RECEIVED: March 4, 2020

REVISED: April 5, 2020

ACCEPTED: April 7, 2020

PUBLISHED: April 27, 2020

A direct link between unflavored leptogenesis and low-energy CP violation via the one-loop quantum corrections

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ABSTRACT: In the type-I seesaw mechanism the Casas-Ibarra (CI) parametrization provides a convenient description of the Dirac neutrino mass matrix in terms of the light and heavy Majorana neutrino masses, the lepton flavor mixing matrix U and an unknown complex orthogonal matrix O . If O is assumed to be real, it will be impossible to generate *unflavored* thermal leptogenesis via the lepton-number-violating and CP-violating decays of the lightest heavy Majorana neutrino. We find that this observation can be invalidated after small but unavoidable quantum corrections to the CI parametrization are taken into account with the help of the one-loop renormalization-group equations (RGEs) between the seesaw and electroweak scales. We illustrate a novel and viable unflavored leptogenesis scenario of this kind based on the RGEs in the seesaw-extended standard model, and show its direct link to the CP-violating phases of U at low energies.

KEYWORDS: CP violation, Neutrino Physics, Renormalization Group

ARXIV EPRINT: [2003.00480](https://arxiv.org/abs/2003.00480)

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1 Introduction

The canonical (type-I) seesaw mechanism [1–6] is theoretically elegant in the sense that it attributes the tiny masses of three known neutrinos naturally to the huge masses of three unknown Majorana neutrinos at a sufficiently high energy scale. Moreover, the lepton-number-violating and CP-violating decays of such seesaw-motivated heavy neutrinos in the early Universe may provide a natural way to account for the observed baryon-antibaryon asymmetry in today’s Universe — a mechanism that is commonly referred to as baryogenesis via thermal leptogenesis [7]. The key points of such a “killing two birds with one stone” picture are briefly summarized as follows.

On the one hand, the standard model (SM) of electroweak interactions is extended by adding three right-handed neutrino fields $N_{\alpha R}$ (for $\alpha = e, \mu, \tau$) and allowing lepton number violation. In this case the gauge-invariant lepton mass terms can be written as

$$-\mathcal{L}_{\text{lepton}} = \bar{\ell}_L Y_l H E_R + \bar{\ell}_L Y_\nu \tilde{H} N_R + \frac{1}{2} \bar{N}_R^c M_R N_R + \text{h.c.} , \quad (1.1)$$

where the relevant field notations are self-explanatory, and M_R is a symmetric Majorana mass matrix. Integrating out the heavy degrees of freedom in eq. (1.1) [8], one is left with the dimension-five Weinberg operator $\mathcal{O}_\nu = \left(\kappa_{\alpha\beta}/2\right) \bar{\ell}_{\alpha L} \tilde{H} \tilde{H}^T \ell_{\beta L}^c$ with $\kappa = Y_\nu M_R^{-1} Y_\nu^T$ for three light neutrinos [9]. After spontaneous gauge symmetry breaking at the Fermi scale (i.e., $\Lambda_{\text{EW}} \sim 10^2 \text{ GeV}$), we obtain the mass terms of three charged leptons and three light Majorana neutrinos

$$-\mathcal{L}'_{\text{lepton}} = \bar{l}_L M_l E_R + \frac{1}{2} \bar{\nu}_L M_\nu \nu_L^c + \text{h.c.} , \quad (1.2)$$

where $l_L = (e, \mu, \tau)_L^T$ and $\nu_L = (\nu_e, \nu_\mu, \nu_\tau)_L^T$, the charged-lepton mass matrix M_l is expressed as $M_l = Y_l v$ with $v \equiv \langle H^0 \rangle \simeq 174 \text{ GeV}$ being the vacuum expectation value of the neutral Higgs field, and the effective Majorana neutrino mass matrix M_ν is given by the famous seesaw formula

$$M_\nu = -v^2 \kappa = -M_D M_R^{-1} M_D^T , \quad (1.3)$$

with $M_D = Y_\nu v$ being the Dirac neutrino mass matrix. The tiny masses m_i (i.e., the singular values of M_ν) of three light neutrinos ν_i can therefore be attributed to the large masses M_i (i.e., the singular values of M_R) of three heavy neutrinos N_i (for $i = 1, 2, 3$) as compared with v . In between the seesaw and Fermi scales, which are characterized respectively by the mass M_1 of the lightest heavy Majorana neutrino N_1 and the value of v , it is in general necessary to consider quantum corrections to M_l and M_ν with the help of the renormalization-group equations (RGEs) [10–15].

On the other hand, the lepton-number-violating decays $N_i \rightarrow \ell_\alpha + H$ and $N_i \rightarrow \bar{\ell}_\alpha + \bar{H}$ may happen via the Yukawa interactions described by eq. (1.1). Such processes are also CP-violating because of the interference between their tree and one-loop amplitudes [7, 16–18]. Considering the case in which $M_1 \ll M_2 < M_3$ holds and all the Yukawa interactions are blind to the lepton flavors (i.e., the temperature of the Universe satisfies $T = M_1 \gtrsim 10^{12}$ GeV [19–24]), one expects that mainly the flavor-independent (or *unflavored*) CP-violating asymmetry

$$\begin{aligned} \varepsilon_1 &\equiv \frac{\sum_\alpha [\Gamma(N_1 \rightarrow \ell_\alpha + H) - \Gamma(N_1 \rightarrow \bar{\ell}_\alpha + \bar{H})]}{\sum_\alpha [\Gamma(N_1 \rightarrow \ell_\alpha + H) + \Gamma(N_1 \rightarrow \bar{\ell}_\alpha + \bar{H})]} \\ &\simeq -\frac{3M_1}{16\pi v^2 (M_D^\dagger M_D)_{11}} \sum_i \left[\frac{\text{Im} \left((M_D^\dagger M_D)_{1i}^2 \right)}{M_i} \right] \end{aligned} \quad (1.4)$$

can survive and give rise to a net lepton-antilepton number asymmetry $Y_L \equiv (n_L - n_{\bar{L}})/s$ with s being the entropy density of the Universe. To subsequently convert Y_L to a net baryon-antibaryon number asymmetry $Y_B \equiv (n_B - n_{\bar{B}})/s$, such an unflavored leptogenesis mechanism should keep taking effect in the temperature range $10^2 \text{ GeV} \lesssim T \lesssim 10^{12} \text{ GeV}$ in which the non-perturbative $(B - L)$ -conserving sphaleron interactions may stay in thermal equilibrium and thus can be very efficient [25–27]. To be explicit, we have $Y_B = -(28/79)Y_L$ in the SM framework [28, 29]. It is then possible to account for the observed baryon-to-photon ratio [30]

$$\eta \equiv \frac{n_B}{n_\gamma} \simeq (6.12 \pm 0.03) \times 10^{-10}, \quad (1.5)$$

with the help of the relation $\eta = sY_B/n_\gamma \simeq 7.04Y_B$ [8]. A comprehensive review of the thermal leptogenesis mechanism with or without flavor effects can be found in refs. [31–33].

Note, however, that the unknown flavor structure of M_D is an obstacle to the calculation of ε_1 in eq. (1.4) [34]. Without invoking any specific seesaw model and without loss of generality, one may follow Casas and Ibarra (CI) to parametrize M_D in the flavor basis where both M_l and M_R are diagonal (i.e., $M_l = D_l \equiv \text{Diag}\{m_e, m_\mu, m_\tau\}$ and $M_R = D_N \equiv \text{Diag}\{M_1, M_2, M_3\}$) [35]:

$$M_D = iU\sqrt{D_\nu} O\sqrt{D_N}, \quad (1.6)$$

where U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix [36–38] used to diagonalize M_ν in the chosen basis (i.e., $U^\dagger M_\nu U^* = D_\nu \equiv \text{Diag}\{m_1, m_2, m_3\}$), and O is an arbitrary complex orthogonal matrix. This CI parametrization of M_D is certainly consistent with the seesaw formula in eq. (1.3), and it has been extensively applied to the studies of various seesaw models, leptogenesis scenarios and lepton-flavor-violating rare decays of charged leptons.

Combining eqs. (1.4) and (1.6), one can immediately draw a conclusion that the unflavored thermal leptogenesis has nothing to do with leptonic CP violation at low energies, simply because ε_1 is independent of U (see, e.g., refs. [39–41] and references therein).¹ Along this line of thought, an interesting way out is to invoke flavor effects by taking $M_1 \lesssim 10^{12}$ GeV and assume O to be real [42–47]. Then the *flavored* CP-violating asymmetry $\varepsilon_{1\alpha}$ (for $\alpha = e, \mu, \tau$) will depend on the CP-violating phases of U in a direct way, making it possible to connect the cosmological baryon-antibaryon asymmetry to CP violation at low energies via flavored thermal leptogenesis.

In this paper we point out a novel way to make a direct link between *unflavored* thermal leptogenesis and CP violation at low energies based on the CI parametrization of M_D and a choice of real O . The key point is to take into account small but important radiative corrections to M_ν , which are equivalent to a slight modification of the expression of M_D on the right-hand side of eq. (1.6), by means of the one-loop RGEs of Y_l and Y_ν between the seesaw and Fermi scales. In this case the low-energy PMNS matrix U cannot be fully cancelled out in the expression of ε_1 given by eq. (1.4), and thus we are left with an unflavored leptogenesis scenario in which the unique source of leptonic CP violation is just the CP-violating phases of U .

At this point it is worth mentioning that we are motivated to assume the orthogonal matrix O to be real for two reasons. On the one hand, we intend to highlight the novel RGE-induced effect on unflavored thermal leptogenesis, which would otherwise be overwhelmed by those contributions originating directly from the imaginary parts of O . On the other hand, switching off the imaginary parts of O makes M_D dependent only upon the CP-violating phases of U in the CI parametrization. This assumption may therefore allow us to directly connect unflavored leptogenesis with low-energy CP violation via the RGE-induced quantum corrections. But, of course, such a simple assumption remains purely phenomenological at this stage.²

The remaining parts of this paper are organized as follows. In section 2 we first show how the CI parametrization in eq. (1.6) is slightly modified by the one-loop RGE effect, and then figure out the explicit expression of the unflavored CP violating asymmetry ε_1 .

¹At this point one should keep in mind that the unitarity of U in the CI parametrization is consistent with the symmetry of M_ν in the leading-order seesaw formula. If a slight departure of U from exact unitarity is taken into account in the type-I seesaw mechanism, one should go beyond eqs. (1.3) and (1.6) to make a self-consistent analysis of thermal leptogenesis and its possible connection to low-energy neutrino masses, flavor mixing and CP violation [39–41].

²One may even wonder whether a viable unflavored thermal leptogenesis scenario can be achieved in this connection by simply assuming O to be the identity matrix. We find that the answer to this question is negative, but it can be affirmative for *resonant* leptogenesis with flavor effects [48].

Section 3 is devoted to illustrating that this RGE-assisted unflavored leptogenesis scenario can work well in accounting for the observed value of η . Finally, we make a brief summary and some concluding remarks in section 4.

2 Quantum corrections

In the framework of the SM with three light Majorana neutrinos, the one-loop RGE for the effective neutrino coupling matrix $\kappa = Y_\nu M_R^{-1} Y_\nu^T$ evolving between the seesaw scale ($\Lambda_{\text{SS}} \sim M_1$) and the Fermi scale ($\Lambda_{\text{EW}} \sim 10^2 \text{ GeV}$) is given by [10–12]³

$$16\pi^2 \frac{d\kappa}{dt} = \alpha_\kappa \kappa + C_\kappa \left[\left(Y_l Y_l^\dagger \right) \kappa + \kappa \left(Y_l Y_l^\dagger \right)^T \right], \quad (2.1)$$

in which $t \equiv \ln(\mu/\Lambda_{\text{EW}})$ with μ being an arbitrary renormalization scale between Λ_{EW} and Λ_{SS} , $C_\kappa = -3/2$ and $\alpha_\kappa \approx -3g_2^2 + 6y_t^2 + \lambda$ with g_2 , y_t and λ standing respectively for the $\text{SU}(2)_L$ gauge coupling, the top-quark Yukawa coupling and the Higgs self-coupling constant. Working in the chosen flavor basis with both Y_l and M_R being diagonal (i.e., $Y_l = \text{Diag}\{y_e, y_\mu, y_\tau\} = D_l/v$ and $M_R = D_N$), one may integrate eq. (2.1) from Λ_{EW} to Λ_{SS} and then arrive at

$$\kappa(\Lambda_{\text{SS}}) = I_0^2 [T_l \cdot \kappa(\Lambda_{\text{EW}}) \cdot T_l], \quad (2.2)$$

where $T_l = \text{Diag}\{I_e, I_\mu, I_\tau\}$, and I_0 and I_α (for $\alpha = e, \mu, \tau$) are defined as

$$\begin{aligned} I_0 &= \exp \left[\frac{1}{32\pi^2} \int_0^{\ln(\Lambda_{\text{SS}}/\Lambda_{\text{EW}})} \alpha_\kappa(t) dt \right], \\ I_\alpha &= \exp \left[\frac{C_\kappa}{16\pi^2} \int_0^{\ln(\Lambda_{\text{SS}}/\Lambda_{\text{EW}})} y_\alpha^2(t) dt \right]. \end{aligned} \quad (2.3)$$

Given the strong hierarchy $y_e^2 \ll y_\mu^2 \ll y_\tau^2 \ll 1$ in the SM [34], it is obvious that $T_l \simeq \text{Diag}\{1, 1, 1 + \Delta_\tau\}$ holds as an excellent approximation, where

$$\Delta_\tau = \frac{C_\kappa}{16\pi^2} \int_0^{\ln(\Lambda_{\text{SS}}/\Lambda_{\text{EW}})} y_\tau^2(t) dt \quad (2.4)$$

is the small τ -flavored correction. The sizes of I_0 and Δ_τ at the seesaw scale are illustrated in figure 1 with $\Lambda_{\text{SS}} \in [10^{12}, 10^{14}] \text{ GeV}$. Although the RGE-induced effect is negligible in most cases, we are going to show that it may play an important role in a specific unflavored thermal leptogenesis scenario if the seesaw scale is high enough.

A combination of eqs. (1.3) and (2.2) leads us to the RGE-corrected version of the CI parametrization at the seesaw scale:

$$M_D(\Lambda_{\text{SS}}) = i I_0 T_l U(\Lambda_{\text{EW}}) \sqrt{D_\nu(\Lambda_{\text{EW}})} O \sqrt{D_N(\Lambda_{\text{SS}})}, \quad (2.5)$$

³Here the coefficient of the Weinberg operator $\mathcal{O}_\nu$ is obtained by means of the tree-level matching condition [13]. The one-loop threshold corrections to $\mathcal{O}_\nu$ at the matching scale should be taken into account in a consistent way [49], but their effects are so small that it is absolutely safe to neglect such next-to-leading-order corrections in our work.

Figure 1. The values of I_0 and Δ_τ against the seesaw scale $\Lambda_{\text{SS}} \in [10^{12}, 10^{14}]$ GeV in the SM.

where U and D_ν are the PMNS neutrino mixing matrix and the diagonal neutrino mass matrix at low energies, respectively. Comparing eq. (2.5) with eq. (1.6), one can see that the flavor structure of M_D at Λ_{SS} is slightly modified by nonzero Δ_τ . As a consequence, the product $M_D^\dagger M_D$ becomes U -dependent. This new observation motivates us to reexamine whether unflavored leptogenesis has something to do with leptonic CP violation at low energies when O is taken to be real.

To be more explicit, let us calculate the elements of $M_D^\dagger M_D$ that appear in eq. (1.4) at Λ_{SS} by using the RGE-corrected CI parametrization in eq. (2.5). We obtain

$$\begin{aligned} \left(M_D^\dagger M_D\right)_{1i} &= \left(I_0^2 \sqrt{D_N} O^\dagger \sqrt{D_\nu} U^\dagger T_l^2 U \sqrt{D_\nu} O \sqrt{D_N}\right)_{1i} \\ &\simeq I_0^2 \sqrt{M_1 M_i} \left[\sum_j (m_j O_{j1}^* O_{ji}) + 2\Delta_\tau \sum_{j,k} \left(\sqrt{m_j m_k} O_{j1}^* O_{ki} U_{\tau j}^* U_{\tau k} \right) \right] + \mathcal{O}(\Delta_\tau^2), \end{aligned} \quad (2.6)$$

in which the Latin subscripts run over $(1, 2, 3)$, and the values of both the neutrino masses and the PMNS matrix elements are from low energies. To minimize the uncertainties associated with the source of CP violation, we assume the orthogonal matrix O to be real from now on. The unflavored CP-violating asymmetry ε_1 in eq. (1.4) turns out to be

$$\varepsilon_1 \simeq -\frac{3\Delta_\tau I_0^2 M_1}{4\pi v^2} \cdot \frac{\sum_{j>k} \sqrt{m_j m_k} (m_k - m_j) O_{j1} O_{k1} \text{Im}(U_{\tau j}^* U_{\tau k})}{\sum_i m_i O_{i1}^2} + \mathcal{O}(\Delta_\tau^2). \quad (2.7)$$

Some immediate comments on the salient features of this result are in order.

- $\varepsilon_1 \propto \Delta_\tau$ is naturally expected, as one can see from eq. (2.5) in the $T_l \simeq \text{Diag}\{1, 1, 1 + \Delta_\tau\}$ approximation. Namely, the third row of M_D is slightly corrected due to the existence of Δ_τ , so are the Yukawa coupling elements $(Y_\nu)_{\tau i}$ (for $i = 1, 2, 3$) at the

seesaw scale. As a result, each vertex involving the τ -flavored lepton doublet in the Feynman diagrams of $N_1 \rightarrow \ell_\alpha + H$ and $N_1 \rightarrow \bar{\ell}_\alpha + \bar{H}$ decays is slightly modified, making it possible to trigger the interference between their tree and one-loop amplitudes at the leading order of Δ_τ and result in the unflavored CP-violating asymmetry ε_1 as shown in eq. (2.7). That is why ε_1 will automatically vanish if the RGE-induced effect between Λ_{EW} and Λ_{SS} is switched off. In other words, the unflavored CP-violating asymmetry ε_1 is actually dependent on the τ -flavored quantum correction.

- Eq. (2.7) provides us with a direct link between unflavored leptogenesis at the seesaw scale and the CP-violating phases of U at low energies, since O has been assumed to be real. In fact, only the elements in the first column of O and those in the third row of U are involved in the expression of ε_1 . So one may simply use two rotation angles θ and ϕ to make the parametrization $(O_{11}, O_{21}, O_{31}) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$ with $\theta \in (0, \pi]$ and $\phi \in (0, 2\pi]$. On the other hand, only two of the three CP-violating phases of U (or two combinations of theirs) take effect in ε_1 given by eq. (2.7).⁴
- Given the phase convention for the PMNS matrix U [51],

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (2.8)$$

in which $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$ (for $ij = 12, 13, 23$) with θ_{ij} lying in the first quadrant, eq. (2.7) tells us that ε_1 only contains three terms (e.g., $jk = 31, 32, 21$) whose Majorana CP phases are ρ , σ and $\rho - \sigma$ respectively. It is easy to see that if the transformations $\rho \rightarrow \rho \pm \pi$ and $\sigma \rightarrow \sigma \pm \pi$ are made, either separately or simultaneously, ε_1 may keep unchanged if a proper transformation of θ or ϕ is accordingly made (e.g., a combination of the transformations $\rho \rightarrow \rho \pm \pi$, $\sigma \rightarrow \sigma \pm \pi$ and $\theta \rightarrow \pi - \theta$ keeps ε_1 invariant). Such properties are pretty useful for us to understand the numerical results for the parameter space of our scenario in section 3.

- One may wonder what will happen if the unknown orthogonal matrix O is taken to be the identity matrix. In this special case, we are left with a much simpler result

$$\varepsilon_1 \simeq -\frac{3\Delta_\tau^2 I_0^2 M_1}{4\pi v^2} \sum_i m_i \text{Im}(U_{\tau 1}^* U_{\tau i})^2 + \mathcal{O}(\Delta_\tau^3), \quad (2.9)$$

which is proportional to Δ_τ^2 and thus strongly suppressed in magnitude.

Needless to say, to make the unflavored leptogenesis scenario under consideration viable in interpreting the observed baryon-antibaryon asymmetry of the Universe, the value of M_1 must be big enough such that both the magnitudes of Δ_τ and ε_1 can be properly enhanced.

⁴This point will be more transparent if one adopts a particular Euler-like parametrization of U proposed by Fritzsch and one of us [50], in which the elements in the third row only involves two Majorana-type CP-violating phases.

3 Unflavored leptogenesis

Given $M_1 \ll M_2 < M_3$ and $T = M_1 \gtrsim 10^{12}$ GeV, the Yukawa interactions described by eq. (1.1) are blind to all the lepton flavors and thus mainly the unflavored CP-violating asymmetry ε_1 given in eq. (1.4) survives and contributes to a net baryon-antibaryon asymmetry via thermal leptogenesis. To be explicit, the final baryon-to-photon ratio η is related to ε_1 as follows [52, 53]:

$$\eta \simeq -9.6 \times 10^{-3} \varepsilon_1 \kappa_f, \quad (3.1)$$

where κ_f is the efficiency factor determined by solving of the Boltzmann equations of heavy Majorana neutrino and lepton number densities, and it measures the washout effects caused by the inverse decays and lepton-number-violating scattering processes. To figure out the value of κ_f , let us first of all define the out-of-equilibrium parameter of N_1 decays as $K_1 \equiv \Gamma_1/H(M_1) = \tilde{m}_1/m_*$, where $\Gamma_1 = \left(Y_\nu^\dagger Y_\nu\right)_{11} M_1/(8\pi)$ denotes the total decay width of N_1 , $H(M_1) = \sqrt{8\pi^3 g_*/90} M_1^2/M_{\text{pl}}$ is the Hubble expansion parameter at temperature $T = M_1$ with $g_* = 106.75$ being the total number of relativistic degrees of freedom in the SM and $M_{\text{pl}} = 1.22 \times 10^{19}$ GeV being the Planck mass, $\tilde{m}_1 = \left(M_D^\dagger M_D\right)_{11}/M_1$ represents the effective neutrino mass, and $m_* = 8\pi v^2 H(M_1)/M_1^2 \simeq 1.08 \times 10^{-3}$ eV stands for the equilibrium neutrino mass. With the help of eq. (2.6), we obtain

$$\tilde{m}_1 \simeq I_0^2 \left[\sum_i m_i O_{i1}^2 + 2\Delta_\tau \sum_{i,j} \sqrt{m_i m_j} O_{i1} O_{j1} \text{Re}(U_{\tau i}^* U_{\tau j}) \right]. \quad (3.2)$$

It is obvious that K_1 controls whether or not the decays of N_1 are in equilibrium. In the far out-of-equilibrium (or weak washout) regime (i.e., $K_1 \ll 1$), κ_f depends heavily on the initial abundance of heavy Majorana neutrinos, and the produced $(B - L)$ -asymmetry is not reduced by washout effects. In the strong washout regime (i.e., $K_1 \gg 1$), however, κ_f is almost independent of the initial conditions and hence the $(B - L)$ -asymmetry produced at high temperatures is efficiently washed out. Given the initial thermal abundance of heavy Majorana neutrinos, the approximate analytical relation between κ_f and K_1 can be expressed as [24, 31]

$$\kappa_f \simeq \frac{2}{K_1 z_B(K_1)} \left[1 - \exp\left(-\frac{1}{2} K_1 z_B(K_1)\right) \right], \quad (3.3)$$

with $z_B(K_1) \simeq 2 + 4K_1^{0.13} \exp(-2.5/K_1)$. A combination of eq. (2.7) and eqs. (3.1)–(3.3) will therefore allow us to estimate the value of η via unflavored leptogenesis and examine its dependence on the CP-violating phases of U at low energies.

Adopting the parametrization of the PMNS matrix U given in eq. (2.8), we find that there are totally twelve parameters involved in our unflavored leptogenesis scenario: the heavy neutrino mass M_1 which determines the values of I_0 and Δ_τ ; three light neutrino masses m_i (for $i = 1, 2, 3$); three lepton flavor mixing angles θ_{12} , θ_{13} and θ_{23} ; three CP-violating phases δ , ρ and σ ; and two free parameters θ and ϕ used to parametrize O_{i1}

(for $i = 1, 2, 3$). For simplicity, here we only input the best-fit values of θ_{12} , θ_{13} , θ_{23} , δ , $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ and $\Delta m_{31}^2 \equiv m_3^2 - m_1^2$ (or $\Delta m_{32}^2 \equiv m_3^2 - m_2^2$) extracted from a recent global analysis of current neutrino oscillation data [54, 55]:

$$\sin^2 \theta_{12} = \begin{cases} 0.310 \\ 0.310 \end{cases}, \quad \sin^2 \theta_{13} = \begin{cases} 0.02241 \\ 0.02261 \end{cases}, \quad \sin^2 \theta_{23} = \begin{cases} 0.558 \\ 0.563 \end{cases}, \quad \delta = \begin{cases} 222^\circ \\ 285^\circ \end{cases}, \quad (3.4)$$

and

$$\Delta m_{21}^2 = \begin{cases} 7.39 \times 10^{-5} \text{ eV}^2 \\ 7.39 \times 10^{-5} \text{ eV}^2 \end{cases}, \quad \begin{cases} \Delta m_{31}^2 = 2.523 \times 10^{-3} \text{ eV}^2 \\ \Delta m_{32}^2 = -2.509 \times 10^{-3} \text{ eV}^2 \end{cases}, \quad (3.5)$$

where both the normal neutrino mass ordering (NMO, upper values) and the inverted mass ordering (IMO, lower values) are taken into account. We also choose a set of typical values of θ and ϕ in our numerical calculations: $(\theta, \phi) = (84.9^\circ, 351.1^\circ)$ for the NMO case and $(\theta, \phi) = (174.9^\circ, 58.0^\circ)$ for the IMO case, which both allow K_1 to satisfy $K_1 \gtrsim 1$ if the best-fit values in eqs. (3.4) and (3.5) and the reasonable ranges of m_1 (or m_3), M_1 , ρ and σ are input. In the $K_1 \gtrsim 1$ regime any lepton-antilepton asymmetries generated by the lepton-number-violating and CP-violating decays of N_2 and N_3 can be efficiently washed out, and thus we are only left with the asymmetry produced by the decays of N_1 . The latter depends only on four unknown parameters: m_1 (or m_3), M_1 , ρ and σ . In the following we shall use the observed value of η given in eq. (1.5) to constrain the parameter space of ρ and σ by allowing m_1 (or m_3) and M_1 to vary in the ranges $[10^{-8}, 10^{-2}] \text{ eV}$ and $[10^{13}, 10^{14}] \text{ GeV}$, respectively; or to constrain the parameter space of m_1 (or m_3) and M_1 by allowing both ρ and σ to vary in the $(0, 2\pi]$ range. Our numerical results are plotted in figure 2 and figure 3 for the NMO and IMO cases, respectively. Some discussions are in order.

- *The NMO case.* As shown in the upper panels of figure 2, the parameter space of ρ and σ is mainly located in the $[0, \pi]$ range. With m_1 increasing or M_1 decreasing, the values of ρ and σ will approach $\pi/2$, indicating an upper boundary for m_1 and a lower boundary for M_1 . Such boundaries are dominated by $\rho = \sigma = \pi/2$, which are described by the black bands in the lower panels of figure 2. It is obvious that $m_1 \lesssim 10^{-3} \text{ eV}$ and $M_1 \gtrsim 3 \times 10^{13} \text{ GeV}$ hold, implying that a nearly degenerate neutrino mass spectrum is not compatible with this unflavored thermal leptogenesis scenario. In fact, a similar conclusion has been drawn in refs. [52, 56, 57]. We find that m_1 may approach zero if $M_1 \gtrsim 4.2 \times 10^{13} \text{ GeV}$ holds. From the lower right panel of figure 2, one can see that all the bands converge at $M_1 \simeq 4.2 \times 10^{13} \text{ GeV}$ when m_1 becomes smaller and smaller. This observation means that in the given parameter setting the value of η is essentially insensitive to the value of ρ , as clearly shown by the yellow band with $m_1 = 1 \times 10^{-6} \text{ eV}$ in the upper right panel of figure 2. The reason is simply that the term $\text{Im}(U_{\tau 3}^* U_{\tau 2})$ dominates the size of ε_1 in eq. (2.7) because of the smallness of m_1 . There is a similar behavior in the lower left panel of figure 2, in which all the bands nearly converge at $m_1 \simeq 1 \times 10^{-3} \text{ eV}$ when $M_1 > 8 \times 10^{13} \text{ GeV}$ holds, and it implies that the sensitivity of η to σ is quite weak in this parameter setting.

Figure 2. A viable unflavored leptogenesis scenario in the *normal* neutrino mass ordering case: the parameter space of ρ and σ (upper panels) with some given values of m_1 and M_1 ; and the parameter space of m_1 and M_1 (lower panels) with some given values of ρ and σ .

- *The IMO case.* The upper panels of figure 3 show that the parameter space of ρ and σ is mainly located in the $\rho \in [0, \pi]$ and $\sigma \in [\pi, 2\pi]$ ranges. As in the NMO case, ρ and σ approach $\pi/2$ and $3\pi/2$, respectively, when M_1 decreases. But as m_3 increases from a small value, (ρ, σ) first go far away from $(\pi/2, 3\pi/2)$ and then approach $(\pi/2, 3\pi/2)$ again, a behavior which is different from the NMO case. Such features mean that there exit a lower boundary for M_1 and both lower and upper boundaries for m_3 which are determined by $\rho = \pi/2$ and $\sigma = 3\pi/2$, as explicitly shown by the black bands in the lower panels of figure 3. The smallest value of M_1 needed to fit the observed value of η is slightly smaller than that in the NMO case, and it is around 2.8×10^{13} GeV as

Figure 3. A viable unflavored leptogenesis scenario in the *inverted* neutrino mass ordering case: the parameter space of ρ and σ (upper panels) with some given values of m_1 and M_1 ; and the parameter space of m_1 and M_1 (lower panels) with some given values of ρ and σ .

one can also see from the lower panels of figure 3. We find that m_3 is constrained to lie in the range 5×10^{-6} eV $\lesssim m_3 \lesssim 1.6 \times 10^{-3}$ eV when $M_1 \leq 10^{14}$ GeV is required. Both the cases of $m_3 \simeq 0$ and $m_1 \approx m_2 \approx m_3$ are incompatible with this unflavored thermal leptogenesis scenario for the given parameter setting.

Before ending this section, let us briefly mention an alternative possibility that the heavy Majorana neutrinos have the initial zero abundance. In this case the efficiency factor κ_f is different from that given by eq. (2.8), especially in the weak washout regime (i.e., $K_1 \ll 1$) [24, 31]. But our preliminary numerical analysis shows that if the same values of θ and ϕ are chosen to assure $K_1 \gtrsim 1$, then the size of κ_f will be only slightly

smaller than that in the situation of the initial thermal abundance. As a consequence, the allowed lower bound of M_1 becomes larger (i.e., $M_1 \gtrsim 10^{14}$ GeV) and the behaviors of the other three parameters (i.e., m_1 , ρ , σ) are quite similar to those in the initial thermal abundance case.

4 Summary

With the help of the RGE-corrected CI parametrization of the Dirac neutrino mass matrix M_D in the canonical seesaw mechanism, we have shown that it is possible to link *unflavored* thermal leptogenesis to CP violation at low energies in a direct way. The point is that the PMNS matrix U can no longer be fully cancelled out in the unflavored CP-violating asymmetry ε_1 , and thus it is the unique source of CP violation if the unknown orthogonal matrix O in the expression of M_D is assumed to be real. As a numerical exercise, we have taken special values for the elements O_{i1} (for $i = 1, 2, 3$) and adopted the best-fit values of six neutrino oscillation parameters to illustrate the dependence of the baryon-to-photon ratio η on both the neutrino masses (i.e., M_1 and m_1 or m_3) and the Majorana-type CP-violating phases of U (i.e., ρ and σ). It is found that such a RGE-assisted unflavored leptogenesis scenario can work well for $M_1 \gtrsim 10^{13}$ GeV in the framework of the seesaw-extended SM.

It is worth remarking that the huge gap between the seesaw scale Λ_{SS} and the Fermi scale Λ_{EW} makes it definitely meaningful to take into account radiative corrections to the original CI parametrization by using the one-loop RGEs, although such quantum effects are very small and even negligible in most cases. If one turns to the framework of the minimal supersymmetric standard model (MSSM) extended with the type-I seesaw mechanism, however, the RGE-induced corrections to M_D are expected to be much more appreciable. In this MSSM case one may similarly explore a direct connection between unflavored leptogenesis and CP violation at low energies based on the RGE-corrected CI parametrization with O being real [58].

Of course, we have only focused on the effect of unflavored leptogenesis induced by the lepton-number-violating and CP-violating decays of the lightest heavy Majorana neutrino with the condition 10^{13} GeV $\lesssim M_1 \ll M_2 < M_3$. As far as *flavored* thermal leptogenesis is concerned, we find that the similar RGE-induced effect (proportional to Δ_τ) will in general become a next-to-leading-order term in the expressions of $\varepsilon_{1\alpha}$ (for $\alpha = e, \mu, \tau$) and hence unimportant. But such a preliminary observation deserves a further and comprehensive study.

In short, a successful leptogenesis mechanism at the seesaw scale is generally unnecessary to have a direct link to lepton flavor mixing and CP violation at low energies [39, 59, 60], but it is always interesting to find a specific and transparent scenario to bridge the gap between such high-scale and low-scale physics. The present work has therefore given a new example of this kind.

Acknowledgments

The original idea of this work came into being in a white night for one of us (Z.Z.X.) during his participation in the workshop entitled “New Physics on the Low-energy Precision Frontier” at CERN, from 28 January to 7 February 2020. We are greatly indebted to Zhenhua Zhao for many useful discussions and a friendly “competition”, and to Shun Zhou for his enlightening comments on leptogenesis. Our current research activities are supported in part by the National Natural Science Foundation of China under grant No. 11775231 and grant No. 11835013.

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