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Naturalness without new particles

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ABSTRACT: We demonstrate that the physics which resolves naturalness problems need not take the form of new particles and can sometimes manifest itself as higher dimensional operators. As a proof of principle, we present a simple model where the scale of new particles is parametrically separated from that estimated via naturalness arguments applied to self-quartic couplings. In this example, new particles appear far above the scale $m/\sqrt{\lambda}$, where m is the mass of the particle and λ is its self-quartic coupling. The shift symmetry responsible for resolving the naturalness problem involves higher dimensional operators rather than new particles.

KEYWORDS: Beyond Standard Model, Effective Field Theories

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The Electroweak Hierarchy problem is one of the driving forces in particle physics (see e.g. ref. [1] for a review of the current state of affairs). When the naturalness paradigm is applied to this problem, it is typically stated that new particles must appear by the TeV scale modulo accidental fine tunings. In this article, we provide an interesting proof of principle that the physics which resolves naturalness problems need not involve new particles. In particular, the symmetry that solves the problem involves the appearance of higher dimensional operators rather than the typically assumed new particles. The presence of higher dimensional operators implies that new particles exist at a higher energy scale, however the scaling of their mass is completely different from those typically expected.

Naturalness is experimentally relevant as it involves an $\mathcal{O}(1)$ prediction of where new physics must appear. To make this concrete, consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4. \quad (1)$$

If one assumes that this theory is UV completed into a theory where m is calculable, then dimensional analysis suggests that there is a contribution to m that scales as

$$\delta m^2 \sim \lambda \Lambda_{\text{NP}}^2, \quad (2)$$

where Λ_{NP} is the scale of new physics and to emphasize the schematic nature of the estimates, we have neglected the factor of $3/16\pi^2$ that is sometimes included in this estimate. Requiring that this contribution be smaller than the observed mass, i.e. requiring that there is no fine tuning, gives

$$\Lambda_{\text{NP}}^2 \lesssim \frac{m^2}{\lambda}. \quad (3)$$

Indeed, if one writes down a supersymmetric UV completion of this model, that is exactly what one finds. In the absence of fine-tuned cancellations, the mass of the fermionic superpartners are necessarily below the scale Λ_{NP} . This argument can be generalized to Yukawa couplings where $\Lambda_{\text{NP}} \lesssim m/y$, to gauge couplings where $\Lambda_{\text{NP}} \lesssim m/g$ or to the cosmological constant where $\Lambda_{\text{NP}} \sim \Lambda_{\text{cc}}$.

When applied to the Standard Model, the previous argument leads one to conclude that there should be new physics at the scale ~ 500 GeV or below. To the extent that there

are no new particles below the TeV scale, we have been led to conclude that the Standard Model Higgs is *necessarily* tuned. Critical to this analysis is the scaling shown in eq. (3). In this paper, we provide a counterexample to this scaling relationship.

1 An explicit counterexample

The UV theory we will consider has a complex scalar Φ which is charged under a $\mathbb{Z}_N$ symmetry so that

$$\Phi \rightarrow e^{\frac{2\pi i}{N}} \Phi. \quad (1.1)$$

The most general Lagrangian allowed by symmetries is

$$V = -f^2 |\Phi|^2 + |\Phi|^4 + \frac{\Phi^N}{\Lambda^{N-4}} + \dots, \quad (1.2)$$

where we take all coefficients to be $\mathcal{O}(1)$ numbers. The scalar Φ obtains a vev $\langle \Phi \rangle \sim f$. The radial mode obtains a mass $m_{\text{radial}} \sim f$, while the pseudo-Goldstone boson ϕ remains light.

In the context of axions, this exact potential has been considered many times when discussing Planck suppressed corrections to the axion potential, see e.g. ref. [2]. At the renormalizable level, there is an accidental U(1) symmetry. The leading-order operator that breaks this symmetry is shown above and is a higher-dimensional operator whose origin is unimportant. Later, we will give a simple example where fermions with a mass $\gtrsim f$ can be integrated out to generate it. We include it because it is the leading source of mass for the pNGB ϕ and will work in the limit where it is extremely small so that higher order terms can be neglected.

From this UV theory, we flow to the IR theory by RG evolving from the UV scale to the mass of the radial mode. There we integrate out the radial mode, including threshold corrections. A spurion argument shows us that all of these effects are multiplicative and their exact values are not relevant for the discussion at hand. Because we are taking $\mathcal{O}(1)$ coupling constants, to accurately determine the anomalous dimensions and the threshold corrections requires the complicated machinery of e.g. ref. [3]. The IR theory we arrive at has the pNGB ϕ with the potential

$$V = -\epsilon^4 \cos \frac{N\phi}{f}, \quad \epsilon^4 \propto \frac{f^N}{\Lambda^{N-4}}. \quad (1.3)$$

At even lower energies, we can expand the potential and obtain the theory with

$$V = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} \phi^4 + \dots \quad (1.4)$$

$$m^2 \sim \frac{\epsilon^4 N^2}{f^2}, \quad \lambda \sim \frac{\epsilon^4 N^4}{f^4}. \quad (1.5)$$

At this stage, we can apply the standard naturalness arguments that led to eq. (3) and obtain

$$\Lambda_{\text{NP}} \lesssim \frac{m}{\sqrt{\lambda}} \sim \frac{f}{N} \sim \frac{m_{\text{radial}}}{N}. \quad (1.6)$$

Note that the mass of the new particle, namely the radial mode, appears at the scale f rather than f/N . Thus by taking N large, we have a proof of principle that the mass scale of new particles can be parametrically separated from where naturalness arguments typically expect them to be. If higher harmonics were generated instead, e.g. $V \sim \cos^m(N\phi/f)$, the expectation would be that new particles are at the scale $f/(N\sqrt{m})$ further separating the scale where new physics is expected and the scale at which it actually appears.

2 An interpretation

In principle, we have now achieved our goal by providing an explicit counterexample to the standard lore. However, it is instructive to examine why the standard argument failed. The two most common arguments for the scaling of eq. (3) involve either dimensional analysis or quadratic divergences. We will show how this counterexample gets around both of these arguments.

One way to obtain eq. (3) is to use dimensional analysis. However, a critical aspect of dimensional analysis is knowing the symmetries of the problem and it fails if one does not know all of the symmetries of the theory. From this point of view, the reason why the IR physicist incorrectly estimated the scale of new physics is because measuring the renormalizable Lagrangian was not enough information to determine the full symmetries of the theory. In particular, the symmetry of interest is the shift symmetry

$$\phi \rightarrow \phi + \frac{2\pi}{N}f. \quad (2.1)$$

The reason why this shift symmetry is critical, is that it provides a new scale, f/N that can replace Λ_{NP} in eq. (2). Because there is a scale in the theory that is *not* associated with the mass of a particle, it has the potential to drastically change typical assertion that Λ_{NP} is the mass of a new particle. In fact this is exactly what happens in this example. The shift symmetry forces the potential to be of the form $V \sim \cos \frac{N\phi}{f}$ where the mass and quartic coupling are related to each other not by the mass of a new particle but instead by the shift symmetry parameter f/N .

In order to discover the existence of this shift symmetry, one would need to discover the higher dimensional operator

$$\frac{\epsilon^4 N^6 \phi^6}{f^6} \sim \frac{\lambda^2 \phi^6}{m^2}. \quad (2.2)$$

It is important to note that this operator becomes strongly coupled at a scale $\gg f$ so that it would be difficult to find experimentally. Upon measuring this operator, the experimentalist would note that this dimension six operator is algebraically related to the quartic coupling and mass terms and would suspect that there is a shift symmetry in the theory. The useful operators of the theory are not ϕ^2 , ϕ^4 and ϕ^6 but instead only a single operator in the theory has been discovered, $\cos \frac{N\phi}{f}$. As the coefficient of this operator, ϵ^4 , goes to zero there is an enhanced full shift symmetry for ϕ . It is thus technically natural to set ϵ^4 , and therefore the mass, small.

Another way in which one can arrive at scaling in eq. (3) is by studying quadratic divergences. We will now study how the one loop divergences operate in our example. The

one loop divergence can be found from the Coleman-Weinberg potential and is

$$V \approx V_{\text{tree}} + V_{1\text{-loop}} = -\epsilon^4 \cos \frac{N\phi}{f} + \frac{\Lambda_{\text{NP}}^2}{32\pi^2} \frac{d^2 V_{\text{tree}}}{d\phi^2} = -\epsilon^4 \left(1 - \frac{N^2 \Lambda_{\text{NP}}^2}{32\pi^2 f^2}\right) \cos \frac{N\phi}{f}. \quad (2.3)$$

We see that the divergence is multiplicative rather than additive! This is similar to the case of the electron mass where corrections to the electron mass are proportional to the electron mass itself.

The impact of the divergences being multiplicative rather than additive cannot be overstated. In the original scenario, the small mass is obtained by the fine tuned cancellation of a large bare mass against the large 1-loop quadratic divergence. However once the full symmetry is known, the small mass is actually a product of a small number ϵ^4 and a large number $\left(1 - \frac{N^2 \Lambda_{\text{NP}}^2}{32\pi^2 f^2}\right)$. Requiring that a small number times a larger number is small is completely natural. Thus a shift symmetry solves this naturalness problem in much the same way that chiral symmetry protects fermion masses.

Another way to interpret eq. (2.3) is that both the mass term and the quartic term have quadratic divergences coming from the quartic and sextic couplings respectively. Using the relation $m^2 = -6\lambda f^2/N^2$, we have

$$\lambda_{\text{obs}} = \lambda + \delta\lambda = \lambda - \lambda \frac{N^2 \Lambda_{\text{NP}}^2}{32\pi^2 f^2} \quad (2.4)$$

$$m_{\text{obs}}^2 = m^2 + \delta m^2 = m^2 + \frac{3\lambda \Lambda_{\text{NP}}^2}{16\pi^2} = -\frac{6\lambda f^2}{N^2} \left(1 - \frac{N^2 \Lambda_{\text{NP}}^2}{32\pi^2 f^2}\right) = -\frac{6\lambda_{\text{obs}} f^2}{N^2}. \quad (2.5)$$

The divergence associated with the mass term is also a divergence that the quartic coupling has. Thus the relationship between the two is independent of the quadratic divergence.

3 Example UV completion of the higher dimensional operator

At this point, we give a quick demonstration of how the higher-dimensional operator in eq. (1.2) can be generated. There are many ways to generate this operator, e.g. from theory space [4] or from discrete symmetries [5]. We will follow the discrete symmetry approach of adding to the theory a set of N vector-like fermions Ψ_j and Ψ_j^c , which transform under a $\mathbb{Z}_N$ symmetry as

$$\Psi_j^{(c)} \rightarrow \Psi_{j+1}^{(c)}, \quad (3.1)$$

with $j = 0$ and $j = N$ identified. Writing down the most general renormalizable Lagrangian, we have

$$\mathcal{L} \supset f^2 |\Phi|^2 - |\Phi|^4 + \sum_{j=1}^N \left(m_f + y e^{2\pi i j/N} \Phi \right) \Psi_j \Psi_j^c. \quad (3.2)$$

Taking $m_f > yf$, we can integrate out the fermions Ψ , which generates the higher-dimensional operator of interest ($y^N \Phi^N / m_f^{N-4}$) as well as some $U(1)$ symmetry preserving operators.

4 Conclusion

We have shown that the physics which resolves naturalness problems could take the form of a symmetry that would be discovered by the observation of higher dimensional operators rather than new particles. The natural question is to wonder if the solution to the Electroweak Hierarchy problem can also take the form of higher dimensional operators. Currently we do not know whether or not this is possible. However it is clear that the moral of the story is that if such a solution were to exist, that one would search for higher dimensional operators rather than new particles. Without an explicit example, it is not clear which higher dimensional operators to look for or what value to expect them to take (e.g. one might expect an operator of the form $\sim \lambda^2 (HH^\dagger)^3 / m_H^2$). The scale that suppresses these higher dimensional operators might be large, but one hopes that they are in reach of current experiments.

There is reason to expect that a solution of this type can be implemented in a realistic model of the Higgs boson. The key ingredient in this approach is a shift symmetry, a feature already present in all Higgs as a pseudo-Goldstone boson models. In fact, even the presence of a higher harmonic cosine potential is already present in the original Little Higgs model. As all of the ingredients are already present, although utilized differently, in existing models, it is extremely plausible that some version of this mechanism will translate over to the Electroweak Hierarchy problem.

To conclude, we have demonstrated through an explicit counterexample that the solution to a naturalness problem need not involve new particles. While this example has nothing to say about gauge and Yukawa quadratic divergences or about the cosmological constant, it would be interesting if similar models could be constructed for these other couplings. This example suggests an interesting new approach to the naturalness paradigm. Perhaps new physics will appear in the form of higher dimensional operators rather than new particles.

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