

A light scalar explanation of $(g - 2)_\mu$ and the KOTO anomaly

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ABSTRACT: The KOTO experiment has recently performed a search for neutral Kaons decaying into neutral pions and a pair of neutrinos. Three events were observed in the KOTO signal region, with an expected background of about 0.05. Since no clear signal of systematic errors have been found, the excess of events in the decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is quite intriguing. One possibility to explain this anomaly would be the presence of a scalar ϕ with mass of the order of the pion mass and inducing decays $K_L \rightarrow \pi^0 \phi$ which mimic the observed signal. A scalar with mass of the order of the pion mass and a coupling to muons of the order of the Standard Model Higgs coupling could also explain the muon anomalous magnetic moment anomaly $(g - 2)_\mu$. We built on these facts to show that a light singlet scalar with couplings to the leptons and quarks as the ones induced by mixing with Higgs states in two Higgs doublet models may lead to an explanation of both anomalies. More specifically, we show that this is the case in the so-called type-X models in which leptons and quarks couple to two different Higgs doublets, and for scalar masses that are in the range between 40 and 70 MeV. Due to the relatively large coupling to leptons required to fit $(g - 2)_\mu$, the scalar lifetime accidentally falls into the sub-nanosecond range which is essential to evade the severe proton beam dump experiments and astrophysical constraints, though it becomes sensitive to constraints from electron beam dump experiments. The additional phenomenological properties of this model are discussed.

KEYWORDS: Beyond Standard Model, Higgs Physics

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1 Introduction

The Standard Model (SM) provides an excellent description of all experimental phenomena [1]. After the discovery of the Higgs, the basic mechanism for electroweak symmetry breaking has been confirmed. Moreover, the Higgs couplings to the gauge bosons, as well as to the third generation quarks and leptons are in very good agreement with the predictions of the SM [2]. Furthermore, no clear signals of new physics have been observed at the LHC, implying that the SM contains all basic ingredients to be the proper effective theory at the weak scale and perhaps at much higher energies.

The SM, however, does not provide an explanation for the observed Dark Matter density in the Universe. More importantly the couplings of the Higgs to the first and second generation quarks and leptons is still unknown and large deviations with respect to the SM predictions may be present in these sectors (see for example, refs. [3–5]). Indeed, it is a priori unlikely that the scalar sector of the theory reduces to a single Higgs doublet. Additional doublets and singlets may be present, some of them light and which have not been detected due to their small couplings to fermions and bosons of the SM.

One example of such scalars is associated with an explanation of the muon $(g - 2)$ anomaly. As has been stressed in many works [6–15] a light singlet scalar with a mass of the order of the pion mass and a coupling to muons of the order of 10^{-3} may lead to an explanation of this anomaly. Due to gauge invariance, however, such a singlet, cannot couple directly to the muons and could couple to leptons via effective operators involving SU(2) Higgs doublets.

Recently, the KOTO experiment looked for the decay of neutral Kaons into neutral pions and a pair of neutrinos [16, 17]. In particular, KOTO is mostly sensitive for the process $K_L \rightarrow \pi^0 \nu \bar{\nu}$. The neutral pions would subsequently decay into photon pairs. KOTO performed a blind search for such events. The SM expected rate is more than two orders of magnitude below the current KOTO sensitivity. When the process was unblinded, however, three events remain in the signal region, with an expected background of about 0.05 [17].

The appearance of such a signal is particularly surprising due to the existence of the so-called Grossman-Nir bound [18], which is based on a simple relationship between the decay widths of charged and neutral Kaons into charged and neutral pions and neutrinos. Using the respective lifetimes of charged and neutral Kaons, one obtains that

$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \lesssim 4.3 \text{ BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \quad (1.1)$$

This bound puts strong constraints on any high energy physics explanation of the KOTO anomaly. However, it is known that a scalar, with mass of about the pion mass and either stable (when the mass is close to the pion mass) or with a lifetime lower than about a nanosecond can provide an explanation of the KOTO anomaly without violating the Grossman-Nir bound. This is due to the experimental sensitivities of the charged and neutral Kaon experiments. The possibility of a stable, light scalar, with mass close to the pion mass, leading to a possible excess at the KOTO experiment without violating the Grossman-Nir bound was first stressed in refs. [19, 20], well before the KOTO excess observation. The possibility that a light scalar, with mass different from the pion mass and with a long lifetime, but not necessarily stable, can also explain the observed KOTO excess without violating the Grossman Nir bound was stressed by several authors, including those of refs. [21–23]. Other possibilities have been discussed in refs. [21, 24–26].

The range of scalar masses necessary to explain the KOTO anomaly is very similar to the one arising in the light scalar explanation of the $(g - 2)_\mu$ anomaly, suggesting a possible common explanation of both physical phenomena. In this article, we propose a simple model, which leads to such an explanation. Our model is based on a singlet scalar mixing with Higgs doublet states as the ones that appear in a type-X two Higgs doublet model (2HDM), in which quarks and leptons couple to different Higgs doublets. The required singlet scalar turns out to be lighter than the muon and pion.

Our model differs from previous proposals to explain the KOTO anomaly [22, 23], in which a singlet scalar mixing with the SM Higgs (SSM) is assumed. The decay of the scalar into electrons in such proposals lead to a lifetime of the order of $c\tau \sim 100$ km due to the small mixing angle $\sin \theta \sim 10^{-3}$ necessary to explain the KOTO anomaly. In our scenario, instead, the lifetime is much shorter. It is indeed a non-trivial fact that the coupling to leptons consistent with the explanation of $(g - 2)_\mu$ leads to the proper scalar lifetime of ~ 0.01 meter that allows to avoid the charged pion decay constraints. Moreover, this fact also allows a solution to the KOTO anomaly in a different mass range compared with previous proposals.

The article is organized as follows. In section 2, we introduce the effective model and show how it can explain both the $(g - 2)_\mu$ and KOTO anomalies. In section 3, we discuss

the ultraviolet (UV) completion of the model and its constraints from collider, beam dump and astrophysics searches. We reserve section 4 for our conclusions.

2 The effective model for the $(g - 2)_\mu$ and KOTO anomalies

The anomalous magnetic moment of the muon is an excellent probe of new physics at the weak scale. It is governed mainly by a dimension five dipole operator, which is chirally violating, whose contribution to $a_\mu = (g - 2)_\mu/2$ is expected to be loop suppressed and proportional to $(m_\mu/\tilde{m})^2$, where \tilde{m} parameterizes the scale of new physics. The current measurement is more than three standard deviations from the expected value in the SM [27], namely

$$\Delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM} = (2.74 \pm 0.73) \times 10^{-9} \tag{2.1}$$

If this deviation were confirmed, it would be a clear indication of new physics at or below the weak scale. A new measurement of a_μ is expected to be reported by the $g - 2$ collaboration at Fermilab within the next few months [28]. Many models of physics beyond the SM can lead to an explanation of this anomaly [29, 30]. In our work, we consider a simple extension of the SM which includes a light scalar ϕ that couples to both the leptons and quarks but with different couplings relative to the SM ones. We assume that the couplings are flavor diagonal and universal within each sector. Thus, we consider the low-energy effective theory defined by

$$\mathcal{L}_{eff} \supset \sum_q \epsilon_q \frac{m_q}{v} \phi \bar{q}q + \sum_\ell \epsilon_\ell \frac{m_\ell}{v} \phi \bar{\ell}\ell + \epsilon_W \frac{2m_W^2}{v} \phi W_\mu^+ W^{\mu-}. \tag{2.2}$$

This setup can occur, for instance, in a lepton-specific 2HDM with an additional singlet. Later, we will argue that $\epsilon_W \approx \epsilon_q$ in some reasonable limit. Thus, comparing with the SSM model, the only difference with our effective model is that ϵ_ℓ is an extra free parameter. We will return to the UV completion of this model in the next section. In [15] it was shown how a light scalar with a generic coupling to the muon can account for the deviation in the muon anomalous magnetic moment. The contribution to $(g - 2)_\mu$ in our effective model is given by [31–33]

$$\Delta a_\mu = \frac{m_\mu^2}{8\pi^2 v^2} \epsilon_\ell^2 \int_0^1 dx \frac{(1-x)^2(1+x)}{(1-x)^2 + x(m_\phi/m_\mu)^2}. \tag{2.3}$$

It is easy to calculate that the requirement to satisfy $(g - 2)_\mu$ fixes $\epsilon_\ell \simeq \mathcal{O}(1)$, for the range of masses m_ϕ that is relevant for the KOTO anomaly.

In a recent paper [21], it was shown that the KOTO anomaly could be explained by new physics involving a particle, X , with lifetime $\tau \sim \mathcal{O}(0.1 - 0.01)$ ns and appearing in decays of Kaons with a neutral Kaon branching ratio $\text{BR}(K_L \rightarrow \pi^0 X) \sim 10^{-2} - 10^{-8}$. In our case, the decay of the Kaon is induced at one-loop through penguin diagrams with the W boson and leads an effective $s - d - \phi$ vertex. The partial decay widths for K_L are then

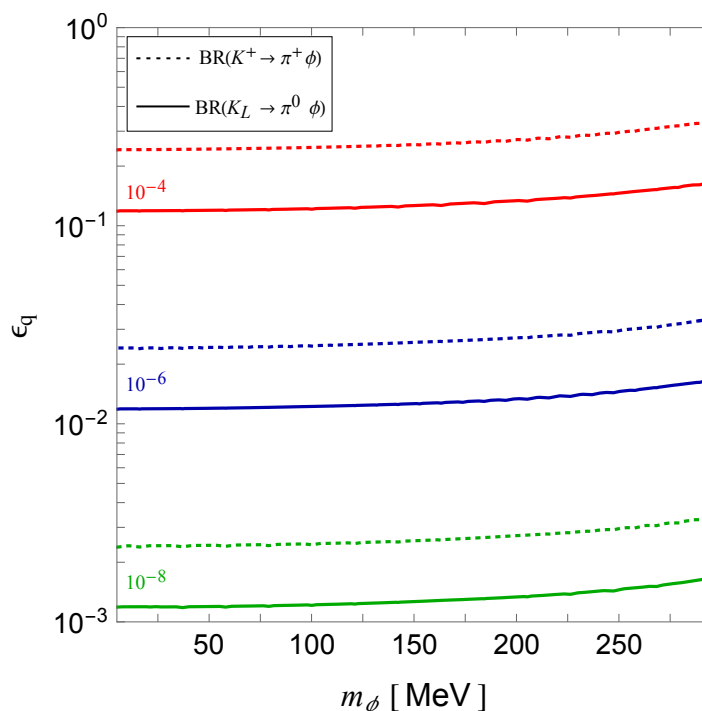


Figure 1. Contours for $\text{BR}(K_L \rightarrow \pi^0 \phi)$ and $\text{BR}(K^+ \rightarrow \pi^+ \phi)$ in the ϵ_q - m_ϕ plane. The solid colored lines indicate the contours for $\text{BR}(K_L \rightarrow \pi^0 \phi)$. Dotted lines of corresponding colors show where $\text{BR}(K^+ \rightarrow \pi^+ \phi)$ achieves the corresponding value.

controlled by ϵ_q and m_ϕ [34, 35],

$$\Gamma(K_L \rightarrow \pi^0 \phi) = \frac{(\text{Re}[g(\epsilon_q)])^2}{16\pi m_K^3} \lambda^{1/2}(m_K^2, m_\pi^2, m_\phi^2), \quad (2.4)$$

$$g(\epsilon_q) = \frac{3m_K^2}{32\pi^2 v^3} \epsilon_q f_+(0) \sum_{q=u,c,t} m_q^2 V_{qd}^* V_{qs}, \quad (2.5)$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$ is the triangle function and $f_+(0)$ is the vector form factor at zero momentum transfer. For the partial width of $K^+ \rightarrow \pi^+ \phi$, one needs to substitute $\text{Re}[g(\epsilon_q)]$ to $|g(\epsilon_q)|$ and change the corresponding mass parameters. The calculation of the decay width involves the evaluation of the appropriate form factor. Following ref. [35] (and references therein) and also validated in ref. [34] using Higgs low-energy theorems, this factor is taken to be close to unity. A more precise calculation from lattice QCD shows $f_+(0) = 0.9709(46)$, which is indeed close to unity [36].

In figure 1, we show the contours for $\text{BR}(K_L \rightarrow \pi^0 \phi)$ and $\text{BR}(K^+ \rightarrow \pi^+ \phi)$ (solid and dotted lines respectively) in the $m_\phi - \epsilon_q$ plane. We see that in the range of scalar masses we consider, the branching ratio is fairly insensitive to m_ϕ and thus determined mostly by ϵ_q . Furthermore, obtaining a branching ratio appropriate for the KOTO anomaly sets $\epsilon_q \ll \epsilon_\ell$, as the latter is of order 1.

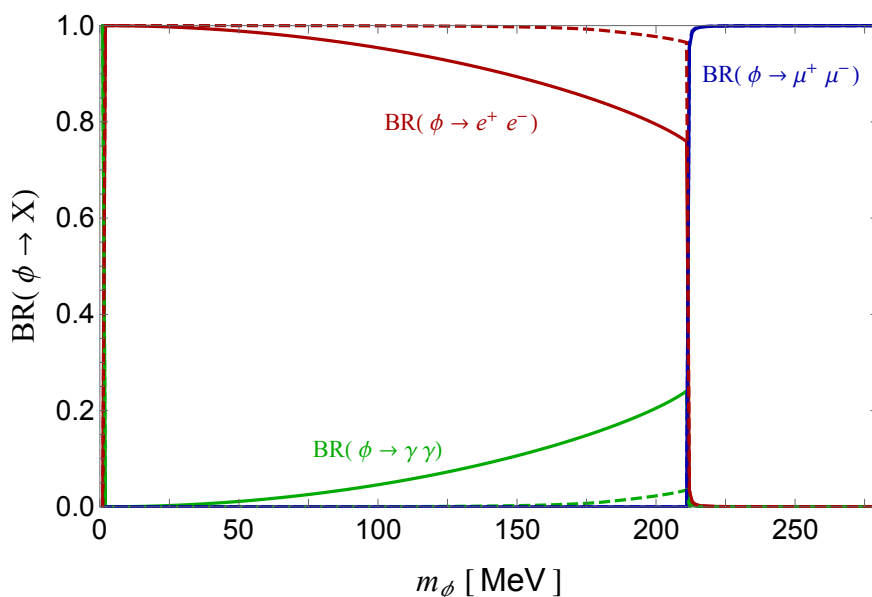


Figure 2. Branching ratios of the allowed ϕ decay modes. The solid lines show branching ratios $\text{BR}(\phi \rightarrow X)$ when the coupling $\epsilon_\ell \sim 1$ is fixed to fit the central value for Δa_μ , and $\epsilon_q = \epsilon_W \sim 6 \times 10^{-3}$ is fixed so that $\text{BR}(K_L \rightarrow \pi^0 \phi) = 10^{-6}$. In dashed lines, the branching ratios for singlet scalar mixing model $\epsilon_q = \epsilon_W = \epsilon_\ell$ are shown. In the latter case, a dramatic cancellation between W loop function and fermion loop function happens for m_ϕ around the pion mass, which leads to a higher electron-positron decay branching ratio even with a much smaller ϵ_ℓ .

With ϵ_ℓ fixed by the requirement to explain $(g - 2)_\mu$ and ϵ_q fixed by the choice of $\text{BR}(K_L \rightarrow \pi^0 \phi)$ to explain the KOTO experiment result, the lifetime of ϕ is then uniquely determined by its mass, once all allowed decay modes are identified. For the range of masses relevant to both experiments, the allowed decay modes are $\phi \rightarrow e^+ e^-$, $\phi \rightarrow \gamma \gamma$, and when $m_\phi > 2m_\mu$ $\phi \rightarrow \mu^+ \mu^-$. The decay widths are given by

$$\Gamma(\phi \rightarrow \ell\ell) = \frac{\epsilon_\ell^2 m_\ell^2}{8\pi v^2} m_\phi (1 - \tau_\ell)^{3/2} \theta(m_\phi^2 - 4m_\ell^2), \quad (2.6)$$

$$\Gamma(\phi \rightarrow \gamma\gamma) = \frac{\alpha^2 m_\phi^3}{1024\pi^3} \left| \sum_q \frac{6\epsilon_q}{v} Q_q^2 A_{1/2}(\tau_q) + \sum_\ell \frac{2\epsilon_\ell}{v} A_{1/2}(\tau_\ell) + \frac{2\epsilon_W}{v} A_1(\tau_W) \right|^2, \quad (2.7)$$

where θ is the step function, $A_{1/2}(\tau_i)$ ($A_1(\tau_i)$) is the usual fermion (vector-boson) loop function and $\tau_i = 4m_i^2/m_\phi^2$ [35].

In figure 2, we show the branching ratios of the allowed decay modes of ϕ . The decay widths are determined by fixing ϵ_ℓ so that Δa_μ is fit to the central value of $(g - 2)_\mu$, and ϵ_q so that $\text{BR}(K_L \rightarrow \pi^0 \phi) = 10^{-6}$. However, the branching ratios are largely unaffected by deviations from these choices so long as $\epsilon_q \ll \epsilon_\ell$. Despite the fact that the diphoton width receives contributions from both quarks and leptons it is loop suppressed, and thus $\phi \rightarrow e^+ e^-$ will always be the dominant decay mode for $m_\phi < 2m_\mu$. Besides the decay branching ratio of our model (solid lines), we also show the corresponding branching ratios

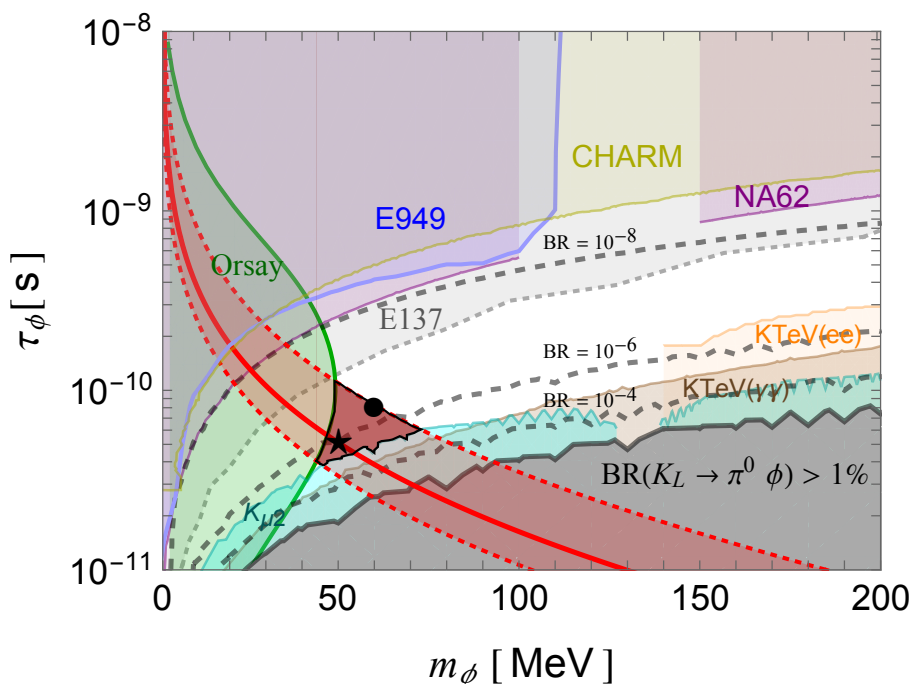


Figure 3. The parameter space for the effective model in the m_ϕ - τ_ϕ plane. For each point on the plane, $\text{BR}(K_L \rightarrow \pi^0 \phi)$ is fixed to fit to the central value measured by the KOTO experiment. We show contours of $\text{BR}(K_L \rightarrow \pi^0 \phi) = 10^{-4}, 10^{-6},$ and 10^{-8} in gray dashed lines. Once the value of ϵ_ℓ is fixed to fit $(g-2)_\mu$, the scalar lifetime shrinks to the red shaded region, where the red solid line corresponds to the central value of Δa_μ and the red dotted lines correspond to the 2σ region. The dark red highlighted region can explain both $(g-2)_\mu$ and KOTO anomalies. The cyan, blue, purple, yellow, light orange, brown, green, and light gray shaded regions are excluded at 95% C.L. by the $K_{\mu 2}$ [37], E949 [38, 39], NA62 [40], CHARM [41], KTeV/E799 ($K_L \rightarrow \pi^0 e^+ e^-$) [42], KTeV ($K_L \rightarrow \pi^0 \gamma \gamma$) [43], Orsay [44], and E137 [45] experiments respectively. The gray shaded region is excluded by untagged K_L decay [1].

in the SSM model (dashed lines) which has $\epsilon_q = \epsilon_\ell = \epsilon_W$. It is interesting to mention that for our model $\epsilon_q = \epsilon_W \sim 10^{-3}$ – 10^{-2} and $\epsilon_\ell \sim 1$, thus it is natural to expect $\text{BR}(\phi \rightarrow e^+ e^-) \simeq 1$. However, due to a surprising cancellation between W loop function and fermion loop function contributions to the di-photon decay amplitude for m_ϕ around the pion mass, the SSM model has a larger $\text{BR}(\phi \rightarrow e^+ e^-)$. Therefore, in some beam dump experiments our model will be more constrained.

For a light scalar to mimic the signal in the KOTO experiment, it must decay outside of the detector. Taking into account the appropriate boost factors and detector efficiencies this leads to an effective branching ratio

$$\text{BR}(K_L \rightarrow \pi^0 \phi; \text{KOTO}) = \epsilon_{\text{eff}} \text{BR}(K_L \rightarrow \pi^0 \phi) e^{-\frac{L}{p_\phi} \frac{m_\phi}{\tau_\phi}}, \quad (2.8)$$

where p_ϕ is the typical momentum of ϕ and L is the detector size in the KOTO experiment. The efficiency factor ϵ_{eff} is included to account for the signal efficiency difference between

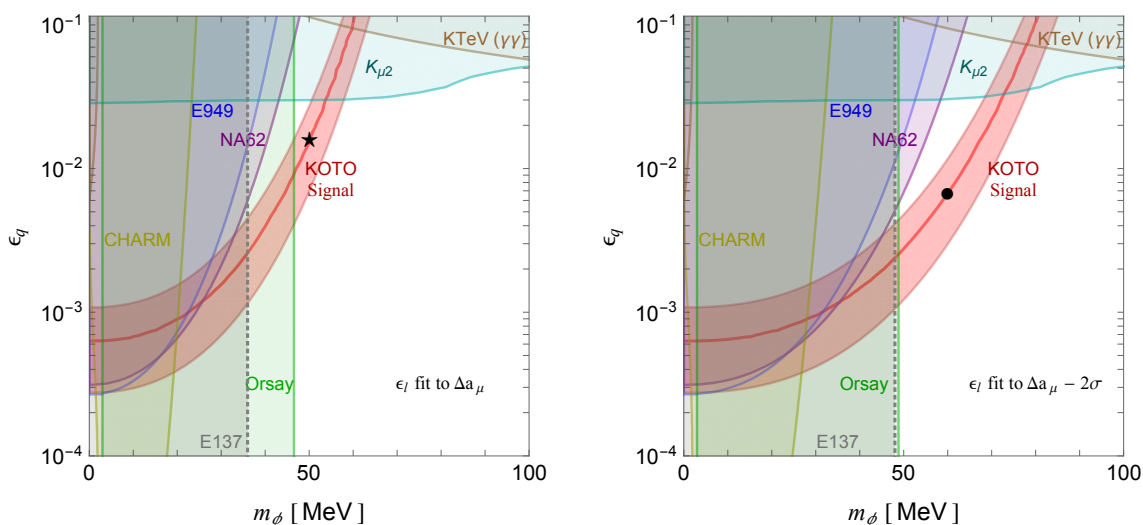


Figure 4. *Left:* the parameter space for the effective model in the m_ϕ - ϵ_q plane when ϵ_ℓ is fixed for the central value of Δa_μ . The red shaded band shows the parameter space where the KOTO signal is achieved at 95% confidential level. *Right:* the parameter space for the effective model in the m_ϕ - ϵ_q plane when ϵ_ℓ is fixed to $\Delta a_\mu - 2\sigma$. All the constraints are shown in the same colors as in figure 3.

$K_L \rightarrow \pi^0 \phi$ and $K_L \rightarrow \pi^0 \bar{\nu} \nu$, which is taken from [16]. Following [21], we fix $L = 3m$ and $E_\phi = 1.5 \text{ GeV}$ for the KOTO experiment.

In our model, there are three free parameters m_ϕ , ϵ_ℓ and $\epsilon_q = \epsilon_W$ where the last two are fixed by the $(g-2)_\mu$ and KOTO experiments, respectively. One can trade them into another set of parameters m_ϕ , τ_ϕ and $\text{BR}(K_L \rightarrow \pi^0 \phi)$. Since $\epsilon_q \ll \epsilon_\ell$, the total width and lifetime of ϕ are dominantly determined by ϵ_ℓ . In this sense, τ_ϕ is insensitive to the precise value of $\text{BR}(K_L \rightarrow \pi^0 \phi)$.

In figure 3, we show the model parameter space in the m_ϕ - τ_ϕ plane. Once ϵ_ℓ is fixed to fit $(g-2)_\mu$, the model parameters shrink to the red shaded region, where the red solid line corresponds to the central value of Δa_μ and the red dashed lines correspond to the 2σ region. The dashed gray lines show contours of $\text{BR}(K_L \rightarrow \pi^0 \phi)$ where the central value of the KOTO signal is achieved (see figure 1 to translate this to values of ϵ_q). The cyan, blue, purple, yellow, light orange, brown, green, and light gray shaded regions are excluded by the $K_{\mu 2}$ [37], E949 [38, 39], NA62 [40], CHARM [41], KTeV/E799 ($K_L \rightarrow \pi^0 e^+ e^-$) [42], KTeV ($K_L \rightarrow \pi^0 \gamma \gamma$) [43], Orsay [44], and E137 [45] experiments respectively. The gray shaded region is excluded by untagged K_L decay [1]. We defer the details of the constraints to section 3. The parameter space which can solve both the $(g-2)_\mu$ and KOTO anomalies is highlighted in the dark red region near $m_\phi \sim 50 \text{ MeV}$.

In figure 4 we show our results in the m_ϕ - ϵ_q plane. In the left plot, we show in the red shaded region where the KOTO signal is achieved at the 95% C.L. when ϵ_ℓ is fixed for the central value of Δa_μ . When ϵ_ℓ is allowed to vary within a 2σ range of Δa_μ , the variation will be reflected in the lifetime of ϕ . This will modify the probability decay factor in both the KOTO effective branching ratio and the corresponding factors in the constraints. The

m_ϕ [MeV]	ϵ_q	ϵ_ℓ	$\text{BR}(K_L \rightarrow \pi^0 \phi)$	τ [s]	$\tan \beta$	$\sin \alpha$	$\sin \theta_{1\phi}$	$\sin \theta_{2\phi}$
50	1.6×10^{-2}	1.22	1.7×10^{-6}	5.1×10^{-11}	100	-0.01	0.0122	1.6×10^{-2}
60	6.8×10^{-3}	0.87	3.2×10^{-7}	8.25×10^{-11}	100	-0.01	0.0087	6.8×10^{-3}

Table 1. Benchmark points of the effective model which satisfies both the KOTO and $(g - 2)_\mu$ anomalies. The benchmark with $m_\phi = 50$ MeV is indicated in figures 3 & 4 by a star. The benchmark with $m_\phi = 60$ MeV is indicated in figures 3 & 4 by a circle. The first three columns are the three free parameters of the effective model, and the fourth and fifth columns are the second set of parameters if one trades the two effective couplings for the Kaon decay branching ratio and ϕ lifetime. The last four columns give the corresponding values of $\tan \beta$ and three mixing angles in the UV model.

right plot of figure 4 shows the result of fixing ϵ_ℓ to $\Delta a_\mu - 2\sigma$, displaying the maximum range allowed for m_ϕ which satisfies the experimental constraints.¹

In figure 3 and 4, we see that the allowed mass range for ϕ which can explain both $(g - 2)_\mu$ and KOTO anomalies fall into the range of $\sim 40 - 70$ MeV. In table 1, we give two benchmark points for the model parameters. Both points are indicated in figures 3 and 4. We have further checked the Barr-Zee diagram contribution from the top quark and the tau lepton for these benchmark points [46, 47]. The results show that they are smaller than 1-loop contribution by 3 orders and 2 orders of magnitudes respectively, due to small ϵ_q and an extra loop suppression. In the following section we give an example of a possible UV completion model and discuss the details of the experimental constraints on the model.

3 The UV completion of the model

3.1 Type-X 2HDM plus singlet scalar model

In this subsection, we discuss the ultra-violet completion of the effective model and the constraints in the next subsection. We shall work on a type-X two Higgs doublet model [48–50], in which the two Higgs doublets couple disjointly to either leptons or quarks. We will denote Φ_2 as the scalar doublet that couples to quarks, and Φ_1 as the one that couples to leptons. We will assume the existence of an additional real singlet scalar ϕ mixing with the CP-even neutral components of these Higgs doublets. This light scalar ϕ is exactly the light degree of freedom in the effective model.

A clear advantage of this framework is that no flavor changing neutral currents associated the scalar fields appear at tree level. The appearance of a coupling of one scalar to the lepton fields independent of the one to quark fields allows to solve the $(g - 2)_\mu$ anomaly simultaneously with the explanation of the observed KOTO excess. Moreover, as emphasized before, the singlet scalar lifetime will be modified by the presence of the lepton couplings and will be far shorter than the one that would be obtained by a singlet mixing with only a SM-like Higgs boson, which allows a solution of the KOTO anomaly in a different mass range than the one obtained in previous works [22, 23].

¹Similarly, fixing ϵ_ℓ to fit $\Delta a_\mu + 2\sigma$ would give an indication of the minimum m_ϕ allowed. However, from figure 3 we see that this range (bottom orange dashed curve) is ruled out by multiple constraints.

The effective Lagrangian density, describing the interactions of the scalar doublets and the fermions is

$$\mathcal{L}_{\text{yuk}} = -\lambda_u \bar{Q} \tilde{\Phi}_2 u_R - \lambda_d \bar{Q} \Phi_2 d_R - \lambda_e \bar{L} \Phi_1 e_R + h.c., \quad (3.1)$$

while the scalar potential can be given as,

$$\begin{aligned} V(\Phi_1, \Phi_2, \phi_0) = & \sum_{i=1,2} -\mu_i^2 \Phi_i^\dagger \Phi_i + \lambda_i (\Phi_i^\dagger \Phi_i)^2 - \mu_\phi^2 \phi_0^2 + \lambda_\phi \phi_0^4 \\ & - (\rho \phi_0 \Phi_1^\dagger \Phi_2 + H.c.) + \lambda_{12} \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2 - \rho_{1\phi} \Phi_1^\dagger \Phi_1 \phi_0 - \rho_{2\phi} \Phi_2^\dagger \Phi_2 \phi_0 \\ & + \dots \end{aligned} \quad (3.2)$$

where the first line determines the masses of the three CP-even neutral scalars, the second line determines the three mixing angles between the CP-even neutral scalars and the third line contains all terms irrelevant to our discussion. The vacuum expectation values (vevs) of the two Higgs doublets are v_1 and v_2 respectively. We will assume that $\tan \beta = v_2/v_1 \gg 1$, implying that the field Φ_2 will have approximately standard interactions with quarks and gauge bosons, and will carry the dominant component of the SM-like Higgs in the alignment limit [50–52]. We will assume that the non-standard Higgs' have masses around several hundreds of GeV.

The CP-even neutral scalars in the two Higgs doublets ($\Phi_{1,2}^0$) will mix with the singlet scalar ϕ_0 through the mixing matrix

$$\begin{pmatrix} \sqrt{2}\text{Re}[\Phi_1^0] \\ \sqrt{2}\text{Re}[\Phi_2^0] \\ \phi_0 \end{pmatrix} \simeq \begin{pmatrix} \cos \alpha & -\sin \alpha & \sin \theta_{1\phi} \\ \sin \alpha & \cos \alpha & \sin \theta_{2\phi} \\ -\sin \theta_{1\phi} & -\sin \theta_{2\phi} & 1 \end{pmatrix} \cdot \begin{pmatrix} H \\ h \\ \phi \end{pmatrix}, \quad (3.3)$$

where we have assumed the mixing angles between the singlet and the CP-even scalars coming from the Higgs doublets are very small. The mass eigenstates are h, H, ϕ , where $h \approx \sqrt{2}\text{Re}[\Phi_2^0]$ is the SM-like Higgs. In terms of mass eigenstates, the Yukawa lagrangian at leading order in the mixing angles is given by

$$\begin{aligned} \mathcal{L}_{\text{yuk}} \simeq & (\sin \alpha H + \cos \alpha h + \sin \theta_{2\phi} \phi) \sum_q \frac{m_q}{v_2} \bar{q} q + (\cos \alpha H - \sin \alpha h + \sin \theta_{1\phi} \phi) \sum_\ell \frac{m_\ell}{v_1} \bar{\ell} \ell \\ & + \frac{i}{v} A [m_d \bar{d} \gamma_5 d \cot \beta - m_u \bar{u} \gamma_5 u \cot \beta - m_\ell \bar{\ell} \gamma_5 \ell \tan \beta] \\ & + \frac{\sqrt{2}}{v} m_d H^+ \bar{u}_L d_R \cot \beta - \frac{\sqrt{2}}{v} m_u H^- \bar{d}_L u_R \cot \beta - \frac{\sqrt{2}}{v} m_\ell H^+ \bar{\nu}_L \ell_R \tan \beta + h.c., \end{aligned} \quad (3.4)$$

where $v_1^2 + v_2^2 = v^2 = (246 \text{ GeV})^2$.

For h to be SM-like, its couplings to leptons requires

$$\sin \alpha / \cos \beta \approx -1, \quad (3.5)$$

which is the same as the usual Higgs alignment requirement [50–52].

Once we apply this requirement, the Yukawa terms can be simplified as

$$\mathcal{L}_{\text{yuk}} \simeq \left(h - \frac{H}{\tan \beta} + \frac{\sin \theta_{2\phi}}{\sin \beta} \phi \right) \sum_q \frac{m_q}{v} \bar{q}q + \left(h + \tan \beta H + \frac{\sin \theta_{1\phi}}{\cos \beta} \phi \right) \sum_\ell \frac{m_\ell}{v} \bar{\ell}\ell \quad (3.6)$$

where we will assume that $\tan \beta \gg 1$. One can read out the relation between UV parameters and the effective model parameters,

$$\epsilon_q \simeq \frac{\sin \theta_{2\phi}}{\sin \beta}, \quad \epsilon_\ell \simeq \frac{\sin \theta_{1\phi}}{\cos \beta}. \quad (3.7)$$

To calculate the diphoton decay BR for ϕ , one also needs the coupling to W gauge boson,

$$\epsilon_W \simeq (\sin \theta_{1\phi} \cos \beta + \sin \theta_{2\phi} \sin \beta) \quad (3.8)$$

$$\approx \epsilon_\ell \cos^2 \beta + \epsilon_q \sin^2 \beta \approx \epsilon_q, \quad (3.9)$$

where in the second line, $\tan^2 \beta \gg \epsilon_\ell/\epsilon_q$ was assumed. One can see that in this limit, we have simplified to $\epsilon_W \approx \epsilon_q$ which is similar to the SSM model case. The charged Higgs can also contribute to the diphoton decay for ϕ . However, there are too many parameters in the scalar potential to uniquely determine the relevant coupling. For example, the mixing angle $\sin \theta_{1\phi}$ can come from $\rho\phi_0\Phi_1^\dagger\Phi_2$ term instead of $\rho_{1\phi}\phi_0\Phi_1^\dagger\Phi_1$. As a result, the coupling of ϕ to the charged Higgs can be small without changing the phenomenology we are interested in. Therefore we neglect the charged Higgs contribution in our results.

A few comments are in order regarding the calculation of $K_L \rightarrow \pi^0\phi$ in the full model. Recall that this decay is induced by a 1-loop $s - d - \phi$ coupling in the effective model. Similar loop induced decays have been calculated for h and H in general 2HDM [53, 54]. Upon adding the W boson, charged Goldstones and charged Higgs diagrams, the UV cancellation and gauge invariance is explicit. The singlet contribution just proceeds from the small mixing with the h and H Higgs bosons. Secondly, our calculation in the effective model assumes that the singlet contribution is approximately given by the one coming from the mixing of the singlet with h , $\epsilon_q = \epsilon_W$. In the full model, this assumption is violated due to the non-standard Higgs contributions, which are however quite small. For instance $\epsilon_q + \epsilon_\ell/\tan^2 \beta \approx \epsilon_W$, implying that the relation $\epsilon_q = \epsilon_W$ is precise at the 10^{-2} level. The contributions coming from the mixing of the singlet with H are suppressed due to the $1/\tan \beta$ suppression of the H coupling to quarks and its suppression to its coupling to the W and Goldstone bosons in the alignment limit, $\cos(\beta - \alpha) \sim 0$. Similarly, the inclusion of the charged Higgs in the diagram is suppressed by three reasons: the $1/\tan \beta$ suppression of the charged Higgs coupling to quarks, the proximity to the alignment limit we are assuming, that suppresses the coupling of the Standard Model Higgs to the charged Higgs and W boson $\cos(\beta - \alpha) \sim 0$, and the heaviness of the charged Higgs. Therefore, we conclude all contributions not contained in our approximate expression are suppressed by at least a $1/\tan \beta$ factor with respect to the ones we considered.

3.2 The various constraints

In the UV model, since $\Phi_{1,2}$ couples to leptons and quarks respectively, there are no FCNC issues at the tree-level. However, there are constraints from collider, beam dump and astrophysical experiments [55–60]. We will discuss them one by one in the following subsections.

3.2.1 Collider constraints

We consider several relevant constraints for the UV model from collider measurements. The first is the modification of the τ leptonic decay branching ratio, because the charged Higgs couples to τ leptons with a large Yukawa coupling. The next is exotic Z decays to lepton final states. The third is the exotic Higgs decay $h \rightarrow \phi\phi$ and the last two are B meson decays $B_s \rightarrow \mu^+\mu^-$ and $B \rightarrow K^{(*)}e^+e^-$.

Firstly, the charged Higgs coupling to leptons is constrained by the τ leptonic decay [50, 61]. Its contribution to the τ leptonic decay width is given as

$$\Gamma_{\text{tree}}^{H^\pm}(\tau \rightarrow \ell \bar{\nu}_\ell \nu_\tau) = \Gamma_0 \left[\frac{m_\tau^2 m_\ell^2 \tan^4 \beta}{4m_{H^\pm}^4} - 2 \frac{m_\ell m_\tau \tan^2 \beta}{m_{H^\pm}^2} \frac{m_\ell}{m_\tau} \kappa \left(\frac{m_\ell^2}{m_\tau^2} \right) \right], \quad (3.10)$$

where Γ_0 is the SM total decay width of τ and

$$\begin{aligned} \kappa(x) &= \frac{g(x)}{f(x)}, & g(x) &= 1 + 9x - 9x^2 - x^3 + 6x(1+x) \ln x, \\ & & f(x) &= 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x. \end{aligned} \quad (3.11)$$

It is clear that if the combination $m_\tau m_\ell \tan^2 \beta \ll m_{H^\pm}^2$, then the deviation from the SM decay width is small. The SM branching ratio for the tau decay to a muon plus a neutrino pair is measured to be $\text{BR}(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau) = 17.39 \pm 0.04\%$ [1]. Therefore, within 1σ , this measurement constrains the combination $\tan^2 \beta / m_{H^\pm}^2 < 0.89 \text{ GeV}^{-2}$. For our benchmark points we have taken $\tan \beta = 100$. Thus, the constraint from tau decays requires that $m_{H^\pm} \gtrsim 100 \text{ GeV}$, which is easily satisfied. On the other hand, the contribution of H^\pm for $\tau \rightarrow e \bar{\nu}_e \nu_\tau$ is suppressed by the electron mass, thus the corresponding constraint is much weaker.

Secondly, since the new Higgs couplings to leptons are enhanced by large $\tan \beta$, one might worry that the branching ratio $Z \rightarrow 4\ell$ might be changed due to new Higgs mediation. The measurement on Z decay branching ratio $Z \rightarrow 4\ell$ is $(4.58 \pm 0.26) \times 10^{-6}$ [62], where ℓ means e, μ . However, in their study, an invariant mass cut larger than 4 GeV is applied for all opposite-sign, same-flavor lepton pairs. Therefore, the exotic decay $Z \rightarrow \mu^+\mu^-\phi$ does not set a significant constraint.

Thirdly, the exotic Higgs can decay to a pair of ϕ particles, $h \rightarrow \phi\phi$. Since ϕ decays to e^+e^- and has a mass around 50 MeV, the electron and positron will be highly collimated and can not be separated by the LHC. Such an exotic event might be identified as $h \rightarrow e^+e^-$ and could be constrained by exotic Higgs decays to two lepton jets measurements [63]. The ATLAS collaboration performed a search for prompt lepton jets at 8 TeV and the model they consider is $h \rightarrow A'A' + X$ where the two A' decay to lepton jets subsequently. For $m_{A'} = 0.4 \text{ GeV}$, the constraint with prompt A' decay is $\text{BR}(h \rightarrow A'A' + X) \lesssim 0.3\%$ at 95% C.L.. Since ϕ decays mostly to an electron pair in our model, it falls into the electron lepton

jet which is less stringent. It is reasonable to say that the constraint for $\text{BR}(h \rightarrow \phi\phi) \lesssim 1\%$. Again, in the scalar potential, there are many free parameters and the relevant terms in the second line of eq. (3.2) will control the exact value of the $\phi - \phi - h$ coupling. Therefore, this constraint may be avoided while keeping the three mixing angles required to explain the KOTO and $(g - 2)_\mu$ anomalies.

Next, ϕ can mediate $B_s \rightarrow \mu^+\mu^-$ decays as it couples, in particular to bottom and strange quarks, and muons. Recently, this process has been measured by CMS and LHCb [64–68] and found to be in agreement with the SM predictions. The constraints for a single-mixing coupling with strength proportional to m_f/v are calculated for a pseudoscalar in [58] and for a scalar in [59, 60, 69, 70], which conservatively require that the new contribution can not exceed the SM branching ratio. Relevant to our scenario, for $m_\phi < 1 \text{ GeV}$, the limit on ϵ_f is $\lesssim 1$. Since the process is proportional to ϵ_f^4 [71], in our model the corresponding factor is $\epsilon_q^2 \epsilon_\ell^2$. Fitting the $(g - 2)_\mu$ anomaly implies $\epsilon_\ell \sim 1$ for the light ϕ , but this constraint becomes quite weak since ϵ_q is about 10^{-3} – 10^{-2} . One might worry about the heavy Higgs H which carries a $\tan\beta$ enhanced lepton coupling. However, its quark coupling is suppressed by the same $\tan\beta$ factor and thus the multiplication of quark and lepton couplings gives, apart from a sign, the same as that for the SM-like Higgs h . Given the fact that the H is heavier than h and the SM contribution is dominated by Z-penguin and box diagram, it is safe to neglect the heavy Higgs boson contribution to this process. Similarly the couplings of fermions to the charged Higgs are suppressed by $1/\tan\beta$ together with the charged Higgs mass, thus we neglect this contribution as well.

Lastly, ϕ can result in the B meson decay $B \rightarrow K^{(*)}\phi$, where ϕ subsequently decays to e^+e^- . The branching ratio measurement of $\text{BR}(B \rightarrow K^{(*)}e^+e^-)$ can generally constrain the 2HDM plus singlet scalar model, see [72]. For our benchmarks, one can calculate the branching ratio of $B \rightarrow K^{(*)}\phi$ to be $\sim 10^{-4}$, following the calculations in [73]. The LHCb measurement of the SM branching ratio $\text{BR}(B^0 \rightarrow K^{(*)0}e^+e^-)$ is about $3.1_{-0.88}^{+0.94} \times 10^{-7}$ in the dilepton mass range from 30 MeV to 1000 MeV [74]. However, the reconstruction of B^0 needs K^{*0} and e^+e^- to form a good quality vertex, where the LHCb vertex resolution is about $\sigma_v^T = \mathcal{O}(10) \mu\text{m}$ in transverse plane and $\sigma_v^L = \mathcal{O}(100) \mu\text{m}$ in longitudinal direction, highly depending on the number of tracks of the vertex [75]. Our benchmarks have a lifetime of $c\tau \sim 1.5 \text{ cm}$, therefore, the probability for ϕ to decay within the vertex resolution L is $1 - e^{-m_\phi L/(p_\phi \tau_\phi)} \sim m_\phi L/(p_\phi \tau_\phi)$. The average momentum of the B meson is about 80 GeV [76], thus the decay probability is $\sim 4 \times 10^{-4}$, conservatively taking $L \sim 5 \text{ mm}^2$ and $p_\phi \sim 40 \text{ GeV}$. Thus, this decay is too small to contribute to the LHCb measurement. Moreover, in the 3-body decay $B^0 \rightarrow K^{(*)0}e^+e^-$ in SM, the angle between e^+e^- can be quite large. However, in our case, e^+e^- is very collimated giving a signature in the detector similar to the SM background $B^0 \rightarrow K^{*0}\gamma$, when the photon later converts into an e^+e^- pair. As a result, we conclude our benchmark is safe from the prompt search of $\text{BR}(B^0 \rightarrow K^{(*)0}e^+e^-)$ due to large lifetime and small scalar mass. It is worth mentioning that the BaBar collaboration has considered $B \rightarrow X_s\phi$ decays where X_s is the strange

²Although the vertex resolution of LHCb is about $\mathcal{O}(100) \mu\text{m}$, the references [56, 58, 77] choose $L \sim 5 \text{ mm}$ as the criteria of prompt decay.

hadronic system, with subsequent displaced decays $\phi \rightarrow e^+e^-$, $\mu^+\mu^-$ and several other possible channels [78], however $m_{ee} > 0.4 \text{ GeV}$ is required.

3.2.2 Beam dump experiments

In this section, we consider various beam dump experiments. In this type of experiment, displaced decays of new particles is critical, otherwise they will be swamped by the SM background. Therefore, the lifetime is a very important factor in the signal analysis as it appears in the exponential form of the decay probability. Some of the beam dump experiments look for new particle decays to visible final states, e.g. electron pairs, while some look for invisible decays. In our scenario, the light scalar ϕ will dominantly decay to visible final states e^+e^- and subdominantly to $\gamma\gamma$. With a finite lifetime, it can always be subject to constraints by either kind of beam dump experiments, depending on decay probability within and without the detector. We will go through the various experiments in the following discussion.

E949 and NA62. The E949 collaboration [38, 39] and NA62 collaboration [40] have measured the process $K^+ \rightarrow \pi^+\bar{\nu}\nu$, which could be mimicked by the $K^+ \rightarrow \pi^+\phi$ decay.

The recent NA62 result combines 2016 and 2017 data, which sets $\text{BR}(K^+ \rightarrow \pi^+\bar{\nu}\nu) < 1.85 \times 10^{-10}$ at 90% C.L. If the process is the two body decay $K^+ \rightarrow \pi^+\pi^0$, they can also constrain $\text{BR}(\pi^0 \rightarrow \text{invisible}) < 4.4 \times 10^{-9}$ at 90% C.L. Since the ϕ has finite lifetime, there is probability for it to decay outside the detector. Therefore, one can use eq. (2.8) to describe the constraint for NA62, with the substitution of detector size $L = 150 \text{ m}$. Since NA62 used the flying Kaon with energy of 75 GeV, one can calculate the energy of ϕ , assuming an isotropic decay of K^+ and further requiring that the π^+ energy falls into their signal box 15–35 GeV. Neglecting the signal efficiency difference between 2-body and 3-body decay of K^+ , we can arrive at a constraint for NA62 on the parameter space, which is the purple shaded region in figure 3 and 4.

For the result from the E949 collaboration [39], the possibility of $K^+ \rightarrow \pi^+\phi$ with ϕ being long-lived has been explicitly explored. The constraints on $\text{BR}(K^+ \rightarrow \pi^+\phi)$ have been given as a function of m_ϕ and its lifetime. We translate the limit into our signal model and the excluded parameter space is shown as blue shaded region in figure 3. We comment on the constraint from the invisible decay for B meson $B \rightarrow K + \text{inv}$. It can exclude $\epsilon_q \lesssim 10^{-2}$ in SSM model [57, 59, 60], however it can not be applied in our case because ϕ is too short-lived.

CHARM. The CHARM experiment measures the displaced decay of neutral particles into $\gamma\gamma$, e^+e^- and $\mu^+\mu^-$. Since our signal can result from ϕ being produced from the decays $K_L \rightarrow \pi^0\phi$ and $K^+ \rightarrow \pi^+\phi$, the CHARM experiment is relevant for long-lived ϕ . Following [58], we have the number of events of this exotic decay for CHARM detector to be

$$N_{\text{det}} \approx N_\phi \left(e^{-\frac{480m}{\gamma_\phi \beta_\phi c \tau_\phi}} - e^{-\frac{480+35m}{\gamma_\phi \beta_\phi c \tau_\phi}} \right) \sum_{X=e,\mu,\gamma} \text{BR}(\phi \rightarrow XX). \quad (3.12)$$

In CHARM experiment, the energy of ϕ is about 10 GeV thus one can calculate the velocity β_ϕ and the boost factor γ_ϕ . The number of ϕ produced in the Kaon decay is

$N_\phi = 2.9 * 10^{17} \sigma_\phi / \sigma_{\pi_0}$ and σ_ϕ is production cross-section [55]

$$\sigma_\phi \approx \sigma_{pp} M_{pp} \chi_s (0.5 \text{ BR}(K^+ \rightarrow \pi^+ \phi) + 0.25 \text{ BR}(K_L \rightarrow \pi^0 \phi)), \quad (3.13)$$

where σ_{pp} is the total proton cross-section, M_{pp} is the total hadron multiplicity and $\chi_s = 1/7$ is the relative parts going into strange flavour [79]. For the pion, there is the relation $\sigma_{\pi_0} \simeq \sigma_{pp} M_{pp} / 3$. In [58], the authors calculated the CHARM limit for ϕ as a pseudoscalar. In [57, 59, 60], the CP-even scalar case is calculated, and the effect of the parity of the scalar is subdominant. Since the CHARM experiment has observed zero event for such exotic decay, one can set 90% confidential level bound by requiring $N_{\text{det}} < 2.3$. The CHARM experiment excludes the parameter space of our model in the yellow shaded region in figure 3 and 4.

$K_{\mu 2}$ experiment. Due to $K^+ \rightarrow \pi^+ \phi$ process, a stopping K^+ decay to π^+ in two body final state is relevant as a by product of the $K_{\mu 2}$ experiment [37]. Because K^+ is stopped in the above 2-body decay, the momentum of π^+ is mono-chromatic with a dependence only on the ϕ mass. Since this energy peak has not been observed, one can set a constraint on $\text{BR}(K^+ \rightarrow \pi^+ \phi)$, regardless of the ϕ decay products [37, 58, 59]. We apply this constraint to our model and translate the constraint on $\text{BR}(K_L \rightarrow \pi^0 \phi)$ to the model parameter ϵ_q . The excluded parameter space is shown as the cyan shaded region in figure 3 and 4.

KTeV/E799 (e^+e^-). Given the ϕ mass we are interested in, ϕ will dominantly decay to e^+e^- . The searches for $K_L \rightarrow \pi^0 e^+e^-$ from KTeV/E799 [42] has set limits $\text{BR}(K_L \rightarrow \pi^0 e^+e^-) < 2.8 \times 10^{-10}$ at 90% C.L., which is relevant for the process $K_L \rightarrow \pi^0 \phi$. Another search $K^+ \rightarrow \pi^+ e^+e^-$ from NA48/2 [80] has measured $\text{BR}(K^+ \rightarrow \pi^+ e^+e^-) = (3.11 \pm 0.12) \times 10^{-7}$, which is also relevant to our signal, but is much less stringent. In these type of searches, an invariant mass cut of $m_{e^+e^-} > 140 \text{ MeV}$ is always applied to suppress the dominant background $K \rightarrow \pi \pi_D^0$, where π_D^0 is the pion Dalitz decay $\pi^0 \rightarrow \gamma e^+e^-$. Therefore it is not applicable for light ϕ with $m_\phi < 140 \text{ MeV}$. Furthermore, the leptons e^+e^- are required to have a common vertex with the Kaon decay vertex. Thus, if ϕ is long-lived the constraint will vanish. Following [58], the vertex resolution of KTeV/E799 is taken to be 4 mm, which corresponds to 1.3×10^{-11} seconds. The probability of $\phi \rightarrow e^+e^-$ being prompt is

$$P_{\text{prompt}} = 1 - e^{-\frac{L_{\text{min}}}{\tau_\phi} \frac{m_\phi}{p_\phi}}, \quad (3.14)$$

where L_{min} is the vertex resolution, p_ϕ is the momentum of ϕ in the laboratory frame in the decay. For the KTeV/E799 experiment, the Kaon has an energy range of 20.3–216 GeV [42], which determines the energy of ϕ when assuming isotropic decay in the Kaon center of mass frame. Further, assuming the Kaon energy has a flat distribution, one can calculate the probability P_{prompt} . We set the KTeV/E799 constraint in the orange shaded region in figure 3.

KTeV ($\gamma\gamma$). The KTeV collaboration has measured the process $K_L \rightarrow \pi^0 \gamma\gamma$ [43], and determined the branching ratio to be $\text{BR}(K_L \rightarrow \pi^0 \gamma\gamma) = (1.29 \pm 0.03 \pm 0.05) \times 10^{-6}$. Since the four photons are measured by the CsI calorimeter, the information is not sufficient to

reconstruct the decay vertex as it needs to assume that the invariant mass of four photons equals the Kaon mass. Therefore, it is not sensitive to whether the $\phi \rightarrow \gamma\gamma$ decay is displaced or prompt in the signal process $K_L \rightarrow \pi^0\phi$. The efficiency difference between 3-body decay and 2-body decay $K_L \rightarrow \pi^0\phi$ with $\phi \rightarrow \gamma\gamma$ is not given in [43]. As a result, we can conservatively assume that the new physics contribution should be as small as $\text{BR}(K_L \rightarrow \pi^0\phi) \text{BR}(\phi \rightarrow \gamma\gamma) \lesssim 10^{-6}$, see [21]. The constraint for KTeV shown in the brown shaded region in figure 3 and 4.

Orsay. Orsay is an electron beam dump experiment which is sensitive to a light scalar decaying to electrons. The process is electron bremsstrahlung $eN \rightarrow eN\phi$ where ϕ subsequently decays to e^+e^- . In [44], a search for light Higgs bosons was performed under the assumption that the light Higgs couplings exclusively to electrons. This constraint, at the 90% C.L., can be directly applied to our model, where the only difference is a tiny $\text{BR}(\phi \rightarrow \gamma\gamma)$ which can be neglected for $m_\phi < 60 \text{ MeV}$. We fit to the 90% C.L. constraint set in [44] and project this limit to the 95% C.L. The results are shown in figure 3 and figure 4 in the green shaded region and the fitting procedure is described in detail in the appendix.

E137. Additional constraints are also relevant from other electron beam dump experiments at SLAC, e.g. E137 [45] and E141 [81]. In [82–84], these experiments were used to constrain the parameter space for dark photons. In particular, the E137 experimental setup provides an accurate upper bound on the dark-photon kinetic mixing. Therefore, we estimate the E137 limit by translating the bound on kinetic mixing to the scalar lifetime by equating the corresponding lifetime of the dark photon, $\tau_{A'} = \tau_\phi$. This limit is shown in figure 3 and figure 4 in light gray shaded region. Indeed, suppose we can split the signal event into $N \approx \sigma P(\tau)$, where σ is the production cross-section, and $P(\tau)$ is the probability to decay within the right volume. Assuming that $\tau_{A'} = \tau_\phi$, the probabilities for decay will be equivalent $P_{A'} = P_\phi$. This would imply that $\epsilon\epsilon > \epsilon_\ell m_e/v$ since

$$\Gamma(A' \rightarrow e^+e^-) \approx \frac{(\epsilon\epsilon)^2}{12\pi} m_{A'}. \tag{3.15}$$

Comparing to eq. (2.7) this gives $\sigma_{A'} > \sigma_\phi$. Though, the difference in cross sections occurs even in the case when the couplings are equivalent due to the collinear enhancement of the vector boson production rate. Thus, the number of events which decay within the detector in the case of scalar particles is strictly smaller than that for dark photons. Hence, while translating the bounds on the dark photon mass for a given lifetime to the scalar case, we are imposing a conservative bound.

We have also compared our results to the limit presented in [12], where the limit is obtained by relating the dark photon kinetic mixing and Higgs-like couplings, $\epsilon\epsilon = \epsilon_\ell m_e/v$. We find that our estimate is slightly weaker than the one obtained by this method. However, we argue that relating the dark photon and scalar lifetimes provides a more accurate bound. Indeed, assuming that $\epsilon\epsilon = \epsilon_\ell m_e/v$ one obtains $\tau_{A'} > \tau_\phi$, or $P_{A'} > P_\phi$. In addition, the larger cross section associated to the dark photon production lead to a further enhancement of their event rate, implying that their bound is too restrictive when applied to the scalar case.

3.2.3 Astrophysical constraints

Astrophysical constraints can set relevant bounds on models with long-lived scalars. For instance, supernova can lose energy by emitting the light scalar outside the neutrino sphere $R_\nu = 40$ km [85]. If the scalar decays inside or is absorbed inside R_ν , the supernova neutrino flux is not affected. The supernova can not stand the instantaneous luminosity of exotic particle emission, which exceeds the neutrino luminosity when the core reaches its peak density $\rho_c \sim 3 \times 10^{14}$ g/cm³ and temperature $T_c \sim 30$ MeV. Otherwise, the duration of the neutrino burst will be shortened by half and the energy spectrum will be incorrect [86]. The typical core radius R_c is about 10 km. Since in our interested region, the scalar ϕ has mass around $\mathcal{O}(30)$ MeV and lifetime around 10^{-11} – 10^{-10} seconds, the decay length is about 0.3–3 cm, and hence much smaller than the size of the core. Naively, such decay length is too small to result in a sizable energy leak due to ϕ emission.

Before discarding this constraint, one should remember that the progenitor star is an electron rich environment, and hence it is necessary to consider the Pauli blocking effect which suppresses ϕ decaying to electron pairs. Inside the core, the chemical potential for the electron is $\mu_e \simeq 100$ MeV [87]. The Pauli blocking factor is

$$\left(e^{\frac{\mu_e - E_e}{T}} + 1 \right)^{-1}, \tag{3.16}$$

where we can conservatively replace the electron energy E_e as $m_\phi/2$. Due to the high T_c in the core, the Pauli blocking factor is about 0.05 which is too small to make ϕ decay outside of the core. Outside of the core region, the Pauli blocking factor can also be calculated. Adopting fiducial model parameters for the progenitor star [85, 86], the density and temperature can be modeled as $\rho(r) = \rho_c(r/R_c)^{-5}$ and $T(r) = T_c(r/R_c)^{-5/3}$. Applying a uniform proton fraction of 0.3, one can obtain the number density of electron n_e , which equals the number density of proton n_p . The temperature within the neutrino sphere R_ν is higher than 3 MeV [85], thus the electrons are relativistic. In the limit of $m_e = 0$, one can have a relation $n_e(T, \mu_e) = (3\pi^2)^{-1} \mu_e (\mu_e^2 + \pi^2 T^2)$ [88]. Solving for the electron chemical potential, we find that the Pauli blocking factor is about 3×10^{-4} for $R_c < r < R_\nu$. Taking account the Pauli blocking, the typical decay length for ϕ becomes 10–100 meters, which is still much smaller than the size of the neutrino sphere R_ν . Therefore, unlike the case of the SSM model in [23], in our scenario the supernova constraint is not relevant due to the short lifetime of ϕ .

Besides the supernova, the light scalar ϕ can also affect the big bang nucleosynthesis (BBN) if it decays to SM particles fairly late. In our scenario, ϕ couples to leptons with similar a Yukawa coupling as the Higgs. Thus, its lifetime is as small as ~ 0.1 nanosecond, which is much smaller than lifetimes larger than $\mathcal{O}(1$ sec) that are constrained by these considerations. As a result, our model will not affect either BBN or the cosmic microwave background.

4 Conclusions

Although the SM accurately describes all experimental data, it is expected to be only an effective field theory. The presence of extra scalar degrees of freedom, beyond the standard

Higgs doublet, is a natural feature that is present in many extensions of the SM. Motivated by experimental data, we have explored the possible extension of the scalar sector by an additional Higgs doublet and a singlet. The singlet plays an essential role in our phenomenological analysis and it couples to quarks, leptons and gauge bosons via the mixing with the Higgs doublets, therefore avoiding tree level flavor changing neutral interactions.

Our goal has been to explain at the same time the observed anomalous magnetic moment of the muon and the excess of events observed at the KOTO experiment. We have shown that for this to happen the singlet should have a mass between about 40 and 70 MeV, and with a coupling to leptons and to quarks that are fixed by the $(g - 2)_\mu$ anomaly and the KOTO excess, respectively. While the couplings to leptons should be approximately of the same order as the Yukawa coupling to the SM Higgs boson, the coupling to quarks should be more than two orders smaller than the SM Yukawa coupling.

Under these conditions, the singlet lifetime is controlled by its decay into electrons, which is by far the dominant decay mode of this scalar. Actually, the singlet decays significantly more promptly than in the previously explored case in which it only mixes with the SM Higgs. The lifetime that is obtained is of the same order as the one needed to invalidate the constraints coming from the charged Kaon decay into charged pions and neutrinos, avoiding therefore the Grossman-Nir bound on the analogous neutral Kaon decays. This happens for a relatively large range of masses, which is a very attractive feature of our scenario.

The requirement of obtaining different values of the coupling to quarks and leptons with respect to the SM Yukawa couplings is obtained by assuming that the 2HDM is of type-X, in which one Higgs doublet couples only to quarks and the other doublet couples to leptons. We assume the system to be close to the alignment limit, implying the presence of a scalar with similar couplings to quark, leptons and gauge bosons as the SM Higgs. We also assume that $\tan\beta$ is large, of the order of 100, with the doublet that couples to leptons acquiring a small vacuum expectation value. This implies that the SM Higgs doublet will be mostly associated with the one that couples to quarks. This also implies that although the singlet has small mixings with the neutral CP-even components of both Higgs doublets, its coupling to leptons will be similar to the SM Yukawa due to the large $\tan\beta$ enhancement of the lepton coupling to the non-standard Higgs doublet.

There are two features that do not have a natural explanation in our model, but depend strongly on details of the model that are not associated with the phenomenological properties discussed in this article. One is the possible decays of the SM Higgs into a pair of singlets. In order to preserve the agreement with Higgs precision measurements, the branching ratio of this decay should be smaller than 10^{-2} . The second feature is associated with the stability of the singlet mass. In our model the singlet mixing angle to the non-standard neutral CP-even Higgs is of the order of 10^{-2} . This mixing may be smaller, but at the cost of increasing the coupling of the non-standard neutral Higgs bosons to tau leptons and inducing a Landau pole on this coupling at too low scales. Although small, this mixing induces corrections to the singlet mass that are much larger than its predicted value.

Beyond these theoretical issues, the model presented here leads to an explanation of both the observed value of $(g - 2)_\mu$ and of the KOTO excess, while avoiding the severe

proton beam dump experiments and astrophysical constraints. Due to the relatively large coupling of the new scalar to leptons, some of the strongest constraints on our model come from electron beam dump experiments. These experiments lead to somewhat weaker bounds for values of $(g - 2)_\mu$ below the current measured value, while values of $(g - 2)_\mu$ more than two standard deviations above the current measured value are firmly ruled out. Therefore, the expected measurement of the muon anomalous magnetic moment at the g-2 experiment at Fermilab will further test this model. Moreover, the KOTO experiment is expected to update its measurement within the next few years. Therefore, this model will be tested in a definitive way by the KOTO and g-2 experiments in the near future.

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A Numerical fit of Orsay result

In this appendix, we provide the details of our estimation for the 95% C.L. Orsay constraint [44]. The signal number for Orsay should have a form close to

$$N_{\text{sig}} \approx \frac{c_0}{\tau_\phi m_\phi^3} \left(e^{-a_1 \frac{L_{\text{sh}}}{c\tau_\phi \beta_\phi} \frac{m_\phi}{E_\phi}} - e^{-a_2 \frac{L_{\text{sh}} + L_{\text{dec}}}{c\tau_\phi \beta_\phi} \frac{m_\phi}{E_\phi}} \right), \tag{A.1}$$

where the term $(\tau_\phi m_\phi^3)^{-1}$ comes from the total signal production, and the term in parentheses is the decay probability for Orsay, and $L_{\text{sh}} = 1$ m, $L_{\text{dec}} = 2$ m [84]. The energy E_ϕ should be within 0.75 GeV and 1.6 GeV, where the upper bound is the electron beam energy and lower bound is the experimental cut [44]. We reserve a_1 and a_2 to be $\mathcal{O}(1)$ factors to compensate the electron beam energy attenuation, geometric setup of experiment and signal efficiency of the experiment. c_0 is an overall factor which fits to the 90% C.L. bound ($N_{\text{sig}} = 2.3$) given in figure 4 of [44]. In figure 5 we show the fitting result using eq. (A.1), and our projection to the 95% C.L. ($N_{\text{sig}} = 3$).

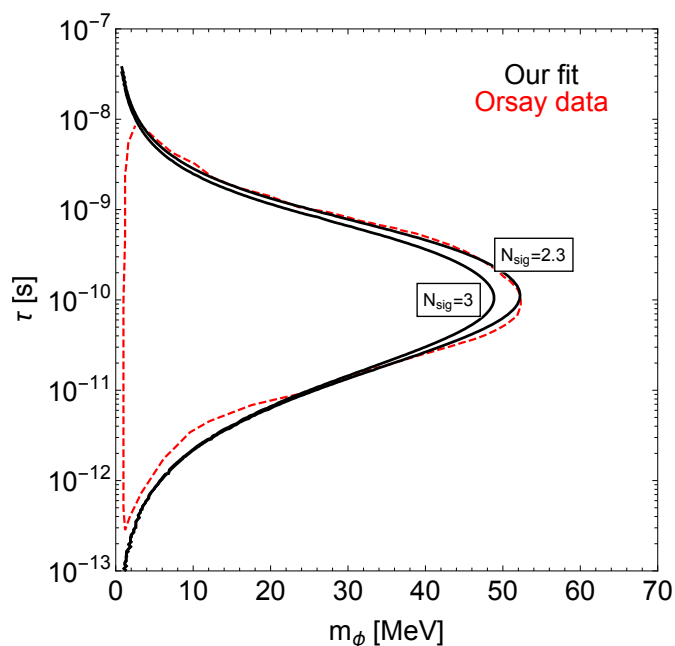


Figure 5. The numerical fit to Orsay constraint.

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