

# F-theory vacua and $\alpha'$ -corrections

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**ABSTRACT:** In this work we analyze F-theory and Type IIB orientifold compactifications to study  $\alpha'$ -corrections to the four-dimensional,  $\mathcal{N} = 1$  effective actions. In particular, we obtain corrections to the Kählermoduli space metric and its complex structure for generic dimension originating from eight-derivative corrections to eleven-dimensional supergravity. We propose a completion of the  $G^2R^3$  and  $(\nabla G)^2R^2$ -sector in eleven-dimensions relevant in Calabi-Yau fourfold reductions. We suggest that the three-dimensional,  $\mathcal{N} = 2$  Kähler coordinates may be expressed as topological integrals depending on the first, second, and third Chern-forms of the divisors of the internal Calabi-Yau fourfold.

The divisor integral Ansatz for the Kähler potential and Kähler coordinates may be lifted to four-dimensional,  $\mathcal{N} = 1$  F-theory vacua. We identify a novel correction to the Kähler potential and coordinates at order  $\alpha'^2$ , which is leading compared to other known corrections in the literature. At weak string coupling the correction arises from the intersection of  $D7$ -branes and  $O7$ -planes with base divisors and the volume of self-intersection curves of divisors in the base. In the presence of the conjectured novel  $\alpha'$ -correction resulting from the divisor interpretation the no-scale structure may be broken. Furthermore, we propose a model independent scenario to achieve non-supersymmetric AdS vacua for Calabi-Yau orientifold backgrounds with negative Euler-characteristic.

**KEYWORDS:** F-Theory, M-Theory

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## 1 Introduction

Four-dimensional minimal super-gravity theories are of particular phenomenological interest. The effective actions are commonly derived by dimensionally reducing ten-dimensional supergravity actions arising in string theory with localized brane sources. The stringy imprint arises in the form of  $\alpha'$ -corrections<sup>1</sup> to the Kähler potential and coordinates of the leading two-derivative action or in form of high-derivative couplings in four dimensions.

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<sup>1</sup>Which is given by  $\alpha' = l_S^2$  with string length  $l_S$ . The canonical convention for the definition of  $\alpha'$  is w.r.t, the string tension  $T$  as  $T^{-1} = 2\pi\alpha'$ .

Such corrections have been shown to be crucial for moduli stabilization and thus to determine the vacuum of the effective theory [1, 2]. However, to compute  $\alpha'$ -corrections in a truly minimal supersymmetric i.e.  $\mathcal{N} = 1$  set-up has been a challenging endeavor. A promising approach is to utilize F-theory which is a formulation of Type IIB string theory with space-time filling seven-branes at varying string coupling [3]. It captures the string coupling dependence in the geometry of an elliptically fibered higher-dimensional manifold. The general effective actions of F-theory compactifications have been studied using the duality with M-theory [4, 5]. A wide range of phenomenologically promising geometric F-theory backgrounds are known to give rise to non-Abelian gauge groups [4, 6, 7].

The starting point of the M/F-theory duality is the long wave length limit of M-theory, i.e. eleven-dimensional supergravity. Higher-derivative or higher-order  $l_M$ -corrections can then be followed through the duality to give rise to  $\alpha'$ -corrections in the resulting four-dimensional  $\mathcal{N} = 1$  theory. In this work we first compactify eleven-dimensional supergravity including the next to leading order eight-derivative or  $l_M^6$ -couplings to three dimensions on a supersymmetry preserving 8-dimensional background. More precisely, we perform a classical Kaluza-Klein reduction of the purely gravitational M-theory  $R^4$ -terms [8–13] on elliptically fibered Calabi-Yau fourfolds. Furthermore, one needs to consider the  $G^2 R^3$  and  $(\nabla G)^2 R^2$ -sector, where  $G$  is the M-theory four-form field strength. One easily verifies that all those couplings carry eight derivatives. We then implement the F-theory limit by decompactifying the three-dimensional theory to four space-time dimensions and interpret the resulting  $\alpha'$ -corrections to the two-derivative effective theory. In particular, we study  $l_M^6$ -corrections to the three-dimensional Kähler potential and Kähler coordinates of the  $\mathcal{N} = 2$  theory, which then modify the four-dimensional Kähler potential and Kähler coordinates in the F-theory limit. We identify a new leading order  $\alpha'^2$ -correction to the Kähler potential and coordinates which may break the no-scale structure. It is then of interest to study its effects in moduli stabilization scenarios.

We start the discussion in section 2 by reviewing the  $G^2 R^3$  and  $(\nabla G)^2 R^2$ -sector. No super-symmetric completions of those sectors are known. In this work we propose a completion of the bosonic terms relevant for Calabi-Yau fourfold reductions. We start from a general basis and fix the coefficients via comparison to controlled theories upon dimensional reduction. In particular, we compactify on Calabi-Yau threefolds and verify compatibility with  $5d, \mathcal{N} = 2$  supergravity. Furthermore, upon reduction on  $S^1 \times K3$  we make use of the Heterotic/IIA-theory duality.

This then allows us to fix the parameters such that we can perform a controlled dimensional reduction on Calabi-Yau fourfolds with a generic number of Kähler deformations in section 3. Also in this section we review our previous results for the one-modulus case for which the integration in a three-dimensional Kähler potential and coordinates can be performed exactly [14].

In section 4 we suggest a proposal for the three-dimensional Kähler potential and coordinates for a generic number of Kähler moduli of the Calabi-Yau fourfold background. The key new approach in contrast to our previous attempts [15, 16] is the formulation of the higher-derivative contributions as divisor integrals, analogous to the discussion of the warp-factor in [15]. We argue in 4.1 that the new formulation can indeed give rise to

all relevant higher-derivative couplings in the reduction result obtained in 3.2. However, to match the reduction result is beyond the aim of this work and we suggest that non-trivial identities relating the higher-derivative objects are needed to perform this tasks. Let us stress that obtaining the correct building blocks from a Kähler potential and Kähler coordinates is a big leap forward as this steps meets heavy obstacles as pointed out in [15]. We then proceed in 4.2 by showing that the divisor integral Kähler coordinates can be re-expressed as topological integrals. This is very intriguing as it will allow for a F-theory interpretation. Lastly, in section 4.3 we show compatibility with the one-modulus case [14].

In section 5 we discuss the F-theory uplift of the three-dimensional  $l_M^6$ -corrected Kähler potential and coordinates to four dimensions. The classical uplift of the topological integrals is well understood and can be performed rigorously. It is expected that the F-theory lift receives loop-corrections which result from integrating out Kaluza-Klein states on the  $4d/3d$  circle at one-loop. As we encounter a  $l_M^6$ -correction to the Kähler coordinates with [14] logarithmic dependence on the Calabi-Yau fourfold volume reminiscent of such a loop correction we comment on a one-loop modification of the F-theory uplift. However, to present a complete analysis of the F-theory uplift at one-loop is beyond the scope of this work. Due to this the resulting  $\alpha'^2$ -corrected four-dimensional Kähler potential and coordinates carry free parameters we are not able to fix. The three-dimensional Kähler coordinates generically lead to a breaking of the no-scale structure which may remain present in four-dimensions. This breaking of the no-scale structure is also consistent with the one-modulus case [14]. However, we conclude that a better understanding of the F-theory uplift at one-loop is required before deciding on the ultimate fate of the  $\alpha'^2$ -correction to the four-dimensional scalar potential.

To give an independent interpretation of the novel  $\alpha'^2$ -correction we take the Type IIB weak string coupling limit [17]. The correction is proportional to the volume of the intersection curve of  $D7$ -branes and the  $O7$ -plane with divisors in the Kähler base of the elliptically fibered Calabi-Yau fourfold. Moreover, it depends on the volume of the self-intersection curves of those divisors in the base. We also identify a second correction which survives the F-theory limit. However it vanishes due to a conspiracy of pre-factors. The latter correction is proportional to the self-intersection of divisors in the base intersecting the  $D7$ -branes and the  $O7$ -plane. Both corrections are expected to arise from tree-level string amplitudes of oriented open strings with the topology of a disk or non-orientable closed strings with the topology of a projective plane analogous to the  $\alpha'^2$ -correction encountered in [18, 19]. We also discuss the latter in this work.

In section 5.4 we discuss the implications of the  $\alpha'^2$ -corrections on moduli stabilization. We propose a scenario to achieve non-supersymmetric AdS vacua for geometric backgrounds with negative Euler-characteristic  $\chi(B_3) < 0$ , where  $B_3$  is the base of the elliptically fibered Calabi-Yau fourfold in F-theory. In the IIB picture thus for Calabi-Yau orientifold backgrounds with negative Euler-characteristic. The vacua are obtained due to an interplay of the Euler-Characteristic correction [20] and the  $\alpha'^2$ -corrections to the scalar potential.<sup>2</sup> We close by emphasizing that the discussion can be performed analogously for

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<sup>2</sup>The form of the scalar potential due to the  $\alpha'^2$ -correction obtained in [18, 19] is similar to the one obtained at order  $\alpha'^3$  in [2, 21].

Calabi-Yau fourfolds with  $\chi(B_3) > 0$  which leads to de Sitter extrema. We thus suggest that the scenarios may suffice to construct an explicit counter example to the recent conjecture by [22]. Let us emphasize that we do not study explicit geometric backgrounds in this work but derive constraints on the topological quantities such that vacua may be obtained.

## 2 Towards a completion of the $G^2 R^3$ and $(\nabla G)^2 R^2$ sectors

In section 2.1 we review the known eleven-dimensional supergravity action at eight-derivatives. In section 2.2 we consider the possibility of having additional  $G^2 R^3$  and  $(\nabla G)^2 R^2$ -terms in the eleven-dimensional action, where  $G$  denotes the M-theory four-form field strength and  $R$  is an abbreviation for the Riemann tensor. We propose a completion of these two sectors relevant for Calabi-Yau fourfold reductions. Due to these potential novel terms one encounters an additional parameter freedom in the reduction result in section 3. However, as we do not make use of this parameter freedom in the remaining work let us stress that this section stands independently. The reader more interested in the three and four-dimensional effective actions can thus safely skip the technical section 2.2 and carry on with section 3.

### 2.1 Higher-derivative corrections in M-theory

In this section we review the eleven-dimensional supergravity action including the relevant eight-derivative terms. Note that we comment on a completion of the  $G^2 R^3$  and  $(\nabla G)^2 R^2$ -sector relevant for a Calabi-Yau fourfold  $CY_4$  reductions in the next section 2.2. The bosonic part of the classical two-derivative  $\mathcal{N} = 1$  action in eleven dimensions is given by

$$2\kappa_{11}^2 S_{11} = \int_{M_{11}} R * 1 - \frac{1}{2} G \wedge * G - \frac{1}{6} C \wedge G \wedge G. \quad (2.1)$$

The purely gravitational sector is corrected at eight-derivatives by  $R^4$ -terms given by

$$2\kappa_{11}^2 S_{R^4} = \int_{M_{11}} \left( t_8 t_8 - \frac{1}{24} \epsilon_{11} \epsilon_{11} \right) \mathcal{R}^4 * 1 - 3^2 2^{13} C \wedge X_8. \quad (2.2)$$

First derived in [23, 24] these terms can be shown to be related to the R-symmetry and conformal anomaly of the world-volume theory of a stack of  $N$  M5-branes [13]. Secondly, the known contributions [25] to the  $G^2 R^3$  and  $(\nabla G)^2 R^2$ -sector of the four-form field strength are given by

$$2\kappa_{11}^2 S_G = \int_{M_{11}} - \left( t_8 t_8 + \frac{1}{96} \epsilon_{11} \epsilon_{11} \right) G^2 R^3 * 1 + s_{18} (\nabla G)^2 R^2 * 1 + 256 ZG \wedge * G. \quad (2.3)$$

The last term in (2.3) was argued to be necessary to ensure Type IIA/M-theory duality when considering Calabi-Yau threefold compactifications [21]. The precise definition of the higher-derivative terms in (2.2) and (2.3) can be found in the appendix in B.3. The detailed index structure of the terms  $(\nabla G)^2 R^2$  in (2.3) can be found in B.3.

## 2.2 Checks on the $G^2R^3$ and $(\nabla G)^2R^2$ -sector

No supersymmetric completions of the eleven-dimensional  $G^2\mathcal{R}^3$ -sector and  $(\nabla G)^2R^2$ -sector are known. The eleven-dimensional eight-derivative terms involving two powers of the four-form field strength are lifted from the corresponding terms in the Type IIA effective action. Those arise at the level of the five point-functions in the Type IIA superstring and partial indirect conclusions can be drawn at the level of the six-point function [25]. However, let us stress that a conclusive study at the level of the six-point function and especially at higher order n-point functions remains absent. In particular a supersymmetric completion of the  $G^2\mathcal{R}^3$ -sector and  $(\nabla G)^2R^2$ -sector employing the Noether coupling method would be of great interest. It is thus desirable to discuss possible extensions of the  $G^2\mathcal{R}^3$  and  $(\nabla G)^2R^2$ -sector beyond the known terms.<sup>3</sup> In this section we accomplish this task and provide a complete maximal extension of the eleven-dimensional  $G^2\mathcal{R}^3$  and  $(\nabla G)^2R^2$ -sector relevant for Calabi-Yau fourfold reductions.<sup>4</sup>

Instead of computing string amplitudes or employing the Noether coupling method we take a more pragmatic way here. In [14] a complete basis of eight-derivative terms of the schematic form  $G^2\mathcal{R}^3$  was constructed. We then compliment this with a basis for the  $(\nabla G)^2R^2$ -sector given in appendix B.3, both of which contribute to the kinetic terms of the three-dimensional vectors upon dimensional reduction. We follow the same logic as in our previous work [14, 21] i.e. we derive constraints on the parameters of the eleven-dimensional Ansatz by verifying compatibility upon dimensional reduction with lower-dimensional supersymmetry. For example, as the  $R^4$ -sector is known to be complete one can fix certain lower-dimensional supersymmetry variables by dimensional reduction, which then can be compared to the ones derived from the  $G^2\mathcal{R}^3$  and the  $(\nabla G)^2R^2$ -sector.

Let us next discuss the general form of the relevant terms in the basis of  $G^2\mathcal{R}^3$  and  $(\nabla G)^2R^2$ . The terms contributing to the three-dimensional effective action are those, which do not contain any Ricci tensors or scalars as these vanish trivially on a Calabi-Yau manifold. Taking into account the first Bianchi identity for the Riemann tensor a minimal basis of these terms is given in appendix B.3. The general expansion of terms which may contribute in addition to (2.3) to the three-dimensional action is then

$$2\kappa_{11}^2 S^{\text{extra, gen}} = \alpha^2 \int_{M_{11}} \sum_{i=1}^{17} C_i \mathcal{B}_i * 1 + \sum_{i=1}^{24} C_{i+17} B_i * 1 \quad (2.4)$$

for some coefficients  $C_i \in \mathbb{R}$ . To restrict the parameters in the Ansatz (2.4) we first take a detour to Calabi-Yau threefold compactifications and furthermore discuss the dimensional reduction on  $K3 \times S^1$ . Thus in particular, we provide the maximal complete extensions

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<sup>3</sup>Note that there are other terms quadratic in  $G$  containing eight derivatives such as e.g.  $(\nabla^3 G)^2$ ,  $(\nabla^2 G)^2 R$  and  $(\nabla^2 G)\nabla^3 R$ . All other terms do not constitute independent degrees of freedom in the eleven-dimensional action. In other words they can be rewritten up to total derivatives in the basis of  $\nabla^2 G_4^2 R^2$  and  $G_4^2 R^3$  by making use of Bianchi identities and the fact that  $G_4$  is totally antisymmetric.

<sup>4</sup>In other words due to the Calabi-Yau condition certain terms in the Ansatz yield zero upon reduction. Those coefficients can not be fixed by our arguments but constitute a complete description relevant for Calabi-Yau fourfold reductions.

of the eleven-dimensional  $G^2\mathcal{R}^3$  and  $(\nabla G)^2R^2$ -sector (2.4), which is compatible upon dimensional reduction with five-dimensional,  $\mathcal{N} = 2$  supersymmetry, i.e. by dimensional reduction on Calabi-Yau threefolds to five dimension for a generic number of Kähler moduli. Moreover, we perform the dimensional reduction on  $K3 \times S^1$  to six dimensions and employ the Heterotic-IIA duality to compare the resulting four-derivative couplings to the well known terms on the Heterotic side of the duality. It turns out that these arguments are very restrictive and allow us to parametrize the  $G^2\mathcal{R}^3$  basis with only five parameters [14]. However, when allowing for an interplay with the  $(\nabla G)^2R^2$ -sector the number of independent parameters reduces from forty-one to thirteen.

Moreover, the above analysis allows us to infer that the  $G^2\mathcal{R}^3$  and  $(\nabla G)^2R^2$ -terms are consistent with the partially known six-point function results [25]. Let us stress that it would be of great interest to study additional constraints on this eleven-dimensional sector by circular reduction to type IIA effective supergravity. Any combination of novel terms need to be vanishing at the level of the five-point one-loop string scattering amplitude with two NS-NS two-form field and three graviton vertex operator insertions. We suggest that such a study will lead to fix the remaining parameter freedom in the eleven-dimensional action.

By dimensionally reducing the extension (2.4) one modifies the kinetic couplings of the three-dimensional vectors and introduces an additional parameter freedom. One may use this to rewrite the reduction result in terms of  $3d$ ,  $\mathcal{N} = 2$  variables. In section 3.2 we perform the dimensional reduction of the  $G^2\mathcal{R}^3$  and  $(\nabla G)^2R^2$ -extensions to three space-time dimensions on Calabi-Yau fourfolds with arbitrary number of Kähler moduli.

**Calabi-Yau threefold checks to  $5d, \mathcal{N} = 2$ .** In the following we derive constraints on the coefficients  $C_i$  in (2.4) by demanding compatibility with  $\mathcal{N} = 2$  supersymmetry in five dimensions upon compactification on a Calabi-Yau threefold. The  $l_M^6$ -corrections give contributions to the five-dimensional vector multiplets of the  $\mathcal{N} = 2$  supergravity which is expressed in terms of a real pre-potential  $\mathcal{F}(X^I)$  and real special coordinates  $X^I$ . Note that physical scalars in the vector multiplets obey

$$\mathcal{F}(X^I) = \frac{1}{3!} C_{IJK} X^I X^J X^K = 1. \quad (2.5)$$

The totally symmetric and constant tensor  $C_{IJK}$  is entirely determined by the U(1) Chern-Simons terms  $\sim C_{IJK} A^I F^J F^K$ , which however do not receive  $l_M^6$ -corrections. One concludes that also the physical scalars  $X^I$  remain uncorrected.

We dimensionally reduce the action (2.4) with general coefficients  $C_i$  on a Calabi-Yau threefold  $Y_3$  to five dimensions. As our focus is on the kinetic terms for the vectors we note that in order to dimensionally reduce one expands

$$G = F_{5D}^i \wedge \omega_i^{CY_3}, \quad (2.6)$$

with the field strength of the five-dimensional vectors  $F_{5D}^i$  and the harmonic  $(1, 1)$ -forms on the Calabi-Yau threefold  $\omega_i^{CY_3}$ ,  $i = 1, \dots, h^{1,1}(CY_3)$ . The constraints imposed by supersymmetry are then inferred by making use of Shouten and total derivative identities on the



internal space  $CY_3$ . The condition one encounters is that novel terms (2.4) may not contribute to the five-dimensional couplings, which is equivalent to the non-renormalisation of (2.5). The computation is in principle straightforward (but tedious) and leads us to impose the relations among the coefficients  $C_1 \dots, C_{41}$ . Details can be found in the appendix (B.21).

**Heterotic and type IIA duality.** In this section we compactify (2.4) on  $K3 \times S^1$ . We first circularly reduce the basis of forty-one  $G^2\mathcal{R}^3$  and  $(\nabla G)^2 R^2$ -terms to ten dimensions on  $\mathbb{R}^{1,9} \times S^1$  to obtain a  $l_M^6$ -modified IIA supergravity theory. The only terms relevant for us are the ones which arise from

$$G_{11MNO} = e^{\frac{\phi}{3}} H_{MNO}, \quad M, N, O = 1, \dots, 10, \quad (2.7)$$

where 11 denotes the direction along  $S^1$  and with  $H$  the field strength of the type IIA Kalb-Ramond tensor field. We then check compatibility of the novel induced  $H^2 R^3$ -terms making use of the IIA-Heterotic duality by dimensional reduction on  $K3$ . Compactifying type IIA on  $K3$  is dual to the Heterotic string on  $\mathbb{T}^4$ . For our purpose it is enough to show that when compactifying the novel  $H^2 R^3$ -terms on  $K3$  those do not induce any  $l_M^6$ -correction to the six-dimensional action. The absence of four-derivative terms is imposed, which results in one further constraint on the parameters. In particular, the additional constraints on the  $C$ 's arise from imposing the vanishing of the four-derivative terms such as e.g.

$$\sim \chi(K3) H^{6D\mu\nu\rho} H^{6D}_{\mu}{}^{\nu_1\rho_1} R^{6D}_{\mu\mu_1\nu\nu_1}, \quad (2.8)$$

with  $\mu, \nu = 1, \dots, 6$ . One then infers the additional constraints on the parameters in (2.4) to be

$$C_2 = 0, \quad C_1 = -\frac{1}{6}(8C_3 + 2C_{31} + C_{35} + 36C_4 + 3C_6). \quad (2.9)$$

This concludes that by fixing the parameter (2.9) the proposed maximal extension of  $G^2\mathcal{R}^3$  and  $(\nabla G)^2 R^2$ -terms in the M-theory effective action is fully consistent with the indirect six-point functions results discussed in [25].

### 3 Three-dimensional effective actions revisited

F-theory may be viewed as a chain of duality maps which allows one to derive controlled IIB orientifold backgrounds at weak string coupling with  $D7$  branes and  $O7$ -planes [3–5]. The starting point of this journey is eleven-dimensional supergravity, which compactified on an appropriate eight-dimensional internal space gives a  $3d, \mathcal{N} = 2$  supergravity theory. Latter is related via the F-theory lift to a  $4d, \mathcal{N} = 1$  supergravity theory. The main objective of this section is the dimensional reduction of eleven-dimensional supergravity including the novel eight-derivative couplings (2.4) on Calabi-Yau fourfolds for a generic number of Kähler moduli in section 3.2. We start our discussion with a review of the generic properties of  $3d, \mathcal{N} = 2$  supergravity theories in section 3.1. Finally, we conclude this section with a review of the one-modulus case in which the warp-factor as well as the higher-derivative couplings can be matched to the  $3d, \mathcal{N} = 2$  variables [14].



**Background solution.** Let us set the stage by reviewing the fourfold solutions including eight-derivative terms studied in [26–28]. The background solution is taken to be an expansion in terms of the dimensionful parameter<sup>5</sup>

$$\alpha^2 = \frac{(4\pi \kappa_{11}^2)^{\frac{2}{3}}}{(2\pi)^4 3^2 \cdot 2^{13}}, \quad 2\kappa_{11}^2 = (2\pi)^5 l_M^9, \quad (3.1)$$

which reduces to the ordinary direct product solution  $\mathbb{R}^{1,2} \times CY_4$  without fluxes and warping to lowest order in  $\alpha$ . At order  $\alpha^2$  a warp-factor  $W^{(2)} = W^{(2)}(z, \bar{z})$  and fluxes are induced. The background solution is known [27, 28] to then take the form

$$\langle ds^2 \rangle = \epsilon^{\alpha^2 \Phi^{(2)}} \left( \epsilon^{-2\alpha^2 W^{(2)}} \eta_{\mu\nu} dx^\mu dx^\nu + 2\epsilon^{\alpha^2 W^{(2)}} g_{m\bar{m}} dz^m d\bar{z}^{\bar{m}} \right), \quad (3.2)$$

$$\langle G \rangle = \alpha G^{(1)} + \text{dvol}_{\mathbb{R}^{1,2}} \wedge d(\epsilon^{-3\alpha^2 W^{(2)}}). \quad (3.3)$$

By solving the eleven-dimensional E.O.M.’s for the metric  $g_{m\bar{m}}$  of the internal space one encounters that it seizes to be Ricci flat i.e. Calabi-Yau [18]. It receives a correction at order  $\alpha^2$  as

$$g_{m\bar{m}} = g_{m\bar{m}}^{(0)} + \alpha^2 g_{m\bar{m}}^{(2)}, \quad g_{m\bar{m}}^{(2)} \sim \partial_m \bar{\partial}_{\bar{m}} *^{(0)} (J^{(0)} \wedge J^{(0)} \wedge F_4), \quad (3.4)$$

where  $g^{(0)}$  is the lowest order, Ricci-flat Calabi-Yau metric and  $J^{(0)}$  is its associated Kähler form and where  $F_4$  the non-harmonic part of the third Chern form. Latter is however irrelevant for the following discussion, as it only contributes couplings to the effective action which are total derivatives [19]. Furthermore, (3.4) includes an overall Weyl factor  $\Phi^{(2)} = -\frac{512}{3} *^{(0)} (c_3^{(0)} \wedge J^{(0)})$ , which was first discussed in [28] and a warp-factor  $W^{(2)}(z, \bar{z})$  satisfying the warp-factor equation

$$\Delta^{(0)} \epsilon^{3\alpha^2 W^{(2)}} \text{dvol}_{Y_4}^{(0)} + \frac{1}{2} \alpha^2 G^{(1)} \wedge G^{(1)} - 3^2 2^{13} \alpha^2 X_8^{(0)} = 0. \quad (3.5)$$

The background value of the four-form field strength (3.3) is given by the sum of the internal flux  $G^{(1)} \in H^4(CY_4)$  and a warp-factor contribution. Due to lowest order supersymmetry constraints the flux is to be self-dual with respect to the lowest order Calabi-Yau metric. Note that we do not discuss the corrections to the gravitino variations at order  $l_M^6$  here but refer the reader to [28] for a detailed discussion. Let us emphasize that the  $l_M^6$ -gravitino variations are not known as a supersymmetric completion of eleven-dimensional supergravity at higher  $l_M$ -order remains elusive. However, it is widely believed that (3.2)–(3.5) constitutes a supersymmetric background.

### 3.1 Three-dimensional gauged $\mathcal{N} = 2$ supergravity

In this section we briefly review  $\mathcal{N} = 2$  gauged supergravity in three dimensions where all shift symmetries are gauged. Shift symmetries corresponds to an isometry of the geometry of the scalar field space. Three-dimensional maximal and non-maximal supergravities are discussed in [29]. For our purpose it is sufficient to consider three-dimensional  $\mathcal{N} = 2$

<sup>5</sup>We follow the conventions of [13].

supergravity coupled to chiral multiplets with complex scalars  $N^a$ , which are gauged along the isometries  $I^{ab}$  and subject to the constant embedding tensor  $\Theta_{ab}$ . One then infers the simply form of the  $\mathcal{N} = 2$  action to be

$$S_{\mathcal{N}=2} = \int_{M_3} \frac{1}{2} R * 1 - K_{a\bar{b}} \nabla N^a \wedge * \nabla \bar{N}^{\bar{b}} - \frac{1}{2} \Theta_{ab} A^a \wedge F^b - (V_D + V_F) * 1, \quad (3.6)$$

where  $K_{a\bar{b}} = \partial_{N^a} \partial_{\bar{N}^{\bar{b}}} K$  is a Kähler metric with Kähler potential  $K$ . The gauge covariant derivative  $\nabla N^a$  is defined by  $\nabla N^a = dN^a + \Theta_{bc} I^{ab} A^c$ . The F-term scalar potential in (3.6) is given by

$$V_F = \epsilon^K (K^{a\bar{b}} D_i W \overline{D_b W} - 4|W|^2), \quad (3.7)$$

with  $K^{a\bar{b}} = (K^{-1})^{a\bar{b}}$  the inverse of the Kähler metric given by a hermitian matrix and  $W$  a holomorphic super potential. Furthermore, one finds that  $V_D = K^{a\bar{b}} \partial_a D \partial_{\bar{b}} D - D^2$  where  $D$  is a real function of the chiral fields  $N^i$ . Lastly, note that the vectors in the Chern-Simons term (3.6) are non-dynamical.

**Dualization of the action.** One may now split the chiral fields as  $N^a = (M^I, T_i)$  and dualizes the chiral multiplets in (3.6) with bosonic component  $T_i$  into vector multiplets [30]. Note that dualization is in general not possible but requires  $\text{Im} T_i$  to admit a shift symmetry. Upon Legendre dualization the theory depends on the kinematic potential  $\tilde{K}$  which is expressed in terms of the quantities of the dual theory as

$$K(M, T) = \tilde{K}(M, L) - \text{Re} T_i L^i, \quad L^i = -\frac{\partial K}{\partial \text{Re} T_i}. \quad (3.8)$$

One then derives the dual action to take the form<sup>6</sup>

$$\begin{aligned} S_{\mathcal{N}=2, \text{dual}} = & \int_{M_3} \frac{1}{2} R * 1 - \tilde{K}_{M^I \bar{M}^J} \mathcal{D} M^I \wedge * \mathcal{D} \bar{M}^{\bar{J}} + \frac{1}{4} \tilde{K}_{L^i L^j} dL^i \wedge * dL^j \\ & + \int_{M_3} \frac{1}{4} \tilde{K}_{L^i L^j} F^i \wedge * F^j + \frac{1}{2} \Theta_{ij} A^i \wedge F^j + F^i \wedge \text{Im} [\tilde{K}_{L^i M^I} \nabla M^I] \\ & - \int_{M_3} (V_D + V_F) * 1, \end{aligned} \quad (3.9)$$

with kinematic couplings given by

$$\tilde{K}_{L^i L^j} = \partial_{L^i} \partial_{L^j} \tilde{K}. \quad (3.10)$$

Note that the scalars  $L^i$  belong to vector multiplets. One may furthermore infer from (3.8) that

$$K_{T_i \bar{T}_j} = -\frac{1}{4} \tilde{K}^{L^i L^j}, \quad \text{Re} T_i = \tilde{K}_{L^j}, \quad \frac{\partial L^i}{\partial T_j} = \frac{1}{2} \tilde{K}^{L^i L^j}. \quad (3.11)$$

---

<sup>6</sup>One may choose a constant embedding tensor such that

$$I^{ij} = -2i dx^{ij}, \quad I^{IJ} = \tilde{I}^{I\bar{J}} = 0, \quad I^{iJ} = 0, \quad \Theta_{IJ} = 0.$$

Left to discuss is the dualization of the scalar potential.<sup>7</sup> The F-term scalar potential in the vector multiplet language is then given by

$$V_F = \epsilon^K \left[ \tilde{K}^{M\bar{I}\bar{M}^{\bar{J}}} D_{M\bar{I}} W \overline{D_{M\bar{J}} W} - (4 + L^i \tilde{K}_{L^i L_j} L^j) |W|^2 \right], \quad (3.13)$$

where we have assumed that the superpotential does not depend on the scalars  $L^i$  in the vector multiplet. This case is relevant when matching to the string theory reduction result in which the superpotential does not depend on the Kähler moduli, i.e. non-perturbative effects such as  $M5$ -brane instantons are absent. For the discussion in this work (3.13) will be sufficient.

### 3.2 Calabi-Yau fourfold reduction for generic $h^{1,1}$

In this section we discuss the reduction result of M-theory involving the eight-derivative action (2.1)–(2.3) and (2.4) on the warped background (3.2)–(3.5) and allow for an arbitrary number of Kähler moduli of the internal manifold. Latter is achieved by deforming the background metric as

$$g_{m\bar{n}} \rightarrow g_{m\bar{n}} + i\delta v^i \omega_{im\bar{n}}^{(0)}, \quad (3.14)$$

where  $\delta v^i = \delta v^i(x)$  are infinitesimal scalar deformations and  $\{\omega_i^{(0)}\}$  are harmonic  $(1,1)$ -forms w.r.t the background Calabi-Yau metric  $g^{(0)}$ , with  $i = 1, \dots, h^{1,1}(CY_4)$ . The non-vanishing contribution for the dynamical three-dimensional vectors  $A_\mu^i$  is derived by<sup>8</sup>

$$G_{\mu\nu m\bar{n}} = F_{\mu\nu}^i \omega_{im\bar{n}}^{(0)}, \quad F^i = dA^i. \quad (3.15)$$

To enhance the readability of the main text in the following we shift the more technical steps to the appendix. To express the reduction result we need to introduce several higher-derivative building blocks. Among them the familiar second and third Chern-forms  $c_2$  and  $c_3$ , respectively, and  $Z, Z_{m\bar{m}}, Z_{m\bar{m}n\bar{n}}$  and  $\mathcal{Y}_{ij}, \Omega_{ij}$ . All higher-derivative objects are w.r.t. the zeroth  $\alpha$ -order Calabi-Yau metric. Their precise definition can be found in appendix A, in particular (A.19)–(A.26). Here let us schematically note that

$$Z, Z_{m\bar{m}}, Z_{m\bar{m}n\bar{n}} \sim (R)^3, \quad \mathcal{Y}_{ij} \sim (\nabla\omega_i)(\nabla\omega_j)(R)^2, \quad \Omega_{ij} \sim (\omega_i)(\omega_j)(R). \quad (3.16)$$

where  $R$  denotes the Riemann tensor on the internal manifold and  $\nabla$  is the covariant derivative w.r.t. the Calabi-Yau metric. The warp-factor dependence can be elegantly captured by introducing the warped volume and warped metric

$$\mathcal{V}_{\mathcal{W}} = \mathcal{V} + 3\mathcal{W}, \quad \mathcal{W} = \int_{Y_4} W^{(2)} *^{(0)} 1, \quad G_{ij}^{\mathcal{W}} = \frac{1}{2\mathcal{V}_{\mathcal{W}}} \int_{Y_4} e^{3\alpha^2 W^{(2)}} \omega_i^{(0)} \wedge *^{(0)} \omega_j^{(0)}, \quad (3.17)$$

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<sup>7</sup>The D-term results in

$$V_D = \tilde{K}^{M\bar{I}\bar{M}^{\bar{J}}} \partial_{M\bar{I}} \mathcal{T} \partial_{\bar{M}\bar{J}} \mathcal{T} - \tilde{K}^{L^i L^j} \partial_{L^i} \mathcal{T} \partial_{L^j} \mathcal{T} - D, \quad D = -\frac{1}{2} L^i \Theta_{ij} L^j. \quad (3.12)$$

<sup>8</sup>Note that in the presence of  $l_M^6$ -correction the deformations (3.14) and (3.15) may receive higher-order corrections as discussed in [15, 16], none of which alter the dynamics of the resulting theory. We thus omit them from the present discussion.

which at zeroth order in  $\alpha$  reduce to  $\mathcal{V}$  and  $G_{ij} = \frac{1}{2\mathcal{V}} \int_{Y_4} \omega_i^{(0)} \wedge *^{(0)} \omega_j^{(0)}$ . We also introduce

$$\mathcal{K}_i^{\mathcal{W}} = i\mathcal{V} \omega_{im}^{(0)m} + \frac{9}{2} \alpha^2 \int_{Y_4} \partial_i W^{(2)} | *^{(0)} 1, \quad (3.18)$$

which at lowest order simply reduces to  $\mathcal{K}_i^{(0)} = i\mathcal{V} \omega_{im}^{(0)m} = \frac{1}{3!} \int_{Y_4} \omega_i^{(0)} \wedge J^{(0)} \wedge J^{(0)} \wedge J^{(0)}$ . Note that we use the notation  $\mathcal{K}_i^{(0)}$  to abbreviate the intersection number evaluated in the background, in contrast to the analogue quantities  $\mathcal{K}_i$  which may vary over the Kähler moduli space. With these definitions we state that the action including the  $l_M^6$ -corrections to the kinetic terms [15, 16] is given by

$$\begin{aligned} S_{\text{kin}} = & \frac{1}{2\kappa_{11}} \int_{\mathcal{M}_3} \left[ R * 1 - (G_{ij}^{\mathcal{W}} + \mathcal{V}^{-2} K_i^{\mathcal{W}} K_j^{\mathcal{W}}) d\delta v^i \wedge * d\delta v^j - \mathcal{V}^2 G_{ij}^{\mathcal{W}} F^i \wedge * F^j \right. \\ & - d\delta v^i \wedge * d\delta v^j \frac{\alpha^2}{\mathcal{V}_0} \int_{CY_4} \left( 768 Z \omega_{im}^{(0)m} \omega_{jn}^{(0)n} - 3072 i Z_{m\bar{n}} \omega_i^{(0)\bar{n}m} \omega_{js}^{(0)s} \right) *^{(0)} 1 \\ & + d\delta v^i \wedge * d\delta v^j \frac{\alpha^2}{\mathcal{V}_0} \int_{CY_4} 3072 Z_{m\bar{n}r\bar{s}} \omega_i^{(0)\bar{n}m} \omega_j^{(0)\bar{s}r} *^{(0)} 1 \\ & - F^i \wedge * F^j \alpha^2 \mathcal{V}_0 \int_{CY_4} \left( -256 Z \omega_{im\bar{n}}^{(0)} \omega_j^{(0)\bar{n}m} + 192(7 - a_1) i Z_{m\bar{n}} \omega_i^{(0)\bar{r}m} \omega_j^{(0)\bar{n}\bar{r}} \right) *^{(0)} 1 \\ & \left. + F^i \wedge * F^j \alpha^2 \mathcal{V}_0 \int_{CY_4} 384(1 + a_1) Z_{m\bar{n}r\bar{s}} \omega_i^{(0)\bar{n}m} \omega_j^{(0)\bar{s}r} *^{(0)} 1 + \Theta_{ij} A^i \wedge F^j \right]. \quad (3.19) \end{aligned}$$

The one parameter freedom  $a_1$  arises from the uncertainty inherent in the  $(\nabla G)^2 R^2$ -sector. From the novel sector [21] we find

$$\delta S_1 = 256 F^i \wedge * F^j \alpha^2 \mathcal{V} \int_{Y_4} Z \omega_{im\bar{n}}^{(0)} \omega_j^{(0)\bar{n}m} *^{(0)} 1. \quad (3.20)$$

Note that (3.20) is precisely cancelled by the same structure in (3.19). Lastly, one performs the dimensional reduction of (2.4) to give the potentially novel terms

$$\begin{aligned} \delta S_2 = & F^i \wedge * F^j \alpha^2 \mathcal{V} \int_{Y_4} \left( 8i(a_3 + a_4) Z_{m\bar{n}} \omega_i^{(0)\bar{n}m} \omega_{js}^{(0)s} * 1 - 8a_3 Z_{m\bar{n}r\bar{s}} \omega_i^{(0)\bar{n}m} \omega_j^{(0)\bar{s}r} \right) *^{(0)} 1 \\ & + F^i \wedge * F^j \alpha^2 \mathcal{V} \int_{Y_4} a_2 c_2 \wedge \Omega_{ij}, \quad (3.21) \end{aligned}$$

with the coefficients  $a_3, a_4$  result from the unfixed eleven dimensional parameters,  $a_3 = -C_{22} + 4C_3$  and  $a_4 = 18C_4$ . Let us close this section with some remarks. Note that in (3.21) one obtains a term proportional to the second Chern-form. In the limit  $h^{1,1} \rightarrow 1$ , i.e. the one-modulus case we see that

$$\delta S_2 \rightarrow a_4 \mathcal{Z}, \quad (3.22)$$

as the term  $\Omega_{ij}$  vanishes. For the physical arguments provided in [14] where the one-modulus case is discussed we infer that  $\delta S_2 \rightarrow 0$  as it would change the physical interpretation else-wise. Hence in the remainder of this work we assume  $C_4 = 0$  and thus  $a_4 = 0$ .<sup>9</sup>

<sup>9</sup>Comparison to five point-scattering and six-point amplitudes can in principle fix the 11-dimensional coefficient of the basis, thus also  $C_4$ .

Furthermore, note that the action (3.19) depends on the infinitesimal deformation  $\delta v^i$ . To establish the connection to the full field space  $v^i$ , i.e. the coordinates on the Kähler moduli space we replace  $\delta v^i \rightarrow v^i$  in the following.<sup>10</sup> This will become relevant for the discussion in section 4.

### 3.3 Review one-modulus Kähler potential and coordinates

The dimensional reduction of the eleven-dimensional supergravity action including higher-derivative terms on a warped Calabi-Yau fourfold background with one Kähler modulus, i.e.  $h^{1,1} = 1$  case was discussed rigorously in [14]. We devote this section to reviewing this discussion, in particular the derivation of the Kähler potential and coordinates of the  $3d, \mathcal{N} = 2$  theory. As a starting point we may take the limit  $h^{1,1} \rightarrow 1$  of the generic Calabi-Yau fourfold reduction result presented in (3.19)–(3.21). One then infers that the  $l_M^6$ -corrected action takes the standard form

$$S^{3d} = \int_{M_3} \frac{1}{2} R * 1 + \frac{1}{4} \tilde{G}_{LL}(L) dL \wedge * dL + \frac{1}{4} \tilde{G}_{LL}(L) F \wedge * F, \quad (3.23)$$

with

$$\tilde{G}_{LL}(L) = -\frac{4}{L^2} \left( 1 - 384 \alpha^2 \tilde{\mathcal{Z}} L \right) = -\frac{4}{L^2} + 1536 \alpha^2 \tilde{\mathcal{Z}} \frac{1}{L}, \quad (3.24)$$

and with the topological coupling depending on the third Chern-form given by

$$\mathcal{Z} = (2\pi^3) \int_{CY_4} c_3 \wedge J, \quad \mathcal{Z} = \mathcal{V}^{\frac{1}{4}} \tilde{\mathcal{Z}}, \quad (3.25)$$

where we have used that  $J = \omega_0 \mathcal{V}^{\frac{1}{4}}$ . We can integrate the metric  $\tilde{G}_{LL}$  to obtain the kinetic potential  $\tilde{K}(L)$  and coordinate

$$\tilde{K} = 4 \log L + 1536 \alpha^2 \tilde{\mathcal{Z}} L (\log(L) - 1) + 4, \quad (3.26)$$

$$L = \mathcal{V}^{-\frac{3}{4}} - 3 \alpha^2 \mathcal{W} \mathcal{V}^{-\frac{7}{4}}, \quad (3.27)$$

where we have chosen the integration constants in a convenient way.

**Determining the Kähler potential.** One may next dualize the vector multiplet to a chiral multiplet, whose metric derives from a Kähler potential. As outlined in section 3.1 this is achieved by a Legendre transformation of the kinetic potential

$$K = \tilde{K} - L \operatorname{Re} T, \quad \operatorname{Re} T = \partial_L \tilde{K}. \quad (3.28)$$

One thus derives the Kähler potential  $K(T + \bar{T})$  to be

$$K = 4 \log L - 1536 \alpha^2 \tilde{\mathcal{Z}} L = -3 \log \left( \mathcal{V} + \alpha^2 (4\mathcal{W} + 512 \mathcal{Z}) \right), \quad (3.29)$$

with corresponding coordinate

$$\operatorname{Re} T = \frac{4}{L} + 1536 \alpha^2 \tilde{\mathcal{Z}} \log L = 4 \mathcal{V}^{\frac{3}{4}} + 12 \alpha^2 \mathcal{V}^{-\frac{1}{4}} \mathcal{W} - 1152 \alpha^2 \tilde{\mathcal{Z}} \log \mathcal{V}. \quad (3.30)$$

Note that all quantities in the Kähler potential (3.29) depend on the one-modulus  $\mathcal{V}$ , i.e. the overall volume.

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<sup>10</sup>Possible obstructions and subtleties to this step for higher-derivative couplings of non-topological nature were discussed in [31].

**The no-scale condition and the scalar potential.** We next argue that the  $\ell_M^6$ -suppressed corrections to the Kähler potential in (3.29) generically lead to a breaking of the no-scale condition and thus generate a  $F$ -term scalar potential. One straightforwardly computes that

$$K_T K^{T\bar{T}} K_{\bar{T}} = \frac{K_T^2}{K_{T\bar{T}}} = 4 - 1536 \frac{\alpha^2}{\mathcal{V}} \mathcal{Z}. \quad (3.31)$$

One may next infer the scalar potential originating from the breaking of the no-scale condition. It enters the effective action via the F-term scalar potential<sup>11</sup>

$$V_F = \epsilon^K \left( K^{T\bar{T}} D_T W \overline{D_{\bar{T}} W} - 4 |W|^2 \right) = -1536 \alpha^2 \frac{|W_0|^2}{\mathcal{V}^4} \mathcal{Z}. \quad (3.32)$$

Note that it exhibits a runaway direction for  $\mathcal{V} \rightarrow \infty$  if  $\int_{Y_4} c_3 \wedge J < 0$ .<sup>12</sup> In (3.32) we assumed that the complex structure moduli are stabilized by the GVW superpotential [33] given by

$$W = \frac{1}{\ell_M^3} \int_{Y_4} G^{(1)} \wedge \Omega, \quad \Omega \in H^{4,0}(Y_4), \quad (3.33)$$

which in the vacuum then takes the constant value  $W_0$ . A critical assessment of this two step procedure is discussed in [34–36]. The runaway behavior of (3.32) for large volume  $\mathcal{V}$  signals an instability of the solution for the case of a non-vanishing  $W_0$  as recently examined in [37].

Let us conclude this section by emphasizing the importance of the one-modulus results in particular the integration into a Kähler potential and coordinates. In a following section we will show compatibility with the generic moduli case which is exceedingly more complicated due to the appearance of non-topological higher-derivative contributions to the Kähler metric.

## 4 Three-dimensional Kähler potential and coordinates

The eleven-dimensional higher-derivative corrections manifest themselves in terms of  $\ell_M^6$ -modifications of the kinematic couplings of the two-derivative three-dimensional supergravity theory as discussed in the previous section 3.2. The objective is to express these  $\ell_M^6$ -modifications to the kinematic couplings in the language of three-dimensional,  $\mathcal{N} = 2$  supergravity. Namely these must result from a  $\ell_M^6$ -correction to the Kähler potential and Kähler coordinates, i.e. fixing the complex structure on the Kähler moduli space. We reviewed this procedure for the one-modulus case, i.e.  $h^{1,1} = 1$  in 3.3. In this section we propose a novel description of the Kähler coordinates in terms of divisor integrals. Due to these specific divisor integrals of the Calabi-Yau fourfold one manages to reproduce all high-derivative structures appearing in the reduction result of the Kähler metric (3.19)–(3.21) which we discuss in section 4.1. To motivate our Ansatz note that the Kähler coordinates

<sup>11</sup>Note that superpotential can not be renormalized perturbatively but may be subject to e.g.  $M5$ -instanton corrections which correspond to  $D3$ -instantons in the F-theory limit [32].

<sup>12</sup>An example with this property and  $h^{1,1} = 1$  is the sextic fourfold. For the sextic one finds  $\int_{Y_4} c_3 \wedge \omega = -420$ .

are expected to linearise the action of M5-brane instantons on divisors  $D_i$ .<sup>13</sup> This implies that the  $T_i$ 's are expected to be integrals over divisors  $D_i$ . In particular the Ansatz depends on the first, second and third Chern-form of the Divisors  $\tilde{c}_{1,2,3} = \tilde{c}_{1,2,3}(D_i)$ . Let us first recall further definitions

$$\mathcal{Z}_i = (2\pi)^3 \int_{CY_4} c_3 \wedge \omega_i = (2\pi)^3 \int_{D_i} c_3, \quad \mathcal{W}_i = \int_{D_i} W^{(2)} * 1, \quad \mathcal{F}_i = 1536 \int_{D_i} F_6 * 1. \quad (4.1)$$

The Ansatz for the Kähler potential and coordinates depends on the real parameters  $\alpha_1, \dots, \alpha_9$  and  $\kappa_1, \dots, \kappa_6$ . We assert the Kähler potential to take the form

$$K = -3 \log \left( \mathcal{V} + \alpha^2 (4\mathcal{W}_i v^i + \kappa_1 \mathcal{Z}_i v^i + \kappa_2 \mathcal{T}_i v^i) \right), \quad (4.2)$$

and for the Kähler coordinates to be<sup>14</sup>

$$\text{Re} T_i = \mathcal{K}_i + \alpha^2 \left( \mathcal{F}_i + 3\mathcal{W}_i + \kappa_3 \frac{\mathcal{K}_i}{\mathcal{V}} \mathcal{Z}_j v^j + \kappa_4 \mathcal{Z}_i \log \mathcal{V} + \kappa_5 \frac{\mathcal{K}_i}{\mathcal{V}} \mathcal{T}_j v^j + \kappa_6 \mathcal{T}_i \right). \quad (4.3)$$

Note that the warp-factor part of this Ansatz was fixed in [16, 39].<sup>15</sup> In (4.3) we introduce a novel divisor integral higher-order correction

$$\begin{aligned} \mathcal{T}_i = & \alpha_1 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{c}_1 + \alpha_2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_2 + \alpha_3 \int_{D_i} \tilde{c}_3 + \alpha_4 \int_{D_i} \mathcal{C}_1 \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{J} \\ & + \alpha_5 \int_{D_i} \mathcal{C}_1^2 \tilde{c}_1 \wedge \tilde{J} \wedge \tilde{J} + \alpha_6 \int_{D_i} \mathcal{C}_1 \tilde{c}_2 \wedge \tilde{J} + \alpha_7 \int_{D_i} *_6(\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_2 \\ & + \alpha_8 \int_{D_i} *_6(\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_1 \wedge \tilde{c}_1 + \alpha_9 \int_{D_i} \mathcal{C}_1 \tilde{c}_1 \wedge *_6 \tilde{c}_1, \end{aligned} \quad (4.4)$$

with  $\mathcal{C}_1 = *_6(\tilde{c}_1 \wedge \tilde{J}^2) = 2R_m^m n^n$  and  $i = 1, \dots, h^{1,1}$  and where  $D_i = PD(\omega_i)$  are the Poincare-dual divisors to the harmonic forms  $\omega_i$  the Calabi-Yau fourfold. Furthermore,  $\tilde{c}_1, \tilde{c}_2, \tilde{c}_3$  are the corresponding Chern-forms of the Divisor and  $\tilde{J} = i^* J$  the pull-back of the Kähler form  $i : D_j \rightarrow CY_4$ . In the following  $c_3$  is the third Chern-form of  $CY_4$ . Note that although  $c_1(CY_4) = 0$  the divisors i.e. sub-manifolds of complex co-dimension one generically have  $c_1(D_i) := \tilde{c}_1 \neq 0$ . Let us use the notation

$$\mathcal{Z} = \mathcal{Z}_i v^i, \quad (4.5)$$

in the following. Furthermore, we choose the normalization

$$\alpha_3 = 1, \quad (4.6)$$

which is argued for in section 4.1. Note that as in the Ansatz (4.3) we allow for additional pre-factors (4.6) can be imposed without loss of generality.

<sup>13</sup>In fact, as discussed in [38] a holomorphic super-potential of the schematic form  $W \propto e^{-T_i}$  can be induced by such instanton effects.

<sup>14</sup>We omit constant shifts such as  $\mathcal{Z}_i$  in the definition of the Kähler coordinates.

<sup>15</sup>Comparison of the warp-factor contribution of the one modulus Kähler coordinates (3.30) and (4.3) suggest that  $\mathcal{F}_i \rightarrow 9\mathcal{W}\mathcal{V}^{-1/4}$  in the one-modulus case.



Let us next briefly outline the logic of this section. In 4.1 we compute the variation of the Ansatz (4.4) w.r.t. Kähler deformations of the Calabi-Yau fourfold and show the correlation with the higher-derivative structures encountered in the reduction result. We will argue in section 4.2 that the Ansatz (4.2) and (4.3) can be rewritten solely in terms of topological quantities of the divisors. All the higher-derivative structures of the reduction result (3.19)–(3.21) appear. This steps fixes the relative factors  $\alpha_1, \dots, \alpha_9$  with one remaining free parameter  $\alpha_2$ . In section 4.3 we discuss the compatibility of this Ansatz with the one-modulus case which can be integrated exactly into a Kähler potential [14] which induces certain relations among the  $\kappa$ 's in the Ansatz. However, a precise determination of the remaining  $\kappa$ -parameters is beyond the aim of this work. We conjecture that the matching of the reduction result is possible with the Ansatz (4.4), (4.2) and (4.3) which then may fix all the parameters uniquely. Lastly, we provide further indirect evidence for this claim by comparison to the newly discovered structures (3.21) proportional to the second Chern form of the Calabi-Yau fourfold which may also be reproduced by the novel Ansatz. This insight however is not used in the direct line of arguments which precedes through the following sections.

#### 4.1 Kähler coordinates as integrals on $CY_4$

To write the integrals (4.4) defined over Divisors  $D_i = PD(\omega_i)$  as integrals over the Calabi-Yau fourfold we note that e.g.

$$\int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{c}_1 = \int_{CY_4} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{c}_1 \wedge \omega_i. \quad (4.7)$$

Note that it is crucial to maintain  $\tilde{c}_1$  instead of  $c_1$  as latter would vanish due to the Calabi-Yau condition. The induced metric on a minimal divisor  $D_i$  inherited from the ambient space is itself Kähler [40, 41] but generically not Calabi-Yau. Let us note that in previous work we considered the correction written in terms of topological quantity namely the third Chern-form of the Calabi-Yau fourfold. One may write the Kähler coordinates (4.4) in terms of a basis of well defined  $CY_4$ -integrals in terms the Calabi-Yau metric and co-variant quantities thereof such as the Riemann tensors if the parameters in (4.4) obey the following relations

$$\begin{aligned} \alpha_5 &= -\frac{1}{8}\alpha_1 + \frac{1}{24} + \frac{1}{4}\alpha_4, \\ \alpha_6 &= \frac{1}{2}\alpha_2 + \frac{1}{2}, \\ \alpha_7 &= \alpha_2 + 1, \\ \alpha_8 &= \frac{1}{2}\alpha_1 - \frac{1}{3} - \alpha_4, \\ \alpha_9 &= -\alpha_1 + \frac{1}{6}. \end{aligned} \quad (4.8)$$

Thus in other words by imposing (4.8) we can rewrite the Kähler coordinates in terms of a higher-derivative density on the Calabi-Yau fourfold, which as we argue in appendix A.1

may take the form

$$\mathcal{T}_i = \int_{CY_4} \omega_i \wedge \mathcal{X}, \quad \mathcal{X} \sim R^3 \quad (4.9)$$

with the higher-derivative (3,3)-form  $\mathcal{X}$  defined in the appendix (A.31). One can easily verify the property

$$\mathcal{T}_i v^i = \mathcal{Z}. \quad (4.10)$$

To compute the Kähler metric we need to take derivatives of the Kähler potential w.r.t. to the Kähler coordinates as

$$K_{ij} = \frac{\partial^2 K}{\partial \text{Re} T_i \partial \text{Re} T_j} = \frac{\partial^2 v^k}{\partial \text{Re} T_i \partial \text{Re} T_j} \frac{\partial K}{\partial v^k} + \frac{\partial v^k}{\partial \text{Re} T_i} \frac{\partial v^l}{\partial \text{Re} T_j} \frac{\partial^2 K}{\partial v^k \partial v^l}, \quad (4.11)$$

with

$$\frac{\partial v^i}{\partial \text{Re} T_j} = \left( \frac{\partial \text{Re} T_j}{\partial v^i} \right)^{-1} = \mathcal{K}^{ij} - \alpha^2 \kappa_5 \mathcal{K}^{ik} \left( \frac{\partial}{\partial v^k} \mathcal{T}_l + \dots \right) \mathcal{K}^{lj}, \quad (4.12)$$

where  $\mathcal{K}^{ij}$  is the inverse intersection number of the Calabi-Yau fourfold defined in the appendix (A.11). The variation of  $\mathcal{T}_i$  w.r.t. to the Kähler moduli fields of the Calabi-Yau fourfold constitutes the crucial new ingredient to generate and match the higher-derivative structures in the reduction result (3.19)–(3.21) of the Kähler metric. Let us next discuss it in more detail.

**Variational derivative of Kähler coordinates.** The aim of this section is to argue that the Ansatz for the Kähler potential (4.2) and Kähler coordinates (4.3) may reproduce the Kähler metric in the Legendre dual variables which are in agreement with the reduction results. In other words we are able to encounter all relevant higher-derivative structures found in the reduction result (3.19)–(3.21). However, let us stress that to precisely match the factors in the reduction result is beyond the aim of this work. It is expected that additional non-trivial identities relating the higher-derivative building blocks (4.14) and (3.19)–(3.21), and (4.16) are required to perform this task.

Let us proceed with the main argument. It is straight forward to compute derivatives of the previously encountered topological objects [19] w.r.t. to the Kähler moduli fields as

$$\frac{\partial}{\partial v^i} \mathcal{Z} = \mathcal{Z}_i, \quad \frac{\partial}{\partial v^j} \mathcal{Z}_j = 0. \quad (4.13)$$

Let us note that due to (4.13) no terms proportional to the logarithm of the volume —  $\log \mathcal{V}$  — appear in the Kähler metric nor in the Legendre dual variables and thus (4.3) and (4.2) are in agreement with the reduction result in this regard.

Let us next compute the variation of  $\mathcal{T}_i$  in (4.9) w.r.t. to the Kähler moduli fields which gives

$$\frac{\partial}{\partial v^j} \mathcal{T}_i = -\frac{3}{\mathcal{V}} \mathcal{K}_j \mathcal{T}_i + \frac{5}{\mathcal{V}} \mathcal{K}_i \mathcal{T}_j + 3 \mathcal{T}_{ij} + \frac{3}{\mathcal{V}} \mathcal{Z}_i \mathcal{K}_j + 4i \int_{CY_4} Z_{m\bar{n}} \omega_i^{\bar{n}s} \omega_{js}^m * 1, \quad (4.14)$$

where

$$\mathcal{T}_{ij} = \int_{CY_4} *_8 (\omega_i \wedge \omega_j \wedge J) \wedge \mathcal{X}. \quad (4.15)$$

To compute (4.14) we make extensive use of the compute Algebra package xTensor [42]. We provide some more technical details in appendix A.3. There we also discuss couplings of the Kähler metric proportional to the second Chern form of the Calabi-Yau fourfold. By using the relation

$$\mathcal{Y}_{ij} = -\frac{1}{6} \int_{Y_4} (iZ_{m\bar{n}}\omega_i^{\bar{r}m}\omega_j^{\bar{n}\bar{r}} + 2Z_{m\bar{n}r\bar{s}}\omega_i^{\bar{n}m}\omega_j^{\bar{s}r}) * 1, \quad (4.16)$$

one infers that (4.14) can be put in relation to  $Z_{m\bar{n}r\bar{s}}\omega_i^{\bar{n}m}\omega_i^{\bar{r}s}$  and  $\mathcal{Y}_{ij}$ . Let us emphasize that establishing the relation of topological Kähler coordinates and the building blocks of the Kähler metric obtained by dimensional reduction  $\sim Z_{m\bar{n}r\bar{s}}\omega_i^{\bar{n}m}\omega_i^{\bar{r}s}$  as well as  $\sim Z_{m\bar{n}}\omega_i^{\bar{n}s}\omega_{js}^m$  has been a long standing problem posed in our previous work [15, 16].

Let us close this section by providing further arguments in favor of our conjecture. By evaluating (4.11) one obtains that the Kähler metric  $K_{ij}$  contains  $\mathcal{V}\mathcal{K}_{kl}\mathcal{K}_{ijk}\mathcal{T}_j$  and  $\mathcal{K}_{(i}\mathcal{T}_{j)}$ .<sup>16</sup> Those structures arise naturally from the variation of the Kähler coordinates (4.14), in particular  $\mathcal{T}_{ij} \sim \mathcal{K}^{kl}\mathcal{K}_{ijk}\mathcal{T}_i + \dots$ . It has been argued for analogous relations in [43, 44]. Concludingly, the divisor integral Ansatz (4.3) manages to reproduce all relevant higher-derivative building blocks which appear in the reduction result (3.19)–(3.21). However, we also find that we have one additional object namely  $\mathcal{Y}_{ij}$  which does not appear in the reduction result but would be generated by our Ansatz. In [15] we had argued for a relation in between the  $\mathcal{F}$  and higher-derivative objects which in the light of this work most certainly is in need of a revision. Let us close this section with remarks on the warp-factor in the Kähler potential and coordinates and its potential connection to the higher-derivative structures. In appendix A.3 we review the integration of the warp-factor into a Kähler potential in particular in (A.51) — (A.62). From the definition (A.20) one immediately infers that  $\mathcal{Y}_{ij}v^j = \mathcal{Y}_{ji}v^j = 0$  and thus it takes special simplified role in the process of matching the reduction result. One may speculate that a relation  $\mathcal{Y}_{ij} \sim \mathcal{F}_{ij}$  can be established to proof the conjectured integration into a Kähler potential which revises the claims of [15].

## 4.2 Topological divisor integrals as Kähler coordinates

In this section we argue that the Ansatz for the Kähler coordinates (4.4) may be rewritten in terms of “topological quantities” by fixing the coefficients in the Ansatz. The quotation marks refer to an abuse of the word as the integrands can be reduced to topological integrands by factorizing out Kähler moduli deformations, e.g. the intersection number of the Calabi-Yau fourfold  $\mathcal{K}_{ijkl}$  is a topological quantity, in contrast to the volume of a complex curve  $\mathcal{K}_{ijk}$ . Latter is not as it depends on the position in moduli space. However one may write it in terms of the topological intersection numbers by factorizing out the Kähler moduli fields as  $\mathcal{K}_{ijk} = \mathcal{K}_{ijkl}v^l$ .

<sup>16</sup>The precise form of the Kähler metric results from (4.11) by inserting our Ansatz (4.3), (4.2) and by using the properties of the intersection numbers listed in equation (A.11). Furthermore, one may use the relations on the higher-derivative building blocks (A.33) and (A.34).

To set the stage note that any closed form such as  $\tilde{c}_1$  may be written in terms of its harmonic part plus a double exact contribution

$$\tilde{c}_1 = H\tilde{c}_1 + \partial\bar{\partial}\lambda, \quad (4.17)$$

where  $\lambda$  is a function on the divisor. From the closure of  $\tilde{c}_1$  and by using inferred relation thereof in appendix A.2 one may show that the Ansatz for the Kähler coordinates (4.4) can be rewritten as

$$\begin{aligned} \mathcal{T}_i = & \alpha_1 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{c}_1 + \alpha_2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_2 + \alpha_3 \int_{D_i} \tilde{c}_3 + \frac{\alpha_4}{\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{J} \\ & + \frac{\alpha_5}{\mathcal{K}_i^2} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 + \frac{\alpha_6}{\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_2 \wedge \tilde{J} \\ & + 2\alpha_6 \int_{D_i} \tilde{*}_6(H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_2 - (2\alpha_4 + 8\alpha_5) \int_{D_i} \tilde{*}_6(H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_1 \wedge \tilde{c}_1 \\ & - \frac{4\alpha_5}{\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge *_6 H\tilde{c}_1, \end{aligned} \quad (4.18)$$

where  $\mathcal{K}_i$  denotes the volume of the divisor  $D_i$ . Note that in order to obtain (4.18) one fixes the coefficients such that

$$\alpha_7 = 2\alpha_6, \quad \alpha_8 = 2\alpha_4 - 8\alpha_5, \quad \alpha_9 = -4\alpha_5. \quad (4.19)$$

Additionally requiring that we can write  $\mathcal{T}_i$  as integrals on the Calabi-Yau fourfold one is led to additional constraints which in combination with (4.8) then impose

$$\begin{aligned} \alpha_1 &= \frac{1}{6}, & \alpha_3 &= 1, & \alpha_4 &= -\frac{1}{12}, & \alpha_5 &= 0, \\ \alpha_6 &= \frac{1}{2} + \frac{1}{2}\alpha_2, & \alpha_7 &= 1 + \alpha_2, & \alpha_8 &= -\frac{1}{6}, & \alpha_9 &= 0. \end{aligned} \quad (4.20)$$

One thus infers from (4.20) the final form of the higher-derivative Kähler coordinate divisor integral to be

$$\begin{aligned} \mathcal{T}_i = & \frac{1}{6} \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{c}_1 + \alpha_2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_2 + \int_{D_i} \tilde{c}_3 - \frac{1}{12\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{J} \\ & + \frac{1}{2}(1 + \alpha_2) \frac{1}{\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_2 \wedge \tilde{J} + (1 + \alpha_2) \int_{D_i} \tilde{*}_6(H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_2 \\ & - \frac{1}{6} \int_{D_i} \tilde{*}_6(H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_1 \wedge \tilde{c}_1. \end{aligned} \quad (4.21)$$

Let us note that (4.21) is in indeed a sum of “topological integrals”. In this sense after factorizing out Kähler moduli deformations one may vary the integrands of (4.21) w.r.t. the induced metric on the divisors  $D_i$  and find that the resulting variation constitutes a total derivative. This follows straightforwardly from the properties of  $\tilde{c}_1, \tilde{c}_2, \tilde{c}_3$  and  $\tilde{J}$ . The integrands involving the hodge star  $\tilde{*}_6$  crucially have it act on only the harmonic part of the first Chern-form  $H\tilde{c}_1$ .

### 4.3 One-modulus compatibility

The one-modulus case can be integrated exactly into a Kähler potential as discussed in section 3.3. Thus in this section we examine the limit  $h^{1,1} \rightarrow 1$  of the generic moduli case 3.2 to impose constraints on the  $\kappa$ -parameters in the Ansatz. We focus on the higher-derivative components and do not discuss the warp-factor contributions  $\mathcal{W}$  and  $\mathcal{F}$  here. Recall that

$$\mathcal{Z}_i = (2\pi)^3 \int_{CY_4} c_3 \wedge \omega_i, \quad \mathcal{Z} = \mathcal{Z}_i v^i. \quad (4.22)$$

We made the Ansatz for the Kähler potential

$$K = -3 \log \left( \mathcal{V} + \alpha^2 (\kappa_1 \mathcal{Z}_i v^i + \kappa_2 \mathcal{T}_i v^i) \right), \quad (4.23)$$

and for the Kähler coordinates

$$\text{Re} T_i = \mathcal{K}_i + \alpha^2 \left( \mathcal{F}_i + 3\mathcal{W}_i + \kappa_3 \frac{\mathcal{K}_i}{\mathcal{V}} \mathcal{Z}_j v^j + \kappa_4 \mathcal{Z}_i \log \mathcal{V} + \kappa_5 \frac{\mathcal{K}_i}{\mathcal{V}} \mathcal{T}_j v^j + \kappa_6 \mathcal{T}_i \right). \quad (4.24)$$

Let us next analyse these expressions (4.23) and (4.24) in the case  $h^{1,1} = 1$ . One finds that

$$\mathcal{K}_i \rightarrow 4\mathcal{V}^{\frac{3}{4}}, \quad \mathcal{K}_{ij} \rightarrow 12\mathcal{V}^{\frac{1}{2}}, \quad \mathcal{K}_{ijk} \rightarrow 24\mathcal{V}^{\frac{1}{4}}, \quad \mathcal{K}_{ijkl} \rightarrow 1, \quad \mathcal{K}^{ij} \rightarrow \frac{1}{12}\mathcal{V}^{-\frac{1}{2}} \quad (4.25)$$

and from the expression (4.9) and (4.10) that in the one-modulus case

$$\mathcal{T}_i \rightarrow \tilde{\mathcal{Z}} \quad \text{with} \quad \tilde{\mathcal{Z}} = (2\pi)^3 \int_{CY_4} c_3 \wedge \omega^0. \quad (4.26)$$

The relation (4.26) follows from (4.4) and (4.18) due to the Calabi-Yau condition which leads to a vanishing of terms proportional to  $\tilde{c}_1$ . One furthermore notes that  $J = \omega_0 \mathcal{V}^{\frac{1}{4}}$  and thus

$$\mathcal{Z} = \mathcal{V}^{\frac{1}{4}} \tilde{\mathcal{Z}} = (2\pi)^3 \int_{CY_4} c_3 \wedge J. \quad (4.27)$$

One concludes that (4.23) and (4.24) in the limit  $h^{1,1} \rightarrow 1$  become

$$K \rightarrow -3 \log \left( \mathcal{V} + \alpha^2 (\kappa_1 + \kappa_2) \mathcal{V}^{\frac{1}{4}} \tilde{\mathcal{Z}} \right), \quad (4.28)$$

and

$$\text{Re} T_i \rightarrow 4\mathcal{V}^{\frac{3}{4}} + \alpha^2 \left( (4\kappa_3 + 4\kappa_5 + \kappa_6) \tilde{\mathcal{Z}} + \kappa_4 \tilde{\mathcal{Z}} \log \mathcal{V} \right), \quad (4.29)$$

where we have used (4.25)–(4.27). Thus one infers by comparison to the one-modulus case (3.29)–(3.30) that

$$\kappa_1 + \kappa_2 = 512, \quad 4\kappa_3 + 4\kappa_5 + \kappa_6 = 0, \quad \kappa_4 = -1152. \quad (4.30)$$

Additionally one aims to match the Legendre dual coordinates to the one modulus case. To proceed one needs to specify the precise form of the Kähler coordinates in terms of Calabi-Yau fourfold integrals. In section 4.1 we emphasized that the match with the divisor

integral form remains ambiguous. Let us proceed with (4.15) for the remainder of this section. One can then use

$$L^i = -\frac{\partial K}{\partial T_i} = -\frac{\partial K}{\partial v^j} \frac{\partial v^j}{\partial T_i}, \quad (4.31)$$

to find

$$L^i = \frac{v^i}{\mathcal{V}} + \frac{\alpha^2}{\mathcal{V}} \left( \kappa_6 \mathcal{K}^{ij} \mathcal{T}_j + (3\kappa_1 - 4\kappa_3 - 4\kappa_5 - \kappa_6) \mathcal{K}^{ij} \mathcal{Z}_j - \frac{v^i}{3\mathcal{V}} (3\kappa_1 - \kappa_3 - \kappa_5 + \kappa_4) \mathcal{Z} \right). \quad (4.32)$$

To compute (4.32) we only used the fact that  $(\frac{\partial}{\partial v^j} \mathcal{T}_i) v^i = -\mathcal{T}_j + \mathcal{Z}_j$  which follows from  $\mathcal{T}_i v^i = \mathcal{Z}$ . Lastly, by imposing (4.30) one infers a match of (3.27) with comparison of the one-modulus limit of (4.32), i.e. the other  $l_M^6$ -contributions vanishes in the limit. One can furthermore compute the scalar potential by evaluating (4.11) which can be performed by using (A.34) and (A.11) contracted with (4.32). One finds for a non vanishing flux-superpotential  $W_0$  that

$$V_F = \frac{|W_0|^2}{\mathcal{V}^4} \frac{4\kappa_4}{3} \mathcal{Z}, \quad (4.33)$$

which by imposing (4.30) matches the one modulus case given in (3.32). Moreover, note that from (4.33) one infers that for the Ansatz (4.2) and (4.3) the no-scale structure is broken due to the imposed compatibility with the one-modulus case.

Let us close this section with a critical remark. In section 4.1 and 4.2 we pointed out that the lift of the divisor integral expressions to integrals on the Calabi-Yau fourfold leaves certain parameters unfixed. In order to compute other quantities such as (4.33) in full generality we suggest that a better understanding of the  $\mathcal{T}_i$  contribution is to be developed. Note that the arguments supporting the  $\mathcal{Z}_i \log \mathcal{V}$  — correction to the Kähler coordinates are more solid.

## 5 F-theory uplift to 4d, $\mathcal{N} = 1$

In this section we utilize the duality between M-theory and F-theory to lift the  $l_M$ -corrections in the three-dimensional theory obtained in the previous section to  $\alpha'$ -corrections to the four-dimensional effective theory arising from F-theory compactified on  $CY_4$ . This requires the Calabi-Yau manifold to be elliptically fibered over a three-dimensional Kähler base  $B_3$ .

In the following we consider the classical result of the F-theory uplift [5]. One may parametrize the shrinking of the torus fiber by the parameter  $\epsilon \rightarrow 0$ . One then infers the scaling of the fields  $v^0 \sim \epsilon$  and  $v^\alpha \sim \epsilon^{-1/2}$ . This leads to an identification of the  $3d, \mathcal{N} = 2$  multiplet field  $L^0 = \frac{v^0}{\mathcal{V}} = \frac{1}{r^2}$  with  $r$  the radius of the  $4d/3d$  circular reduction. To keep the base volume finite in the limit one finds

$$2\pi v_b^\alpha = \sqrt{v^0} v^\alpha. \quad (5.1)$$

For simplicity, let us restrict to a smooth Weierstrass model, i.e. a geometry without non-Abelian singularities, that can be embedded in an ambient fibration with typical fibers being the weighted projective space  $W\mathbb{P}_{231}$ . This implies having just two types of divisors

$D_i$ ,  $i = 1, \dots, h^{1,1}(CY_4)$ . There is the horizontal divisor corresponding to the zero-section  $D_0$ , and the vertical divisors  $D_\alpha$ ,  $\alpha = 1, \dots, h^{1,1}(B_3)$ , corresponding to elliptic fibrations over base divisors  $D_\alpha^b$ . Denoting the Poincare-dual two-forms to the divisors by  $\omega_i = (\omega_0, \omega_\alpha)$ , one expands the Kähler form as

$$J = v^0 \omega_0 + v^\alpha \omega_\alpha, \quad (5.2)$$

where  $v^0$  is the volume of the elliptic fiber, and we choose the harmonic representatives of the class. We are now in a position to discuss the F-theory uplift of the individual terms in

$$\begin{aligned} \mathcal{T}_i = & \frac{1}{6} \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{c}_1 + \alpha_2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_2 + \int_{D_i} \tilde{c}_3 - \frac{1}{12\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{J} \\ & + \frac{1}{2}(1 + \alpha_2) \frac{1}{\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_2 \wedge \tilde{J} + (1 + \alpha_2) \int_{D_i} \tilde{*}_6(H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_2 \\ & - \frac{1}{6} \int_{D_i} \tilde{*}_6(H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_1 \wedge \tilde{c}_1, \end{aligned} \quad (5.3)$$

and

$$\mathcal{Z}_\alpha = (2\pi)^3 \int_{CY_4} c_3 \wedge \omega_\alpha, \quad (5.4)$$

where  $\mathcal{K}_i$  is the volume of the divisor  $D_i$ . Latter was discussed already in [18, 19] however, we review these results in section 5.2. Note that the relation between the eleven-dimensional Planck length  $l_M$  and the string length  $l_s$  by the M/F-theory duality is obtained as

$$2\pi l_s = \mathcal{V}^{\frac{1}{2}} l_M. \quad (5.5)$$

As in the F-theory limit one sends  $v^0 \rightarrow 0$  decompactifying the fourth dimension by sending to infinity the radius of the  $4d/3d$  circle  $r \sim \mathcal{V}^{3/2} \rightarrow \infty$ . Thus after the limit all volumes of the base  $B_3$  are expressed in terms of the string units  $l_s$ . In the following we omit the warp-factor  $\mathcal{W}$  and thus  $\mathcal{F}$  from the discussion.

In section 5.1 we shortly comment on the uplift of F-theory involving one-loop corrections resulting from integrating out massive KK-modes at one-loop in the circular reduction from four to three dimensions. As those results are not well studied in the literature we present an superficial discussion. Let us stress however, that as we are not able to fix all parameters in the  $3d$ ,  $\mathcal{N} = 2$  coordinates the ambiguity of the “one-loop” up-lift can be hidden in the following section in the uncertainty of the parameters. In section 5.2 we then analyse the terms in the Kähler potential (4.2) and Kähler metric (4.3) surviving the F-theory uplift. Finally, in section 5.3 we then combine the conclusions of sections 5.1 and 5.2 to discuss the  $4d$ ,  $\mathcal{N} = 1$  Kähler potential and Kähler metric. In particular we give a string theory interpretation of the novel corrections and discuss the breaking of the no-scale structure and the  $\alpha'^2$ -modified scalar potential.

## 5.1 The F-theory uplift

In this section we review the supergravity perspective of the F-theory lift identifying the connection in-between the four and three-dimensional fields and their kinematic couplings [5]. Note that by compactifying a general four-dimensional,  $\mathcal{N} = 1$  supergravity



theory on a circle one matches the original four-dimensional Kähler potential with the three-dimensional Kähler potential  $K$  or kinetic potential  $\tilde{K}$ . The resulting kinetic potential arising in the  $4d/3d$  circular dimensional reduction takes the form

$$\tilde{K}(r, T_\alpha) = -\log(r^2) + K^F(T_\alpha). \quad (5.6)$$

To match (5.6) with the natural three-dimensional multiplets one may split  $L^i$  and  $T_i$  such that

$$L^i = (L^0 \equiv R, L^\alpha), \quad T_i = (T_0, T_\alpha). \quad (5.7)$$

One is then led to identify that  $R$  is given by  $R = r^{-2}$ , where  $r$  is the radius of the  $4d/3d$  circle [5]. Furthermore, the fields  $T_\alpha$  remain complex scalars in four dimensions whilst  $T_0$  should be dualized already in three dimensions into vector multiplets with  $(R, A^0)$  and then uplifted to four dimensions as it arises from the four-dimensional metric. Note that one computes the dualized kinetic potential  $\tilde{K}(R, \text{Re } T_\alpha)$  by Legendre dualization as discussed in 3.1. In the F-theory limit one then identifies

$$L_b^\alpha = L^\alpha|_{\epsilon=0}, \quad T_\alpha^b = T_\alpha|_{\epsilon=0}, \quad (5.8)$$

where we denote the four-dimensional fields  $L_b^\alpha$  and  $T_\alpha^b$  due to the fact that they correspond to fields with couplings related to the base  $B_3$  representing the Calabi-Yau orientifold in the IIB picture, i.e. in the F-theory limit. Let us next review the classical analysis to determine  $K^F(T_\alpha^b)$ . Evaluating the intersection numbers  $\mathcal{K}_{ijkl}$  for an elliptic fibration the non-vanishing couplings are given by

$$\mathcal{K}_{0\alpha\beta\gamma} = \mathcal{K}_{\alpha\beta\gamma}^b, \quad \mathcal{K}_{\alpha\beta\gamma}^b = \int_{B_3} \omega_\alpha \wedge \omega_\beta \wedge \omega_\gamma. \quad (5.9)$$

The kinetic potential and coordinates take the following form for an elliptic fibration

$$\tilde{K}(L^i) = \log(R) - 2 \log(\mathcal{V}_b + \mathcal{O}(R)) + 4, \quad (5.10)$$

$$\text{Re } T_\alpha = \mathcal{K}_\alpha^b + \mathcal{O}(R), \quad \mathcal{V}_b = \frac{1}{3!} \mathcal{K}_{\alpha\beta\gamma} v_b^\alpha v_b^\beta v_b^\gamma, \quad (5.11)$$

or equivalently

$$\tilde{K}(L^i) = \log(R) + \log\left(\frac{1}{3!} \mathcal{K}_{\alpha\beta\gamma}^b L_b^\alpha L_b^\beta L_b^\gamma + \mathcal{O}(R)\right) + 4, \quad (5.12)$$

$$\text{Re } T_\alpha = \frac{1}{2!} \frac{\mathcal{K}_{\alpha\beta\gamma}^b L_b^\beta L_b^\gamma}{\hat{\mathcal{V}}_b(L_b)} + \mathcal{O}(R), \quad \hat{\mathcal{V}}_b(L_b) = \frac{1}{3!} \mathcal{K}_{\alpha\beta\gamma} L_b^\alpha L_b^\beta L_b^\gamma, \quad (5.13)$$

where we have replaced the  $L^\alpha$  with  $L_b^\alpha$  by means of (5.8) and made use of the relation  $\hat{\mathcal{V}}^b(L_b) = (\mathcal{V}^b)^{-2}$ . Performing a Legendre transformation in order to express everything in terms of  $T_\alpha^b$  and comparing the result with (5.6) with  $R = r^{-2}$  one encounters in the limit  $r \rightarrow \infty$  that

$$K^F(T_\alpha^b) = -2 \log(\mathcal{V}_b) = \log(\hat{\mathcal{V}}_b(L_b)), \quad \text{Re } T_\alpha^b = \mathcal{K}_\alpha^b = \frac{1}{2!} \frac{\mathcal{K}_{\alpha\beta\gamma}^b L_b^\beta L_b^\gamma}{\hat{\mathcal{V}}_b(L_b)}, \quad (5.14)$$

where one has to solve  $T_\alpha^b$  for  $L_b^\alpha(T_\alpha^b)$  and insert the result into  $K^F$ .

Let us next comment on the case present in this work namely where one encounters higher-order  $l_M^6$ -corrections to the three-dimensional fields. As suggested by the generic  $4d/3d$  circular reduction result and one infers for the corrected the Kähler coordinates that

$$\text{Re}T_\alpha \rightarrow \text{Re}T_\alpha^b, \quad (5.15)$$

where we analyse  $\text{Re}T_\alpha^b$  in the next section 5.2. The corrected Kähler potential (4.2) can be re-written as

$$K = -\log(R) - 2\log\left(\mathcal{V}_b\left(1 + \alpha^2 \frac{3}{2\mathcal{V}_b}((512 - \kappa_2)\mathcal{Z}_\alpha^b v_b^\alpha + \kappa_2 \mathcal{T}_\alpha^b v_b^\alpha)\right) + \mathcal{O}(R)\right), \quad (5.16)$$

by making use of the sub-leading order of  $\alpha^2$ . Thus in the limit  $r \rightarrow \infty$  one encounters

$$K^F(T_\alpha^b) = -2\log\left(\mathcal{V}_b + \alpha^2((768 - \tilde{\kappa}_2)\mathcal{Z}_\alpha^b v_b^\alpha + \tilde{\kappa}_2 \mathcal{T}_\alpha^b v_b^\alpha)\right), \quad (5.17)$$

where is  $\mathcal{Z}_\alpha^b$  the F-theory limit of  $\mathcal{Z}_\alpha$  derived in the following section 5.2 and  $\tilde{\kappa}_2 = \frac{3}{2}\kappa_2$ . The identification of the dependence  $T_\alpha^b$  is implicit.

Let us close this section with remarks on one-loop corrections to the F-theory limit resulting from integrating out massive KK-modes which is expected to modify the relation (5.15). The  $\log \mathcal{V}$ -correction to the Kähler coordinates (4.3) is reminiscent of such a one loop correction. To see this one is to perform a dimensional reduction of a general  $4d$ ,  $\mathcal{N} = 1$  supergravity theory on the circle to three dimensions where massive KK-modes are integrated out at one-loop. The case for pure supergravity is discussed in [45] which yields the three-dimensional Kähler coordinates

$$\text{Re}T_0^{1\text{-loop}} = \frac{2\pi^2}{R} - \frac{7}{48}\log(R). \quad (5.18)$$

However, we are interested in a theory with additional chiral multiplets and vector multiplets which will lead to a modification of the purely gravitational result (5.18). We are not aware of such a discussion in the literature and thus have no rigorous tool to argue for the up-lift of the  $\mathcal{Z}_\alpha \log \mathcal{V}$  correction in F-theory except the comments made in [14]. Let us assume in the following that the  $\log \mathcal{V}$ -correction in the Kähler coordinates (4.3) is absorbed entirely by the F-theory uplift. This leads us to write

$$\text{Re}T_i = \mathcal{K}_i + \alpha^2 \kappa_4 \mathcal{Z}_i \log \mathcal{V}, \quad (5.19)$$

where for simplicity we only write the logarithmic correction to the Kähler coordinates. Considering (5.19) on the elliptically fibered Calabi-Yau fourfold one finds

$$\begin{aligned} \text{Re}T_0 &= \frac{1}{R} - \frac{\alpha^2 \kappa_4}{3} \mathcal{Z}_0^b \log R - \frac{\alpha^2 \kappa_4}{3} \mathcal{Z}_0^b \log((\mathcal{V}^b)^{-3} + \mathcal{O}(R)), \\ \text{Re}T_\alpha &= \mathcal{K}_\alpha^b - \frac{\alpha^2 \kappa_4}{3} \mathcal{Z}_\alpha^b \log(R) - \frac{\alpha^2 \kappa_4}{3} \mathcal{Z}_\alpha^b \log((\mathcal{V}^b)^{-3} + \mathcal{O}(R)). \end{aligned} \quad (5.20)$$

The assumption that it is absorbed in the uplift immediately leads us to a revision of (5.15) to

$$\begin{aligned} \text{Re}T_0^{1\text{-loop}} &\rightarrow \frac{1}{R} - q_0 \log R + \frac{1}{2} q_0 K^{4d}, \\ \text{Re}T_\alpha^{1\text{-loop}} &\rightarrow \text{Re}T_\alpha^{b\text{tree}} - q_\alpha \log R + \frac{1}{2} q_\alpha K^{4d}, \end{aligned} \quad (5.21)$$

where  $K^{4d}$  is the four-dimensional classical Kähler potential. By matching (5.20) and (5.21) one fixes the charges to  $q_i = -\frac{\alpha^2 \kappa_4}{3} \mathcal{Z}_i^b + \mathcal{O}(R)$ . Note that (5.21) and (5.20) are very sensitive to a conspiracy of factors. Thus at this stage as the one-loop F-theory uplift remains elusive it cannot be excluded that a finite contribution in (5.20) may survive the F-theory uplift such as

$$\text{Re}T_\alpha^b = \mathcal{K}_\alpha^b + \alpha^2 \kappa_4 \mathcal{Z}_\alpha^b \log(\mathcal{V}_b). \quad (5.22)$$

Let us stress that different assumptions lead us to find (5.21) and (5.22) but a honest one-loop computation needs to be performed to decide their validity. Note that (5.21) would imply that by integrating out massive Kaluza-Klein modes only the three-dimensional Kähler coordinates receive modifications whilst the Kähler potential remains uncorrected.

**Critical comments on Kaluza Klein one-loop corrections.** As discussed in this section the fate of the correction (5.22) in the F-theory approach can't be determined with certainty at this stage. The study of Kaluza-Klein induced loop corrections to the Kähler potential of  $3d, \mathcal{N} = 2$  and  $3d, \mathcal{N} = 1$  of string effective supergravity actions remains an open problem. Not only in the reduction from four to three dimensions but also from the reduction from eleven to three dimensions KK induced one-loop terms may be generated.

## 5.2 Topological integrals on elliptic Calabi-Yau fourfolds

In this section we discuss the F-theory uplift of the higher-order  $l_M$ -corrections appearing in (5.3) and (5.4) resulting in  $\alpha'$ -corrections. For topological integrals we can use adjunction formulae to express Chern-classes of  $CY_4$  and the divisors  $D_\alpha$  in terms of Chern-classes of the base  $B_3$ . For details of the derivation of the adjunction formulae see appendix B.2. One infers that

$$\begin{aligned} \tilde{c}_3(D_\alpha) &= c_3(B_3) - c_1(B_3) \wedge c_2(B_3) - 60c_1^3(B_3) - 60c_1^2(B_3) \wedge \omega_0 - \tilde{c}_2(D_\alpha) \wedge \omega_\alpha, \\ \tilde{c}_2(D_\alpha) &= c_2(B_3) + 11c_1^2(B_3) + 12c_1(B_3) \wedge \omega_0 + \omega_\alpha^2, \\ \tilde{c}_1(D_\alpha) &= -\omega_\alpha, \end{aligned} \quad (5.23)$$

where the  $c_{i=1,2,3}(B_3)$  on the r.h.s. of these expressions denote the Chern classes of  $B_3$  pulled-back to  $CY_4$  restricted to  $D_\alpha$ . Note that the Poincare duals of the harmonic  $(1,1)$ -forms in (5.23) are given by  $PD(\omega_0) = B_3$ , and  $PD(\omega_\alpha) = D_\alpha$ . We choose to omit the pull-back map in expressions in this section for notational simplicity. One furthermore finds that

$$\omega_0^2 = -c_1(B_3) \wedge \omega_0. \quad (5.24)$$

Note that the new contribution to the Kähler coordinates  $\mathcal{T}_i$  is expressed as integrals on the divisors  $D_i$  where the Kählerform is inherited from the ambient  $CY_4$ . One may thus use the decomposition (5.2) as well for  $\tilde{J}$ . In the F-theory limit one finds the scalings discussed at the beginning of this section to imply

$$v^\alpha \sim \epsilon^{-\frac{1}{2}}, \quad v^0 \sim \epsilon \quad \Rightarrow \quad \mathcal{V}_\alpha = \mathcal{K}_\alpha \sim \epsilon^0. \quad (5.25)$$

Using (5.23) and (5.25) one infers the contributions in (4.3) and (4.2) which survive the F-theory limit. For the object defined as the third Chern-form of the Calabi-Yau fourfold (5.4) one finds in the limit

$$\mathcal{Z}_\alpha \longrightarrow \mathcal{Z}_\alpha^b = -60 (2\pi)^2 \int_{D_\alpha^b} c_1(B_3) \wedge c_1(B_3). \quad (5.26)$$

The leading order contributions which are non vanishing in the limit must scale as  $\mathcal{T}_\alpha \sim \mathcal{O}(\epsilon^0)$ . The integrals in (5.4) which thus contribute are

$$\begin{aligned} \int_{D_\alpha} \tilde{c}_1 \wedge \tilde{c}_2 &\longrightarrow -12 \int_{D_\alpha^b} c_1(B_3) \wedge \omega_\alpha^b, \\ \int_{D_\alpha} \tilde{c}_3 &\longrightarrow -60 \int_{D_\alpha^b} c_1(B_3) \wedge c_1(B_3) - 12 \int_{D_\alpha^b} c_1(B_3) \wedge \omega_\alpha^b, \\ \int_{D_\alpha} \tilde{*}_6(H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_2 &\longrightarrow -\frac{12}{\mathcal{K}_\alpha^b} \int_{D_\alpha^b} \omega_\alpha^b \wedge J^b \int_{D_\alpha^b} c_1(B_3) \wedge J^b + 12 \int_{D_\alpha^b} c_1(B_3) \wedge \omega_\alpha^b, \\ \frac{1}{\mathcal{K}_\alpha} \int_{D_\alpha} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_\alpha} \tilde{c}_2 \wedge \tilde{J} &\longrightarrow -\frac{12}{\mathcal{K}_\alpha^b} \int_{D_\alpha^b} \omega_\alpha^b \wedge J^b \int_{D_\alpha^b} c_1(B_3) \wedge J^b, \end{aligned} \quad (5.27)$$

where we used (A.14) and where  $D_\alpha^b$  are the divisors of the base such that their pre-image w.r.t. the projection  $\pi : CY_4 \rightarrow B_3$  gives the vertical divisors of the Calabi-Yau fourfold as  $D_\alpha = \pi^{-1}(D_\alpha^b)$ .<sup>17</sup> One thus infers the divisor integral contribution of the Kähler coordinates in the limit to take the form

$$\mathcal{T}_\alpha \longrightarrow \mathcal{T}_\alpha^b = \mathcal{Z}_\alpha^b - 18(1 + \alpha_2) (2\pi)^2 \frac{1}{\mathcal{K}_\alpha^b} \int_{D_\alpha^b} \omega_\alpha^b \wedge J^b \int_{D_\alpha^b} c_1(B_3) \wedge J^b \quad (5.28)$$

with  $\mathcal{K}_\alpha^b = \frac{1}{2!} \int_{B_3} \omega_\alpha^b \wedge J^{b2}$  the volume of the divisor  $D_\alpha^b$  and the Kähler form  $J^b = \omega_\alpha^b v_b^\alpha$ . For further use let us define

$$\mathcal{W}_\alpha^b := \frac{(2\pi)^2}{\mathcal{K}_\alpha^b} \int_{D_\alpha^b} \omega_\alpha^b \wedge J^b \int_{D_\alpha^b} c_1(B_3) \wedge J^b, \quad (5.29)$$

$$\mathcal{U}_\alpha^b := (2\pi)^2 \int_{D_\alpha^b} c_1(B_3) \wedge \omega_\alpha^b. \quad (5.30)$$

The contribution (5.29) takes a special role as it depends on the Kähler form of the divisor and thus is non-vanishing upon taking derivatives w.r.t. Kähler moduli fields. Note that the F-theory uplift absorbs two-derivatives along the fiber thus the resulting corrections are of order  $\alpha'^2$ . It would be interesting to establish a connection to the  $\alpha'^2$ -corrections to the Kähler potential predicted in the Heterotic string [46]. The  $\mathcal{U}_\alpha^b$ -correction (5.30) vanishes from (5.28) due to a vanishing pre-factor. As one may find that our constraints imposed are too restrictive this correction may survive if an additional parameter freedom

<sup>17</sup>Note that in order to rewrite the integrals we note that e.g.

$$\int_{B_3} c_1(B_3) \wedge \omega_\alpha^b \wedge \omega_\alpha^b = \int_{D_\alpha^b} c_1(B_3) \wedge \omega_\alpha^b,$$

where we again omit the pull-back map on  $c_1(B_3)$  in the r.h.s. of the equality.

is somehow introduced in the present discussion of divisor integrals. In the following we thus as well comment on its potential origin and interpretation.

Let us next comment on some special cases before providing a Type IIB string interpretation of the  $\alpha'$ -corrections in (5.26) and (5.28). Firstly, for a trivial elliptic fibration, i.e.  $CY_4 = CY_3 \times T^2$  with  $CY_3$  a Calabi-Yau threefold, one infers that  $c_i(CY_4) = c_i(CY_3)$ ,  $i = 1, 2, 3$ , in particular  $c_1(CY_3) = 0$ . Furthermore, the divisors relevant in the Kähler coordinates (4.3) are a direct product and obey  $c_1(D_\alpha^b \times T^2) = c_1(D_\alpha^b)$ ,  $c_2(D_\alpha^b \times T^2) = c_2(D_\alpha^b)$  and  $c_3(D_\alpha^b \times T^2) = 0$ , see appendix B.2 for details. One infers that in this case all corrections in (5.27) and (5.26) go to zero due to their scaling behavior in the limit  $v^0 \rightarrow 0$  and thus the  $\alpha'$ -corrections in the resulting  $4d, \mathcal{N} = 2$  theory are absent.

Secondly, one may study other  $\mathcal{N} = 2$  F-theory vacua by taking  $Y_4 = K3 \times K3$ , a configuration discussed in [47] with a focus on  $\alpha'$ -corrections. In this case  $c_3(Y_4) = 0$  and thus the  $\mathcal{Z}^b$ -correction (5.26) vanishes identically. The corrections resulting from the divisors (5.27) vanish due to analogous arguments as in the above case. Concludingly, the  $\alpha'$ -corrections discussed in this work vanish in these  $\mathcal{N} = 2$  set-ups.

Finally, let us stress that there are several additional  $l_M$ -corrections to the fourfold volume surviving the F-theory limit. Let us again go back to the example of the product geometry  $Y_4 = X_3 \times T^2$ , without  $D7$ -branes. The  $\alpha'$ -corrections involving the Type IIB axio-dilaton  $\tau$  have been computed by integrating out the whole tower of  $T^2$  Kaluza-Klein modes of the 11d supergravity multiplet [9], which results in  $v^{0-\frac{1}{2}} \chi(CY_3) E_{3/2}(\tau, \bar{\tau})$  with  $E_{3/2}$  the non-holomorphic Eisenstein series. Note that it obeys the correct scaling behavior to survive the F-theory limit. One expects that the proper treatment of the KK-modes in a generic elliptic fibration is crucial to encounter the Euler-characteristic  $\alpha'^3$ -correction [20] to the  $4d, \mathcal{N} = 1$  Kähler potential inside the F-theory framework.<sup>18</sup>

### 5.3 $4d, \mathcal{N} = 1$ Kähler potential and coordinates

The discussion of the uplift of the  $\alpha'$ -corrections in the previous sections 5.1 and 5.2 enables us to infer the resulting  $4d, \mathcal{N} = 1$  Kähler potential and coordinates. Let us use the dimensionless coefficients from now on, where all dimensionful quantities, e.g.  $\alpha'$ -corrections are expressed in terms of the string length  $l_s$ , we thus write

$$\alpha^2 \rightarrow \frac{1}{3^2 \cdot 2^{13}}. \quad (5.31)$$

One infers that

$$\mathcal{K}^{4d, \mathcal{N}=1} = -2 \log \left( \mathcal{V}_b + \alpha^2 ((768 - \tilde{\kappa}_2) \mathcal{Z}_\alpha^b v_b^\alpha + \tilde{\kappa}_2 \mathcal{T}_\alpha^b v_b^\alpha) \right), \quad (5.32)$$

and

$$\text{Re} T_\alpha^b = \mathcal{K}_\alpha^b + \alpha^2 \left( \kappa_3 \frac{\mathcal{K}_\alpha^b}{\mathcal{V}_b} \mathcal{Z}_\alpha^v v_b^\alpha + \kappa_5 \frac{\mathcal{K}_\alpha^b}{\mathcal{V}_b} \mathcal{T}_\alpha^v v_b^\alpha - 4(\kappa_3 + \kappa_5) \mathcal{T}_\alpha^b \right). \quad (5.33)$$

Note that we have argued in 5.1 that the logarithmic term may be absorbed in the F-theory uplift as a one-loop correction and is thus not present in (5.33). To verify this assumption

<sup>18</sup>An alternative approach was taken in [48].

is of great interest. One may however perform the uplift of this term which then leads to a modification of (5.33) of the form

$$\alpha^2 \kappa_4 \mathcal{Z}_\alpha^b \log \mathcal{V}_b, \quad (5.34)$$

as suggested in (5.22). Note that (5.32) and (5.33) depend on four unfixed parameters, due to the additional freedom in (5.28). Further studies are required to proof the existence of the  $\alpha'$ -corrections in (5.32) and (5.33) and especially (5.34). Nevertheless, let us proceed by giving a string theory interpretation of the novel corrections to the four-dimensional Kähler potential and coordinates.

**String theory interpretation and weak string-coupling limit.** We follow the weak string-coupling limit by Sen [17] which is performed in the complex structure moduli space of  $CY_4$  to give a weakly coupled description of F-theory in terms of Type IIB string theory on a Calabi-Yau threefold with an  $O7$ -plane and  $D7$ -branes. Where the  $CY_3$  is a double cover of the base  $B_3$  branched along the  $O7$ -plane. Let us stress that the class of this branching locus is the pull-back of  $c_1(B_3)$  to  $CY_3$ . In section 5.2 we considered the topological divisor integrals on the geometries described by the smooth Weierstrass model i.e. non-Abelian singularities are absent. In this case Sen's limit contains a single recombined  $D7$ -brane wrapping a divisor of class  $8c_1(B_3)$ . This follows from the seven-brane tadpole cancellation condition. As was noted in [49, 50] this  $D7$ -brane is of the characteristic Whitney-umbrella shape. It would be interesting to extend the study to geometries with non-Abelian singularities analogously to [19].

The  $\mathcal{Z}_\alpha^b$ -correction (5.26) was discussed extensively in [18, 19] and we refer the reader to this work for details. Let us mention here however, that in more generic geometries it morally counts the number of self-intersections of stacks of  $D7$ -branes and the  $O7$ -plane. It should arise at tree-level in string theory and is of order  $\alpha'^2$ . In the geometry studied in this work this can be checked by identifying

$$\mathcal{V}_{D7 \cap O7} = 8 \int_{CY_3} c_1^2(B_3) \wedge J_b, \quad (5.35)$$

where we omitted the pull-back map from  $B_3$  to its double cover  $CY_3$  in the integrand. To give the string theory interpretation one identifies the string amplitude capturing it by considering the Einstein-Hilbert term of the four-dimensional action in the string frame<sup>19</sup>

$$S_{(4)} \supset \frac{1}{(2\pi)^7 l_s^2 g_{IIB}^2} \int \left( \mathcal{V}_b^s - \frac{5\pi^2}{2} g_{IIB} \mathcal{V}_{D7 \cap O7}^s \right) R_{sc}^s *_4^s 1. \quad (5.36)$$

Let us recall the general formula for the Euler number of Riemann surfaces, possibly non-orientable and with boundaries, is

$$\chi(\Sigma) = 2 - 2g - b - c, \quad (5.37)$$

where  $g, b, c$  denote the genus, the number of boundaries, and the number of cross caps, respectively. One thus infers that the correction in (5.36) arises from a string amplitude

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<sup>19</sup>The superscript  $^s$  denotes quantities computed using the string frame metric.

that involves the sum over two topologies, namely the disk  $g = c = 0, b = 1$  and the projective plane  $g = b = 0, c = 1$ . These are tree-level amplitudes of the orientable open strings and non-orientable closed strings which is in agreement with the property that the correction is intrinsically  $\mathcal{N} = 1$ , i.e. its presence is constrained by having  $D7$ -branes intersecting with an  $O7$ -plane.

Let us next give a string theory interpretation of (5.28). In fact, at weak string coupling one infers that

$$\int_{D_\alpha^b} c_1(B_3) \wedge J^b \sim (\mathcal{V}_{D7 \cap D_\alpha^b} + 4\mathcal{V}_{O7 \cap D_\alpha^b}), \quad (5.38)$$

where  $\mathcal{V}_{D7}$  and  $\mathcal{V}_{O7}$  are the volumes of the  $D7$ -brane and the  $O7$ -plane in  $CY_3$ , respectively. Both volumes are in the Einstein frame and in units of  $l_s$ . By tadpole cancellation one infers  $\mathcal{V}_{D7} = 8\mathcal{V}_{O7}$ . It follows that

$$\int_{D_\alpha^b} c_1(B_3) \wedge \omega_\alpha^b \sim D7 \cap D_\alpha^b \cap D_\alpha^b + 4 O7 \cap D_\alpha^b \cap D_\alpha^b, \quad (5.39)$$

are the self-intersection curves of the base Divisors  $D_\alpha^b$  intersected with the  $D7$ -brane and the  $O7$ -plane in  $CY_3$ , respectively. Lastly, the  $\mathcal{W}_\alpha^b$ -correction in (5.29) which is of order  $\alpha'^2$  and depends on the volume the self intersection curve of  $D_\alpha^b$

$$\int_{D_\alpha^b} \omega_\alpha \wedge J^b \sim \mathcal{V}_{D_\alpha^b \cap D_\alpha^b}, \quad (5.40)$$

and furthermore

$$\int_{D_\alpha^b} c_1(B_3) \wedge J^b \sim \mathcal{V}_{D7 \cap D_\alpha^b} + 4\mathcal{V}_{O7 \cap D_\alpha^b}, \quad (5.41)$$

the volume to the intersection curves of the  $D7$ -branes and  $O7$ -planes with the base divisors  $D_\alpha^b$ . One concludes that (5.29) is a product of the curve volumes (5.40) and (5.41) weighted over the volume of the divisor  $D_\alpha^b$ . It is of the same order in the string coupling as (5.36) and is thus expected to arise equivalently from a tree-level amplitude of the orientable open string or non-orientable closed string amplitude in an orientifold background.

**4d scalar potential and no-scale condition.** We next comment on the scalar potential resulting from (5.32) and (5.33). We assume that the complex structure moduli have been fixed and thus the superpotential remains to be a function of the Kähler moduli. The F-term scalar potential of a  $4d, \mathcal{N} = 1$  theory is well known

$$V_F^{4d} = e^K \left( K_{\alpha\beta} D^\alpha W \overline{D^\beta W} - 3|W|^2 \right), \quad (5.42)$$

with the superpotential  $W = W(\text{Re}T_\alpha)$  and the Kähler covariant derivative given by

$$D^\alpha W = \frac{\partial K}{\partial \text{Re}T_\alpha} W + \frac{\partial W}{\partial \text{Re}T_\alpha}, \quad K_{\alpha\beta} = (\partial_{\text{Re}T_\alpha} \partial_{\text{Re}T_\beta} K)^{-1}. \quad (5.43)$$

Let us next discuss the special case in which the superpotential is generated by fluxes in F-theory (3.33) and non-perturbative effects are absent. We denote  $W_0$  as the vacuum expectation value of the superpotential resulting after stabilizing the complex structure



moduli. One then infers that for the Kähler potential (5.32) and Kähler coordinates (5.33) the F-term scalar potential (5.42) the resulting corrections at order  $\alpha'^2$  vanish. This cancellation was observed in [51, 52]. The terms in the Kähler coordinates (5.33) admit a functional structure which remarkably never breaks the no-scale condition. Thus the sole contribution to the scalar potential at order  $\alpha'^2$  may arise from the speculative logarithm term (5.34) to be

$$V_F = -\alpha'^2 \frac{3|W_0|^2}{2\mathcal{V}_b^3} \kappa_4 \mathcal{Z}_\alpha^b v_b^\alpha. \quad (5.44)$$

Let us emphasize that this is due to an assumption on the F-theory uplift. As the uplift of this one-loop term remains elusive note that the pre-factor of the  $\mathcal{Z}_\alpha^b$ -correction in (5.44) might be subject to change and may vanish. In the further context of this work however it is suggested that the  $\alpha'^2$ -correction breaks the no-scale structure as seen from (5.44) as it is interesting to study potential phenomenological consequences.<sup>20</sup> Let us emphasize that the corrections  $\mathcal{Z}_\alpha^b$ -correction (5.44) is of order  $\alpha'^2$  and thus leading with respect to the well known Euler-characteristic correction [20]. It is of interest to study the connection to the correction discussed in [53].

Let us close this section with two critical remarks. Firstly, the F-theory lift is performed by shrinking the fiber i.e. making the geometry singular and thus other higher-order corrections may become relevant. However, let us emphasize that all the corrections discussed in this work are of topological nature and thus are expected to be protected in the F-theory limit. Secondly, let us stress that we did not aim to prove the integration into  $3d, \mathcal{N} = 2$  variables of the reduction result. However, we suggest an Ansatz for the Kähler coordinates and Kähler potential which allow to obtain all the higher-derivative couplings in the Kähler metric obtained by dimensional reduction from the  $l_M^6$  eight-derivative couplings to eleven-dimensional supergravity. This is a necessary but not sufficient step, and it thus remains to ultimately decide on the fate of the  $\alpha'$ -corrections  $\mathcal{W}_\alpha^b$  and  $\mathcal{Z}_\alpha^b$ . In particular, this applies to the phenomenologically interesting correction to the scalar potential (5.44).

## 5.4 Moduli stabilisation

In this section we comment on the vacuum structure of the potential generated by the novel conjectured  $\alpha'$ -correction (5.44). Furthermore, we study the interplay with the well known Euler-characteristic  $\alpha'^3$ -correction to the Kähler potential

$$\xi = -g_s^{-\frac{3}{2}} (2\pi)^3 \frac{\zeta(3)}{4} \chi(CY_3), \quad (5.45)$$

with  $\chi(CY_3)$  Euler-characteristic of  $CY_3$ . Note that it is of order  $\mathcal{O}(\alpha'^3)$  and it depends on the Type IIB string coupling.<sup>21</sup> It is obtained from the parent  $\mathcal{N} = 2$  theory arising from compactification of Type IIB on Calabi-Yau orientifolds [20, 54].<sup>22</sup> Note that Calabi-Yau

<sup>20</sup>Not at least to increase the interest in the tedious study of  $\alpha'$ -corrections.

<sup>21</sup>The correction is known to depend on the dilaton  $e^{-\Phi}$ . We assume that the dilaton is stabilized by the flux background and we thus encounter the string coupling constant  $g_s = \langle e^\Phi \rangle$ .

<sup>22</sup>To compute the correction to the scalar potential resulting from (5.45) we use the Kähler potential and coordinates obtained in [54].

threefold in IIB is the double cover of the base  $B_3$  branched along the  $O7$ -plane. Thus in particular we find that  $\chi(CY_3) = \chi(B_3)$ . As we discuss intrinsic  $\mathcal{N} = 1$  vacua in this work we continue in the terminology of F-theory. We comment on the potential F-theoretic origin of the correction (5.45) in section 5.2. Note that the string coupling dependence of (5.45) makes it parametrically relevant although being sub-leading in  $\alpha'$  compared to (5.44).

Let us remark on the stability of the following scenarios in regard to higher-order corrections in  $\alpha'$  and  $g_s$  in the light of [55]. The classical correction to the scalar potential vanishes due to the no-scale condition and thus the leading order  $g_s$  and  $\alpha'$ -correction determine the vacuum. Higher-order  $\alpha'$ -corrections are parametrically under control as one stabilizes the internal space at large volumes. Moreover the string coupling constant  $g_s$  may be achieved to be parametrically small thus higher-order string loop corrections may be safely neglected. Details depend however on the relative pre-factors of the perturbative terms given by topological quantities of the internal space.

**Extrema in the generic moduli case.** In this section we discuss a scenario in which all Kähler moduli might be stabilized in a non-supersymmetric anti-de Sitter minimum for manifolds with  $\chi(B_3) < 0$ . We argue for a model independent extremum and provide a sufficient condition for the existence of a local minimum in generic geometric backgrounds for Kähler cone coordinates  $v_b^\alpha > 0$  to be,

$$\langle \mathcal{K}_\alpha^b \rangle \langle \mathcal{K}_\beta^b \rangle v_b^\alpha v_b^\beta > -\frac{1}{2} \langle \mathcal{K}_{\alpha\beta}^b \rangle \langle \mathcal{V}^b \rangle v_b^\alpha v_b^\beta, \quad \forall v_b^\alpha. \quad (5.46)$$

However, to show that is a true local minimum further studies in explicit geometries are required. The stabilization is achieved by an interplay of the correction proportional to  $\mathcal{Z}_\alpha^b$  in (5.44) with the  $\alpha'^3$  Euler-characteristic correction (5.45) to the Kähler potential [20, 54]. To achieve positivity of the four-cycle volumes in the vacuum the  $\alpha'$ -corrections additionally needs to obey strict positivity and negativity conditions, i.e. the geometric background must be suitable. Note that due to a similar potential all Kähler moduli may be stabilized for  $\chi(B_3) > 0$  as discussed in [2]. Let us emphasize that the we do not require non-perturbative effects which are generically exponentially suppressed by the volume of the cycles. In future work [56] we study a modified scenario by additionally considering the  $\alpha'^3 g_s^{-3/2}$ -correction to the scalar potential discussed in [2, 21]. The resulting potential in the large volume limit then takes the form<sup>23</sup>

$$V_F = \frac{3g_s |W_0|^2}{4 \mathcal{V}_b^3} \left( \xi + \hat{\kappa}_4 \mathcal{Z}_\alpha^b v_b^\alpha \right), \quad (5.47)$$

where  $\hat{\kappa}_4 = -\frac{1}{64}$ . We note that the functional structure is similar to the  $\alpha'$ -correction discussed in [2, 21].<sup>24</sup> One finds the AdS vacua where all four-cycle volumes  $\mathcal{K}_\alpha^b$  are stabilized at

$$\langle \mathcal{K}_\alpha^b \rangle = \Lambda^2 \mathcal{Z}_\alpha^b, \quad \text{with} \quad \Lambda = \frac{9\xi}{8|\hat{\kappa}_4|} \cdot \frac{1}{\mathcal{Z}_\alpha^b \langle v_0^\alpha \rangle}, \quad (5.48)$$

<sup>23</sup>We refer to the large volume limit to the regime at large volumes  $\mathcal{V}^b$  and weak string coupling such that higher order  $\alpha'$  and  $g_s$ -corrections can be neglected.

<sup>24</sup>The overall factor  $g_s/2$  in (5.47) stems from the dilation dependence of the Kähler potential.

where  $\langle v_0^\alpha \rangle = \frac{1}{\Lambda} \langle v_b^\alpha \rangle$  is the expectation value of the fields such that

$$\mathcal{K}_{\alpha\beta\gamma}^b \langle v_0^\beta \rangle \langle v_0^\gamma \rangle = \mathcal{Z}_\alpha^b. \quad (5.49)$$

This scaling is required to ensure that  $\mathcal{Z}_\alpha^b \langle v_b^\alpha \rangle = 9\xi / (8|\hat{\kappa}_4|)$ . In other words this additional condition can always be satisfied as one concludes from (5.49) which fixes  $\langle v_0^\alpha \rangle$  uniquely, and thus implies (5.48). One infers the volume in the extremum to be

$$\langle \mathcal{V}^b \rangle = \frac{3\xi\Lambda^2}{8|\hat{\kappa}_4|} \sim g_s^{-\frac{9}{2}}, \quad (5.50)$$

and moreover that the value of the potential in the extremum takes the form

$$\langle V^F \rangle = -\frac{3g_s\xi}{32} \cdot \frac{|W_0|^2}{\langle \mathcal{V}^b \rangle^3} \sim -g_s^{13} |W_0|^2 < 0. \quad (5.51)$$

Note that since  $\chi(B_3) < 0$  one infers that  $\xi > 0$ . In the weakly coupled string regime  $g_s < 1$  one generically achieves a large positive overall volume  $\langle \mathcal{V}^b \rangle > 0$  in (5.50). Moreover, positivity of all four-cycles volumes  $\langle \mathcal{K}_\alpha^b \rangle > 0$  for  $\mathcal{Z}_\alpha^b > 0$  for all  $\alpha = 1, \dots, h^{1,1}(B_3)$  in (5.48) and (5.49).<sup>25</sup> From (5.51) one finds that one may achieve small values of  $\langle V^F \rangle$  also for a moderately large  $|W_0|$  due to the strong string coupling suppression. By analyzing the matrix of second derivatives in the extremum one infers

$$\left\langle \frac{\partial^2 V_F}{\partial v_b^\alpha \partial v_b^\beta} \right\rangle = \frac{3g_s |W_0|^2 \Lambda^2}{4 \langle \mathcal{V}^b \rangle^5} \left( \gamma_1 \mathcal{K}_{\alpha\beta}^b + \gamma_2 \mathcal{Z}_\alpha^b \mathcal{Z}_\beta^b \right), \quad \gamma_1 = \frac{9}{64|\hat{\kappa}_4|} \xi^2, \quad \gamma_2 = \frac{3}{4} \xi \Lambda^2, \quad (5.52)$$

where one concludes that  $\gamma_1 > 0$  and  $\gamma_2 > 0$ . The matrix  $\gamma_2 \mathcal{Z}_\alpha^b \mathcal{Z}_\beta^b$  is positive semi-definite, however it was argued in [57] that  $\mathcal{K}_{\alpha\beta}^b$  is of signature  $(1, h^{1,1}(B_3))$ , i.e. it exhibits one positive eigenvalue in the direction of the vector  $\langle v_b^\alpha \rangle$ . Thus to argue for a local minimum one needs to analyse (5.52) in explicit models. One may rewrite (5.52) to be in the form

$$\left\langle \frac{\partial^2 V_F}{\partial v_b^\alpha \partial v_b^\beta} \right\rangle = \frac{9g_s |W_0|^2 \xi}{16 \langle \mathcal{V}^b \rangle^4} \left( \frac{1}{\langle \mathcal{V}^b \rangle} \langle \mathcal{K}_\alpha^b \rangle \langle \mathcal{K}_\beta^b \rangle + \frac{1}{2} \langle \mathcal{K}_{\alpha\beta}^b \rangle \right), \quad (5.53)$$

from which one infers a sufficient condition on the geometry for positive semi-definiteness of (5.53) and thus for the existence of a local minimum to be

$$\langle \mathcal{K}_\alpha^b \rangle \langle \mathcal{K}_\beta^b \rangle v_b^\alpha v_b^\beta > -\frac{1}{2} \langle \mathcal{K}_{\alpha\beta}^b \rangle \langle \mathcal{V}^b \rangle v_b^\alpha v_b^\beta, \quad \forall v_b^\alpha. \quad (5.54)$$

Note that in this paragraph we have assumed that the self-intersection numbers are vanishing to argue for the vanishing of the  $\mathcal{W}_\alpha^b$  in the scalar potential (5.47). Thus (5.54) is automatically satisfied by the non-vanishing of all four-cycle volumes in the vacuum. Thus one encounters a local minimum for those geometries. Let us next compare the gravitino mass with the string and Kaluza-Klein scale [58] for which one finds that

$$m_S \sim \langle \mathcal{V}^b \rangle^{-\frac{1}{2}}, \quad m_{KK} \sim \langle \mathcal{V}^b \rangle^{-\frac{2}{3}}, \quad m_{3/2} \sim \frac{|W_0|}{\langle \mathcal{V}^b \rangle}. \quad (5.55)$$

---

<sup>25</sup>Note that the mechanism could also be applied for different sign of the pre-factor of the  $\mathcal{Z}_\alpha^b$ -correction in (5.47) and would then lead to  $\mathcal{Z}_\alpha^b > 0$  for all  $\alpha = 1, \dots, h^{1,1}(B_3)$  with opposite overall sign in (5.48) and (5.49).

Thus one infers by using (5.50) that

$$\frac{m_{3/2}}{m_S} \sim \frac{|W_0|}{2|\Lambda|} \sqrt{\frac{3}{\xi}} \sim g_s^{\frac{9}{4}} |W_0|, \quad \frac{m_{3/2}}{m_{KK}} \sim g_s^{\frac{3}{2}} |W_0|, \quad \frac{m_{KK}}{m_S} \sim g_s^{\frac{3}{4}}. \quad (5.56)$$

Thus for a weakly coupled string regime the hierarchies in (5.56) can be satisfied accordingly. Let us conclude by noting that this mechanism might lead to a stabilization for all four-cycles for geometric backgrounds with  $\chi(B_3) < 0$ . This is achieved solely by an interplay of the Euler-Characteristic  $\alpha^3$ -correction [20] with the  $\alpha^2$ -correction [18, 19]. As the volume can be stabilized at sufficiently large values higher-order  $\alpha'$ -corrections are under control i.e. the vacuum may not be shifted.

Let us close this section with some remarks concerning the recent conjecture by [22] which in particular implies the absence of local de Sitter extrema in any controlled string theory set-up. Note that the discussion in this section can be performed analogously for  $\chi(B_3) > 0$  which then leads to a de Sitter extremum as seen by equation (5.51) with  $\xi < 0$ . To achieve a positive overall volume  $\mathcal{V}^b > 0$  in (5.50) and positivity of all four-cycles volumes  $\mathcal{K}_\alpha^b > 0$  one infers that  $\mathcal{Z}_\alpha^b < 0$  for all  $\alpha = 1, \dots, h^{1,1}(B_3)$  and the opposite overall sign choice in (5.48) and (5.49) which as well constitutes a solution. It would be interesting to study explicit geometries where  $\mathcal{Z}_\alpha^b$  takes values such that a de Sitter extremum is obtained. Let us close this section by emphasizing that the scenario in this section might thus suffice as the starting point for a concrete counter example to the conjecture [22].

## 6 Conclusions

In this work we established a connection in between eight-derivative  $l_M^6$ -couplings in eleven-dimensional supergravity i.e. the low wave length limit of M-theory, and  $\alpha'$ -corrections to the Kähler potential and Kähler coordinates of four-dimensional  $\mathcal{N} = 1$  supergravity theories. The derivation relies on the M/F-theory duality. In particular we argue for two potential novel corrections to the Kähler coordinates and potential at order  $\alpha'^2$ . Noteworthy, one of them breaks the no-scale structure. However, we are not able to ultimately decide on the fate of the proposed correction as a more complete analysis of the  $3d, \mathcal{N} = 2$  variables is required. Furthermore, it was the intention of the author to review our previous work to allow to emphasize the open questions in this research program in a self-contained way. We provide the completion of the eleven-dimensional  $G^2 R^3$  and  $(\nabla G)^2 R^2$ -sectors relevant for Calabi-Yau fourfold reductions. We suggest that it would be of great interest to match our proposal against 5-point and 6-point scattering amplitudes. Furthermore, we provide the reduction result of the  $G^2 R^3$  and  $(\nabla G)^2 R^2$ -sectors for Calabi-Yau fourfolds with an arbitrary number of Kähler moduli.

We conjecture a divisor integral basis for the three-dimensional Kähler coordinates at higher-order in  $l_M$ . This allows us to derive the non-topological higher-derivative couplings obtained in the dimensional reduction from the novel Ansatz for the Kähler potential and Kähler coordinates. We suggest that in order to prove the integration into the proposed  $3d, \mathcal{N} = 2$  variables additional non-trivial identities relating the higher-derivative building blocks are required. Then this amounts to fixing the remaining parameters in our Ansatz.

We are able to fix several parameters by ensuring compatibility with the one-modulus case in which the Kähler potential and Kähler coordinates can be determined exactly as no non-trivial higher-derivative couplings appear.

To connect the  $l_M^6$ -corrections in the three-dimensional Kähler coordinates and Kähler potential to the  $\alpha'^2$ -corrections in the  $4d, \mathcal{N} = 1$  theory we employ the well understood classical F-theory uplift. Although it is expected that a one-loop modification of the F-theory lift is needed we argue that in particular the novel  $\log(\mathcal{V})$ -contribution to the Kähler potential and coordinates. It would be interesting to perform a dimensional reduction of a generic  $4d, \mathcal{N} = 1$  supergravity theory in particular with vector and chiral multiplets where the Kaluza Klein-modes are integrated out at one-loop extending the work of [45]. The novel divisor integral contribution in four-dimensions is of order  $\alpha'^2$ . Let us stress that the ultimate fate of the novel  $\alpha'$ -corrections to the scalar potential shall be decided on in a forthcoming work. Let us continue with a critical remark. The F-theory lift is performed by shrinking the fiber of the Calabi-Yau fourfold, i.e. the geometry becomes singular in this process. In this limit other higher-order UV-corrections may become relevant and modify the uplift. However, the corrections discussed in this work are of topological nature and are thus expected to be protected in the F-theory limit.

Although the resulting  $\alpha'^2$ -corrected scalar potential arises from a conjectured correction to the Kähler coordinates it is of interest to study possible scenarios to obtain stable vacua. We discuss a scenario in which the  $\mathcal{Z}_\alpha^b$ -correction at order  $\alpha'^2$  interplays with the  $\alpha'^3$  Euler-characteristic correction to achieve a non-supersymmetric anti-de Sitter minimum for geometric backgrounds with  $\chi(B_3) < 0$ . Moreover constraints on the topological quantities of the geometric backgrounds are derived such that a minimum may be obtained. It would be of great interest to realize our constraints in explicit examples of elliptically fibered Calabi-Yau fourfolds. Furthermore, we note that the scenarios provide a model independent de Sitter extremum for geometric backgrounds with  $\chi(B_3) > 0$ . One may extend the present analysis [56] by additionally considering the  $\alpha'^3$ -correction to the scalar potential discussed in [2, 21]. Lastly, let us point the reader to an obvious extension of the present work. Our analysis of geometries does not allow for non-Abelian singularities, i.e. no non-Abelian four-dimensional gauge fields are present. It would be highly desirable to analyse the uplift of the Kähler potential and Kähler coordinates for such backgrounds.

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## A Conventions, definitions, and identities

In this work we denote the eleven-dimensional space indices by capital Latin letters  $M, N = 0, \dots, 10$  and the external ones by  $\mu, \nu = 0, 1, 2$ , and the internal complex ones by  $m, n, p = 1, \dots, 4$  and  $\bar{m}, \bar{n}, \bar{p} = 1, \dots, 4$ . The metric signature of the eleven-dimensional space is  $(-, +, \dots, +)$ . Furthermore, the convention for the totally anti-symmetric tensor in Lorentzian space in an orthonormal frame is  $\epsilon_{012\dots 10} = \epsilon_{012} = +1$ . The epsilon tensor in  $d$  dimensions then satisfies

$$\epsilon^{R_1 \dots R_p N_1 \dots N_{d-p}} \epsilon_{R_1 \dots R_p M_1 \dots M_{d-p}} = (-1)^s (d-p)! p! \delta^{N_1}_{[M_1} \dots \delta^{N_{d-p}}_{M_{d-p}]}, \quad (\text{A.1})$$

where  $s = 0$  if the metric has Riemannian signature and  $s = 1$  for a Lorentzian metric. We adopt the following conventions for the Christoffel symbols and Riemann tensor

$$\begin{aligned} \Gamma^R_{MN} &= \frac{1}{2} g^{RS} (\partial_M g_{NS} + \partial_N g_{MS} - \partial_S g_{MN}), & R_{MN} &= R^R_{MRN}, \\ R^M_{NRS} &= \partial_R \Gamma^M_{SN} - \partial_S \Gamma^M_{RN} + \Gamma^M_{RT} \Gamma^T_{SN} - \Gamma^M_{ST} \Gamma^T_{RN}, & R &= R_{MN} g^{MN}, \end{aligned} \quad (\text{A.2})$$

with equivalent definitions on the internal and external spaces. Written in components, the first and second Bianchi identity are

$$\begin{aligned} R^O_{PMN} + R^O_{MNP} + R^O_{NPM} &= 0 \\ (\nabla_L R)^O_{PMN} + (\nabla_M R)^O_{PNL} + (\nabla_N R)^O_{PLM} &= 0. \end{aligned} \quad (\text{A.3})$$

Differential  $p$ -forms are expanded in a basis of differential one-forms as

$$\Lambda = \frac{1}{p!} \Lambda_{M_1 \dots M_p} dx^{M_1} \wedge \dots \wedge dx^{M_p}. \quad (\text{A.4})$$

The wedge product between a  $p$ -form  $\Lambda^{(p)}$  and a  $q$ -form  $\Lambda^{(q)}$  is given by

$$(\Lambda^{(p)} \wedge \Lambda^{(q)})_{M_1 \dots M_{p+q}} = \frac{(p+q)!}{p!q!} \Lambda^{(p)}_{[M_1 \dots M_p} \Lambda^{(q)}_{M_1 \dots M_q]}. \quad (\text{A.5})$$

Furthermore, the exterior derivative on a  $p$ -form  $\Lambda$  results in

$$(d\Lambda)_{NM_1 \dots M_p} = (p+1) \partial_{[N} \Lambda_{M_1 \dots M_p]}, \quad (\text{A.6})$$

while the Hodge star of  $p$ -form  $\Lambda$  in  $d$  real coordinates is given by

$$(*_d \Lambda)_{N_1 \dots N_{d-p}} = \frac{1}{p!} \Lambda^{M_1 \dots M_p} \epsilon_{M_1 \dots M_p N_1 \dots N_{d-p}}. \quad (\text{A.7})$$

Moreover,

$$\Lambda^{(1)} \wedge *_d \Lambda^{(2)} = \frac{1}{p!} \Lambda^{(1)}_{M_1 \dots M_p} \Lambda^{(2) M_1 \dots M_p} *_1, \quad (\text{A.8})$$

which holds for two arbitrary  $p$ -forms  $\Lambda^{(1)}$  and  $\Lambda^{(2)}$ .

Let us next define the intersection numbers, where  $\{\omega_i\}$  are harmonic w.r.t. to the Calabi- Yau metric  $g_{m\bar{n}}$

$$\begin{aligned}\mathcal{K}_{ijkl} &= \int_X \omega_i \wedge \omega_j \wedge \omega_k \wedge \omega_l, & \mathcal{K}_{ij} &= \mathcal{K}_{ijkl} v^l, & \mathcal{K}_{ij} &= \frac{1}{2} \mathcal{K}_{ijkl} v^k v^l, \\ \mathcal{K}_i &= \frac{1}{3!} \mathcal{K}_{ijkl} v^j v^k v^l, & \mathcal{V} &= \frac{1}{4!} \mathcal{K}_{ijkl} v^i v^j v^k v^l.\end{aligned}\quad (\text{A.9})$$

Let us review well known identities such as

$$\int \omega_i \wedge * \omega_j = -\mathcal{K}_{ij} + \frac{1}{\mathcal{V}} \mathcal{K}_i \mathcal{K}_j. \quad (\text{A.10})$$

Let us note that the intersection numbers obey the properties

$$\begin{aligned}\mathcal{K}_i v^i &= 4\mathcal{V}, & \mathcal{K}_{ij} v^j &= 3\mathcal{K}_i, & \mathcal{K}_{ijk} v^k &= 2\mathcal{K}_{ij}, \\ \mathcal{K}_{ijkl} v^l &= \mathcal{K}_{ijk}, & \mathcal{K}^{ik} \mathcal{K}_{jk} &= \delta_j^i, & \mathcal{K}^{ik} \mathcal{K}_k &= \frac{1}{3} v^i \\ \left(\frac{\partial}{\partial v^k} \mathcal{K}^{ij}\right) \mathcal{K}_j &= -\frac{2}{3} \delta_k^i, & \left(\frac{\partial}{\partial v^k} \mathcal{K}^{ij}\right) \mathcal{K}_{jl} &= -\mathcal{K}^{ij} \mathcal{K}_{kl},\end{aligned}\quad (\text{A.11})$$

with the inverse intersection matrix  $\mathcal{K}^{ij}$ . The intersection numbers for the Kähler base are given by

$$\begin{aligned}\mathcal{K}_{\alpha\beta\gamma}^b &= \int_{B_3} \omega_\alpha \wedge \omega_\beta \wedge \omega_\gamma, & \mathcal{K}_{\alpha^b\beta} &= \mathcal{K}_{\alpha\beta\gamma}^b v_b^\gamma, & \mathcal{K}_\alpha^b &= \frac{1}{2} \mathcal{K}_{\alpha\beta\gamma}^b v_b^\beta v_b^\gamma, \\ \mathcal{V}^b &= \frac{1}{3!} \mathcal{K}_{\alpha\beta\gamma}^b v_b^\alpha v_b^\beta v_b^\gamma.\end{aligned}\quad (\text{A.12})$$

One may show that for a six-dimensional Kähler manifold

$$*_6 (\omega_\alpha^b \wedge J^b) = \frac{\mathcal{K}_\alpha^b}{\mathcal{V}^b} J^b - \omega_\alpha^b, \quad (\text{A.13})$$

with intersection numbers defined analogously to (A.9). In particular, this implies the analogous relation

$$\int_{D_\alpha} *_6 (H \tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_2 = \frac{1}{\mathcal{K}_\alpha} \int_{D_\alpha} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_\alpha} \tilde{c}_2 \wedge \tilde{J} - \int_{D_\alpha} \tilde{c}_1 \wedge \tilde{c}_2, \quad (\text{A.14})$$

which holds due to the harmonicity of  $H \tilde{c}_1(D_\alpha)$ .

We define the curvature two-form for Hermitian manifolds to be

$$\mathcal{R}^m{}_n = R^m{}_{nr\bar{s}} dz^r \wedge d\bar{z}^{\bar{s}}, \quad (\text{A.15})$$

and

$$\begin{aligned}\text{Tr} \mathcal{R} &= R^m{}_{mr\bar{s}} dz^r \wedge d\bar{z}^{\bar{s}}, \\ \text{Tr} \mathcal{R}^2 &= R^m{}_{nr\bar{s}} R^n{}_{mr_1\bar{s}_1} dz^r \wedge d\bar{z}^{\bar{s}} \wedge dz^{r_1} \wedge d\bar{z}^{\bar{s}_1}, \\ \text{Tr} \mathcal{R}^3 &= R^m{}_{nr\bar{s}} R^n{}_{n_1 r_1 \bar{s}_1} R^{n_1}{}_{m r_2 \bar{s}_2} dz^r \wedge d\bar{z}^{\bar{s}} \wedge dz^{r_1} \wedge d\bar{z}^{\bar{s}_1} \wedge dz^{r_2} \wedge d\bar{z}^{\bar{s}_2}.\end{aligned}\quad (\text{A.16})$$



The Chern forms can be expressed in terms of the curvature two-form as

$$\begin{aligned}
 c_1 &= i\text{Tr}\mathcal{R}, \\
 c_2 &= \frac{1}{2}(\text{Tr}\mathcal{R}^2 - (\text{Tr}\mathcal{R})^2), \\
 c_3 &= \frac{1}{3}c_1c_2 + \frac{1}{3}c_1 \wedge \text{Tr}\mathcal{R}^2 - \frac{i}{3}\text{Tr}\mathcal{R}^3, \\
 c_4 &= \frac{1}{24}(c_1^4 - 6c_1^2\text{Tr}\mathcal{R}^2 - 8ic_1\text{Tr}\mathcal{R}^3) + \frac{1}{8}((\text{Tr}\mathcal{R}^2)^2 - 2\text{Tr}\mathcal{R}^4).
 \end{aligned} \tag{A.17}$$

The Chern classes of a  $n$  complex-dimensional Calabi-Yau manifold  $CY_n$  reduce to

$$c_3(CY_{n \geq 3}) = -\frac{i}{3}\text{Tr}\mathcal{R}^3 \quad \text{and} \quad c_4(CY_{n \geq 4}) = \frac{1}{8}((\text{Tr}\mathcal{R}^2)^2 - 2\text{Tr}\mathcal{R}^4), \tag{A.18}$$

with  $\text{Tr}\mathcal{R}^4$  defined analogous as in (A.16). Let us next define a set of higher-derivative building blocks identified in [16] as

$$Z_{m\bar{m}n\bar{n}} = \frac{1}{4!}\epsilon_{m\bar{m}m_1\bar{m}_1m_2\bar{m}_2m_3\bar{m}_3}\epsilon_{n\bar{n}n_1\bar{n}_1n_2\bar{n}_2n_3\bar{n}_3}R^{(0)\bar{m}_1m_1\bar{n}_1n_1}R^{(0)\bar{m}_2m_2\bar{n}_2n_2}R^{(0)\bar{m}_3m_3\bar{n}_3n_3}, \tag{A.19}$$

and

$$\begin{aligned}
 Y_{ijm\bar{n}} &= \frac{1}{4!}\epsilon_{m\bar{m}m_1\bar{m}_1m_2\bar{m}_2m_3\bar{m}_3}\epsilon_{n\bar{n}n_1\bar{n}_1n_2\bar{n}_2n_3\bar{n}_3}\nabla^{(0)n}\omega_i^{(0)\bar{m}_1m_1}\nabla^{(0)\bar{m}}\omega_j^{(0)\bar{n}_1n_1} \\
 &\quad \times R^{(0)\bar{m}_2m_2\bar{n}_2n_2}R^{(0)\bar{m}_3m_3\bar{n}_3n_3}.
 \end{aligned} \tag{A.20}$$

It turns out that the tensor  $Z_{m\bar{m}n\bar{n}}$  given in (A.19) plays a central role in the following and is related to the key topological quantities on  $Y_4$ . It satisfies the identities

$$Z_{m\bar{m}n\bar{n}} = Z_{n\bar{m}m\bar{n}} = Z_{m\bar{n}n\bar{m}}, \quad \nabla^{(0)m}Z_{m\bar{m}n\bar{n}} = \nabla^{(0)\bar{m}}Z_{m\bar{m}n\bar{n}} = 0. \tag{A.21}$$

It is related to the third Chern-form  $c_3^{(0)}$  via

$$\begin{aligned}
 Z_{m\bar{m}} &= i2Z_{m\bar{m}n\bar{n}}{}^n = (2\pi)^3 \frac{1}{2}(*^{(0)}c_3^{(0)})_{m\bar{m}}, \\
 Z &= i2Z_m{}^m = (2\pi)^3 *^{(0)}(J^{(0)} \wedge c_3^{(0)}), \quad (2\pi)^3 *^{(0)}(c_3^{(0)} \wedge \omega_i^{(0)}) = -2Z_{m\bar{n}}\omega_i^{(0)\bar{n}m},
 \end{aligned} \tag{A.22}$$

and yields the fourth Chern-form  $c_4^{(0)}$  by contraction with the Riemann tensor as

$$Z_{m\bar{m}n\bar{n}}R^{(0)\bar{m}m\bar{n}n} = (2\pi)^4 *^{(0)}c_4^{(0)}. \tag{A.23}$$

We note that  $Y_{ijm\bar{n}}$  is also related to  $Z_{m\bar{m}n\bar{n}}$  upon integration as

$$\int_{Y_4} Y_{ijm\bar{n}}{}^m *^{(0)}1 = -\frac{1}{6} \int_{Y_4} (iZ_{m\bar{n}}\omega_i^{(0)\bar{r}m}\omega_j^{(0)\bar{n}}{}_{\bar{r}} + 2Z_{m\bar{n}r\bar{s}}\omega_i^{(0)\bar{n}m}\omega_j^{(0)\bar{s}r}) *^{(0)}1, \tag{A.24}$$

where the right hand side represents the same linear combination that will be relevant in 4.1. Let us for further use define

$$\mathcal{Y}_{ij} := \int_{Y_4} Y_{ijm\bar{n}}{}^m *^{(0)}1. \tag{A.25}$$

Lastly in this work we encounter a new (2,2)-form object

$$\Omega_{ij} = R_{m\bar{n}r\bar{s}}^{(0)}\omega_i^{(0)r}{}_{\bar{t}}\omega_j^{(0)\bar{s}}{}_{\bar{u}} dz^m \wedge dz^t \wedge d\bar{z}^{\bar{n}} \wedge d\bar{z}^{\bar{u}}. \tag{A.26}$$

### A.1 Divisor integrals in terms of $CY_4$ integrals

We define an arbitrary basis of higher-derivative  $(1, 1)$  -forms convenient for the computations in this work

$$\begin{aligned}
 X_1 &= R_m^{m_2 m_5 n_2} R_{m_2}^{n_3 n_4} R_{n_3 \bar{m} n_4}^{m_5} dz^m \wedge d\bar{z}^{\bar{m}} \\
 X_2 &= R_m^{m_2 m_5 n_2} R_{m_2 \bar{m} n_3}^{n_4} R_{n_2}^{m_5 n_4 n_3} dz^m \wedge d\bar{z}^{\bar{m}} \\
 X_3 &= R_{m \bar{m} m_2}^{m_5} R_{m_5}^{n_2 n_3 n_4} R_{n_2}^{m_2 n_4 n_3} dz^m \wedge d\bar{z}^{\bar{m}} \\
 X_4 &= g_{m \bar{m}} R_{m_1}^{m_2 m_5 n_2} R_{m_2}^{n_3 n_4} R_{n_3}^{m_1 n_4 m_5} dz^m \wedge d\bar{z}^{\bar{m}} \\
 X_5 &= g_{m \bar{m}} R_{m_1}^{m_2 m_5 n_2} R_{m_2}^{m_1 n_3 n_4} R_{n_2}^{m_5 n_4 n_3} dz^m \wedge d\bar{z}^{\bar{m}}
 \end{aligned} \tag{A.27}$$

These  $(1, 1)$ -forms can be expressed as integrals on Calabi-Yau fourfolds which admit an interpretation as integrals on divisors  $D_i$  of a Calabi-Yau fourfold as

$$\int_{CY_4} (*_8 X_{k=1, \dots, 5}) \wedge \omega_i = \int_{D_i} *_8 X_{k=1, \dots, 5}, \tag{A.28}$$

where the r.h.s. is to be seen as pulled back to the divisor. Let us now recall the fact [40] that any complex sub-manifold of a Kähler manifold  $M$  is itself Kähler with induced metric and Kähler form  $g, J$  of  $M$ . Thus in particular we find for the Divisors  $i : D_i \hookrightarrow CY_4$  the Kähler metric and form  $*ig$  and  $*iJ$ , respectively, which are pulled back from the Calabi-Yau fourfold. One may thus as well restrict Riemann tensors on the Calabi-Yau fourfold to divisors  $D_i$  expressed by the induced metric which generically obeys  $c_1(D_i) \neq 0$ . In particular contractions of the Riemann tensors which do not vanish on the Calabi-Yau manifold due to the Calabi-Yau conditions may be pulled back to the divisors and expressed in terms of Riemann tensors in terms of the induced metric on  $D_i$ . Note that the  $(1, 1)$ -forms in (A.27) expressed as integrals on divisors (A.28) are of this form. By

We may write the Kähler coordinates as (4.4) in terms as the new basis on  $CY_4$  in the following way if the coefficients obey the following relations

$$\begin{aligned}
 \alpha_5 &= -\frac{1}{8}\alpha_1 + \frac{1}{24}\alpha_3 + \frac{1}{4}\alpha_4, \\
 \alpha_6 &= \frac{1}{2}\alpha_2 + \frac{1}{2}\alpha_3, \\
 \alpha_7 &= \alpha_2 + \alpha_3, \\
 \alpha_8 &= \frac{1}{2}\alpha_1 - \frac{1}{3}\alpha_3 - \alpha_4, \\
 \alpha_9 &= -\alpha_1 + \frac{1}{6}\alpha_3,
 \end{aligned} \tag{A.29}$$

one then infers that

$$\begin{aligned}
 \mathcal{T}_i &= -\frac{i}{3} \int_{CY_4} \omega_i \wedge *_8 \Big( (\alpha_3 + 3\gamma_2 + 6\gamma_3)X_1 + (\alpha_3 + 3\gamma_1 - 12\gamma_3)X_2 + 3(\gamma_3 - \alpha_3)X_3 \\
 &\quad + 3(-\gamma_2 + \gamma_4 + \gamma_5)X_4 - 3(\gamma_1 + 2\gamma_4 + 2\gamma_5)X_5 \Big),
 \end{aligned} \tag{A.30}$$

where  $X_{i=1,2,3,4,5}$  are defined in (A.27), and where the freedom in the real parameters  $\gamma_1, \dots, \gamma_5$  results due to total derivatives which take different form on the divisors integrals and Calabi-Yau fourfold integrals, respectively. The simplest choice in this work for coefficients  $\gamma_1, \gamma_2, \gamma_3$  defines the higher-derivative (3, 3)-form to be

$$\mathcal{X} = -\frac{i}{3} *_8 (X_1 + X_2 - X_3), \quad (\text{A.31})$$

and thus the Kähler coordinate modification is

$$\mathcal{T}_i = -\frac{i}{3} \int_{CY_4} \omega_i \wedge *_8 (X_1 + X_2 - X_3). \quad (\text{A.32})$$

Note that  $\alpha_3 = 1$  which is in agreement with the divisor integral one-modulus limit.<sup>26</sup> Note that the choice of fixing  $\alpha_1$  does not limit the Ansatz for the Kähler coordinates as it amounts only to an overall coefficients which is anyway taken into account for in (4.3). One may easily show that thus

$$\mathcal{T}_i v^i = \int_{CY_4} J \wedge \mathcal{X} = \mathcal{Z}, \quad (\text{A.33})$$

and from this property (A.33) that

$$\begin{aligned} \left( \frac{\partial}{\partial v^j} \mathcal{T}_i \right) v^i &= -\mathcal{T}_j + \mathcal{Z}_j \\ \left( \frac{\partial^2}{\partial v^k \partial v^j} \mathcal{T}_i \right) v^i &= - \left( \frac{\partial}{\partial v^j} \mathcal{T}_k \right) - \left( \frac{\partial}{\partial v^k} \mathcal{T}_j \right). \end{aligned} \quad (\text{A.34})$$

Let us comment on (A.31). the combination of basis elements  $X_{i=1,2,3,4,5}$  is a choice compatible with the match to six-dimensional divisor integrals. In section A.3 we discuss the variation of  $\mathcal{T}_i$  w.r.t. to the Kähler deformations.

As the matching of the correction to the Kähler coordinates in terms of  $CY_4$  integrals to the divisor integral expression is not unique, let us close this section on remarks other possible choices of  $\gamma_1, \gamma_2, \gamma_3$ . Due to (A.33) the Ansatz (4.2) and (4.3) cannot depend separately on  $\mathcal{T}_i v^i$ . It is interesting to study the possible where (4.2) is modified by this expression as well and (4.3) by  $\frac{1}{\mathcal{V}} \mathcal{K}_i \mathcal{T}_j v^j$ . Let us close this section by discussing a caveat to the Ansatz in this work namely that our choice for  $\mathcal{T}_i$  (A.31) may be rewritten by splitting integrals using the harmonicity of  $\omega_i$

$$\mathcal{T}_i = \frac{1}{3} \left( -\mathcal{Z}_i + \frac{1}{\mathcal{V}} \mathcal{K}_i \mathcal{Z} \right). \quad (\text{A.35})$$

Let us emphasize that the insights of this work is that the higher-derivative structures derived in dimensional Calabi-Yau fourfold reductions for  $h^{1,1} > 1$  can be obtained by variation of  $\mathcal{T}_i$  before applying the integral split (A.35) which suggests an interpretation in terms of divisor integrals. One infers that by imposing (A.35) first the Ansatz for the Kähler coordinates (4.3) does not carry any new information, i.e. those to steps seem not

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<sup>26</sup>The coefficients in (A.30) are chosen as  $\gamma_1 = 8/3$ ,  $\gamma_2 = -4/3$ ,  $\gamma_3 = 2/3$ ,  $\gamma_5 = -4/3 - \gamma_4$ .

to commute. However, by choosing a more involved combination for the correction to the Kähler coordinate in terms of Calabi-Yau fourfold quantities in (A.30) this caveat can be prevented as then no analogous relation for (A.35) holds. Generically we expect the form  $\mathcal{T}_i + \mathcal{T}_i^0$  where in the one-modulus limit  $\mathcal{T}_i \rightarrow \tilde{\mathcal{Z}}$  and  $\mathcal{T}_i^0 \rightarrow 0$ . This suggests that one might need to extend the basis (A.27) to also contain terms with explicit covariant derivatives such as e.g.  $\sim (\nabla R)^2$ . Moreover, one may not expect to capture the information of topological quantities of divisors entirely by local covariant integral densities on the entire space but may need to include additional global obstructions to succeed in the matching.

## A.2 3d Kähler coordinates as topological divisor integrals

In this section we argue that the Ansatz for the Kähler coordinates (4.4) may be rewritten in terms of topological integrals by fixing the coefficients in the Ansatz. Any closed form on such as  $\tilde{c}_1$  may be written in terms of its harmonic part plus a double exact contribution

$$\tilde{c}_1 = H\tilde{c}_1 + \partial\bar{\partial}\lambda, \quad (\text{A.36})$$

where  $\lambda$  is a function on the divisor. From the closure of  $\tilde{c}_1$  we infer that

$$\nabla_{[m}\tilde{R}_{n]\bar{n}r}{}^r = 0. \quad (\text{A.37})$$

But equivalently one may use that

$$\begin{aligned} \nabla_m \tilde{R}_n{}^n{}_r{}^r &= \nabla_m \nabla_n \nabla^n \lambda, \\ \nabla^m \tilde{R}_n{}^n{}_r{}^r &= \nabla^m \nabla_n \nabla^n \lambda, \\ \nabla_m \tilde{R}_n{}^m{}_r{}^r &= \nabla_m \nabla_n \nabla^m \lambda, \\ \nabla^m \tilde{R}_m{}^n{}_r{}^r &= \nabla^m \nabla_n \nabla^m \lambda. \end{aligned} \quad (\text{A.38})$$

Using the above set of equations one may show that the Ansatz for the Kähler coordinates (4.4) can be written as

$$\begin{aligned} \mathcal{T}_i &= \alpha_1 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{c}_1 + \alpha_2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_2 + \alpha_3 \int_{D_i} \tilde{c}_3 + \frac{\alpha_4}{\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{c}_1 \wedge \tilde{J} \\ &+ \frac{\alpha_5}{\mathcal{K}_i^2} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 + \frac{\alpha_6}{\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_2 \wedge \tilde{J} \\ &+ 2\alpha_6 \int_{D_i} \tilde{*}_6(H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_2 - (2\alpha_4 + 8\alpha_5) \int_{D_i} \tilde{*}_6(H\tilde{c}_1 \wedge \tilde{J}) \wedge \tilde{c}_1 \wedge \tilde{c}_1 \\ &- \frac{4\alpha_5}{\mathcal{K}_i} \int_{D_i} \tilde{c}_1 \wedge \tilde{J}^2 \int_{D_i} \tilde{c}_1 \wedge \tilde{*}_6 H\tilde{c}_1, \end{aligned} \quad (\text{A.39})$$

where  $\mathcal{K}_i$  denotes the volume of the divisor  $D_i$ . Note that in order to obtain (4.18) one fixes the coefficients such that

$$\alpha_7 = 2\alpha_6, \quad \alpha_8 = 2\alpha_4 + 8\alpha_5, \quad \alpha_9 = -4\alpha_5. \quad (\text{A.40})$$

Additionally requiring that we can write  $\mathcal{T}_i$  as integrals on the Calabi-Yau fourfold i.e. the constraints (4.8) then imposes

$$\begin{aligned} \alpha_1 &= \frac{1}{6}, & \alpha_3 &= 1, & \alpha_4 &= -\frac{1}{12}, & \alpha_5 &= 0, \\ \alpha_6 &= \frac{1}{2} + \frac{1}{2}\alpha_2, & \alpha_7 &= 1 + \alpha_2, & \alpha_8 &= -\frac{1}{6}, & \alpha_9 &= 0. \end{aligned} \quad (\text{A.41})$$

Note that this coordinate (A.41) depends on the free parameter  $\alpha_2$ . It would be interesting to determine it by imposing some other constraint.

### A.3 Variation w.r.t. Kähler moduli fields

To compute the variation of covariant integral densities such as (3.12) w.r.t. Kähler moduli fields we deform the Calabi-Yau fourfold metric  $g_{m\bar{n}}$  in complex coordinates by

$$g_{m\bar{n}} \rightarrow g_{m\bar{n}} + i\delta v^i \omega_{im\bar{n}} \quad \text{and} \quad g^{\bar{n}m} \rightarrow g^{\bar{n}m} - i\delta v^i \omega_i^{\bar{n}m}. \quad (\text{A.42})$$

The determinant of the metric subject to (A.42) derives to

$$\sqrt{-g} \rightarrow \sqrt{-g} + i\sqrt{-g} v^i \omega_{im}^{\bar{m}}. \quad (\text{A.43})$$

Note that we are only interested in linear deformations here thus we need to expand the expression to  $\mathcal{O}(\delta v^i)$ . The Riemann tensors variation compute to

$$R_{m\bar{m}n\bar{n}} \rightarrow R_{m\bar{m}n\bar{n}} + i\delta v^i \nabla_m \nabla_{\bar{m}} \omega_{i\bar{n}\bar{n}} + \frac{i}{2} \delta v^i R_{m\bar{m}n}{}^r \omega_{ir\bar{m}} + \frac{i}{2} \delta v^i R_{m\bar{m}n}{}^r \omega_{ir\bar{n}}. \quad (\text{A.44})$$

To evaluate the variation of higher-derivative object a computer algebra package such as xTensor [42] is highly desirable. One may employ its power to generate a complete set of Shouten identities, Bianchi identities and total derivatives to show that the variation of (A.32) can be written as

$$\frac{\partial}{\partial v^j} \mathcal{T}_i = -\frac{3}{\mathcal{V}} \mathcal{K}_j \mathcal{T}_i + \frac{5}{\mathcal{V}} \mathcal{K}_i \mathcal{T}_j + 3 \mathcal{T}_{ij} + \Lambda_{ij}, \quad (\text{A.45})$$

where

$$\mathcal{T}_{ij} = \int_{CY_4} *_8 (\omega_i \wedge \omega_j \wedge J) \wedge \mathcal{X}, \quad (\text{A.46})$$

and

$$\begin{aligned} \Lambda_{ij} &= 4i \int_{CY_4} Z_{m\bar{n}} \omega_i^{\bar{n}s} \omega_{js}{}^m * 1 - 6i \int_{CY_4} Z_{m\bar{n}} \omega_j^{\bar{m}n} \omega_{is}{}^s * 1 \\ &= 4i \int_{CY_4} Z_{m\bar{n}} \omega_i^{\bar{n}s} \omega_{js}{}^m * 1 + \frac{3}{\mathcal{V}} \mathcal{Z}_j \mathcal{K}_i. \end{aligned} \quad (\text{A.47})$$

Let us stress that in order to compute (4.14) we make extensive use of the computer algebra package [42], and a non-publicly self-developed extension for complex manifolds and tools to perform the above computation. By using the relation

$$\mathcal{Y}_{ij} = -\frac{1}{6} \int_{Y_4} (i Z_{m\bar{n}} \omega_i^{\bar{r}m} \omega_j^{\bar{n}}{}_{\bar{r}} + 2 Z_{m\bar{n}r\bar{s}} \omega_i^{\bar{n}m} \omega_j^{\bar{s}r}) * 1. \quad (\text{A.48})$$

We note in section A.1 that the we are not able to fix  $\mathcal{T}_i$  precisely in this work. Thus let us present here the variation of a different possible choice of the parameter freedom in (A.30) which one may show then leads to analogous expression as (A.45). It is intriguing to note that one can obtain also the novel higher-derivative structure in (3.21) by variation of the alternative Kähler coordinates

$$\frac{\partial}{\partial v^j} \mathcal{T}_i^{alt} \supset \int_{CY_4} c_2 \wedge \Omega_{ij} + \int_{CY_4} c_2 \wedge J \wedge \Omega_{ij}^1 + \int_{CY_4} c_2 \wedge J \wedge \Omega_{ij}^2 \quad (\text{A.49})$$

with  $\Omega_{ij}$  defined in (A.26) and with the (1,1)-forms

$$\Omega_{ij\,m\bar{n}}^1 := (\nabla_m \nabla_{\bar{n}} \omega_{i\bar{r}\bar{s}}) \omega_j^{\bar{s}r} \quad \Omega_{ij\,m\bar{n}}^2 := (\nabla_r \nabla^r \omega_{im\bar{s}}) \omega_j^{\bar{s}\bar{n}}. \quad (\text{A.50})$$

Note that the second Chern-form  $c_2$  appears in this case (A.49) in particular in the combination as in (3.21). Note that (A.49) is of schematic form and we do not specify the factors in this work.

**Warp-factor and the Kähler potential.** Let us next review the integration of the warp factor into a Kähler potential following [15]. To begin with, let us reduce our Ansatz (4.3) and (4.2) to the warp factor related quantities which gives

$$K = -3 \log \left( \mathcal{V} + 4\alpha^2 \mathcal{W}_i v^i \right) \quad (\text{A.51})$$

We therefore suggest that they take the form

$$\text{Re} T_i = \mathcal{K}_i + \alpha^2 \left( \mathcal{F}_i + 3\mathcal{W}_i \right) \quad (\text{A.52})$$

where  $D_i$  are  $h^{1,1}(Y_4)$  divisors of  $Y_4$  that span the homology  $H_2(Y_4, \mathbb{R})$ . The six-form  $F_6$  in this expression is a function of degrees of freedom associated with the internal space metric. It is constrained by a relation to the fourth Chern form  $c_4$  such that  $F_6$  determines the non harmonic part of  $c_4$  as

$$c_4 = H c_4 + i \partial \bar{\partial} F_6. \quad (\text{A.53})$$

Note that (A.53) leaves the harmonic and exact part of  $F_6$  unfixed and we will discuss constraints on these pieces in more detail below. The justification of the first term in  $\text{Re} T_i$  is simpler. Remarkably, this definition of the Kähler coordinates as  $D_i$  integrals will help us to obtain the couplings  $\int e^{3\alpha^2 W^{(2)}} J \wedge J \wedge \omega_i \wedge \omega_j$ , which, as we stressed in our previous work [16], cannot be obtained as  $v^i$ -derivatives of the considered  $CY_4$ -integrals. In order to evaluate the derivatives of  $T_i$  with respect to  $v^i$  and to make contact with the Kähler metric found in the reduction result (3.19), we have to rewrite the integrals over  $D_i$  into integrals over  $CY_4$ . Due to the appearance of the warp-factor and the non-closed form  $F_6$  in (A.52) this is not straightforward. In particular, one cannot simply use Poincaré duality and write  $T_i$  as an integral over  $CY_4$  with inserted  $\omega_i$ . Of course, it is always possible to write  $T_i$  as a  $CY_4$  integral when inserting a delta-current localized on  $D_i$ , i.e.

$$\text{Re} T_i = \int_{CY_4} \left( \frac{1}{3!} e^{3\alpha^2 W^{(2)}} J \wedge J \wedge J + 1536 \alpha^2 F_6 \right) \wedge \delta_i, \quad (\text{A.54})$$

where  $\delta_i$  is the (1,1)-form delta-current that restricts to the divisor  $D_i$ . Appropriately extending the notion of cohomology to include currents [59], we can now ask how much  $\delta_i$  differs from the harmonic form  $\omega_i$  in the same class. In fact, any current  $\delta_i$  is related to the harmonic element of the same class  $\omega_i$  by a doubly exact piece as

$$\delta_i = \omega_i + i\partial\bar{\partial}\lambda_i. \quad (\text{A.55})$$

This equation should be viewed as relating currents. Importantly, as we assume  $D_i$  and hence  $\delta_i$  to be  $v^i$ -independent, the  $v^i$  dependence of the harmonic form  $\omega_i$  and the current  $\lambda_i$  has to cancel such that  $\partial_j\omega_i = -i\partial\bar{\partial}\partial_j\lambda_i$ . Importantly, once we determine  $\partial_j\text{Re}T_j$  we can express the result as  $Y_4$ -integrals without invoking currents. We therefore need to understand how each part of  $T_i$  varies under a change of moduli. This will also fix the numerical factor in front of  $F_6$  in (A.52).

In order to take derivatives of  $T_i$  we first use the fact that  $D_i$  and hence  $\delta_i$  are independent of the moduli  $v^i$ , which implies

$$\partial_j\text{Re}T_i = \int_{Y_4} \left( \frac{1}{2}e^{3\alpha^2 W^{(2)}}\omega_j \wedge J \wedge J + \frac{1}{2}\alpha^2\partial_j W^{(2)}J \wedge J \wedge J + 1536\alpha^2\partial_j F_6 \right) \wedge \delta_i. \quad (\text{A.56})$$

We next claim that we can replace  $\delta_i$  with  $\omega_i$  such that finally

$$\partial_j\text{Re}T_i = \frac{1}{2} \int_{Y_4} e^{3\alpha^2 W^{(2)}}\omega_i \wedge \omega_j \wedge J \wedge J + \frac{1}{2}\alpha^2 \int_{Y_4} \partial_j W^{(2)}\omega_i \wedge J \wedge J \wedge J + 1536\alpha^2 \int_{Y_4} \omega_i \wedge \partial_j F_6. \quad (\text{A.57})$$

Note that by using (A.55) the two expressions (A.56) and (A.57) only differ by a term involving  $\partial\bar{\partial}\lambda_i$ . By partial integration this term is proportional to

$$\begin{aligned} & \int_{Y_4} \lambda_i \partial\bar{\partial} \left( \frac{1}{2}e^{3\alpha^2 W^{(2)}}\omega_j \wedge J \wedge J + \frac{1}{2}\alpha^2\partial_j W^{(2)}J \wedge J \wedge J + 1536\alpha^2\partial_j F_6 \right) \\ &= \int_{Y_4} \lambda_i \left( \frac{1}{2}\partial\bar{\partial}(e^{3\alpha^2 W^{(2)}})\omega_j \wedge J \wedge J + \frac{1}{2}\alpha^2\partial\bar{\partial}(\partial_j W^{(2)})J \wedge J \wedge J + 1536\alpha^2\partial\bar{\partial}\partial_j F_6 \right). \end{aligned} \quad (\text{A.58})$$

It is now straightforward to see that the terms multiplying  $\lambda_i$  are simply the  $\partial_j$  derivative of the warp-factor equation (3.5). One first writes (3.5) as

$$d^\dagger de^{3\alpha^2 W^{(2)}} *_8 1 - \alpha^2 Q_8 = -\frac{1}{3}i\partial\bar{\partial}(e^{3\alpha^2 W^{(2)}}) \wedge J \wedge J \wedge J - \alpha^2 Q_8. \quad (\text{A.59})$$

Then one takes the  $v^j$ -derivative of (A.59) by using the fact that  $Q_8$  is given via

$$Q_8 = -\frac{1}{2}G^{(1)} \wedge G^{(1)} - 3^2 2^{13} X_8^{(0)}, \quad (\text{A.60})$$

which can easily be inferred by comparison to (3.5) and (A.53). The moduli dependence of  $Q_8$  only arises from the term involving  $F_6$ , i.e. one has  $\partial_i Q_8 = 3072 i \partial\bar{\partial}\partial_i F_6$ . Hence one finds exactly the terms in (A.58) such that this  $\lambda_i$  dependent part of the  $T_i$  variation vanishes due to the warp-factor equation (3.5). The final expression (A.57) is then written as

$$\partial_j\text{Re}T_i = \frac{1}{2} \int_{Y_4} e^{3\alpha^2 W^{(2)}}\omega_i \wedge \omega_j \wedge J \wedge J + 3\alpha^2 \mathcal{K}_i \mathcal{W}_j + 1536\alpha^2 \int_{Y_4} \omega_i \wedge \partial_j F_6. \quad (\text{A.61})$$

Evaluating (4.11) effective action will depend on the quantities

$$\int_{Y_4} \omega_i \wedge \partial_j F_6| \quad \text{and} \quad \int_{Y_4} J \wedge \partial_i \partial_j F_6|, \quad (\text{A.62})$$

in order for the results to match the reduction result those terms need to interact with the higher-derivative building blocks. One may use the freedom in the definition (A.53) to accomplish this task. A concise match with the reduction result is beyond the scope of this work.

## B Higher-derivatives and F-theory

### B.1 11d higher-derivative terms

The terms  $t_8 t_8 R^4$  and  $t_8 t_8 G^2 R^3$  in require the definition

$$\begin{aligned} t_8^{N_1 \dots N_8} = & \frac{1}{16} \left( -2 \left( g^{N_1 N_3} g^{N_2 N_4} g^{N_5 N_7} g^{N_6 N_8} + g^{N_1 N_5} g^{N_2 N_6} g^{N_3 N_7} g^{N_4 N_8} + g^{N_1 N_7} g^{N_2 N_8} g^{N_3 N_5} g^{N_4 N_6} \right) \right. \\ & + 8 \left( g^{N_2 N_3} g^{N_4 N_5} g^{N_6 N_7} g^{N_8 N_1} + g^{N_2 N_5} g^{N_6 N_3} g^{N_4 N_7} g^{N_8 N_1} + g^{N_2 N_5} g^{N_6 N_7} g^{N_8 N_3} g^{N_4 N_1} \right) \\ & \left. - (N_1 \leftrightarrow N_2) - (N_3 \leftrightarrow N_4) - (N_5 \leftrightarrow N_6) - (N_7 \leftrightarrow N_8) \right). \end{aligned} \quad (\text{B.1})$$

Let us now discuss the various eight-derivative couplings in more detail. We recall the definition

$$X_8 = \frac{1}{192} \left( \text{Tr} \mathcal{R}^4 - \frac{1}{4} (\text{Tr} \mathcal{R}^2)^2 \right), \quad (\text{B.2})$$

where  $\mathcal{R}$  is the eleven-dimensional curvature two-form  $\mathcal{R}_N^M = \frac{1}{2} R^M_{NPQ} dx^P \wedge dx^Q$ , and

$$\begin{aligned} \epsilon_{11} \epsilon_{11} R^4 &= \epsilon_{R_1 R_2 R_3 M_1 \dots M_8} \epsilon_{R_1 R_2 R_3 N_1 \dots N_8} R^{N_1 N_2}_{M_1 M_2} R^{N_3 N_4}_{M_3 M_4} R^{N_5 N_6}_{M_5 M_6} R^{N_7 N_8}_{M_7 M_8}, \\ t_8 t_8 R^4 &= t_8^{M_1 \dots M_8} t_8^{N_1 \dots N_8} R^{N_1 N_2}_{M_1 M_2} R^{N_3 N_4}_{M_3 M_4} R^{N_5 N_6}_{M_5 M_6} R^{N_7 N_8}_{M_7 M_8}, \end{aligned} \quad (\text{B.3})$$

where  $\epsilon_{11}$  is the eleven-dimensional totally anti-symmetric epsilon tensor and  $t_8$  is given explicitly in (B.1). Using  $\epsilon_{11}$  and  $t_8$  the explicit form for the terms in section 2.1 are precisely given by

$$\begin{aligned} \epsilon_{11} \epsilon_{11} G^2 R^3 &= \epsilon^{R M_1 \dots M_{10}} \epsilon_{R N_1 \dots N_{10}} G^{N_1 N_2}_{M_1 M_2} G^{N_3 N_4}_{M_3 M_4} \\ &\quad \times R^{N_5 N_6}_{M_5 M_6} R^{N_7 N_8}_{M_7 M_8} R^{N_9 N_{10}}_{M_9 M_{10}}, \\ t_8 t_8 G^2 R^3 &= t_8^{M_1 \dots M_8} t_8^{N_1 \dots N_8} G^{N_1}_{M_1 R_1 R_2} G^{N_2}_{M_2 R_1 R_2} R^{N_3 N_4}_{M_3 M_4} R^{N_5 N_6}_{M_5 M_6} R^{N_7 N_8}_{M_7 M_8}. \end{aligned} \quad (\text{B.4})$$

Finally, we need to introduce the tensor  $s_{18}^{N_1 \dots N_{18}}$ , however its precise form not known. Significant parts of it may be fixed following [60]. We argue for an extension in 2.1 of this work. In order to express the kyon parts we use the basis  $B_i$ ,  $i = 1, \dots, 24$  of [60], that labels all unrelated index contractions in  $s_{18}(\nabla G)^2 R^2$ . The basis  $\{B_i\}$  is explicitly given in section B.3. The result can then be expressed in terms of a four-point amplitude



contribution  $\mathcal{A}$  and a linear combination of six contributions  $\mathcal{S}_{i=1,\dots,6}$  which do not affect the 4-point amplitude as

$$\begin{aligned} s_{18}(\nabla G)^2 R^2 &= s_{18}^{N_1\dots N_{18}} R_{N_1\dots N_4} R_{N_5\dots N_8} \nabla_{N_9} G_{N_{10}\dots N_{13}} \nabla_{N_{14}} G_{N_{15}\dots N_{18}} \\ &= \mathcal{A} + \sum_{n=1}^2 a_n \mathcal{S}_n + \sum_{n=3}^6 \hat{a}_n \mathcal{S}_n. \end{aligned} \quad (\text{B.5})$$

The combinations  $\mathcal{A}$  and  $\mathcal{S}_n$  are then given in terms of the basis elements as

$$\begin{aligned} \mathcal{A} &= -24B_5 - 48B_8 - 24B_{10} - 6B_{12} - 12B_{13} + 12B_{14} + 8B_{16} - 4B_{20} \\ &\quad + B_{22} + 4B_{23} + B_{24}, \\ \mathcal{S}_1 &= 48B_1 + 48B_2 - 48B_3 + 36B_4 + 96B_6 + 48B_7 - 48B_8 + 96B_{10} \\ &\quad + 12B_{12} + 24B_{13} - 12B_{14} + 8B_{15} + 8B_{16} - 16B_{17} + 6B_{19} + 2B_{22} + B_{24}, \\ \mathcal{S}_2 &= -48B_1 - 48B_2 - 24B_4 - 24B_5 + 48B_6 - 48B_8 - 24B_9 - 72B_{10} \\ &\quad - 24B_{13} + 24B_{14} - B_{22} + 4B_{23}, \\ \mathcal{S}_3 &= 12B_1 + 12B_2 - 24B_3 + 9B_4 + 48B_6 + 24B_7 - 24B_8 + 24B_{10} \\ &\quad + 6B_{12} + 6B_{13} + 4B_{15} - 4B_{17} + 3B_{19} + 2B_{21}, \\ \mathcal{S}_4 &= 12B_1 + 12B_2 - 12B_3 + 9B_4 + 24B_6 + 12B_7 - 12B_8 + 24B_{10} + 3B_{12} \\ &\quad + 6B_{13} + 4B_{15} - 4B_{17} + 2B_{20}, \\ \mathcal{S}_5 &= 4B_3 - 8B_6 - 4B_7 + 4B_8 - B_{12} - 2B_{14} + 4B_{18}, \\ \mathcal{S}_6 &= B_4 + 2B_{11}. \end{aligned} \quad (\text{B.6})$$

Note that  $\mathcal{S}_3$  to  $\mathcal{S}_6$  vanish both on the considered Calabi-Yau fourfold background solution.

## B.2 Adjunction of Chern-classes

Let us next discuss the adjunction of Chern-classes of divisors on an elliptically fibered Calabi-Yau fourfold  $CY_4$  which is a hyper-surface in a  $\mathbb{P}_{321}$  bundle of the Kähler base  $B_3$  denoted by  $\mathbb{P}_{321}(\mathcal{L})$  given by the vanishing locus of the Weierstrass equation

$$y^2 - (x^3 + fxz^4 + gz^6) = 0, \quad (\text{B.7})$$

with  $f, g$  holomorphic sections of  $\mathcal{L}^4$  and  $\mathcal{L}^6$ , respectively. The  $\text{SL}(2, \mathbb{Z})$  line bundle  $\mathcal{L}$  over  $B$  together with the choice of  $f, g$  defines the elliptic fibration. One may show that the first Chern class is given by

$$c_1(CY_3) = c_1(B_3) - c_1(\mathcal{L}), \quad (\text{B.8})$$

where the r.h.s is pulled back to  $CY_3$ . Then the total Chern class is given by

$$c(\mathbb{P}_{321}(\mathcal{L})) = c(B_3)(1 + 2\omega_0 + 2c_1(B_3))(1 + 3\omega_0 + 3c_1(B_3))(1 + \omega_0) \quad (\text{B.9})$$

where  $\omega_0$  is the harmonic  $(1, 1)$ -form such that  $PD(\omega_0) = B$ .<sup>27</sup> Using adjunction formulae for the

$$c(CY_4) = \frac{c(\mathbb{P}_{321}(\mathcal{L}))}{(1 + \mathcal{L})} \quad (\text{B.10})$$

<sup>27</sup>We are using abuse of notation in the following using  $\omega_0$  and  $c_{1,2,3}$  in the context of a concrete representative of the class as well as the class itself.

with

$$\mathcal{L} = 6\omega_0 + 6c_1(B_3) \quad (\text{B.11})$$

one then derives

$$\begin{aligned} c_3(CY_4) &= c_3(B_3) - c_1(B_3) \wedge c_2(B_3) - 60c_1^3(B_3) - 60c_1^2(B_3) \wedge \omega_0 \\ c_2(CY_4) &= c_2(B_3) + 11c_1^2(B_3) + 12c_1(B_3) \wedge \omega_0 \\ c_1(CY_4) &= 0 \end{aligned} \quad (\text{B.12})$$

and furthermore

$$\omega_0^2 = -c_1(B_3) \wedge \omega_0, \quad (\text{B.13})$$

where the  $c_{i=1,2,3}(B_3)$  on the r.h.s. of these expressions denote the Chern classes of  $B$  pulled-back to  $CY_4$ .

One may next iterate the adjunction formulae to find The Chern-forms of the vertical divisors  $D_\alpha$  of the Calabi-Yau fourfold which are pullbacks of divisors of the base  $D_\alpha^b$ . Thus we denote the class of such divisors via its representatives of harmonic  $(1,1)$ -forms  $\omega_\alpha$ ,  $\alpha = 1, \dots, h^{1,1}$ . Thus one may use adjunction to write

$$c(D_\alpha) = \frac{c(\mathbb{P}_{321}(\mathcal{L}))}{(1 + \mathcal{L})(1 + \omega_\alpha)}, \quad (\text{B.14})$$

with which one then derives

$$\begin{aligned} c_3(D_\alpha) &= c_3(B_3) - c_1(B_3) \wedge c_2(B_3) - 60c_1^3(B_3) - 60c_1^2(B_3) \wedge \omega_0 - c_2(D_\alpha) \wedge \omega_\alpha \\ c_2(D_\alpha) &= c_2(B_3) + 11c_1^2(B_3) + 12c_1(B_3) \wedge \omega_0 + \omega_\alpha^2 \\ c_1(D_\alpha) &= -\omega_\alpha, \end{aligned} \quad (\text{B.15})$$

where  $c_{i=1,2,3}(B_3)$  on the r.h.s of the above equality are pulled back to the divisor  $D_\alpha$ , which amounts to a simply restriction to the subspace  $D_\alpha \subset CY_4$ . In particular we find that the self intersection of divisors  $[D_\alpha] \cdot [D_\alpha]$  is generically non-vanishing.

Let us close this section by analyzing the case where the Calabi-Yau fourfold is a direct product manifold e.g.  $CY_4 = CY_3 \times T^2$  or  $CY_4 = K3 \times K3$ . The Chern-character on product spaces  $X = Y \times Z$  obeys  $c(X) = c(Y)c(Z)$ . Thus we find for the Chern-forms

$$\begin{aligned} c_3(X) &= c_1(Y) \wedge c_2(Z) + c_2(Y) \wedge c_1(Z) + c_3(Y) + c_3(Z), \\ c_2(X) &= c_1(Y) \wedge c_1(Z) + c_2(Y) + c_2(Z), \\ c_1(X) &= c_1(Y) + c_1(Z). \end{aligned} \quad (\text{B.16})$$

Furthermore, on may apply adjunction to compute the Chern-forms of  $CY_3$  in terms of Chern-forms Divisors  $D_\alpha^b$  pulled back to  $CY_3$  which results in

$$c_1(D_\alpha^b) = \omega_\alpha^b, \quad c_2(D_\alpha^b) = c_2(CY_3), \quad (\text{B.17})$$

where we have used the Calabi-Yau condition  $c_1(CY_3) = 0$ . Divisors inside  $CY_4 = CY_3 \times T^2$  wrapping the torus are as well a direct product of  $D_\alpha^b \times T^2$ . Thus by combining (B.16) and (B.17) one can straightforwardly infer their Chern-forms.

### B.3 Basis of the $G^2 R^3$ and $(\nabla G)^2 R^2$ -sector

**Basis of the  $G^2 R^3$ -sector.** The complete eleven-dimensional  $G^2 R^3$  terms may be written in terms of the basis [21] The basis for the potentially relevant eight-derivative terms involving the four-form field strength is

$$\begin{aligned}
 \mathcal{B}_1 &= G_{M_5}^{M_7 M_8 M_9} G_{M_6 M_7 M_8 M_9} R_{M M_2}^{M_4 M_5} R^{M M_1 M_2 M_3} R_{M_1 M_3 M_4}^{M_6} \\
 \mathcal{B}_2 &= G_{M_4 M_6}^{M_8 M_9} G_{M_5 M_7 M_8 M_9} R_{M M_2}^{M_4 M_5} R^{M M_1 M_2 M_3} R_{M_1 M_3}^{M_6 M_7} \\
 \mathcal{B}_3 &= G_{M_4 M_5}^{M_8 M_9} G_{M_6 M_7 M_8 M_9} R_{M M_2}^{M_4 M_5} R^{M M_1 M_2 M_3} R_{M_1 M_3}^{M_6 M_7} \\
 \mathcal{B}_4 &= G_{M_6 M_7 M_8 M_9} G_{M_4}^{M_6 M_7 M_8 M_9} R_{M M_2}^{M_4 M_5} R^{M M_1 M_2 M_3} R_{M_1 M_4 M_3 M_5} \\
 \mathcal{B}_5 &= G_{M_6 M_7 M_8 M_9} G_4^{M_6 M_7 M_8 M_9} R_M^{M_4} M_2^{M_5} R^{M M_1 M_2 M_3} R_{M_1 M_4 M_3 M_5} \\
 \mathcal{B}_6 &= G_{M_5}^{M_7 M_8 M_9} G_{M_6 M_7 M_8 M_9} R_{M M_2}^{M_4 M_5} R^{M M_1 M_2 M_3} R_{M_1 M_4 M_3}^{M_6} \\
 \mathcal{B}_7 &= G_{M_5}^{M_7 M_8 M_9} G_{M_6 M_7 M_8 M_9} R_M^{M_4} M_2^{M_5} R^{M M_1 M_2 M_3} R_{M_1 M_4 M_3}^{M_6} \\
 \mathcal{B}_8 &= G_{M_3 M_6}^{M_8 M_9} G_{M_5 M_7 M_8 M_9} R_{M M_2}^{M_4 M_5} R^{M M_1 M_2 M_3} R_{M_1 M_4}^{M_6 M_7} \\
 \mathcal{B}_9 &= G_{M_3 M_5}^{M_8 M_9} G_{M_6 M_7 M_8 M_9} R_{M M_2}^{M_4 M_5} R^{M M_1 M_2 M_3} R_{M_1 M_4}^{M_6 M_7} \\
 \mathcal{B}_{10} &= G_{M_3 M_6}^{M_8 M_9} G_{M_5 M_7 M_8 M_9} R_M^{M_4} M_2^{M_5} R^{M M_1 M_2 M_3} R_{M_1 M_4}^{M_6 M_7} \\
 \mathcal{B}_{11} &= G_{M_3 M_5}^{M_8 M_9} G_{M_6 M_7 M_8 M_9} R_M^{M_4} M_2^{M_5} R^{M M_1 M_2 M_3} R_{M_1 M_4}^{M_6 M_7} \\
 \mathcal{B}_{12} &= G_{M_4 M_7}^{M_8 M_9} G_{M_5 M_6 M_8 M_9} R_M^{M_4} M_2^{M_5} R^{M M_1 M_2 M_3} R_{M_1}^{M_6} M_3^{M_7} \\
 \mathcal{B}_{13} &= G_{M_3 M_7}^{M_8 M_9} G_{M_5 M_6 M_8 M_9} R_{M M_2}^{M_4 M_5} R^{M M_1 M_2 M_3} R_{M_1}^{M_6} M_4^{M_7} \\
 \mathcal{B}_{14} &= G_{M_3 M_7}^{M_8 M_9} G_{M_5 M_6 M_8 M_9} R_M^{M_4} M_2^{M_5} R^{M M_1 M_2 M_3} R_{M_1}^{M_6} M_4^{M_7} \\
 \mathcal{B}_{15} &= G_{M_5}^{M_7 M_8 M_9} G_{M_6 M_7 M_8 M_9} R_{M M_1 M_2}^{M_4} R^{M M_1 M_2 M_3} R_{M_3}^{M_5} M_4^{M_6} \\
 \mathcal{B}_{16} &= G_{M_4 M_6}^{M_8 M_9} G_{M_5 M_7 M_8 M_9} R_{M M_1 M_2}^{M_4} R^{M M_1 M_2 M_3} R_{M_3}^{M_5} M_6 M_7 \\
 \mathcal{B}_{17} &= G_{M_4 M_6}^{M_8 M_9} G_{M_5 M_7 M_8 M_9} R_{M M_1 M_2 M_3} R^{M M_1 M_2 M_3} R^{M_4 M_5 M_6 M_7}. \tag{B.18}
 \end{aligned}$$

**Basis of the  $(\nabla G)^2 R^2$ -sector.** The complete eleven-dimensional  $(\nabla G)^2 R^2$  terms may be written in terms of the basis [60]. In order to discuss the term  $s_{18}$  appearing in (2.4) and (B.5) we introduce the basis

$$\begin{aligned}
 B_1 &= R_{M_1 M_2 M_3 M_4} R_{M_5 M_6 M_7 M_8} \nabla^{M_5} G^{M_1 M_7 M_8}_{M_9} \nabla^{M_3} G^{M_2 M_4 M_6 M_9}, \\
 B_2 &= R_{M_1 M_2 M_3 M_4} R_{M_5 M_6 M_7 M_8} \nabla^{M_5} G^{M_1 M_3 M_7}_{M_9} \nabla^{M_8} G^{M_2 M_4 M_6 M_9}, \\
 B_3 &= R_{M_1 M_2 M_3 M_4} R_{M_5 M_6 M_7 M_8} \nabla^{M_5} G^{M_1 M_3 M_7}_{M_9} \nabla^{M_6} G^{M_2 M_4 M_8 M_9}, \\
 B_4 &= R_{M_1 M_2 M_3 M_4} R_{M_5 M_6 M_7 M_8} \nabla_{M_9} G^{M_3 M_4 M_7 M_8} \nabla^{M_6} G^{M_9 M_1 M_2 M_5}, \\
 B_5 &= R_{M_1 M_2 M_3 M_4} R_{M_5 M_6 M_7}^{M_4} \nabla^{M_1} G^{M_2 M_3}_{M_8 M_9} \nabla^{M_5} G^{M_6 M_7 M_8 M_9}, \\
 B_6 &= R_{M_1 M_2 M_3 M_4} R_{M_5 M_6 M_7}^{M_4} \nabla^{M_1} G^{M_2 M_5}_{M_8 M_9} \nabla^{M_3} G^{M_6 M_7 M_8 M_9}, \\
 B_7 &= R_{M_1 M_2 M_3 M_4} R_{M_5 M_6 M_7}^{M_4} \nabla^{M_1} G^{M_2 M_5}_{M_8 M_9} \nabla^{M_7} G^{M_3 M_6 M_8 M_9},
 \end{aligned}$$

$$\begin{aligned}
 B_8 &= R_{M_1 M_2 M_3 M_4} R_{M_5 M_6 M_7}^{M_4} \nabla^{M_1} G^{M_3 M_5}{}_{M_8 M_9} \nabla^{M_2} G^{M_6 M_7 M_8 M_9}, \\
 B_9 &= R_{M_1 M_2 M_3 M_4} R_{M_5 M_6 M_7}^{M_4} \nabla^{M_1} G^{M_3 M_5}{}_{M_8 M_9} \nabla^{M_6} G^{M_2 M_7 M_8 M_9}, \\
 B_{10} &= R_{M_1 M_2 M_3 M_4} R_{M_5 M_6 M_7}^{M_4} \nabla_{M_9} G^{M_3 M_5 M_7 M_8} \nabla^{M_9} G^{M_1 M_2 M_6 M_8}, \\
 B_{11} &= R_{M_1 M_2 M_3 M_4} R_{M_5 M_6 M_7}^{M_4} \nabla_{M_8} G^{M_1 M_2 M_6}{}_{M_9} \nabla^{M_9} G^{M_3 M_5 M_7 M_8}, \\
 B_{12} &= R_{M_1 M_2 M_3 M_4} R_{M_5 M_6 M_7}^{M_4} \nabla^{M_3} G^{M_5 M_6}{}_{M_8 M_9} \nabla^{M_7} G^{M_2 M_1 M_8 M_9}, \\
 B_{13} &= R_{M_1 M_2 M_3 M_4} R_{M_5}^{M_1}{}_{M_6}^{M_3} \nabla_{M_9} G^{M_2 M_6}{}_{M_7 M_8} \nabla^{M_9} G^{M_4 M_5 M_7 M_8}, \\
 B_{14} &= R_{M_1 M_2 M_3 M_4} R_{M_5}^{M_1}{}_{M_6}^{M_3} \nabla_{M_9} G^{M_2 M_4}{}_{M_7 M_8} \nabla^{M_9} G^{M_5 M_6 M_7 M_8}, \\
 B_{15} &= R_{M_1 M_2 M_3 M_4} R_{M_5}^{M_1}{}_{M_6}^{M_3} \nabla^{M_2} G^{M_6}{}_{M_7 M_8 M_9} \nabla^{M_5} G^{M_4 M_7 M_8 M_9}, \\
 B_{16} &= R_{M_1 M_2 M_3 M_4} R_{M_5}^{M_1}{}_{M_6}^{M_3} \nabla^{M_2} G^{M_4}{}_{M_7 M_8 M_9} \nabla^{M_5} G^{M_6 M_7 M_8 M_9}, \\
 B_{17} &= R_{M_1 M_2 M_3 M_4} R_{M_5}^{M_1}{}_{M_6}^{M_3} \nabla^{M_2} G^{M_5}{}_{M_7 M_8 M_9} \nabla^{M_4} G^{M_6 M_7 M_8 M_9}, \\
 B_{18} &= R_{M_1 M_2 M_3 M_4} R_{M_5}^{M_1}{}_{M_6}^{M_3} \nabla_{M_9} G^{M_5 M_6}{}_{M_7 M_8} \nabla^{M_4} G^{M_2 M_7 M_8 M_9}, \\
 B_{19} &= R_{M_1 M_2 M_3 M_4} R_{M_5 M_6}^{M_3 M_4} \nabla_{M_9} G^{M_1 M_5}{}_{M_7 M_8} \nabla^{M_9} G^{M_2 M_6 M_7 M_8}, \\
 B_{20} &= R_{M_1 M_2 M_3 M_4} R_{M_5 M_6}^{M_3 M_4} \nabla^{M_1} G^{M_5}{}_{M_7 M_8 M_9} \nabla^{M_2} G^{M_6 M_7 M_8 M_9}, \\
 B_{21} &= R_{M_1 M_2 M_3 M_4} R_{M_5 M_6}^{M_3 M_4} \nabla^{M_1} G^{M_5}{}_{M_7 M_8 M_9} \nabla^{M_6} G^{M_2 M_7 M_8 M_9}, \\
 B_{22} &= R_{M_1 M_2 M_3 M_4} R_{M_5}^{M_1 M_3 M_4} \nabla^{M_2} G_{M_6 M_7 M_8 M_9} \nabla^{M_5} G^{M_6 M_7 M_8 M_9}, \\
 B_{23} &= R_{M_1 M_2 M_3 M_4} R_{M_5}^{M_1 M_3 M_4} \nabla_{M_9} G^{M_2}{}_{M_6 M_7 M_8} \nabla^{M_9} G^{M_5 M_6 M_7 M_8}, \\
 B_{24} &= R_{M_1 M_2 M_3 M_4} R^{M_1 M_2 M_3 M_4} \nabla_{M_5} G_{M_6 M_7 M_8 M_9} \nabla^{M_6} G^{M_5 M_7 M_8 M_9}. \tag{B.19}
 \end{aligned}$$

The contributions to  $s_{18}(\nabla G)^2 R^2$  are then formed from the linear combinations described in (B.5). We write the eleven-dimensional action as

$$2\kappa_{11}^2 S^{\text{extra, gen}} = \alpha^2 \int_{M_{11}} \sum_{i=1}^{17} C_i \mathcal{B}_i * 1 + \sum_{i=1}^{24} C_{i+17} B_i * 1 \tag{B.20}$$

with real parameters  $C_1, \dots, C_{41}$  which are fixed by the reduction on a Calabi-Yau threefold and compatibility with  $5d, \mathcal{N}_2$  super symmetry to

$$\begin{aligned}
 C_5 &= -\frac{1}{2}C_4, & C_7 &= -C_1 - \frac{1}{2}C_6, \\
 C_9 &= 4C_3, & C_{10} &= -3C_1 - 2C_2 - 8C_3 - 18C_4 - \frac{3}{2}C_6 - C_8, \\
 C_{11} &= -4C_3, & C_{12} &= 4C_3, \\
 C_{13} &= -8C_3, & C_{14} &= -6C_1 - 2C_2 - 12C_3 - 36C_4 - 3C_6 - C_8, \\
 C_{15} &= \frac{1}{3}C_2 + 3C_4, & C_{16} &= -2C_2 - 4C_3, \\
 C_{17} &= \frac{1}{4}C_2, & C_{25} &= 2C_{22} - C_{24}, \\
 C_{29} &= \frac{1}{4}C_{22} + \frac{1}{4}C_{23} - \frac{1}{4}C_{24} + \frac{1}{4}C_{26}, \\
 C_{32} &= -C_1 - \frac{1}{3}C_{22} - \frac{4}{3}C_3 + \frac{2}{3}C_{30} - 6C_4 - \frac{1}{2}C_6,
 \end{aligned}$$

$$\begin{aligned}
C_{33} &= C_1 - \frac{1}{3}C_{22} + \frac{4}{3}C_3 + 6C_4 + \frac{1}{2}C_6, \\
C_{37} &= -C_1 - \frac{4}{3}C_3 - \frac{1}{3}C_{31} - \frac{1}{2}C_{34} - \frac{1}{6}C_{35} - \frac{2}{3}C_{36} - 6C_4 - \frac{1}{2}C_6, \\
C_{38} &= \frac{1}{3}C_{30} + \frac{1}{3}C_{31} + \frac{1}{2}C_{34} + \frac{1}{6}C_{35} + \frac{2}{3}C_{36}, \\
C_{39} &= \frac{1}{4}C_1 - \frac{1}{24}C_{22} + \frac{1}{3}C_3 + \frac{3}{2}C_4 + \frac{1}{8}C_6, \\
C_{40} &= \frac{1}{2}C_1 + \frac{2}{3}C_3 + \frac{1}{3}C_{31} + \frac{1}{6}C_{35} + 3C_4 + \frac{1}{4}C_6, \\
C_{41} &= \frac{1}{4}C_1 + \frac{1}{3}C_3 + \frac{1}{12}C_{31} + \frac{1}{24}C_{35} + \frac{3}{2}C_4 + \frac{1}{8}C_6,
\end{aligned} \tag{B.21}$$

We then check compatibility of the novel induces  $H^2R^3$  terms making use of the IIA-Heterotic duality. Compactifying type IIA on  $K3$  is dual to the Heterotic string on  $\mathbb{T}^4$ . One finds that additionally

$$C_2 = 0, \quad C_1 = -\frac{1}{6}(8C_3 + 2C_{31} + C_{35} + 36C_4 + 3C_6), \tag{B.22}$$

where more details can be found in section 2.2.

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