

# Exploring charm decays with missing energy in leptoquark models

Gaber Faisel,<sup>a</sup> Jhih-Ying Su<sup>b</sup> and Jusak Tandean<sup>c,d</sup>

<sup>a</sup>*Department of Physics, Faculty of Arts and Sciences, Süleyman Demirel University, Isparta 32260, Turkey*

<sup>b</sup>*Department of Physics, University of Massachusetts Amherst, Massachusetts 01003, U.S.A.*

<sup>c</sup>*Department of Physics, National Taiwan University, Taipei 106, Taiwan*

<sup>d</sup>*Physics Division, National Center for Theoretical Sciences, Hsinchu 300, Taiwan*

*E-mail:* [gaberfaisel@sdu.edu.tr](mailto:gaberfaisel@sdu.edu.tr), [jhihyingsu@umass.edu](mailto:jhihyingsu@umass.edu), [jtandean@yahoo.com](mailto:jtandean@yahoo.com)

**ABSTRACT:** We investigate the possibility that scalar leptoquarks generate consequential effects on the flavor-changing neutral-current decays of charmed hadrons into final states with missing energy ( $\cancel{E}$ ) carried away by either standard model or sterile neutrinos. We focus on scenarios involving the  $R_2$ ,  $\tilde{R}_2$ , and  $\tilde{S}_1$  leptoquarks and take into account various pertinent constraints, learning that meson-mixing ones and those inferred from collider searches can be of significance. We find in particular that the branching fractions of charmed meson decays  $D \rightarrow M\cancel{E}$ ,  $M = \pi, \rho$ , and  $D_s \rightarrow K^{(*)}\cancel{E}$  and singly charmed baryon decays  $\Lambda_c^+ \rightarrow p\cancel{E}$  and  $\Xi_c \rightarrow \Sigma\cancel{E}$  are presently allowed to attain the  $10^{-7}$ - $10^{-6}$  levels if induced by  $R_2$  and that the impact of  $\tilde{R}_2$  is comparatively much less. In contrast, the contributions of  $\tilde{S}_1$ , which couples to right-handed up-type quarks and the sterile neutrinos, could lead to branching fractions as high as order  $10^{-3}$ . This suggests that these charmed hadron decays might be within reach of the BESIII and Belle II experiments or future super charm-tau factories and could serve as potentially promising probes of leptoquark interactions with sterile neutrinos.

**KEYWORDS:** Beyond Standard Model, Heavy Quark Physics, Effective Field Theories, Neutrino Physics

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## 1 Introduction

The flavor-changing neutral-current (FCNC) decays of charmed hadrons into a lighter hadron plus missing energy ( $\cancel{E}$ ) have long been anticipated in the literature [1–14] to be among likely environments in which to discover hints of new physics (NP) beyond the standard model (SM). In the SM these processes arise at short distance from the quark transition  $c \rightarrow u\nu\bar{\nu}$ , which emits unobserved neutrinos ( $\nu\bar{\nu}$ ) and is greatly suppressed because it proceeds from loop diagrams and is subject to very efficient Glashow-Iliopoulos-Maiani cancellation [1]. The effects of long-distance physics on these decays have also been estimated to be tiny [1]. Beyond the SM there could be extra ingredients causing modifications to the SM component and/or yield additional channels with one or more invisible nonstandard particles, which might translate into substantially amplified rates detectable by upcoming quests.

Experimentally, there still has not been a lot of activity to look for charmed hadron decays of this kind [15]. At the moment the sole result available is a limit on the branching fraction of charmed meson decay  $D^0 \rightarrow \cancel{E}$ , which has been set by the Belle Collaboration [16]. Due to the importance of these processes as valuable tools in the quest for NP, it is hoped that dedicated efforts will increasingly be made to pursue them. Since a clean environment and sizable luminosity are crucial for such endeavors, it is then timely that there are now heavy-flavor factories which are running and expectedly well-suited for them, namely BESIII [17] and Belle II [18]. In the future, further measurements with improved sensitivity would presumably be feasible, such as at the proposed super charm-tau factories [19, 20] and Circular Electron Positron Collider (CEPC) operated as a  $Z$ -boson factory [21].

The foregoing motivates us in this paper to explore these FCNC charm transitions with missing energy in the contexts of relatively simple NP scenarios. We entertain specifically the possibility that heavy leptoquarks (LQs) with spin 0 mediate the NP contributions to the FCNCs. Over the last several years LQs have attracted a good deal of attention because suggested models containing them are among those that could offer the preferred explanations for the so-called  $B$ -physics anomalies [7, 22]. While more data are awaited in order to clarify whether or not these anomalies are attributable to NP, it is therefore of interest to investigate if LQs can give rise to appreciable manifestations in the charm sector too. There have been various analyses in the past, such as refs. [6–8, 23–33], looking into the effects of LQs on FCNC charm processes, but  $c \rightarrow u\cancel{E}$  was covered by only a few [6–8]. The outcomes of our work would be complementary to those of the latter.

Besides the scalar LQs, we will incorporate light right-handed sterile neutrinos into the theory. They are singlets under the SM gauge groups and in the presence of the LQs can have renormalizable links to SM quarks. Our inclusion of right-handed neutrinos is well motivated for two reasons. First, their existence will be necessary if measurements in the future establish that neutrinos are Dirac in nature. Second, as will be demonstrated later on, relative to those with SM fermions alone the LQ-mediated FCNCs that involve the sterile neutrinos and SM quarks might exert considerably enhanced influence on the charm decays of concern, especially if the quarks are also right-handed. This latter LQ feature has not been treated previously in the  $c \rightarrow u\cancel{E}$  context [6–8].

With respect to the predictions of the models to be studied shortly, it will be useful to have some idea about the extent to which they might be accessible by the aforementioned experiments. Here we address this question briefly, in light of the currently scant details obtainable from the literature. For  $D^0 \rightarrow \pi^0\cancel{E}$ , its branching fraction could be tested by BESIII down to  $10^{-4}$  or lower [34], after it gathers a data sample of  $20\text{fb}^{-1}$  at center-of-mass energy  $\sqrt{s} \simeq 3.77\text{GeV}$ . No corresponding information exists for Belle II as far as we can tell, but a rough estimate may be made based on the projected prospects of these ongoing efforts to discover  $D^0 \rightarrow \cancel{E}$ . Thus, since it is expected to improve on the Belle bound  $\mathcal{B}(D^0 \rightarrow \cancel{E}) < 9.4 \times 10^{-5}$  [16] by a factor of seven [18] and BESIII could strengthen it further to  $10^{-6}$  with its final charm dataset [34], the Belle II sensitivity to  $\mathcal{B}(D^0 \rightarrow \pi^0\cancel{E})$  may reach only around  $(10^{-4}/10^{-6})(9.4 \times 10^{-5}/7) \sim 10^{-3}$  if the ratios of efficiencies of these machines to reconstruct the two modes are alike. Nevertheless, it is hoped that Belle II will ultimately be able to exceed this naive expectation. Later in the future, the super charm-tau factories [19, 20] are planned to attain total luminosities 100 times that of BESIII, and so they could have the capability to access  $\mathcal{B}(D^0 \rightarrow \pi^0\cancel{E}) \sim 10^{-5}$  or better. On the other hand, the CEPC running at the  $Z$  pole [21] is proposed to produce a few times more charmed hadrons than Belle II and hence might be somewhat superior to the latter for probing this decay. Given that the  $D^0$  amount collected at these different facilities [18, 21, 34] is bigger than those of  $D_{(s)}^+$  and charmed baryons, their sensitivity to the other FCNC charmed-hadron transitions we will look at would probably be similar or less.

The organization of the remainder of the paper is as follows. In section 2 we describe the interactions of the new particles, namely the scalar LQs and the light sterile neutrinos,

with the SM fermions. Concentrating on three distinct scalar LQs, in sections 3, 4, and 5 we examine three separate scenarios, in each of which merely one of the LQs is responsible for  $c \rightarrow u\cancel{E}$ . In these sections, before presenting our numerical results, we first deal with the pertinent limitations on the Yukawa parameters and masses of the LQs. We point out particularly that meson-mixing constraints in the charm and down-type quark sectors are potentially relevant, but may be avoided in certain instances. Furthermore, we take into account restrictions inferred from collider searches. Subsequently, we evaluate a variety of FCNC decays of the lightest charmed pseudoscalar-mesons and singly charmed baryons manifesting the LQ effects on  $c \rightarrow u\cancel{E}$ . In section 6 we draw our conclusions. In the appendix, we provide general expressions for the rates of the hadron decay modes under consideration. For the meson channels we show that, assuming the invisible particles to have negligible masses, the formulas for the LQ-generated branching fractions can be determined with the aid of data on the corresponding semileptonic modes and without relying on the form factors in the mesonic matrix elements.

## 2 Leptoquark couplings to fermions

Among LQs that can have renormalizable interactions with SM fermions without violating the conservations of baryon and lepton numbers and the SM gauge symmetries, there are four which possess spin 0 and can at tree level contribute to the quark transition  $c \rightarrow u\cancel{E}$  where the missing energy is carried away by either SM or sterile neutrinos [7].<sup>1</sup> In the nomenclature of ref. [7], these scalar LQs, with their assignments under the SM gauge groups  $SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y$ , are denoted by  $R_2(3, 2, 7/6)$ ,  $\tilde{R}_2(3, 2, 1/6)$ ,  $\bar{S}_1(\bar{3}, 1, -2/3)$ , and  $S_3(\bar{3}, 3, 1/3)$ . Here we pay attention to the first three because we have found that the couplings of  $S_3$  are comparatively more restrained than those of the other three LQs. In terms of their components,

$$R_2 = \begin{pmatrix} R_2^{5/3} \\ R_2^{2/3} \end{pmatrix}, \quad \tilde{R}_2 = \begin{pmatrix} \tilde{R}_2^{2/3} \\ \tilde{R}_2^{-1/3} \end{pmatrix}, \quad \bar{S}_1 = \bar{S}_1^{-2/3}, \quad (2.1)$$

where the superscripts refer to their electric charges.

As for the right-handed neutrinos, we assume that there are three of them ( $N_1, N_2$ , and  $N_3$ ) and that they are of Dirac nature. Moreover, we suppose that they have masses which may be unequal but are sufficiently small to be neglected in the charmed-hadron processes of concern. In addition, we take  $N_{1,2,3}$  to be long-lived enough that they do not decay inside detectors.

We express the Lagrangian for the renormalizable interactions of  $R_2, \tilde{R}_2$ , and  $\bar{S}_1$  with SM fermions plus  $N_{1,2,3}$  as

$$\mathcal{L}_{\text{LQ}} = Y_{2,jy}^{\text{RL}} \bar{u}_j R_2^T i\tau_2 P_L l_y + \tilde{Y}_{2,jy}^{\text{LR}} \bar{q}_j \tilde{R}_2 P_R N_y + \bar{Y}_{1,jy}^{\text{RR}} \bar{u}_j^c P_R N_y \bar{S}_1 + \text{H.c.}, \quad (2.2)$$

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<sup>1</sup>In the recent literature covering the impact of NP on  $c \rightarrow u\cancel{E}$ , the missing energy could alternatively be carried away by a single particle such as QCD axion [10] or massless dark photon [11–13].

where  $Y_{2,jy}^{\text{RL}}$ ,  $\tilde{Y}_{2,jy}^{\text{LR}}$ , and  $\bar{Y}_{1,jy}^{\text{RR}}$  are generally complex elements of the LQ Yukawa matrices, summation over family indices  $j, y = 1, 2, 3$  is implicit,  $q_j$  ( $l_y$ ) and  $u_j$  symbolize a left-handed quark (lepton) doublet and right-handed up-type quark singlet,  $\tau_2$  is the second Pauli matrix,  $P_{L,R} = (1 \mp \gamma_5)/2$ , and the superscript C indicates charge conjugation. In eq. (2.2), we introduce only the minimal ingredients which serve our purposes pertaining to the  $c \rightarrow u\bar{E}$  transitions to be studied. Now we entertain three distinct possibilities each involving one of the LQs, taken to be heavy, with the couplings specified above.

### 3 $R_2$ model

Expanding the  $R_2$  portion of eq. (2.2), we have

$$\mathcal{L}_{R_2} = Y_{2,jy}^{\text{RL}} \bar{U}_j P_L (\ell_y R_2^{5/3} - \nu_{\ell_y} R_2^{2/3}) + \text{H.c.}, \quad (3.1)$$

where  $\mathcal{U}_{1,2,3} = u, c, t$  and  $\ell_{1,2,3} = e, \mu, \tau$  represent mass eigenstates. Given that the ordinary neutrinos in the decays of interest have vanishing masses and are not detected experimentally, we can work with the states  $\nu_{\ell_y}$  associated with  $\ell_y$ .

From  $\mathcal{L}_{R_2}$ , one can derive effective  $|\Delta C| = 1$  quark-lepton operators which at low energies are expressible as

$$\mathcal{L}_{ucff'} = -\sqrt{2} G_F K_{\ell_x \ell_y} \bar{u} \gamma_\beta P_R c (\bar{\nu}_{\ell_x} \gamma^\beta P_L \nu_{\ell_y} + \bar{\ell}_x \gamma^\beta P_L \ell_y) + \text{H.c.}, \quad (3.2)$$

where  $G_F$  is the Fermi constant,  $x, y = 1, 2, 3$  are summed over, and

$$K_{\ell_x \ell_y} = \frac{\hat{v}^2 Y_{2,1y}^{\text{RL}} Y_{2,2x}^{\text{RL}*}}{2m_{R_2}^2} \quad (3.3)$$

is a dimensionless coefficient, with  $\hat{v} = 2^{-1/4} G_F^{-1/2} \simeq 246$  GeV and  $m_{R_2}$  being the mass of  $R_2$ . They induce FCNC charmed-hadron decays with missing energy as well as those with charged leptons in the final state. Before treating the former processes, we look at some potentially important constraints on the LQ parameters in eq. (3.3).

It is long known that scalar-LQ interactions could influence the mixing of charmed mesons  $D^0$  and  $\bar{D}^0$  via  $|\Delta C| = 2$  four-quark operators arising from box diagrams [6, 7, 23–29]. In the presence of  $\mathcal{L}_{R_2}$  in eq. (3.1), the loops contain the SM charged and neutral leptons, besides  $R_2$ . This results in the effective Hamiltonian [7]

$$\mathcal{H}_{|\Delta C|=2}^{R_2} = \frac{(\sum_x Y_{2,1x}^{\text{RL}} Y_{2,2x}^{\text{RL}*})^2}{64\pi^2 m_{R_2}^2} \bar{u} \gamma^\beta P_R c \bar{u} \gamma_\beta P_R c + \text{H.c.} \quad (3.4)$$

It affects the mixing observable  $\Delta m_D = |\langle \bar{D}^0 | \mathcal{H}_{|\Delta C|=2} | D^0 \rangle| \tilde{r} / m_{D^0}$ , where  $\tilde{r} = 0.74$  accounts for the renormalization-group running of the coefficient in eq. (3.4) from the scale  $m_{R_2} = 2$  TeV down to 3 GeV [26]. Our choice for  $m_{R_2}$  is consistent with the negative outcome of a recent direct search at the LHC for scalar LQs decaying fully into a quark and an electron (muon), which has excluded the mass region below 1.8 (1.7) TeV [35].

Employing  $\langle \bar{D}^0 | \bar{u} \gamma^\kappa P_R c \bar{u} \gamma_\kappa P_R c | D^0 \rangle = 0.0805(57) \text{ GeV}^4$  at the scale of 3 GeV from a lattice QCD computation [36] and the empirical value  $\Delta m_D^{\text{exp}} = (95_{-44}^{+41}) \times 10^8 / \text{s}$  [15], and

assuming that the  $R_2$  contribution saturates the latter, as the SM prediction suffers from a sizable hadronic uncertainty [26], we then get the  $2\sigma$  upper-limit

$$\frac{|\sum_x Y_{2,1x}^{\text{RL}} Y_{2,2x}^{\text{RL}*}|}{m_{R_2}} < \frac{1.6 \times 10^{-2}}{\text{TeV}}. \quad (3.5)$$

Barring cancellations among the terms in the summation over  $x$ , for  $m_{R_2} = 2\text{TeV}$  this corresponds to  $|K_{\ell_x \ell_x}| < 2.4 \times 10^{-4}$ , which is more stringent than  $|K_{ee,\mu\mu,\tau\tau}| \lesssim (4, 2, 7) \times 10^{-3}$  at 95% CL estimated from data on the high-invariant-mass tails of the dilepton reactions  $pp \rightarrow \ell^+ \ell^-$  at the LHC and than weaker bounds from  $D \rightarrow \pi e^+ e^-, \pi \mu^+ \mu^-$  measurements [37].

Interestingly, we notice that the condition in eq. (3.5) no longer matters if the nonzero elements of the first and second rows of the  $Y_2^{\text{RL}}$  matrix do not share same columns [38], implying that  $x \neq y$  in  $K_{\ell_x \ell_y}$ . However, in that case there are restrictions inferred from quests for flavor-violating  $pp \rightarrow \ell^+ \ell^-$  at the LHC, namely  $(|K_{e\mu, e\tau, \mu\tau}|^2 + |K_{\mu e, \tau e, \tau\mu}|^2)^{1/2} < (2.0, 5.8, 6.4) \times 10^{-3}$  at the  $2\sigma$  level [39], the first of which is stronger than  $|K_{e\mu, \mu e}| \lesssim (0.01, 0.009)$  from hunts for rare semileptonic  $D_{(s)}$  decays manifesting lepton-flavor violation (LFV), as discussed in the appendix.

Accordingly, we can suppose that the only nonvanishing couplings are  $K_{e\mu, \tau\mu}$  and demand that they comply with  $|K_{e\mu}| < 2.0 \times 10^{-3}$  and  $|K_{\tau\mu}| < 6.4 \times 10^{-3}$ . This can be realized with a Yukawa matrix having the texture

$$Y_2^{\text{RL}} = \begin{pmatrix} 0 & y_{u\mu} & 0 \\ y_{ce} & 0 & y_{c\tau} \\ 0 & 0 & 0 \end{pmatrix}, \quad (3.6)$$

which can satisfy the limitations from other LFV searches [39] and escapes the  $D$ -mixing restraint. To see the consequences for  $c \rightarrow u \not{E}$  explicitly, we incorporate the above  $|K_{e\mu, \tau\mu}|$  into the expressions displayed in eqs. (A.11) and (A.16) in the appendix for the branching fractions of various FCNC decays of charmed mesons and baryons with missing energy. Thus, we obtain

$$\begin{aligned} \mathcal{B}(D^+ \rightarrow \pi^+ \not{E})_{R_2} &< 1.6 \times 10^{-6}, & \mathcal{B}(D^+ \rightarrow \rho^+ \not{E})_{R_2} &< 8.3 \times 10^{-7}, \\ \mathcal{B}(D^0 \rightarrow \pi^0 \not{E})_{R_2} &< 3.2 \times 10^{-7}, & \mathcal{B}(D^0 \rightarrow \eta \not{E})_{R_2} &< 9.6 \times 10^{-8}, \\ \mathcal{B}(D^0 \rightarrow \rho^0 \not{E})_{R_2} &< 1.9 \times 10^{-7}, & \mathcal{B}(D^0 \rightarrow \omega \not{E})_{R_2} &< 1.5 \times 10^{-7}, \\ \mathcal{B}(D^0 \rightarrow \eta' \not{E})_{R_2} &< 1.7 \times 10^{-8}, & & \\ \mathcal{B}(D_s^+ \rightarrow K^+ \not{E})_{R_2} &< 7.5 \times 10^{-7}, & \mathcal{B}(D_s^+ \rightarrow K^{*+} \not{E})_{R_2} &< 4.7 \times 10^{-7}, \end{aligned} \quad (3.7)$$

$$\begin{aligned} \mathcal{B}(\Lambda_c^+ \rightarrow p \not{E})_{R_2} &< 9.3 \times 10^{-7}, & \mathcal{B}(\Xi_c^+ \rightarrow \Sigma^+ \not{E})_{R_2} &< 1.1 \times 10^{-6}, \\ \mathcal{B}(\Xi_c^0 \rightarrow \Sigma^0 \not{E})_{R_2} &< 3.6 \times 10^{-7}. & & \end{aligned} \quad (3.8)$$

where each entry is a sum of branching fractions of the modes with  $\nu_e \bar{\nu}_\mu$  and  $\nu_\tau \bar{\nu}_\mu$  in the final states, the former making up merely about 10% of the total. Also, we find that including  $K_{e\tau, \tau e}$  would barely increase the preceding results because the associated  $Y_2^{\text{RL}}$

elements would have to fulfill other significant requirements. Moreover, the aforesaid  $D$ -mixing requisite  $|\kappa_{\ell_x \ell_x}| \lesssim 2.4 \times 10^{-4}$  in the lepton-flavor conserving case would translate into numbers that are smaller by roughly three orders of magnitude. Comparing eqs. (3.7)–(3.8) to the sensitivity reach of ongoing and future experiments addressed in section 1, we can conclude that these  $R_2$ -scenario predictions probably will not be testable anytime soon.

## 4 $\tilde{R}_2$ model

From the  $\tilde{R}_2$  part of eq. (2.2), in the mass basis of the down-type quarks we have

$$\mathcal{L}_{\tilde{R}_2} = \tilde{Y}_{2,jy}^{\text{LR}} \left( \mathcal{V}_{kj} \overline{\mathcal{U}}_k \tilde{R}_2^{2/3} + \overline{\mathcal{D}}_j \tilde{R}_2^{-1/3} \right) P_R N_y + \text{H.c.} \quad (4.1)$$

where  $\mathcal{V} \equiv \mathcal{V}_{\text{CKM}}$  designates the Cabibbo-Kobayashi-Maskawa mixing matrix and  $\mathcal{D}_{1,2,3} = d, s, b$  refer to the mass eigenstates. At low energies, from  $\mathcal{L}_{\tilde{R}_2}$  proceed the  $|\Delta C| = 1$  effective four-fermion interactions specified by

$$\mathcal{L}_{qq'NN'} = -\sqrt{2} G_F \left( \tilde{K}_{N_x N_y}^{\tilde{R}_2} \bar{u} \gamma_\beta P_L c + \tilde{K}_{jkxy} \overline{\mathcal{D}}_j \gamma_\beta P_L \mathcal{D}_k \right) \bar{N}_x \gamma^\beta P_R N_y + \text{H.c.}, \quad (4.2)$$

where  $j, k, x, y = 1, 2, 3$  are implicitly summed over,

$$\tilde{K}_{N_x N_y}^{\tilde{R}_2} = \frac{\hat{v}^2 (\mathcal{V} \tilde{Y}_2^{\text{LR}})_{1y} (\mathcal{V} \tilde{Y}_2^{\text{LR}})_{2x}^*}{2m_{\tilde{R}_2}^2}, \quad \tilde{K}_{jkxy} = \frac{\hat{v}^2 \tilde{Y}_{2,jy}^{\text{LR}} \tilde{Y}_{2,kx}^{\text{LR}*}}{2m_{\tilde{R}_2}^2}. \quad (4.3)$$

As in the last section, this brings about the FCNC decays of charmed hadrons with missing energy, but now it is the right-handed neutrinos that act as the invisibles. Furthermore,  $\mathcal{L}_{qq'NN'}$  can induce analogous transitions among down-type quarks.

From eq. (4.1), one can calculate the box diagrams, with  $\tilde{R}_2$  and  $N_y$  running around the loops, which affects  $D^0$ - $\bar{D}^0$  mixing, like in the  $R_2$  scenario. This leads to the effective Hamiltonian

$$\mathcal{H}_{|\Delta C|=2}^{\tilde{R}_2} = \frac{\left[ \sum_x (\mathcal{V} \tilde{Y}_2^{\text{LR}})_{1x} (\mathcal{V} \tilde{Y}_2^{\text{LR}})_{2x}^* \right]^2}{128\pi^2 m_{\tilde{R}_2}^2} \bar{u} \gamma^\beta P_L c \bar{u} \gamma_\beta P_L c + \text{H.c.} \quad (4.4)$$

However, differently from before, there are additionally contributions to its kaon and  $b$ -meson ( $B_d$  and  $B_s$ ) counterparts, described by

$$\mathcal{H}_{|\Delta S|=2}^{\tilde{R}_2} = \frac{\left( \sum_x \tilde{Y}_{2,2x}^{\text{LR}} \tilde{Y}_{2,1x}^{\text{LR}*} \right)^2}{128\pi^2 m_{\tilde{R}_2}^2} \bar{s} \gamma^\beta P_L d \bar{s} \gamma_\beta P_L d + \text{H.c.} \quad (4.5)$$

and similar formulas in the  $B_{d,s}$ -mixing cases. Since we cannot avoid all the mixing constraints at the same time, we can opt instead to do so in the down-type sector alone. It is evident from eq. (4.5) that this is attainable if the nonzero elements of  $\tilde{Y}_2^{\text{LR}}$  lie in separate columns. Accordingly, for simplicity we can pick

$$\tilde{Y}_2^{\text{LR}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \tilde{y}_{s3} \\ 0 & 0 & 0 \end{pmatrix}, \quad (4.6)$$

and so we have

$$\mathcal{Y}\tilde{Y}_2^{\text{LR}} = \begin{pmatrix} 0 & 0 & \lambda \\ 0 & 0 & 1 - \frac{1}{2}\lambda^2 \\ 0 & 0 & -\lambda^2 A \end{pmatrix} \tilde{Y}_{s3}, \quad (4.7)$$

up to  $\mathcal{O}(\lambda^2)$ , where  $\lambda$  and  $A$  are Wolfenstein parameters. To this order in  $\lambda \sim 0.23$  the nonvanishing coefficients in eq. (4.3) are

$$\begin{aligned} K_{N_3 N_3}^{\tilde{R}_2} &= \frac{\hat{v}^2 (\mathcal{Y}\tilde{Y}_2^{\text{LR}})_{13} (\mathcal{Y}\tilde{Y}_2^{\text{LR}})_{23}^*}{2m_{\tilde{R}_2}^2} = \frac{\lambda \hat{v}^2 |\tilde{Y}_{s3}|^2}{2m_{\tilde{R}_2}^2}, \\ \tilde{K}_{2233} &= \frac{\hat{v}^2 |\tilde{Y}_{2,23}^{\text{LR}}|^2}{2m_{\tilde{R}_2}^2} = \frac{\hat{v}^2 |\tilde{Y}_{s3}|^2}{2m_{\tilde{R}_2}^2}. \end{aligned} \quad (4.8)$$

Using  $\langle \bar{D}^0 | \bar{u}\gamma^\kappa P_L c \bar{u}\gamma_\kappa P_L c | D^0 \rangle = 0.0805(57) \text{ GeV}^4$  from lattice QCD work [36] and demanding again that the  $\tilde{R}_2$  contribution saturate  $\Delta m_D^{\text{exp}}$ , we get

$$\frac{|\sum_x (\mathcal{Y}\tilde{Y}_2^{\text{LR}})_{1x} (\mathcal{Y}\tilde{Y}_2^{\text{LR}})_{2x}^*|}{m_{\tilde{R}_2}} < \sqrt{\frac{128\pi^2 \Delta m_D^{\text{exp}} m_{D^0}}{\langle \bar{D}^0 | \bar{u}\gamma^\kappa P_L c \bar{u}\gamma_\kappa P_L c | D^0 \rangle \tilde{r}}} = \frac{2.3 \times 10^{-2}}{\text{TeV}} \quad (4.9)$$

at the  $2\sigma$  level. For  $m_{\tilde{R}_2} = 2 \text{ TeV}$ , this translates into

$$|K_{N_3 N_3}^{\tilde{R}_2}| < 3.4 \times 10^{-4}. \quad (4.10)$$

With this coupling value, the charmed-hadron decay channels with missing energy listed in eqs. (3.7) and (3.8) would turn out to have branching fractions about two orders of magnitude smaller than the corresponding numbers displayed therein. We further find that having more nonzero elements in, say, the first two rows of  $\tilde{Y}_2^{\text{LR}}$  would produce little change to this conclusion because they would be subject mainly to the meson-mixing requisites and/or stringent bounds inferred from  $K \rightarrow \pi \cancel{E}$  measurements.

## 5 $\bar{S}_1$ model

From the  $\bar{S}_1$  portion of eq. (2.2),

$$\mathcal{L}_{\bar{S}_1} = \bar{Y}_{1, jy}^{\text{RR}} \bar{U}_j^c P_R N_y \bar{S}_1^{-2/3} + \text{H.c.}, \quad (5.1)$$

we derive

$$\mathcal{L}_{ucNN'} = -\sqrt{2} G_F K_{N_x N_y}^{\bar{S}_1} \bar{u}\gamma^\beta P_R c \bar{N}_x \gamma_\beta P_R N_y + \text{H.c.}, \quad (5.2)$$

where

$$K_{N_x N_y}^{\bar{S}_1} = \frac{-\hat{v}^2 \bar{Y}_{1,1x}^{\text{RR}*} \bar{Y}_{1,2y}^{\text{RR}}}{2m_{\bar{S}_1}^2}. \quad (5.3)$$



This again gives rise to  $c \rightarrow uN_x\bar{N}_y$ , with  $N_x\bar{N}_y$  emitted invisibly, and affects  $\Delta m_D$ , the latter via

$$\mathcal{H}_{|\Delta C|=2}^{\bar{S}_1} = \frac{(\sum_x \bar{Y}_{1,1x}^{\text{RR}*} \bar{Y}_{1,2x}^{\text{RR}})^2}{128\pi^2 m_{\bar{S}_1}^2} \bar{u}\gamma^\beta P_R c \bar{u}\gamma_\beta P_R c + \text{H.c.} \quad (5.4)$$

Hence the mixing requirement is escapable if the contributing elements of the first and second rows of  $\bar{Y}_1^{\text{RR}}$  belong to different columns, as in this simple example:

$$\bar{Y}_1^{\text{RR}} = \begin{pmatrix} 0 & \bar{y}_{u2} & 0 \\ \bar{y}_{c1} & 0 & \bar{y}_{c3} \\ 0 & 0 & 0 \end{pmatrix}. \quad (5.5)$$

With  $x \neq y$  in  $K_{N_x N_y}^{\bar{S}_1}$ , the remaining consequential limitation on the  $\bar{Y}_1^{\text{RR}}$  elements is that from the perturbativity condition:  $|\bar{Y}_{1,ix}^{\text{RR}}| < \sqrt{4\pi}$ . As for the allowed range of the  $\bar{S}_1$  mass, the latest quest by the CMS Collaboration [40] for scalar LQs decaying fully into a quark and neutrino has ruled out masses up to 1.1 TeV at 95% CL. Since this is applicable to the possibility that the neutrino is a right-handed one, we can set  $m_{\bar{S}_1} > 1.2$  TeV. These parameters also enter loop diagrams involving  $\bar{S}_1$  and the  $u$  and  $c$  quarks and modifying the invisible partial width of the  $Z$  boson, but we have checked that their impact is insignificant. Incorporating these numbers into eq. (5.3) yields, for  $x \neq y$ ,

$$|K_{N_x N_y}^{\bar{S}_1}| < 0.26. \quad (5.6)$$

To illustrate the implications for the aforementioned charmed-hadron decays, we adopt the Yukawa matrix in eq. (5.5), in which case only  $K_{N_2 N_1}^{\bar{S}_1}$  and  $K_{N_2 N_3}^{\bar{S}_1}$  are present. Assuming that they each have the maximal value in eq. (5.6) and putting them together with eqs. (A.11) and (A.16), we then arrive at

$$\begin{aligned} \mathcal{B}(D^+ \rightarrow \pi^+ \cancel{E})_{\bar{S}_1} &< 4.9 \times 10^{-3}, & \mathcal{B}(D^+ \rightarrow \rho^+ \cancel{E})_{\bar{S}_1} &< 2.5 \times 10^{-3}, \\ \mathcal{B}(D^0 \rightarrow \pi^0 \cancel{E})_{\bar{S}_1} &< 9.7 \times 10^{-4}, & \mathcal{B}(D^0 \rightarrow \eta \cancel{E})_{\bar{S}_1} &< 2.9 \times 10^{-4}, \\ \mathcal{B}(D^0 \rightarrow \rho^0 \cancel{E})_{\bar{S}_1} &< 5.7 \times 10^{-4}, & \mathcal{B}(D^0 \rightarrow \omega \cancel{E})_{\bar{S}_1} &< 4.4 \times 10^{-4}, \\ \mathcal{B}(D^0 \rightarrow \eta' \cancel{E})_{\bar{S}_1} &< 5.2 \times 10^{-5}, & & \\ \mathcal{B}(D_s^+ \rightarrow K^+ \cancel{E})_{\bar{S}_1} &< 2.2 \times 10^{-3}, & \mathcal{B}(D_s^+ \rightarrow K^{*+} \cancel{E})_{\bar{S}_1} &< 1.4 \times 10^{-3}, \end{aligned} \quad (5.7)$$

$$\begin{aligned} \mathcal{B}(\Lambda_c^+ \rightarrow p \cancel{E})_{\bar{S}_1} &< 2.8 \times 10^{-3}, & \mathcal{B}(\Xi_c^+ \rightarrow \Sigma^+ \cancel{E})_{\bar{S}_1} &< 3.2 \times 10^{-3}, \\ \mathcal{B}(\Xi_c^0 \rightarrow \Sigma^0 \cancel{E})_{\bar{S}_1} &< 1.1 \times 10^{-3}, & & \end{aligned} \quad (5.8)$$

where each entry is a combination of branching fractions of the modes with  $N_2\bar{N}_1$  and  $N_2\bar{N}_3$  carrying away the missing energy in the final states. These numbers are considerably higher than their counterparts in the models containing  $R_2$  and  $\tilde{R}_2$ . This is attributable to the fact that  $\bar{S}_1$  does not have any direct couplings to the SM lepton and quark doublets. Additionally, one can see that some of the results in eqs. (5.7)–(5.8) are within the sensitivity reach of BESIII and Belle II described in section 1, suggesting that they might soon discover one or more of these  $\bar{S}_1$ -mediated processes or, if not, set useful bounds on them.

It is worth noting that our selection above for the Yukawa couplings of  $\bar{S}_1$  can be explained in terms of flavor symmetry imposed on the interactions of the sterile neutrinos. Specifically, supposing that  $N_2^\dagger$  and  $N_{1,3}^\dagger$  carry, respectively, what may be called “upness” and “charmness” quantum numbers associated with the right-handed mass-eigenstates of the  $u$  and  $c$  quarks, we can see that  $\mathcal{L}_{\bar{S}_1}$  in eq. (5.1) with  $\bar{Y}_1^{\text{RR}}$  picked to be of the form in eq. (5.5) conserves these numbers, as do  $c \rightarrow uN_2\bar{N}_1$  and  $c \rightarrow uN_2\bar{N}_3$  following from it.<sup>2</sup> At the same time, this choice prevents  $N_{1,2,3}$  from affecting  $D^0$ - $\bar{D}^0$  mixing via the Hamiltonian in eq. (5.4), which violates the symmetry.

## 6 Conclusions

We have explored the FCNC decays of charmed hadrons into a lighter hadron and missing energy carried away by a pair of either SM or right-handed sterile neutrinos in LQ scenarios, concentrating on the influence of the  $R_2$ ,  $\tilde{R}_2$ , and  $\bar{S}_1$  scalar LQs. We take into account various relevant constraints and learn that the meson-mixing ones and those inferred from LHC searches are especially important. Nevertheless, we point out that the meson-mixing restrictions may be evaded in certain situations. Additionally, we demonstrate that the contributions of these LQs to the branching fractions of  $D^+ \rightarrow \mathcal{M}^+\not{E}$ ,  $\mathcal{M} = \pi, \rho$ , of  $D^0 \rightarrow \tilde{\mathcal{M}}\not{E}$ ,  $\tilde{\mathcal{M}} = \pi^0, \eta, \rho^0, \omega, \eta'$ , and of  $D_s^+ \rightarrow K^{(*)+}\not{E}$  can be evaluated without knowing the details of the mesonic form factors associated with the quark currents if the invisibles have vanishing masses, by employing the data on the corresponding semileptonic modes and assuming isospin symmetry. As a consequence, the calculated  $D_{(s)}$  rates are free from the uncertainties attendant in form-factor estimation.

Our numerical work indicates that several of these charmed-meson decays and their baryon counterparts  $\Lambda_c^+ \rightarrow p\not{E}$  and  $\Xi_c^{+,0} \rightarrow \Sigma^{+,0}\not{E}$  are currently permitted to have branching fractions reaching the  $10^{-7}$ - $10^{-6}$  levels if  $R_2$  is responsible for the underlying operators. On the other hand, the effects of  $\tilde{R}_2$  are comparatively much less. By contrast, the contributions of  $\bar{S}_1$ , which has fermionic couplings exclusively to right-handed up-type quarks and the sterile neutrinos, could produce substantially bigger branching fractions, up to a few times  $10^{-3}$ . This is understandable because  $R_2$  and  $\tilde{R}_2$  are directly linked to the SM left-handed lepton and quark doublets, respectively, implying relatively stronger restraints on the Yukawa-matrix elements of these LQs. We therefore conclude that the charmed hadron decays we have studied are potentially promising as probes of LQ interactions involving sterile neutrinos. Lastly, a number of our predictions, notably in the  $\bar{S}_1$  scenario, may be large enough to be testable in the ongoing BESIII and Belle II experiments.

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<sup>2</sup>Here we have taken the light  $N_1$  and  $N_3$  to have unequal masses. If these are the same instead, the  $N_{1,3}$  fields can be rotated such that only one of their linear combinations participates in  $c \rightarrow u\not{E}$  along with  $N_2$ .

## A Branching ratio formulas

The effective Lagrangian for the  $c \rightarrow u$  transitions of interest has the form

$$\mathcal{L}_{ucff'} = -\bar{u}\gamma^\kappa c \bar{\mathbf{f}}\gamma_\kappa (\mathbf{C}_{\mathbf{f}\mathbf{f}'}^V + \gamma_5 \mathbf{C}_{\mathbf{f}\mathbf{f}'}^A) \mathbf{f}' - \bar{u}\gamma^\kappa \gamma_5 c \bar{\mathbf{f}}\gamma_\kappa (\tilde{\mathbf{C}}_{\mathbf{f}\mathbf{f}'}^V + \gamma_5 \tilde{\mathbf{C}}_{\mathbf{f}\mathbf{f}'}^A) \mathbf{f}' + \text{H.c.}, \quad (\text{A.1})$$

where  $\mathbf{f}$  and  $\mathbf{f}'$  are either SM leptons or SM-gauge-singlet fermions. This gives rise to  $D \rightarrow \mathbb{P}\mathbf{f}\bar{\mathbf{f}}'$  and  $D \rightarrow \mathbb{V}\mathbf{f}\bar{\mathbf{f}}'$ , where  $D$  stands for a charmed pseudoscalar meson and  $\mathbb{P}$  and  $\mathbb{V}$  designate charmless pseudoscalar and vector mesons, respectively. The amplitudes  $\mathcal{M}_{D \rightarrow \mathbb{P}\mathbf{f}\bar{\mathbf{f}}'}$  and  $\mathcal{M}_{D \rightarrow \mathbb{V}\mathbf{f}\bar{\mathbf{f}}'}$  for these decays depend on the mesonic matrix elements [7, 41]

$$\begin{aligned} \langle \mathbb{P} | \bar{u}\gamma^\alpha c | D \rangle &= F_+ \tilde{p}^\alpha + F_- \tilde{q}^\alpha, & \langle \mathbb{V} | \bar{u}\gamma^\alpha c | D \rangle &= \frac{V}{\tilde{m}_+} \epsilon^{\alpha\rho\sigma\tau} \epsilon_\rho^* \tilde{q}_\sigma \tilde{p}_\tau, \\ \langle \mathbb{V} | \bar{u}\gamma^\alpha \gamma_5 c | D \rangle &= i \left[ \frac{2A_0 m_V}{\tilde{q}^2} \tilde{q}^\alpha - \frac{A_2}{\tilde{m}_+} \left( \tilde{p}^\alpha - \frac{\tilde{m}_+ \tilde{m}_-}{\tilde{q}^2} \tilde{q}^\alpha \right) \right] \epsilon_\kappa^* \tilde{q}^\kappa + i A_1 \tilde{m}_+ \left( \epsilon^{*\alpha} - \frac{\epsilon_\kappa^* \tilde{q}^\kappa}{\tilde{q}^2} \tilde{q}^\alpha \right), \end{aligned} \quad (\text{A.2})$$

where  $m_X$  and  $p_X$  are the mass and momentum of  $X$ , respectively,

$$\tilde{m}_\pm = m_D \pm m_V, \quad \tilde{p} = p_D + p_M, \quad \tilde{q} = p_D - p_M, \quad M = \mathbb{P}, \mathbb{V}, \quad (\text{A.3})$$

$\epsilon$  denotes the polarization vector of  $\mathbb{V}$ , and  $F_\pm$ ,  $V$ , and  $A_{0,1,2}$  symbolize form factors which are functions of  $\tilde{q}^2$ . In this paper, we focus on the possibility that the  $\mathbf{f}$  and  $\mathbf{f}'$  masses,  $m_{\mathbf{f}}$  and  $m_{\mathbf{f}'}$ , are sufficiently small to be negligible.<sup>3</sup> It follows that

$$\mathcal{M}_{D \rightarrow \mathbb{P}\mathbf{f}\bar{\mathbf{f}}'} = 2F_+ \bar{u}_{\mathbf{f}} \not{p}_{\mathbb{P}} (\mathbf{C}_{\mathbf{f}\mathbf{f}'}^V + \gamma_5 \mathbf{C}_{\mathbf{f}\mathbf{f}'}^A) v_{\bar{\mathbf{f}}'}, \quad (\text{A.4})$$

$$\begin{aligned} \mathcal{M}_{D \rightarrow \mathbb{V}\mathbf{f}\bar{\mathbf{f}}'} &= i \left( A_1 \tilde{m}_+ \epsilon_\alpha^* - \frac{A_2}{\tilde{m}_+} \epsilon_\kappa^* \tilde{q}^\kappa \tilde{p}_\alpha \right) \bar{u}_{\mathbf{f}} \gamma^\alpha (\tilde{\mathbf{C}}_{\mathbf{f}\mathbf{f}'}^V + \gamma_5 \tilde{\mathbf{C}}_{\mathbf{f}\mathbf{f}'}^A) v_{\bar{\mathbf{f}}}' \\ &\quad + \frac{V}{\tilde{m}_+} \epsilon^{\alpha\rho\sigma\tau} \epsilon_\rho^* \tilde{q}_\sigma \tilde{p}_\tau \bar{u}_{\mathbf{f}} \gamma_\alpha (\mathbf{C}_{\mathbf{f}\mathbf{f}'}^V + \gamma_5 \mathbf{C}_{\mathbf{f}\mathbf{f}'}^A) v_{\bar{\mathbf{f}}}', \end{aligned} \quad (\text{A.5})$$

where  $u_{\mathbf{f}}$  and  $v_{\bar{\mathbf{f}}}'$  represent the fermions' Dirac spinors and the contributions of the terms with  $\tilde{q}^\alpha = p_{\mathbf{f}}^\alpha + p_{\bar{\mathbf{f}}'}^\alpha$  in eq. (A.2) have dropped out upon contraction with the  $\mathbf{f}\bar{\mathbf{f}}'$  current due to  $m_{\mathbf{f},\mathbf{f}'} \simeq 0$ . For  $\mathbf{f} \neq \mathbf{f}'$ , these amplitudes translate into the differential rates

$$\frac{d\Gamma_{D \rightarrow \mathbb{P}\mathbf{f}\bar{\mathbf{f}}'}}{d\hat{s}} = \frac{\tilde{\lambda}_{D\mathbb{P}}^{3/2} F_+^2}{192\pi^3 m_D^3} (|\mathbf{C}_{\mathbf{f}\mathbf{f}'}^V|^2 + |\mathbf{C}_{\mathbf{f}\mathbf{f}'}^A|^2), \quad (\text{A.6})$$

$$\begin{aligned} \frac{d\Gamma_{D \rightarrow \mathbb{V}\mathbf{f}\bar{\mathbf{f}}'}}{d\hat{s}} &= \frac{\tilde{\lambda}_{D\mathbb{V}}^{3/2}}{768\pi^3 m_D^3} \left\{ \left[ A_1^2 \tilde{m}_+^2 \left( 1 + \frac{12m_V^2 \hat{s}}{\tilde{\lambda}_{D\mathbb{V}}} \right) + \tilde{\zeta} A_1 A_2 + \frac{\tilde{\lambda}_{D\mathbb{V}} A_2^2}{\tilde{m}_+^2} \right] \frac{|\tilde{\mathbf{C}}_{\mathbf{f}\mathbf{f}'}^V|^2 + |\tilde{\mathbf{C}}_{\mathbf{f}\mathbf{f}'}^A|^2}{m_V^2} \right. \\ &\quad \left. + \frac{8V^2 \hat{s}}{\tilde{m}_+^2} (|\mathbf{C}_{\mathbf{f}\mathbf{f}'}^V|^2 + |\mathbf{C}_{\mathbf{f}\mathbf{f}'}^A|^2) \right\}, \end{aligned} \quad (\text{A.7})$$

where

$$\hat{s} = (p_{\mathbf{f}} + p_{\bar{\mathbf{f}}'})^2, \quad \tilde{\lambda}_{XY} = (m_X^2 - m_Y^2 - \hat{s})^2 - 4m_Y^2 \hat{s}, \quad \tilde{\zeta} = 2\hat{s} - 2\tilde{m}_+ \tilde{m}_-. \quad (\text{A.8})$$

<sup>3</sup>In that case  $D^0 \rightarrow \mathbf{f}\bar{\mathbf{f}}'$  arising from  $\mathcal{L}_{ucff'}$  is chirally suppressed and hence has a tiny rate.

For the LQ-mediated operators in eqs. (3.2), (4.2), and (5.2) containing constants of the form  $K_{\mathbf{f}\mathbf{f}'}$ , we can then apply eqs. (A.6) and (A.7) by setting  $|\mathbf{C}_{\mathbf{f}\mathbf{f}'}^{V,A}| = |\tilde{\mathbf{C}}_{\mathbf{f}\mathbf{f}'}^{V,A}| = G_F |K_{\mathbf{f}\mathbf{f}'}|/\sqrt{8}$ .

Now, the semileptonic transitions  $D^0 \rightarrow \mathcal{M}^- \nu e^+$  with  $\mathcal{M} = \pi, \rho$  receive SM contributions described by  $\mathcal{L}_{dc\nu e}^{\text{SM}} = -\sqrt{8} G_F V_{cd}^* \bar{d} \gamma^\alpha P_L c \bar{\nu}_e \gamma_\alpha P_L e + \text{H.c.}$  Comparing this to  $\mathcal{L}_{uc\mathbf{f}\mathbf{f}'}$  in eq. (A.1) and ignoring the leptons' masses, we can see that the expressions for the differential rates of  $D^0 \rightarrow \mathcal{M}^- \nu e^+$  with  $\mathcal{M} = \pi, \rho$  in the SM are equal to those in eqs. (A.6) and (A.7), respectively, but with coefficients given by  $|\mathbf{C}_{\nu e}^{V,A}| = |\tilde{\mathbf{C}}_{\nu e}^{V,A}| = G_F |V_{cd}|/\sqrt{2}$ . For the rate of LQ-induced  $D^+ \rightarrow \mathcal{M}^+ \mathbf{f}\bar{\mathbf{f}'}$ , neglecting small isospin-breaking effects we then arrive at  $4\Gamma_{D^+ \rightarrow \mathcal{M}^+ \mathbf{f}\bar{\mathbf{f}'}}^{\text{LQ}} |V_{cd}|^2 = \Gamma_{D^0 \rightarrow \mathcal{M}^- \nu e^+}^{\text{SM}} |K_{\mathbf{f}\mathbf{f}'}|^2$  without having to know how  $F_+$ ,  $V$ , and  $A_{1,2}$  depend on  $\hat{s}$ . As the LQ interactions in eq. (2.2) do not directly affect  $D^0 \rightarrow \mathcal{M}^- \nu e^+$ , we can replace  $\Gamma_{D^0 \rightarrow \mathcal{M}^- \nu e^+}^{\text{SM}}$  with their experimental values. This implies the branching-fraction relation

$$\mathcal{B}(D^+ \rightarrow \mathcal{M}^+ \mathbf{f}\bar{\mathbf{f}'})_{\text{LQ}} = \frac{\tau_{D^+}}{\tau_{D^0}} \frac{\mathcal{B}(D^0 \rightarrow \mathcal{M}^- \nu e^+)_{\text{exp}}}{4|V_{cd}|^2} |K_{\mathbf{f}\mathbf{f}'}|^2, \quad (\text{A.9})$$

where  $\tau_{D^{+(0)}}$  is the measured  $D^{+(0)}$  lifetime. It is straightforward to write down analogous formulas for other modes, particularly  $D^0 \rightarrow \tilde{\mathcal{M}} \mathbf{f}\bar{\mathbf{f}'}$ ,  $\tilde{\mathcal{M}} = \pi^0, \eta, \rho^0, \omega, \eta'$ , and  $D_s^+ \rightarrow K^{(*)+} \mathbf{f}\bar{\mathbf{f}'}$ . Clearly, the outcomes of this procedure do not suffer from the uncertainties inherent in the estimation of hadronic matrix elements.

To proceed, we need the empirical information on the relevant semileptonic modes [15]:

$$\begin{aligned} \mathcal{B}(D^0 \rightarrow \pi^- \nu e^+)_{\text{exp}} &= 2.91(4), & \mathcal{B}(D^0 \rightarrow \rho^- \nu e^+)_{\text{exp}} &= 1.50(12), \\ \mathcal{B}(D^+ \rightarrow \pi^0 \nu e^+)_{\text{exp}} &= 3.72(17), & \mathcal{B}(D^+ \rightarrow \eta \nu e^+)_{\text{exp}} &= 1.11(7), \\ \mathcal{B}(D^+ \rightarrow \rho^0 \nu e^+)_{\text{exp}} &= 2.18_{-0.25}^{+0.17}, & \mathcal{B}(D^+ \rightarrow \omega \nu e^+)_{\text{exp}} &= 1.69(11), \\ \mathcal{B}(D^+ \rightarrow \eta' \nu e^+)_{\text{exp}} &= 0.20(4), & & \\ \mathcal{B}(D_s^+ \rightarrow K^0 \nu e^+)_{\text{exp}} &= 3.4(4), & \mathcal{B}(D_s^+ \rightarrow K^{*0} \nu e^+)_{\text{exp}} &= 2.15(28) \end{aligned} \quad (\text{A.10})$$

all in units of  $10^{-3}$ . Using their central values and the CKM matrix element  $|V_{cd}| = 0.22636(48)$  [15], we then find

$$\begin{aligned} \mathcal{B}(D^+ \rightarrow \pi^+ \mathbf{f}\bar{\mathbf{f}'})_{\text{LQ}} &= 3.60 \times 10^{-2} |K_{\mathbf{f}\mathbf{f}'}|^2, \\ \mathcal{B}(D^+ \rightarrow \rho^+ \mathbf{f}\bar{\mathbf{f}'})_{\text{LQ}} &= 1.86 \times 10^{-2} |K_{\mathbf{f}\mathbf{f}'}|^2, \\ \mathcal{B}(D^0 \rightarrow \pi^0 \mathbf{f}\bar{\mathbf{f}'})_{\text{LQ}} &= 7.16 \times 10^{-3} |K_{\mathbf{f}\mathbf{f}'}|^2, \\ \mathcal{B}(D^0 \rightarrow \eta \mathbf{f}\bar{\mathbf{f}'})_{\text{LQ}} &= 2.14 \times 10^{-3} |K_{\mathbf{f}\mathbf{f}'}|^2, \\ \mathcal{B}(D^0 \rightarrow \rho^0 \mathbf{f}\bar{\mathbf{f}'})_{\text{LQ}} &= 4.19 \times 10^{-3} |K_{\mathbf{f}\mathbf{f}'}|^2, \\ \mathcal{B}(D^0 \rightarrow \omega \mathbf{f}\bar{\mathbf{f}'})_{\text{LQ}} &= 3.25 \times 10^{-3} |K_{\mathbf{f}\mathbf{f}'}|^2, \\ \mathcal{B}(D^0 \rightarrow \eta' \mathbf{f}\bar{\mathbf{f}'})_{\text{LQ}} &= 3.8 \times 10^{-4} |K_{\mathbf{f}\mathbf{f}'}|^2, \\ \mathcal{B}(D_s^+ \rightarrow K^+ \mathbf{f}\bar{\mathbf{f}'})_{\text{LQ}} &= 1.7 \times 10^{-2} |K_{\mathbf{f}\mathbf{f}'}|^2, \\ \mathcal{B}(D_s^+ \rightarrow K^{*+} \mathbf{f}\bar{\mathbf{f}'})_{\text{LQ}} &= 1.05 \times 10^{-2} |K_{\mathbf{f}\mathbf{f}'}|^2. \end{aligned} \quad (\text{A.11})$$

The numbers in eq. (A.11) have relative errors approximately equal to those of the corresponding data in eq. (A.10).

We can apply the preceding results to extract bounds on  $\mathcal{K}_{e\mu,\mu e}$  defined in eq. (3.2) from hunts for  $D_{(s)} \rightarrow \pi(K)e^\pm\mu^\mp$ , the channels with the tau lepton being kinematically closed. In light of eq. (A.11) and the available limits on the pertinent modes [15], the strongest restraints come from  $\mathcal{B}(D^+ \rightarrow \pi^+e^-\mu^+)_{\text{exp}} < 3.6 \times 10^{-6}$  and  $\mathcal{B}(D^+ \rightarrow \pi^+e^+\mu^-)_{\text{exp}} < 2.9 \times 10^{-6}$ , both at 90% CL [15], which translate into

$$|\mathcal{K}_{e\mu}| < 1.0 \times 10^{-2}, \quad |\mathcal{K}_{\mu e}| < 9.0 \times 10^{-3}, \quad (\text{A.12})$$

respectively. These turn out to be less stringent than those implied by  $D^0 \rightarrow e^\pm\mu^\mp$  data [15] and inferred from quests for  $pp \rightarrow e^\pm\mu^\mp$  at the LHC [39].

It is worth mentioning that when using eq. (A.11) for the models in sections 3–5, where  $\mathbf{f}$  and  $\mathbf{f}'$  are invisible, we ignore the SM contributions, which are highly suppressed [26]. We also note that  $D_{(s)}^+ \rightarrow \mathcal{M}_{(s)}^+\mathbf{f}\mathbf{f}'$  with  $\mathcal{M}_{(s)} = \pi, \rho(K, K^*)$  and invisible  $\mathbf{f}$  and  $\mathbf{f}'$  have SM backgrounds from the sequential decays  $D_{(s)}^+ \rightarrow \tau^+\nu$  and  $\tau^+ \rightarrow \mathcal{M}_{(s)}^+\nu$  [1]. Their impact can be removed by implementing kinematical cuts such as  $\hat{s}_{\text{min}} = (m_D^2 - m_\tau^2)(m_\tau^2 - m_{\mathcal{M}}^2)/m_\tau^2$  [42]. Our  $D_{(s)}^+$  numbers in eq. (A.11) do not yet incorporate them, and we suppose that they will be taken into account in the experimental searches.

The LQ-induced operators in eqs. (3.2), (4.2), and (5.2) bring about analogous transitions in the charmed-baryon sector. Here we look at those of the singly charmed baryons  $\Lambda_c^+, \Xi_c^+$ , and  $\Xi_c^0$ , which have spin parity  $J^P = 1/2^+$ , make up a flavor SU(3) antitriplet, and decay weakly [15]. Specifically, we examine the modes  $\Lambda_c^+ \rightarrow p\cancel{E}$  and  $\Xi_c^{+,0} \rightarrow \Sigma^{+,0}\cancel{E}$ . The baryonic matrix elements pertaining to the former are [15, 43]

$$\begin{aligned} \langle p|\bar{u}\gamma^\kappa c|\Lambda_c^+\rangle &= \bar{u}_p \left\{ f_\perp \left[ \gamma^\kappa - \frac{M_+\hat{p}^\kappa - M_-\hat{q}^\kappa}{\hat{\sigma}_+} \right] + f_+ \left[ \hat{p}^\kappa - \frac{M_+M_-\hat{q}^\kappa}{\hat{q}^2} \right] \frac{M_+}{\hat{\sigma}_+} + f_0 \frac{M_-\hat{q}^\kappa}{\hat{q}^2} \right\} u_{\Lambda_c}, \\ \langle p|\bar{u}\gamma^\kappa\gamma_5 c|\Lambda_c^+\rangle &= \bar{u}_p \left\{ g_\perp \left[ \gamma^\kappa + \frac{M_-\hat{p}^\kappa - M_+\hat{q}^\kappa}{\hat{\sigma}_-} \right] - g_+ \left[ \hat{p}^\kappa - \frac{M_+M_-\hat{q}^\kappa}{\hat{q}^2} \right] \frac{M_-}{\hat{\sigma}_-} - g_0 \frac{M_+\hat{q}^\kappa}{\hat{q}^2} \right\} \gamma_5 u_{\Lambda_c}, \end{aligned} \quad (\text{A.13})$$

where

$$\hat{p} = p_{\Lambda_c} + p_p, \quad \hat{q} = p_{\Lambda_c} - p_p, \quad M_\pm = m_{\Lambda_c} \pm m_p, \quad \hat{\sigma}_\pm = M_\pm^2 - \hat{s}, \quad (\text{A.14})$$

and  $f_{\perp,+0}$  and  $g_{\perp,+0}$  are form factors depending on  $\hat{q}^2$ . With eq. (A.13), we derive the amplitude for  $\Lambda_c^+ \rightarrow p\mathbf{f}\bar{\mathbf{f}}'$  due to  $\mathcal{L}_{uc\mathbf{f}\mathbf{f}'}$  in eq. (A.1). Subsequently, with  $|\mathbf{c}_{\mathbf{f}\mathbf{f}'}^{V,A}| = |\bar{\mathbf{c}}_{\mathbf{f}\mathbf{f}'}^{V,A}| = G_F|\mathcal{K}_{\mathbf{f}\mathbf{f}'}|/\sqrt{8}$  as before, we arrive at the differential decay rate

$$\frac{d\Gamma_{\Lambda_c \rightarrow p\mathbf{f}\bar{\mathbf{f}}'}}{d\hat{s}} = \frac{\tilde{\lambda}_{\Lambda_c p}^{1/2} G_F^2 |\mathcal{K}_{\mathbf{f}\mathbf{f}'}|^2}{768\pi^3 m_{\Lambda_c}^3} \left[ \hat{\sigma}_- (f_+^2 M_+^2 + 2f_\perp^2 \hat{s}) + \hat{\sigma}_+ (g_+^2 M_-^2 + 2g_\perp^2 \hat{s}) \right] \quad (\text{A.15})$$

for  $m_{\mathbf{f},\mathbf{f}'} \simeq 0$ , in which case the  $f_0$  and  $g_0$  terms drop out from the rate as well. Its  $\Xi_c^{+,0} \rightarrow \Sigma^{+,0}\mathbf{f}\bar{\mathbf{f}}'$  counterparts are similar in form. Given that the empirical information on  $\Lambda_c^+ \rightarrow n\nu e^+$  and  $\Xi_c^{+,0} \rightarrow \Sigma^{0,-}\nu e^+$  is still unavailable [15], we cannot implement a

procedure like that followed for the meson modes above and must instead rely on theoretical estimates for the baryonic matrix elements. Thus, numerically, for the  $\Lambda_c^+ \rightarrow p$  form factors we adopt the results of the lattice QCD calculation in ref. [43], while for  $\Xi_c^{+,0} \rightarrow \Sigma^{+,0}$  we employ those computed with light-cone QCD sum rules in ref. [44] and assume isospin symmetry. Putting things together, we then obtain

$$\begin{aligned} \mathcal{B}(\Lambda_c^+ \rightarrow p \bar{f} \bar{f}')_{\text{LQ}} &= 2.07 \times 10^{-2} |K_{\bar{f}f'}|^2, \\ \mathcal{B}(\Xi_c^+ \rightarrow \Sigma^+ \bar{f} \bar{f}')_{\text{LQ}} &= 2.39 \times 10^{-2} |K_{\bar{f}f'}|^2, \\ \mathcal{B}(\Xi_c^0 \rightarrow \Sigma^0 \bar{f} \bar{f}')_{\text{LQ}} &= 8.01 \times 10^{-3} |K_{\bar{f}f'}|^2, \end{aligned} \tag{A.16}$$

where the  $\Lambda_c^+$  and  $\Xi_c^{+,0}$  results have uncertainties of order 10% and 30%, respectively [43, 44], and the difference between the  $\Xi_c^{+,0}$  numbers is ascribable mainly to  $\tau_{\Xi_c^+} = 3.9 \tau_{\Xi_c^0}$ .

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