RECEIVED: July 11, 2019

REVISED: December 22, 2019

ACCEPTED: April 7, 2020

PUBLISHED: April 28, 2020

Magnetogenesis from a rotating scalar: à la scalar chiral magnetic effect

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ABSTRACT: The chiral magnetic effect (CME) is a phenomenon in which an electric current is induced parallel to an external magnetic field in the presence of chiral asymmetry in a fermionic system. In this paper, we show that the electric current induced by the dynamics of a pseudo-scalar field which anomalously couples to electromagnetic fields can be interpreted as closely analogous to the CME. In particular, the velocity of the pseudo-scalar field, which is the phase of a complex scalar, indicates that the system carries a global U(1) number asymmetry as the source of the induced current. We demonstrate that an initial kick to the phase-field velocity and an anomalous coupling between the phase-field and gauge fields are naturally provided, in a set-up such as the Affleck-Dine mechanism. The resulting asymmetry carried by the Affleck-Dine field can give rise to instability in the (electro)magnetic field. Cosmological consequences of this mechanism are also investigated.

KEYWORDS: Beyond Standard Model, Cosmology of Theories beyond the SM, Supersymmetric Standard Model

ARXIV EPRINT: [1905.06966](https://arxiv.org/abs/1905.06966)

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1 Introduction

In the chiral magnetic effect (CME) [1, 2], electric currents are induced by magnetic fields in the presence of chiral asymmetry. Since the CME originates from quantum anomalies [3, 4], which are ubiquitous in quantum systems regardless of their energy scales, it can play an important role in a variety of settings: relativistic heavy-ion collisions [2, 5–12], Weyl semimetals [13–19], astrophysical objects such as neutron stars [20–24], supernovae [25, 26], etc. Moreover, it has been argued that in the early Universe, when the chiral asymmetry was well preserved [27], the CME can cause a tachyonic instability in the (hyper)magnetic fields [28, 29]. This “chiral plasma instability” has recently been studied with full magnetohydrodynamic simulations [30–33],¹ which showed that a maximal transfer of chiral asymmetry to magnetic helicity is likely to occur. One implication is that these maximally helical (hyper)magnetic fields may be the source of the baryon asymmetry of the Universe [35–41].²

¹See also ref. [34] for the lattice study on the chiral plasma instability.

²This mechanism can be used to explain the proposed intergalactic magnetic fields from blazar observations [42–49]. However, since baryons may be overproduced in this case [40, 41], more careful analysis is required around the period of electroweak symmetry breaking.

In this scenario the chiral asymmetry, the origin of the CME, is usually carried by light fermions. On the other hand, even if there are no light fermions, such an effect can also be observed in the low-energy effective theory of axions [50–53], where it takes the form of an anomalous coupling between the axion and gauge bosons [54].³ In that case the background dynamics of the axion field also induces an electric current, similar to the CME. Accordingly, it has been argued that the chiral asymmetry can be interpreted as an axion-like scalar degree of freedom [56–58]. Indeed, the cosmological coherent dynamics of axion-like fields has been utilized to show how cosmological magnetic fields could be generated during inflation [59–62] and after inflation [63–66], or later times [67].

In this paper, we more directly investigate the analogy between the magnetic field amplification from the axion-like field and that from the CME. In particular, we find that the dynamics of the axion-like field gives rise to a non-vanishing chemical potential for the global $U(1)_{PQ}$ symmetry,⁴ similar to the chiral chemical potential. The difference between these two scenarios lies in the conservation of the chirality/global $U(1)_{PQ}$ charge. In the CME case, after chiral asymmetry is generated, chiral symmetry is approximately restored so that a maximal transfer of chiral asymmetry to magnetic helicity is possible. In contrast, in the axion case, the full dynamics is driven by a scalar potential which explicitly breaks $U(1)_{PQ}$ symmetry. Hence, the $U(1)_{PQ}$ charge is not conserved when the magnetic fields are amplified. From this perspective, the magnetic field amplification from axion-like fields reported in the literatures is not maximally efficient.

Notably, the anomalous coupling of axion-like fields is not limited to QCD or string-theoretical axions but is common to pseudo-scalar fields in general. One example in cosmology that takes advantage of the pseudo-scalar dynamics is the Affleck-Dine (AD) mechanism for baryogenesis [68, 69]. In this mechanism, a complex scalar field with baryonic (leptonic) charge acquires a large expectation value in the early Universe, and a non-vanishing velocity in the phase direction (i.e. the Nambu-Goldstone mode) is provided by the explicit baryon (lepton) number-violating interactions. Baryon asymmetry is also generated at the same time. In implementations of the AD mechanism, the baryon (lepton) number-violating interactions quickly become ineffective, such that a baryon (lepton) number is conserved after its generation. The interesting consequence is that the complex scalar exhibits a coherent rotation in the field space with a constant angular velocity.

In this paper, we point out that if a complex scalar field of the AD mechanism is charged under the $U(1)_{PQ}$, maximally helical magnetic fields can be easily obtained. Thanks to the anomalous coupling of the phase-field to the $U(1)$ gauge fields and a conserved current induced by a *constant* $U(1)_{PQ}$ asymmetry, only one helicity mode of magnetic fields is amplified. Thus, our mechanism is more efficient than previous realizations of magnetogenesis through the axion-like field dynamics and is similar to the chiral plasma instability.

This idea has several important implications for model building in early Universe cosmology. First of all, if axion-like fields such as Peccei-Quinn-Weinberg-Wilczek (PQWW) axions [50–53] experience cosmological evolution like the AD mechanism, it leads to a new

³See also the recent discussions in ref. [55].

⁴We use the notation $U(1)_{PQ}$ to denote a global $U(1)$ symmetry which is broken anomalously by gauge interactions, as first proposed by Peccei and Quinn.

mechanism of magnetogenesis. We also note that such magnetic field amplification can even occur in the usual AD mechanism, i.e., even in the minimal supersymmetric standard model (MSSM) and other supersymmetric extensions of the Standard Model of particle physics (SM). Indeed, we shall show that in some flat directions of the supersymmetric SM, the phase-fields of the complex AD fields have anomalous couplings to the unbroken U(1) gauge symmetry, and this new mechanism can be naturally realized. This may change the cosmological consequences of the AD mechanism, such as Q -ball formation [70–76]. One may wonder if our idea may spoil the AD mechanism as a mechanism for baryogenesis. Indeed, even if the rotating AD field generates both baryon (B) and lepton (L) asymmetries while maintaining $B - L = 0$, these asymmetries are efficiently transferred to the magnetic helicity, so they become smaller. However, as shown in ref. [40], the baryon asymmetry is regenerated through the transfer of the magnetic helicity during the electroweak phase transition. Thus, the AD mechanism can still be responsible for the baryon asymmetry, albeit indirectly, like the case discussed in ref. [41].

This paper is organized as follows. In section 2, we shall study the cosmological consequences of the complex scalar field with the anomalous coupling to the unbroken U(1) gauge symmetry. We identify its evolution, like the AD mechanism, and determine the resultant magnetic field properties generated by this new mechanism. Then, in section 3 we discuss how our mechanism may be realized and embedded within well-motivated extensions of the SM. Finally, in section 4, we provide some concluding remarks and future prospects for this mechanism.

2 Magnetogenesis from a rotating scalar in the field space

2.1 Axion-induced current as the scalar chiral magnetic effect

First, we study a toy model as a low energy effective theory and investigate its cosmological consequences. In the next section, we will discuss realizations of the scenario in realistic models of physics beyond the SM. Let us consider a simple model of a complex scalar field (à la AD field) with an approximate global U(1)_A symmetry and a massless U(1) gauge field,

$$\begin{aligned}
 -\frac{\mathcal{L}}{\sqrt{-g}} = & \partial_\mu \phi^* \partial^\mu \phi + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (m_0^2 - c_H H^2) |\phi|^2 + b_\phi (\phi^2 + \text{h.c.}) \\
 & + \frac{(a_\phi \phi^n + \text{h.c.})}{n M^{n-3}} + \frac{|\phi|^{2n-2}}{M^{2n-6}} + c_F \frac{e^2}{16\pi^2} \theta F_{\mu\nu} \tilde{F}^{\mu\nu},
 \end{aligned} \tag{2.1}$$

motivated by the AD mechanism [68, 69]. This is enough to catch the essence of our idea. The scalar field ϕ is neutral under the U(1) gauge interaction. Here we adopt the metric convention $g_{\mu\nu} = (-, +, +, +)$ and consider the Friedmann background $ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2$ with $H = \dot{a}/a$ being the Hubble parameter. We use the dot as the derivative with respect to the physical time t . m_0 is the zero-temperature mass, c_H is a numerical coefficient of the order of the unity that parameterizes the negative Hubble induced mass, b_ϕ and a_ϕ parameterize the small global U(1)_A symmetry breaking terms (b_ϕ and a_ϕ -terms, respectively), and M is the cutoff scale of the higher-dimensional operators. We assume

that the scalar field receives the negative Hubble induced mass squared during and after inflation and the value of c_H does not change significantly. b_ϕ is taken to be real while a_ϕ is taken to be complex without loss of generality. $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}/2\sqrt{-g}$ is the dual tensor with $\epsilon^{\mu\nu\rho\sigma}$ being the Levi-Civita symbol, $\epsilon^{0123} = 1$, $\theta = \theta(x)$ is the phase-field of the complex scalar $\phi(x)$, e is the gauge coupling constant, and c_F is the numerical coefficient of the order of unity for the anomalous coupling. As the phase-field θ is the (pseudo) Nambu Goldstone boson associated with spontaneous symmetry breaking of the $U(1)_A$, it can be also regarded as an axion-like field.

When the Hubble parameter is much larger than the zero-temperature mass, the net mass squared term is negative and the scalar field gets an expectation value as

$$\phi = \frac{\varphi(t)}{\sqrt{2}} e^{-i\theta(x)}. \quad (2.2)$$

Once the phase of the scalar field acquires a non-zero velocity, $\dot{\theta} \neq 0$, this yields the $U(1)_A$ asymmetry in the system,

$$n_A = i(\dot{\phi}^* \phi - \dot{\phi} \phi^*) = \varphi^2 \dot{\theta}. \quad (2.3)$$

The non-zero velocity of the angular field originates from the rotation of the complex scalar in the field space due to the $U(1)_A$ breaking a_ϕ -term, as in the case of the AD mechanism [68, 69]. Taking this configuration as the background, it can easily be seen that the equations of motion for the gauge field are given by

$$\partial_\mu(\sqrt{-g}F^{\mu\nu}) + \sqrt{-g}c_F \frac{e^2}{4\pi^2}(\partial_\mu\theta)\tilde{F}^{\mu\nu} = 0 \Rightarrow -\frac{1}{a^2} \frac{d}{dt}(a^2 E^i) + \frac{\epsilon^{ijk}}{a(t)} \frac{\partial B_k}{\partial x^j} - c_F \frac{e^2}{4\pi^2} \dot{\theta} B^i = 0. \quad (2.4)$$

Thus we determine that the current induced by the number density of the $U(1)_A$ asymmetry,

$$J_{\text{ind}}^i = c_F \frac{e^2}{4\pi^2} \dot{\theta} B^i = c_F \frac{e^2}{8\pi^2} \frac{n_A(t)}{a^3(t)\varphi^2(t)} B^i, \quad (2.5)$$

mimics the chiral magnetic effect [1, 2] with a correspondence

$$\mu_5 \leftrightarrow c_F \frac{n_A}{4a^3(t)\varphi^2}. \quad (2.6)$$

Here the physical electric and magnetic fields are defined as

$$E^i = a(t)F^{0i}, \quad E_i = a^{-1}(t)F_{i0}, \quad B_i = \frac{a^2(t)}{2}\epsilon_{ijk}F^{jk}, \quad B^i = \frac{a^{-2}(t)}{2}\epsilon^{ijk}F_{jk}. \quad (2.7)$$

This effective current is nothing but an axion-induced current in the axion electromagnetism. In the literature it has been argued that the chiral magnetic effect is understood to be an effective axion field [56–58]. Here we just emphasize that by relating the axion velocity $\dot{\theta}$ to the number density of the $U(1)_A$ asymmetry, the correspondence between the chiral magnetic effect and the axion-induced current is clearer. Note that the number density of the chiral asymmetry at a high temperature T is given in terms of the chiral chemical potential by $n_5 = \mu_5 T^2/6$.

2.2 Generation of $U(1)_A$ asymmetry and magnetogenesis in the early Universe

The axion-induced current causes tachyonic instability on the gauge fields, which is the essence of axionic inflationary magnetogenesis [59–61]. In that case, the non-zero axion velocity $\dot{\theta}$ is driven by a (time-independent) axion scalar potential, and the gauge field is mainly produced just after inflation, i.e. during several oscillations of the axion field [64–66]. Therefore, the corresponding current ($\propto \dot{\theta}$) is not a constant and even changes sign during the magnetic field amplification process. In this sense, the magnetic field amplification is less efficient, and the process is somehow different from the chiral plasma instability [28–32, 41]. In contrast, if the rotation in a complex scalar field space is induced by a $U(1)_A$ breaking term that is no longer effective after its onset, the dynamics of the phase-field becomes different from that of the axion mentioned above. The barrier of the scalar potential along the axion direction θ decreases over time and disappears so that the axion does not oscillate and $\dot{\theta}$ can be taken as a constant until the backreaction becomes important. In this case, the process is quite similar to the chiral plasma instability. In the following, we investigate the mechanism that generates the $U(1)_A$ asymmetry in a similar way to the AD mechanism [68, 69], and the resulting magnetogenesis.

Suppose the Universe undergoes inflation, followed by a matter-dominated era due to inflaton oscillations. Here we adopt the model with eq. (2.1), assuming $m_0 \sim |a_\phi| \gg \sqrt{b_\phi}$.⁵ When the Hubble parameter is large during inflation and the period of inflaton oscillation ($H > m_0/\sqrt{c_H}$), the ϕ field follows the (time-dependent) potential minimum generated by the balance between the negative quadratic and positive $|\phi|^{2n-6}$ term, $\varphi \simeq (HM^{n-3})^{1/(n-2)}$, with a spatially homogeneous distribution. Thanks to inflation, we naturally suppose that the phase-field θ is also spatially homogeneous. As the Hubble parameter decreases, eventually the potential minimum disappears at $H_{\text{osc}} \simeq m_0/\sqrt{c_H}$ and the ϕ field starts oscillation around the origin. At the onset of oscillation, the a_ϕ -term also gives a kick in the phase direction so that the non-zero number density of the $U(1)_A$ charge,

$$n_A \simeq \varphi_{\text{osc}}^2 \dot{\theta}, \quad \text{with} \quad \varphi \simeq \varphi_{\text{osc}} \equiv (m_0 M^{n-3})^{1/(n-2)}, \quad \dot{\theta} \simeq m_0, \quad (2.8)$$

is generated and the trajectory of the scalar field in the complex field space is an ellipse with a small eccentricity for $a_\phi \sim m_0$ [69]. Here the subscript “osc” indicates that the quantity is evaluated at the onset of the scalar field oscillation. The scalar field evolve as

$$\varphi \propto a^{-3/2}, \quad \dot{\theta} \simeq m_0 = \text{const.} \quad (2.9)$$

The former in eq. (2.9) comes from the fact that both the real and imaginary parts of the scalar field are harmonic oscillators in the matter-dominated Universe and damp in proportion to t^{-1} , and the latter is derived from the comoving number density conservation, $a^3 n_A = a^3 \dot{\theta} \varphi^2 = \text{const.}$ During the evolution, the $U(1)_A$ -breaking a_ϕ -term potential decays

⁵If a_ϕ is much larger than m_0 , unwanted symmetry-breaking vacua appear, hence we have to be more careful to ensure that ϕ is not trapped in the false vacua [77]. If a_ϕ is much smaller than m_0 , the trajectory of ϕ becomes highly elliptical with a large eccentricity so that $\dot{\theta}$ strongly oscillates. In this case, magnetic fields would be generated through $\theta F_{\mu\nu} \tilde{F}^{\mu\nu}$ coupling but quantitative estimates get more complicated, which is beyond the scope of this study.

Figure 1. Schematic picture of the evolution of ϕ field.

in proportion to $a^{-3n/2}$ whereas the quadratic term scales as a^{-3} . Hence, roughly in Hubble time after the onset of oscillation, the $U(1)_A$ breaking term becomes ineffective and n_A is nearly preserved as long as the b_ϕ -term is negligible. Figure 1 shows the schematic picture of the evolution of the ϕ field.

Before proceeding, let us comment about an issue omitted in the discussion above. Indeed, to show our idea simply and clearly, we do not take thermal effects into account [78, 79]. In principle, there should be thermal corrections to the scalar potential even before the completion of reheating, since the partial decay of inflaton quanta generates a high temperature plasma as a subdominant component of the Universe. The absence of such thermal corrections are valid if, e.g., the inflaton decays mainly into a hidden sector and the SM particles are not significantly produced. If thermal corrections to the scalar field potential exist, they induce an early onset of the scalar field oscillation, whose eccentricity is larger.

Concretely, this requirement is satisfied if the thermal correction is smaller than the bare mass term at the onset of ϕ oscillation. Typically the thermal potential is given as [78, 79]

$$V_{\text{th}}(\varphi) \simeq \begin{cases} \frac{1}{2} T_{\text{SM}}^2 \varphi^2 & \text{for } T > \varphi, \\ T_{\text{SM}}^4 \log\left(\frac{\varphi^2}{T^2}\right) & \text{for } T < \varphi, \end{cases} \quad (2.10)$$

where T_{SM} is the temperature of the Standard Model plasma. We did not explicitly write the coupling constants of order of unity that give the dominant contribution for the thermal potential (typically gauge couplings or the top Yukawa coupling). The upper one is the thermal mass which is active when the fields directly coupled to the ϕ field are light

enough to be in thermal equilibrium. If the masses of the coupled fields are heavier than the temperature, $M_f \simeq \varphi > T_{\text{SM}}$, as will be discussed in section 3.1, a two-loop contribution gives the lower one, the thermal logarithmic potential [79]. If we suppose that the inflaton decay rate is much smaller than the inflaton mass, the reheating process is well described by the perturbative decays of the inflaton field. The temperature of the hidden sector T_{hid} as well as that of the Standard Model sector before the completion of reheating are given by [80]

$$T_{\text{hid}} \simeq (M_{\text{pl}}^2 H \Gamma_{\text{hid}})^{1/4}, \quad T_{\text{SM}} \simeq (M_{\text{pl}}^2 H \Gamma_{\text{SM}})^{1/4}, \quad (2.11)$$

where M_{pl} is the reduced Planck mass and Γ_{hid} (Γ_{SM}) is the decay rate of the inflaton into the hidden sector (the Standard Model sector). Note that the former of eq. (2.11) is related to the reheating temperature of the hidden sector as $\Gamma_{\text{hid}} \simeq T_{\text{RH}}^2/M_{\text{pl}}$. Since T_{SM} decreases more slowly than the Hubble parameter, it is important to evaluate the thermal potential at around $H \simeq m_0$. For $m_0 \ll \varphi_{\text{osc}}$ which is the case of our interest, the thermal logarithmic potential must be smaller than the Hubble induced potential. Thus, $T_{\text{SM}} \ll \sqrt{\varphi_{\text{osc}} m_0}$ and

$$\Gamma_{\text{SM}} \ll 2 \times 10^{-18} \text{ GeV} \left(\frac{m_0}{10^3 \text{ GeV}} \right) \left(\frac{\varphi_{\text{osc}}}{10^8 \text{ GeV}} \right)^2. \quad (2.12)$$

In terms of the branching ratio, the constraint reads

$$\text{Br}_{\text{SM}} \ll 4 \times 10^{-16} \left(\frac{m_0}{10^3 \text{ GeV}} \right) \left(\frac{\varphi_{\text{osc}}}{10^8 \text{ GeV}} \right)^2 \left(\frac{T_{\text{RH}}}{10^8 \text{ GeV}} \right)^{-2}. \quad (2.13)$$

The small branching ratio might be achieved by tiny couplings ($g_{\text{SM}} \ll g_{\text{hidden}}$) or by kinematics ($M_{\text{hidden}} \ll M_{\text{inflaton}} \ll M_f \sim \varphi_{\text{osc}}$) during magnetogenesis. A detailed implementation of viable reheating models is an interesting problem, but beyond the scope of the paper, so we leave it as future work.

Now let us examine how the gauge fields are amplified due to the tachyonic instability and how they backreact to the scalar field dynamics. The equations of motion for the phase direction of the scalar field and gauge fields are given by

$$\partial_\mu (\sqrt{-g} \varphi^2(t) \partial^\mu \theta) - \sqrt{-g} c_F \frac{e^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} = 0, \quad (2.14)$$

$$\partial_\mu (\sqrt{-g} F^{\mu\nu}) + \sqrt{-g} c_F \frac{e^2}{4\pi^2} (\partial_\mu \theta) \tilde{F}^{\mu\nu} = 0. \quad (2.15)$$

The latter exhibits the instability of the gauge fields for the non-zero background $\dot{\theta}$. It can be explicitly seen as follows. As long as the phase-field evolves with a homogeneous constant velocity, $\partial_\mu \theta \simeq (\dot{\theta}, 0, 0, 0)$, with a negligible backreaction, we can take them as a background for the evolution of the gauge fields. Switching from the physical time to the conformal time so that $ds^2 = a^2(\tau)(-d\tau^2 + d\mathbf{x}^2)$, the equations of motion for the gauge fields read

$$-\frac{\partial^2}{\partial \tau^2} A_i + \sum_j \frac{\partial^2}{\partial x_j^2} A_i + c_F \frac{e^2 a(\tau)}{4\pi^2} \dot{\theta} \sum_{j,k} \epsilon_{ijk} \partial_j A_k = 0, \quad (2.16)$$

where we work in the radiation gauge $\nabla \cdot \mathbf{A} = 0$, $A_0 = 0$. To solve the equations of motion, it is convenient to work in the momentum space by performing a Fourier transformation,

$$A_i(\tau, \mathbf{x}) = \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^{3/2}} \left[A_\lambda(\tau, \mathbf{k}) \epsilon_{i,\lambda}(\mathbf{k}) e^{i\mathbf{k}\mathbf{x}} + \text{h.c.} \right], \quad (2.17)$$

with $\epsilon_{i,h}(\mathbf{k})$ being the circular polarization tensor that satisfies

$$\epsilon_{i,\lambda}(\mathbf{k}) \cdot \epsilon_{\lambda'}^{*i}(\mathbf{k}) = \delta_{\lambda,\lambda'}, \quad k^i \epsilon_{i,\lambda}(\mathbf{k}) = 0, \quad i\epsilon^{ijk} k_j \epsilon_{k,\lambda}(\mathbf{k}) = \lambda k \epsilon_{i,\lambda}(\mathbf{k}). \quad (2.18)$$

With these decomposition the equations of motion for the Fourier modes are rewritten as

$$-\frac{\partial^2}{\partial \tau^2} A_\lambda(\tau, \mathbf{k}) - k^2 A_\lambda(\tau, \mathbf{k}) + c_F \frac{e^2 a(\tau)}{4\pi^2} \dot{\theta} \lambda k A_\lambda(\tau, \mathbf{k}) = 0. \quad (2.19)$$

We see that the last term acts as a tachyonic mass term for $\dot{\theta} \lambda > 0$ ($\lambda = \pm 1$) and triggers the instability of the gauge fields. For the inflaton oscillation epoch with $a(t) \propto t^{2/3} \propto \tau^2$, the $\pm$ mode of the gauge field feels unstable for $\dot{\theta} \gtrless 0$ at $k_{\text{ins}}/a(\tau_{\text{ins}}) \simeq c_F e^2 \dot{\theta}/4\pi^2$ around $a(\tau_{\text{ins}})\tau_{\text{ins}} \simeq 4\pi^2/(c_F e^2 \dot{\theta})$, equivalently $H_{\text{ins}} \simeq c_F e^2 \dot{\theta}/8\pi^2 \simeq c_F e^2 m_0/8\pi^2$. As a result, for a given sign of $\dot{\theta}$ just one mode grows exponentially, so maximally helical gauge fields are obtained. Here the subscript “ins” indicates that the quantity is evaluated at the time when the instability starts to grow. Here we assume that there is no thermal plasma in the Standard Model sector, which includes relevant U(1) gauge charged particles, as has also been discussed in the ϕ field dynamics.

It also modifies the equations of motion of the gauge fields (2.19), by introducing a friction term $-\sigma_{\text{SM}} \partial A_\lambda / \partial \tau$ with $\sigma_{\text{SM}} \simeq 100T$ [81, 82] being the electric conductivity induced by the SM plasma. Light charged degrees of freedom would also induce electric currents like the Schwinger effect. Then the magnetic field amplification would become less efficient [62] and the light particles may be thermalized [83]. As a result, the process of gauge field amplification becomes more involved. In light of this, here we assume that there are no light charged degrees of freedom, which can be satisfied if all charged degrees of freedom acquire their masses from the AD field. Investigation of this effect is left for a future study. See also the discussion at the end of section 3.2.

The amplification of the gauge fields stops when the backreaction from the gauge field production becomes non-negligible. By taking a spatial average, eq. (2.14) can be understood as the conservation law for the sum of chiral asymmetry and magnetic helicity,

$$\frac{\partial}{\partial \tau} \left(a^3(\tau) \varphi^2 \dot{\theta} + c_F \frac{e^2}{8\pi^2} h \right) = 0, \quad h = \frac{1}{V} \int_V d^3x \epsilon^{ijk} A_i \partial_j A_k. \quad (2.20)$$

Therefore we can estimate that when

$$\begin{aligned} c_F \frac{e^2}{8\pi^2} h_{\text{sat}} &= \frac{c_F}{V} \frac{e^2}{8\pi^2} \int_V d^3x \epsilon^{ijk} A_i \partial_j A_k \simeq a^3(\tau_{\text{ins}}) \varphi^2(\tau_{\text{ins}}) \dot{\theta} \simeq a^3(\tau_{\text{osc}}) \varphi^2(\tau_{\text{osc}}) \dot{\theta} \\ \Leftrightarrow h_{\text{sat}} &= \frac{8\pi^2}{c_F e^2} a^3(\tau_{\text{osc}}) \varphi_{\text{osc}}^2 \dot{\theta} \end{aligned} \quad (2.21)$$

the amplification of the gauge fields saturates. In other words, magnetic field amplification stops when the maximal transfer from chiral asymmetry to magnetic helicity is completed. Here the subscript “sat” indicates that the quantity is evaluated at the time when the gauge field amplification becomes saturated. Since the instability induces an exponential growth, we approximate $\tau_{\text{sat}} \simeq \tau_{\text{ins}}$. Focusing on the magnetic fields, by approximating

$$h_{\text{sat}} \simeq a^2(\tau_{\text{sat}})A_{\text{sat}}B_{\text{sat}} = a^3(\tau_{\text{ins}})\frac{B_{\text{sat}}^2}{k_{\text{ins}}/a(\tau_{\text{ins}})}, \quad \text{where} \quad B_{\text{sat}} = \frac{k_{\text{ins}}}{a^2(\tau_{\text{ins}})}A_{\text{sat}}, \quad (2.22)$$

the magnetic field strength B and coherence length λ_B are obtained

$$\begin{aligned} B_{\text{sat}} &\simeq \sqrt{\frac{8\pi^2}{c_F e^2}} \left(\frac{a(\tau_{\text{osc}})}{a(\tau_{\text{ins}})} \right)^{3/2} \varphi_{\text{osc}} \sqrt{\frac{k_{\text{ins}} \dot{\theta}}{a(\tau_{\text{ins}})}} \simeq \sqrt{2} \left(\frac{a(\tau_{\text{osc}})}{a(\tau_{\text{ins}})} \right)^{3/2} \varphi_{\text{osc}} \dot{\theta} \\ &\simeq \sqrt{2} \left(\frac{H_{\text{ins}}}{H_{\text{osc}}} \right) \varphi_{\text{osc}} \dot{\theta} \simeq 2 \times 10^{12} \text{ GeV}^2 \left(\frac{\varphi_{\text{osc}}}{10^{12} \text{ GeV}} \right) \left(\frac{\dot{\theta}}{10^3 \text{ GeV}} \right), \end{aligned} \quad (2.23)$$

$$\lambda_{B,\text{sat}} \simeq \lambda_B(\tau_{\text{ins}}) \simeq 2\pi \left(\frac{k_{\text{sat}}}{a(\tau_{\text{sat}})} \right)^{-1} \simeq 2\pi \left(c_F \frac{e^2}{4\pi^2} \dot{\theta} \right)^{-1} \simeq c_F^{-1} \left(\frac{3}{1 \text{ GeV}} \right) \left(\frac{\dot{\theta}}{10^3 \text{ GeV}} \right)^{-1} \quad (2.24)$$

at the time when the gauge field amplification gets saturated. Here we take $e \simeq 0.3$. It is noted that B_{sat} is independent of c_F while $\lambda_{B,\text{sat}}$ is inversely proportional to c_F . Note also that in the absence of thermal plasma, electric fields in amounts similar to the magnetic fields are produced at the same time.

2.3 Cosmological evolution of magnetic fields

So far we have not specified the relationship between the U(1) gauge symmetry discussed in the previous sections and the U(1) in the SM. Let us investigate the cosmological consequences when the gauge fields are those of the U(1) gauge symmetry in the SM. After the saturation of the gauge field amplification, the physical magnetic field (as well as the electric field) evolves adiabatically, $B \propto a^{-2}$ and $\lambda_B \propto a$, until the SM particles are thermalized and the magnetohydrodynamics becomes important for their evolution [38, 86]. Once the SM particles are thermalized, the electric fields are screened due to the thermal effect, while the magnetic fields retain their properties. The magnetic fields induce the fluid dynamics and the fluid develops a turbulence. Then both the magnetic fields and velocity fields start to co-evolve according to the magnetohydrodynamic equations and follow the inverse cascade process once the eddy turnover scale of the fluid catches up with the magnetic field coherence length, $\lambda_B \simeq v_A t \simeq B/\sqrt{\rho}H$, where v_A is the Alfvén velocity [84, 85]. The magnetic field further evolves until today according to the magnetohydrodynamics, which determines the linear relation between the magnetic field strength and coherence length today as [84]

$$\lambda_B(t_0) \sim 1\text{pc} \left(\frac{B(t_0)}{10^{-14} \text{ G}} \right), \quad (2.25)$$

where t_0 is the present physical time. On the other hand, thermal plasma induces a large electric conductivity, which ensures the comoving magnetic helicity is a good conserved

quantity. Since it is also conserved during adiabatic evolution, we have the relation

$$a(t_0)^3 \lambda_B(t_0) B(t_0)^2 \simeq a(\tau_{\text{ins}})^3 \lambda_B(\tau_{\text{ins}}) B(\tau_{\text{ins}})^2. \quad (2.26)$$

Then we have

$$\begin{aligned} \lambda_B(t_0) B(t_0)^2 &= \left(\frac{a_{\text{RH}}}{a(t_0)} \right)^3 \left(\frac{a_{\text{ins}}}{a_{\text{RH}}} \right)^3 \lambda_{\text{ins}} B_{\text{ins}}^2 \\ &= \frac{g_{*s}^0 T_0^3}{g_{*s}^{\text{RH}} T_{\text{RH}}^3} \left(\frac{H_{\text{RH}}}{H_{\text{ins}}} \right)^2 \left(\frac{12 \times 10^{24} \text{ GeV}^3}{c_F} \right) \left(\frac{\varphi_{\text{osc}}}{10^{12} \text{ GeV}} \right)^2 \left(\frac{\dot{\theta}}{10^3 \text{ GeV}} \right) \\ &= 9 \times 10^{-68} \left(\frac{T_{\text{RH}}}{10^8 \text{ GeV}} \right) \left(\frac{H_{\text{ins}}}{\text{GeV}} \right)^{-2} \left(\frac{12 \times 10^{24} \text{ GeV}^3}{c_F} \right) \left(\frac{\varphi_{\text{osc}}}{10^{12} \text{ GeV}} \right)^2 \left(\frac{\dot{\theta}}{10^3 \text{ GeV}} \right) \\ &= \left(\frac{10^{-35} \text{ pcG}^2}{c_F} \right) \left(\frac{T_{\text{RH}}}{10^8 \text{ GeV}} \right) \left(\frac{H_{\text{ins}}}{\text{GeV}} \right)^{-2} \left(\frac{\varphi_{\text{osc}}}{10^{12} \text{ GeV}} \right)^2 \left(\frac{\dot{\theta}}{10^3 \text{ GeV}} \right), \end{aligned} \quad (2.27)$$

where we have used $g_{*s} = 3.91$, $T_0 = 2.3 \times 10^{-13} \text{ GeV}$, $1 \text{ pc} = 1.56 \times 10^{32} \text{ GeV}^{-1}$, and $1 \text{ G} = 1.95 \times 10^{-20} \text{ GeV}^2$ (in natural Lorentz-Heaviside units) and assumed that at $H = H_{\text{RH}}$, the Universe is filled with relativistic particles with the effective temperature T_{RH} , and the energy density and entropy are given by $\rho = (\pi^2 g_{*s}^{\text{RH}}/30) T_{\text{RH}}^4$, $s = (2\pi^2 g_{*s}^{\text{RH}}/45) T_{\text{RH}}^3$. We have also assumed that the Universe is eventually filled with the SM radiation without additional entropy production. Combining it with eq. (2.25), and assuming $c_F \simeq 1$, we obtain the present magnetic field properties,

$$B(t_0) \simeq 10^{-16} \text{ G} \left(\frac{T_{\text{RH}}}{10^8 \text{ GeV}} \right)^{1/3} \left(\frac{H_{\text{ins}}}{\text{GeV}} \right)^{-2/3} \left(\frac{\dot{\theta}}{10^3 \text{ GeV}} \right)^{1/3} \left(\frac{\varphi_{\text{osc}}}{10^{12} \text{ GeV}} \right)^{2/3}, \quad (2.28)$$

$$\lambda_B(t_0) \simeq 10^{-2} \text{ pc} \left(\frac{T_{\text{RH}}}{10^8 \text{ GeV}} \right)^{1/3} \left(\frac{H_{\text{ins}}}{\text{GeV}} \right)^{-2/3} \left(\frac{\dot{\theta}}{10^3 \text{ GeV}} \right)^{1/3} \left(\frac{\varphi_{\text{osc}}}{10^{12} \text{ GeV}} \right)^{2/3}. \quad (2.29)$$

This suggests that the detection of intergalactic magnetic fields with maximal helicity can be a trace of this scenario.

Moreover, we note that the set of fiducial values is suitable for baryogenesis [40]. This is not surprising because if there is not a magnetic field amplification and the asymmetry is conserved, the asymmetry-to-entropy ratio is

$$\frac{n}{s} = \left(\frac{a_{\text{osc}}}{a_{\text{RH}}} \right)^3 \frac{\dot{\theta} \varphi_{\text{osc}}^2}{(2\pi^2 g_{*s}^{\text{RH}}/45) T_{\text{RH}}^3} \sim 10^{-9}, \quad (2.30)$$

for the fiducial values. In this scenario, if the generated magnetic fields are those of hyper-gauge interaction, the asymmetry produced by the scalar field dynamics is first transferred to hypermagnetic helicity. It is eventually transferred back to the baryon asymmetry at the electroweak phase transition, without large loss in the sum of magnetic helicity and $U(1)_A$ asymmetry, similar to the case studied in ref. [41]. Even if electroweak symmetry is broken down to the electromagnetism by the expectation values of the scalar field and the electromagnetic fields are produced in this scenario, they transform into the hypermagnetic fields once the scalar field decays. Then the same process follows for baryogenesis.

2.4 Comment on the b_ϕ -term

The effect of the b_ϕ -term has been ignored to avoid the time variation in the $U(1)_A$ asymmetry. However, from the phenomenological point of view, this term is unavoidable in some realizations. We discuss how small this term should be for successful magnetogenesis.

Let us examine the evolution of the scalar fields in more depth after the onset of oscillation. Taking into account the b_ϕ -term, the masses of the real and imaginary parts of the complex scalar field differ as

$$m_{\text{re/im}} = \sqrt{m_0^2 \pm 2b_\phi} \equiv m_0 \pm \Delta m. \quad (2.31)$$

When b_ϕ is hierarchically smaller than m_0^2 , $\Delta m \simeq b_\phi/m_0 \ll m_0$. The evolution of the scalar fields is given by

$$\begin{aligned} \phi_R(t) &\equiv \text{Re}(\phi(t)) \simeq \sqrt{m_0 M} \left(\frac{\cos(m_{\text{re}} t)}{m_0 t} \right), \\ \phi_I(t) &\equiv \text{Im}(\phi(t)) \simeq \sqrt{m_0 M} \left(\frac{\sin(m_{\text{im}} t)}{m_0 t} \right), \end{aligned} \quad (2.32)$$

which yields

$$\dot{\theta} = \frac{\phi_R(t)\dot{\phi}_I(t) - \dot{\phi}_R(t)\phi_I(t)}{\phi_R^2(t) + \phi_I^2(t)} \simeq \frac{m_0 \cos(2\Delta m t) - \Delta m \cos(2m_0 t)}{1 - \sin(2m_0 t) \sin(2\Delta m t)}. \quad (2.33)$$

$\dot{\theta}$ evolves with the combination of the oscillation with a longer period $\Delta t_L \simeq (\Delta m)^{-1}$ and the one with a shorter period $\Delta t_S \simeq (m_0)^{-1}$. This means that after the onset of oscillation, the trajectory of the scalar in the field space is approximately a circle, as long as $t \ll \Delta t_L$, so that we can take $\dot{\theta}$ as a constant. Let us adopt an ansatz that $\dot{\theta}$ is regarded as a constant if $0.9m_0 \lesssim \dot{\theta} \lesssim 1.1m_0$. This corresponds to $\sin(\Delta m t) \lesssim 0.1$. Thus we take

$$\Delta t_c = 0.1(\Delta m)^{-1} \quad (2.34)$$

as the criteria for the duration during which $\dot{\theta}$ can be regarded as a constant. For $t \gtrsim \Delta t_c$, eventually it becomes decoherent and $\dot{\theta}$ cannot be taken as a constant any longer.

Since the magnetic field amplification occurs within the time scale

$$\Delta t \sim H_{\text{ins}}^{-1} \simeq \left(c_F \frac{e^2}{16\pi^2} \dot{\theta} \right)^{-1} \simeq \left(c_F \frac{e^2}{16\pi^2} m_0 \right)^{-1}, \quad (2.35)$$

requiring that this is shorter than Δt_c , we obtain the constraint on Δm as

$$\Delta m < c_F \frac{e^2}{160\pi^2} m_0 \quad \text{or} \quad b_\phi < c_F \frac{e^2}{160\pi^2} m_0^2 \simeq 5 \times 10^{-5} c_F m_0^2. \quad (2.36)$$

This gives a constraint on the b_ϕ -term in the phenomenological model building during magnetogenesis.

3 Realization

In this section, we describe how the low energy effective Lagrangian Eq (2.1) is realized in the well-motivated models. The idea is completely analogous to the couplings between axions and gauge fields. Namely, for large values of $\varphi \gg m_0 \sim a_\phi = \mathcal{O}(0.1 - 1 \text{ TeV})$, U(1) charged fields get heavy, $M_f \sim y_f \varphi$, from the interactions like $y_f \phi \bar{\psi}_L \psi_R + \text{h.c.}$. After integrating out those heavy fermions, an anomalous coupling in the form $(e^2 \theta / 16\pi^2) F_{\mu\nu} \tilde{F}^{\mu\nu}$ is induced by the triangle diagrams. Note that the relevant light degrees of freedom are θ , and U(1) gauge field, A_μ . We will demonstrate several examples in well-motivated models of the physics beyond the SM as proofs of concept. This suggests that such an anomalous coupling and magnetogenesis are general features of the AD mechanism and other similar cosmological scenarios.

3.1 Two Higgs doublet model

The first (clear) example is that the phase-field θ is the angular direction of the Higgs field in the type-II two Higgs doublet model (2HDM). Since it is nothing but the PQWW axion or the CP-odd Higgs field, by mapping the global U(1)_A to the approximate Peccei-Quinn symmetry U(1)_{PQ} ($H_1 H_2 \rightarrow e^{-i\beta} H_1 H_2$), we obtain the unbroken gauge symmetry U(1)_{em} for the large Higgs expectation values. In this case, all U(1)_{em} charged SM fermions and vector bosons get heavy, and the anomalous coupling between the light CP-odd Higgs and the U(1)_{em} gauge field is generated at low energies. Therefore, we expect the effective Lagrangian in the form of eq. (2.1). Let us see in more depth how to realize our situation of interest in the type-II 2HDM, and especially, how to realize the coherent motion of the Higgs fields and the vanishingly small b -term as discussed in section 2.4.

3.1.1 Scalar potential

Let us first investigate how to construct the scalar potential that allows the Higgs fields to develop large expectation values during inflation. The SM gauge charges and PQ charges for the SM fields in the type-II 2HDM are given in table 1, which allow us to determine the Yukawa couplings as

$$-\frac{\mathcal{L}_{\text{Yuk}}}{\sqrt{-g}} = (y_d)_{ij} \bar{Q}_{Li} d_{Rj} H_1 + (y_u)_{ij} \bar{Q}_{Li} u_{Rj} H_2 + (y_e)_{ij} \bar{L}_{Li} e_{Rj} H_1 + \text{h.c.} \quad (3.1)$$

For the Lagrangian of the Higgs sector

$$-\frac{\mathcal{L}_{\text{Higgs}}}{\sqrt{-g}} = |D_\mu H_1|^2 + |D_\mu H_2|^2 + V(H_1, H_2), \quad (3.2)$$

the form of the scalar potential $V(H_1, H_2)$ is important to realize our setup. Note that the PQ symmetry is anomalous under the hypergauge interaction.

There are eight degrees of freedom of the Higgs fields in total, which are characterized in terms of the four complex scalars as

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^0 \\ H_2^- \end{pmatrix}. \quad (3.3)$$

Fields	Q_{Li}	u_{Ri}	d_{Ri}	L_{Li}	e_{Ri}	H_1	H_2
$SU(2)_L$	2	1	1	2	1	2	2
$U(1)_Y$	1/6	2/3	-1/3	-1/2	-1	1/2	-1/2
$U(1)_{PQ}$	1	-1	-1	1	-1	2	2

Table 1. The $SU(2)_L \times U(1)_Y$ and PQ charge assignment in the SM.

Indeed, we can construct scalar potentials with a flat direction using a complex scalar degree of freedom among the four, while the other six degrees of freedom are heavy enough along the flat direction. To realize such a feature, some key ideas can be borrowed from the supersymmetric (SUSY) extension of the Standard Model for illustration. Two Higgs doubles are naturally introduced, and there are three contributions to the Higgs potential, namely, from the D -term, F -term, and soft breaking terms. Assuming that the PQ symmetry of the Higgs sector is broken only by the following higher dimensional superpotential,

$$W_{\text{PQB}} = \frac{(H_1 H_2)^2}{M}, \quad (3.4)$$

the scalar potential of H_1 and H_2 is obtained as

$$V_{\text{AD}}(H_1, H_2) = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + \left(\frac{a_H}{M} (H_1 H_2)^2 + \text{h.c.} \right) + \frac{4(|H_1|^2 + |H_2|^2)|H_1 H_2|^2}{M^2} \\ + \frac{g^2 + g'^2}{8} (|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2} |H_1^+ H_2^{0*} + H_1^0 H_2^{-*}|^2. \quad (3.5)$$

Here $H_1 H_2 \equiv \epsilon_{ab} H_1^a H_2^b$ and M is the cutoff scale. The first three terms in the r.h.s. in eq. (3.5) are the soft SUSY breaking terms with $m_1 \sim m_2 \sim a_H = \mathcal{O}(0.1 - 1 \text{ TeV})$. The last term in the first line is the F -term contribution. The quartic potential in the second line is the D -term potential, which gives the approximate flat direction:

$$(H_1)_{D\text{-flat}} = \begin{pmatrix} 0 \\ \frac{1}{2}\varphi e^{-i\theta} \end{pmatrix}, \quad (H_2)_{D\text{-flat}} = \begin{pmatrix} \frac{1}{2}\varphi e^{-i\theta} \\ 0 \end{pmatrix}. \quad (3.6)$$

Note that once the Higgs fields develop the expectation values along the flat direction, the coupled charged Higgs get heavy and their expectation values vanish, and hence there is no F -term (and D -term) contribution from them. Focusing on the flat direction, parameterized by the fields φ and θ , we obtain the effective potential of the à la AD field (φ and θ) in the form of eq. (2.1) (without the Hubble induced mass).

Let us check whether the other six degrees of freedom become sufficiently heavy along the flat direction. Along this direction, taking $\varphi \gg m_0$, we can see the splitting of the mass spectrum into heavy modes with masses of $\mathcal{O}(\varphi)$, and light modes as follows. As $SU(2)_L \times U(1)_Y$ is spontaneously broken to $U(1)_{\text{em}}$, and denoting the fields along the flat direction as δH , three scalar degrees, $G^0 \equiv \text{Im}(\delta H_1^0 - \delta H_2^0)$ and $G^+ \equiv \delta H_1^+ - \delta H_2^{+*}$, $G^- \equiv G^{+*}$ are eaten by Z^0 and $W^\pm$ and become heavy with masses $g\varphi/2$ and $\sqrt{g^2 + g'^2}\varphi/2$, respectively. One of the CP-even Higgs degrees of freedom $H^0 \equiv \text{Re}(\delta H_1^0 - \delta H_2^0)$ and the

charged Higgs components $H^+ \equiv \delta H_1^+ + \delta H_2^{-*}$, $H^- \equiv H^{+*}$ are also heavy with masses $\sqrt{g^2 + g'^2}\varphi/2$ and $g\varphi/2$ at the leading order. The scalar fields φ and θ get masses only from soft terms and a higher dimensional operator, so they are much lighter than the above six scalar degrees of freedom. It is clear that the θ field is the CP-odd Higgs/the PQWW axion.

The negative Hubble induced mass terms for H_1 and H_2 can be added as

$$\Delta V_{\text{Hubble}} = -c_1 H^2 |H_1|^2 - c_2 H^2 |H_2|^2, \quad (3.7)$$

by supposing, e.g., the non-minimal couplings to gravity, $-\xi_1 R |H_1|^2 - \xi_2 R |H_2|^2$ with R being the Ricci scalar, or non-trivial Kähler potential between the inflaton and the Higgs doublets in the supersymmetric case [69, 87]. Note that the Ricci scalar is $R = \mathcal{O}(H^2)$ during inflation and a matter-dominated Universe.

In eq. (3.5), we did not address the b_H -term potential presented in eq. (2.1) (i.e. $\Delta V = b_H H_1 H_2 + \text{h.c.}$). A sizable b_H -term is disadvantageous for generating magnetic fields as discussed in section 2.4. However, if b_H is much smaller than $m_1^2 + m_2^2$ as required, the value of $\langle |H_2^0| \rangle$ in the present Universe is too small to be realistic, because $\langle |H_2^0| \rangle / \langle |H_1^0| \rangle \simeq |b_H| / (m_1 + m_2)^2$. This leads to non-perturbatively large Yukawa couplings to obtain the correct masses of down quarks and charged leptons, $m_{d/e} = y_{d/e} \langle |H_2^0| \rangle$. One way to avoid this problem and give more freedom to the b_H -term is to consider the case where the b_H -term in the present Universe is dominated by the vacuum expectation value of a scalar field as $b_H \sim \langle S^2 \rangle = \mathcal{O}(m_1^2 + m_2^2)$, by introducing a gauge singlet PQ charged complex scalar field, S , while the PQ breaking bare b_H -term is vanishingly small. Let us consider the following potential for the S field,

$$\Delta V_{b\text{-term}} = (m_S^2 + \kappa_1 |H_1|^2 + \kappa_2 |H_2|^2) |S|^2 + (\kappa H_1 H_2 S^2 + a_S^3 S + \text{h.c.}) + \frac{\lambda_S}{4} |S|^4. \quad (3.8)$$

Here $|m_S| \sim |a_S| = \mathcal{O}(m_1^2 + m_2^2)$, $\kappa, \kappa_1, \kappa_2$, and λ_S are parameters on the order of the unity, and a_S is the soft PQ breaking parameter, which allows the S field to develop the vacuum expectation value on the order of 0.1 – 1 TeV in the present Universe. When the Higgs field develops the expectation values along the flat direction, $H_1 \simeq H_2 \sim \varphi \gg |m_S|$, S becomes heavy with a mass of $\mathcal{O}(\varphi)$, and its vacuum value shifted by the a_S -term is quite suppressed as $\langle S \rangle \sim a_S^3 / \varphi^2 \ll m_0 = \sqrt{(m_1^2 + m_2^2)/2}$. The resulting $b_H = \kappa S^2 \sim a_S^6 / \varphi^4$ is much smaller than m_0^2 , and satisfies eq. (2.36). As the φ value decreases and becomes $\mathcal{O}(m_0)$, then $\langle S \rangle \sim m_0$, and $b_H \sim m_0^2$, so the PQWW axion becomes heavy with a mass of $\mathcal{O}(m_0)$, which is safe from various astrophysical/collider constraints.

We would like to emphasize that the scalar potential we suggest in this section is a proof of concept, in which a flat direction ($|H_1| = |H_2|$) exists and b_ϕ -term is dynamical, which is suitable for our magnetogenesis scenario. Clever ideas are welcome and desirable in order to provide a more natural set-up for our mechanism. See appendix A for a concrete example to realize the $H_1 H_2$ flat direction without a bare b_H -term in a supersymmetric extension of the SM ($H_1 \rightarrow i\sigma_2 H_d^*$, and $H_2 \rightarrow i\sigma_2 H_u^*$).

3.1.2 Effective action with light degrees of freedom

Let us now see how the anomalous coupling $\sim (e^2/16\pi^2)\theta F_{\mu\nu}\tilde{F}^{\mu\nu}$ is obtained in the low energy effective Lagrangian. Here we focus on the non-supersymmetric theory although we use the SUSY-inspired potential. When the Higgs fields obtain large field values along the flat direction $\varphi \gg m_0$, we can divide the fields, not only the Higgs field described in the above but also the matter and gauge fields, into heavy fields whose masses are proportional to φ , and light fields which are massless or obtain masses at most with the soft breaking scales. The former includes the quarks, charged leptons, weak gauge bosons, and heavy Higgs fields, as well as the singlet scalar S , if any, and the latter includes the gluons, (electromagnetic) photons, neutrinos, and the light Higgs field (the à la AD field). In the unitary gauge, the Lagrangian density for the light fields is

$$-\frac{\mathcal{L}_{\text{light}}}{\sqrt{-g}} = \frac{1}{2}\text{Tr}G_{\mu\nu}G^{\mu\nu} + i\bar{\nu}_L\sigma^\mu\partial_\mu\nu_L + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu\varphi)^2 + \frac{\varphi^2}{2}(\partial_\mu\theta)^2 \\ + \frac{1}{2}(m_0^2 - c_H H^2)\varphi^2 + \frac{|a_\phi|}{8M}\varphi^4\cos(4\theta - \theta_A) + \frac{\varphi^6}{8M^2}, \quad (3.9)$$

where $a_\phi = |a_\phi|e^{i\theta_A}$, $m_0^2 = (m_1^2 + m_2^2)/2$, $c_H = -(c_1 + c_2)/2$. m_0^2 can be naturally positive even if $m_1^2 m_2^2 < 0$ in order for electroweak symmetry breaking in the present Universe. The Lagrangian density for the heavy fields up to the quadratic order is given as

$$-\frac{\mathcal{L}_{\text{heavy}}}{\sqrt{-g}} = \frac{1}{2}(\partial_\mu H^0)^2 + \frac{1}{2}\left(\frac{g^2\varphi^2}{4}\right)(H^0)^2 + (\partial_\mu H^+)(\partial^\mu H^-) + \left(\frac{(g^2 + g'^2)\varphi^2}{4}\right)H^+H^- \\ + \frac{1}{2}(\partial_\mu S_R)^2 + \frac{1}{2}\left(m_S^2 + \frac{(\kappa_1 + \kappa_2 + \kappa)\varphi^2}{4}\right)S_R^2 + \sqrt{2}a_S^3 S_R \\ + \frac{1}{2}(\partial_\mu S_I)^2 + \frac{1}{2}\left(m_S^2 + \frac{(\kappa_1 + \kappa_2 - \kappa)\varphi^2}{4}\right)S_I^2 + \bar{\psi}_{ui}\left(i\gamma^\mu D_\mu + \frac{y_{ui}}{2}\varphi e^{i\gamma_5\theta}\right)\psi_{ui} \\ + \bar{\psi}_{di}\left(i\gamma^\mu D_\mu + \frac{y_{di}}{2}\varphi e^{i\gamma_5\theta}\right)\psi_{di} + \bar{\psi}_{ei}\left(i\gamma^\mu D_\mu + \frac{y_{ei}}{2}\varphi e^{i\gamma_5\theta}\right)\psi_{ei} \\ + \frac{1}{2}W_{\mu\nu}^+W^{\mu\nu-} + \frac{g^2\varphi^2}{4}W_\mu^+W^{\mu-} + \frac{1}{4}Z_{\mu\nu}^0Z^{\mu\nu 0} + \frac{(g^2 + g'^2)\varphi^2}{8}Z_\mu^0Z^{\mu 0}, \quad (3.10)$$

where Dirac fermions are constructed as

$$\psi_{ui} = \begin{pmatrix} u_{Li} \\ u_{Ri} \end{pmatrix}, \quad \psi_{di} = \begin{pmatrix} d_{Li} \\ d_{Ri} \end{pmatrix}, \quad \psi_{ei} = \begin{pmatrix} e_{Li} \\ e_{Ri} \end{pmatrix}, \quad (3.11)$$

by using the chiral representation for the Dirac matrices, $S = (S_R + iS_I)/\sqrt{2}$, and for simplicity κ and a_S are taken to be real. The unbroken gauge group is $\text{SU}(3)_C \times \text{U}(1)_{\text{em}}$, and the corresponding covariant derivative is given by

$$D_\mu = \partial_\mu - ig_s T_\psi^a G_\mu^a - ie q_\psi A_\mu, \quad (3.12)$$

where T_ψ^a is the generator of $\text{SU}(3)_C$, and q_ψ is the EM charge for a given fermion ψ . For quarks, $T_{u,d}^a = \lambda^a/2$, where λ_{ij}^a are Gell-Mann matrices, and for charged leptons, $T_e^a = 0$.

The EM charges (q_ψ) are $q_u = 2/3$, $q_d = -1/3$, and $q_e = -1$. We ignore the interaction between θ and S because it does not have any effect on our interest.

For low energy scales much less than φ , the effective action can be obtained by integrating out heavy fields. Since the expectation values of heavy fields vanish, basically they do not leave any traces except anomalous couplings and threshold corrections. While the latter can be absorbed by the redefinition of model parameters, the former should be added explicitly to the Lagrangian. This is derived by calculating one-loop triangle diagrams mediated by heavy fermions (ψ_{u_i} , ψ_{d_i} , ψ_{e_i}) so that

$$\begin{aligned} -\frac{\mathcal{L}_{\text{anom}}}{\sqrt{-g}} &= \frac{2N_f g_s^2}{16\pi^2} \theta \text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{N_f e^2}{16\pi^2} (3q_u^2 + 3q_d^2 + q_e^2) \theta F_{\mu\nu} \tilde{F}^{\mu\nu} \\ &= \frac{3g_s^2}{8\pi^2} \theta \text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{e^2}{2\pi^2} \theta F_{\mu\nu} \tilde{F}^{\mu\nu}. \end{aligned} \quad (3.13)$$

Here N_f is the number of heavy families, and we take $N_f = 3$ for the SM. The appearance of such anomalous terms can be understood by noting that the flat direction is charged under PQ symmetry, which is anomalous under the $\text{SU}(3)_C$ and $\text{U}(1)_{\text{em}}$, and all PQ charged fermions are heavy along the flat direction. Then, we arrive at the low energy effective Lagrangian density eq. (2.1), and conclude that magnetogenesis is successful in the type-II 2HDM. We do not worry about the current induced by the Schwinger effect because all $\text{U}(1)_{\text{em}}$ charged particles are massive.

3.2 LH_u flat direction in supersymmetric SM

In the previous section, we utilized some of the properties of the supersymmetric SM just to justify a part of the form of the scalar potential in the type-II 2HDM, but did not take into account any SUSY partners. In this section, we shall consider the supersymmetric extension of the SM more seriously, as is adopted in the AD mechanism. In the MSSM, or extended supersymmetric SMs, there are many scalar fields, namely, the SUSY partners of the SM fermions such as squarks and sleptons, which exhibit many flat directions [88], along which the scalar potential vanishes except for the SUSY-breaking effects and contributions from non-renormalizable operators. Scalar fields can develop expectation values along a flat direction to cause the AD mechanism.

As a proof of concept, let us focus on the LH_u flat direction, which has been often used for the AD leptogenesis [89]. In order to make the scalar dynamics simpler, we will consider a flat direction only governed by a slepton with a single flavor f , $\tilde{L}_{Lf}$, and H_u ,⁶ while H_d and other scalar fields do not develop non-zero field values. Hereafter we use the tilde for supersymmetric partners. Such a condition can be easily realized, e.g., in the next-to-minimal-supersymmetric Standard Model (NMSSM) with a superpotential,

$$W_{\text{NMSSM}} = y_u Q_L u_R^c H_u + y_d Q_L d_R^c H_d + y_e L_L e_R^c H_d + \lambda S H_u H_d + \frac{1}{2} m_S S^2 + \frac{1}{3} \kappa S^3. \quad (3.14)$$

⁶Note that multiple flavors of sleptons [90] as well as the H_d field [91] can co-exist with the $L_{Lf} H_u$ flat direction, which lead to possible multiple field dynamics in the AD mechanism. Indeed, in the MSSM, due to the μ -term and $B\mu$ -term, the expectation value of H_d field is induced along the $L_{Lf} H_u$ flat direction. See appendix B for the detail.

It can be easily seen that with the configuration

$$(H_u)_{D\text{-flat}} = \begin{pmatrix} 0 \\ \frac{1}{2}\varphi e^{-i\theta} \end{pmatrix}, \quad (\tilde{L}_{Lf})_{D\text{-flat}} = \begin{pmatrix} \frac{1}{2}\varphi e^{-i\theta} \\ 0 \end{pmatrix}, \quad (3.15)$$

the D -term potential as well as the F -term potential for the φ and θ fields vanish and their potential is lifted only from the SUSY breaking effects as well as the Hubble induced terms which give them “small” masses, on the order of the soft SUSY-breaking mass, $m_{\text{soft}} = \mathcal{O}(\text{TeV})$, and the Hubble parameter, respectively. The scalar fields H_d and S acquire “large” masses of $\mathcal{O}(\lambda\varphi)$ along the F -flat direction so that we can integrate them out for low energy effective theory.

Taking $H_d = S = 0$ while keeping the H_u and $\tilde{L}_{Lf}$ fields explicitly, the form of their scalar potential is the same as in eq. (3.5) by replacing (H_1, H_2) to $(H_u, \tilde{L}_{Lf})$. Since H_d and L_L have the same SM gauge charges, this clearly shows that the expectation value of φ breaks $\text{SU}(2)_L \times \text{U}(1)_Y$ symmetry down to the $\text{U}(1)_{\text{em}}$ so that the three scalar modes in the H_u and $\tilde{L}_{Lf}$ fields other than the φ and θ fields are absorbed by vector bosons. Similarly, one CP even and one complex field also become also heavy with the masses of $\mathcal{O}(g\varphi)$ from the D -term potentials. As a result their field values can be safely set to be zero, and, again, they can be integrated out. The low energy effective scalar potential along the D -flat direction, parameterized by the φ and θ fields, is the same as that of eq. (3.9).⁷

The difference compared to the non-supersymmetric type-II 2HDM studied in the previous section is the additional fermionic degrees of freedom: Higgsinos $(\tilde{H}_u, \tilde{H}_d)$ and gauginos $(\tilde{W}^a, \tilde{B})$, and a pattern of the fermion mass splitting. While all the charged fermions get massive in the 2HDM case, massless charged fermions remain along the $L_{Lf}H_u$ flat direction. The Yukawa interactions are given by

$$\left(y_{ui} H_u^0 u_{Li} u_{Ri}^c + g(H_u^0)^* \tilde{W}^- \tilde{H}_u^+ + y_{ef} \tilde{L}_{Lf}^0 \tilde{H}_d^- e_{Rf}^c + g(\tilde{L}_{Lf}^0)^* e_{Lf} \tilde{W}^+ \right) + \text{h.c.}, \quad (3.16)$$

for the charged fermions who get masses from the expectation values of H_u and $\tilde{L}_{Lf}$. In the unitary gauge, the corresponding Lagrangian density for the heavy fermions is written as

$$\begin{aligned} -\frac{\mathcal{L}_{\text{heavy}}}{\sqrt{-g}} = & \bar{\psi}_{ui} \left(i\gamma^\mu D_\mu + \frac{y_{ui}}{2} \varphi e^{i\gamma_5 \theta} \right) \psi_{ui} + \bar{\psi}_{Hu} \left(i\gamma^\mu D_\mu + \frac{g}{2} \varphi e^{-i\gamma_5 \theta} \right) \psi_{Hu} \\ & + \bar{\psi}_{Hd} \left(i\gamma^\mu D_\mu + \frac{y_{ef}}{2} \varphi e^{i\gamma_5 \theta} \right) \psi_{Hd} + \bar{\psi}_W \left(i\gamma^\mu D_\mu + \frac{g}{2} \varphi e^{-i\gamma_5 \theta} \right) \psi_W \end{aligned} \quad (3.17)$$

where the Dirac fermions ψ are defined as

$$\psi_{ui} = \begin{pmatrix} u_{Li} \\ u_{Ri}^{c\dagger} \end{pmatrix}, \quad \psi_{Hu} = \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_u^{+\dagger} \end{pmatrix}, \quad \psi_{Hd} = \begin{pmatrix} \tilde{H}_d^- \\ e_{Rf}^{c\dagger} \end{pmatrix}, \quad \psi_W = \begin{pmatrix} e_{Lf} \\ \tilde{W}^{+\dagger} \end{pmatrix}. \quad (3.18)$$

Note that ψ_{Hd} and ψ_W become heavy due to the non-zero $\langle \tilde{L}^f \rangle = \varphi/2$. They have the same electromagnetic charges ($q_e = -1$), but couple to the axion oppositely, so integrating

⁷The absence of other flavors of sleptons can be justified by supposing a positive Hubble induced mass for them.

Fields	Q_{Li}	u_{Ri}^c	d_{Ri}^c	L_{Li}	e_{Ri}^c	H_u	H_d	S
$SU(2)_L$	2	1	1	2	1	2	2	1
$U(1)_Y$	1/6	-2/3	1/3	-1/2	1	1/2	-1/2	0
$U(1)_{PQ}$	1	1	1	1	1	-2	-2	4

Table 2. The $SU(2)_L \times U(1)_Y$ and PQ charge assignment in the NMSSM.

them out does not yield low energy coupling between the axion and photons. This is consistent with the fact that lepton number is not anomalous under $U(1)_{em}$. There is no such cancellation between ψ_{ui} and ψ_{Hu} ($q_u = 2/3, q_{Hu} = q_e = -1$), providing the low energy couplings as

$$\begin{aligned}
 -\frac{\mathcal{L}_{anom}}{\sqrt{-g}} &= \frac{N_f g_s^2}{16\pi^2} \theta \text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{e^2}{16\pi^2} (N_f 3q_u^2 - q_e^2) \theta F_{\mu\nu} \tilde{F}^{\mu\nu} \\
 &= \frac{3g_s^2}{16\pi^2} \theta \text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu} + \frac{3e^2}{16\pi^2} \theta F_{\mu\nu} \tilde{F}^{\mu\nu}.
 \end{aligned} \tag{3.19}$$

Once more, we have used $N_f = 3$. Since $\langle H_d \rangle = 0$ during the evolution of φ , three d -quark pairs $\psi_{di=1,2,3} = (d_{Li} \ d_{Ri}^{c\dagger})$, and two charged lepton pairs $\psi_{ei \neq f} = (e_{Li} \ e_{Ri}^{c\dagger})$ are massless. Because in our field basis those light charged fermions only couple to H_d , not H_u and $\tilde{L}_f$, there is no coupling between the axion and massless fermions. This can be seen by assigning $U(1)_{A'}$ charges to the fermion fields and the axion θ as

$$Q_\psi \equiv -\frac{5}{4}B - \frac{7}{4}L - Y - q_{em} + \frac{1}{4}q_{PQ}, \tag{3.20}$$

so that the axion charge is one, $Q_\theta = 1$, but the electromagnetic charged massless fermions are neutral. The PQ charge assignments q_{PQ} are given in table 2. Since this $U(1)_{A'}$ contains the PQ charge, it is also anomalous under $SU(3)_C \times U(1)_{em}$.

By supposing higher dimensional operators

$$W_{NM} = \frac{(L_{Lf} H_u)^2}{M_f} + \dots, \tag{3.21}$$

and the negative Hubble induced quadratic terms for $\tilde{L}_f$ and H_u in the same way as the 2HDM case, the final low energy Lagrangian density for the light fields is given by

$$\begin{aligned}
 -\frac{\mathcal{L}_{eff}}{\sqrt{-g}} &= \frac{1}{2} \text{Tr} G_{\mu\nu} G^{\mu\nu} + i\bar{\nu}\sigma^\mu \partial_\mu \nu + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (\partial_\mu \varphi)^2 + \frac{\varphi^2}{2} (\partial_\mu \theta)^2 \\
 &+ \frac{1}{2} (m_0^2 - c_H H^2) \varphi^2 + \frac{|a_\phi|}{8M_f} \varphi^4 \cos(4\theta - \theta_A) + \frac{\varphi^6}{8M_f^2} \\
 &+ \sum_{i=1}^3 \bar{\psi}_{di} i\gamma^\mu D_\mu \psi_{ui} + \sum_{i=1}^2 \bar{\psi}_{ei} i\gamma^\mu D_\mu \psi_{ei} - \frac{\mathcal{L}_{anom}}{\sqrt{-g}}.
 \end{aligned} \tag{3.22}$$

We have imposed the a_ϕ -term while the b_ϕ -term is absent since lepton number breaking is prohibited at the renormalizable level. Thus we reach the effective Lagrangian in the form of eq. (2.1), but massless $U(1)_{em}$ charged particles also exist.

We can take a different field basis, by θ dependent chiral transformation of d -quarks and charged leptons. Then the axion photon couplings can be removed through the chiral anomaly. Instead, axion-current interactions are generated.⁸ Therefore this coupling is important for both the generation of gauge fields and the helicity of the fermions, which has also been discussed in the context of inflationary magnetogenesis in ref. [62]. Fermion production through the axion-current interaction has also been studied recently in refs. [95–98].

Because W_{NM} breaks lepton number and becomes the source of the neutrino masses as the Weinberg operator, there is the lower bound on M from the upper bound on the neutrino masses $\sum m_\nu < \mathcal{O}(0.1\text{eV})$. On the other hand, since we do not know the lower bound on the lightest neutrino mass, a very large value of M_f is allowed. For example, in order to have the fiducial value for magnetogenesis, $\varphi_{\text{osc}} \simeq 10^{12} \text{ GeV}$, studied in section 2 for $m_0 \simeq 10^4 \text{ GeV}$, $M_f \simeq 10^{20} \text{ GeV}$, we require a tiny neutrino mass $m_{\nu_f} \sim 10^{-7} \text{ eV}$.

Let us comment about the effects of massless charged particles on magnetogenesis. Through chiral anomaly, once helical magnetic fields are generated from the dynamics of the rotating scalar, fermions with chiral asymmetry will be also generated, by satisfying $\Delta h \simeq (e^2/16\pi^2)\Delta n_5$, with n_5 being the number density of the chiral asymmetry. Moreover, through the Schwinger effect, non-chiral particles can be also generated, which can lead to thermalization of the charged particles [83]. As is discussed in ref. [62], these effects will suppress the efficiency of magnetogenesis. Thus we might not have as much magnetic helicity as much as evaluated in section 2.

However, in the case of standard chiral plasma instability, the numerical MHD studies have shown that full transfer of the chiral asymmetry to the magnetic helicity is possible even in the fully thermalized system [30–33]. From these observations, we expect that even in our case the full transfer of the scalar asymmetry to the magnetic helicity can be accomplished in the existence of light particles as well as the thermal plasma. For a concrete conclusion, nevertheless further investigation is needed, which is left for a future study.

In this subsection we have focused on the LH_u flat direction as a concrete example for proof of concept, but we expect that similar effects can be seen in other flat directions in the supersymmetric SM, including the MSSM, because it is often the case that an unbroken $U(1)$ gauge symmetry remains along the flat directions. For example, in the case of udd flat direction, a linear combination of the hyper gauge field, and the third and eighth gluons is unbroken and its anomalous coupling to the angular direction of the complex flat direction is expected.

In this section, we show that the new mechanism of magnetogenesis studied in section 2 can be naturally realized in the PQWW axion dynamics as well as in the usual AD mechanism. As described in the introduction, our findings have two important messages. Namely, 1) by supposing a cosmic history like the AD mechanism, axions can generate magnetic fields efficiently. 2) In some cases, the AD mechanism also generates magnetic fields, which requires careful analysis of the scenario. Since we have only studied some of simplified situations to show the proof of concept of the idea, further studies are needed to give precise and quantitative consequences of this effect.

⁸The coupling between the phase of the AD field and currents has been used for the realization of spontaneous baryogenesis in refs. [92–94]. Our discussion suggests that magnetic fields are also produced in these setups.

4 Discussion

In this work, we studied the evolution of $U(1)$ gauge fields that have an anomalous coupling to the phase of a rotating complex scalar field, which is often realized in cosmology in the context of the AD mechanism. The existence of such an anomalous coupling is not surprising since the phase of the AD field can be identified as an axion. Compared to other existing scenarios of magnetogenesis from axion dynamics, in which the axion oscillates around the PQ breaking potential, our magnetogenesis is novel in the sense that the PQ breaking effects are important only at the onset of dynamics in the phase direction, and are absent during most of the evolution of the phase-field. As a result, only one helicity mode of the gauge fields is continuously subject to the tachyonic instability. This allows the full transfer of the asymmetry from the scalar field to magnetic helicity, making magnetogenesis more efficient. The mechanism studied in this work is analogous to the chiral plasma instability, in which the chiral magnetic effect induces an instability in the magnetic fields.

We have shown that our mechanism can be realized in well-motivated extensions of the SM. As a proof of concept, we have demonstrated that the PQWW axion in the type-II 2HDM, as well as the phase of the complex LH_u flat direction in the AD leptogenesis, can act as a phase-field of the rotating scalar in this new magnetogenesis scenario. We also note that magnetogenesis induced by anomalous couplings is a general phenomenon of the AD mechanism, which has not been recognized before.

In order to evaluate the consequences of magnetogenesis, we employed a relatively simplified setup. Namely, we have assumed a negligible thermal plasma in the scalar field dynamics and omitted the effects of possible light charged particles. The inclusion of the thermal effect would trigger an early onset of the scalar field rotation, making $\dot{\theta}$ vary during oscillations. The effect of light particles is the induction of an electric current, which corresponds to the Schwinger effect in a vacuum and an ohmic current in thermal plasma. It will screen the electric field and suppress the efficiency of the magnetogenesis. Estimating the induced current in the presence of the chiral anomaly is a considerably involved task which we leave for future work. A natural consideration is whether the anomalous coupling of the AD field can play an important role at later times. In particular, one may expect that the coupling can introduce a new channel for Q -ball decay, since this process breaks the global $U(1)$ symmetry that guarantees the stability of Q balls. However, while the size of a Q ball is inversely related to the velocity of the phase-field, the instability scale is larger than that by a factor of $1/\alpha$, where α is the fine structure constant. Therefore, at first glance, we expect that Q -ball decay triggered by anomalous coupling is not so efficient, but this effect still could be interesting to explore in more depth.

Acknowledgments

We are grateful to Jeff Kost for careful reading of the manuscript. KK thanks IBS-CTPU for kind hospitality, where this work was initiated. The work of KK was supported by JSPS KAKENHI, Grant-in-Aid for Scientific Research JP19K03842 and Grant-in-Aid for Scientific Research on Innovative Areas 19H04610. KK and CS were supported by IBS under the project code, IBS-R018-D1.

Fields	Q_{Li}	u_{Ri}^c	d_{Ri}^c	L_{Li}	e_{Ri}^c	H_u	H_d	S	S^c
$SU(2)_L$	2	1	1	2	1	2	2	1	1
$U(1)_Y$	1/6	-2/3	1/3	-1/2	1	1/2	-1/2	0	0
$U(1)_{PQ}$	1	1	1	1	1	-2	-2	2	-6

Table 3. The $SU(2)_L \times U(1)_Y$ and PQ charge assignment.

A $H_u H_d$ flat direction in supersymmetric SM

In the MSSM, the $H_u H_d$ scalar field configuration is not a good flat direction for magnetogenesis, because the constant $B\mu$ -term explicitly breaks the PQ symmetry at the renormalizable level. The corresponding quadratic scalar potential ($\Delta V \sim B\mu H_u H_d + \text{h.c.}$) strongly disturbs a long time rotation in the complex field space. In the NMSSM discussed in section 3.2, because of the quartic potential induced by the F -term ($\Delta V \sim \lambda^2 |H_u H_d|^2$), the $H_u H_d$ is not even a flat direction. In this appendix, we construct a viable supersymmetric extension of the SM in which the $H_u H_d$ field configuration exhibits a flat direction.

Let us introduce two gauge singlet chiral superfields S and S^c with the PQ charge assignment in table 3. Then the relevant superpotential can have the form

$$W = y_u Q_L u_R^c H_u + y_d Q_L d_R^c H_d + y_e L_L e_R^c H_d + \frac{S^2 H_u H_d}{M_1} + \frac{S^3 S^c}{M_2} + \frac{(H_u H_d)^2}{M_3}, \quad (\text{A.1})$$

where M_1 , M_2 , and M_3 are very large constants compared to the weak scales. The bare μ and $B\mu$ terms are forbidden at the renormalizable level. However, we allow higher dimensional operators which explicitly break the PQ symmetry such as $(H_u H_d)^2/M_3$. Including soft SUSY-breaking terms, the scalar potential of the neutral scalar fields is given by

$$\begin{aligned} V = & \left(m_S^2 + \frac{4|h_u^0 h_d^0|^2}{M_1^2} \right) |S|^2 + \frac{9|S|^6}{M_2^2} + \left(m_{S^c}^2 + \frac{|S|^4}{M_2^2} \right) |S^c|^2 + \left[\left(\frac{A_2}{M_2} S^3 \right) S^c + \text{h.c.} \right] \\ & + \left[\frac{A_1}{M_1} S^2 + \frac{6|S|^2 (SS^c)^*}{M_1 M_2} + \frac{2(|h_u^0|^2 + |h_d^0|^2)(S^2)^*}{M_1 M_3} \right] h_u^0 h_d^0 + \text{h.c.} \\ & + \left(m_{H_d}^2 + \frac{|S|^4}{M_1^2} \right) |h_d^0|^2 + \left(m_{H_u}^2 + \frac{|S|^4}{M_1^2} \right) |h_u^0|^2 + \frac{A_3}{M_3} (h_u^0 h_d^0)^2 \\ & + \frac{4(|h_u^0|^2 + |h_d^0|^2)|h_u^0 h_d^0|^2}{M_3^2} + \frac{g^2 + g'^2}{8} (|h_u^0|^2 - |h_d^0|^2)^2. \end{aligned} \quad (\text{A.2})$$

In the present Universe, large vacuum values of S and S^c are developed by the following scalar potential which is relevant for the dynamics of S and S^c , assuming that $m_S^2 < 0$, $m_{S^c}^2 > 0$.

$$\begin{aligned} V(S) = & -|m_S|^2 |S|^2 + \frac{9|S|^6}{M_2^2} + m_{S^c}^2 |S^c|^2 + \frac{2A_2 \langle |S|^3 \rangle \cos(3\theta_S + \theta_{S^c})}{M_2} |S^c| + \dots \\ \Rightarrow \langle |S| \rangle = & \mathcal{O}(\sqrt{|m_S| M_2}), \quad \langle |S^c| \rangle \sim \mathcal{O}\left(\frac{A_2 |m_S|}{m_{S^c}^2} \langle |S| \rangle \right). \end{aligned} \quad (\text{A.3})$$

Here, the nonzero expectation values of the S and S^c are induced by the negative mass squared term of S . The large value of M_2 ensures the large expectation values of these fields, which justify the omission of the Higgs fields in evaluating them. Then the expectation value of S dynamically induces μ -term and $B\mu$ -term as

$$\mu = \frac{\langle S^2 \rangle}{M_1} \sim |m_S| \frac{M_2}{M_1}, \quad B\mu = A_1 \mu \sim A_1 m_S \frac{M_2}{M_1}. \quad (\text{A.4})$$

We naturally assume $M_1 \sim M_2$, so that $\mu \sim |m_S| = \mathcal{O}(m_{\text{soft}})$, $B\mu \sim A_1 |m_S| = \mathcal{O}(m_{\text{soft}}^2)$ are realized in the present Universe.

On the other hand, for the large expectation values of the Higgs fields as $|h_u^0| = |h_d^0| = \varphi/2 \sim \sqrt{m_0 M_3} \gg m_{\text{soft}}$ with the assumption of $M_1^2 \sim M_2^2 \ll M_3^2$, the large positive mass squared term of S is induced by the scalar potential $\varphi^4 |S|^2 / M_1^2 \sim (M_3/M_1)^2 m_0^2 |S|^2 \gg |m_S^2| |S|^2$. Consequently, the S field is trapped at the origin during magnetogenesis. Since the $B\mu$ -term is dynamically absent in this period, we get a sizable amount of magnetic fields through a long time evolution of the $H_u H_d$ phase field.

B H_d field configuration along the LH_u flat direction in the MSSM

In this appendix, we study the configuration of H_d field along the LH_u flat direction in the MSSM, in which μ and $B\mu$ -terms are given by constants. We show that H_d field gets expectation value induced by the LH_u flat direction, which makes the anomalous coupling of the phase direction of the LH_u flat direction to the photon vanish.

We consider the following superpotential of the MSSM and lepton number violating higher dimensional operators,

$$W = y_u Q_L u_R^c H_u + y_d Q_L d_R^c H_d + y_e Q_L e_R^c H_d + \mu H_u H_d + \frac{(L_{Lf} H_u)^2}{M}. \quad (\text{B.1})$$

Keeping in mind that LH_u flat direction and $H_u H_d$ flat direction can coexist [88, 91], let us parameterize the relevant scalar degrees of freedom along the LH_u flat direction as [91]

$$\tilde{L}_{Lf} = \begin{pmatrix} \tilde{\nu} \\ 0 \end{pmatrix}, \quad H_u = \begin{pmatrix} 0 \\ \tilde{h}_u^0 \end{pmatrix}, \quad H_d = \begin{pmatrix} \tilde{h}_d^0 \\ 0 \end{pmatrix}. \quad (\text{B.2})$$

Then the scalar potential from D -terms, F -terms, soft breaking terms, and the Hubble induced mass term is

$$\begin{aligned} V = & (m_L^2 + c_L H^2) |\tilde{\nu}|^2 + (m_{H_u}^2 + \mu^2 + c_u H^2) |h_u^0|^2 + (m_{H_d}^2 + c_d H^2) |h_d^0|^2 \\ & + \left(\frac{A_\nu}{M} \tilde{\nu}^2 h_u^{02} + B\mu h_u^0 h_d^0 + \text{h.c.} \right) + \frac{4|h_u^0|^4 |\tilde{\nu}|^2}{M^2} + \left| \frac{2\tilde{\nu}^2 h_u^0}{M} + \mu h_d^0 \right|^2 \\ & + \frac{g^2 + g'^2}{8} (|h_u^0|^2 - |h_d^0|^2 - |\tilde{\nu}|^2)^2, \end{aligned} \quad (\text{B.3})$$

where $|c_u|, |c_d|, |c_L| = \mathcal{O}(1)$. We can easily see that the field configuration of the pure LH_u flat direction ($|h_u^0| = |\tilde{\nu}|$) with $|h_d^0| = 0$ is impossible since there is the tadpole potential

for h_d^0 , induced by SUSY breaking ($B\mu\langle h_u^0\rangle h_d^0$) and supersymmetric ($\mu\langle\tilde{\nu}^2 h_u^0\rangle^* h_d^0/M$) contributions. Therefore we have to estimate how $\langle h_d\rangle$ can be large along the LH_u direction.

For the large values of $|h_u^0|$ and $|\tilde{\nu}|$ compared to soft SUSY breaking masses, the D -term potential makes one scalar degree heavy, so we can integrate out the corresponding field through the equations of motion. Parameterizing the scalar field amplitudes as

$$|h_u^0| = \varphi_u, \quad |h_d^0| = \varphi_d, \quad |\tilde{\nu}| = \varphi_l. \quad (\text{B.4})$$

for $m^2, H^2 \ll \varphi_d^2 \ll \varphi_u^2 \sim \varphi_l^2 \ll M^2$, which is realized for the negative Hubble induced mass for L and H_u , φ_l is determined by the D -flat condition,

$$\varphi_l^2 \simeq \varphi_u^2 - \varphi_d^2. \quad (\text{B.5})$$

By imposing this D -flat condition, the potential for φ_u and φ_d as well as the gauge invariant phase fields, θ_H and θ_L , defined as

$$h_u^0 h_d^0 = \varphi_u \varphi_d e^{-i\theta_H}, \quad \tilde{\nu} h_u^0 = \varphi_u \varphi_l e^{-i\theta_L}, \quad (\text{B.6})$$

is given by

$$\begin{aligned} V_{\text{eff}}(\varphi_u, \varphi_d) = & \left[m_{H_d}^2 + \mu^2 - m_L^2 + (c_d - c_L)H^2 \right] \varphi_d^2 \\ & + \left[2B\mu\varphi_d \cos\theta_H - \frac{4\mu\varphi_d^3 \cos(\theta_H - 2\theta_L)}{M} \right] \varphi_u \\ & + \left[m_L^2 + m_{H_u}^2 + \mu^2 + (c_L + c_u)H^2 - \frac{2A_\nu\varphi_d^2 \cos(2\theta_L)}{M} + \frac{4\varphi_d^4}{M^2} \right] \varphi_u^2 \\ & + \left[\frac{4\mu\varphi_d \cos(\theta_H - 2\theta_L)}{M} \right] \varphi_u^3 + \left[\frac{2A_\nu \cos 2\theta_L}{M} - \frac{12\varphi_d^2}{M^2} \right] \varphi_u^4 + \frac{8\varphi_u^6}{M^2}. \end{aligned} \quad (\text{B.7})$$

Here we have assumed that all constant parameters are real for simplicity.

For $m \ll H$, with a reasonable assumption:

$$m_L^2 + m_{H_u}^2 + \mu^2 + (c_L + c_u)H^2 < 0, \quad (\text{B.8})$$

φ_u gets a finite vacuum value as

$$V(\varphi_u) \sim (c_L + c_u)H^2\varphi_u^2 + \frac{8}{M^2}\varphi_u^6 \Rightarrow \langle\varphi_u\rangle = c\sqrt{HM}, \quad (\text{B.9})$$

with $c = \mathcal{O}(1)$ whereas $\varphi_d \ll \varphi_u$. By inserting this to the potential, supposing $c_d - c_L - 12c^4 > 0$, the dominant contribution for the vacuum value of φ_d is given by

$$V(\varphi_d) \sim (c_d - c_L - 12c^4)H^2\varphi_d^2 + \frac{4\mu\langle\varphi_u^3\rangle}{M} \cos(\theta_H - 2\theta_L)\varphi_d \Rightarrow \langle\varphi_d\rangle = \mathcal{O}\left(\frac{\mu}{H}\langle\varphi_u\rangle\right). \quad (\text{B.10})$$

Note that the contribution from the μ -term is stronger than that from the $B\mu$ -term. The angular field, θ_H also get a mass squared of $\mathcal{O}(\mu H\langle\varphi_u\rangle/\langle\varphi_d\rangle) \sim H^2$, so that θ_H is also heavy and follows the slow-rolling θ_L as $\langle\theta_H\rangle = 2\theta_L + \pi$.

As H decreases and crosses the value of $\mathcal{O}(\mu)$, the field value of φ_u becomes around $\sqrt{\mu M}$. Then the contribution of $B\mu$ -term is no longer negligible for the potential of the φ_d field so that

$$V(\phi_d) \sim (m_{H_d}^2 + \mu^2 - m_L^2)^2 \varphi_d^2 - (B\mu\varphi_u(t) \cos \theta_H) \varphi_d \Rightarrow \langle \varphi_d \rangle \sim \varphi_u(t). \quad (\text{B.11})$$

Now the dynamics of θ_H is governed by the $B\mu$ -term, which gives a constant heavy mass of $\mathcal{O}(\sqrt{B\mu})$, so θ_H will exhibit the damped oscillation around π . Therefore, while the phase of LH_u rotates in the same way as the usual AD leptogenesis, the $H_u H_d$ rotation will be quickly damped away. Since all the massless electromagnetic charged fermions in the pure LH_u flat direction, such as d quarks, acquire heavy masses from the H_d field value, the anomalous coupling between the phase of LH_u flat direction and photons is cancelled in the low energy effective theory. Now we have found that the dynamical phase θ_L does not have the anomalous coupling to photons and another phase θ_H , which has the anomalous coupling, no longer shows the constant velocity, we conclude that in the MSSM with a bare $B\mu$ -term the magnetogenesis does not happen unless the $B\mu$ -term is sufficiently suppressed as discussed in section 2.4. Note that in ref. [91] the $B\mu$ -term is not taken into account. This is the reason why $\dot{\theta}_H$ becomes constant and is not damped after the onset of scalar field oscillations around the origin there.

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