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Erratum: Soft-collinear gravity beyond the leading power

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This erratum corrects a number of typos and equations in the original article [1].

1) In eq. (2.26), the position argument of $A_{s\mu}$ should read $x_- + s(x - x_-)$.

2) In the definition of $\mathcal{A}_s$ in eq. (2.31) a term is missing. The correct expression is

$$\mathcal{A}_s^\mu(x) \equiv R^\dagger \left(A_s^\mu(x) - n_- A_s(x_-) \frac{n_+^\mu}{2} \right) R + \frac{i}{g} R^\dagger \left[D_s^\mu \, , R \right] . \tag{1}$$

3) Below eqs. (2.57) and (2.59), the non-locality of the building blocks for the N-jet operators is stated to be in the n_{i-}^μ-direction. The collinear fields are non-local in the n_{i+}^μ-direction, as shown explicitly in eq. (2.60).

4) The Einstein-Hilbert Lagrangian $\mathcal{L}^{(1)}$ in eq. (3.24) is missing some terms. Adding them, the correct expression simplifies and reads

$$\begin{aligned} \mathcal{L}^{(1)} = & -\frac{1}{2} h^{\alpha\beta} \Big(h \partial_\alpha \partial_\beta h + 2 \partial_\mu h^{\mu\nu} \partial_\nu h_{\alpha\beta} + \partial_\alpha h_{\mu\nu} \partial_\beta h^{\mu\nu} + h_{\alpha\beta} \Box h + 2 \partial^\mu h_{\mu\alpha} \partial_\beta h \\ & + \partial_\alpha h_{\beta\mu} \partial^\mu h - h \partial^\mu \partial_\alpha h_{\beta\mu} - h_\alpha^\mu \Box h_{\mu\beta} + 2 \partial_\mu h_{\beta\nu} \partial^\nu h_\alpha^\mu + 4 \partial^\mu \partial^\nu h_{\alpha\mu} h_{\beta\nu} \Big) \\ & - \frac{1}{4} h \partial_\mu h \partial^\mu h + \frac{1}{4} h \partial_\alpha h_{\mu\nu} \partial^\alpha h^{\mu\nu} . \end{aligned} \tag{2}$$

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5) Eq. (4.26) is the equation of motion for the trace in the absence of matter. If a scalar matter field is present as is assumed in section 4.4, the complete expression for the trace reads

$$\mathfrak{h} = \frac{1}{2}\left(\mathfrak{h}_{\alpha_\perp\beta_\perp}\mathfrak{h}^{\alpha_\perp\beta_\perp} - \frac{1}{\partial_+^2}\left[\partial_+\mathfrak{h}_{\alpha_\perp\beta_\perp}\partial_+\mathfrak{h}^{\alpha_\perp\beta_\perp} - \partial_+\chi_c\partial_+\chi_c\right]\right) + \mathcal{O}(\lambda^3)\,, \tag{3}$$

where χ_c is the gauge-invariant scalar building block.

6) Eqs. (5.83) and (5.84) related to the soft covariant derivative contain errors. The corrected eq. (5.83) reads

$$\hat{g}_s^{\mu\nu}(x)\partial_\mu\varphi\partial_\nu\varphi = \partial_{\mu_\perp}\varphi\partial^{\mu_\perp}\varphi + n_+\partial\varphi n_- D_{s,\varphi}\varphi\,, \tag{4}$$

and eq. (5.84) must be replaced by

$$\begin{aligned} n_- D_{s,\varphi} &= \frac{1}{4}\hat{g}_s^{--}(x)n_+\partial + \hat{g}_s^{\mu_\perp -}(x)\partial_{\mu_\perp} + \frac{1}{2}\hat{g}_s^{+-}(x)n_-\partial \\ &= \partial_- - \frac{1}{2}s_-{}^\mu\partial_\mu + \frac{1}{8}s_{-\alpha}s^{\alpha\mu}\partial_\mu + \frac{1}{8}s_{-+}s^\mu{}_-\partial_\mu + \frac{1}{2}\left[\Omega_-\right]_{\mu\nu}J^{\mu\nu} + \mathcal{O}(\lambda^5)\,, \end{aligned} \tag{5}$$

where the last term proportional to the spin-connection comes with $+\frac{1}{2}$ instead of $-\frac{1}{4}$. This correction affects eq. (5.97), where accordingly the last term should read $+\frac{1}{2}\left[\Omega_-\right]_{\mu\nu}J^{\mu\nu}$. The soft covariant derivative can be compactly expressed via the emergent soft background vierbein [2]

$$n_- D_{s,\varphi} = \hat{E}_s{}^\mu{}_-(x)\partial_\mu\,, \tag{6}$$

where

$$\hat{E}_s{}^\mu{}_-(x) = n_-^\mu - \frac{1}{2}s_-{}^\mu + \frac{1}{8}s_{-\alpha}s^{\alpha\mu} + \frac{1}{8}s_{-+}s^\mu{}_- - \left[\Omega_-\right]^\mu{}_\rho\,(x-x_-)^\rho + \mathcal{O}(\lambda^5)\,. \tag{7}$$

The final Lagrangian given in eqs. (5.129)–(5.133) was given correctly and is not affected by the above corrections.

7) The final Einstein-Hilbert Lagrangian $\mathcal{L}_{\rm EH}^{(1)}$ in (5.138) is affected by the mistake in eq. (3.24), corrected by (2) here. It should read

$$\begin{aligned} \mathcal{L}_{\rm EH}^{(1)} = &-\frac{\kappa}{2}\hat{h}^{\alpha\beta}\Big(\hat{h}\partial_\alpha\partial_\beta\hat{h} + 2\partial_\mu\hat{h}^{\mu\nu}\partial_\nu\hat{h}_{\alpha\beta} + \partial_\alpha\hat{h}_{\mu\nu}\partial_\beta\hat{h}^{\mu\nu} + \hat{h}_{\alpha\beta}\partial_\mu\partial^\mu\hat{h} + 2\partial^\mu\hat{h}_{\mu\alpha}\partial_\beta\hat{h} \\ &\qquad + \partial_\alpha\hat{h}_{\beta\mu}\partial^\mu\hat{h} - \hat{h}\partial^\mu\partial_\alpha\hat{h}_{\beta\mu} - \hat{h}^\mu_\alpha\partial_\nu\partial^\nu\hat{h}_{\mu\beta} + 2\partial_\mu\hat{h}_{\beta\nu}\partial^\nu\hat{h}^\mu_\alpha + 4\partial^\mu\partial^\nu\hat{h}_{\alpha\mu}\hat{h}_{\beta\nu}\Big) \\ &- \frac{\kappa}{4}\hat{h}\partial_\mu\hat{h}\partial^\mu\hat{h} + \frac{\kappa}{4}\hat{h}\partial_\alpha\hat{h}_{\mu\nu}\partial^\alpha\hat{h}^{\mu\nu} - \frac{\kappa}{4}s_{-\mu_\perp}\partial^{\mu_\perp}\hat{h}^{\alpha\beta}\partial_+\hat{h}_{\alpha\beta} + \frac{\kappa}{4}s_{-\mu_\perp}\partial^{\mu_\perp}\hat{h}\partial_+\hat{h} \\ &- \frac{\kappa}{8}\left[\partial_\alpha s_{--} - \partial_- s_{\alpha-}\right]x_\perp^\alpha\partial_+\hat{h}_{\mu\nu}\partial_+\hat{h}^{\mu\nu} + \frac{\kappa}{8}\left[\partial_\alpha s_{--} - \partial_- s_{\alpha-}\right]x_\perp^\alpha\partial_+\hat{h}\partial_+\hat{h} \\ &+ \frac{\kappa^2}{8}\hat{h}^{\alpha\beta}(\hat{h}_{\alpha\beta}s_{--}\partial_+^2\hat{h} - \hat{h}^\mu_\alpha s_{--}\partial_+^2\hat{h}_{\mu\beta}) + \frac{\kappa^2}{16}\hat{h}s_{--}\partial_+\hat{h}\partial_+\hat{h} \\ &- \frac{\kappa^2}{16}\hat{h}s_{--}\partial_+\hat{h}_{\mu\nu}\partial_+\hat{h}^{\mu\nu}\,. \end{aligned} \tag{8}$$

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