

Supersymmetric black holes in AdS_4 from very special geometry

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ABSTRACT: Supersymmetric black holes in AdS spacetime are inherently interesting for the AdS/CFT correspondence. Within a four dimensional gauged supergravity theory coupled to vector multiplets, the only analytic solutions for regular, supersymmetric, static black holes in AdS_4 are those in the STU-model due to Cacciatori and Klemm. We study a class of $U(1)$ -gauged supergravity theories coupled to vector multiplets which have a cubic prepotential, the scalar manifold is then a *very special Kähler* manifold. When the resulting very special Kähler manifold is a homogeneous space, we find analytic solutions for static, supersymmetric AdS_4 black holes with vanishing axions. The horizon geometries of our solutions are constant curvature Riemann surfaces of arbitrary genus.

KEYWORDS: Black Holes in String Theory, AdS-CFT Correspondence, Supergravity Models

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1 Introduction

Black holes in AdS space have been studied extensively since the development of the AdS/CFT correspondence [1–3] but supersymmetric black holes in AdS spacetime have proved to be rare finds. In four dimensional gauged supergravity coupled to n_v vector multiplets, the only analytic solutions of regular, static, supersymmetric black holes are due to Cacciatori and Klemm (CK) [4] and preserve two real supercharges. These solutions are found within the so-called STU supergravity model, which is standard nomenclature for a model with $n_v = 3$. In this work we use the tools of special geometry and the general structure of the CK solution to find solutions within a particular infinite family of $\mathcal{N} = 2$ gauged supergravity theories.

The Lagrangian of four dimensional $\mathcal{N} = 2$ supergravity coupled to n_v -vector multiplets is governed by special Kähler geometry [5–7]. When this geometry is in turn derived from a cubic prepotential

$$F = -\frac{d_{ijk}X^iX^jX^k}{X^0} \tag{1.1}$$

it is called *very special Kähler geometry* and this is focus of our work. Before gauging, such supergravity theories can be obtained by dimensional reduction from $\mathcal{N} = 2$ supergravity in five dimensions [8].

There is an additional simplification we will employ which facilitates the calculations, namely we require the scalar manifold, \mathcal{M}_v , to be a homogeneous space. The central utility of this assumption is that it ensures the existence of a constant tensor \hat{d}^{ijk} (see appendix B for its definition and numerous identities which it satisfies). We will find that with this assumption the supersymmetric black hole equations are solvable in quite some generality. In fact the homogeneous, very special Kähler geometries have been classified some time ago in the nice work by de Wit and Van Proeyen [9–11] and includes several infinite families as well as certain sporadic geometries related to the dimensional reduction of the magical supergravity theories in five dimensions.

The R-symmetry of four dimensional $\mathcal{N} = 2$ supergravity is

$$SU(2)_R \times U(1)_R \tag{1.2}$$

and we are interested in gauging a $U(1)$ subgroup embedded as

$$U(1)_g \subset SU(2)_R . \tag{1.3}$$

This goes by the moniker FI-supergravity since the gauge couplings generate a potential much like Fayet-Iliopoulos terms in field theory [12]. A useful feature of this abelian gauging is that the scalar fields of the vector multiplets remain neutral under the gauged $U(1)_g$ vector.¹ The fermionic fields are minimally coupled and acquire a charge under the gauge field so that in addition to the abelian charges of the black hole (p^Λ, q_Λ) , there are now additional parameters determining the theory, proportional to the gauging coupling g .

To provide a duality covariant treatment we will consider the general case where the gauging is specified by a symplectic vector containing both electric and magnetic parameters

$$\mathcal{G}^T = (g^\Lambda, g_\Lambda) . \tag{1.4}$$

The supersymmetric Lagrangian for gauged $\mathcal{N} = 2$ Supergravity has been constructed for electric gauging [13], and it has been extended to magnetic gauging in the formalism of conformal supergravity [14]. In order to have a standard Lagrangian formulation that includes magnetic gauging, under which the fermions will be minimally coupled, one must also introduce auxiliary tensor fields. Our strategy is to work with a symplectic completion of the BPS equations which results from electrically gauged models [15].²

Early work on supersymmetric black holes in AdS space were suggestive of a no-go theorem prohibiting regular, half-BPS, asymptotically AdS₄ black holes [17–19], for example the black hole of [20] has a naked singularity. An early workaround was found in [19] where it was found that one could analytically continue AdS-Schwarzschild and construct a quarter-BPS solution of $\mathcal{N} = 2$ gauged supergravity with constant scalar fields

¹In addition, if hypermultiplets are present they remain decoupled.

²See also [16] where a similiar formalism has been used.

with the proviso that the horizon is hyperbolic.³ Some time later, Cacciatori and Klemm [4] successfully demonstrated that by allowing for non-constant scalar fields, the solution of [19] admits a vast generalization within the STU -model of $\mathcal{N} = 2$ FI-gauged supergravity including solutions with spherical and flat horizons (see also [15, 21] for additional analysis of these BPS black holes⁴). In a particular symplectic frame which will be elaborated on below, the CK solutions have four magnetic charges for the four gauge fields and the BPS Dirac quantization condition reduces this to three independent magnetic charges. The absence of electric charges is ultimately tantamount to the absence of axions in the CK solutions. The far-reaching work of Maldacena and Nunez [30] provides a framework to understand the M-theory embedding of the CK solutions.

The central result of our current work is to derive analytic solutions for quarter-BPS black holes in AdS_4 which generalize the CK solution from the STU -model to models whose scalar manifold is a homogeneous very special Kähler manifold. Our new solutions also have vanishing axions and, in the symplectic frame where the gaugings are electric, the charges are all magnetic. A first step towards this result was the work [31] where these models were studied and the general solution for supersymmetric horizon geometries of the form $AdS_2 \times \Sigma_g$ was found. That solution allows for generic gaugings and both electric and magnetic charges, whereas the black hole solutions of the current work will be far more restrictive. Regardless, the results of [31] constitute a solution to the IR boundary conditions for our black holes. In the current work we also analyze the UV AdS_4 boundary conditions and find that they are equivalent to the supersymmetric attractor equations in *ungauged* supergravity, before proceeding to solve for the entire black hole. A key step in our argument is to show that for the static black hole ansatz, a solution with vanishing axions puts strong constraints on the allowed gauging parameters.

Our paper is organised as follows. In section 2 we review some basic facts about $\mathcal{N} = 2$ gauged supergravity in the formalism of [15], the black hole ansatz and the resulting BPS equations. In section 3 we solve the UV boundary conditions, providing the explicit solution for AdS_4 vacuum in $\mathcal{N} = 2$ FI-gauged supergravity. In section 4 we perform our central calculation: we present an analytic solution for axion-free black holes in models whose scalar manifold is a homogeneous very special Kähler geometry. Section 5 contains some comments about the IR boundary conditions and regularity of the solutions.

2 Generalities of $\frac{1}{4}$ -BPS static black holes in AdS_4

The Lagrangian of $\mathcal{N} = 2$ gauged supergravity coupled to n_v vector multiplets is

$$S_{4d} = \int d^4x \sqrt{-g} \left(\frac{1}{2} R - g_{i\bar{j}} \partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} + \mathcal{L}_{\Lambda\Sigma} F_{\mu\nu}^\Lambda F^{\Sigma\mu\nu} + \frac{1}{2\sqrt{-g}} \mathcal{R}_{\Lambda\Sigma} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^\Lambda F_{\rho\sigma}^\Sigma - V_g \right). \tag{2.1}$$

³This can then be quotiented by a discrete group to give a Riemann surface of genus $g > 1$.

⁴Further work has been done extending these solutions to non-BPS and non-extremal black holes [22–27]. There has also been recent work on supersymmetric AdS_4 black holes with hypermultiplets [28, 29] where the resulting solutions are numerical.

We will work in the symplectically covariant formulation of [15] which, for the black hole ansatz we use, provides a covariant form of the BPS equations. Our spacetime ansatz is that of a static black hole with constant curvature horizon:

$$ds_4^2 = -e^{2U} dt^2 + e^{-2U} dr^2 + e^{2(V-U)} d\Sigma_g^2, \tag{2.2}$$

where $d\Sigma_g$ is the uniform metric on $\Sigma_g = \{S^2, T^2, \mathbb{H}^2/\Gamma\}$ of curvature $\kappa = \{1, 0, -1\}$ respectively.⁵ The gauge fields are chosen such that

$$p^\Lambda = \frac{1}{\text{vol}(\Sigma_g)} \int_{\Sigma_g} F^\Lambda, \quad q_\Lambda = \frac{1}{\text{vol}(\Sigma_g)} \int_{\Sigma_g} G_\Lambda \tag{2.3}$$

where

$$G_\Lambda = \frac{\delta L_{Ad}}{\delta F^\Lambda} = \mathcal{R}_{\Lambda\Sigma} F^\Sigma - \mathcal{I}_{\Lambda\Sigma} * F^\Sigma \tag{2.4}$$

is the dual field strength and $\text{vol}(\Sigma_g)$ is the volume of Σ_g . In fact the BPS equations are independent of the precise profiles for the gauge fields, they depend only on the charges (p^Λ, q_Λ) . For a static and spherically symmetric configuration the metric and the scalar fields depend only on the radial co-ordinate $z^i = z^i(r)$. We will restrict to this assumptions throughout the paper.

As mentioned in the introduction the gauging is parametrized by a symplectic vector \mathcal{G} , corresponding to the gravitino charge under the $U(1)$ field of the gauging. Thus, one can organize the data into a pair of symplectic vectors:

$$\mathcal{Q} = \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix}, \quad \mathcal{G} = \begin{pmatrix} g^\Lambda \\ g_\Lambda \end{pmatrix}. \tag{2.5}$$

In our notation, the symplectic section over the scalar manifold \mathcal{M}_v is denoted⁶ \mathcal{V} :

$$\mathcal{V} = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix} = e^{K/2} \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix} \tag{2.6}$$

and we have used the symplectic inner product between two vectors $A = (A^\Lambda, A_\Lambda)$ and $B = (B^\Lambda, B_\Lambda)$

$$\langle A, B \rangle \equiv A^T \Omega B = B^\Lambda A_\Lambda - A^\Lambda B_\Lambda \tag{2.7}$$

to produce the invariants

$$\mathcal{Z} = \langle \mathcal{Q}, \mathcal{V} \rangle, \quad \mathcal{L} = \langle \mathcal{G}, \mathcal{V} \rangle, \quad \mathcal{Z}_i = \langle \mathcal{Q}, D_i \mathcal{V} \rangle, \quad \mathcal{L}_i = \langle \mathcal{G}, D_i \mathcal{V} \rangle. \tag{2.8}$$

The BPS equations for preservation of at least two supercharges were derived in [4] for electric gaugings and [15] for general dyonic gaugings. We use the results of [15] which

⁵The discrete group Γ is a Fuchsian group and its precise form does not alter this local analysis.

⁶We use the notation and conventions of [12] as much as possible, apart from the signature of space-time which we take to be *mostly plus*.

were found by reducing (2.1) to one dimension and re-writing the resulting action as a sum of squares. One then arrives at the BPS equations in the form:

$$2e^{2V} \partial_r \left[\text{Im} \left(e^{-i\psi} e^{-U} \mathcal{V} \right) \right] = 8e^{2(V-U)} \text{Re} \left(e^{-i\psi} \mathcal{L} \right) \text{Re} \left(e^{-i\psi} \mathcal{V} \right) - \mathcal{Q} - e^{2(V-U)} \Omega \mathcal{M} \mathcal{G}, \quad (2.9)$$

$$\partial_r (e^V) = 2e^{V-U} \text{Im} \left(e^{-i\psi} \mathcal{L} \right), \quad (2.10)$$

$$\psi' = -A_r - 2e^{-U} \text{Re} \left(e^{-i\psi} \mathcal{L} \right). \quad (2.11)$$

Notice that ψ , the phase of the Killing spinor, appears explicitly in the equations. The connection A_μ is given by

$$A_\mu = \text{Im} \left(\partial_\mu z^i \partial_i K \right) \quad (2.12)$$

and the matrix \mathcal{M} is defined in (A.10). When \mathcal{M} is contracted with the symplectic form Ω it gives a complex structure on the $Sp(2n_v + 2, \mathbb{R})$ bundle over \mathcal{M}_v :

$$\Omega \mathcal{M} \mathcal{V} = -i\mathcal{V}, \quad \Omega \mathcal{M} (D_i \mathcal{V}) = iD_i \mathcal{V}. \quad (2.13)$$

While (2.9) may seem cumbersome, it is just a repackaging of the first order equations for the scalar fields z^i and the metric function e^U . This repackaging is useful since much like the ungauged $\mathcal{N} = 2$ supersymmetric black holes [32, 33], the analytic black hole solutions are particularly simple when expressed in terms of this data. By re-deriving a version of these equations in a frame with electric gaugings using the formulae of [12] one can establish that the resulting black holes preserve two out of eight real supercharges along the flow and four at the horizon.

Notice that (2.11) is the equation for the phase ψ of the supersymmetry parameter. This is not a new degree of freedom and in fact one can show [15] that this is given algebraically by the phase of a superpotential $W = e^U e^{-i\psi} (\mathcal{Z} - ie^{2(V-U)} \mathcal{L})$, or equivalently

$$e^{2i\psi} = \frac{\mathcal{Z} - ie^{2(V-U)} \mathcal{L}}{\mathcal{Z} + ie^{2(V-U)} \mathcal{L}}. \quad (2.14)$$

Using this definition the flow equation (2.11) follows from (2.9) and (2.10). Since the gravitino is charged, there is a Dirac quantization condition

$$\langle \mathcal{G}, \mathcal{Q} \rangle \in \mathbb{Z} \quad (2.15)$$

and the supersymmetry conditions fix this integer to be the curvature of the horizon:

$$\langle \mathcal{G}, \mathcal{Q} \rangle = -\kappa. \quad (2.16)$$

It is interesting to note that (2.16) is the only place where the curvature of the horizon geometry appears. Pragmatically this means that solutions are independent of the curvature of Σ_g but the regularity conditions do depend on κ .

The single center, static black holes we consider in this work interpolate between AdS_4 at large r and $AdS_2 \times \Sigma_g$ at some finite positive $r = r_h$. The metric functions for these spaces are

$$AdS_4 : e^U = \frac{r}{R}, \quad e^V = \frac{r^2}{R}, \quad (2.17)$$

$$AdS_2 \times \Sigma_g : e^U = \frac{r}{R_2}, \quad e^V = \frac{rR_2}{R_1}, \quad (2.18)$$

where R is the curvature radius of AdS_4 , and R_1, R_2 are the curvature radii of Σ_g and AdS_2 respectively; the scalar fields and the phase ψ are constant. We leave the analysis of the AdS_4 solutions as a function of the gauging parameters to the next section.

The spectrum of horizon geometries (2.18) as a function of both the gaugings (g^Λ, g_Λ) and charges (p^Λ, q_Λ) was solved in [31].

3 UV boundary conditions from very special geometry

In this section we solve the BPS equations (2.9) and (2.10) for AdS_4 geometries (2.17). We look for a supersymmetric vacuum AdS_4 with constant scalars in absence of gauge fields, so the constraint (2.16) does not have to be imposed. The equations for the vacuum identify the subspace of gauging parameters which support black hole solutions with vanishing axions.

3.1 General AdS_4 solutions

We first analyze the boundary conditions in the UV, and we obtain the exact solution to the BPS equations. For AdS_4 , the metric functions are given by (2.17), the scalars and the phase ψ are constant and the charges are zero:

$$z = x_0 + iy_0, \quad \psi = \psi_0, \quad \mathcal{Q} = 0. \tag{3.1}$$

With this ansatz, the equations give

$$\mathcal{G} = -2\text{Im} [\bar{\mathcal{L}}\mathcal{V}] \tag{3.2}$$

$$\mathcal{L} = \text{Re}\mathcal{L} + i\text{Im}\mathcal{L} = \frac{i}{R}e^{i\psi_0} \tag{3.3}$$

These equations are in fact identical to the attractor equations for solutions of the form $AdS_2 \times S^2$ in *ungauged* $\mathcal{N} = 2$ supergravity [34] with the obvious replacement of the gauging parameters \mathcal{G} with charges \mathcal{Q} .

When \mathcal{M}_v is a very special geometry, these equations are quite tractable and have been analyzed in [35]. In special coordinates (3.2) amounts to

$$g^0 = 2e^{K/2} \text{Im}\mathcal{L}, \tag{3.4}$$

$$g_0 = g^0 d_{ijk}(x^i x^j x^k - 3x^i y^j y^k) + 2 \text{Re}\mathcal{L} e^{K/2} d_{ijk}(y^i y^j y^k - 3y^i x^j x^k), \tag{3.5}$$

$$g^i = g^0 x^i - 2e^{K/2} \text{Re}\mathcal{L} y^i, \tag{3.6}$$

$$g_i = 3g^0 d_{ijk}(y^j y^k - x^j x^k) + 12 \text{Re}\mathcal{L} e^{K/2} d_{ijk} x^j y^k \tag{3.7}$$

and the solution requires inverting these and expressing the scalars (x^i, y^i) and ($\text{Re}\mathcal{L}, \text{Im}\mathcal{L}$) in terms of the gaugings (g^Λ, g_Λ).

If one makes the assumption that $g^0 = 0$ the general solution is quite straightforward to obtain:

$$\frac{1}{R^2} = \sqrt{-4d_g g_0 + \frac{1}{3}(d_g^{-1})^{ij} g_i g_j} \quad (3.8)$$

$$x^i = -\frac{1}{6}(d_g^{-1})^{ij} g_j, \quad (3.9)$$

$$y^i = \frac{g^i}{2d_g} \frac{1}{R^2}, \quad (3.10)$$

$$\text{Re}\mathcal{L} = \frac{\epsilon}{R}, \quad (3.11)$$

$$\text{Im}\mathcal{L} = 0, \quad (3.12)$$

$$\psi_0 = -\epsilon \frac{\pi}{2}. \quad (3.13)$$

In the above equations d_g denotes the contraction of the d -tensor with the gauging parameters (as defined in eq. A.6 of appendix A) and $\epsilon = \pm 1$ is a conventional sign, consequence of the identity (2.14) which defines the phase ψ up to a factor of π .

Up to obtaining an expression for d_g^{-1} , which is always well defined for homogeneous scalar manifolds, this comprises an explicit solution.

With $g^0 \neq 0$ the general solution requires solving the set of n_v real, quadratic equations

$$\Delta_i = d_{ijk} \tilde{y}^j \tilde{y}^k \quad (3.14)$$

where

$$\Delta_i = 3d_{ijk} g^j g^k + g^0 g_i, \quad \tilde{y}^i = \sqrt{12} |\mathcal{L}| e^{K/2} y^i. \quad (3.15)$$

The general solution to (3.14) is not known and, since they are real equations, they are not in general guaranteed to have real solutions. If we assume \mathcal{M}_v to be a homogeneous space in addition to a very special Kähler geometry, then we can solve (3.14) using the *constant* tensor

$$\hat{d}^{ijk} = \frac{g^{il} g^{jm} g^{kn} d_{lmn}}{d_y^2} \quad (3.16)$$

and the corresponding identities detailed in appendix B. The solution to (3.4)–(3.7) is then given by

$$\frac{1}{R^2} = \sqrt{\mathcal{I}_4(g^\Lambda, g_\Lambda)} \quad (3.17)$$

$$\text{Re}\mathcal{L} = \frac{2\mathcal{I}_2(g^\Lambda, g_\Lambda) \mathcal{I}_4(g^\Lambda, g_\Lambda)^{1/4}}{\sqrt{\mathcal{I}_4(g^\Lambda, g_\Lambda) + 4\mathcal{I}_2(g^\Lambda, g_\Lambda)^2}} \quad (3.18)$$

$$\text{Im}\mathcal{L} = \frac{[\mathcal{I}_4(g^\Lambda, g_\Lambda)]^{3/4}}{\sqrt{\mathcal{I}_4(g^\Lambda, g_\Lambda) + 4\mathcal{I}_2(g^\Lambda, g_\Lambda)^2}} \quad (3.19)$$

$$y^i = \frac{3}{32} \frac{1}{(g^0)^2} \frac{\mathcal{I}_4(g^\Lambda, g_\Lambda)^{1/2}}{\mathcal{I}_4(g^\Lambda, g_\Lambda) + 4\mathcal{I}_2(g^\Lambda, g_\Lambda)^2} \hat{d}^{ijk} \Delta_j \Delta_k \quad (3.20)$$

$$x^i = \frac{g^i}{g^0} + \frac{3}{16} \frac{\mathcal{I}_2(g^\Lambda, g_\Lambda)}{(g^0)^2} \frac{\hat{d}^{ijk} \Delta_j \Delta_k}{\mathcal{I}_4(g^\Lambda, g_\Lambda) + 4\mathcal{I}_2(g^\Lambda, g_\Lambda)^2} \quad (3.21)$$

where we have used the identity

$$\widehat{d}_\Delta = 16(g^0)^2 [\mathcal{I}_4(g^\Lambda, g_\Lambda) + 4\mathcal{I}_2(g^\Lambda, g_\Lambda)^2] \tag{3.22}$$

and the invariants $(\mathcal{I}_2, \mathcal{I}_4)$ are defined in (B.11) and (B.12). Note that even though we derived (3.17)–(3.21) assuming $g^0 \neq 0$, they have a smooth $g^0 \rightarrow 0$ limit which agrees with (3.8)–(3.13).

3.2 AdS₄ Solutions with vanishing axions

The black holes we will study in this paper have vanishing axions, thus we first focus our analysis on AdS₄ solutions with imaginary scalar fields. These will serve as the asymptotic UV boundary conditions for the black hole solutions.

At spatial infinity the vacuum is parametrized by the gauging constants, namely the $2n_v + 2$ entries of the vector $\mathcal{G} = (g^\Lambda, g_\Lambda)$, since the gauge fields fall-off at infinity. In addition, the requirement of vanishing axions are n_v constraints $x^i = 0$ which, from (3.4)–(3.7), can be written in the form

$$d_g g_i = -3g_0 g^0 d_{g,i}. \tag{3.23}$$

In conclusion, we expect that the general space of zero-axion AdS₄ solutions is $n_v + 2$ dimensional. Assuming $g_0 \neq 0$ the explicit solution is given by

$$y^i = -\sqrt{-\frac{g_0}{d_g}} g^i, \quad \text{Re } \mathcal{L} = \sqrt{2}(-g_0 d_g)^{1/4}, \quad \text{Im } \mathcal{L} = \frac{\sqrt{2}g_0 g^0}{(-g_0 d_g)^{1/4}}. \tag{3.24}$$

The co-dimension one subspace with $(g^0, g_i) = (0, 0)$ will be the focus of our work in the next section.

4 Black holes from very special geometry

In this section we solve for supersymmetric black holes with vanishing axions. We restrict to black holes with $g^0 = g_i = 0$. We first demonstrate that for this class of black holes the phase ψ is constant. We then proceed to make an ansatz for the scalar fields and the metric, that will allow us to solve the BPS equations analytically. To describe our ansatz we first fix the Kähler gauge by choosing special coordinates

$$X^\Lambda = \begin{pmatrix} 1 \\ z^i \end{pmatrix}, \tag{4.1}$$

where $z^i = x^i + iy^i$. Having done this, we assume that the axions vanish

$$x^i = 0. \tag{4.2}$$

Note that since we have fixed that Kähler gauge, we cannot shift ψ .

4.1 Constant ψ

We now explore which configurations of gauge couplings result in a constant supersymmetry phase when the axions are set to zero. Combining (3.3) with (2.11) we see that

$$\psi'|_{\infty} = 0 . \tag{4.3}$$

We then proceed by induction in order of derivatives on ψ .

The condition $x^i = 0$ implies that \mathcal{A}_r is zero along the whole flow but this is not enough to show that ψ is constant. In order to ensure that, we need to assume one of the two configurations⁷

$$1. \quad g^0 = g_i = 0 , \tag{4.4}$$

$$2. \quad g_0 = g^i = 0 . \tag{4.5}$$

For the case (4.4), then, (2.11) reduces to

$$\psi' = -2e^{-U(r)}\mathcal{L}(r)\cos(\psi(r)) . \tag{4.6}$$

Since the differential equation is of the form

$$\begin{aligned} \psi(r)' &= a(r)\cos\psi(r) \\ \psi(r \rightarrow \infty)' &= 0 , \end{aligned} \tag{4.7}$$

every n -th derivative of ψ depends only on terms which are

- terms proportional to $\cos\psi(r)$, which vanishes at infinity since $\psi_{\infty} = \pm\pi/2$,
- terms containing derivatives of $\psi(r)$ up to the order $n - 1$, which vanish at infinity by assumption.

This means that all derivatives calculated at infinity are zero, and thus the phase ψ is constant throughout the entire spacetime. The latter case of (4.5) goes through similarly but with $\text{Re}(\mathcal{L}) = 0$ throughout.

This does not exhaust the possible black holes with vanishing axions in these models since the UV asymptotics given by (4.4) and (4.5) are co-dimension one in the space defined by (3.23). In simple examples we have found that in the UV we can use a duality transformation to generate a general AdS_4 solution satisfying (3.23) from one satisfying (3.23) and (4.4) but such a transformation does generate axions in the bulk of the flow. It would be interesting to solidify these observations and determine unambiguously whether or not vanishing axions implies a constant spinor for this entire class of black holes.

⁷In both these configurations one can use a duality transformation to set the remaining gauge couplings equal in magnitude, for example in the STU-model to the frame with $g_0 = -g^i = g$ which has an M-theory lift. We find it simpler to refrain from making this transformation as it allows us to more easily maintain covariant formulae.

4.2 The ansatz

With a constant phase ψ , the BPS equations (2.9) and (2.10) simplify to

$$2e^{2V} \partial_r \left[\text{Im} \left(e^{-i\psi_0} e^{-U} L^\Lambda \right) \right] = -p^\Lambda + e^{2(V-U)} \mathcal{I}^{\Lambda\Sigma} g_\Sigma, \quad (4.8)$$

$$2e^{2V} \partial_r \left[\text{Im} \left(e^{-i\psi_0} e^{-U} M_\Lambda \right) \right] = -q_\Lambda - e^{2(V-U)} \mathcal{I}_{\Lambda\Sigma} g^\Sigma, \quad (4.9)$$

$$\partial_r(e^V) = 2e^{V-U} (g_0 L^0 - g^i M_i). \quad (4.10)$$

Despite the fact that in the UV we could solve the full $g^0 = 0$ solution space in all generality, to proceed further with the black hole solution we make the simplifying assumption that \mathcal{M}_v is a homogeneous space. So we assume that (4.4) holds and then continue by solving for (L^0, M_i) , we find the equations (4.8)–(4.10) become

$$2e^V \partial_r(\tilde{L}^0) - 2\partial_r(e^V) \tilde{L}^0 = -p^0 - 8g_0(\tilde{L}^0)^2 \quad (4.11)$$

$$2e^V \partial_r(\tilde{M}_i) - 2\partial_r(e^V) \tilde{M}_i = -q_i - \left[\frac{9}{4} d_{ijk} \hat{d}^{klm} \tilde{M}_l \tilde{M}_m - 8\tilde{M}_i \tilde{M}_j \right] g^j \quad (4.12)$$

$$\partial_r(e^V) = 2 \left[g_0 \tilde{L}^0 - g^i \tilde{M}_i \right], \quad (4.13)$$

where we have defined the rescaled sections

$$\tilde{L}^\Lambda = e^{V-U} L^\Lambda, \quad \tilde{M}_\Lambda = e^{V-U} M_\Lambda \quad (4.14)$$

and have used the following data (valid for $x^i = 0$)

$$L^\Lambda = e^{K/2} \begin{pmatrix} 1 \\ iy^i \end{pmatrix}, \quad M_\Lambda = e^{K/2} \begin{pmatrix} -idy \\ 3d_{y,i} \end{pmatrix}, \quad (4.15)$$

$$\Omega\mathcal{M} = \begin{pmatrix} 0 & -\mathcal{I}^{-1} \\ \mathcal{I} & 0 \end{pmatrix}, \quad \mathcal{I}_{\Lambda\Sigma} = -d_y \begin{pmatrix} 1 & 0 \\ 0 & 4g_{ij} \end{pmatrix}, \quad (4.16)$$

$$d_y g_{ij} = -\frac{9}{16} d_{ijk} \hat{d}^{klm} M_l M_m + 2M_i M_j. \quad (4.17)$$

Due to our assumption that $g^0 = 0$, we see that $\text{Re}(e^{-i\psi} \mathcal{L}) = 0$ and, from (2.11), that $\psi = \psi_0 = -\pi/2$ is constant throughout the flow. Another consequence of constant ψ and $x^i = 0$ is that from (4.8) and (4.9) it follows that $p_i = q^0 = 0$.

The scalar fields and the warp factor e^U can be expressed as functions of $e^V, \tilde{L}^0, \tilde{M}_i, \psi_0$:

$$y^i = \frac{3}{8} \hat{d}^{ijk} M_j M_k, \quad (4.18)$$

$$1 = L^0 \hat{d}^{ijk} M_i M_j M_k, \quad (4.19)$$

which gives

$$y^i = \frac{3}{8} \frac{\hat{d}^{ijk} \tilde{M}_j \tilde{M}_k}{\sqrt{\tilde{L}^0 \hat{d}^{lmp} \tilde{M}_l \tilde{M}_m \tilde{M}_p}}, \quad (4.20)$$

$$e^{4U} = \frac{e^{4V}}{\tilde{L}^0 \hat{d}^{ijk} \tilde{M}_i \tilde{M}_j \tilde{M}_k}. \quad (4.21)$$

Thus, once we find the set of functions $(e^V, \tilde{L}^0, \tilde{M}_i, \psi_0)$ which solve (4.11)–(4.13), the black hole solution is fully determined. We proceed to do that in the next section.

4.3 The solution

Taking the solution of [4] as inspiration, we make the ansatz

$$e^V = \frac{r^2}{R} - v_0, \tag{4.22}$$

$$\widetilde{L}^0 = \alpha^0 r + \beta^0, \tag{4.23}$$

$$\widetilde{M}_i = \alpha_i r + \beta_i. \tag{4.24}$$

This is a rather enlightened ansatz which is difficult to motivate in advance. In principle the UV boundary conditions fix (R, α^0, α_i) while the IR ones fix (v_0, β^0, β_i) . The flow equations will then highly constrain the system and the fact that a BPS solution of such simpler form exists, as we will show in this section, is a nontrivial result of yet unexplained origin.

From (4.13) we get

$$\frac{2}{R} = 2\alpha^0 g_0 - 2\alpha_i g^i \tag{4.25}$$

$$0 = g_0 \beta^0 - g^i \beta_i \tag{4.26}$$

We find from (4.11)

$$\frac{2\alpha^0}{R} - \frac{4\alpha^0}{R} = -8g_0(\alpha^0)^2 \tag{4.27}$$

$$-\frac{4}{R}\beta^0 = -16g_0\alpha^0\beta^0 \tag{4.28}$$

$$-2v_0\alpha^0 = -p^0 - 8g_0(\beta^0)^2 \tag{4.29}$$

and we immediately see that

$$\alpha^0 = \frac{1}{4g_0 R} \tag{4.30}$$

$$\beta^0 = \frac{\epsilon_0}{g_0} \sqrt{\frac{1}{8} \left(\frac{v_0}{2R} - g_0 p^0 \right)} \tag{4.31}$$

where $\epsilon_0 = \pm 1$.

Then from (4.12) we get

$$-\frac{2\alpha_i}{R} = -\left[\frac{9}{4} d_{ijk} \widehat{d}^{klm} g^j \alpha_l \alpha_m - 8\alpha_i \alpha_j g^j \right] \tag{4.32}$$

$$-\frac{4}{R} \beta_i = -\left[\frac{9}{2} d_{ijk} \widehat{d}^{klm} g^j \alpha_l \beta_m - 8(\alpha_i \beta_j + \beta_i \alpha_j) g^j \right] \tag{4.33}$$

$$-2v_0 \alpha_i = -q_i - \left[\frac{9}{4} d_{ijk} \widehat{d}^{klm} g^j \beta_l \beta_m - 8\beta_i \beta_j g^j \right] \tag{4.34}$$

and from (4.32) we find that

$$\alpha_i = -\frac{3}{4R d_g} d_{ijk} g^j g^k \quad \Rightarrow \quad \alpha_i g^i = -\frac{3}{4R}, \tag{4.35}$$

which agrees with the UV analysis in section 3. We can immediately see that (4.25) is satisfied and with some effort (using identities in appendix B) one can also compute that (4.33) is automatically satisfied.

It now remains to use (4.26) and (4.34) to solve for (v_0, β_i) . This is $(n_v + 1)$ -equations for $(n_v + 1)$ -parameters and should thus admit a solution. From (4.34) we obtain an expression for β_i :

$$-\frac{4}{9}(d_g^{-1})^{ij}q_j - \frac{2v_0}{3Rd_g}g^i = \widehat{d}^{ilm}\beta_l\beta_m - \frac{32}{9}(d_g^{-1})^{ij}\beta_j\beta_kg^k \quad (4.36)$$

Now using \widehat{d}^{ijk} we have an explicit expression for d_g^{-1}

$$(d_g^{-1})^{ij} = \frac{1}{d_g} \left[\frac{27}{16} \widehat{d}^{ijk} d_{g,k} - 3g^i g^j \right] \quad (4.37)$$

and (4.36) becomes

$$-\frac{4}{9}(d_g^{-1})^{ij}q_j - \frac{2v_0}{3Rd_g}g^i = \widehat{d}^{ilm} \left[\beta_l - \frac{3}{d_g} d_{g,l} \beta_j g^j \right] \left[\beta_m - \frac{3}{d_g} d_{g,m} \beta_j g^j \right] - \frac{32}{3d_g} g^i (\beta_i g^i)^2. \quad (4.38)$$

By using (4.26) and (4.31) we know that

$$\beta_i g^i = \epsilon_0 \sqrt{\frac{1}{8} \left(\frac{v_0}{2R} - g_0 p^0 \right)} \quad (4.39)$$

and thus we can define an object Π^i which depends only on \mathcal{G} and \mathcal{Q} :

$$\Pi^i = -\frac{3}{4d_g} \widehat{d}^{ijk} d_{g,k} q_j + g^i \frac{4}{3d_g} (q_m g^m - g_0 p^0) \quad (4.40)$$

so that (4.38) becomes a familiar type of equation (see eq. (B.8))

$$\Pi^i = \widehat{d}^{ilm} \left[\beta_l - \frac{3}{d_g} d_{g,l} \beta_i g^i \right] \left[\beta_m - \frac{3}{d_g} d_{g,m} \beta_i g^i \right]. \quad (4.41)$$

This can be solved explicitly yielding an expression for β_i in terms of v_0 along with the charges and gaugings:

$$\beta_i = \epsilon \sqrt{\frac{27}{64} \frac{d_{ijk} \Pi^j \Pi^k}{\sqrt{d_\Pi}}} + \frac{3\beta_j g^j}{d_g} d_{g,i}, \quad (4.42)$$

where again $\epsilon = \pm 1$. It remains to solve for v_0 , which is done as follows. Contracting (4.42) with g^i gives

$$\beta_i g^i = -\frac{\epsilon}{2} \sqrt{\frac{27}{64} \frac{d_{ijk} g^i \Pi^j \Pi^k}{\sqrt{d_\Pi}}} \quad (4.43)$$

which when combined with (4.39) gives the solution for v_0 purely in terms of the charges and gaugings:

$$v_0 = 2R \left[g_0 p^0 + \frac{27(d_{ijk} g^i \Pi^j \Pi^k)^2}{32d_\Pi} \right] \quad (4.44)$$

so that

$$\beta_i = \epsilon \sqrt{\frac{27}{64 d_\Pi}} \left[d_{ijk} \Pi^j \Pi^k - \frac{3}{2 d_g} d_{g,i} d_{lmn} g^l \Pi^m \Pi^n \right] \quad (4.45)$$

and

$$\epsilon = -\epsilon_0. \quad (4.46)$$

4.4 Constant scalar flows

We conclude this analysis by writing out the universal black hole with constant scalar fields. This is well known to require a hyperbolic horizon [15, 19] and we confirm that result here.

When one further restricts the BPS flow equations to constant scalars, one obtains

$$\beta^0 = 0 \quad \Rightarrow \quad v_0 = 2Rg_0p^0, \quad (4.47)$$

$$\beta_i = 0 \quad \Rightarrow \quad \Pi^i = 0, \quad (4.48)$$

and the constraint $\Pi^i = 0$ gives

$$q_i = -\frac{3g_0p^0}{d_g} d_{g,i}. \quad (4.49)$$

Contracting with g^i and using (2.16) gives

$$g^i q_i = -3g_0p^0 \quad \Rightarrow \quad \kappa = -4g_0p^0, \quad (4.50)$$

which must be positive by (4.47) and therefore these solutions require

$$\kappa = -1 \quad \Rightarrow \quad \Sigma_g = \mathbb{H}^2/\Gamma. \quad (4.51)$$

4.5 Summary of the solution

It may provide some clarity to summarize the entire solution in one place. The rescaled sections $(\tilde{L}^0, \tilde{M}_i)$ are given by (4.23) and (4.24) in terms of $(\alpha^0, \beta_0, \alpha_i, \beta_i)$ which in turn are given in (4.30), (4.31), (4.35), (4.45). The metric function e^V is given by (4.22) and (4.44). To obtain the scalar fields y^i and the metric function e^U one uses (4.21) and (4.20).

If one chooses $n_v = 3$, $d_{123} = \frac{1}{6}$ and $\hat{d}^{123} = \frac{32}{3}$ (and symmetric permutations) one obtains the so-called STU-model, the model employed in [4]. If in addition one sets

$$g_0 = -g^i = g \quad (4.52)$$

the AdS₄ black holes of the STU-model can be embedded into M-theory compactified on S^7 [20, 36, 37]. For more general very special Kähler manifolds which are homogeneous spaces, one can find the explicit form of the corresponding d_{ijk} -tensor in section 5 of [10]. The embedding of these models into M-theory or string theory remains an important outstanding problem.

4.6 Rotation to electric gaugings

If the reader is for whatever reason uncomfortable with the use of magnetic gauging parameters, one can rotate the solutions of this paper to a frame where the gaugings are electric. Precisely, one finds that from the prepotential (A.1) and the gauging parameters (g_0, g^i) one can rotate to a new symplectic frame using

$$\mathcal{S} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad A = D = \text{diag}\{1, 0, \dots, 0\}, \quad B = -C = \text{diag}\{0, 1, \dots, 1\} \quad (4.53)$$

to find a new prepotential

$$\tilde{F} = -i\sqrt{\frac{1}{16}}\sqrt{\tilde{X}^0 \hat{d}^{ijk}(\delta_{il} \tilde{X}^l)(\delta_{jm} \tilde{X}^m)(\delta_{kn} \tilde{X}^n)} \quad (4.54)$$

with new gaugings

$$\tilde{g}_i = -g^i, \quad \tilde{g}_0 = g_0. \quad (4.55)$$

The space-time metric and scalar fields remain invariant under this symplectic transformation. For the STU model this frame has $\tilde{F} = -2i\sqrt{\tilde{X}^0 \tilde{X}^1 \tilde{X}^2 \tilde{X}^3}$ and this is the frame which is used to embed the STU model into the de Wit-Nicolai theory [38] and thus into M-theory.

5 AdS₂ × Σ_g: IR boundary conditions

We now make some brief statements about regularity of our solutions. To map out the subspace of regular solutions from section 4, one needs to ensure that the scalar fields do not vanish before the horizon is reached:

$$r_h > r_i, \quad r_h > r_0, \quad (5.1)$$

where

$$r_h = \sqrt{v_0 R}, \quad r_0 = -\frac{\beta^0}{\alpha^0}, \quad r_i = -\frac{\beta^i}{\alpha^i}. \quad (5.2)$$

From $r_h > r_0$ we get

$$\sqrt{v_0 R} > -\epsilon_0 \sqrt{v_0 R - g_0 p^0}, \quad (5.3)$$

which is satisfied automatically if $\epsilon_0 = 1$ whereas if $\epsilon_0 = -1$ it requires $g_0 p^0 > 0$. More generally $r_h > r_i$ is a rather complicated expression which puts bounds on the allowed charges. It is difficult to analyze in complete generality but manageable in any given example.

Assuming that these conditions are satisfied we can use the results of [31] to analyze regular horizon geometries. Consider a black hole configuration with charges (p^0, q_i) and gauging parameters (g^i, g_0) , in homogeneous d -geometries. The attractor equations give an expression of the horizon radius in terms of the charges and the d -tensor as derived in [31],

which reads

$$\begin{aligned}
 R_2^4 &= \frac{-a_4 \pm \sqrt{a_4^2 - 4a_0a_8}}{2a_8} & (5.4) \\
 a_0 &= -\frac{1}{16}p^0\widehat{d}^{ijk}q_iq_jq_k \\
 a_4 &= \frac{9}{16}d_{g,i}\widehat{d}^{ilm}q_lq_m - (p^0g_0 + g^iq_i)^2 \\
 a_8 &= 4g_0d_g = (\ell_{\text{AdS}})^{-4} > 0 \\
 p^0g_0 - q_i g^i &= 1. & (5.5)
 \end{aligned}$$

Still, a necessary condition for the existence of the horizon is that $R_2^2 > 0$. Notice that the sign of g_0 is chosen accordingly to the condition that $a_8 > 0$. If we use the last constraint on the charges, we can write

$$\begin{aligned}
 -4a_0a_8 &= d_g\widehat{d}_q(1 + q_i g^i), \\
 a_4 &= \frac{9}{16}d_{g,i}\widehat{d}_q^i - (1 + 2q_i g^i)^2. & (5.6)
 \end{aligned}$$

Notice that if $-4a_0a_8 > 0$ there is always a choice of sign in (5.4) for which the radius is positive. But the sign of $-4a_0a_8$ can be driven to positive or negative by the choice of charges q_i . In that case, whatever the sign of a_4 , there will always be a solution of the attractor equations for which the radius is positive. Indeed, let us consider a rescaling of all the q 's charges of the solution by a factor $\alpha \geq 0$. This leads to

$$\begin{aligned}
 (-4a_0a_8)_\alpha &= \alpha^3 d_g\widehat{d}_q(1 + \alpha q_i g^i), \\
 (a_4)_\alpha &= \alpha^2 \frac{9}{16}d_{g,i}\widehat{d}_q^i - (1 + 2\alpha q_i g^i)^2. & (5.7)
 \end{aligned}$$

Independently on what is the sign of $(a_4)_\alpha$, then, we can chose a small enough $\alpha \in \mathbb{R}$ for which $1 + \alpha q_i g^i > 0$. Then, depending on the sign of \widehat{d}_q , we can fix the sign of $(-4a_0a_8)_\alpha$ to be positive by requiring $\alpha \leq 0$. This is enough to ensure that there is a root in the attractor equation corresponding to a positive R_2^4 .

6 Conclusions

In this work we have studied four dimensional $\mathcal{N} = 2$ FI-gauged supergravity theories where the scalar manifold is a homogeneous very special Kähler geometry. In these models we have found quarter-BPS black hole solutions with vanishing axions and constant phase ψ .

There are numerous interesting and outstanding questions regarding this variety of AdS black holes. In the work [31] models with arbitrary gauging parameters and arbitrary dyonic charges were considered. It was found that that solution space of supersymmetric horizon geometries has real dimension $2n_v$; there are $2n_v + 2$ charges and two constraints. It seems to be a well posed and reasonable open problem to solve for the most general quarter-BPS black hole in these theories with complex scalar fields . This would answer

in the affirmative or otherwise whether every supersymmetric horizon geometry can be completed to a UV AdS₄ solution. This is very difficult to attack numerically but should one obtain the general analytic result, it would seem to be a reasonable question.

A key step in pursuing such an objective is a better understanding of black holes where the phase of the supersymmetry parameter is non-constant. In the current work we have only found black holes with constant phase but the full space of static BPS, AdS₄ black holes will surely include those with non-constant spinors. This could be quite challenging since, for example, all known solutions with hypermultiplets have non-trivial axions; there are certain solutions with constant phase ψ [28, 29], while the general solution has varying ψ [29] and in the latter case the analysis is significantly more complicated. We have argued that all black holes with trivial axions will satisfy either (4.4) or (4.5) and thus have constant ψ , but have not found a proof of this statement.

A more modest objective could hopefully be realized using just the results of the current work. That is to determine whether every horizon geometry from [31] with vanishing axions and which satisfies (4.4) arises as the IR of the black holes in section 4.

At least to these humble authors, the origin of the ansatz (4.22)–(4.24) and the ansatz in [4] appears fairly mysterious. We have shown that such assumption allows for solutions to be found but we would certainly be comforted to have a deeper understanding of *why* it works. It is natural to speculate that a dimensional reduction to three dimensions [39] could aid this understanding, since such a reduction clarifies various issues for ungauged supergravity black holes [40]. Another challenging approach would be to explicitly integrate the BPS equations rather than making the ansatz (4.22)–(4.24).

Hopefully these results will be a few steps along the road to a complete solution of supersymmetric static black holes in four dimensional gauged supergravity.

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A Special geometry conventions

We include in this section standard results in order to make our conventions clear, with particular attention to numerical factors.

The prepotential we consider is

$$F = -d_{ijk} \frac{X^i X^j X^k}{X^0} \tag{A.1}$$

and we use special coordinates

$$X^\Lambda = \begin{pmatrix} 1 \\ z^i \end{pmatrix}, \quad z^i = x^i + iy^i. \tag{A.2}$$

From this we obtain that the dual sections $F_\Lambda = \partial_\Lambda F$ are

$$F_\Lambda = \begin{pmatrix} d_{ijk} z^i z^j z^k \\ -3d_{z,i} \end{pmatrix} \quad (\text{A.3})$$

and the Kähler potential is

$$e^{-K} = 8d_{ijk} y^i y^j y^k \quad (\text{A.4})$$

so that the moduli space is constrained by $y^i > 0$, $i = 1, 2, 3$. The symplectic form is given by

$$\Omega = \begin{pmatrix} 0 & -\mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}. \quad (\text{A.5})$$

We use the following shorthand for contraction with the symmetric tensors d_{ijk} and \widehat{d}^{ijk} of any component g_i and g^i taken from the matter couplings in any symplectic vector (g^0, g^i, g_0, g_i) :

$$\begin{aligned} d_g &= d_{ijk} g^i g^j g^k, & d_{g,i} &= d_{ijk} g^j g^k, & d_{g,ij} &= d_{ijk} g^k, \\ \widehat{d}_g &= \widehat{d}^{ijk} g_i g_j g_k, & \widehat{d}_g^i &= \widehat{d}^{ijk} g_j g_k, & \widehat{d}_g^{ij} &= \widehat{d}^{ijk} g_k. \end{aligned} \quad (\text{A.6})$$

For homogeneous spaces the matrix $d_{g,ij}$ is invertible. The results of this paper do not need the explicit form of its inverse: we simply write $(d_g^{-1})^{ij}$ for the matrix that satisfies

$$(d_g^{-1})^{ij} d_{g,jk} = \delta_k^i. \quad (\text{A.7})$$

The metric on \mathcal{M}_v is given by

$$g_{ij} = -\frac{3}{2} \frac{d_{y,ij}}{d_y} + \frac{9}{4} \frac{d_{y,i} d_{y,j}}{d_y^2} \quad (\text{A.8})$$

and its inverse by

$$g^{ij} = -\frac{2}{3} d_y (d_y^{-1})^{ij} + 2y^i y^j. \quad (\text{A.9})$$

The following matrix is used in the presentation of the BPS equations

$$\mathcal{M} = \begin{pmatrix} 1 & -\mathcal{R} \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \mathcal{I} & 0 \\ 0 & \mathcal{I}^{-1} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\mathcal{R} & 1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad (\text{A.10})$$

with

$$A = \mathcal{I} + \mathcal{R}\mathcal{I}^{-1}\mathcal{R}, \quad D = \mathcal{I}^{-1}, \quad B = C^T = -\mathcal{R}\mathcal{I}^{-1} \quad (\text{A.11})$$

and where

$$\mathcal{N}_{\Lambda\Sigma} = \mathcal{R}_{\Lambda\Sigma} + i\mathcal{I}_{\Lambda\Sigma} \quad (\text{A.12})$$

is the symplectic matrix such that

$$M_\Lambda = \mathcal{N}_{\Lambda\Sigma} L^\Sigma. \quad (\text{A.13})$$

In addition $(\mathcal{R}, \mathcal{I})$ give the vector kinetic and topological terms in the Lagrangian (2.1). One can quite easily check that \mathcal{M} satisfies the identity

$$\Omega \mathcal{M} \mathcal{V} = -i\mathcal{V}. \quad (\text{A.14})$$

B Homogeneous very special Kähler spaces

For a homogeneous very special Kähler geometry we have the constant tensor

$$\widehat{d}^{ijk} = \frac{g^{il}g^{jm}g^{kn}d_{lmn}}{d_y^2} \tag{B.1}$$

which satisfies the relations

$$\widehat{d}^{ijk}d_{j(lm}d_{np)k} = \frac{64}{27}\delta_{(l}^i d_{mnp)}, \tag{B.2}$$

$$d_{ijk}\widehat{d}^{j(lm}\widehat{d}^{np)k} = \frac{64}{27}\delta_i^{(l}\widehat{d}^{mnp)}. \tag{B.3}$$

One can re-write (B.2) and (B.3) in an equivalent manner

$$\widehat{d}^{ijk}d_{j(lm}d_{np)k} = \frac{16}{27}\left[\delta_l^j d_{mnp} + 3\delta_{(m}^i d_{np)l}\right], \tag{B.4}$$

$$d_{ijk}d^{j(lm}d^{np)k} = \frac{16}{27}\left[\delta_i^l \widehat{d}^{mnp} + 3\delta_i^{(m} \widehat{d}^{np)l}\right], \tag{B.5}$$

and easily check that both sides are symmetric in all four indices $(lmnp)$. This allows one to easily establish the veracity of the following expressions

$$(d_g^{-1})^{ij} = \frac{1}{d_g}\left[\frac{27}{16}\widehat{d}^{ijk}d_{g,k} - 3g^i g^j\right], \tag{B.6}$$

$$(\widehat{d}_g^{-1})^{ij} = \frac{1}{\widehat{d}_g}\left[\frac{27}{16}d_{ijk}\widehat{d}_g^k - 3g_i g_j\right] \tag{B.7}$$

which are used several times in this article. Using this we can solve the following equation which often appears in our work

$$F^i = \widehat{d}^{ijk}G_j G_k \quad \Rightarrow \quad G_i = \pm\sqrt{\frac{27}{64}\frac{d_{ijk}F^j F^k}{\sqrt{d_F}}}, \tag{B.8}$$

$$G_i = d_{ijk}F^j F^k \quad \Rightarrow \quad F^i = \pm\sqrt{\frac{27}{64}\frac{\widehat{d}^{ijk}G_j G_k}{\sqrt{\widehat{d}_G}}}. \tag{B.9}$$

One can also use \widehat{d}^{ijk} to express the complex scalar fields in terms of the sections

$$z^i = \frac{3}{8}\frac{d_y}{d_z}\widehat{d}^{ijk}M_j M_k \tag{B.10}$$

The quadratic and quartic invariants are given by

$$\mathcal{I}_2(a^\Lambda, b_\Lambda) = -\frac{d_{ijk}a^i a^j a^k}{a^0} - \frac{1}{2}a^\Lambda b_\Lambda \tag{B.11}$$

$$\mathcal{I}_4(a^\Lambda, b_\Lambda) = -(a^\Lambda b_\Lambda)^2 + \frac{1}{16}a^0 \widehat{d}^{ijk}b_i b_j b_k - 4b_0 d_{ijk}a^i a^j a^k + \frac{9}{16}d_{ijk}\widehat{d}^{ilm}a^j a^k b_l b_m \tag{B.12}$$

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