,8)/\mathrm{SO}(8)\times\mathrm{SO}(8)$ coset manifold as in [16], we will use the parametrization similar to that of [25]. In this parametrization, it is more convenient to determine the residual gauge symmetry while the parametrization used in [16] gives a simple action of the translation generators $\mathbf{T}^{12}$ on scalar fields.

It has been argued in [14] that this theory is an effective theory of ten dimensional supergravity on $AdS_3\times S^3\times S^3\times S^1$, or nine dimensional supergravity on $AdS_3\times S^3\times S^3$ for small S^1 radius, and describes the coupling of two massive spin-1 multiplets, containing twelve vectors, to the non-propagating supergravity multiplet of the reduction. All together, the resulting theory is $N = 8$ gauged supergravity with the scalar manifold $\mathrm{SO}(8,8)/\mathrm{SO}(8)\times\mathrm{SO}(8)$ and $(\mathrm{SO}(4)\times\mathrm{SO}(4))\ltimes\mathbf{T}^{12}$ gauge group.

The whole construction is similar to that given in [16] and [25]. We will work in the $\mathrm{SO}(8)$ R-symmetry covariant formulation of [12] with some relevant formulae and details explicitly given in appendix A. We first introduce the basis for a $GL(16,\mathbb{R})$ matrices

$$(e_{mn})_{pq} = \delta_{mp}\delta_{nq}, \qquad m,n,p,q = 1,\ldots,16\,. \tag{2.1}$$

The compact generators of $\mathrm{SO}(8,8)$ are then given by

$$\begin{aligned} &\mathrm{SO}(8)^{(1)}: \qquad && J_1^{IJ} = e_{JI} - e_{IJ}, \qquad && I,J = 1,\ldots,8,\\ &\mathrm{SO}(8)^{(2)}: \qquad && J_2^{rs} = e_{s+8,r+8} - e_{r+8,s+8}, \qquad && r,s = 1,\ldots,8\,. \end{aligned} \tag{2.2}$$

The non-compact generators corresponding to 64 scalars are identified as

$$Y^{Kr} = e_{K,r+8} + e_{r+8,K}, \qquad K,r = 1,\ldots,8\,. \tag{2.3}$$

In the formulation of [12], scalars transform as a spinor under $\mathrm{SO}(8)_R$ R-symmetry. It can be easily seen from the above equation that Y^{Kr} transform as a vector under $\mathrm{SO}(8)_R$ identified with $\mathrm{SO}(8)^{(1)}$ with generators J_1^{IJ}. We define the following $\mathrm{SO}(8)_R$ generators in a spinor representation by

$$T^{IJ} = \begin{pmatrix} \Gamma^{IJ} & 0 \\ 0 & 0 \end{pmatrix} \tag{2.4}$$

constructed from the 8×8 SO(8) gamma matrices Γ^I. We have defined

$$\Gamma^{IJ} = -\frac{1}{4}\left(\Gamma^I(\Gamma^J)^T - \Gamma^J(\Gamma^I)^T\right) \tag{2.5}$$

with the 8×8 gamma matrices Γ^I are given in appendix A.

The gauge group $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$ is embedded in SO(8,8) as follow. We first form a diagonal subgroup of $\mathrm{SO}(8) \times \mathrm{SO}(8)$ with generators

$$\mathrm{SO}(8)_{\mathrm{diag}}: \qquad J^{AB} = J_1^{AB} + J_2^{AB}, \qquad A, B = 1, \ldots, 8\,. \tag{2.6}$$

The $\mathrm{SO}(4) \times \mathrm{SO}(4)$ part is generated by

$$\begin{aligned} &\mathrm{SO}(4)^+: \qquad && j_1^{ab} = J^{ab}, && \\ &\mathrm{SO}(4)^-: \qquad && j_2^{\hat{a}\hat{b}} = J^{\hat{a}+4,\hat{b}+4}, && a, b, \hat{a}, \hat{b} = 1, \ldots, 4\,. \end{aligned} \tag{2.7}$$

The "hat" indices refer to $\mathrm{SO}(4)^-$. We now construct the translational generators $\mathbf{T}^{28}$ as in [25]

$$t^{AB} = J_1^{AB} - J_2^{AB} + Y^{BA} - Y^{AB} \tag{2.8}$$

and identify $\mathbf{T}^{12} \sim \mathbf{T}^6 \times \mathbf{T}^6$ generators as

$$t_1^{ab} = t^{ab}, \qquad t_2^{\hat{a}\hat{b}} = t^{\hat{a}+4,\hat{b}+4}, \qquad a, b, \hat{a}, \hat{b} = 1, \ldots, 4\,. \tag{2.9}$$

The gauge group is embedded in SO(8,8) with a specific form of the embedding tensor. As shown in [26], there is no coupling among the $\mathrm{SO}(4)^\pm$. The gauging is very similar to the $\mathrm{SO}(4) \ltimes \mathbf{T}^6$ gauged supergravity constructed in [15] with two factors of $\mathrm{SO}(4) \ltimes \mathbf{T}^6$. The embedding tensor is simply given by two copies of that given in [15]. We end up with two independent coupling constants

$$\Theta = g_1\Theta_1 + g_2\Theta_2\,. \tag{2.10}$$

where $\Theta_{1,2}$ describe the embedding of each $\mathrm{SO}(4) \ltimes \mathbf{T}^6$ factor of the full gauge group. We should note that supersymmetry allows for four independent couplings namely between the moment maps $g_1'(\mathcal{V}(j_1^{ab}), \mathcal{V}(t_1^{ab}))$, $g_2'(\mathcal{V}(t_1^{ab}), \mathcal{V}(t_1^{ab}))$, $g_3'(\mathcal{V}(j_2^{ab}), \mathcal{V}(t_2^{ab}))$ and $g_4'(\mathcal{V}(t_2^{ab}), \mathcal{V}(t_2^{ab}))$ in the T-tensor, see [15] and [16]. We have used a shorthand notation for $\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}$. However, the requirement that the theory admits a maximally supersymmetric vacuum at the origin of the scalar manifold imposes two conditions on the original four couplings. In more detail, the two conditions require $g_2' = -g_1'$ and $g_4' = -g_3'$. After rename the relevant couplings, we end up with the embedding tensor

$$\Theta_{abcd} = g_1\epsilon^+_{abcd} + g_2\epsilon^-_{\hat{a}\hat{b}\hat{c}\hat{d}}\,. \tag{2.11}$$

This embedding tensor together with the formulae in appendix A and an explicit parametrization of the coset representative of $\mathrm{SO}(8,8)/\mathrm{SO}(8) \times \mathrm{SO}(8)$ can be used to compute the scalar potential. We will analyze the resulting potential on submanifolds of $\mathrm{SO}(8,8)/\mathrm{SO}(8) \times \mathrm{SO}(8)$ invariant under some subgroups of $\mathrm{SO}(4) \times \mathrm{SO}(4)$ in the next section.

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Before looking at the critical points, we give a review of the relation between $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$, $N = 8$ gauged supergravity and $N = (4, 4)$ SCFT. The semisimple part of the gauge group $\mathrm{SO}(4)^+ \times \mathrm{SO}(4)^-$ corresponds to the isometry of $S^3 \times S^3$. Together with the usual $\mathrm{SO}(2, 2)$ isometry of AdS_3, they constitute the bosonic subgroup $\mathrm{SO}(2, 1)_L \times \mathrm{SU}(2)^+_L \times \mathrm{SU}(2)^-_L \times \mathrm{SO}(2, 1)_R \times \mathrm{SU}(2)^+_R \times \mathrm{SU}(2)^-_R$ of the superconformal group $D^1(2, 1; \alpha) \times D^1(2, 1; \alpha)$ via the isomorphisms $\mathrm{SO}(2, 2) \sim \mathrm{SO}(2, 1)_L \times \mathrm{SO}(2, 1)_R$ and $\mathrm{SO}(4)^\pm \sim \mathrm{SU}(2)^\pm_L \times \mathrm{SU}(2)^\pm_R$. The α parameter is identified with the ratio of the coupling constant $g_2 = \alpha g_1$. For positive α, the theory describes the dimensional reduction of nine dimensional supergravity on $S^3 \times S^3$. For negative α, it may possibly describe the reduction on $S^3 \times H^3$ where H^3 is a hyperbolic space in three dimensions.

The translational part $\mathbf{T}^{12}$ of the gauge group describes twelve massive vector fields [26]. The massive vector fields will show up in the vacuum of the theory via twelve massless scalars in the adjoint representation of $\mathrm{SO}(4)\times\mathrm{SO}(4)$. These are Goldstone bosons for the $\mathbf{T}^{12}$ symmetry since the vacuum is invariant only under $\mathrm{SO}(4)^+ \times \mathrm{SO}(4)^-$ not the full gauge group. We will see this when we compute the mass spectrum of scalar fields.

3 Some critical points of $N = 8$, $(\mathrm{SO}(4) \ltimes \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$ gauged supergravity

We now look for critical points of the $N = 8$ gauged supergravity constructed in the previous section. Analyzing the scalar potential on the full 64-dimensional scalar manifold $\mathrm{SO}(8, 8)/\mathrm{SO}(8) \times \mathrm{SO}(8)$ is beyond our reach with the present-time computer. We then employ an effective method given in [27] to find some interesting critical points on a submanifold invariant under some subgroup of the gauge group. A group theoretical argument guarantees that the corresponding critical points are critical points of the scalar potential on the full scalar manifold. Even on these truncated manifolds, the explicit form of the potential is still very complicated. Therefore, in most cases, we refrain from giving the full expression for the potential.

At the trivial critical point with all scalars vanishing, the full gauge group $(\mathrm{SO}(4) \times \mathrm{SO}(4))\ltimes\mathbf{T}^{12}$ is broken down to its maximal compact subgroup $\mathrm{SO}(4)\times\mathrm{SO}(4)$ corresponding to the isometry of $S^3 \times S^3$. The 64 scalars transform under $\mathrm{SO}(8) \times \mathrm{SO}(8) \subset \mathrm{SO}(8, 8)$ as $(\mathbf{8}, \mathbf{8})$. Then, under the $\mathrm{SO}(4)^+ \times \mathrm{SO}(4)^- \subset \mathrm{SO}(8)_{\mathrm{diag}}$, they transform as

$$\begin{aligned}\mathbf{8} \times \mathbf{8} &= \left[(\mathbf{4}^+, \mathbf{1}^+) + (\mathbf{1}^-, \mathbf{4}^-)\right] \times \left[(\mathbf{4}^+, \mathbf{1}^+) + (\mathbf{1}^-, \mathbf{4}^-)\right] \\ &= (\mathbf{1}^+ + \mathbf{6}^+ + \mathbf{9}^+, \mathbf{1}^+) + (\mathbf{1}^-, \mathbf{1}^- + \mathbf{6}^- + \mathbf{9}^-) + (\mathbf{4}^+, \mathbf{4}^-) + (\mathbf{4}^-, \mathbf{4}^+).\end{aligned} \tag{3.1}$$

We can further decompose the above representations into $\mathrm{SU}(2)^+_L \times \mathrm{SU}(2)^+_R \times \mathrm{SU}(2)^-_L \times \mathrm{SU}(2)^-_R$ representations labeled by $(\ell^+_L, \ell^+_R; \ell^-_L, \ell^-_R)$ as follow:

$$\begin{aligned}\mathbf{8} \times \mathbf{8} &= (\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}; \mathbf{1}, \mathbf{1}) + (\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{1}) + (\mathbf{3}, \mathbf{3}; \mathbf{1}, \mathbf{1}) \\ &\quad +(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{1}; \mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}; \mathbf{3}, \mathbf{3}) \\ &\quad +(\mathbf{2}, \mathbf{2}; \mathbf{2}, \mathbf{2}) + (\mathbf{2}, \mathbf{2}; \mathbf{2}, \mathbf{2}).\end{aligned} \tag{3.2}$$

h_L \ h_R	$\frac{\alpha}{1+\alpha}$	$\frac{3\alpha+1}{2(1+\alpha)}$	$\frac{2\alpha+1}{1+\alpha}$
$\frac{\alpha}{1+\alpha}$	$(0,1;0,1)$	$(0,1;\frac{1}{2},\frac{1}{2})$	$(0,1;0,0)$
$\frac{3\alpha+1}{2(1+\alpha)}$	$(\frac{1}{2},\frac{1}{2};0,1)$	$(\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2};0,0)$
$\frac{2\alpha+1}{1+\alpha}$	$(0,0;0,1)$	$(0,0;\frac{1}{2},\frac{1}{2})$	$(0,0,0,0)$

Table 1. The massive spin-1 multiplet $(0,1;0,1)_{\rm S}$.

h_L \ h_R	$\frac{1}{1+\alpha}$	$\frac{3+\alpha}{2(1+\alpha)}$	$\frac{2+\alpha}{1+\alpha}$
$\frac{1}{1+\alpha}$	$(1,0;1,0)$	$(1,0;\frac{1}{2},\frac{1}{2})$	$(1,0;0,0)$
$\frac{3+\alpha}{2(1+\alpha)}$	$(\frac{1}{2},\frac{1}{2};1,0)$	$(\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2};0,0)$
$\frac{2+\alpha}{1+\alpha}$	$(0,0;1,0)$	$(0,0;\frac{1}{2},\frac{1}{2})$	$(0,0,0,0)$

Table 2. The massive spin-1 multiplet $(1,0;1,0)_{\rm S}$.

$\mathrm{SO}(4)^+ \times \mathrm{SO}(4)^-$	m^2L^2
$(\mathbf{1},\mathbf{1})$	$\frac{4g_1(2g_1+g_2)}{(g_1+g_2)^2}$
$(\mathbf{6},\mathbf{1})$	0
$(\mathbf{9},\mathbf{1})$	$-\frac{4g_1g_2}{(g_1+g_2)^2}$
$(\mathbf{1},\mathbf{1})$	$\frac{4g_2(2g_2+g_1)}{(g_1+g_2)^2}$
$(\mathbf{1},\mathbf{6})$	0
$(\mathbf{1},\mathbf{9})$	$-\frac{4g_1g_2}{(g_1+g_2)^2}$
$(\mathbf{4},\mathbf{4})$	$\frac{3g_2^2-2g_1g_2-g_1^2}{(g_1+g_2)^2}$
$(\mathbf{4},\mathbf{4})$	$\frac{3g_1^2-2g_1g_2-g_2^2}{(g_1+g_2)^2}$

Table 3. The mass spectrum of the trivial critical point.

The result precisely agrees with the representation content obtained from the $AdS_3 \times S^3 \times S^3$ reduction [8]. For conveniences, we also repeat the massive spin-1 supermultiplets $(0,1;0,1)_{\rm S}$ and $(1,0;1,0)_{\rm S}$ of the $AdS_3 \times S^3 \times S^3$ reduction in table 1 and 2.

We can now compute the scalar potential by using the formulae in appendix A. After expanding the potential around $L = \mathbf{I}$, we find the scalar mass spectrum at the maximally supersymmetric vacuum as shown in table 3. The AdS_3 radius is given by $L = \frac{1}{\sqrt{-V_0}}$, and the value of the potential at this point is $V_0 = -64(g_1+g_2)^2$. Using the relation $m^2L^2 = \Delta(\Delta-2)$ and $\Delta = h_L + h_R$, we can verify that the mass spectrum agrees with the values of h_R and h_L in table 1 and 2. As mentioned before, there are twelve massless Goldstone bosons transforming in the adjoint representation $(\mathbf{1},\mathbf{6})+(\mathbf{6},\mathbf{1})$ of $\mathrm{SO}(4)\times\mathrm{SO}(4)$. Note also that there is a Minkowski vacuum at $g_2 = -g_1$ or $\alpha = -1$.

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3.1 Critical points on the $SO(4)_{\text{diag}}$ invariant manifold

We first consider scalars which are singlets under the diagonal subgroup $SO(4)_{\text{diag}} \subset SO(4) \times SO(4)$. To obtain representations of the scalars under this subgroup, we take a tensor product in the last line of (3.1). We find that there are four singlets, two from the obvious ones $(\mathbf{1}^+ \times \mathbf{1}^+, \mathbf{1}^- \times \mathbf{1}^-)$ and the other two from the product $(\mathbf{4}^+ \times \mathbf{4}^-, \mathbf{4}^- \times \mathbf{4}^+)$. They correspond to the following non-compact generators

$$\begin{aligned} \tilde{Y}_1 &= Y^{11} + Y^{22} + Y^{33} + Y^{44}, & \tilde{Y}_2 &= Y^{55} + Y^{66} + Y^{77} + Y^{88}, \\ \tilde{Y}_3 &= Y^{51} + Y^{62} + Y^{73} + Y^{84}, & \tilde{Y}_4 &= Y^{15} + Y^{26} + Y^{37} + Y^{48}\,. \end{aligned} \tag{3.3}$$

The coset representative is accordingly parametrized by

$$L = e^{a_1 \tilde{Y}_1} e^{a_2 \tilde{Y}_2} e^{a_3 \tilde{Y}_3} e^{a_4 \tilde{Y}_4}\,. \tag{3.4}$$

Apart from the trivial critical point at $a_1 = a_2 = a_3 = a_4 = 0$, we find the following critical points.

- A non-supersymmetric AdS_3 is given by $a_1 = \frac{1}{2} \ln \frac{\sqrt{g_1 - 4g_3} - \sqrt{g_1}}{2\sqrt{g_1}}$ and $a_2 = a_3 = a_4 = 0$. The cosmological constant is

$$V_0 = -32 \left[g_1^2 + 4g_2^2 - 6g_1 g_2 + (4g_2 - g_1)\sqrt{g_1(g_1 - 4g_3)} \right]. \tag{3.5}$$

  a_1 is real for $g_1 > 0$ and $g_2 < 0$, and the critical point is AdS_3, $V_0 < 0$, for $g_1 > 0$ and $g_2 < -\frac{\sqrt{2}+1}{2} g_1$. An equivalent critical point is given by $a_2 \neq 0$ and $a_1 = a_3 = a_4 = 0$ but with $g_1 \leftrightarrow g_2$. For later reference, we will call this critical point P_1.

- Another non supersymmetric critical point is at $a_4 = \ln \frac{\sqrt{g_1} + \sqrt{3g_2}}{\sqrt{g_1} - \sqrt{3g_2}}$ with $g_2 = \frac{1}{9}\left(\sqrt{13} - 2\right) g_1$ and $V_0 = -\frac{8}{3}\left(43 + 13\sqrt{13}\right) g_1^2$. In this case, only a specific value of α gives a critical point. The residual gauge symmetry in this case is $SO(4)_{\text{diag}}$. We will label this critical point as P_2.

The full scalar potential for the four scalars is given in appendix B.

We now analyze the scalar masses at the above critical points to check their stability. For critical point P_1, it is useful to classify the 64 scalars according to their representations under the residual symmetry $SO(4) \times SO(4)$. The result is shown in table 4. Similar to the trivial critical point, there are 12 massless scalars corresponding to the broken $\mathbf{T}^{12}$ symmetry. The stability bound, or BF bound $m^2 L^2 \geq -1$, is satisfied by $-\frac{13+9\sqrt{2}}{2} g_1 < g_2 < -\frac{1+\sqrt{1+\sqrt{2}}}{2} g_1$.

For critical point P_2, we can compute all scalar masses as shown in table 5. It is easily seen that all masses satisfy the BF bound. There are 18 massless Goldstone bosons corresponding to the symmetry breaking $(SO(4) \times SO(4)) \ltimes \mathbf{T}^{12} \to SO(4)$.

We end this subsection by noting an interesting result discovered in [17] but with a compact gauge group $SO(4) \times SO(4)$. This solution describes a marginal deformation of $N = (4,4)$ SCFT to $N = (3,3)$ SCFT and has an interpretation in term of a reconnection of D5-branes in the double D1-D5 system. The solution is also encoded in our present

$\mathrm{SO}(4)^+ \times \mathrm{SO}(4)^-$	m^2L^2
$(\mathbf{1},\mathbf{1})$	$\frac{12g_2}{g_2+\sqrt{g_1(g_1-4g_2)}}$ $-\frac{16g_2^2+20g_1g_2-6g_1^2+2(g_1+2g_2)\sqrt{g_1(g_1-4g_2)}}{g_1^2-4g_1g_2-4g_2^2}$ $\frac{4g_2^2+14g_1g_2-3g_1^2+(4g_2-g_1)\sqrt{g_1(g_1-4g_2)}}{2(g_1^2-4g_1g_2-4g_2^2)}$ $-\frac{3g_1^2-30g_1g_2+12g_2^2+3(3g_1-4g_2)\sqrt{g_1(g_1-4g_2)}}{2(g_1^2-4g_1g_2-4g_2^2)}$
$(\mathbf{6},\mathbf{1})$	0
$(\mathbf{9},\mathbf{1})$	$\frac{8g_1g_2}{g_1^2-6g_1g_2+(2g_2-g_1)\sqrt{g_1(g_1-4g_2)}}$
$(\mathbf{1},\mathbf{1})$	$\frac{4g_2(2g_2+g_1)}{(g_1+g_2)^2}$
$(\mathbf{1},\mathbf{6})$	0
$(\mathbf{1},\mathbf{9})$	$-\frac{4g_1^2-24g_1g_2-8g_2^2+4(g_1-g_2)\sqrt{g_1(g_1-4g_2)}}{g_1^2-4g_1g_2-4g_2^2}$
$(\mathbf{4},\mathbf{4})$	$\frac{4g_2^2+14g_1g_2-3g_1^2+(4g_2-g_1)\sqrt{g_1(g_1-4g_2)}}{2(g_1^2-4g_1g_2-g_2^2)}$
$(\mathbf{4},\mathbf{4})$	$-\frac{12g_2^2-30g_1g_2+3g_1^2+(9g_1-12g_2)\sqrt{g_1(g_1-4g_2)}}{2(g_1^2-4g_1g_2-g_2^2)}$

Table 4. The scalar mass spectrum of the $\mathrm{SO}(4) \times \mathrm{SO}(4)$ critical point P_1.

$\mathrm{SO}(4)$	m^2L^2
$\mathbf{1}$	13.6358, 6.0931, 3.3703, 3.1180
$\mathbf{6}$	$0_{(\times 18)}$
$\mathbf{9}$	$\frac{8}{29}(7\sqrt{13}-12)_{(\times 9)}$, $\frac{4}{29}(5\sqrt{13}-21)_{(\times 9)}$, $\frac{4}{29}(8+5\sqrt{13})_{(\times 9)}$, $\frac{4}{87}(19\sqrt{13}-74)_{(\times 9)}$

Table 5. The scalar mass spectrum of the $\mathrm{SO}(4)$ critical point P_2 for $g_2 = \frac{\sqrt{13}-2}{9}g_1$.

framework. In this case, we must set $g_2 = g_1$, or equivalently setting $\alpha = 1$ in order to get massless (marginal) scalars preserving the $\mathrm{SO}(4)$ diagonal subgroup of $\mathrm{SO}(4) \times \mathrm{SO}(4)$.

Follow [17], we further truncate the four scalars to two via

$$a_2 = a_1, \qquad a_4 = -a_3\,. \tag{3.6}$$

This is a consistent truncation for $g_2 = g_1$ since it corresponds to a fixed point of an inner automorphism that leaves the embedding tensor invariant [17]. We find a critical point at

$$e^{a_1+a_3} = 1 + \sqrt{1-e^{2a_1}}, \qquad V_0 = -256g_1^2 \tag{3.7}$$

with the corresponding A_1 tensor given by

$$A_1^{IJ} = \mathrm{diag}\left(-8g_1, -8g_1, -8g_1, 8g_1, 8g_1, 8g_1, -8g_1\sqrt{4e^{-2a_1}-3}, 8g_1\sqrt{4e^{-2a_1}-3}\right). \tag{3.8}$$

We can see that as long as $a_1 \neq 0$, the $N = (4,4)$ supersymmetry is broken to $N = (3,3)$. We refer the reader to [17] for the full discussion of this vacuum.

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3.2 Critical points on the $SO(2)_{diag} \times SO(2)_{diag}$ invariant manifold

We now proceed to consider a smaller residual symmetry $SO(2)_{diag} \times SO(2)_{diag} \subset SO(4)_{diag}$. Under $SO(2) \times SO(2)$, the SO(4) fundamental representation $\mathbf{4}$ decomposes according to $\mathbf{4} \to (\mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{2})$. Substituting this decomposition for $\mathbf{4}^+$ and $\mathbf{4}^-$ in (3.1) and taking the product to form a diagonal subgroup, we find that there are sixteen singlets given by the non-compact generators

$$\begin{aligned} &\bar{Y}_1 = Y^{11} + Y^{22}, \quad &&\bar{Y}_2 = Y^{33} + Y^{44}, \quad &&\bar{Y}_3 = Y^{55} + Y^{66}, \quad &&\bar{Y}_4 = Y^{77} + Y^{88}, \\ &\bar{Y}_5 = Y^{15} + Y^{26}, &&\bar{Y}_6 = Y^{37} + Y^{48}, &&\bar{Y}_7 = Y^{51} + Y^{62}, &&\bar{Y}_8 = Y^{73} + Y^{84}, \\ &\bar{Y}_9 = Y^{12} - Y^{21}, &&\bar{Y}_{10} = Y^{34} - Y^{43}, &&\bar{Y}_{11} = Y^{56} - Y^{65}, &&\bar{Y}_{12} = Y^{78} - Y^{87}, \\ &\bar{Y}_{13} = Y^{16} - Y^{25}, &&\bar{Y}_{14} = Y^{38} - Y^{47}, &&\bar{Y}_{15} = Y^{52} - Y^{61}, &&\bar{Y}_{16} = Y^{74} - Y^{83}. \end{aligned} \tag{3.9}$$

The coset representative can be parametrized by

$$L = \prod_{i=1}^{16} e^{a_i \bar{Y}_i}. \tag{3.10}$$

Unlike the previous case, the scalar potential is so complicated that it is not possible to make the full analysis. However, with some ansatz, we find one non-trivial critical point at

$$\begin{aligned} &a_1 = a_2 = \frac{1}{2}\ln 2, \qquad a_3 = -a_4 = \frac{1}{2}\ln\frac{g_2 - 6g_1 + \sqrt{36g_1^2 - 12g_1g_2 - 3g_2^2}}{2g_2}, \\ &V_0 = 64(8g_1^2 - g_2^2). \end{aligned} \tag{3.11}$$

a_3 and a_4 are real for $g_1 > 0$ and $g_2 \geq -6g_1$. In this range, we find $V_0 < 0$ if $g_2 < -2\sqrt{2}g_1$. Therefore, it is possible to have an AdS_3 critical point. The residual symmetry is $SO(4) \times SO(2) \times SO(2)$. We will denote this critical point by P_3 for later reference.

The stability of this critical point can be verified from the scalar mass spectrum given in table 6 in which α_i are eigenvalues of the submatrix

$$\frac{1}{8g_1^2 - g_2^2}\begin{pmatrix} -80g_1^2 & x_1 & x_2 \\ x_1 & -\frac{g_2^2}{3} & -\frac{2g_2^2}{3} \\ x_2 & -\frac{2g_2^2}{3} & -\frac{g_2^2}{3} \end{pmatrix} \tag{3.12}$$

with the following elements

$$\begin{aligned} &x_1 = 2\sqrt{2}g_1\left(6g_1 + g_2 - \sqrt{36g_1^2 - 12g_1g_2 - 3g_2^2}\right) \\ \text{and} \quad &x_2 = 2\sqrt{2}g_1\left(6g_1 + g_2 + \sqrt{36g_1^2 - 12g_1g_2 - 3g_2^2}\right). \end{aligned} \tag{3.13}$$

Their numerical values can be obtained upon specifying the values of g_1 and g_2.

For all but $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{1}, \mathbf{2})$ scalars, the masses are above the BF bound for $-6g_1 < g_2 < -2\sqrt{2}g_1$. The mass squares of $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ scalars are above the BF bound for $-6g_1 < g_2 < -4.47g_1$. For $(\mathbf{1}, \mathbf{1}, \mathbf{2})$ scalars, the mass squares are above the BF bound

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$\mathrm{SO}(4)\times\mathrm{SO}(2)\times\mathrm{SO}(2)$	m^2L^2
$(\mathbf{4},\mathbf{2},\mathbf{1})$	$-\frac{60g_1^2-14g_1g_2+g_2^2+(6g_1-3g_2)\sqrt{36g_1^2-12g_1g_2-3g_2^2}}{16g_1^2-2g_2^2}$
$(\mathbf{4},\mathbf{1},\mathbf{2})$	$-\frac{60g_1^2-24g_1g_2+g_2^2+(3g_2-6g_1)\sqrt{36g_1^2-12g_1g_2-3g_2^2}}{16g_1^2-2g_2^2}$
$(\mathbf{4},\mathbf{2},\mathbf{1})$	$-\frac{124g_1^2-3g_2^2+(g_2+6g_1)\sqrt{36g_1^2-12g_1g_2-3g_2^2}}{16g_1^2-2g_2^2}$
$(\mathbf{4},\mathbf{1},\mathbf{2})$	$-\frac{124g_1^2-3g_2^2-(g_2+6g_1)\sqrt{36g_1^2-12g_1g_2-3g_2^2}}{16g_1^2-2g_2^2}$
$(\mathbf{1},\mathbf{2},\mathbf{1})$	$\frac{6g_2^2+24g_1g_2-72g_1^2+2(g_2-6g_1)\sqrt{36g_1^2-12g_1g_2-3g_2^2}}{8g_1^2-g_2^2}$
$(\mathbf{1},\mathbf{1},\mathbf{2})$	$\frac{6g_2^2+24g_1g_2-72g_1^2-2(g_2-6g_1)\sqrt{36g_1^2-12g_1g_2-3g_2^2}}{8g_1^2-g_2^2}$
$(\mathbf{9},\mathbf{1},\mathbf{1})$	$\frac{48g_1^2}{g_2^2-8g_1^2}$
$(\mathbf{6},\mathbf{1},\mathbf{1})$	0
$2\times(\mathbf{1},\mathbf{2},\mathbf{2})$	0
$2\times(\mathbf{1},\mathbf{1},\mathbf{1})$	0
$(\mathbf{1},\mathbf{1},\mathbf{1})$	$\alpha_1,\ \alpha_2,\ \alpha_3$

Table 6. The scalar mass spectrum of the $\mathrm{SO}(4)\times\mathrm{SO}(2)\times\mathrm{SO}(2)$ critical point P_3.

for $-6g_1<g_2<\mathcal{X}$ with $\mathcal{X}$ being the first root of $p(\mathcal{X})=1088g_1^4-384g_1^3\mathcal{X}+352g_1^2\mathcal{X}^2-144g_1\mathcal{X}^3-37\mathcal{X}^4=0$. This can be translated to the value of α by setting $\mathcal{X}=\alpha g_1$. The equation $p(\mathcal{X})=0$ gives the value of $\alpha=-5.93479$. The stability is obtained in the range $-6g_1<g_2<-5.93479g_1$ which is very narrow. Notice that for $g_2=-6g_1$, we find $a_3=a_4=0$, and the symmetry is enhanced to $\mathrm{SO}(4)\times\mathrm{SO}(4)$. It can be checked that this critical point indeed becomes critical point P_1 with $g_2=-6g_1$.

3.3 Critical points on the $\mathbf{SU(2)}_L^+\times\mathbf{SU(2)}_L^-$ invariant manifold

One interesting deformation of $N=(4,4)$ SCFT is the chiral supersymmetry breaking $(4,4)\to(4,0)$. The realization of this breaking in the D1-D5 system has been studied in [28]. Another gravity dual of $N=(4,0)$ SCFT from string theory has been studied in [29], and the marginal perturbation driving $N=(4,4)$ SCFT to the $N=(4,0)$ SCFT has been identified in [30]. This supersymmetry breaking is not possible in the compact $\mathrm{SO}(4)\times\mathrm{SO}(4)$ gauging of [13] since there are no scalars which are singlets under a non-trivial subgroup of $\mathrm{SO}(4)\times\mathrm{SO}(4)$ in order to become the R-symmetry of $N=(4,0)$.

This is however possible in the present gauging. According to (3.2), we see that there are eight singlets under $\mathrm{SU}(2)_L^+\times\mathrm{SU}(2)_L^-$ given by

$$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{3};\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{3}). \tag{3.14}$$

They correspond to the following non-compact generators

$$\begin{aligned}
\hat{Y}_1&=Y^{11}+Y^{22}+Y^{33}+Y^{44}, & \hat{Y}_2&=Y^{12}-Y^{21}+Y^{34}-Y^{43},\\
\hat{Y}_3&=Y^{13}-Y^{31}-Y^{24}+Y^{42}, & \hat{Y}_4&=Y^{14}-Y^{41}+Y^{23}-Y^{32},\\
\hat{Y}_5&=Y^{55}+Y^{66}+Y^{77}+Y^{88}, & \hat{Y}_6&=Y^{56}-Y^{65}+Y^{78}-Y^{87},\\
\hat{Y}_7&=Y^{57}-Y^{75}-Y^{68}+Y^{86}, & \hat{Y}_8&=Y^{58}-Y^{85}+Y^{67}-Y^{76}\,.
\end{aligned} \tag{3.15}$$

We can parametrize the coset representative accordingly

$$L = e^{b_1\hat{Y}_1}e^{a_2\hat{Y}_2}e^{a_3\hat{Y}_3}e^{a_4\hat{Y}_4}e^{b_5\hat{Y}_5}e^{a_6\hat{Y}_6}e^{a_7\hat{Y}_7}e^{a_8\hat{Y}_8} \tag{3.16}$$

in which b_1 and b_5 denote the SO(4) $\times$ SO(4) singlets. We find one non-supersymmetric AdS_3 critical point characterized by

$$\begin{aligned} a_2 &= \cosh^{-1}\sqrt{\frac{g_1+\sqrt{g_1(g_1-4g_2)}}{4g_1}}, \\ V_0 &= -32\left[g_1^2+4g_2^2-6g_1g_2+(4g_2-g_1)\sqrt{g_1(g_1-4g_2)}\right]. \end{aligned} \tag{3.17}$$

The cosmological constant is the same as P_1, but the residual gauge symmetry is just $\mathrm{SO}(4)^-\times\mathrm{SU}(2)^+_L\times\mathrm{U}(1)^+_R$ in which $\mathrm{U}(1)^+_R\subset\mathrm{SU}(2)^+_R$.

3.4 Critical points on the $\mathrm{SU}(2)_{L\mathrm{diag}}$ invariant manifold

We further reduce the residual symmetry to $\mathrm{SU}(2)_{L\mathrm{diag}}\subset\mathrm{SU}(2)^+_L\times\mathrm{SU}(2)^-_L$. Under $\mathrm{SO}(4)_{\mathrm{diag}}$, we already know that the 64 scalars transform as four copies of $\mathbf{1}+\mathbf{6}+\mathbf{9}$. We can then further truncate to $\mathrm{SU}(2)_{L\mathrm{diag}}$ and find sixteen singlets given by four copies of $(\mathbf{1},\mathbf{1})+(\mathbf{1},\mathbf{3})$ under $\mathrm{SU}(2)_{L\mathrm{diag}}\times\mathrm{SU}(2)_{R\mathrm{diag}}$. They can be parametrized by the coset representative

$$L=\prod_{i=1}^{16}e^{a_i\mathcal{Y}_i} \tag{3.18}$$

in which the non-compact generators are defined by

$$\begin{aligned}
\mathcal{Y}_1 &= \frac{1}{2}\left(Y^{15}+Y^{26}+Y^{37}+Y^{48}\right), & \mathcal{Y}_2 &= \frac{1}{2}\left(Y^{16}-Y^{25}+Y^{38}-Y^{47}\right), \\
\mathcal{Y}_3 &= \frac{1}{2}\left(Y^{17}-Y^{35}-Y^{28}+Y^{46}\right), & \mathcal{Y}_4 &= \frac{1}{2}\left(Y^{18}-Y^{45}+Y^{27}-Y^{36}\right), \\
\mathcal{Y}_5 &= \frac{1}{2}\left(Y^{51}+Y^{62}+Y^{73}+Y^{84}\right), & \mathcal{Y}_6 &= \frac{1}{2}\left(Y^{52}-Y^{61}+Y^{74}-Y^{83}\right), \\
\mathcal{Y}_7 &= \frac{1}{2}\left(Y^{53}-Y^{71}-Y^{64}+Y^{82}\right), & \mathcal{Y}_8 &= \frac{1}{2}\left(Y^{54}-Y^{81}+Y^{63}-Y^{72}\right), \\
\mathcal{Y}_9 &= \frac{1}{2}\left(Y^{11}+Y^{22}+Y^{33}+Y^{44}\right), & \mathcal{Y}_{10} &= \frac{1}{2}\left(Y^{12}-Y^{21}+Y^{34}-Y^{48}\right), \\
\mathcal{Y}_{11} &= \frac{1}{2}\left(Y^{13}-Y^{31}-Y^{24}+Y^{42}\right), & \mathcal{Y}_{12} &= \frac{1}{2}\left(Y^{14}-Y^{41}+Y^{23}-Y^{32}\right), \\
\mathcal{Y}_{13} &= \frac{1}{2}\left(Y^{55}+Y^{66}+Y^{77}+Y^{88}\right), & \mathcal{Y}_{14} &= \frac{1}{2}\left(Y^{56}-Y^{65}+Y^{78}-Y^{87}\right), \\
\mathcal{Y}_{15} &= \frac{1}{2}\left(Y^{57}-Y^{75}-Y^{68}+Y^{86}\right), & \mathcal{Y}_{16} &= \frac{1}{2}\left(Y^{58}-Y^{85}+Y^{67}-Y^{76}\right).
\end{aligned} \tag{3.19}$$

From a very complicated potential, we find one non-supersymmetric AdS_3 critical point given by

$$\begin{aligned} a_6 &= \ln\frac{\sqrt{g_2}-\sqrt{3g_1}}{\sqrt{g_2}+\sqrt{3g_1}}, \qquad g_2=(2+\sqrt{13})g_1, \\ V_0 &= -8(469+131\sqrt{13})g_1^2 \end{aligned} \tag{3.20}$$

which is invariant under SU(2) $\times$ U(1) symmetry.

Apart from P_1, P_2 and P_3, we have not given the complete mass spectra for other AdS_3 critical points since the computation is much more involved. A partial check shows that at least the scalar masses for the singlets in each sector satisfy the BF bound. It could happen that some other scalars might have masses violating the bound. However, similar to the three stable critical points studied above, it is likely that the other critical points are stable for some values of α or $g_{1,2}$.

4 Deformations of the $N = (4, 4)$ SCFT

In this section, we will study supersymmetric flows of the maximally supersymmetric $SO(4) \times SO(4)$ critical point in the UV to non-conformal field theories in the IR and half-supersymmetric domain walls. At the end of this section, we will discuss some RG flow solutions interpolating between the UV $N = (4, 4)$ SCFT and some of the non-supersymmetric critical points identified in the previous section.

4.1 Supersymmetric deformations

We begin with supersymmetric solutions which can be obtained by finding solutions of the associated BPS equations. We have not found any supersymmetric critical point apart from the trivial one at $L = \mathbf{I}$, so we only expect to find flow solutions to non-conformal field theories. In these flows, the solutions interpolate between the UV point at which all scalars vanish and the IR with infinite values of scalar vev's [31]. Since supersymmetric solutions are of interest here, we need the supersymmetry transformations of fermions which in the present case are given by the non-propagating gravitini ψ^I_μ and the spin-$\frac{1}{2}$ fields χ^{iI}. Their supersymmetry transformations are given by, see [12] for more details and conventions,

$$\delta\psi^I_\mu = \mathcal{D}_\mu\epsilon^I + gA_1^{IJ}\gamma_\mu\epsilon^J, \tag{4.1}$$

$$\delta\chi^{iI} = \frac{1}{2}(\delta^{IJ}\mathbf{1} - f^{IJ})^i{}_j\not{\mathcal{D}}\phi^j\epsilon^J - gNA_2^{JIi}\epsilon^J\,. \tag{4.2}$$

These equations will be used to find supersymmetric solutions in the next subsections.

4.1.1 A supersymmetric flow to $SO(4) \times SO(4)$ non-conformal field theory

We first look for a simple solution preserving $SO(4) \times SO(4)$ symmetry. Accordingly, only a_1 and a_2 in equation (3.4) are turned on in order to preserve the full $SO(4)\times SO(4)$. Using the standard domain wall ansatz for the metric

$$ds^2 = e^{2A}dx^2_{1,1} + dr^2 \tag{4.3}$$

with A depending only on the radial coordinate r, we find the BPS equations

$$a_1' + 8g_1e^{2a_1}\left(e^{2a_1} - 1\right) = 0, \tag{4.4}$$

$$a_2' + 8g_2e^{2a_2}\left(e^{2a_2} - 1\right) = 0, \tag{4.5}$$

$$A' + 8\left[g_1e^{2a_1}\left(e^{2a_1} - 2\right) + g_2e^{2a_2}\left(e^{2a_2} - 2\right)\right] = 0 \tag{4.6}$$

where we have imposed the projector $\gamma_r\epsilon^I = -\epsilon^I$, $I = 2, 4, 5, 8$ and $\gamma_r\epsilon^I = \epsilon^I$, $I = 1, 3, 6, 7$. The $'$ denotes the r-derivative. The resulting solution is then half-supersymmetric with $N =$

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(4, 4) Poincare supersymmetry in the dual two dimensional field theory. Equations (4.4) and (4.5) can be solved for a_1 and a_2 as an implicit function of r. The result is

$$r = c_1 - \frac{1}{16g_1}\left[e^{-2a_1} + \ln\left(1 - e^{-2a_1}\right)\right], \tag{4.7}$$

$$r = c_2 - \frac{1}{16g_2}\left[e^{-2a_2} + \ln\left(1 - e^{-2a_2}\right)\right] \tag{4.8}$$

with integration constants c_1 and c_2. Equation (4.6) can immediately be integrated to give A as a function of a_1 and a_2. The result is

$$A = 2(a_1 + a_2) - \frac{1}{2}\ln(1 - e^{2a_1}) - \frac{1}{2}\ln(1 - e^{2a_2}). \tag{4.9}$$

In the UV, the dual field theory is conformal with $a_1 = a_2 = 0$. Near this point, the scalars behave as $a_1 \approx e^{-16g_1 r} = e^{-\frac{2g_1}{g_1+g_2}\frac{r}{L_{UV}}}$ and $a_2 \approx e^{-16g_2 r} = e^{-\frac{2g_2}{g_1+g_2}\frac{r}{L_{UV}}}$. We see that $a_{1,2} \to 0$ as $r \to \infty$. In this limit, we find $A' \approx 8(g_1 + g_2) = \frac{1}{L_{UV}}$ or $A \approx \frac{r}{L_{UV}}$ which gives the maximally supersymmetric AdS_3.

As $a_1, a_2 \to \infty$, we find $r \to$ constant as it should. Near $a_1, a_2 \to \infty$, equations (4.7) and (4.8) give $a_1 \approx -\frac{1}{4}\ln(32g_1 r)$ and $a_2 \approx -\frac{1}{4}\ln(32g_2 r)$. From equation (4.9), we find $A \approx a_1 + a_2 = -\frac{1}{4}\ln\left[(32r)^2 g_1 g_2\right]$. Accordingly, the metric becomes a domain wall in the IR

$$ds^2 = \frac{1}{32r\sqrt{g_1 g_2}}dx_{1,1}^2 + dr^2. \tag{4.10}$$

The full bosonic symmetry is $\mathrm{ISO}(1,1) \times \mathrm{SO}(4) \times \mathrm{SO}(4)$ corresponding to non-comformal field theory with $N = (4, 4)$ supersymmetry.

However, flows of this type generally involve singularities. Various types of possible singularities have been classified in [32]. According to the result of [32], physical singularities are the ones at which the scalar potential is bounded from above. However, with the solution given above, the potential becomes infinite in this case. Therefore, the corresponding flow solution is not physically acceptable by the criterion of [32]. Since the framework we have used could be uplifted to ten dimensions via $S^3 \times S^3 \times S^1$ reduction, it is interesting to investigate whether this singularity is resolved in the full string theory.

4.1.2 A half-supersymmetric domain wall

We then look for a more general supersymmetric solution. The scalar sector of interest here is the $\mathrm{SU}(2)^+_L \times \mathrm{SU}(2)^-_L$ invariant one given in (3.16). We first relabel the scalars $(a_2, a_3, a_4, a_6, a_7, a_8)$ to $(b_2, b_3, b_4, b_6, b_7, b_8)$ in order to work with a uniform notation.

We begin with the BPS equations given by $\delta\chi^{iI} = 0$

$$b_1' = -16g_1 e^{b_1}\left(e^{b_1} - \mathrm{sech}b_2\mathrm{sech}b_3\mathrm{sech}b_4\right), \tag{4.11}$$

$$b_2' = -16g_1 e^{b_1}\left(e^{b_1}\cosh b_2 - \mathrm{sech}b_3\mathrm{sech}b_4\right)\sinh b_2, \tag{4.12}$$

$$b_3' = -16g_1 \cosh b_2 \sinh b_3 e^{b_1}\left(e^{b_1}\cosh b_2\cosh b_3 - \mathrm{sech}b_4\right), \tag{4.13}$$

$$b_4' = -16g_1 \cosh b_2\cosh b_3\sinh b_4 e^{b_1}\left(e^{b_1}\cosh b_2\cosh b_3\cosh b_4 - 1\right), \tag{4.14}$$

$$b_5' = -16g_2 e^{b_5}\left(e^{b_5} - \text{sech}b_6\text{sech}b_7\text{sech}b_8\right), \tag{4.15}$$
$$b_6' = -16g_2 \sinh b_6 e^{b_5}\left(e^{b_5}\cosh b_6 - \text{sech}b_7\text{sech}b_8\right), \tag{4.16}$$
$$b_7' = -16g_2 \cosh b_6 \sinh b_7 e^{b_5}\left(e^{b_5}\cosh b_6\cosh b_7 - \text{sech}b_8\right), \tag{4.17}$$
$$b_8' = -16g_2 \cosh b_6\cosh b_7\sinh b_8 e^{b_5}\left(e^{b_5}\cosh b_6\cosh b_7\cosh b_8 - 1\right). \tag{4.18}$$

where we have used the projection conditions $\gamma_r\epsilon^I = -\epsilon^I$, $I = 2,4,5,8$ and $\gamma_r\epsilon^I = \epsilon^I$, $I = 1,3,6,7$ as in the previous case. The gravitino variation $\delta\psi^I_\mu$, $\mu = 0,1$, gives

$$\begin{aligned} A' &= -8g_1 e^{b_1}\cosh b_2\cosh b_3\cosh b_4\left(e^{b_1}\cosh b_2\cosh b_3\cosh b_4 - 2\right) \\ &\quad -8g_2 e^{b_5}\cosh b_6\cosh b_7\cosh b_8\left(e^{b_5}\cosh b_6\cosh b_7\cosh b_8 - 2\right). \end{aligned} \tag{4.19}$$

From these equations, we see that apart from the maximally supersymmetric point at $b_i = 0$, $i = 1,\ldots,8$, there is a flat direction of the potential given by

$$e^{-b_1} = \cosh b_2\cosh b_3\cosh b_4, \qquad e^{-b_5} = \cosh b_6\cosh b_7\cosh b_8 \tag{4.20}$$

which leads to $V_0 = -64(g_1+g_2)^2$. Equation (4.19) gives $A' = 8(g_1+g_2)$ or $A = 8(g_1+g_2)r$ which is the AdS_3 solution with radius $L = \frac{1}{8(g_1+g_2)}$. It can also be verified that the full $(4,4)$ supersymmetry is preserved. This should correspond to a marginal deformation of the $N = (4,4)$ SCFT. There are no other supersymmetric critical points in this sector. Therefore, the flow breaking supersymmetry from $(4,4)$ to $(4,0)$ is not possible.

However, there is a half-supersymmetric domain wall solution similar to the dilatonic p-brane solutions of $N = 1$, $D = 7$ and $N = 2$, $D = 6$ gauged supergravities studied in [33]. It is remarkable that the full set of the above equations admits an analytic solution. The strategy to find the solution is as follow. We first determine $b_{2,3,4}$ as functions of b_1 and similarly determine $b_{6,7,8}$ as functions of b_5. b_1 and b_5 are determined as functions of r and can be solved explicitly. From (4.11) and (4.12), we find

$$\frac{db_2}{db_1} = \cosh b_2\sinh b_2 \tag{4.21}$$

which can be solved for b_2 as a function of b_1 giving rise to

$$b_2 = \coth^{-1} e^{-b_2-2c_1}. \tag{4.22}$$

Using (4.11) and (4.13) together with b_2 solution from (4.22), we find

$$\frac{db_3}{db_1} = \frac{\sinh(2b_3)}{2\left(1 - e^{2b_1+4c_1}\right)} \tag{4.23}$$

whose solution is given by

$$b_3 = \tanh^{-1}\frac{e^{b_1+2c_2}}{\sqrt{1 - e^{2b_1+4c_1}}}. \tag{4.24}$$

Combining (4.11) and (4.14) and substituting for b_2 and b_3 solutions give

$$\frac{db_4}{db_1} = -\frac{\cosh b_4\sinh b_4}{\left(e^{4c_1}+e^{4c_2}\right)e^{b_1} - 1}. \tag{4.25}$$

We then find the solution for b_4

$$b_4 = \tanh^{-1} \frac{e^{b_1+2c_3}}{\sqrt{1-e^{2b_1}\left(e^{4c_1}+e^{4c_2}\right)}}. \tag{4.26}$$

With solutions for b_2, b_3 and b_4, equation (4.11) becomes

$$b_1' = 16g_1 e^{b_1}\left(\sqrt{1-e^{2b_1}\left(e^{4c_1}+e^{4c_2}+e^{4c_3}\right)}-e^{b_1}\right). \tag{4.27}$$

This can be solved for b_1 as an implicit function of r. The solution is

$$\begin{aligned} r = & -\frac{1}{32g_1}\left[2e^{-b_1}\sqrt{1-\beta_1 e^{2b_1}}+\ln\left[e^{-2b_1}\left((\beta_1-1)e^{2b_1}-1+2e^{b_1}\sqrt{1-\beta_1 e^{2b_1}}\right)\right]\right] \\ & +\text{constant} \end{aligned} \tag{4.28}$$

where $\beta_1 = e^{4c_1}+e^{4c_2}+e^{4c_3}$.

We can solve (4.15) to (4.18) by the same procedure. The resulting solutions are given by

$$\begin{aligned} b_6 &= \tanh^{-1} e^{b_5+2c_4}, \qquad b_7 = \tanh^{-1}\frac{e^{b_5+2c_5}}{\sqrt{1-e^{2b_5+4c_4}}}, \\ b_8 &= \tanh^{-1}\frac{e^{b_5+3c_6}}{\sqrt{1-e^{b_5}\left(e^{4c_4}+e^{4c_5}\right)}}, \\ r &= -\frac{1}{32g_2}\left[2e^{-b_5}\sqrt{1-\beta_2 e^{2b_5}}+\ln\left[e^{-2b_5}\left((\beta_2-1)e^{2b_5}-1+2e^{b_5}\sqrt{1-\beta_2 e^{2b_5}}\right)\right]\right] \\ & \quad +\text{constant} \end{aligned} \tag{4.29}$$

where $\beta_2 = e^{4c_4}+e^{4c_5}+e^{4c_6}$.

After substituting all of the b_i solutions for $i = 2,3,4,6,7,8$ in (4.19), we obtain

$$A' = \frac{16g_1 e^{b_1}}{\sqrt{1-\beta_1 e^{2b_1}}} - \frac{8g_1 e^{2b_1}}{1-\beta_1 e^{2b_1}} + \frac{16g_2 e^{b_5}}{\sqrt{1-\beta_2 e^{2b_5}}} - \frac{8g_2 e^{2b_5}}{1-\beta_2 e^{2b_5}} \tag{4.30}$$

whose solution in terms of b_1 and b_5 is readily found by a direct integration using (4.11) and (4.15) including the solutions for the other b_i's. The resulting solution is given by

$$\begin{aligned} A = & \ b_1 + b_5 + \frac{1}{2}\tanh^{-1}\frac{e^{b_1}}{\sqrt{1-\beta_1 e^{2b_1}}} + \frac{1}{2}\tanh^{-1}\frac{e^{b_5}}{\sqrt{1-\beta_2 e^{2b_5}}} - \ln\left[1-\beta_1 e^{2b_1}\right] \\ & -\ln\left[1-(1+\beta_1)e^{2b_1}\right] - \ln\left[1-\beta_2 e^{2b_5}\right] - \ln\left[1-(1+\beta_2)e^{2b_5}\right]. \end{aligned} \tag{4.31}$$

As $b_1, b_5 \to 0$, other scalars do not vanish for finite c_i. We then find that the solution will not have an interpretation in terms of the usual holographic RG flows. The solution is rather of the 1-brane soliton type, see [33] for a general discussion of $(D-2)$-brane solitons in D dimensions. It can also be verified that the $\delta\psi^I_r = 0$ condition precisely gives the Killing spinors for the unbroken supersymmetry $\epsilon^I = e^{\frac{A}{2}}\epsilon^I_0$ with the constant spinor ϵ^I_0 satisfying $\gamma_r\epsilon^I_0 = -\epsilon^I_0$, $I = 2,4,5,8$ and $\gamma_r\epsilon^I_0 = \epsilon^I_0$, $I = 1,3,6,7$.

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4.2 Non-supersymmetric deformations

We now briefly discuss non-supersymmetric RG flow solutions interpolating between the $N = (4, 4)$ SCFT in the UV and some critical points found in the previous section. The solutions are essentially non-supersymmetric since they connect a supersymmetric to a non-supersymmetric critical point. Finding the corresponding solutions involve solving the full second order field equations for both the scalars and the metric in contrast to solving the first order BPS equations in the supersymmetric case. Although there are some examples of analytic supersymmetric flow solutions in three dimensions, in general, analytic solutions with many active scalars, even for the supersymmetric case, can be very difficult to find. Therefore, we will not expect to find any analytic solutions in the non-supersymmetric case but rather look for numerical flow solutions.

In all cases, the interpolating solutions generally exist and can be obtained by a similar procedure used in [34]. In solving the second-order field equations for scalars and the metric function, two types of asymptotic behavior of scalars arise near the UV fixed point. One of them corresponds to a deformation by turning on a dual operator while the other corresponds to a vacuum expectation value (vev). The second-order equations lead to an ambiguity between these two possibilities. One way to solve this ambiguity is to recast the second-order field equations into a first-order form by introducing the generating function W [35, 36]. Like supersymmetric solutions obtained from first-order BPS equations, only one possibility is singled out from these new first-order equations.

In the present case, numerical analyses show that non-supersymmetric flows to P_1 and P_2 are driven by turning on relevant operators. These describe true deformations of the UV SCFT rather than vev deformations. The flow to P_3 involves four active scalars and is more difficult to find. However, the flow is expected to be driven by a scalar transforming as $(\mathbf{1}, \mathbf{1})$ under $\mathrm{SO}(4) \times \mathrm{SO}(4)$ at the UV point. From the value of g_1 and g_2 in the stability range, it can be checked that only the deformation dual to this scalar is relevant. The deformations corresponding to the remaining active scalars are given by vacuum expectation values of irrelevant operators since these scalars have positive mass squares.

5 Conclusions and discussions

In this paper, we have studied $N = 8$ gauged supergravity in three dimensions with a non-semisimple gauge group $(\mathrm{SO}(4) \times \mathrm{SO}(4)) \ltimes \mathbf{T}^{12}$. The ratio of the coupling constants of the two SO(4)'s is given by a parameter α. For positive α, the theory describes an effective theory of ten dimensional supergravity reduced on $S^3 \times S^3 \times S^1$. For negative α, on the other hand, the theory may describe a similar reduction on $S^3 \times H^3 \times S^1$ in which H^3 is a three-dimensional hyperbolic space. With $\alpha = -1$, the cosmological constant is zero. This solution should describe a ten dimensional background $M_3 \times S^3 \times H^3 \times S^1$ where M_3 is the three-dimensional Minkowski space.

We have studied the scalar potential and found a number of non-supersymmetric AdS_3 critical points. The trivial critical point with maximal supersymmetry is identified with the dual large $N = (4, 4)$ SCFT in two dimensions. We have explicitly checked the stability of some non-supersymmetric critical points by computing the full scalar mass spectra at these critical points. They are perturbatively stable for some values of α parameter in the sense that all scalar masses are above the BF bound. It is also interesting to see whether other critical points are stable or not. We have investigated RG flows, interpolating between the large $N = (4, 4)$ SCFT in the UV and non-supersymmetric IR fixed points with $SO(4) \times SO(4)$, $SO(4) \times SO(2) \times SO(2)$ and $SO(4)$ symmetries, and also commented on the operators driving these flows.

Another result of this paper is half-supersymmetric domain wall solutions to $N = 8$ gauged supergravity. For the domain wall preserving $SO(4)\times SO(4)$ symmetry, the solution describes an RG flow from $N = (4, 4)$ SCFT in the UV to a non-conformal $N = (4, 4)$ field theory in the IR. The solution has however a bad singularity according to the criterion of [32]. For the solution preserving $SU(2)\times SU(2)$ symmetry, the holographic interpretation is not clear. In the point of view of a $(D - 2)$-brane soliton, the solution should describe a 1-brane soliton in three dimensions according to the general discussion in [33]. When uplifted to ten dimensions, the solution might describe some configuration of D1-branes. Hopefully, the solutions obtained in this paper might be useful in string/M theory context, black hole physics and the AdS/CFT correspondence. The uplifted solution of the non-conformal flow preserving $SO(4) \times SO(4)$ symmetry is also necessary for the resolution of its singularity if the full ten-dimensional solution turns out to be non-singular.

Finally, the chiral supersymmetry breaking $(4, 4) \to (4, 0)$ found in [28] cannot be implemented in the framework of $N = 8$ gauged supergravity studied here. It would probably require a larger theory of $N = 16$ gauged supergravity with $(SO(4) \times SO(4)) \ltimes (\mathbf{T}^{12}, \hat{\mathbf{T}}^{34})$ gauge group studied in [14]. It would be very interesting to find the flow solution of [28] explicitly in the three dimensional framework. We hope to come back to these issues in future research.

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A Useful formulae and details

For completeness, we include a short review of gauged supergravity in three dimensions in the formulation of [12]. The theory is a gauged version of a supersymmetric non-linear sigma model coupled to non-propagating supergravity fields. N-extended supersymmetry requires the presence of $N - 1$ almost complex structures f^P, $P = 2, \ldots, N$ on the scalar manifold. The tensors $f^{IJ} = f^{[IJ]}$, generating the $SO(N)$ R-symmetry in a spinor representation under which scalar fields transform, play an important role. In the case of symmetric scalar manifolds of the form $G/SO(N) \times H'$, they can be written in terms of

SO(N) gamma matrices. In our case, we use the 16×16 Dirac gamma matrices of SO(8)

$$\gamma^I = \begin{pmatrix} 0 & \Gamma^I \\ (\Gamma^I)^T & 0 \end{pmatrix}. \tag{A.1}$$

The 8×8 gamma matrices are explicitly given by

$$\begin{aligned} \Gamma_1 &= \sigma_4\otimes\sigma_4\otimes\sigma_4, & \Gamma_2 &= \sigma_1\otimes\sigma_3\otimes\sigma_4, \\ \Gamma_3 &= \sigma_4\otimes\sigma_1\otimes\sigma_3, & \Gamma_4 &= \sigma_3\otimes\sigma_4\otimes\sigma_1, \\ \Gamma_5 &= \sigma_1\otimes\sigma_2\otimes\sigma_4, & \Gamma_6 &= \sigma_4\otimes\sigma_1\otimes\sigma_2, \\ \Gamma_7 &= \sigma_2\otimes\sigma_4\otimes\sigma_1, & \Gamma_8 &= \sigma_1\otimes\sigma_1\otimes\sigma_1 \end{aligned} \tag{A.2}$$

where

$$\begin{aligned} \sigma_1 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, & \sigma_2 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\ \sigma_3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, & \sigma_4 &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \end{aligned} \tag{A.3}$$

According to our normalization, we find

$$f^{IJ}_{Kr,Ls} = -\mathrm{Tr}(Y_{Ls}\,[T^{IJ}, Y_{Kr}]). \tag{A.4}$$

Generally, the $d=\dim(G/H)$ scalar fields ϕ^i, $i=1,\ldots,d$ can be described by a coset representative L. The useful formulae for a coset space are

$$L^{-1}t^{\mathcal{M}}L = \frac{1}{2}\mathcal{V}^{\mathcal{M}}{}_{IJ}T^{IJ} + \mathcal{V}^{\mathcal{M}}{}_{\alpha}X^{\alpha} + \mathcal{V}^{\mathcal{M}}{}_{A}Y^{A}, \tag{A.5}$$

$$L^{-1}\partial_i L = \frac{1}{2}Q_i^{IJ}T^{IJ} + Q_i^{\alpha}X^{\alpha} + e_i^{A}Y^{A} \tag{A.6}$$

where e_i^A, Q_i^{IJ} and Q_i^α are the vielbein on the coset manifold and SO(N) $\times H'$ composite connections, respectively. X^α's denote the H' generators.

Any gauging can be described by a symmetric and gauge invariant embedding tensor satisfying the so-called quadratic constraint

$$\Theta_{\mathcal{PL}}f^{\mathcal{KL}}{}_{(\mathcal{M}}\Theta_{\mathcal{N})\mathcal{K}} = 0, \tag{A.7}$$

and the projection constraint

$$\mathbb{P}_{R_0}\Theta_{\mathcal{MN}} = 0\,. \tag{A.8}$$

The first condition ensures that the gauge symmetry forms a proper symmetry algebra while the second condition guarantees the consistency with supersymmetry.

The T-tensor given by the moment map of the embedding tensor by scalar matrices $\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}$, obtained from (A.5), is defined by

$$T_{\mathcal{AB}} = \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}\Theta_{\mathcal{MN}}\mathcal{V}^{\mathcal{N}}{}_{\mathcal{B}}\,. \tag{A.9}$$

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Only the components $T^{IJ,KL}$ and $T^{IJ,A}$ are relevant for computing the scalar potential. With our SO(8, 8) generators, we obtain the following $\mathcal{V}$ maps

$$\begin{aligned}
\mathcal{V}_{\mathcal{A}1}^{ab,IJ} &= -\frac{1}{2}\mathrm{Tr}(L^{-1}J_1^{ab}T^{IJ}), & \mathcal{V}_{\mathcal{B}1}^{ab,IJ} &= -\frac{1}{2}\mathrm{Tr}(L^{-1}t_1^{ab}T^{IJ}), \\
\mathcal{V}_{\mathcal{A}1}^{ab,Kr} &= \frac{1}{2}\mathrm{Tr}(L^{-1}J_1^{ab}Y^{Kr}), & \mathcal{V}_{\mathcal{B}1}^{ab,Kr} &= \frac{1}{2}\mathrm{Tr}(L^{-1}t_1^{ab}Y^{Kr}), \\
\mathcal{V}_{\mathcal{A}2}^{\hat{a}\hat{b},IJ} &= -\frac{1}{2}\mathrm{Tr}(L^{-1}J_2^{\hat{a}\hat{b}}T^{IJ}), & \mathcal{V}_{\mathcal{B}2}^{\hat{a}\hat{b},IJ} &= -\frac{1}{2}\mathrm{Tr}(L^{-1}t_2^{\hat{a}\hat{b}}T^{IJ}), \\
\mathcal{V}_{\mathcal{A}2}^{\hat{a}\hat{b},Kr} &= \frac{1}{2}\mathrm{Tr}(L^{-1}J_2^{\hat{a}\hat{b}}Y^{Kr}), & \mathcal{V}_{\mathcal{B}2}^{\hat{a}\hat{b},Kr} &= \frac{1}{2}\mathrm{Tr}(L^{-1}t_2^{\hat{a}\hat{b}}Y^{Kr})
\end{aligned} \tag{A.10}$$

where we have followed the convention of calling the semisimple part $\mathrm{SO}(4) \times \mathrm{SO}(4)$ and the nilpotent part $\mathbf{T}^{12} \sim \mathbf{T}^6 \times \mathbf{T}^6$ as $\mathcal{A}$ and $\mathcal{B}$ types, respectively. We then compute the T-tensor components

$$\begin{aligned}
T^{IJ,KL} &= g_1\left(\mathcal{V}_{\mathcal{A}1}^{ab,IJ}\mathcal{V}_{\mathcal{B}1}^{cd,KL} + \mathcal{V}_{\mathcal{B}1}^{ab,IJ}\mathcal{V}_{\mathcal{A}1}^{cd,KL} - \mathcal{V}_{\mathcal{B}1}^{ab,IJ}\mathcal{V}_{\mathcal{B}1}^{cd,KL}\right)\epsilon_{abcd} \\
&\quad + g_2\left(\mathcal{V}_{\mathcal{A}2}^{\hat{a}\hat{b},IJ}\mathcal{V}_{\mathcal{B}2}^{\hat{c}\hat{d},KL} + \mathcal{V}_{\mathcal{B}2}^{\hat{a}\hat{b},IJ}\mathcal{V}_{\mathcal{A}2}^{\hat{c}\hat{d},KL} - \mathcal{V}_{\mathcal{B}2}^{\hat{a}\hat{b},IJ}\mathcal{V}_{\mathcal{B}2}^{\hat{c}\hat{d},KL}\right)\epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}},
\end{aligned} \tag{A.11}$$

$$\begin{aligned}
T^{IJ,Kr} &= g_1\left(\mathcal{V}_{\mathcal{A}1}^{ab,IJ}\mathcal{V}_{\mathcal{B}1}^{cd,Kr} + \mathcal{V}_{\mathcal{B}1}^{ab,IJ}\mathcal{V}_{\mathcal{A}1}^{cd,Kr} - \mathcal{V}_{\mathcal{B}1}^{ab,IJ}\mathcal{V}_{\mathcal{B}1}^{cd,Kr}\right)\epsilon_{abcd} \\
&\quad + g_2\, \mathcal{V}_{\mathcal{A}2}^{\hat{a}\hat{b},IJ}\mathcal{V}_{\mathcal{B}2}^{\hat{c}\hat{d},Kr} + \mathcal{V}_{\mathcal{B}2}^{\hat{a}\hat{b},IJ}\mathcal{V}_{\mathcal{A}2}^{\hat{c}\hat{d},Kr} - \mathcal{V}_{\mathcal{B}2}^{\hat{a}\hat{b},IJ}\mathcal{V}_{\mathcal{B}2}^{\hat{c}\hat{d},Kr}\Big)\epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}\,.
\end{aligned} \tag{A.12}$$

The scalar potential can be computed by using the formula

$$V = -\frac{4}{N}\left(A_1^{IJ}A_1^{IJ} - \frac{1}{2}Ng^{ij}A_{2i}^{IJ}A_{2j}^{IJ}\right) \tag{A.13}$$

in which the metric g_{ij} is related to the vielbein by $g_{ij} = e_i^A e_j^A$. The A_1 and A_2 tensors appearing in the gauged Lagrangian as fermionic mass-like terms are given by

$$A_1^{IJ} = -\frac{4}{N-2}T^{IM,JM} + \frac{2}{(N-1)(N-2)}\delta^{IJ}T^{MN,MN}, \tag{A.14}$$

$$A_{2j}^{IJ} = \frac{2}{N}T^{IJ}{}_j + \frac{4}{N(N-2)}f^{M(Im}{}_j T^{J)M}{}_m + \frac{2}{N(N-1)(N-2)}\delta^{IJ}f^{KL}{}_j{}^m T^{KL}{}_m\,. \tag{A.15}$$

Finally, we repeat the condition for supersymmetric critical points. The residual supersymmetry is generated by the eigenvectors of the A_1^{IJ} tensor with eigenvalues equal to $\pm\sqrt{\frac{-V_0}{4}}$.

B Explicit forms of the scalar potential

For $\mathrm{SO}(4)_{\mathrm{diag}}$ invariant scalars, the potential is given by

$$\begin{aligned}
V &= 4e^{6a_1}g_1^2\cosh^2(a_3-a_4)\cosh^2(a_3+a_4)\,[5\cosh[2(a_1-2a_3)] + 8\cosh(4a_3) \\
&\quad +5\cosh[2(a_1+2a_3)] - 4\cosh(2a_1)\,(7+2\cosh(2a_3)\cosh(2a_4)) + 2\cosh(4a_4)\,\times \\
&\quad (\cosh a_1 - 3\sinh a_1)^2 - 6\,(\cosh(4a_3) - 4\cosh(2a_3)\cosh(2a_4) - 6)\sinh(2a_1)\Big]
\end{aligned}$$

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$$
\begin{aligned}
&+4e^{6a_2}g_2^2\cosh^2(a_3-a_4)\cosh^2(a_3+a_4)\,[5\cosh[2(a_2-2a_3)]-8\cosh(4a_3)\\
&+5\cosh[2(a_2+2a_3)]-4\cosh(2a_2)\,(7+2\cosh(2a_3)\cosh(2a_4))+2\cosh(4a_4)\times\\
&(\sinh a_2-3\cosh a_2)^2-6\,(\cosh(4a_3)-4\cosh(2a_3)\cosh(2a_4)-6)\sinh(2a_2)\Big]\\
&-2e^{a_1+a_2+6(a_3+a_4)}g_1g_2\,[86\cosh(a_1+a_2)-64\cosh(a_1-a_2)\cosh(2a_3)+\cosh(2a_3)\times\\
&\cosh(6a_4)\,(\cosh a_1-3\sinh a_1)\,(3\cosh a_2-\sinh a_2)+16\cosh a_1\cosh(4a_3)\sinh a_2\\
&+\cosh(2a_4)\,[-64\cosh(a_1-a_2)+\cosh(6a_3)\,(3\cosh a_1-\sinh a_1)\times\\
&(\cosh a_2-3\sinh a_2)+2\cosh(2a_3)\,(37\cosh(a_1+a_2)-19\sinh(a_1+a_2))]\\
&-66\sinh(a_1+a_2)+2\cosh(4a_4)\,[8\cosh a_2\sinh a_1+\cosh(4a_3)\,(\sinh(a_1+a_2)\\
&-3\cosh(a_1+a_2))]+[25\cosh(a_1+a_2)-27\cosh a_2\sinh a_1+2\cosh(4a_3)\times\\
&(3\cosh a_1-\sinh a_1)\,(\cosh a_2-3\sinh a_2)-35\cosh a_1\sinh a_2]\sinh(2a_3)\sinh(2a_4)\\
&+2\,(\sinh(a_1+a_2)-3\cosh(a_1+a_2))\sinh(4a_3)\sinh(4a_4)+\sinh(2a_3)\sinh(6a_4)\times\\
&(3\cosh a_2-\sinh a_2)\,(\cosh a_1-3\sinh a_1)]\,. \qquad \text{(B.1)}
\end{aligned}
$$

The potential for $\mathrm{SU}(2)^+_L\times\mathrm{SU}(2)^-_L$ invariant scalars is given by, in notation of section 4,

$$
\begin{aligned}
V\;=\;128\,\Big[&g_1^2e^{2b_1}\cosh^2 b_2\cosh^2 b_3\cosh^2 b_4\left(e^{b_1}\cosh b_2\cosh b_3\cosh b_4-1\right)^2\\
&+g_2^2e^{2b_5}\cosh^2 b_6\cosh^2 b_7\cosh^2 b_8\left(e^{b_5}\cosh b_6\cosh b_7\cosh b_8-1\right)^2\Big]\,. \qquad \text{(B.2)}
\end{aligned}
$$

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