Published for SISSA by Springer

Received: *January 17, 2017*
Accepted: *March 7, 2017*
Published: *April 4, 2017*

On timelike supersymmetric solutions of gauged minimal 5-dimensional supergravity

Samuele Chimento and Tomás Ortín

Instituto de Física Teórica UAM/CSIC,
C/Nicolás Cabrera, 13–15, C.U. Cantoblanco, E-28049 Madrid, Spain

E-mail: Samuele.Chimento@csic.es, Tomas.Ortin@csic.es

Abstract: We analyze the timelike supersymmetric solutions of minimal gauged 5-dimensional supergravity for the case in which the Kähler base manifold admits a holomorphic isometry and depends on two real functions satisfying a simple second-order differential equation. Using this general form of the base space, the equations satisfied by the building blocks of the solutions become of, at most, fourth degree and can be solved by simple polynomic ansatzs. In this way we construct two 3-parameter families of solutions that contain almost all the timelike supersymmetric solutions of this theory with one angular momentum known so far and a few more: the (singular) supersymmetric Reissner-Nordström-AdS solutions, the three exact supersymmetric solutions describing the three near-horizon geometries found by Gutowski and Reall, three 1-parameter asymptotically-AdS_5 black-hole solutions with those three near-horizon geometries (Gutowski and Reall's black hole being one of them), three generalizations of the Gödel universe and a few potentially homogenous solutions. A key rôle in finding these solutions is played by our ability to write AdS_5's Kähler base space ($\overline{\mathbb{CP}}^2$ or $SU(1,2)/U(2)$) is three different, yet simple, forms associated to three different isometries. Furthermore, our ansatz for the Kähler metric also allows us to study the dimensional compactification of the theory and its solutions in a systematic way.

Keywords: Black Holes, Black Holes in String Theory, Supergravity Models, Superstring Vacua

ArXiv ePrint: 1611.09383

doi:10.1007/JHEP04(2017)017

JHEP04(2017)017

Contents

JHEP04(2017)017

Introduction. The search for exact solutions of theories of gravity has been, and still is, one of the most fruitful areas of work in gravitational physics. Symmetry has probably been the main tool in this search and, therefore, it is not surprising that, in gravity theories invariant under supersymmetry transformations (theories of supergravity), unbroken supersymmetry has become the main tool as well.[1]

Unbroken supersymmetry is, indeed, a very powerful tool because, beyond the fact that it implies the existence of ordinary symmetry (standard isometries of the metric which also leave invariant the matter fields), relates in non-trivial ways all the fields of the theory and, in particular, it relates all the bosonic matter fields to the metric. This implies that all the fields of a given solution with unbroken supersymmetry (*a.k.a.* supersymmetric or BPS solution) can be constructed from a common set of building blocks (functions, 1-forms, metrics in some submanifold that satisfy simple equations or geometrical conditions) using different combinations or rules. These combinations and rules are characteristic of each supergravity theory and, identifying them, the building blocks and conditions they satisfy makes it possible to construct large families of interesting solutions and discover properties which cannot manifest themselves in single members of the family. The attractor mechanism [2–6] is, perhaps, the best known example of this kind of properties and their relevance: only the knowledge of families of black-hole solutions with different charges and values of the scalars at infinity can one realize that their near-horizon values (and, hence, the entropy formulae) only depend on the charges. The latter being quantized, a microscopic interpretation of the entropy is, in principle, possible.

The systematic characterization or "classification" of supersymmetric solutions was pioneered by Gibbons and Hull ref. [7] and, specially, by Tod ref. [8] who showed that the requirement of existence of just one unbroken supersymmetry in pure $\mathcal{N} = 2, d = 4$ supergravity was strong enough to identify a reduced number of building blocks satisfying simple equations in terms of which all the components of the fields of the supersymmetric solutions could be written. Shortly, Kowalski-Glikman found all the solutions of the same theory admitting the maximal number of unbroken supersymmetries (that is: 8) in ref. [9].

However, since most of the solutions found by Tod were already known[2] and he worked using the Newman-Penrose formalism, it was not until it was realized that the Killing spinor equations could be rewritten as equations on tensors constructed as spinor bilinears (a language much better understood by the superstring community) that this line of research took off. This method was successfully applied to the complete characterization of the supersymmetric solutions of minimal 5-dimensional supergravity in ref. [15] leading to the discovery of a host of new and interesting solutions. This procedure was immediately applied to ever more complex cases. In the framework of $\mathcal{N} = 2, d = 4$ supergravity theories, it was applied to

[1] For a comprehensive review of supersymmetric solutions of supergravity theories with many references see, e.g. ref. [1].

[2] The bosonic sector of pure, ungauged, $\mathcal{N} = 2, d = 4$ supergravity is the well-known and much studied Einstein-Maxwell theory. Then, it is no surprise that, for instance, the timelike supersymmetric solutions corresponded to the Perjés-Israel-Wilson family [10, 11] which, as proven by Hartle and Hawking in ref. [12], only contains as regular non-trivial subfamily the Majumdar-Papapetrou solutions [13, 14] which describe extremal Reissner-Nordström black holes in static equilibrium.

JHEP04(2017)017

- Gauged, pure supergravity in ref. [16].
- Ungauged but coupled to vector multiplets in ref. [17].
- Ungauged but coupled to vector multiplets and hypermultiplets in ref. [18].
- Coupled only to vector multiplets with Abelian gaugings in refs. [19–21].
- Coupled to vector multiplets with non-Abelian gaugings (excluding SU(2) Fayet-Iliopoulos terms)in ref. [22].
- Coupled to vector multiplets and hypermultiplets with the most general gauging (Abelian or not, with Fayet-Iliopoulos terms or not) in ref. [23].[3]

In the $\mathcal{N} = 1, d = 5$ supergravity theories in which we are interested here it has been applied to

- Gauged, pure supergravity in ref. [24].
- Coupled to vector multiplets with Abelian gaugings in ref. [25] for the timelike case (the results for the ungauged case were derived from those of the gauged one in ref. [26]) and in ref. [27] for the null case.
- Ungauged but coupled to vector multiplets and hypermultiplets in ref. [28].
- Coupled to vector multiplets and hypermultiplets with the most general gauging in ref. [29].
- Coupled to vector and tensor multiplets and hypermultiplets with the most general gauging in ref. [30].

A feature of the 5-dimensional case, as compared with 4-dimensional one is that, even in the simplest theory, some of the building blocks are not defined by differential or algebraic equations but by geometrical conditions whose general solution is not known. In particular, the most fundamental building block of the 5-dimensional timelike supersymmetric solutions (which are the ones we will be interested in here) is the so-called *base-space metric*, which is a 4-dimensional Euclidean metric that enters in the construction of the 5-dimensional spacetime metric and on which the differential equations satisfied by the rest of the building blocks are defined, is required to be hyperKähler in the ungauged case (with no hypers, as we will assume form now on to be the case) or just Kähler when there is an Abelian gauging. These geometrical conditions are too general: we do not know how to write a general 4-dimensional Kähler hyperKähler metric in terms of a set of functions, forms or lower-dimensional metrics satisfying simple equations. This problem was solved in ref. [15] by considering only 4-dimensional hyperKähler spaces admitting triholomorphic isometries, which have Gibbons-Hawking metrics [31, 32], a constraint that still allows for many interesting solutions like rotating and static asymptotically-flat[4] (multi) black holes

[3] Only the timelike supersymmetric solutions have been characterized in the most general case.

[4] The magic of the Gibbons-Hawking ansatz is that the additional isometry is compatible with spherically-symmetric (SO(4)-invariant) black-hole solutions and it does not restrict us to work with black strings.

JHEP04(2017)017

and black rings. These metrics are defined by a single building block: a function harmonic in $\mathbb{E}^3$, customarily called H, and, on them, the rest of the supersymmetry conditions can be solved completely in terms of another three harmonic functions. As a bonus, upon dimensional reduction along the additional isometry one finds 4-dimensional supersymmetric black holes.

The same ansatz has recently been used in theories of $\mathcal{N} = 1, d = 5$ with vector multiplets and non-Abelian gaugings (but no Fayet-Iliopoulos terms), or $\mathcal{N} = 1, d = 5$ Super-Einstein-Yang-Mills (SEYM) theories [33]. The general form of the timelike supersymmetric solutions is a particular case of that found in ref. [29] and the base space is also hyperKähler. A piece of the non-Abelian 1-form field is an anti-selfdual instanton on the hyperKähler base space. If one assumes that this space is Gibbons-Hawking one can then use Kronheimer's results [34] to solve the instanton equation on that space in terms of BPS monopole solutions to the Bogomol'nyi equation on $\mathbb{E}^3$ [35]. For the gauge group SU(2) all the spherically symmetric solutions of the Bogomol'nyi equation were found by Protogenov in ref. [36] and one can profit from this result to construct anti-selfdual instantons in the 4-dimensional hyperKähler base space.[5] Somewhat surprisingly, the only monopoles that give rise to regular instantons (the BPST one [37], in fact) in the simplest setup belong to an intriguing class which has vanishing asymptotic charge and a singularity at the origin and which give rise to regular 4-dimensional non-Abelian black holes whose entropy, nevertheless, depends on the non-Abelian field [38]. These black holes were called *coloured black holes* in ref. [39]. They exist for more general gauge groups (because the corresponding *coloured monopoles* also exist in more general gauge groups, as shown in ref. [40]) and are associated to 4-dimensional coloured black holes in which the non-Abelian field configuration is the regular instanton associated to the corresponding coloured monopole.

Given the success of this approach, it is a bit of a mystery that a similar ansatz (i.e. assuming that the Kähler base space has a holomorphic isometry) has not yet been used to simplify the Abelian-gauged case,[6] which is known to lead to complicated sixth-order differential equations [44]. In that reference, Gutowski and Reall managed to find a supersymmetric asymptotically-AdS_5 black-hole solution with a squashed-S^3 near-horizon geometry plus two additional possible non-compact near-horizon geometries. However, given the complexity of the problem, they could not identify other supersymmetric asymptotically-AdS_5 black-hole solutions with the alternative near-horizon geometries. Given the connections between this kind of solutions and the AdS/CFT conjecture, finding them constitutes an important open problem that could have been addressed by making use of the aforementioned ansatz. Furthermore, as explained at the beginning of this introduction, finding general families of solutions (or extending the ones already known) is, by itself, an important goal.

[5] Again, the magic of the Gibbons-Hawking ansatz is that the instantons built from the monopoles, which are only spherically symmetric in $\mathbb{E}^3$ (SO(3)) will be spherically symmetric in the 4-dimensional base space if we make the simplest choice $\mathbb{R}^4_{\{0\}}$.

[6] It should be noted however that less general ansatzs have been used in the literature, namely toric [41] and orthotoric [42] Kähler base spaces. Actually many of the solutions we find here were already included in those works, either explicitly or as particular cases of more general solutions.

JHEP04(2017)017

In a recent paper [45] we have shown how to write any Kähler metric with a holomorphic isometry in a generalized Gibbons-Hawking form that depends on just two real functions H, W the first of which satisfies a W-deformed Laplace equation on $\mathbb{E}^3$. In this paper we are going to use this ansatz to simplify the equations and find more supersymmetric solutions of minimal gauged 5-dimensional supergravity. We start by reviewing this theory in section 1 to introduce our notation and conventions. Its bosonic sector is described in section 1.1 and the conditions found in ref. [24] for a field configuration to be a timelike supersymmetric solution will be reviewed in our notation in section 1.2. Then in section 1.3 we study the particular case in which the base space of the timelike supersymmetric solution (a 4-dimensional Kähler space) has a holomorphic isometry, using the general ansatz found in ref. [45], finding a simpler set of equations to be solved. Before we try to solve them, we have found it useful to rewrite in section 2 some well-known timelike supersymmetric solutions (Reissner-Nordström-AdS$_5$ and AdS$_5$ itself) in a form and coordinates adapted to our ansatz for the base space. We show three different ways of writing AdS$_5$ in a timelike supersymmetric form, each of them associated to a different form of writing the common base space $\overline{\mathbb{CP}}^2$ or SU(1, 2)/U(2). In its turn, each of these forms of AdS$_5$ will inspire a different ansatz for asymptotically-AdS$_5$ timelike supersymmetric solutions. This will allow us to obtain in section 3 two families of solutions characterized by the parameter ϵ that constitute the main result of this paper. The $\epsilon = 1$ family, studied in section 3.1, describes, among others, two kinds of solutions: asymptotically-AdS$_5$ rotating black holes with the three possible near-horizon geometries found in ref. [44] and the three near-horizon geometries as proper timelike supersymmetric solutions. The $\epsilon = 0$ family, studied in section 3.2, describes a large number of non-asymptotically-AdS$_5$ solutions of difficult interpretation. There are three simple solutions in this class that are generalizations of the Gödel universe. As in the ungauged and non-Abelian-gauged cases, all the solutions found by using our ansatz can be immediately reduced to $d = 4$ dimensions and related to the solutions of some of the theories of $\mathcal{N} = 2, d = 4$ Abelian-gauged supergravity classified in refs. [19–21]. In the case we are considering in this paper (minimal gauged supergravity), the corresponding 4-dimensional theory is the Abelian-gauged T^3 model and, in section 4 we study the solutions of this theory that arise from the 5-dimensional solutions discussed in the previous sections. Section 5 contains our conclusions and directions for future work. Finally, appendices A, B and C contain the connection and curvature of the 3-, 4- and 5-dimensional metrics that occur in this problem and appendix D contains a review of the construction of the AdS$_5$ metrics used in the body of the paper.

Note added. In the $\epsilon = 1$ solutions ω_z is determined up to an arbitrary constant d. However this constant d should also appear in ω and, therefore, once we shift the coordinate t to set ω to zero, the d in ω_z is automatically subtracted. In other words: it is not necessary to set $d = 0$ by hand to get asymptotically AdS$_5$ solutions since d is always zero.

1 Minimal gauged $\mathcal{N} = 1, d = 5$ supergravity

In this section we give a brief description of minimal gauged $\mathcal{N} = 1, d = 5$ supergravity and its timelike supersymmetric solutions.

JHEP04(2017)017

Minimal (*pure*) $\mathcal{N}=1, d=5$ supergravity contains the supergravity multiplet, only. This multiplet consists of the graviton $e^a{}_\mu$, the gravitino $\psi^i{}_\mu$ and the graviphoton 1-form A_μ. The spinor $\psi^i{}_\mu$ is a symplectic Majorana spinor and i is a fundamental SU(2) (R-symmetry) index.[7]

Since only one 1-form is available, and there are no scalars, at most a U(1) subgroup of the SU(2) R-symmetry group can be gauged. This is done by adding a Fayet-Iliopoulos (FI) term gn^r, $r=1,2,3$ where n^r is a constant unitary vector which selects the $\mathfrak{u}(1)$ generator in $\mathfrak{su}(2)$ that is going to be gauged: if $\{T_r\}$ are a basis of the $\mathfrak{su}(2)$ Lie algebra, the generator of the U(1) symmetry being gauged will be $T\equiv n^r T_r$. g is the gauge coupling constant and only occurs in the bosonic action as a negative (AdS) cosmological constant as we are going to see.

1.1 The bosonic sector

The bosonic action of minimal gauged 5-dimensional supergravity takes the form of a cosmological Einstein-Maxwell theory supplemented by a Chern-Simons term:

$$S=\int d^5x\sqrt{g}\left\{R+4g^2-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+\frac{1}{12\sqrt{3}}\frac{\varepsilon^{\mu\nu\rho\sigma\alpha}}{\sqrt{g}}F_{\mu\nu}F_{\rho\sigma}A_\alpha\right\}, \tag{1.1}$$

where $F_{\mu\nu}=2\partial_{[\mu}A_{\nu]}$ and g is the U(1) coupling constant. The cosmological constant Λ is given in the above action by[8]

$$\Lambda=-\frac{4}{3}g^2\,, \tag{1.4}$$

and this value as well as the coefficient of the Chern-Simons term are fixed by supersymmetry.

The equations of motion for the bosonic fields that follow from the above action are

$$G_{\mu\nu}-\frac{1}{2}\left(F_\mu{}^\rho F_{\nu\rho}-\frac{1}{4}g_{\mu\nu}F^{\rho\sigma}F_{\rho\sigma}\right)-2g^2g_{\mu\nu}=0\,, \tag{1.5}$$

$$\nabla_\nu F^{\nu\mu}+\tfrac{1}{4\sqrt{3}}\frac{\varepsilon^{\mu\nu\rho\sigma\alpha}}{\sqrt{g}}F_{\nu\rho}F_{\sigma\alpha}=0\,. \tag{1.6}$$

1.2 Timelike supersymmetric configurations

The general form of the solutions of minimal, gauged, 5-dimensional supergravity admitting a timelike Killing spinor[9] was found in ref. [24]. In what follows we are going to review

[7] Our conventions are those in refs. [28, 29] which are those of ref. [46] with minor modifications.

[8] Our definition of the cosmological constant is such that it occurs in the d-dimensional Einstein-Hilbert action as

$$S=\int d^dx\sqrt{|g|}\left\{R-(d-2)\Lambda\right\}\,, \tag{1.2}$$

giving rise to the equations

$$G_{\mu\nu}=-\frac{(d-2)}{2}\Lambda g_{\mu\nu}\,,\quad\text{and}\quad R_{\mu\nu}=\Lambda g_{\mu\nu}\,. \tag{1.3}$$

[9] A timelike (commuting) spinor ϵ^i is, by definition, such that the real vector bilinear constructed from it $iV_\mu\sim\bar\epsilon_i\gamma_\mu\epsilon^i$ is timelike.

JHEP04(2017)017

it using the notation and results of ref. [29] in which the most general gauged theory was considered.

The building blocks of the timelike supersymmetric solutions are the scalar function $\hat{f}$, the 4-dimensional spatial metric $h_{\underline{mn}}$,[10] an anti-selfdual almost hypercomplex structure $\hat{\Phi}^{(r)}{}_{mn}$,[11] a 1-form $\hat{\omega}_{\underline{m}}$, and the 1-form potential $\hat{A}_{\underline{m}}$. All these fields are defined on the 4-dimensional spatial manifold usually called "base space". They are time-independent and must satisfy a number of conditions:

1. The anti-selfdual almost hypercomplex structure $\hat{\Phi}^{(r)}{}_{mn}$, the 1-form potentials $\hat{A}^I{}_{\underline{m}}$ and the base space metric $h_{\underline{mn}}$ (through its Levi-Civita connection) satisfy the equation
$$\hat{\nabla}_m \hat{\Phi}^{(r)}{}_{np} + g\varepsilon^{rst} n^s \hat{A}_m \hat{\Phi}^{(t)}{}_{np} = 0\,. \tag{1.9}$$

2. The selfdual part of the spatial vector field strength $\hat{F} \equiv d\hat{A}$ must be related to the function $\hat{f}$, the 1-form $\hat{\omega}$ by
$$\hat{F}^+ = \frac{2}{\sqrt{3}}(\hat{f} d\hat{\omega})^+\,, \tag{1.10}$$

3. while the anti-selfdual part is related to the almost hypercomplex structure by
$$\hat{F}^- = -2\hat{f}^{-1} n^r \hat{\Phi}^{(r)}\,. \tag{1.11}$$

4. Finally, all the building blocks are related by the equation
$$\hat{\nabla}^2 \hat{f}^{-1} - \frac{1}{6}\hat{F}\cdot\hat{\star}\hat{F} - \frac{1}{2\sqrt{3}}\hat{F}\cdot(\hat{f} d\hat{\omega})^- = 0\,, \tag{1.12}$$
where the dots indicate standard contraction of all the indices of the tensors.

Once the building blocks that satisfy the above conditions have been found, the physical 5-dimensional fields can be built out of them as follows:

1. The 5-dimensional (conformastationary) metric is given by
$$ds^2 = \hat{f}^{\,2}(dt + \hat{\omega})^2 - \hat{f}^{\,-1} h_{\underline{mn}} dx^m dx^n\,. \tag{1.13}$$

2. The complete 5-dimensional 1-form field is given by
$$A = -\sqrt{3}\,\hat{f}(dt + \hat{\omega}) + \hat{A}\,, \tag{1.14}$$

[10] $m, n, p = 1, \cdots, 4$ will be tangent space indices and $\underline{m}, \underline{n}, \underline{p} = 1, \cdots, 4$ will be curved indices. We are going to denote with hats all objects that naturally live in this 4-dimensional space.

[11] That is: the 2-forms $\hat{\Phi}^{(r)}{}_{mn}$ $r, s, t = 1, 2, 3$ satisfy
$$\hat{\Phi}^{(r)\,mn} = -\frac{1}{2}\varepsilon^{mnpq}\hat{\Phi}^{(r)}{}_{pq}\,, \qquad \text{or} \qquad \hat{\Phi}^{(r)} = -\star_4\,\hat{\Phi}^{(r)}\,, \tag{1.7}$$
$$\hat{\Phi}^{(r)\,m}{}_n \hat{\Phi}^{(s)\,n}{}_p = -\delta^{rs}\delta^m{}_p + \varepsilon^{rst}\hat{\Phi}^{(t)\,m}{}_p\,. \tag{1.8}$$

JHEP04(2017)017

so that the spatial components are

$$A_{\underline{m}} = \hat{A}_{\underline{m}} - \sqrt{3}\hat{f}\hat{\omega}_{\underline{m}}\,, \tag{1.15}$$

and the 5-dimensional field strength is

$$F = -\sqrt{3}d[\hat{f}(dt + \hat{\omega})] + \hat{F}\,. \tag{1.16}$$

As it has already been observed in ref. [24], from eq. (1.9) if follows that there is one complex structure (generically given by $n^r\hat{\Phi}^{(r)}$) which is covariantly constant in the base space

$$\hat{\nabla}_m(n^r\hat{\Phi}^{(r)}{}_{np}) = 0\,, \tag{1.17}$$

which, in its turn, implies that the base space metric $h_{\underline{mn}}$ is Kähler with respect to the complex structure $\hat{J}_{mn} \equiv n^r\hat{\Phi}^{(r)}{}_{np}$ (see, e.g. ref. [47]).

It is convenient to choose, for instance, $n^r = \delta^r{}_1$. With this choice, eq. (1.9) splits into

$$\hat{\nabla}_m\hat{\Phi}^{(1)}{}_{np} = 0\,, \tag{1.18}$$

$$\hat{\nabla}_m\hat{\Phi}^{(2)}{}_{np} = g\hat{A}_m\hat{\Phi}^{(3)}{}_{np}\,, \tag{1.19}$$

$$\hat{\nabla}_m\hat{\Phi}^{(3)}{}_{np} = -g\hat{A}_m\hat{\Phi}^{(2)}{}_{np}\,. \tag{1.20}$$

The first equation is just eq. (1.17) for our particular choice of FI term, which implies the choice of complex structure $\hat{J}_{mn} \equiv \hat{\Phi}^{(1)}{}_{np}$. Taking this fact into account,[12] the integrability condition of the other two equations is[13]

$$\hat{\mathfrak{R}}_{mn} = -g\hat{F}_{mn}\,. \tag{1.25}$$

This equation must be read as a constraint on the 1-form potential $\hat{A}_{\underline{m}}$ posed by the choice of base space metric.

Eq. (1.11) takes a simpler form as well:

$$\hat{F}^- = -2g\hat{f}^{-1}\hat{J}\,, \tag{1.26}$$

[12] We use the integrability condition of eq. (1.18)

$$\hat{R}_{mnpq} = \hat{R}_{mnrs}\hat{J}^r{}_p\hat{J}^s{}_q\,, \tag{1.21}$$

which leads to the relation between the Ricci and Riemann tensors

$$\hat{R}_{mn} = -\frac{1}{2}\hat{R}_{mprq}\hat{J}^{rq}\hat{J}^p{}_n\,. \tag{1.22}$$

The Ricci 2-form, defined as

$$\hat{\mathfrak{R}}_{mn} \equiv \hat{R}_{mp}\hat{J}^p{}_n\,, \tag{1.23}$$

is, therefore, related to the Riemann tensor by

$$\hat{\mathfrak{R}}_{mn} = \frac{1}{2}\hat{R}_{mnpq}\hat{J}^{pq}\,. \tag{1.24}$$

[13] If gA_m vanishes (for instance, in the ungauged case), then we have a covariantly constant hyper-Kähler structure and, then, the base space is hyperKähler.

JHEP04(2017)017

Tracing the first of these equations and eq. (1.25) with $\hat{J}^{mn}$ one finds a simple relation between the Ricci scalar of the base space metric and the function $\hat{f}$:

$$\hat{R} = 8g^2 \hat{f}^{-1} \,. \tag{1.27}$$

The last equation to be simplified by our choice is eq. (1.12). Substituting it in eq. (1.26) one finds

$$\hat{\nabla}^2 \hat{f}^{-1} - \frac{1}{6}\hat{F}\cdot\hat{\star}\hat{F} + \frac{1}{\sqrt{3}} g \hat{J}\cdot(d\hat{\omega}) = 0 \,. \tag{1.28}$$

1.3 Timelike supersymmetric solutions with one additional isometry

In order to make progress we need to make assumptions about the base space Kähler metric so we can write it explicitly in terms of a small number of functions that satisfy certain equations. In the ungauged [15, 28] and the non-Abelian gauged cases [33] in which the base space is hyper-Kähler it has proven very useful to assume that the base space metric has an additional triholomorphic isometry because, then, the metric is a Gibbons-Hawking metric [31, 32] that depends on only one independent function customarily denoted by H which is harmonic in $\mathbb{E}^3$. Writing the metric in terms of H and other derived functions simplifies the equations that depend on the metric so much that in the ungauged case the complete solution can be written in terms of several functions harmonic on $\mathbb{E}^3$.

It is natural to try the same strategy in the case at hands. We have shown in ref. [45] that the most general Kähler metric admitting a holomorphic isometry can be written as[14]

$$ds^2 = H^{-1}(dz+\chi)^2 + H\left\{(dx^2)^2 + W^2(\vec{x})[(dx^1)^2+(dx^3)^2]\right\} \,, \tag{1.30}$$

with the functions H and W, and the 1-form χ, depending only on the three coordinates x^i and satisfying the constraints

$$\begin{aligned} (d\chi)_{\underline{12}} &= \partial_{\underline{3}} H \,, \\ (d\chi)_{\underline{23}} &= \partial_{\underline{1}} H \,, \\ (d\chi)_{\underline{31}} &= \partial_{\underline{2}}\left(W^2 H\right) \,, \end{aligned} \tag{1.31}$$

whose integrability condition is

$$\mathfrak{D}^2 H \equiv \partial_{\underline{1}}\partial_{\underline{1}} H + \partial_{\underline{2}}\partial_{\underline{2}}\left(W^2 H\right) + \partial_{\underline{3}}\partial_{\underline{3}} H = 0 \,. \tag{1.32}$$

As shown in ref. [45], imposing different conditions on W one can recover more restricted classes of metrics. In particular, when $W = 1$ the 3-dimensional metric is flat and the constraint eqs. (1.31) reduce to

$$d\chi = \star_3 dH \,, \tag{1.33}$$

[14] The associated complex structure has been chosen to be the anti-selfdual

$$(J_{mn}) \equiv \begin{pmatrix} 0_{2\times 2} & \mathbb{1}_{2\times 2} \\ -\mathbb{1}_{2\times 2} & 0_{2\times 2} \end{pmatrix} \,. \tag{1.29}$$

We will identify it with $\hat{\Phi}^{(1)}$.

JHEP04(2017)017

which implies that H is harmonic on $\mathbb{E}^3$ and the metric eq. (1.30) is a Gibbons-Hawking metric.

The curvature of these metrics has been computed in appendix B using the results of appendix A and we have also computed the curvature of the 5-dimensional metric eq. (1.13) for the above base space in appendix C. In what follows we use the frames defined in the appendices.

The simplest non-trivial example of Kähler manifold admitting a holomorphic isometry is the non-compact symmetric space $\overline{\mathbb{CP}}^2 = \mathrm{SU}(1,2)/\mathrm{U}(2)$ which is the base space of AdS_5.[15] Written in the conformastationary form eq. (1.13), AdS_5 is a U(1) bundle over $\overline{\mathbb{CP}}^2$ [48], the non-compact version of the Hopf fibrations studied in ref. [49]. For the convenience of the reader, we revisit this example in appendix D, giving the functions H and W corresponding to $\overline{\mathbb{CP}}^2$ and describing how to rewrite this metric in more standard coordinates.

Assuming our base space is of the above form, then, we can continue our analysis of the equations that determine the supersymmetric solutions of minimal gauged supergravity.

To start with, if we choose a particular form for the complex structures $\hat{\Phi}^{(2,3)}$ we can solve for $\hat{A}_m$ in eqs. (1.19) and (1.20).

In the frame given by eq. (B.2) and taking into account the choice of $\hat{\Phi}^{(1)}$ already made in footnote 14, we can choose[16]

$$(\hat{\Phi}^{(2)}{}_{mn}) = \begin{pmatrix} i\sigma_2 & 0_{2\times 2} \\ 0_{2\times 2} & -i\sigma_2 \end{pmatrix}, \qquad (\hat{\Phi}^{(3)}{}_{mn}) = \begin{pmatrix} 0_{2\times 2} & -i\sigma_2 \\ -i\sigma_2 & 0_{2\times 2} \end{pmatrix}. \tag{1.34}$$

Then, we find that the flat components of $\hat{A}$ are given by

$$g\hat{A}_\sharp = -H^{-1/2}\overline{\omega}_{112}\,, \qquad g\hat{A}_i = -H^{-1/2}\overline{\omega}_{i13}\,, \tag{1.35}$$

and, taking into account the 3-dimensional metric at hands, we find that we can write all the components of $\hat{A}_m$ in the compact form

$$g\hat{A}_m = \hat{J}_m{}^n\,\partial_n \log W\,, \tag{1.36}$$

and, thus, we have solved the three eqs. (1.18)–(1.20) (or, equivalently, the original eq. (1.9)) in terms of the functions that define the base space.

The consistency of this solution can be checked through the relation between the field strength $\hat{F}_{mn}$ and the Ricci 2-form $\hat{\mathfrak{R}}_{mn}$ Eq. (1.25): using this relation, we get

$$\hat{\mathfrak{R}}_{mn} = -g\hat{F}_{mn} = 2\hat{\nabla}_{[m|}\hat{\nabla}_p \log W \hat{J}^p{}_{|n]}\,, \tag{1.37}$$

$$\hat{R}_{mn} = \hat{\nabla}_m\hat{\nabla}_n \log W + \hat{J}_m{}^p\hat{J}_n{}^q\hat{\nabla}_p\hat{\nabla}_q \log W\,, \tag{1.38}$$

$$\hat{R} = \hat{\nabla}^2 \log W^2\,. \tag{1.39}$$

[15] Actually, it is the only possible base space for AdS_5 [25].

[16] The most general possible form for these matrices would be $\hat{\Phi}^{(2)\prime} = \cos\theta\,\hat{\Phi}^{(2)} + \sin\theta\,\hat{\Phi}^{(3)}$ and $\hat{\Phi}^{(3)\prime} = \cos\theta\,\hat{\Phi}^{(3)} - \sin\theta\,\hat{\Phi}^{(2)}$, for some function θ, in which case $\hat{A} \to \hat{A} - \frac{1}{g}d\theta$, which amounts to just a gauge transformation of the gauge fields.

JHEP04(2017)017

These expressions can be compared with the direct computation of the Ricci tensor and scalar in appendix B. The expression of the Ricci scalar can be used in eq. (1.27) to obtain a direct expression of the metric function $\hat{f}$ in terms of the functions that define the base space:

$$\hat{f}^{-1} = \frac{1}{8g^2}\hat{\nabla}^2 \log W^2 \,. \tag{1.40}$$

Now eq. (1.26) (or, equivalently, the original eq. (1.11)) is also completely solved by eqs. (1.36) and (1.40), and the only equations that remain to be solved are eqs. (1.10) and (1.28). Observe that, since both $\hat{f}^{-1}$ and $\hat{F}_{mn}$ are given by second-order derivatives, the remaining equations will be, at most, of fourth order in derivatives, instead of of sixth order as in ref. [25]. We are going to try to rewrite them in a simpler form as in the ungauged case.

Every (anti-)selfdual 2-form $\mathcal{F}^{\pm}$ on the four dimensional Kähler base space can be written in terms of a 1-form living on the 3-dimensional space $\vartheta = \vartheta_{\underline{i}} dx^i$ as

$$\mathcal{F}^{\pm} = e^{\sharp} \wedge \vartheta \pm \frac{1}{2} H \star_3 \vartheta \,. \tag{1.41}$$

The 2-forms we consider here are also z-independent and so will the components of the corresponding 1-forms be. Thus, we introduce the z-independent 3-dimensional 1-forms Λ, Σ, and $\Omega_{\pm}$ defined by

$$\hat{F}^{+} = -\frac{1}{2}(dz + \chi) \wedge \Lambda - \frac{1}{2} H \star_3 \Lambda \,, \tag{1.42}$$

$$\hat{F}^{-} = -\frac{1}{2}(dz + \chi) \wedge \Sigma + \frac{1}{2} H \star_3 \Sigma \,, \tag{1.43}$$

$$(d\hat{\omega})^{\pm} = (dz + \chi) \wedge \Omega^{\pm} \pm H \star_3 \Omega^{\pm} \,, \tag{1.44}$$

Comparing the expression of $\hat{F}^{-}$ with eq. (1.26) and those of $\hat{F}^{+}$ and $(d\omega)^{+}$ with eq. (1.10) we conclude that

$$\Sigma = 4g\hat{f}^{-1} dx^2 \,, \tag{1.45}$$

$$\Omega^{+} = -\frac{\sqrt{3}}{4}\hat{f}^{-1}\Lambda \,. \tag{1.46}$$

Requiring the closure of $\hat{F} = \hat{F}^{+} + \hat{F}^{-}$ one gets

$$d(\Lambda + \Sigma) = 0 \,, \tag{1.47}$$

which means that, locally,

$$\Lambda = d(K/H) - \Sigma \,, \tag{1.48}$$

for some functions K.

From the same condition, using eq. (1.32) and the definition of the operator $\mathfrak{D}^2$ in that equation, one also gets

$$\mathfrak{D}^2 K = 8g\, \partial_{\underline{2}} \left(H W^2 \hat{f}^{-1} \right) . \tag{1.49}$$

Using eq. (1.25) and the equations in the appendices to compute the Ricci 2-form for a metric of the kind we are considering here, one finds

$$gK = \partial_{\underline{2}} \log W^2 + \kappa H \,, \tag{1.50}$$

where κ is an arbitrary constant that reflects the possibility of adding to the solution of the inhomogeneous equation (1.49) solutions of the homogeneous equation. This expression for K, together with eq. (1.40), automatically solves the second-order equation eq. (1.49). It is convenient to rewrite $\hat{\omega}$ as

$$\hat{\omega} = \omega_z \, (dz + \chi) + \omega \,, \qquad \omega = \omega_{\underline{i}} dx^i \,, \tag{1.51}$$

in terms of which

$$\Omega^{\pm} = \pm \frac{1}{2} H^{-1} \, (\omega_z \star_3 d\chi + \star_3 d\omega) - \frac{1}{2} d\omega_z \,. \tag{1.52}$$

From eqs. (1.46) and (1.48) we find that

$$\Omega^{+} = -\frac{\sqrt{3}}{4} \hat{f}^{-1} \left[d \left(K/H \right) - \Sigma \right] \,, \tag{1.53}$$

and, then, from eq. (1.52), we find that

$$\Omega^{-} = -\Omega^{+} - d\omega_z = \frac{\sqrt{3}}{4} \hat{f}^{-1} \left[d \left(K/H \right) - \Sigma \right] - d\omega_z \,. \tag{1.54}$$

Using either of the last two equations in eq. (1.52) one gets an equation for ω:

$$d\omega = H \star_3 d\omega_z - \omega_z d\chi - \frac{\sqrt{3}}{2} \hat{f}^{-1} H \star_3 \left[d \left(K/H \right) - \Sigma \right] \,. \tag{1.55}$$

Before calculating its integrability condition it is convenient to make a change of variables (identical to the one made in the ungauged case) to (partially) "symplectic-diagonalize" the right-hand side. Thus, we define L and M through

$$\begin{aligned} \hat{f}^{-1} &\equiv L + \frac{1}{12} K^2/H \,, \\ \omega_z &\equiv M + \frac{\sqrt{3}}{4} LK/H + \frac{1}{24\sqrt{3}} K^3/H^2 \,. \end{aligned} \tag{1.56}$$

Substituting these two expressions into eq. (1.55) and using the relation between the 1-form χ and the functions H and W, eqs. (1.31), the equation for ω takes the form[17]

$$d\omega = \star_3 \left\{ HdM - MdH + \frac{\sqrt{3}}{4} \left(KdL - LdK \right) - H \left(\omega_z \partial_{\underline{2}} \log W^2 - 2\sqrt{3} g \hat{f}^{-2} \right) dx^2 \right\} \,, \tag{1.57}$$

and its integrability equation is just[18]

$$\begin{aligned} & H \overline{\nabla}^2 M - M \overline{\nabla}^2 H + \frac{\sqrt{3}}{4} \left(K \overline{\nabla}^2 L - L \overline{\nabla}^2 K \right) \\ & \qquad - \frac{1}{W^2} \partial_{\underline{2}} \left\{ H W^2 \left(\omega_z \partial_{\underline{2}} \log W^2 - 2\sqrt{3} g \hat{f}^{-2} \right) \right\} = 0 \,. \end{aligned} \tag{1.58}$$

[17] We have left one ω_z in order to get a more compact expression.

[18] One has $\star_3 d \star_3 d = \overline{\nabla}^2$.

JHEP04(2017)017

This equation can be simplified by using the equations satisfied by the functions H and K (1.32) and (1.49), respectively. We postpone doing this until we derive the equation for L, which follows from eq. (1.28). First of all, observe that, with our choice of complex structure eq. (1.29)

$$\hat{J}\cdot(d\hat{\omega}) = 4(d\hat{\omega})^{-}_{02} = 4\Omega^{-}_{\underline{2}} = \sqrt{3}\hat{f}^{-1}\left[\partial_{\underline{2}}\left(K/H\right) - 4g\hat{f}^{-1}\right] - \partial_{\underline{2}}\omega_z\,. \tag{1.59}$$

On the other hand, we have

$$\begin{aligned}
\hat{\nabla}^2\hat{f}^{-1} &= H^{-1}\overline{\nabla}^2\hat{f}^{-1}\,,\\
\hat{F}\cdot\hat{\star}\hat{F} &= \Lambda_m\Lambda_m - \Sigma_m\Sigma_m = \partial_m(K/H)\partial_m(K/H) - 2\Sigma_m\partial_m(K/H)\,,\\
H\partial_m(K/H)\partial_m(K/H) &= \overline{\nabla}^2\left(\frac{K^2}{2H}\right) + \frac{K^2}{2H^2}\overline{\nabla}^2 H - \frac{K\overline{\nabla}^2 K}{H}\,,
\end{aligned} \tag{1.60}$$

and, using all these partial results into eq. (1.28), and (not everywhere, for the sake of simplicity) the new variables eqs. (1.56), we arrive at

$$\begin{aligned}
\overline{\nabla}^2 L - \frac{1}{12}(K/H)^2\overline{\nabla}^2 H + \frac{1}{6}(K/H)\overline{\nabla}^2 K + \frac{7}{3}gH\hat{f}^{-1}\partial_{\underline{2}}(K/H)\\
-\frac{4}{\sqrt{3}}gH\partial_{\underline{2}}\omega_z - 4g^2H\hat{f}^{-2} = 0\,.
\end{aligned} \tag{1.61}$$

We can now use the relation between the 3-dimensional Laplacian and the $\mathfrak{D}^2$ operator and the equations for the functions H and K (1.32) and (1.49)

$$\begin{aligned}
\overline{\nabla}^2 H &= \frac{\mathfrak{D}^2 H}{W^2} - \partial_{\underline{2}}H\frac{\partial_{\underline{2}}W^2}{W^2} - H\frac{\partial^2_{\underline{2}}W^2}{W^2} = -\partial_{\underline{2}}H\frac{\partial_{\underline{2}}W^2}{W^2} - H\frac{\partial^2_{\underline{2}}W^2}{W^2}\,,\\
\overline{\nabla}^2 K &= \frac{\mathfrak{D}^2 K}{W^2} - \partial_{\underline{2}}K\frac{\partial_{\underline{2}}W^2}{W^2} - K\frac{\partial^2_{\underline{2}}W^2}{W^2} = \frac{8g}{W^2}\partial_{\underline{2}}(HW^2\hat{f}^{-1}) - \partial_{\underline{2}}K\frac{\partial_{\underline{2}}W^2}{W^2} - K\frac{\partial^2_{\underline{2}}W^2}{W^2}\,,
\end{aligned} \tag{1.62}$$

and, setting $\kappa = 0$ for simplicity from now on, the equation for L becomes

$$\overline{\nabla}^2 L = 4H\left(gL\right)^2 - \frac{2}{3}L\left(gK\right)^2 - \frac{4}{3}gL\partial_{\underline{2}}K - \frac{1}{3}gK\partial_{\underline{2}}L + \frac{4}{\sqrt{3}}Hg\partial_{\underline{2}}M\,. \tag{1.63}$$

Using this equation in the integrability condition for the ω equation, eq. (1.58) we get

$$\overline{\nabla}^2 M = -\sqrt{3}gL\left(gKL + 2\partial_{\underline{2}}L\right)\,. \tag{1.64}$$

While the appearance of these equations is quite compact, we have to take into account that the functions appearing in them are not totally independent. Using eqs. (1.32), (1.40), (1.50) and (1.56) we find the following equations that have to be added to these:

$$\mathfrak{D}^2 H = 0\,, \tag{1.65}$$

$$gK = \partial_{\underline{2}}\log W^2\,, \tag{1.66}$$

$$L = \frac{1}{8g^2H}\left\{\overline{\nabla}^2\log W^2 - \frac{2}{3}\left(\partial_{\underline{2}}\log W^2\right)^2\right\}\,. \tag{1.67}$$

JHEP04(2017)017

Substituting them in the other two, we get fourth order differential equations for H, M, W.

As was first noted in ref. [41] not every Kähler base space can give rise to a supersymmetric solution. This can be seen here as follows: multiplying eq. (1.64) by W^2, differentiating with respect to x^2, eliminating $\partial_{\underline{2}} M$ from the resulting equation with eq. (1.63), and using eqs. (1.66) and (1.67) one gets a sixth order differential equation involving only H and W^2, which are the functions that determine the Kähler base space. This is then a constraint on the admissible base spaces, and while we did not check this explicitly it is likely to be equivalent to the constraint found in ref. [42] for an arbitrary Kähler base space.

2 Examples

Before we set out to solve the equations, in order to gain some insight, it is convenient to rewrite some simple and well-known supersymmetric solutions in the form we are proposing here.

2.1 Reissner-Nordström-AdS$_5$

Thus, let us consider the asymptotically AdS$_5$ Reissner-Nordström (RN-AdS$_5$) solutions, which are given by the metric and vector field

$$\begin{aligned} ds^2 &= [k + h(r) + \tfrac{1}{3}g^2 r^2]dt^2 - \frac{dr^2}{[k + h(r) + \tfrac{1}{3}g^2 r^2]} - r^2 d\Omega^2_{(3,k)}\,, \\ A &= \frac{3q}{r^2}dt\,, \\ h(r) &= -\frac{2M}{r^2} + \frac{3q^2}{r^4} \end{aligned} \tag{2.1}$$

where M is the mass, q, the electric charge that we will assume to be positive for the sake of simplicity,[19] $k = 1, 0, -1$ the curvature of the 3-dimensional metric $d\Omega^2_{(3,k)}$. More explicitly, for $k = 1$ $d\Omega^2_{(3,1)} \equiv d\Omega^2_{(3)}$ is the metric of the round sphere of unit radius

$$d\Omega^2_{(3)} = \frac{1}{4}\left[(d\psi + \cos\theta\, d\varphi)^2 + d\Omega^2_{(2)}\right]\,, \qquad d\Omega^2_{(2)} = d\theta^2 + \sin^2\theta\, d\varphi^2\,, \tag{2.4}$$

for $k = 0$ $d\Omega^2_{(3,0)}$ is the metric of $\mathbb{E}^3$ with the normalization

$$d\Omega^2_{(3,0)} = \frac{1}{4}\left[d\psi^2 + d\Omega^2_{(2,0)}\right]\,, \qquad d\Omega^2_{(2,0)} = (dx^1)^2 + (dx^3)^2\,, \tag{2.5}$$

[19] The mass and charge are defined in units in which

$$\frac{16\pi G^{(5)}_N}{3\omega_{(3)}} = 2\,, \tag{2.2}$$

where $G^{(5)}_N$ is the 5-dimensional Newton constant and $\omega_{(3)}$ the volume of the round 3-sphere of unit radius, for the $k = 1$ case. Equivalently, we have chosen units such that

$$\frac{3\pi}{4G^{(5)}_N} = 1\,. \tag{2.3}$$

JHEP04(2017)017

and for $k=-1$ $d\Omega^2_{(3,-1)}$ is the metric of $\mathbb{H}_3$. We have not succeeded in writing this metric in the form of a fibration over another 2-dimensional space and, therefore, we will not be able to rewrite the corresponding solution in the form required by supersymmetry. Actually, it is well known that supersymmetry requires the following relation between the mass, the charge and k:[20]

$$M^2 = 3kq^2\,, \tag{2.6}$$

and, therefore, we do not expect supersymmetric solutions for $k=-1$, except pure AdS_5 space. However, pure AdS_5 space cannot be described in the form required by $k=-1$.

We are going to rederive this result by rewriting the metric in the canonical form eqs. (1.13), (1.30) and (1.31) we are proposing, identifying the functions $\hat{f}, H, W, \omega_z$ and the 1-forms χ, ω and checking that they satisfy the equations that we have derived from supersymmetry.

First, we transform the coordinate $\psi = z - \frac{2}{\sqrt{3}}gt$ and perform a gauge transformation of the vector field to get

$$\begin{aligned} ds^2 &= (k+h)\left[dt + \frac{1}{2\sqrt{3}}g\frac{r^2}{k+h}(dz+\chi_{(k)})\right]^2 - \frac{r^2}{4(k+h)}\left[k+h(r)+\frac{1}{3}g^2r^2\right](dz+\chi_{(k)})^2 \\ &\quad - \frac{dr^2}{[k+h(r)+\frac{1}{3}g^2r^2]} - \frac{1}{4}r^2\,d\Omega^2_{(2,k)}\,, \\ A &= -\sqrt{3}\left(\delta - \frac{\sqrt{3}q}{r^2}\right)\left[dt + \frac{1}{2\sqrt{3}}g\frac{r^2}{k+h}(dz+\chi_{(k)})\right] + \frac{1}{2}g\frac{\delta r^2 - \sqrt{3}q}{k+h}(dz+\chi_{(k)})\,, \end{aligned} \tag{2.7}$$

where $\chi_{(1)} = \cos\theta d\varphi$, $\chi_{(0)} = 0$ and δ is an arbitrary constant. Observe that, for $h=0$ (pure AdS_5) this transformation can only be made for $k=1$. We will have to study more carefully the asymptotic behaviour of the transformed solution for $k=0$.

We also need to rewrite $d\Omega^2_{(2,k)}$ and, correspondingly $\chi_{(k)}$ as in eq. (D.34). $\hat{f}, \omega_z, \chi, H$ and W can be read immediately from g_{tt}, g_{tz}, g_{zz} and $g_{\underline{11}} = g_{\underline{33}}$, respectively. g_{rr} should be given by $\hat{f}^{-1}H$, but this only happens after a change of coordinates $r = 2\varrho^{1/2}$.[21] The final result is

$$\begin{aligned} \hat{f} &= [k+h(\varrho)]^{1/2}\,, \\ H &= \frac{[k+h(\varrho)]^{1/2}}{\varrho[k+h(\varrho)+\frac{4}{3}g^2\varrho]}\,, \\ W^2 &= \varrho^2\left[k+h(\varrho)+\frac{4}{3}g^2\varrho\right]\Phi(x^1,x^3)\,, \\ \omega_z &= \frac{2}{\sqrt{3}}g\frac{\varrho}{k+h(\varrho)}\,, \\ \chi &= \chi_{(k)}\,, \end{aligned}$$

[20] The supersymmetric $k=1$ RN-AdS_5 solution was first found in ref. [43]. In ref. [42] it was shown that it is the only supersymmetric solution with $\mathbb{R}\times SO(4)$ isometry group. Here , we present it in the canoncial supersymmetric form. The $k=0,-1$ cases have isometry groups $\mathbb{R}\times ISO(3)$ and $\mathbb{R}\times SO(2,2)$, respectively.

[21] We denote by ϱ the coordinate x^2 in eq. (1.30).

JHEP04(2017)017

$$\omega = 0\,,$$
$$A = -\sqrt{3}\left(\delta - \frac{\sqrt{3}q}{4\varrho}\right)[dt + \omega_z(dz + \chi_{(k)})] + \frac{1}{2}g\frac{4\delta\varrho - \sqrt{3}q}{k+h}(dz + \chi_{(k)})\,, \tag{2.8}$$

where, now

$$h(\varrho) = -\frac{M}{2\varrho} + \frac{3q^2}{16\varrho^2}\,, \tag{2.9}$$

and $\Phi_{(k)}(x^1, x^3)$ and $\chi_{(k)}$ have been defined in eqs. (D.34).

Eq. (1.65) is satisfied if

$$k + h = \left(k - \frac{\sqrt{3}q}{4\varrho}\right)^2\,, \tag{2.10}$$

which implies the supersymmetry relation eq. (2.6).

The 1-form potential coincides with the one in eq. (1.14) if $\delta = k$. In particular, and eq. (1.36) is satisfied up to a gauge transformation.

The rest of the equations are also satisfied.

To summarize, the supersymmetric RN-AdS$_5$ solutions for $k = 0, 1$ are given by

$$\begin{aligned}
\hat{f} &= k - \frac{\sqrt{3}q}{4\varrho}\,,\\
H &= \frac{k\varrho - \frac{\sqrt{3}}{4}q}{\frac{4}{3}g^2\varrho^3 + k^2\varrho^2 - \frac{\sqrt{3}}{2}q\varrho + \frac{3}{16}q^2}\,,\\
W^2 &= \left[\frac{4}{3}g^2\varrho^3 + k^2\varrho^2 - \frac{\sqrt{3}}{2}q\varrho + \frac{3}{16}q^2\right]\Phi(x^1, x^3)\,,\\
\omega_z &= \frac{2}{\sqrt{3}}g\frac{\varrho^3}{k^2\varrho^2 - \frac{\sqrt{3}}{2}q\varrho + \frac{3}{16}q^2}\,,\\
\chi &= \chi_{(k)}\,,\\
\omega &= 0\,,\\
A &= -\sqrt{3}\hat{f}[dt + \omega_z(dz + \chi_{(k)})] + 2g\varrho\hat{f}^{-1}(dz + \chi_{(k)})\,.
\end{aligned} \tag{2.11}$$

Setting $q = 0$ in the $k = 1$ case we get AdS$_5$ written in the canonical supersymmetric form

$$ds^2 = \left[dt + \frac{2}{\sqrt{3}}g\varrho(dz + \cos\theta d\varphi)\right]^2 - \varrho\left[1 + \frac{4}{3}g^2\varrho\right](dz + \cos\theta d\varphi)^2 - \frac{d\varrho^2}{\varrho[1 + \frac{4}{3}g^2\varrho]} - \varrho\, d\Omega^2_{(2)}\,. \tag{2.12}$$

In appendix D we discuss the relation between this form of AdS$_5$ and more popular forms of the same metric with $g = \sqrt{3}$. As it is shown there, the base space is the symmetric Kähler space $\overline{\mathbb{CP}}^2$. In ref. [44] it has been shown that this is the only possible base space for AdS$_5$. However, $\overline{\mathbb{CP}}^2$ can be written in different ways, using the metric of S^2, $\mathbb{E}^2$ or $\mathbb{H}_2$, and we are going to see in the next example that there are 3 associated canonical metrics

JHEP04(2017)017

for AdS_5 that can be used to construct more general solutions. The construction of these metrics is explained in appendix D.

In the $k = 0$ case we also get AdS_5, but in different (non-canonical) coordinates:

$$ds^2 = \varrho\left[\frac{4}{\sqrt{3}}g dt dz + (dx^1)^2 + (dx^3)^2\right] - \frac{d\varrho^2}{\frac{4}{3}g^2\varrho^2}\,. \tag{2.13}$$

The $\varrho \to \infty$ limit is, in these solutions, equivalent to setting $q = 0$. In the $\varrho \to 0$ limits both solutions give the following singular geometries

$$\begin{aligned} ds^2 &= \frac{3q^2}{4\varrho^2}dt^2 - \varrho\left[\frac{16}{3q^2}d\varrho^2 + 4d\Omega^2_{(3)}\right]\,, \\ ds^2 &= \frac{3q^2}{4\varrho^2}dt^2 - \varrho\left[\frac{16}{3q^2}d\varrho^2 + dz^2 + (dx^1)^2 + (dx^3)^2\right]\,, \end{aligned} \tag{2.14}$$

which are also examples of supersymmetric solutions written in the canonical form.

Finally, in the $k = 1$ case, the supersymmetric Killing vector becomes null at $\varrho = \frac{\sqrt{3}}{4}q$, indicating the possible existence of a Killing horizon which would also be a candidate to event horizon. It is convenient to work with the shifted coordinate $\varrho' = \varrho - \frac{\sqrt{3}}{4}q$, which is zero at the point of interest. The radial coordinate of the solutions that we are going to present in the next section also vanishes at the same point and, in order to ease the comparison between the solutions, we rewrite here the $k = 1$ RN-AdS_5 solution in the shifted radial coordinate (suppressing the primes):

$$\begin{aligned} \hat{f} &= \varrho\left(\varrho + \frac{\sqrt{3}}{4}q\right)^{-1}\,, \\ H &= \varrho\left[\frac{4g^2}{3}\varrho^3 + (1+\sqrt{3}g^2q)\varrho^2 + \frac{3g^2q^2}{4}\varrho + \frac{\sqrt{3}g^2q^3}{16}\right]^{-1}\,, \\ W^2H &= \varrho\,\Phi(x^1, x^3)\,, \\ \omega_z &= \frac{2g}{\sqrt{3}}\varrho^{-2}\left(\varrho + \frac{\sqrt{3}q}{4}\right)^3\,, \end{aligned} \tag{2.15}$$

and, in the $\varrho \to 0$ limit, the metric takes the form

$$ds^2 = \frac{16}{3q^2}\varrho^2 dt^2 + gq dt(dz + \chi_{(1)}) - \frac{4}{g^2q^2}d\varrho^2 - \sqrt{3}q d\Omega^2_{(3)}\,, \tag{2.16}$$

and does not coincide with any of the near-horizon metrics constructed in ref. [25]. In particular, observe that the metric of the hypersurface $\varrho = 0$ has rank four, which means that it cannot be null. It is a well-known fact that the supersymmetric RN-AdS_5 solution has a naked singularity.

2.2 AdS_5

We have found three interesting ways of writing AdS_5 in the supersymmetric canonical form:

$$ds^2 = \left[dt + \frac{2}{\sqrt{3}}g\varrho(dz + \chi_{(k)})\right]^2 - \varrho\left(k + \frac{4}{3}g^2\varrho\right)(dz + \chi_{(k)})^2 - \frac{d\varrho^2}{\varrho(k + \frac{4}{3}g^2\varrho)} - \varrho d\Omega^2_{(2,k)}\,, \tag{2.17}$$

where $d\Omega^2_{(2,k)}$ (the metric of the unit 2-sphere, plane and hyperplane for, respectively, $k = 1, 0$ and -1) and $\chi_{(k)}$ are given by eqs. (D.34). The case $k = 1$ has been given in eq. (2.12). The base space has a metric of the form eq. (1.30) with

$$H^{-1} = \varrho\left(k + \frac{4}{3}g^2\varrho\right), \qquad W^2 H = \varrho\Phi_{(k)}(x^1, x^3) \qquad \chi = \chi_{(k)}\,. \tag{2.18}$$

It is, by construction, a Kähler space with one holomorphic isometry. In agreement with ref. [44], this metric is that of $\overline{\mathbb{CP}}^2$ for the three values of k as shown in appendix D. In the full 5-dimensional metric, the coordinate z has a different causal character in each case: the norm of the Killing vector ∂_z is $g_{zz} = -k\varrho$, and, since ϱ has to be positive in all cases, the coordinate z turns out to be null for $k = 0$ and timelike for $k = -1$.

For $k = 1$, as shown in appendix D, we can go to an unrotating coordinate system with the change $z = \psi + \frac{2}{\sqrt{3}}gt$

$$ds^2 = \left(1 + \frac{4}{3}g^2\varrho\right)dt^2 - \varrho(d\psi + \chi_{(1)})^2 - \frac{d\varrho^2}{\varrho(1 + \frac{4}{3}g^2\varrho)} - \varrho d\Omega^2_{(2,1)}\,, \tag{2.19}$$

which is well defined for all positive values of ϱ. For $k = -1$, changing $z = \psi - \frac{2}{\sqrt{3}}gt$ we get

$$ds^2 = \varrho(d\psi + \chi_{(-1)})^2 - \left(\frac{4}{3}g^2\varrho - 1\right)dt^2 - \frac{d\varrho^2}{\varrho(\frac{4}{3}g^2\varrho - 1)} - \varrho d\Omega^2_{(2,-1)}\,, \tag{2.20}$$

which is well defined for $\varrho > \frac{3}{4g^2}$ and shows the timelike character of ψ and the spacelike character of t. For $k = 0$ there is no analogous transformation. It is worth stressing that the ϱ coordinates of these three AdS_5 metrics are different as the z and t coordinates are.

In the next section we are going to find two families of solutions using ansatzs adapted to these three forms of AdS_5. Only by using them the equations of motion become tractable. Actually, rewriting the solutions found using the $k = 0, -1$ ansatzs in the $k = 1$ coordinates (more conventional and better understood) although possible, leads to very complicated metrics. Thus, it would be rather convenient to be able to analyze the asymptotic behaviour of the $k = 0, -1$ solutions and compute their conserved charges directly in the $k = 0, -1$ coordinates.

Indeed, naively, some of the $k = 0, -1$ solutions we are going to present seem to approach the above $k = 0, -1$ forms of the AdS_5 metric or the naive asymptotic limit of those metrics. However, given the many subtleties that arise in the study of asymptotically-AdS solutions, a more rigorous analysis using Penrose's conformal techniques [50], as in ref. [51] is required.

Let us first study the three AdS_5 metrics since, as we just discussed, their asymptotic limits appear in the asymptotic limits of the most general solutions.

In the $k = 1$ case the only spacelike coordinate which is not compact is $\varrho \in [0, +\infty)$ and some components of the metric diverge in the $\varrho \to +\infty$ limit. Thus, we make the coordinate transformation $\varrho \equiv \tan^2\xi$, which brings the metric eq. (2.19) to the form

$$ds^2 = 4\cos^{-2}\xi\, d\tilde{s}^2\,, \tag{2.21}$$

JHEP04(2017)017

where

$$\tilde{ds}^2 = \left(\frac{1}{4}\cos^2\xi + (g/\sqrt{3})^2\sin^2\xi\right)dt^2 - \frac{d\xi^2}{4\left(\frac{1}{4}\cos^2\xi + (g/\sqrt{3})^2\sin^2\xi\right)} - \sin^2\xi\, d\Omega^2_{(3)}\,. \quad (2.22)$$

$\tilde{ds}^2$ is regular at $\xi = \pi/2$ ($\varrho \to +\infty$) and becomes, at that point

$$\tilde{ds}^2(\xi = \pi/2) = (g/\sqrt{3})^2 dt^2 - \frac{1}{4}d\Omega^2_{(3)}\,. \quad (2.23)$$

This space is just $\mathbb{R}\times S^3$, whose conformal isometry group is $\mathrm{SO}(2,4)$. Since the conformal factor relating the metrics $\Omega = \cos\xi$ vanishes on the boundary $\xi = \pi/2$ but $\nabla_a\Omega$ does not, according to the definition of ref. [51], the AdS_5 metric, in the $k = 1$ form is asymptotically AdS_5 in the direction $\rho \to \infty$.[22]

In the $k = -1$ case there are two non-compact spacelike coordinates: ϱ and θ. We make in the metric eq. (2.20) the following changes of coordinates:

$$\varrho \equiv \tan^2\xi\,, \qquad \psi \equiv \alpha + \beta\,, \qquad \varphi = \alpha - \beta\,, \qquad \sinh(\theta/2) = \tan\eta\,, \quad (2.24)$$

finding

$$ds^2 = 4\cos^{-2}\xi\cos^{-2}\eta\,\tilde{ds}^2\,, \quad (2.25)$$

with

$$\begin{aligned}\tilde{ds}^2 = {} & \sin^2\xi\left[d\alpha^2 - \eta^2 - \sin^2\eta d\beta^2\right] \\ & - \cos^2\eta\left[\left((g/\sqrt{3})^2\sin^2\xi - \frac{1}{4}\cos^2\xi\right)dt^2 + \frac{d\xi^2}{4\left((g/\sqrt{3})^2\sin^2\xi - \frac{1}{4}\cos^2\xi\right)}\right]\,. \quad (2.26)\end{aligned}$$

In these coordinates, the boundary lies where the conformal factor $\Omega = \cos\xi\cos\eta$ vanishes and it seems to correspond to two different pieces: $\xi = \pi/2$ and $\eta = \pi/2$. The first piece has the induced metric

$$\tilde{ds}^2(\xi = \pi/2) = d\alpha^2 - [d\eta^2 + \sin^2\eta\, d\beta^2 + \cos^2\eta\, d(gt/\sqrt{3})^2]\,, \quad (2.27)$$

which is the metric of $\mathbb{R}\times S^3$, but now it is α the coordinate that plays the rôle of time while t is an angle.

The second piece, though, has a singular metric. To understand the reason for the existence of an apparent second piece of the boundary we can look at the relation between the θ, ϱ coordinates of the $k = 1$ and $k = -1$ case since, in the $k = 1$ case the boundary coincides exactly with the $\varrho \to \infty$ limit. This relation can be inferred from the $k = 1$ and $k = -1$ parametrizations of $\overline{\mathbb{CP}^2}$ in appendix D and takes the form

$$\varrho = \bar{\varrho}\cosh\bar{\theta}/2 - \frac{3}{4g^2}\,, \qquad \tan\theta/2 = \frac{\frac{2}{\sqrt{3}}g\bar{\varrho}}{\sqrt{\frac{4}{3}g^2\bar{\varrho} - 1}}\sinh\bar{\theta}/2\,, \quad (2.28)$$

where the barred coordinates correspond to the $k = -1$ case. While $\bar{\varrho} \to \infty$ limit covers the same region as the $\varrho \to \infty$ limit (the boundary), a subspace of the same $\varrho \to \infty$ region

[22] The Weyl tensor of AdS_5 of course vanishes identically.

JHEP04(2017)017

with $\theta = \pm\pi$ can be also reached in the limits $\bar{\theta} \to \pm\infty$. This subspace is covered twice. We could, then, ignore the $\eta = \pi/2$ $\xi \neq \pi/2$ piece of the boundary and consider just the $\xi = \pi/2$ one. However, the derivative of the conformal factor vanishes on the boundary at $\eta = \pi/2$. We could exclude these points to avoid this problem and add the second piece of the boundary, but, as we have seen, the induced metric is not regular there. Thus, at best, in the $k = -1$ coordinates we can only describe part of the boundary and the solutions that use these coordinates asymptotically will have the same problem.

Things are much more complicated in the $k = 0$ case. It is convenient to proceed in two steps. First, we redefine the ϱ coordinate as in the preceding cases in terms of ξ and set $\xi = \pi/2$, getting

$$\tilde{ds}^2(\xi = \pi/2) = (g/\sqrt{3})dt[dz + 2(ydx - xdy)] - dx^2 - dy^2\,. \tag{2.29}$$

Then, we redefine $x = \frac{\zeta}{2}\cos\varphi$ and $y = \frac{\zeta}{2}\sin\varphi$, getting

$$\tilde{ds}^2(\xi = \pi/2) = (g/\sqrt{3})dt\left[dz - \frac{\zeta^2}{2}d\varphi\right] - \frac{1}{4}[d\zeta^2 + \zeta^2 d\varphi^2]\,, \tag{2.30}$$

and shift the t coordinate

$$dt \to dt - 4\frac{(2+\zeta^2)dz - 2z\zeta d\zeta}{16z^2 + (2+\zeta^2)^2}\,, \tag{2.31}$$

which can be done since the added part is a closed 1-form. Finally, we make the coordinate transformation[23]

$$\begin{aligned}
\psi &= \frac{2\cos\eta\sin(\delta - gt/\sqrt{3})}{3 - 4\cos(\delta - gt/\sqrt{3})\cos\eta + \cos 2\eta}\,,\\
\zeta^2 &= \frac{4\sin^2\eta}{3 - 4\cos(\delta - gt/\sqrt{3})\cos\eta + \cos 2\eta}\,,\\
\varphi &= \gamma - gt/\sqrt{3} - \operatorname{arccot}\left[\cot(\delta - gt/\sqrt{3}) - \frac{1}{\cos\eta\sin(\delta - gt/\sqrt{3})}\right],
\end{aligned} \tag{2.32}$$

getting

$$\tilde{ds}^2 = \frac{1}{3 - 4\cos(\delta - gt/\sqrt{3})\cos\eta + \cos 2\eta}\left[d(gt/\sqrt{3})^2 - d\eta^2 - \cos^2\eta\, d\delta^2 - \sin^2\eta\, d\gamma^2\right], \tag{2.33}$$

which again is conformal to $\mathbb{R}\times S^3$. The total conformal factor is now

$$\Omega = \cos\xi\sqrt{3 - 4\cos(\delta - gt/\sqrt{3})\cos\eta + \cos 2\eta}\,, \tag{2.34}$$

and, again, leads to a description of the boundary in two separate pieces. The analysis if this case is much more involved and we will leave it for future work.

[23] To obtain these coordinate changes one can consider the embedding of AdS_5 in $\mathbb{C}^{1,2}$ in terms of complex coordinates Z^0, Z^1, Z^2. The "correct" limit giving the asymptotic boundary of AdS is the one obtained by sending $|Z^0|$ to infinity while leaving Z^1, Z^2 and the phase of Z^0 independent. For any fixed value of $|Z^0|$, Z^1 and Z^2 parametrize a 3-sphere. One then wants to choose coordinates such that, at infinity, $Z^1 \sim \sin\eta\, e^{i\gamma}$ and $Z^2 \sim \cos\eta\, e^{i\delta}$. In this way one manifestly recovers the wanted $\mathbb{R}\times S^3$ structure, where the S^3 is parametrized by Z^1 and Z^2 in terms of the coordinates η, δ and γ, while the $\mathbb{R}$ factor is parametrized by the phase t of Z^0. The shift eq. (2.31) is necessary because, for $k = 0$, the t coordinate in eq. (2.29) is not the phase of Z^0, but was shifted to remove an additional term in χ.

JHEP04(2017)017

3 Solutions

In this section we are going to try to solve eqs. (1.63)–(1.67) to find supersymmetric solutions of minimal gauged 5-dimensional supergravity.

We are going to search for solutions in which the functions H, L, M, K only depend on the coordinate x^2 which will play the rôle of "radial" coordinate and will be denoted by ϱ as in the previous section. This is possible if W^2 factorizes as follows

$$W^2 = \Psi(\varrho)\Phi(x^1, x^3)\,, \tag{3.1}$$

where, in order to solve eq. (1.65) Ψ must take the form

$$H = (\alpha\varrho + \beta)/\Psi\,, \tag{3.2}$$

for some constants α and β. When $\alpha \neq 0$ we can eliminate β by shifting ϱ and we can set α to 1 by rescaling ϱ. However, if $\alpha = 0$, we cannot eliminate completely β: at most we can set it to 1 by rescaling Ψ. Thus, there are two possible cases to be considered that we can parametrize with $\epsilon = 0, 1$:

$$H = \varrho^\epsilon/\Psi\,. \tag{3.3}$$

If we assume that the metric function $\hat{f}$ is a function of ϱ only, then it follows from eq. (1.40) that Φ is a solution of Liouville's equation

$$\left(\partial_{\underline{1}}^2 + \partial_{\underline{3}}^2\right)\log\Phi = -2k\,\Phi\,, \tag{3.4}$$

so that, for $k = 1, 0, -1$ it is given by the first of eqs. (D.34), then eqs. (1.64)–(1.67) simplify considerably: first, eq. (1.66) gives

$$gK = \Psi'/\Psi\,, \tag{3.5}$$

where primes denote derivation with respect to ϱ. Then, eq. (1.64) M can be integrated once to give

$$M' = \frac{\alpha}{\Psi} - \sqrt{3}gL^2\,. \tag{3.6}$$

This result can be used to eliminate M from eq. (1.63), giving

$$L'' + \frac{4}{3}L'\frac{\Psi'}{\Psi} + \frac{4}{3}L\frac{\Psi''}{\Psi} - \frac{2}{3}L\left(\frac{\Psi'}{\Psi}\right)^2 - \frac{4}{\sqrt{3}}\alpha g\frac{\varrho^\epsilon}{\Psi^2} = 0\,. \tag{3.7}$$

This equation has to be supplemented by eq. (1.67), that now takes the form

$$L = \frac{\Psi}{8g^2\varrho^\epsilon}\left\{-\frac{2k}{\Psi} - \frac{2}{3}\left(\frac{\Psi'}{\Psi}\right)^2 + \frac{\Psi''}{\Psi}\right\}\,. \tag{3.8}$$

Using the last equation to eliminate L from the previous one, we get the promised fourth order differential equation in Ψ

$$\begin{aligned} -96\sqrt{3}\alpha g^3\varrho^{2+2\epsilon} &+ 4\varrho(\Psi')^2\left(3k\varrho - \epsilon\Psi'\right) \\ &+ 6\Psi\left[-\epsilon(1+\epsilon)(\Psi')^2 - 4k\varrho^2\Psi'' + 2\epsilon\varrho\Psi'\left(2k + \Psi''\right)\right] \\ &+ 9\Psi^2\left\{\epsilon\left(1+\epsilon\right)\Psi'' - 2\epsilon\left[k(1+\epsilon) + \varrho\Psi'''\right] + \varrho^2\Psi''''\right\} = 0\,. \end{aligned} \tag{3.9}$$

JHEP04(2017)017

It is convenient to study the $\epsilon = 0$ and $\epsilon = 1$ cases separately. The respective equations take the form

$$\epsilon = 0\,, \quad \Rightarrow \quad -32\sqrt{3}\alpha g^3 + 4k(\Psi')^2 - 8k\Psi\Psi'' + 3\Psi^2\Psi'''' = 0\,, \tag{3.10}$$

and

$$\begin{aligned}\epsilon = 1\,, \quad \Rightarrow \quad & 96\sqrt{3}\alpha g^3\varrho^4 + 4\varrho(\Psi')^2\left(-3k\varrho + \Psi'\right) \\ & +12\Psi\left(2k\varrho - \Psi'\right)\left(-\Psi' + \varrho\Psi''\right) + 9\Psi^2\left(4k - 2\Psi'' + 2\varrho\Psi''' - \varrho^2\Psi''''\right) = 0\,.\end{aligned} \tag{3.11}$$

Our experience with the RN-AdS$_5$ solutions in section 2.1 suggests the use of a polynomic Ansatz to solve eqs. (3.10) and (3.11):

$$\Psi = \sum_{n=0}^{N} c_n \varrho^n\,. \tag{3.12}$$

In both equation, the term of highest order in ϱ is always proportional to the coefficient c_N term in Ψ and this term only vanishes if $c_N = 0$ or if $N \leq 3$, implying that Ψ is at most of 3rd order.

Let us analyze the $\epsilon = 1$ and $\epsilon = 0$ cases separately.

3.1 The $\epsilon = 1$ case: rotating black holes

Eq. (3.11) ($\epsilon = 1$) is only solved if either $c_0 = c_1 = 0$ or if $c_2 = k + \frac{{c_1}^2}{3c_0}$. A parametrization of the solution in terms of three parameters a, b, c that covers both possibilities is[24]

$$\Psi = \frac{1}{a}\left[c\varrho^3 + \varrho^2 + b\varrho + \frac{b^2}{3(1-ak)}\right]\,, \tag{3.13}$$

and the constant α in eq. (3.6) is constrained to take the value

$$\alpha = \frac{1 + 3ak + 3bc\left[\frac{3bc}{1-ak} - 2\left(1 - 2ak\right)\right]}{24\sqrt{3}a^3 g^3}\,. \tag{3.14}$$

Given the above values of Ψ and α, one can immediately compute W^2 using eqs. (3.1) and (D.34), H using eq. (3.3) (with $\epsilon = 1$), L using eq. (3.8), K using eq. (3.5) and M using eq. (3.6). The latter, in particular, being the solution of a first-order differential equation, contains an additional integration constant that we call d.

[24] For $k = 1$ this is the same as a solution found in [42] for a particular case of a scaling limit of the orthotoric base ansatz the authors use. The other particular case of this limit analyzed in the same paper leads to a solution which was already known [41] and that includes all known supersymmetric black hole solutions. As it turns out, this very general solution also includes our solution for $b = 0$ and all three values of k, even if this was not shown explicitly in those papers. Our approach is in any case more systematic, since we have shown that these are the only possible solutions with polynomial Ψ compatible with the assumptions we made.

JHEP04(2017)017

The functions that appear in the metric and 1-form field are

$$\begin{aligned}
\hat{f} &= \frac{4a}{c}(g/\sqrt{3})^2 \frac{\varrho}{\varrho + \frac{(1-ak)}{3c}}\,, \\
H &= \frac{a\varrho}{c\varrho^3 + \varrho^2 + b\varrho + \frac{b^2}{3(1-ka)}}\,, \\
W^2 H &= \varrho\, \Phi_{(k)}\,, \\
\chi &= \chi_{(k)}\,, \\
\omega_z &= d + \frac{b\,[9c\varrho + 2(1-ak)] + 3\varrho\left[6c^2\varrho^2 + 3c\varrho(2-ak) + (1-ak)^2\right]}{16\sqrt{3}g^3a^2\varrho^2}\,, \\
\omega &= -\frac{3\sqrt{3}ck}{16g^3a}\chi_{(k)}\,.
\end{aligned} \tag{3.15}$$

Notice that, since ω is given by a constant times $\chi_{(k)}$, it can be reabsorbed in ω_z with a shift in the t coordinate, so that

$$\omega = 0\,, \qquad \omega_z = d + \frac{18c^2\varrho^3 + 18c(1-ak)\varrho^2 + [9bc + 3(1-ak)^2]\varrho + 2b(1-ak)}{16\sqrt{3}g^3a^2\varrho^2}\,. \tag{3.16}$$

Notice also that the full 5-dimensional metric is invariant under the rescaling $t \to t/\alpha$, $\varrho \to \alpha\varrho$, $b \to \alpha b$, $c \to c/\alpha$, $d \to d/\alpha$. This allows to set one of the constants b, c, d to 1, provided it is not zero, leaving only three independent parameters. Then, assuming $c \neq 0$ (the $c = 0$ case will be dealt with later) we can use this freedom to normalize the metric so that $\hat{f} \to 1$ for large values of ϱ, setting

$$\frac{4a}{c}(g/\sqrt{3})^2 = 1\,. \tag{3.17}$$

Eliminating in this way c from the non-vanishing functions that define the family of solutions, we get

$$\begin{aligned}
\hat{f} &= \varrho\left[\varrho + \frac{(1-ak)}{4ag^2}\right]^{-1} \\
H &= \varrho\left\{\frac{4g^2}{3}\varrho^3 + \frac{1}{a}\varrho^2 + \frac{b}{a}\varrho + \frac{b^2}{3a(1-ka)}\right\}^{-1}\,, \\
W^2 H &= \varrho\, \Phi_{(k)}\,, \\
\chi &= \chi_{(k)}\,, \\
\omega_z &= d + \varrho^{-2}\left\{\frac{2g}{\sqrt{3}}\varrho^3 + \frac{\sqrt{3}(1-ak)}{2ag}\varrho^2 + \left[\frac{\sqrt{3}b}{4ag} + \frac{\sqrt{3}(1-ak)^2}{16a^2g^3}\right]\varrho + \frac{b(1-ak)}{8\sqrt{3}a^2g^3}\right\}.
\end{aligned} \tag{3.18}$$

Comparing this family of solutions with the supersymmetric RN-AdS$_5$ solution in eqs. (2.15) we find that the latter are included in the former for the following values of the independent integration constants:

$$k = 1\,, \qquad a = (1 + \sqrt{3}g^2q)^{-1}\,, \qquad b = \frac{3g^2q^2}{4(1 + \sqrt{3}g^2q)}\,, \qquad d = 0\,. \tag{3.19}$$

JHEP04(2017)017

Since the integration constant d is independent of the rest, we could extend the RN-AdS$_5$ solution by switching it on. The resulting solutions are no longer asymptotically AdS$_5$. This is true for the whole family of solutions presented here and, henceforth, we will set $d = 0$ in what follows.

Taking

$$a^{-1} \to k\,, \qquad b = d = 0\,, \tag{3.20}$$

we get the 3 different forms of the AdS$_5$ metrics eq. (2.17).

Perhaps more interestingly, for

$$k = 1\,, \qquad a = \frac{1}{4\alpha^2}\,, \qquad b = d = 0\,, \tag{3.21}$$

one recovers the asymptotically-AdS$_5$, supersymmetric, charged, rotating black holes found in ref. [44].[25] The mass M, the non-vanishing angular momentum J and the electric charge q of these solutions are given in terms of the only independent parameter a by[26]

$$M = R_0^2 + \frac{g^2}{2}R_0^4 + \frac{2g^4}{27}R_0^6\,, \qquad J = \frac{g}{2\sqrt{3}}R_0^4 + \frac{g^3}{9\sqrt{3}}R_0^6\,, \qquad q = \frac{1}{\sqrt{3}}R_0^2 + \frac{g}{6\sqrt{3}}R_0^4\,, \tag{3.22}$$

where $R_0^2 = (1-a)/(ag^2)$, so that

$$M - \frac{2g}{\sqrt{3}}|J| = \sqrt{3}|q|\,. \tag{3.23}$$

These black-hole solutions have a regular near-horizon geometry, and the horizon is a squashed 3-sphere. This is just one of the three possible near-horizon geometries found in ref. [44]. We are going to show that there are $k = 0, -1$ solutions which have the other two near-horizon geometries. In particular, for

$$k = 0\,, \qquad b = d = 0\,, \tag{3.24}$$

the remaining parameter a can be set to 1 with a rescaling of the coordinates and one gets the solution obtained in ref. [44] as the "large black-hole limit" ($R_0 \to \infty$) of the $k = 1$ solution.

3.1.1 Near-horizon geometries

The event horizon, if it exists, must be placed at $\varrho = 0$. When the parameter $b \neq 0$, our experience with the supersymmetric RN-AdS$_5$ solution suggests that there is no event horizon and the $\varrho \to 0$ limit is not a near-horizon geometry even if it is a regular one. Therefore, we are going to study separately the $b = 0$ and $b \neq 0$ cases.

[25] To compare our solution with the solution described in section 4.1 of ref. [44] we first have to identify the constants $g/\sqrt{3} = \ell^{-1}$, transform our radial coordinate $x^2 \equiv \varrho = \alpha^2\ell^2 \sinh(\rho/\ell)$ and our isometric coordinate $z = \phi$. Furthermore, we have to make the usual coordinate transformation to go from conformally flat coordinates x^1, x^3 to spherical ones (θ, ψ in ref. [44]) on the 2-sphere: $x^1 + ix^3 = \tan(\theta/2)\, e^{i\psi}$.

[26] Our normalization of the electric charge differs by a factor of 2 from that of ref. [44], so, for $J = 0$, we get eq. (2.6). Furthermore, we remind the reader that we have chosen units such that eq. (2.3) holds.

JHEP04(2017)017

When $b = 0$, defining first the coordinates u, v

$$\begin{aligned} dt &= \mp \frac{(1-ak)^{1/2}(1+3ak)^{1/2}}{4ag}\left[du - \frac{(1-ak)}{4g^2}\frac{d\varrho}{\varrho^2}\right], \\ dz &= dv \pm \sqrt{3}a\frac{(1-ak)^{1/2}}{(1+3ak)^{1/2}}\frac{d\varrho}{\varrho}, \end{aligned} \tag{3.25}$$

the near-horizon geometry can be written in a form that generalizes the one obtained for the $k = 1$ case in ref. [44]

$$\begin{aligned} ds^2 = \Delta^2\varrho^2 du^2 - 2du d\varrho + \frac{6k\Delta}{\ell(\Delta^2 - 3\ell^{-2})}\varrho du(dv + \chi_{(k)}) \\ - \frac{k}{\Delta^2 - 3\ell^{-2}}\left[\frac{k\Delta^2}{\Delta^2 - 3\ell^{-2}}(dv + \chi_{(k)})^2 + d\Omega^2_{(2,k)}\right], \end{aligned} \tag{3.26}$$

where $3\ell^{-2} = g^2$ and we have defined

$$\Delta^2 = \frac{1+3ak}{1-ak}g^2\,. \tag{3.27}$$

Observe that the combination $k/(\Delta^2 - 3\ell^{-2}) = (1-ak)/(4ag^2)$ does not vanish for $k = 0$. it does not become negative for $k = -1$ either.

For the $k = 0$ case, a rescaling of the coordinates $v \equiv 4g\omega$, $x^1 \equiv gx$, $x^3 \equiv gy$ brings the above near-horizon metric into the form

$$\begin{aligned} ds^2 = \frac{3}{\ell^2}\varrho^2 du^2 - 2du d\varrho + \frac{6}{\ell}\varrho du\left[dv' + \frac{\sqrt{3}}{2\ell}(ydx - xdy)\right] \\ - \left[dv' + \frac{\sqrt{3}}{2\ell}(ydx - xdy)\right]^2 - dx^2 - dy^2\,, \end{aligned} \tag{3.28}$$

which, at $\varrho = 0$ gives the standard metric of the homogeneous *Nil* group manifold and which, upon dimensional reduction along v' gives the metric of $\mathrm{AdS}_2 \times \mathbb{E}^2$.[27]

For $k = -1$ we rescale $v \equiv -(\Delta^2 - 3\ell^{-2})v'/\Delta$ to obtain

$$\begin{aligned} ds^2 = \Delta^2\varrho^2 du^2 - 2du d\varrho + \frac{6}{\ell}\varrho du\left[dv' - \frac{\Delta}{\Delta^2 - 3\ell^{-2}}\chi_{(k)}\right] \\ - \left[dv' - \frac{\Delta}{\Delta^2 - 3\ell^{-2}}\chi_{(k)}\right]^2 + \frac{\Delta}{\Delta^2 - 3\ell^{-2}}d\Omega^2_{(2,-1)}\,, \end{aligned} \tag{3.29}$$

[27]The solution in eqs. (4.62) of ref. [44], which corresponds to our $k = b = d = 0$ solution, obtained as the "large black-hole limit" ($R_0 \to \infty$) of the $k = 1$ solution, has a horizon with precisely this near-horizon geometry. However, the solution was considered by the authors to be not asymptotically-AdS_5 because, the asymptotic geometry, written in eq. (4.63) of that reference, was interpreted as a supersymmetric plane-fronted wave. It is not difficult to see that, actually, is the $k = 0$ form of AdS_5 as given in eq. (2.17) upon the coordinate change $\varrho = S^2$. Thus, the "large black-hole limit" of the $k = +1$ black hole gives the $k = 0$ black hole.

JHEP04(2017)017

which, upon dimensional reduction along v' gives the metric of $AdS_2 \times \mathbb{H}_2$ and which, for $\Delta = 0$, gives the metric of $AdS_3 \times \mathbb{H}_2$ which arises as the near-horizon geometry of the black strings of ref. [52].

When $b \neq 0$ we obtain in the $\varrho \to 0$ limit a completely regular geometry that does not correspond to a horizon:

$$\begin{aligned} ds^2 = & \frac{16a^2g^4\varrho^2}{(1-ak)^2}dt^2 + \frac{4bg}{\sqrt{3}(1-ak)}dt(dz+\chi_{(k)}) - \frac{3(1-ak)^2}{4b^2g^2}d\varrho^2 \\ & -\frac{1-ak}{4ag^2}\left\{\frac{9b^2c^2+(1-ak)^3(1+3ak)-6bc(1-ak)^2}{4a(1-ak)^3}(dz+\chi_{(k)})^2 + d\Omega^2_{(2,k)}\right\}. \end{aligned} \tag{3.30}$$

3.1.2 Asymptotic limits

The naive asymptotic limits of these solutions are the different forms of AdS_5 presented in section 2.2. However, these limits are very subtle and must be analyzed using the same methods we used for the different forms of AdS_5 at the end of section 2.2. We can use exactly the same changes of coordinates. Then, it can be seen that only for $c \neq 0$ and $d = 0$ these solutions can be asymptotically AdS_5.

Notice however that for $k = 0, -1$ the conformal 4-dimensional metric presents a singularity where the conformal factor multiplying the $\mathbb{R}\times S^3$ metric diverges. For pure AdS_5 this problem can of course be solved by simply changing to the usual ($k = 1$) global coordinates, which amounts to taking a slightly different asymptotic limit. For the full solutions however this is not the case: we have verified that for $k = -1$ a simple $k = 1$ AdS_5-like coordinate transformation leads to a Weyl tensor that diverges in $\eta = \frac{\pi}{2}$ for all values of ξ, while it vanishes in $\xi = \frac{\pi}{2}$ if $\eta \neq \frac{\pi}{2}$. The situation can be improved with a modified coordinate transformation giving a regular Weyl tensor, which however still diverges as one approaches $(\xi, \eta) = (\frac{\pi}{2}, \frac{\pi}{2})$. This could indicate that these solutions do not asymptote to AdS_5 globally, but only locally.

The $k = 0$ case is more complicated, due to the more involved transformation between the correspondent AdS parametrization and the $k = 1$ one, and we have not studied this case in detail. One could expect however a similar behaviour as in the $k = -1$ case.

3.1.3 The conserved charges

For $k = 1$ we can compute the conserved charges of the solutions following the prescription given in [51]. The mass is given by the conserved charge associated to the Killing vector pointing along the time direction of the conformal boundary $\mathbb{R}\times S^3$ metric, and is

$$M = \frac{-31a^4 + a^3(43 - 76g^2b) + a^2(3 + 44g^2b - 64g^4b^2) + a(-11 + 32g^2b) - 4}{54g^2a^3(a-1)}. \tag{3.31}$$

The angular momenta associated to ∂_ψ and ∂_ϕ are

$$J_\psi = \frac{\left[a^2 - 2a(1+2g^2b) + 1\right]\left[7a^2 + a(-5+8g^2b) - 2\right]}{18\sqrt{3}a^3g^3(a-1)}, \qquad J_\phi = 0. \tag{3.32}$$

JHEP04(2017)017

The electric charge can be computed integrating the Hodge dual of the gauge field strength over a 3-sphere at infinity, and is given by

$$Q = \frac{-5a^2 + 4a(1 - g^2 b) + 1}{6\sqrt{3}a^2 g^2} \,. \tag{3.33}$$

It is straightforward to verify that these expressions reduce to the expected values for the Gutowski-Reall black hole and for RN-AdS$_5$, and that the BPS bound eq. (3.23) is satisfied for all values of the parameters a and b.

3.1.4 A homogeneous solution?

Besides the three different parametrizations of AdS$_5$ mentioned above, there is another choice of the parameters giving a solution apparently free of curvature singularities, for which in particular the Ricci and Kretschmann scalars and the Ricci tensor fully contracted with itself are all constant. It is given by $k = 0$, $d = 0$ and $b = \frac{1}{3c} = \frac{1}{4ag^2}$, and its metric, after a rescaling of the coordinates, is given by

$$ds^2 = \frac{3}{4g^2}\left[\frac{\varrho^2}{(1+\varrho)^2}dt^2 + 2(1+\varrho)dt(dz + \chi_{(0)}) - \frac{d\varrho^2}{(1+\varrho)^2} - (1+\varrho)d\Omega^2_{(0)}\right] \tag{3.34}$$

with gauge field strength

$$F = -\frac{3}{2g}\frac{d\varrho \wedge dt}{(1+\varrho)^2} \,. \tag{3.35}$$

Since $b \neq 0$ the solution is horizonless, and since $d = 0$ it is asymptotically, at least locally, AdS$_5$.

In terms of a Vielbein

$$F = -2g\frac{e^2 \wedge (e^0 - e^1)}{\varrho} \,. \tag{3.36}$$

3.1.5 The $c = 0$ solutions

The $c = 0$ solutions (with $d = b = 0$) can be seen to coincide identically with the near-horizon geometries recovered in section 3.1.1: setting $b = c = d = 0$ in eqs. (3.15) and (3.16) we get a metric that coincides exactly with that determined by the leading terms in the $\varrho \to 0$ limit. The change of coordinates eq. (3.25) and the replacement of the parameter a by Δ defined in eq. (3.27) brings it into the form eq. (3.26). The near-horizon configuration is a supersymmetric solution in its own right ad it is included in the general solution that we have presented.

3.2 The $\epsilon = 0$ case: Gödel universes

First of all, in this case, eq. (1.31) implies $d\chi = 0$, and one can set $\chi = 0$. Thus, we can absorb any constant term in ω_z in a redefinition on t. Furthermore, the integration constant α in eq. (3.6) must take the value

$$\alpha = k\frac{{c_1}^2 - 4c_0c_2}{8\sqrt{3}g^3} \,, \tag{3.37}$$

JHEP04(2017)017

and one has to distinguish between the $k = 0$ case, in which Ψ can be an arbitrary 3rd order polynomial, and the $k \neq 0$ one, in which c_3 must vanish, meaning that Ψ must be of just 2nd order.

For $\epsilon = 0$ and $k = 0$ this gives (after the integration to obtain M)

$$\begin{aligned}
\hat{f}^{-1} &= \frac{c_2 + 3c_3\varrho}{4g^2}\,, \\
H^{-1} &= c_0 + c_1\varrho + c_2\varrho^2 + c_3\varrho^3\,, \\
W^2 H &= \Phi_{(0)}\,, \\
\chi &= 0\,, \\
\omega_z &= \frac{\sqrt{3}}{16g^3}\left[({c_2}^2 + 3c_1c_3)\varrho + 6c_2c_3\varrho^2 + 6c_3^2\varrho^3\right]\,, \\
\omega &= -\sqrt{3}\frac{{c_2}^2 - 3c_1c_3}{16g^3}\chi_{(0)}\,,
\end{aligned} \tag{3.38}$$

so that, in particular, the metric takes the form

$$\begin{aligned}
ds^2 = \frac{16g^4}{(c_2+3c_3\varrho)^2}\Bigg\{ & dt + \frac{\sqrt{3}}{16g^3}\left[({c_2}^2 + 3c_1c_3)\varrho + 6c_2c_3\varrho^2 + 6c_3^2\varrho^3\right]dz \\
& + \frac{\sqrt{3}}{8g^3}(c_2^2 - 3c_1c_3)(x\,dy - y\,dx)\Bigg\}^2 - \frac{1}{4g^2}(c_2+3c_3\varrho)(c_0 + c_1\varrho + c_2\varrho^2 + c_3\varrho^3)dz^2 \\
& - \frac{1}{4g^2}\frac{(c_2+3c_3\varrho)}{(c_0 + c_1\varrho + c_2\varrho^2 + c_3\varrho^3)}d\varrho^2 - \frac{1}{g^2}(c_2+3c_3\varrho)(dx^2+dy^2)\,.
\end{aligned} \tag{3.39}$$

For $\epsilon = 0$ and $k \neq 0$, the functions that define the solution are given by

$$\begin{aligned}
\hat{f}^{-1} &= \frac{c_2 - k}{4g^2}\,, \\
H^{-1} &= c_0 + c_1\varrho + c_2\varrho^2\,, \\
W^2 H &= \Phi_{(k)}\,, \\
\chi &= 0\,, \\
\omega_z &= -\frac{(3k - c_2)\,(k + 3c_2)}{16\sqrt{3}g^3}\varrho\,, \\
\omega &= \frac{(k - 3c_2)\,(3k + c_2)}{16\sqrt{3}g^3}\chi_{(k)}\,,
\end{aligned} \tag{3.40}$$

so that the metric, in particular, takes the form

$$\begin{aligned}
ds^2 = \frac{16g^4}{(c_2-k)^2}\Bigg\{ & dt - \frac{(3k-c_2)(k+3c_2)}{16\sqrt{3}g^3}\varrho dz + \frac{(3k+c_2)(k-3c_2)}{16\sqrt{3}g^3}\chi_{(k)}\Bigg\}^2 \\
& - \frac{(c_2-k)}{4g^2}(c_0 + c_1\varrho + c_2\varrho^2)dz^2 - \frac{(c_2-k)}{4g^2(c_0 + c_1\varrho + c_2\varrho^2)}d\varrho^2 - \frac{(c_2-k)}{4g^2}d\Omega^2_{(k)}\,.
\end{aligned} \tag{3.41}$$

The parameters of the solutions above can be reduced by shifting and rescaling ϱ. The remaining independent possibilities are:

JHEP04(2017)017

1. $k=0$, $c_3=1$, $c_2=0$, c_1 and c_0 arbitrary.

2. $k=0,\pm1$, $c_3=0$, $c_2\neq 0$, $c_2>k$, $c_1=0$ and $c_0=0$.

3. $k=0,\pm1$, $c_3=0$, $c_2\neq 0$, $c_2>k$, $c_1=0$ and $c_0=1$.

4. $k=-1$, $c_3=0$, $c_2=0$, $c_1=1$ and $c_0=0$.

5. $k=-1$, $c_3=0$, $c_2=0$, $c_1=0$ and $c_0=1$.

Furthermore, for the cases 2. and 3., if $k=0$, it is possible to set $c_2=1$.

Case 1. is in general of difficult interpretation, however if $c_1=c_0=0$ the solution after a rescaling of the t coordinate takes the form

$$ds^2 = \frac{3}{4g^2}\left[\frac{dt^2}{\varrho^2}+2\varrho dt dz-\frac{d\varrho^2}{\varrho^2}-\varrho d\Omega^2_{(0)}\right] \tag{3.42}$$

$$F = \frac{3}{2g}\frac{d\varrho}{\varrho^2}\wedge dt\,. \tag{3.43}$$

The Ricci and Kretschmann scalars and the Ricci tensor fully contracted with itself are constant for this metric, suggesting it may represent a homogeneous space. The gauge field strength is constant if expressed in terms of a Vielbein, and represents a homogeneous electric field directed along ϱ.

In all the remaining cases the abovementioned curvature scalars are constant.

Cases 2. and 3.:

$$ds^2 = \left(\frac{4g^2}{c_2-k}\right)^2\left[dt-\frac{(3k-c_2)(k+3c_2)}{16\sqrt{3}g^3c_2}\chi_{(-1)}+\frac{(k-3c_2)(3k+c_2)}{16\sqrt{3}g^3}\chi_{(k)}\right]^2 - \frac{c_2-k}{4g^2c_2}\left[d\Omega^2_{(-1)}+c_2 d\Omega^2_{(k)}\right]. \tag{3.44}$$

Cases 4. and 5.:

$$ds^2 = 16g^4\left[dt+\frac{\sqrt{3}}{16g^3}\left(\chi_{(-1)}-\chi_{(0)}\right)\right]^2-\frac{1}{4g^2}\left[d\Omega^2_{(0)}+d\Omega^2_{(-1)}\right]. \tag{3.45}$$

The general expression of the gauge field strength for $c_3=0$ is

$$F=\frac{1}{4g(c_2-k)}\left[\left(3k^2+4c_2k+c_2^2\right)d\varrho\wedge dz+\left(k^2+4kc_2+3c_2^2\right)\Phi_{(k)}dx^3\wedge dx^1\right]. \tag{3.46}$$

Notice that the metric and gauge field for cases 4. and 5. can actually be seen as the particular case $k=0$ of the ones for cases 2. and 3., so that all cases with $c_3=0$ have metric (3.44) and gauge field strength that can be rewritten as

$$F=\frac{1}{4gc_2(c_2-k)}\left[\left(3k^2+4c_2k+c_2^2\right)d\chi_{(-1)}+\left(k^2+4kc_2+3c_2^2\right)c_2 d\chi_{(k)}\right]. \tag{3.47}$$

These solutions are 5-dimensional supersymmetric generalizations of the 4-dimensional Gödel's rotating universe [53], which also solves Einstein's equations with a cosmological

JHEP04(2017)017

constant and contain the 2-dimensional metric $d\Omega^2_{(-1)}$ and the associated 1-form $\chi_{(-1)}$. As in that case and also in the case of the 5-dimensional Gödel solution of the ungauged theory [15, 54], the solution contains closed timelike curves. Those solutions are also homogeneous spaces and it would be interesting to know if the three solutions presented share this property, as the constancy of their curvature invariants seems to indicate. In the ungauged 5-dimensional case [55], the dimensional reduction over the time direction gives rise to a solution of Euclidean $\mathcal{N} = 2, d = 4$ supergravity with an anti-selfdual Abelian instanton field and a geometry which, instead of $\mathbb{E}^4$ is given by $\mathbb{H}_2 \times (S^2, \mathbb{E}^2, \mathbb{H}_2)$ geometry. It is also likely that these 3 Gödel solutions can be obtained from the 3 near-horizon geometries discussed above by the limiting procedure proposed in ref. [55], since the standard Penrose limit cannot be used in gauged supergravity.[28]

4 Reduction to $d = 4$

The dimensional reduction over a circle of the theory of minimal 5-dimensional supergravity gives a theory of $\mathcal{N} = 2, d = 4$ supergravity coupled to one vector multiplet and determined by the cubic prepotential $\mathcal{F} = -(\mathcal{X}^1)^3/\mathcal{X}^0$. The complex scalar $t \equiv -\mathcal{X}^1/\mathcal{X}^0$ parametrizes an $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$ σ-model with Kähler potential $e^{\mathcal{K}} = (\Im\mathfrak{m}\, t)^3$. The relation between this and the rest of the 4-dimensional fields and the 5-dimensional ones (for which we use hats here: $\hat{g}_{\hat{\mu}\hat{\nu}}$ and $\hat{A}_{\hat{\mu}}$, where $\hat{\mu} = \mu, z$) is given by

$$g_{\mu\nu} = k\left(\hat{g}_{\mu\nu} + \frac{\hat{g}_{\mu\underline{z}}\hat{g}_{\nu\underline{z}}}{k^2}\right), \tag{4.1}$$

$$A^0{}_\mu = -\frac{1}{2\sqrt{2}}\frac{\hat{g}_{\mu\underline{z}}}{k^2}, \tag{4.2}$$

$$A^1{}_\mu = -\frac{1}{2\sqrt{6}}\hat{A}_\mu + \frac{1}{\sqrt{3}}\hat{A}_{\underline{z}}A^0{}_\mu, \tag{4.3}$$

$$t = \frac{1}{2\sqrt{3}}\hat{A}_{\underline{z}} + \frac{i}{2}k, \tag{4.4}$$

where

$$k^2 = -\hat{g}_{\underline{z}\underline{z}}, \tag{4.5}$$

is the Kaluza-Klein (KK) scalar measuring the local size of the compactification circle. It is assumed to be positive so the isometric coordinate z is spacelike.

The dimensional reduction of bosonic sector of the minimal, gauged, 5-dimensional theory of supergravity gives exactly the same action with the same relations between the 5- and the 4-dimensional fields except for an additional term corresponding to the 5-dimensional constant.[29] In $d = 4$ it appears multiplied by the KK scalar and becomes a negative-definite (but unbound) scalar potential. Taking into account the relation between the 5- and the 4-dimensional gauge coupling constants $g = -g_4/\sqrt{24}$, the 4-dimensional scalar potential is

$$V_4 = -(g_4/\sqrt{3})^2(\Im\mathfrak{m}\, t)^{-1}. \tag{4.6}$$

[28] We thank P. Meessen for comments on this point.

[29] In presence of hypermultiplets one can get additional terms in the scalar potential using generalized dimensional reduction [56].

JHEP04(2017)017

This potential does not have any extremum at regular points of the scalar manifold and, therefore, the theory does not admit an AdS_4 vacuum.[30] The most symmetric supersymmetric vacuum solution is probably the one obtained by dimensional reduction of the AdS_5 which we are going to review shortly. Since AdS_5 is the only maximally supersymmetric solution of minimal, gauged, 5-dimensional supergravity, this is only solution that could be maximally supersymmetric in the 4-dimensional theory.[31] All the asymptotically-AdS_5 solutions become 4-dimensional solutions that have the same asymptotic behaviour as that solution.

Using the above rules for the dimensional reduction, the metric and 2-form potential of the timelike supersymmetric solutions give rise to the following 4-dimensional fields:

$$ds^2 = e^{2U}(dt+\omega)^2 - e^{-2U}\gamma_{\underline{rs}}dx^r dx^s\,, \tag{4.8}$$

$$A^0 = \frac{1}{2\sqrt{2}}\left\{-\frac{\hat{f}^2\omega_z}{k^2}(dt+\omega)+\chi\right\}\,, \tag{4.9}$$

$$\begin{aligned} A^1 = &-\frac{1}{2\sqrt{6}}\left\{\frac{\hat{f}^2\omega_z}{k^2}\left[-\sqrt{3}\hat{f}\omega_z+\frac{\partial_{\underline{2}}\log W^2}{2gH}\right](dt+\omega)\right\}\,,\\ &-\frac{1}{2g}\left(\partial_{\underline{1}}\log W^2 dx^3-\partial_{\underline{3}}\log W^2 dx^1\right)\,, \end{aligned} \tag{4.10}$$

$$t = \frac{1}{2}\left[-\hat{f}\omega_z+\frac{\partial_{\underline{2}}\log W^2}{2\sqrt{3}gH}\right]+\frac{i}{2}k\,, \tag{4.11}$$

where

$$k^2 = \hat{f}^{-1}H^{-1}-\hat{f}^2\omega_z^2\,, \tag{4.12}$$

$$\gamma_{\underline{rs}}dx^r dx^s = (dx^2)^2+W^2[(dx^1)^2+(dx^3)^2]\,, \tag{4.13}$$

$$e^{-2U} = k\hat{f}^{-1}H = \sqrt{HL^3+\frac{1}{16}L^2K^2-M^2H^2-\frac{\sqrt{3}}{2}MLKH+\frac{1}{12\sqrt{3}}MK^3}\,, \tag{4.14}$$

and H, K, L, M, W, ω and χ are the same functions and 1-form that occur in the 5-dimensional metric. The functions H, K, L, M can be identified with the building blocks of the 4-dimensional timelike supersymmetric solutions (harmonic functions on $\mathbb{E}^3$ in the ungauged case).

The most interesting examples we can apply these relations to are AdS_5 and the Gutowski-Reall black hole.[32]

[30] In the dimensional reduction of a non-minimal gauged theory with vector supermultiplets and a 5-dimensional scalar potential $V_5(\phi)$ we obtain a 4-dimensional scalar potential which will always be of the form

$$V_4 = k^{-1}V_5(\phi)\,, \tag{4.7}$$

and analogous observations apply as well.

[31] Any maximally supersymmetric solution of the 4-dimensional solution must necessarily correspond to a maximally supersymmetric solution of the 5-dimensional theory. The converse is not true.

[32] The dimensional reduction of the supersymmetric Reissner-Nordström-AdS_5 solution gives a singular solution.

JHEP04(2017)017

4.1 Reduction of AdS$_5$

Applying the above relations to the $k = 1$ supersymmetric form of AdS$_5$ in eq. (2.12) we get the 4-dimensional solution

$$ds^2 = \varrho^{1/2}(1 + \frac{1}{18}g_4^2\varrho)dt^2 - \frac{d\varrho^2}{\varrho^{1/2}(1 + \frac{1}{18}g_4^2\varrho)} - \varrho^{3/2}d\Omega^2_{(2,1)}\,, \tag{4.15}$$

$$A^0 = \frac{1}{2\sqrt{2}}\chi_{(1)}\,, \tag{4.16}$$

$$A^1 = \frac{1}{g_4}\chi_{(1)}\,, \tag{4.17}$$

$$t = -\frac{2}{g_4} + \frac{i}{2}\varrho^{1/2}\,. \tag{4.18}$$

This solution is singular at $\varrho = 0$ In particular, the imaginary part of the scalar t vanishes there. The underlying reason is that the compactification circle's radius, measured by the KK scalar, shrinks to zero at $\rho = 0$. Asymptotically, the metric is conformal to that of $\mathbb{R} \times S^2$, but it cannot be considered asymptotically-AdS$_4$ because the Weyl tensor diverges in this limit [51]. This asymptotic behaviour is shared by all the asymptotically-AdS$_5$ solutions written in the $k = 1$ form, such as the Gutowski-Reall black hole.

Typically, some supersymmetry is always broken in the dimensional reduction of AdS$_5$. This will happen if the 5-dimensional Killing vector depends on the isometric coordinate z. To find whether this is the case and how much supersymmetry can be preserved in 4 dimensions one has to solve explicitly the Killing spinor equation which, for vanishing vector field strength, with our choice of FI term, and setting $g = \sqrt{3}$, is given by

$$\delta_\epsilon \psi^i{}_\mu = \nabla_\mu \epsilon^i + \frac{i}{2}\sigma^1{}^i{}_j\gamma_\mu\epsilon^j = 0\,, \tag{4.19}$$

where σ^1 is the first Pauli matrix.

The t component of this equation is

$$\left\{\partial_t + \frac{1}{4}\hat{J}_{mn}\gamma^{mn} + \frac{i}{2}\gamma^0\sigma^1\right\}\epsilon = 0\,, \tag{4.20}$$

and is solved by

$$\epsilon = e^{-\{\frac{1}{4}\hat{J}_{mn}\gamma^{mn} + \frac{i}{2}\gamma^0\sigma^1\}t}\,\eta(\varrho, z, x^1, x^3)\,. \tag{4.21}$$

The ϱ component of the Killing spinor equation reduces to the following equation for the t-independent spinor η:

$$\left\{\partial_\varrho - H^{1/2}\frac{1}{2}\left(\gamma^{0\sharp} + i\gamma^2\sigma^1\right)\right\}\eta = 0\,, \tag{4.22}$$

where $H^{-1} = \varrho(1 + 4\varrho)$, which is solved by

$$\eta = e^{\int d\varrho H^{1/2}\frac{1}{2}(\gamma^{0\sharp} + i\gamma^2\sigma^1)}\xi(z, x^1, x^3)\,. \tag{4.23}$$

The z component, then, reduces to

$$\{\partial_z + A\}\,\xi = 0\,, \tag{4.24}$$

JHEP04(2017)017

where

$$A = -e^{-B}\left\{\tfrac{1}{8}\hat{J}_{mn}\gamma^{mn} + \left(2\varrho + H^{-1/2}\gamma^{0\sharp}\right)\tfrac{1}{2}\left(\gamma^{\sharp 2} - i\gamma^{0}\sigma^{1}\right)\right\}e^{B}\,,$$
$$B = \int d\varrho H^{1/2}\tfrac{1}{2}\left(\gamma^{0\sharp} + i\gamma^{2}\sigma^{1}\right)\,. \tag{4.25}$$

Since the Killing spinor equations are integrable, we know that A is ϱ-independent, but its actual value is important to determine whether ξ, and hence ϵ, is z-dependent or not. A long calculation gives $A = -\frac{1}{8}\hat{J}_{mn}\gamma^{mn}$ and

$$\xi = e^{\frac{1}{8}\hat{J}_{mn}\gamma^{mn}z}\,\zeta(x^1, x^3)\,. \tag{4.26}$$

The z-independent part of this spinor (and of the whole Killing spinor ϵ) is the one satisfying the projection

$$\frac{1}{2}\hat{J}_{mn}\gamma^{mn}\epsilon = \gamma^{\sharp 2}\frac{1}{2}(1+\gamma^{\sharp 123})\epsilon = \gamma^{\sharp 2}\,\frac{1}{2}(1+\gamma^{0})\epsilon = 0\,, \tag{4.27}$$

which is the condition generically satisfied by the timelike Killing spinors of $\mathcal{N}=2, d=4$ theories. Most of the timelike supersymmetric solutions of the minimal, gauged 5-dimensional supergravity must satisfy this condition as well.

4.2 Reduction of the Gutowski-Reall black hole

The Gutowski-Reall black hole is determined by

$$\hat{f} = \frac{\varrho}{\varrho + \frac{4\alpha^2-1}{4g^2}}\,, \qquad H^{-1} = \frac{4}{3}\varrho(3\alpha^2 + g\varrho)\,, \qquad W^2 = \varrho H^{-1}\Phi_{(1)}\,,$$
$$\omega_z = \frac{3(4\alpha^2-1)^2 + 24(4\alpha^2-1)g^2\varrho + 32g^4\varrho^2}{16\sqrt{3}\varrho}\,, \qquad \omega = 0\,,$$

and, according to the general rules, we get a 4-dimensional in which the two 1-form fields and the scalar field take non-trivial expressions. We are just interested in the metric function and the KK scalar, which take the form

$$k^2 = \frac{3(4\alpha^2-1)^3 + 64(4\alpha^2-1)^2(\alpha^2+2)g^2\varrho + 576(4\alpha^2-1)g^4\varrho^2 + 768g^6\varrho^3}{48g^2[(4\alpha^2-1)+4g^2\varrho]^2}\,, \tag{4.28}$$

$$e^{-2U} = \frac{3k[(4\alpha^2-1)+4g^2\varrho]}{16g^2\varrho^2(3\alpha^2+g\varrho)}\,, \tag{4.29}$$

$$e^{-2U}W^2 = k\left[\varrho + \frac{4\alpha^2-1}{4g^2}\right]\Phi_{(1)}\,. \tag{4.30}$$

In the $\varrho \to \infty$ limit the metric of this solution has the same behaviour as that of the previous one. More interestingly, in the $\varrho \to 0$ limit

$$k^2 \sim \frac{4\alpha^2-1}{16g^2} \equiv k^2_{\rm fix}\,, \qquad e^{-2U} \sim \frac{k^3_{\rm fix}}{\alpha^2}\frac{1}{\varrho^2}\,, \qquad e^{-2U}W^2 \sim 4k^3_{\rm fix}\,, \tag{4.31}$$

corresponding to an $\mathrm{AdS}_2\times\mathrm{S}^2$ near-horizon geometry in which the two factor spaces have different radii.

Thus, the Gutowski-Reall black hole reduces to a static, extremal, 4-dimensional black hole with exotic asymptotics.

JHEP04(2017)017

5 Conclusions

In this paper we have shown how the metric ansatz of ref. [45] simplifies the equations the determine the timelike supersymmetric solutions of 5-dimensional minimal gauged supergravity and allows one to find quite general families of interesting solutions such as the black holes with non-compact horizons and the Gödel-like solutions.

Our ansatz was inspired by the Gibbons-Hawking ansatz for the base space made in ref. [15] in the ungauged theory. However, there is a very important difference between the gauged and ungauged cases (beyond the Kähler and hyper-Kähler nature of the base spaces): in the ungauged case, given a choice of base space, it is possible to construct many different solutions which can be seen as "excitations" over the vacuum defined by the choice: the choice of metric function $\hat{f}$ and of the harmonic function H that determines the Gibbons-Hawking metric are independent. In the gauged case the situation is much more complicated because the base space is different for each different solution: the functions H and W that define the Kähler metric with one isometry depend on the metric function $\hat{f}$ and there is a different Kähler geometry for each solution. Of course, this also happens for other Kähler metric ansatzs. With our ansatz this dependence can be controlled more efficiently and it is possible to generate systematically all the required Kähler solutions. The search for new solutions is necessarily the search for new Kähler geometries or new forms for the same Kähler geometries.

Another surprise we have found (in particular, in the study of the vacuum solutions AdS_5) is the convenience (or even necessity) of using different forms of the same base and how the coordinates of the base space (all Euclidean in the base space) can have very different causal characters in the full 5-dimensional metric.

The scope of our investigations was restricted to the simplest solutions with an event horizon. These are black holes with only one independent angular momentum. However, supersymmetric rotating black-hole solutions with more independent angular momenta have also been constructed in ref. [57] and, associated to the general form of their base space which can be adapted to our ansatz, we expect to find other families of solutions. Furthermore, we would like to extend our results to matter-coupled theories to reproduce and extend the results found in ref. [58]. Work in these directions is in progress [59].

Acknowledgments

TO would like to thank I. Bena, M. Dunajski, D. Klemm, P. Meessen, and C. Shahbazi for interesting and inspiring conversations. This work has been supported in part by the Spanish Ministry of Science and Education grants FPA2012-35043-C02-01 and FPA2015-66793-P, the Centro de Excelencia Severo Ochoa Program grant SEV-2012-0249, and the Spanish Consolider-Ingenio 2010 program CPAN CSD2007-00042. The work of SC has been supported by the Spanish Ministry of Science and Education grant FPA2012-35043-C02-01. TO wishes to thank M.M. Fernández for her permanent support.

JHEP04(2017)017

A 3-d metrics

Let us consider 3-dimensional Riemannian metrics of the form

$$d\bar{s}^2 = \gamma_{\underline{ij}} dx^i dx^j = (dx^2)^2 + W^2[(dx^1)^2 + (dx^3)^2]\,, \tag{A.1}$$

where W depends on the three coordinates x^i, $i = 1, 2, 3$ in an arbitrary way. A convenient basis of Dreibeins is

$$\begin{cases} v^{1,3} = W dx^{1,3}\,, \\ v^2 = dx^2\,, \end{cases} \qquad \begin{cases} v_{1,3} = W^{-1}\partial_{\underline{1},\underline{3}}\,, \\ v_2 = \partial_{\underline{2}}\,. \end{cases} \tag{A.2}$$

The non-vanishing components of the spin connection are

$$\overline{\omega}_{112} = \overline{\omega}_{332} = -\partial_2 \log W\,, \qquad \overline{\omega}_{113} = -\partial_3 \log W\,, \qquad \overline{\omega}_{331} = -\partial_1 \log W\,, \tag{A.3}$$

and those of the Riemann curvature tensor are

$$\begin{aligned} \overline{R}_{1212} &= \overline{R}_{2323} = W^{-1}\partial_{\underline{2}}^2 W\,, & \overline{R}_{1213} &= W^{-1}\partial_{\underline{2}}\partial_{\underline{3}} \log W\,, \\ \overline{R}_{1313} &= W^{-2}\left(\partial_{\underline{1}}^2 + \partial_{\underline{3}}^2\right)\log W + \left(\partial_{\underline{2}} W\right)^2\,, & \overline{R}_{1323} &= W^{-1}\partial_{\underline{2}}\partial_{\underline{1}} \log W\,, \end{aligned} \tag{A.4}$$

those of the Ricci tensor are

$$\begin{aligned} &\overline{R}_{11} = \frac{1}{2}W^{-2}\left(\partial_{\underline{1}}^2 + \partial_{\underline{3}}^2 + W^2\partial_{\underline{2}}^2\right)\log W^2 + \frac{1}{2}\left(\partial_{\underline{2}} \log W^2\right)^2\,, \ \overline{R}_{22} = W^{-1}\partial_{\underline{2}}^2 W\,, \\ &\overline{R}_{12} = \frac{1}{2}W^{-1}\partial_{\underline{1}}\partial_{\underline{2}} \log W^2\,, \qquad\qquad \overline{R}_{23} = \frac{1}{2}W^{-1}\partial_{\underline{3}}\partial_{\underline{2}} \log W^2\,, \\ &\overline{R}_{33} = \overline{R}_{11}\,. \end{aligned} \tag{A.5}$$

The Ricci scalar is given by

$$\overline{R} = W^{-2}\left(\partial_{\underline{1}}^2 + \partial_{\underline{3}}^2 + 2W^2\partial_{\underline{2}}^2\right)\log W^2 + 2\left(\partial_{\underline{2}} \log W^2\right)^2\,. \tag{A.6}$$

B 4-d Euclidean metrics with one isometry

Any 4-dimensional Euclidean metric admitting one isometry can be written in the form

$$d\hat{s}^2 = H^{-1}(dz + \chi)^2 + H\gamma_{\underline{ij}} dx^i dx^j\,, \tag{B.1}$$

where $z = x^\sharp$ is the coordinate adapted to the isometry and where the 3-dimensional function H, the 1-form $\chi = \chi_{\underline{i}} dx^i$ and the metric $\gamma_{\underline{ij}} dx^i dx^j$, $i, j = 1, 2, 3$ are z-independent and orthogonal to the Killing vector $k^{\underline{m}} = \delta_z{}^{\underline{m}}$. We denote the world indices by $\{\underline{m}\} = \{z, \underline{i}\}$ and the flat indices by $\{m\} = \{\sharp, i\}$. We will denote 3-dimensional structures (connection, curvature etc.) by an overline, as in the previous appendix. For the moment, the 3-dimensional structures will be completely general and only later on we will assume the 3-dimensional metric to have the form eq. (A.1) and H, χ and W to be related by th

JHEP04(2017)017

W-deformed monopole equation (1.31) which holds when the 4-dimensional metric above is a Kähler metric with respect to the complex structure eq. (1.29).

A convenient basis of Vierbeins is

$$\begin{cases} \hat{V}^\sharp = H^{-1/2}(dz+\chi)\,, \\ \hat{V}^i = H^{1/2}v^i\,, \end{cases} \qquad \begin{cases} \hat{V}_\sharp = H^{1/2}\partial_z\,, \\ \hat{V}_i = H^{-1/2}(\partial_i - \chi_i\partial_z)\,, \end{cases} \tag{B.2}$$

where $v^i = v^i{}_{\underline{j}}dx^{\underline{j}}$ are Dreibeins of the metric $\gamma_{\underline{ij}}$, $\partial_i \equiv v_i{}^{\underline{j}}\partial_{\underline{j}}$ and $\chi_i \equiv v_i{}^{\underline{j}}\chi_{\underline{j}}$.

The non-vanishing components of the spin connection 1-form, defined through the structure equation $\mathcal{D}e^m \equiv de^m - \varpi^m{}_n \wedge e^n = 0$ are

$$\begin{aligned} \varpi_{\sharp\sharp i} &= \frac{1}{2}H^{-3/2}\partial_i H\,, & \varpi_{\sharp ij} &= \frac{1}{2}H^{-3/2}(d\chi)_{ij}\,, \\ \varpi_{i\sharp j} &= \varpi_{\sharp ij}\,, & \varpi_{kij} &= H^{-1/2}\overline{\omega}_{kij} + H^{-3/2}\partial_{[i}H\delta_{j]k}\,, \end{aligned} \tag{B.3}$$

where $(d\chi)_{ij} = 2v_i{}^{\underline{k}}v_j{}^{\underline{l}}\partial_{[\underline{j}}\chi_{\underline{l}]}$ and $\overline{\omega}_{kij}$ is the 3-dimensional connection defined by $\overline{\mathcal{D}}v^i = dv^i - \overline{\omega}^i{}_j \wedge v^j = 0$.

Those of the curvature 2-form, defined through $\hat{R}^m{}_n \equiv d\varpi^m{}_n - \varpi^m{}_p \wedge \varpi^p{}_n$, are

$$\begin{aligned} \hat{R}_{\sharp i\sharp j} &= -\frac{1}{2}H^{-2}\overline{\nabla}_j\partial_i H + \frac{1}{4}H^{-3}\left[5\partial_i H\partial_j H - \delta_{ij}(\partial H)^2 - (d\chi)_{jl}(d\chi)_{il}\right]\,, \\ \hat{R}_{kj\sharp i} &= \overline{\nabla}_k[H^{-2}(d\chi)_{ji}] + \frac{1}{2}H^{-3}\left[2\partial_{[k}H(d\chi)_{j]i} + \partial_l H(d\chi)_{l[k}\delta_{j]i}\right]\,, \\ \hat{R}_{klij} &= H^{-1}\Big\{\overline{R}_{klij} + 2H^{-1}\overline{\nabla}_{[k}\partial^{[i}H\delta^{j]}{}_{l]} + 3H^{-2}\partial_{[i}H\delta_{j][k}\partial_{l]}H \\ &\qquad + \frac{1}{2}H^{-2}(\partial H)^2\delta_{ij\,,kl} + \frac{1}{2}H^{-2}\left[(d\chi)_{ij}(d\chi)_{kl} - (d\chi)_{i[k}(d\chi)_{l]j}\right]\Big\}\,. \end{aligned} \tag{B.4}$$

The components of the Ricci tensor are

$$\begin{aligned} \hat{R}_{\sharp\sharp} &= -\frac{1}{2}H^{-2}\overline{\nabla}^2 H + \frac{1}{2}H^{-3}(\partial H)^2 - \frac{1}{4}H^{-3}(d\chi)^2\,, \\ \hat{R}_{\sharp i} &= \frac{1}{2}\overline{\nabla}_j\left[H^{-2}(d\chi)_{ji}\right]\,, \\ \hat{R}_{ij} &= H^{-1}\overline{R}_{ij} + \frac{1}{2}\delta_{ij}H^{-2}\overline{\nabla}^2 H + \frac{1}{2}H^{-3}\left[\partial_i H\partial_j H - \delta_{ij}(\partial H)^2 + (d\chi)_{ik}(d\chi)_{jk}\right]\,, \end{aligned} \tag{B.5}$$

and the Ricci scalar is given by

$$\hat{R} = H^{-1}\overline{R} + H^{-2}\overline{\nabla}^2 H - \frac{1}{2}H^{-3}\left[(\partial H)^2 - \frac{1}{2}(d\chi)^2\right]\,. \tag{B.6}$$

Observe that if the conditions

$$\overline{R}_{ij} = 0\,, \qquad (d\chi)_{ij} = \varepsilon_{ijk}\partial_l H\,, \tag{B.7}$$

are satisfied the metric eq. (B.1) is a Gibbons-Hawking metric (a hyperKähler metric admitting a triholomorphic isometry) [31, 32] and it is Ricci-flat. If the metric is Kähler with respect to the complex structure eq. (1.29) so that the 3-dimensional metric has the

JHEP04(2017)017

form eq. (A.1) and H, χ and W are related by the W-deformed monopole equation (1.31), then we can use the results in appendix A to find that the non-vanishing components of the Ricci tensor are given by

$$\begin{aligned}
\hat{R}_{\sharp\sharp} = \hat{R}_{22} &= \frac{1}{2}\partial_{\underline{2}}\left(H^{-1}\partial_{\underline{2}}\log W^2\right)\,,\\
\hat{R}_{11} = \hat{R}_{33} &= \frac{1}{2}H^{-1}W^{-2}\left(\partial_{\underline{1}}^2+\partial_{\underline{3}}^2\right)\log W^2+\frac{1}{2}H^{-1}\left(\partial_{\underline{2}}\log W^2\right)^2\\
&\quad+\frac{1}{2}H^{-2}\partial_{\underline{2}}H\partial_{\underline{2}}\log W^2\,,\\
\hat{R}_{01} = \hat{R}_{23} &= -\frac{1}{2}H^{-2}W^{-1}\partial_{\underline{3}}H\partial_{\underline{2}}\log W^2+\frac{1}{2}H^{-1}W^{-1}\partial_{\underline{3}}\partial_{\underline{2}}\log W^2\,,\\
\hat{R}_{03} = -\hat{R}_{12} &= \frac{1}{2}H^{-2}W^{-1}\partial_{\underline{1}}H\partial_{\underline{2}}\log W^2-\frac{1}{2}H^{-1}W^{-1}\partial_{\underline{1}}\partial_{\underline{2}}\log W^2\,.
\end{aligned}\tag{B.8}$$

Exactly the same result is obtained by using eq. (1.38).

Finally, the Ricci scalar is given by

$$\begin{aligned}
\hat{R} &= \hat{\nabla}^2\log W^2 = H^{-1}\overline{\nabla}^2\log W^2\\
&= H^{-1}W^{-2}\left\{\left(\partial_{\underline{1}}^2+\partial_{\underline{3}}^2\right)\log W^2+\partial_{\underline{2}}\left(W^2\partial_{\underline{2}}\log W^2\right)\right\}\,.
\end{aligned}\tag{B.9}$$

C 5-d metrics

Let us consider the time-independent 5-dimensional Lorentzian conformastationary metric

$$ds^2 = \hat{f}^2\left(dt+\hat{\omega}\right)^2-\hat{f}^{-1}h_{\underline{mn}}dx^m dx^n\,,\qquad\qquad m,n=\sharp,1,2,3\,.\tag{C.1}$$

The function $\hat{f}$ and the 1-form $\hat{\omega}=\hat{\omega}_{\underline{m}}dx^m$ can be understood as objects living in the 4-dimensional Euclidean metric $h_{\underline{mn}}$. We will denote this kind of objects with hats.

We choose the Vielbein basis

$$\begin{aligned}
e^0 &= \hat{f}(dt+\hat{\omega})\,, &\qquad e_0 &= \hat{f}^{-1}\partial_t\,,\\
e^m &= \hat{f}^{-1/2}\hat{V}^m\,, &\qquad e_m &= \hat{f}^{1/2}(\partial_m-\hat{\omega}_m\partial_t)\,.
\end{aligned}\tag{C.2}$$

where the $\hat{V}_{\underline{m}}{}^{\underline{p}}$s are a Vierbein for the 4-dimensional Euclidean metric $h_{\underline{mn}}$ and, just as we did with the 3- and 4-dimensional metrics studied before, all the objects in the r.h.s. of all the equations refer to the 4-dimensional metric $h_{\underline{mn}}$ and the Vierbein basis $\hat{V}^{\underline{p}}$ ($\partial_m = V_m{}^{\underline{p}}\partial_{\underline{p}}$).

With this choice of Vielbein, the non-vanishing components of the spin connection are

$$\begin{aligned}
\omega_{00m} &= -2\partial_m\hat{f}^{1/2}\,, &\qquad \omega_{0mn} &= \frac{1}{2}\hat{f}^2\left(d\hat{\omega}\right)_{mn}\,,\\
\omega_{m0n} &= \frac{1}{2}\hat{f}^2\left(d\hat{\omega}\right)_{mn}\,, &\qquad \omega_{mnp} &= -\hat{f}^{1/2}\varpi_{mnp}-2\delta_{m[n}\partial_{p]}\hat{f}^{1/2}\,,
\end{aligned}\tag{C.3}$$

where we are denoting by ϖ_{mnp} the 4-dimensional spin connection.

JHEP04(2017)017

The non-vanishing components of the Ricci tensor are

$$\begin{aligned} R_{00} &= -\hat{\nabla}^2\hat{f} + \hat{f}^{-1}(\partial\hat{f})^2 - \frac{1}{4}\hat{f}^4(d\hat{\omega})^2\,, \\ R_{0m} &= -\frac{1}{2}\hat{f}^{-1/2}\hat{\nabla}_n[\hat{f}^3(d\hat{\omega})_{nm}]\,, \\ R_{mn} &= \hat{f}\hat{R}_{mn} - \frac{1}{2}(d\hat{\omega})_{mp}(d\hat{\omega})_{np} + \frac{3}{2}\hat{f}^{-1}\partial_m\hat{f}\partial_n\hat{f} - \frac{1}{2}\delta_{mn}[\nabla^2\hat{f} - \hat{f}^{-1}(\partial\hat{f})^2]\,, \end{aligned} \tag{C.4}$$

and the Ricci scalar is given by

$$R = -\hat{f}\hat{R} + \frac{1}{4}(d\hat{\omega})^2 + \hat{\nabla}^2\hat{f} - \frac{5}{2}\hat{f}^{-1}(\partial\hat{f})^2\,. \tag{C.5}$$

D $\mathbf{AdS_5}$

It is well known that (the unit radius) AdS_5 can be embedded in $\mathbb{R}^{2,4}$ or equivalently in $\mathbb{C}^{1,2}$ as the set of points satisfying

$$Z^0 Z^{*\,0} - Z^i Z^{*\,i} = 1\,, \qquad i = 1,2 \tag{D.1}$$

with its metric being induced from the ambient metric

$$ds^2 = dZ^0 dZ^{*\,0} - dZ^i dZ^{*\,i}\,. \tag{D.2}$$

Setting $Z^0 = |Z^0|e^{it}$, $Z^i = Z^0\zeta^i$ we can solve for Z^0 in terms of t and ζ^i

$$|Z^0|^{-2} = 1 - \zeta^i\zeta^{*\,i^*}\,, \tag{D.3}$$

and the induced metric takes the form

$$ds^2 = (dt + \mathcal{Q})^2 - 2\mathcal{G}_{ij^*}d\zeta^i d\zeta^{*\,j^*}\,, \tag{D.4}$$

where

$$2\mathcal{G}_{ij^*} = \frac{\delta_{ij^*}}{1 - \zeta^k\zeta^{*\,k^*}} + \frac{\zeta^{*\,i^*}\zeta^j}{(1 - \zeta^k\zeta^{*\,k^*})^2}\,, \tag{D.5}$$

is the metric of the Kähler space $\overline{\mathbb{CP}}^2 = \mathrm{SU}(1,2)/\mathrm{U}(2)$ and

$$\mathcal{Q} = \frac{i}{2}\frac{\zeta^{*\,i^*}d\zeta^i - \zeta^i d\zeta^{*\,i^*}}{1 - \zeta^i\zeta^{*\,i^*}}\,, \tag{D.6}$$

is its corresponding Kähler 1-form connection. The Kähler 2-form is given by

$$\mathcal{J}_{ij^*} = \partial_i\mathcal{Q}_{j^*} - \partial_{j^*}\mathcal{Q}_i = 2i\mathcal{G}_{ij^*}\,. \tag{D.7}$$

This form of the metric makes manifest that AdS_5 can be seen as a U(1) fibration over the Kähler manifold $\overline{\mathbb{CP}}^2$. As shown in ref. [44] this is the only base space that can be used to construct AdS_5 as a supersymmetric solution of minimal gauged 5-dimensional supergravity. There are different ways of writing $\overline{\mathbb{CP}}^2$ in the canonical form eqs. (1.30) and (1.31), associated to the different holomorphic Killing vectors of the manifold which, being the symmetric space $\mathrm{SU}(2,1)/\mathrm{U}(2)$, are 8. We are not going to explore all of them here. We will content ourselves with those in which the metric contains the metric of a 2-dimensional space of constant curvature k that we will denote by $d\Omega_{(2,k)}$, where $k = 1, 0, -1$ for, respectively, S^2, $\mathbb{E}^2$ or $\mathbb{H}_2$.

JHEP04(2017)017

D.1 $k = 1$

In the $k = 1$ case we can use the real coordinates

$$\zeta^1 = \tanh\rho\cos\frac{\theta}{2}\,e^{-\frac{i}{2}(z+\varphi)}\,, \qquad \zeta^2 = \tanh\rho\sin\frac{\theta}{2}\,e^{-\frac{i}{2}(z-\varphi)}\,, \tag{D.8}$$

for which the metric of $\overline{\mathbb{CP}}^2$ and the Kähler 1-form connection are given by

$$\begin{aligned} ds^2_{\overline{\mathbb{CP}}^2} &= d\rho^2 + \frac{1}{4}\sinh^2\rho\cosh^2\rho\,(dz+\cos\theta d\varphi)^2 + \frac{1}{4}\sinh^2\rho d\Omega^2_{(2,1)}\,, \\ \mathcal{Q}_{\overline{\mathbb{CP}}^2} &= \frac{1}{2}\sinh^2\rho\,(dz+\cos\theta\,d\varphi)\,. \end{aligned} \tag{D.9}$$

where

$$d\Omega^2_{(2,1)} = d\theta^2 + \sin^2\theta\,d\varphi^2\,, \tag{D.10}$$

is the metric of S^2.

The metric for the four-dimensional base space can be cast in the form eq. (1.30) by defining the new coordinates

$$x^1 = \tan\frac{\theta}{2}\cos\varphi\,, \qquad x^2 = \frac{1}{4}\sinh^2\rho\,, \qquad x^3 = \tan\frac{\theta}{2}\sin\varphi\,, \tag{D.11}$$

so that the functions H, W and 1-form $\chi_{(1)}$ that define it are given by[33]

$$\begin{aligned} H^{-1} &= x^2(1+4x^2)\,, \\ W^2 &= \frac{4x^2}{H[1+(x^1)^2+(x^3)^2]^2}\,, \\ \chi = \chi_{(1)} &\equiv \frac{[1-(x^1)^2-(x^3)^2]}{[1+(x^1)^2+(x^3)^2]}\frac{x^1dx^3-x^3dx^1}{(x^1)^2+(x^3)^2}\,, \end{aligned} \tag{D.12}$$

and[34]

$$d\chi_{(1)} = -\frac{4}{[1+(x^1)^2+(x^3)^2]^2}dx^1\wedge dx^3\,. \tag{D.14}$$

From these expressions it is trivial to verify that the constraints (1.31) are satisfied.

Using the parametrization (D.8) for $\overline{\mathbb{CP}}^2$ we find the following line element of AdS_5

$$\begin{aligned} ds^2 = &\left[dt + \frac{1}{2}\sinh^2\rho\,(dz+\cos\theta\,d\varphi)\right]^2 \\ &-d\rho^2 - \frac{1}{4}\sinh^2\rho\cosh^2\rho\,(dz+\cos\theta d\varphi)^2 - \frac{1}{4}\sinh^2\rho d\Omega^2_{(2,1)}\,. \end{aligned} \tag{D.15}$$

[33] These functions have been determined for $\overline{\mathbb{CP}}^2$ in ref. [60].

[34] The 1-form $\chi_{(1)}$ is defined up to a total derivative that can be absorbed in a redefinition of the coordinate z. The expression given above for $\chi_{(1)}$ is exactly the one that appears in the metric. A simpler expression is

$$\chi_{(1)} = \frac{x^3dx^1 - x^1dx^3}{1+(x^1)^2+(x^3)^2}\,. \tag{D.13}$$

JHEP04(2017)017

The off-diagonal components can be eliminated by redefining the angular coordinate $z = \psi + 2t$:

$$ds^2 = \cosh^2\rho\, dt^2 - d\rho^2 - \sinh^2\rho\, d\Omega^2_{(3,1)}\,, \tag{D.16}$$

where

$$d\Omega^2_{(3,1)} = \frac{1}{4}\left[\left(d\psi' + \cos\theta\, d\varphi\right)^2 + d\Omega^2_{(2,1)}\right] \tag{D.17}$$

is the metric of the round 3-sphere of unit radius. This is one of the standard expressions for the metric of AdS_5 in global coordinates. The coordinates used in the supersymmetric form (rotating frame $\psi \to z$) also cover the whole AdS_5 spacetime.

Redefining the radial coordinate $r = \sinh\rho$ the metric takes the standard form

$$ds^2 = (1+r^2)dt^2 - \frac{dr^2}{1+r^2} - r^2 d\Omega^2_{(3)}\,. \tag{D.18}$$

Using the results in the previous appendices one finds that the Ricci tensor of this metric is $R_{ab} = -4\eta_{ab}$. In order to get a metric satisfying $R_{ab} = \Lambda\eta_{ab}$ (for $\Lambda < 0$) where Λ is the cosmological constant as defined in footnote 8 we just have to multiply the whole metric by $4/|\Lambda|$. In particular, if we multiply the AdS_5 metric in eq. (D.16) by that factor and make the coordinate redefinitions $r = \sqrt{4/|\Lambda|}\sinh\rho$ and $t' = \sqrt{4/|\Lambda|}\,t$ we get, instead of eq. (D.18)

$$ds^2 = \left(1 + \frac{|\Lambda|}{4}r^2\right)dt'^2 - \left(1 + \frac{|\Lambda|}{4}r^2\right)^{-1} dr^2 - r^2 d\Omega^2_{(3)}\,. \tag{D.19}$$

D.2 $k = 0$

In the $k = 0$ case the real coordinates one has to use for $\overline{\mathbb{CP}}^2$ are essentially the ones customarily used to parametrize the universal hypermultiplet:[35]

$$\zeta^1 = \frac{1-S}{1+S}\,, \qquad \zeta^2 = \frac{2C}{1+S}\,, \qquad \text{with} \qquad \begin{cases} S = \dfrac{1}{x^2} + 4iz + CC^*\,, \\ C = 2(x^1 + ix^3)\,. \end{cases} \tag{D.20}$$

In terms of these coordinates, the metric of $\overline{\mathbb{CP}}^2$ and the Kähler 1-form connection are given by

$$\begin{aligned} ds^2_{\overline{\mathbb{CP}}^2} &= \frac{(dx^2)^2}{4(x^2)^2} + 4(x^2)^2\left[dz + 2(x^3dx^1 - x^1dx^3)\right]^2 + x^2 d\Omega^2_{(2,0)}\,, \\ \mathcal{Q}_{\overline{\mathbb{CP}}^2} &= 2x^2\left[dz + 2(x^3dx^1 - x^1dx^3)\right]\,. \end{aligned} \tag{D.21}$$

where

$$d\Omega^2_{(2,0)} = 4[(dx^1)^2 + (dx^3)^2]\,, \tag{D.22}$$

is the metric of $\mathbb{E}^2$ with a convenient normalization.

[35] See, for instance, ref. [61] and references therein.

JHEP04(2017)017

This metric is already in the form eq. (1.30) and so that the functions H, W and 1-form χ that define it are given by[36]

$$\begin{aligned} H^{-1} &= 4(x^2)^2\,, \\ W^2 &= \frac{x^2}{H}\Phi_{(0)}\,, \\ \chi &= \chi_{(0)} \equiv 2(x^3dx^1 - x^1dx^3)\,. \end{aligned} \tag{D.23}$$

Using these coordinates for $\overline{\mathbb{CP}}^2$ we find the following line element of AdS_5

$$\begin{aligned} ds^2 &= \left\{dt + 2x^2\left[dz + 2(x^3dx^1 - x^1dx^3)\right]\right\}^2 \\ &\quad -\frac{(dx^2)^2}{4(x^2)^2} - 4(x^2)^2\left[dz + 2(x^3dx^1 - x^1dx^3)\right]^2 - x^2 d\Omega^2_{(2,0)}\,. \end{aligned} \tag{D.24}$$

In this case we cannot eliminate the off-diagonal components of the metric with a simple coordinate transformation.

D.3 $k = -1$

In the $k = -1$ case we can use the real coordinates

$$\zeta^1 = \tanh(\theta/2)\, e^{i\varphi}\,, \qquad \zeta^2 = \frac{\tanh\rho}{\cosh(\theta/2)}\, e^{-\frac{i}{2}(z-\varphi)}\,, \tag{D.25}$$

for which the metric of $\overline{\mathbb{CP}}^2$ and the Kähler 1-form connection are given by

$$\begin{aligned} ds^2_{\overline{\mathbb{CP}}^2} &= d\rho^2 + \frac{1}{4}\sinh^2\rho\cosh^2\rho\,(dz - \cosh\theta d\varphi)^2 + \frac{1}{4}\cosh^2\rho\, d\Omega^2_{(2,-1)}\,, \\ \mathcal{Q}_{\overline{\mathbb{CP}}^2} &= \frac{1}{2}\cosh^2\rho\,(dz - \cosh\theta\, d\varphi)\,. \end{aligned} \tag{D.26}$$

where

$$d\Omega^2_{(2,-1)} = d\theta^2 + \sinh^2\theta\, d\varphi^2\,, \tag{D.27}$$

is the metric of the $\mathbb{H}_2$. Observe that now θ is a non-compact coordinate.

To cast the above metric in the form eq. (1.30) we define

$$x^1 = \tanh\frac{\theta}{2}\cos\varphi\,, \qquad x^2 = \frac{1}{4}\cosh^2\rho\,, \qquad x^3 = \tanh\frac{\theta}{2}\sin\varphi\,. \tag{D.28}$$

Then, the functions H, W and 1-form χ that define it are given by[37]

$$\begin{aligned} H^{-1} &= x^2(-1 + 4x^2)\,, \\ W^2 &= \frac{4x^2}{H[1 - (x^1)^2 - (x^3)^2]^2}\,, \\ \chi &= \chi_{(-1)} \equiv \frac{[1 + (x^1)^2 + (x^3)^2]}{[1 - (x^1)^2 - (x^3)^2]}\frac{x^1dx^3 - x^3dx^1}{(x^1)^2 + (x^3)^2}\,. \end{aligned} \tag{D.30}$$

[36] These functions have been determined for $\overline{\mathbb{CP}}^2$ with $k = 1$ in ref. [60].

[37] Again, the expression given above for $\chi_{(-1)}$ is exactly the one that appears in the metric. A simpler expression is

$$\chi_{(-1)} = \frac{x^3dx^1 - x^1dx^3}{1 - (x^1)^2 - (x^3)^2}\,. \tag{D.29}$$

JHEP04(2017)017

The line element for AdS_5 corresponding to the choice of coordinates (D.25) is

$$\begin{aligned} ds^2 = & \left[dt + \frac{1}{2}\cosh^2\rho\,(dz - \cosh\theta\, d\varphi)\right]^2 \\ & -d\rho^2 - \frac{1}{4}\sinh^2\rho\cosh^2\rho\,(dz - \cosh\theta d\varphi)^2 - \frac{1}{4}\cosh^2\rho\, d\Omega^2_{(2,-1)}\,. \end{aligned} \tag{D.31}$$

Observe that, if we eliminate the $dtdz$ terms in the $k=-1$ metric using the same trick as in the $k=1$ case, namely shifting the z coordinate $z = \psi - 2t$, we get the metric

$$ds^2 = -\sinh^2\rho dt^2 + \frac{1}{4}\cosh^2\rho\,(d\psi + \cosh\theta d\varphi)^2 - d\rho^2 - \frac{1}{4}\cosh^2\rho\, d\Omega^2_{(2,-1)}\,, \tag{D.32}$$

in which t and ψ have interchanged their rôles.

The functions corresponding to the three different canonical metrics for $\overline{\mathbb{CP}}^2$ can be written in a unified form:

$$\begin{aligned} H^{-1} &= x^2(k+4x^2)\,, \\ W^2 &= \frac{x^2}{H}\Phi_{(k)}\,, \\ \chi &= \chi_{(k)} \end{aligned} \tag{D.33}$$

with

$$\begin{aligned} d\Omega^2_{(2,k)} &= \frac{4[(dx^1)^2 + (dx^3)^2]}{\{1 + k[(x^1)^2 + (x^3)^2]\}^2} \equiv \Phi_{(k)}(x^1, x^3)[(dx^1)^2 + (dx^3)^2]\,, \\ \chi_{(k)} &= \frac{2[x^3 dx^1 - x^1 dx^3]}{1 + k[(x^1)^2 + (x^3)^2]}\,. \end{aligned} \tag{D.34}$$

Then, the metric of AdS_5 in the supersymmetric canonical form is given by

$$ds^2 = \left[dt + 2x^2(dz + \chi_{(k)})\right]^2 - x^2(k+4x^2)(dz+\chi_{(k)})^2 - \frac{(dx^2)^2}{x^2(k+4x^2)} - x^2 d\Omega^2_{(2,k)}\,. \tag{D.35}$$

References

[1] T. Ortín, *Gravity and strings*, 2nd edition, Cambridge University Press, Cambridge U.K. (2015).

[2] S. Ferrara, R. Kallosh and A. Strominger, $N = 2$ *extremal black holes*, *Phys. Rev.* **D 52** (1995) R5412 [hep-th/9508072] [INSPIRE].

[3] A. Strominger, *Macroscopic entropy of $N = 2$ extremal black holes*, *Phys. Lett.* **B 383** (1996) 39 [hep-th/9602111] [INSPIRE].

[4] S. Ferrara and R. Kallosh, *Supersymmetry and attractors*, *Phys. Rev.* **D 54** (1996) 1514 [hep-th/9602136] [INSPIRE].

JHEP04(2017)017

[5] S. Ferrara and R. Kallosh, *Universality of supersymmetric attractors*, *Phys. Rev.* **D 54** (1996) 1525 [hep-th/9603090] [INSPIRE].

[6] S. Ferrara, G.W. Gibbons and R. Kallosh, *Black holes and critical points in moduli space*, *Nucl. Phys.* **B 500** (1997) 75 [hep-th/9702103] [INSPIRE].

[7] G.W. Gibbons and C.M. Hull, *A bogomolny bound for general relativity and solitons in $N = 2$ supergravity*, *Phys. Lett.* **109B** (1982) 190.

[8] K.P. Tod, *All metrics admitting supercovariantly constant spinors*, *Phys. Lett.* **121B** (1983) 241.

[9] J. Kowalski-Glikman, *Positive energy theorem for eleven-dimensional Kaluza-Klein supergravity*, *Phys. Lett.* **166B** (1986) 149.

[10] Z. Perjes, *Solutions of the coupled Einstein Maxwell equations representing the fields of spinning sources*, *Phys. Rev. Lett.* **27** (1971) 1668 [INSPIRE].

[11] W. Israel and G.A. Wilson, *A class of stationary electromagnetic vacuum fields*, *J. Math. Phys.* **13** (1972) 865 [INSPIRE].

[12] J.B. Hartle and S.W. Hawking, *Solutions of the Einstein-Maxwell equations with many black holes*, *Commun. Math. Phys.* **26** (1972) 87 [INSPIRE].

[13] S.D. Majumdar, *A class of exact solutions of Einstein's field equations*, *Phys. Rev.* **72** (1947) 390 [INSPIRE].

[14] A. Papaetrou, *A Static solution of the equations of the gravitational field for an arbitrary charge distribution*, *Proc. Roy. Irish Acad.* **A 51** (1947) 191 [INSPIRE].

[15] J.P. Gauntlett, J.B. Gutowski, C.M. Hull, S. Pakis and H.S. Reall, *All supersymmetric solutions of minimal supergravity in five-dimensions*, *Class. Quant. Grav.* **20** (2003) 4587 [hep-th/0209114] [INSPIRE].

[16] M.M. Caldarelli and D. Klemm, *All supersymmetric solutions of $N = 2$, $D = 4$ gauged supergravity*, *JHEP* **09** (2003) 019 [hep-th/0307022] [INSPIRE].

[17] P. Meessen and T. Ortín, *The Supersymmetric configurations of $N = 2$, $D = 4$ supergravity coupled to vector supermultiplets*, *Nucl. Phys.* **B 749** (2006) 291 [hep-th/0603099] [INSPIRE].

[18] M. Huebscher, P. Meessen and T. Ortín, *Supersymmetric solutions of $N = 2$ $D = 4$ SUGRA: the whole ungauged shebang*, *Nucl. Phys.* **B 759** (2006) 228 [hep-th/0606281] [INSPIRE].

[19] S.L. Cacciatori, D. Klemm, D.S. Mansi and E. Zorzan, *All timelike supersymmetric solutions of $N = 2$, $D = 4$ gauged supergravity coupled to abelian vector multiplets*, *JHEP* **05** (2008) 097 [arXiv:0804.0009] [INSPIRE].

[20] D. Klemm and E. Zorzan, *All null supersymmetric backgrounds of $N = 2$, $D = 4$ gauged supergravity coupled to abelian vector multiplets*, *Class. Quant. Grav.* **26** (2009) 145018 [arXiv:0902.4186] [INSPIRE].

[21] D. Klemm and E. Zorzan, *The timelike half-supersymmetric backgrounds of $N = 2$, $D = 4$ supergravity with Fayet-Iliopoulos gauging*, *Phys. Rev.* **D 82** (2010) 045012 [arXiv:1003.2974] [INSPIRE].

[22] M. Hübscher, P. Meessen, T. Ortín and S. Vaulà, *$N = 2$ Einstein-Yang-Mills's BPS solutions*, *JHEP* **09** (2008) 099 [arXiv:0806.1477] [INSPIRE].

[23] P. Meessen and T. Ortín, *Supersymmetric solutions to gauged $N = 2$ $D = 4$ SUGRA: the full timelike shebang*, *Nucl. Phys.* **B 863** (2012) 65 [arXiv:1204.0493] [INSPIRE].

JHEP04(2017)017

[24] J.P. Gauntlett and J.B. Gutowski, *All supersymmetric solutions of minimal gauged supergravity in five-dimensions*, *Phys. Rev.* **D 68** (2003) 105009 [*Erratum ibid.* **D 70** (2004) 089901] [hep-th/0304064] [INSPIRE].

[25] J.B. Gutowski and H.S. Reall, *General supersymmetric AdS_5 black holes*, *JHEP* **04** (2004) 048 [hep-th/0401129] [INSPIRE].

[26] J.P. Gauntlett and J.B. Gutowski, *General concentric black rings*, *Phys. Rev.* **D 71** (2005) 045002 [hep-th/0408122] [INSPIRE].

[27] J.B. Gutowski and W. Sabra, *General supersymmetric solutions of five-dimensional supergravity*, *JHEP* **10** (2005) 039 [hep-th/0505185] [INSPIRE].

[28] J. Bellorín, P. Meessen and T. Ortín, *All the supersymmetric solutions of $N = 1$, $d = 5$ ungauged supergravity*, *JHEP* **01** (2007) 020 [hep-th/0610196] [INSPIRE].

[29] J. Bellorín and T. Ortín, *Characterization of all the supersymmetric solutions of gauged $N = 1$, $D = 5$ supergravity*, *JHEP* **08** (2007) 096 [arXiv:0705.2567] [INSPIRE].

[30] J. Bellorín, *Supersymmetric solutions of gauged five-dimensional supergravity with general matter couplings*, *Class. Quant. Grav.* **26** (2009) 195012 [arXiv:0810.0527] [INSPIRE].

[31] G.W. Gibbons and S.W. Hawking, *Gravitational multi-instantons*, *Phys. Lett.* **B 78** (1978) 430 [INSPIRE].

[32] G.W. Gibbons and S.W. Hawking, *Classification of gravitational instanton symmetries*, *Commun. Math. Phys.* **66** (1979) 291 [INSPIRE].

[33] P. Meessen, T. Ortín and P.F. Ramírez, *Non-Abelian, supersymmetric black holes and strings in 5 dimensions*, *JHEP* **03** (2016) 112 [arXiv:1512.07131] [INSPIRE].

[34] P.B. Kronheimer, *Monopoles and Taub-NUT spaces*, M.Sc. dissertation, Oxford University, Oxford U.K. (1995).

[35] E.B. Bogomolny, *Stability of classical solutions*, *Sov. J. Nucl. Phys.* **24** (1976) 449 [*Yad. Fiz.* **24** (1976) 861] [INSPIRE].

[36] A.P. Protogenov, *Exact classical solutions of Yang-Mills sourceless equations*, *Phys. Lett.* **B 67** (1977) 62 [INSPIRE].

[37] A.A. Belavin, A.M. Polyakov, A.S. Schwartz and Yu.S. Tyupkin, *Pseudoparticle solutions of the Yang-Mills equations*, *Phys. Lett.* **B 59** (1975) 85 [INSPIRE].

[38] P. Bueno, P. Meessen, T. Ortín and P.F. Ramírez, *Resolution of* SU(2) *monopole singularities by oxidation*, *Phys. Lett.* **B 746** (2015) 109 [arXiv:1503.01044] [INSPIRE].

[39] P. Meessen, *Supersymmetric coloured/hairy black holes*, *Phys. Lett.* **B 665** (2008) 388 [arXiv:0803.0684] [INSPIRE].

[40] P. Meessen and T. Ortín, *$\mathcal{N} = 2$ super-EYM coloured black holes from defective Lax matrices*, *JHEP* **04** (2015) 100 [arXiv:1501.02078] [INSPIRE].

[41] P. Figueras, C.A.R. Herdeiro and F. Paccetti Correia, *On a class of 4D Kähler bases and AdS_5 supersymmetric black holes*, *JHEP* **11** (2006) 036 [hep-th/0608201] [INSPIRE].

[42] D. Cassani, J. Lorenzen and D. Martelli, *Comments on supersymmetric solutions of minimal gauged supergravity in five dimensions*, *Class. Quant. Grav.* **33** (2016) 115013 [arXiv:1510.01380] [INSPIRE].

JHEP04(2017)017

[43] L.A.J. London, *Arbitrary dimensional cosmological multi-black holes*, *Nucl. Phys.* **B 434** (1995) 709 [INSPIRE].

[44] J.B. Gutowski and H.S. Reall, *Supersymmetric AdS_5 black holes*, *JHEP* **02** (2004) 006 [hep-th/0401042] [INSPIRE].

[45] S. Chimento and T. Ortín, *On 2-dimensional Kähler metrics with one holomorphic isometry*, arXiv:1610.02078 [INSPIRE].

[46] E. Bergshoeff et al., *$N = 2$ supergravity in five-dimensions revisited*, *Class. Quant. Grav.* **21** (2004) 3015 [*Class. Quant. Grav.* **23** (2006) 7149] [hep-th/0403045] [INSPIRE].

[47] D. Joyce, *Compact manifolds with special holonomy*, Oxford University Press, Oxford U.K. (2000).

[48] G.W. Gibbons, *Anti-de-Sitter spacetime and its uses*, arXiv:1110.1206 [INSPIRE].

[49] A. Trautman, *Solutions of the Maxwell and Yang-Mills equations associated with Hopf fibrings*, *Int. J. Theor. Phys.* **16** (1977) 561 [INSPIRE].

[50] R. Penrose, *Zero rest mass fields including gravitation: asymptotic behavior*, *Proc. Roy. Soc. Lond.* **A 284** (1965) 159.

[51] A. Ashtekar and S. Das, *Asymptotically Anti-de Sitter space-times: conserved quantities*, *Class. Quant. Grav.* **17** (2000) L17 [hep-th/9911230] [INSPIRE].

[52] D. Klemm and W.A. Sabra, *Supersymmetry of black strings in $D = 5$ gauged supergravities*, *Phys. Rev.* **D 62** (2000) 024003 [hep-th/0001131] [INSPIRE].

[53] K. Godel, *An example of a new type of cosmological solutions of Einstein's field equations of graviation*, *Rev. Mod. Phys.* **21** (1949) 447 [INSPIRE].

[54] A.A. Tseytlin, *Extreme dyonic black holes in string theory*, *Mod. Phys. Lett.* **A 11** (1996) 689 [hep-th/9601177] [INSPIRE].

[55] P. Meessen and T. Ortín, *Godel space-times, Abelian instantons, the graviphoton background and other flacuum solutions*, *Nucl. Phys.* **B 684** (2004) 235 [hep-th/0401005] [INSPIRE].

[56] N. Gaddam, A. Gnecchi, S. Vandoren and O. Varela, *Rholography, black holes and Scherk-Schwarz*, *JHEP* **06** (2015) 058 [arXiv:1412.7325] [INSPIRE].

[57] Z.W. Chong, M. Cvetič, H. Lü and C.N. Pope, *Five-dimensional gauged supergravity black holes with independent rotation parameters*, *Phys. Rev.* **D 72** (2005) 041901 [hep-th/0505112] [INSPIRE].

[58] H.K. Kunduri, J. Lucietti and H.S. Reall, *Supersymmetric multi-charge AdS_5 black holes*, *JHEP* **04** (2006) 036 [hep-th/0601156] [INSPIRE].

[59] S. Chimento and T. Ortín, work in progress.

[60] M. Dunajski, J. Gutowski and W. Sabra, *Enhanced Euclidean supersymmetry, $11D$ supergravity and $SU(\infty)$ Toda equation*, *JHEP* **10** (2013) 089 [arXiv:1301.1896] [INSPIRE].

[61] S.V. Ketov, *Universal hypermultiplet metrics*, *Nucl. Phys.* **B 604** (2001) 256 [hep-th/0102099] [INSPIRE].

[62] A. Ashtekar and A. Magnon, *Asymptotically Anti-de Sitter space-times*, *Class. Quant. Grav.* **1** (1984) L39 [INSPIRE].

JHEP04(2017)017