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The T_7 flavor symmetry in 3-3-1 model with neutral leptons

V.V. Vien[a] and H.N. Long[b]

[a] *Department of Physics, Tay Nguyen University, 567 Le Duan, Buon Ma Thuot, Vietnam*
[b] *Institute of Physics, VAST, 10 Dao Tan, Ba Dinh, Hanoi, Vietnam*

E-mail: wvienk16@gmail.com, hnlong@iop.vast.ac.vn

Abstract: We construct a 3-3-1 model based on non-Abelian discrete symmetry T_7 responsible for the fermion masses. Neutrinos get masses from only anti-sextets which are in triplets $\underline{3}$ and $\underline{3}^*$ under T_7. The flavor mixing patterns and mass splitting are obtained without perturbation. The tribimaximal form obtained with the breaking $T_7 \to Z_3$ in charged lepton sector and both $T_7 \to Z_3$ and $Z_3 \to \{\text{Identity}\}$ must be taken place in neutrino sector but only apart in breakings $Z_3 \to \{\text{Identity}\}$ (without contribution of σ'), and the upper bound on neutrino mass $\sum_{i=1}^{3} m_i$ at the level is presented. The Dirac CP violation phase δ is predicted to either $\frac{\pi}{2}$ or $\frac{3\pi}{2}$ which is maximal CP violation. From the Dirac CP violation phase we obtain the relation between Euler's angles which is consistent with the experimental in PDG 2012. On the other hand, the realistic lepton mixing can be obtained if both the direction for breakings $T_7 \to Z_3$ and $Z_3 \to \{\text{Identity}\}$ are taken place in neutrino sectors. The CKM matrix is the identity matrix at the tree-level.

Keywords: Beyond Standard Model, Nonperturbative Effects, CP violation, Discrete and Finite Symmetries

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1 Introduction

Despite the great success of the Standard Model (SM) of the elementary particle physics, the origin of flavor structure, masses and mixings between generations of matter particles are unknown yet. The neutrino mass and mixing is one of the most important evidence of beyond Standard Model physics. Many experiments show that neutrinos have tiny masses and their mixing is sill mysterious [1, 2].

The tri-bimaximal form for explaining the lepton mixing scheme was first proposed by Harrison-Perkins-Scott (HPS), which apart from the phase redefinitions, is given by [3–6]

$$U_{\text{HPS}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \tag{1.1}$$

can be considered as a good approximation for the recent neutrino experimental data.

The most recent data are a clear sign of rather large value θ_{13} [7] . The data in PDG2012 [8–12] imply:

$$\sin^2(2\theta_{12}) = 0.857 \pm 0.024, \qquad \sin^2(2\theta_{13}) = 0.098 \pm 0.013,\ \sin^2(2\theta_{23}) > 0.95,$$
$$\Delta m^2_{21} = (7.50 \pm 0.20) \times 10^{-5}\mathrm{eV}^2, \qquad \Delta m^2_{32} = (2.32^{+0.12}_{-0.08}) \times 10^{-3}\mathrm{eV}^2. \tag{1.2}$$

These large neutrino mixing angles are completely different from the quark mixing ones defined by the Cabibbo- Kobayashi-Maskawa (CKM) matrix [13, 14] . This has stimulated work on flavor symmetries and non-Abelian discrete symmetries are considered to be the most attractive candidate to formulate dynamical principles that can lead to the flavor mixing patterns for quarks and lepton. There are many recent models based on the non-Abelian discrete symmetries, such as A_4 [15–32, 34], A_5 [35–47], S_3 [48–87], S_4 [88–116], D_4 [117–128], D_5 [129, 130], T' [131–140], T_7 [141–147] and so forth.

Among the possible extensions of SM, a curious choice are the 3-3-1 models which encompass a class of models based on the gauge group $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ [148–163], that is at first spontaneously broken to the SM group $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ and then undergoes the spontaneously broken to $\mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Q$. The extension of the gauge group with respect to SM leads to interesting consequences. The first one is that the requirement of anomaly cancelation together with that of asymptotic freedom of QCD implies that the number of generations must necessarily be equal to the number of colors, hence giving an explanation for the existence of three generations. Furthermore, quark generations should transform differently under the action of $\mathrm{SU}(3)_L$. In particular, two quark generations should transform as triplets, one as an antitriplet.

A fundamental relation holds among some of the generators of the group [146, 147]:

$$Q = T_3 + \beta T_8 + X \tag{1.3}$$

where Q indicates the electric charge, T_3 and T_8 are two of the SU(3) generators and X is the generator of $\mathrm{U}(1)_X$. β is a key parameter that defines a specific variant of the model.

The model thus provides a partial explanation of the family number, as also required by flavor symmetries such as T_7 for 3-dimensional representations. In addition, due to the anomaly cancelation one family of quarks has to transform under $\mathrm{SU}(3)_L$ differently from the two others. T_7 can meet this requirement with three inequivalent representations $\underline{1}, \underline{1}', \underline{1}''$. Note that T_7 has not been considered before in the kind of the 3-3-1 model.

There are two typical variants of the 3-3-1 models as far as lepton sectors are concerned. In the minimal version, three $\mathrm{SU}(3)_L$ lepton triplets are (ν_L, l_L, l^c_R), where l_R are ordinary right-handed charged-leptons [148–152] . In the second version, the third components of lepton triplets are the right-handed neutrinos, (ν_L, l_L, ν^c_R) [152–157]. To have a model with the realistic neutrino mixing matrix, we should consider another variant of the form (ν_L, l_L, N^c_R) where N_R are three new fermion singlets under standard model symmetry with vanishing lepton-numbers [164–167].

In our previous works [164–167] , the discrete symmetries have been explored to the 3-3-1 models. In ref. [165] we have studied the 3-3-1 model with neutral fermions based on S_4 group, in which most of the Higgs multiplets are in triplets under S_4 except χ lying in

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a singlet,[1] and the exact tribimaximal form [3–6] is obtained, where $\theta_{13} = 0$. As we know, the recent considerations have implied $\theta_{13} \neq 0$ [15–32, 48–116], but small as given in (1.2). This problem has been improved in this type of the model in ref. [166, 167] by adding new $\mathrm{SU}(3)_L$ multiplets and one of them is regarded as a small perturbation. The model therefore contains up to eight Higgs multiplets, and the scalar potential of the model is quite complicated [166, 167].

CP violation plays a crucial role in our understanding of the observed baryon asymmetry of the Universe [168]. In the SM, CP symmetry is violated due to a complex phase in the CKM matrix [13, 14]. However, since the extent of CP violation in the SM is not enough for achieving the observed BAU, we need new source of CP violation for successful BAU. On the other hand, CP violations in the lepton sector are imperative if the BAU could be realized through leptogenesis. So, any hint or observation of the leptonic CP violation can strengthen our belief in leptogenesis [168].

The violation of the CP symmetry is a crucial ingredient of any dynamical mechanism which intends to explain both low energy CP violation and the baryon asymmetry. Renormalizable gauge theories are based on the spontaneous symmetry breaking mechanism, and it is natural to have the spontaneous CP violation as an integral part of that mechanism. Determining all possible sources of CP violation is a fundamental challenge for high energy physics. In theoretical and economical viewpoints, the spontaneous CP breaking necessary to generate the baryon asymmetry and leptonic CP violation at low energies brings us to a common source which comes from the phase of the scalar field responsible for the spontaneous CP breaking at a high energy scale [168].

In this paper, we investigate another choice with T_7, the smallest group with two non-equivalent 3-dimensional irreducible representations, contains two triplet irreducible representations and three singlets which play a crucial role in consistently reproducing fermion masses and mixing. As we will see, T_7 model has some new features since fewer Higgs multiplets are needed in order to allow the fermions to gain masses and to break symmetries and the physics we will see is different from the former. The CP violation is the first time considered under $\mathrm{SU}(3)_L \times \mathrm{U}(1)_X$ model based on T_7 flavor symmetry in which the T_7 symmetry avoids the mass degeneracy of lepton masses. The light neutrino masses can be generated at tree level, and the vacuum alignment problem which arises in the presence of two T_7-triplet scalar fields $\underline{3}, \underline{3}^*$ can naturally explain the measured value of θ_{13} and thereby the hierarchy of neutrino masses. The seesaw mechanism can explain the smallness of the measured neutrino masses and the maximal Dirac CP violation.

The rest of this work is organized as follows. In section 2 and 3 we present the necessary elements of the 3-3-1 model with the T_7 symmetry as well as introducing necessary Higgs fields responsible for the charged lepton masses. In section 4, we discuss on quark sector. Section 5 is devoted for the neutrino mass and mixing. We summarize our results and make conclusions in the section 7. Appendix A presents a brief of the T_7 theory. Appendix B provides the lepton number (L) and lepton parity (P_l) of particles in the model.

[1] χ is the unique singlet under S_4 defined in expression (25) in ref. [165].

2 Fermion content

The gauge symmetry is based on $\mathrm{SU}(3)_C \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$, where the electroweak factor $\mathrm{SU}(3)_L \otimes \mathrm{U}(1)_X$ is extended from those of the Standard Model (SM), and the strong interaction sector is retained. Each lepton family includes a new neutral fermion (N_R) with vanishing lepton number $L(N_R) = 0$ arranged under the $\mathrm{SU}(3)_L$ symmetry as a triplet (ν_L, l_L, N^c_R) and a singlet l_R. The residual electric charge operator Q is therefore related to the generators of the gauge symmetry by

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X, \tag{2.1}$$

where T_a $(a = 1, 2, \ldots, 8)$ are $\mathrm{SU}(3)_L$ charges with $\mathrm{Tr} T_a T_b = \frac{1}{2}\delta_{ab}$ and X is the $\mathrm{U}(1)_X$ charge. This means that the model under consideration does not contain exotic electric charges in the fundamental fermion, scalar and adjoint gauge boson representations.

Since the particles in the lepton triplet have different lepton number (1 and 0), so the lepton number in the model does not commute with the gauge symmetry unlike the SM. Therefore, it is better to work with a new conserved charge $\mathcal{L}$ commuting with the gauge symmetry and related to the ordinary lepton number by diagonal matrices [164–167, 169]

$$L = \frac{2}{\sqrt{3}} T_8 + \mathcal{L}. \tag{2.2}$$

The lepton charge arranged in this way, i.e. $L(N_R) = 0$, is in order to prevent unwanted interactions due to $\mathrm{U}(1)_\mathcal{L}$ symmetry and breaking due to the lepton parity to obtain the consistent lepton and quark spectra. By this embedding, exotic quarks U, D as well as new non-Hermitian gauge bosons X^0, $Y^\pm$ possess lepton charges as of the ordinary leptons: $L(D) = -L(U) = L(X^0) = L(Y^-) = 1$.

Under the $[\mathrm{SU}(3)_L, \mathrm{U}(1)_X, \mathrm{U}(1)_\mathcal{L}, \underline{T}_7]$ symmetries as proposed, the fermions of the model transform as follows

$$\begin{aligned}
&\psi_L \equiv \psi_{1,2,3L} = (\nu_L \quad l_L \quad N^c_R)^T \sim [3, -1/3, 2/3, \underline{3}],\\
&l_{1R} \sim [1, -1, 1, \underline{1}], \qquad l_{2R} \sim [1, -1, 1, \underline{1}'], \qquad l_{3R} \sim [1, -1, 1, \underline{1}''],\\
&Q_{1L} \equiv (d_{1L} \quad -u_{1L} \quad D_{1L})^T \sim [3^*, 0, 1/3, \underline{1}'],\\
&Q_{2L} \equiv (d_{2L} \quad -u_{2L} \quad D_{2L})^T \sim [3^*, 0, 1/3, \underline{1}''],\\
&Q_{3L} = (u_{3L} \quad d_{3L} \quad U_L)^T \sim [3, 1/3, -1/3, \underline{1}],\\
&u_R \sim u_{1,2,3R} = [1, 2/3, 0, \underline{3}], \qquad d_R \sim [1, -1/3, 0, \underline{3}^*],\\
&U_R \sim [1, 2/3, -1, \underline{1}], \qquad D_{1R} \sim [1, -1/3, 1, \underline{1}''], \qquad D_{2R} \sim [1, -1/3, 1, \underline{1}'].
\end{aligned} \tag{2.3}$$

where the subscript numbers on field indicate to respective families which also in order define components of their T_7 multiplets. U and $D_{1,2}$ are exotic quarks carrying lepton numbers $L(U) = -1$ and $L(D_{1,2}) = 1$, known as leptoquarks. In the following, we consider possibilities of generating the masses for the fermions. The scalar multiplets needed for the purpose are also introduced.

3 Charged lepton masses

The charged lepton masses arise from the couplings of $\bar{\psi}_L l_{1R}, \bar{\psi}_L l_{2R}$ and $\bar{\psi}_L l_{3R}$ to scalars, where $\bar{\psi}_L l_{iL}\,(i = 1,2,3)$ transforms as 3^* under $\mathrm{SU}(3)_L$ and $\underline{3}^*$ under T_7. To generate masses for charged leptons, we need a $\mathrm{SU}(3)_L$ Higgs triplets that lying in $\underline{3}$ under T_7 and transforms as 3 under $\mathrm{SU}(3)_L$,

$$\phi_i = \begin{pmatrix} \phi^+_{i1} \\ \phi^0_{i2} \\ \phi^+_{i3} \end{pmatrix} \sim [3, 2/3, -1/3, \underline{3}] \quad (i = 1,2,3). \tag{3.1}$$

Following the potential minimization conditions, we have the followings alignments:

(1) The first alignment: $\langle\phi_1\rangle = \langle\phi_2\rangle = \langle\phi_3\rangle$ then T_7 is broken into Z_3 consisting of the elements $\{e, b, b^2\}$.

(2) The second alignment: $\langle\phi_1\rangle \neq \langle\phi_2\rangle \neq \langle\phi_3\rangle$ or $\langle\phi_1\rangle \neq \langle\phi_2\rangle = \langle\phi_3\rangle$ or $\langle\phi_2\rangle \neq \langle\phi_1\rangle \neq \langle\phi_3\rangle$ or $\langle\phi_3\rangle \neq \langle\phi_1\rangle \neq \langle\phi_2\rangle$ then T_7 is broken into {Identity}.

(3) The third alignment: $0 = \langle\phi_1\rangle \neq \langle\phi_2\rangle = \langle\phi_3\rangle \neq 0$ or $0 = \langle\phi_2\rangle \neq \langle\phi_3\rangle = \langle\phi_1\rangle \neq 0$ or $0 = \langle\phi_3\rangle \neq \langle\phi_1\rangle = \langle\phi_2\rangle \neq 0$ then T_7 is broken into {Identity}.

(4) The fourth alignment: $0 = \langle\phi_1\rangle \neq \langle\phi_2\rangle \neq \langle\phi_3\rangle \neq 0$ or $0 = \langle\phi_2\rangle \neq \langle\phi_1\rangle \neq \langle\phi_3\rangle \neq 0$ or $0 = \langle\phi_3\rangle \neq \langle\phi_2\rangle \neq \langle\phi_1\rangle \neq 0$ then T_7 is broken into {Identity}.

(5) The fifth alignment: $0 = \langle\phi_1\rangle = \langle\phi_2\rangle \neq \langle\phi_3\rangle \neq 0$ or $0 = \langle\phi_1\rangle = \langle\phi_3\rangle \neq \langle\phi_2\rangle \neq 0$ or $0 = \langle\phi_2\rangle = \langle\phi_3\rangle \neq \langle\phi_1\rangle \neq 0$ then T_7 is broken into {Identity}.

In this work, we argue that only the first alignment of VEV in charged - lepton sector is taken place, i.e, $T_7 \to Z_3$, and this can be achieved by the Higgs triplet ϕ with the VEV alignment $\langle\phi\rangle = (\langle\phi_1\rangle, \langle\phi_1\rangle, \langle\phi_1\rangle)$ under T_7, where

$$\langle\phi_1\rangle = (0 \quad v \quad 0)^T. \tag{3.2}$$

The Yukawa interactions are

$$\begin{aligned} -\mathcal{L}_l &= h_1(\bar{\psi}_L\phi)_{\underline{1}} l_{1R} + h_2(\bar{\psi}_L\phi)_{\underline{1}''} l_{2R} + h_3(\bar{\psi}_{iL}\phi)_{\underline{1}'} l_{3R} + \text{H.c.} \\ &= h_1(\bar{\psi}_{1L}\phi_1 + \bar{\psi}_{2L}\phi_2 + \bar{\psi}_{3L}\phi_3) l_{1R} \\ &\quad + h_2(\bar{\psi}_{1L}\phi_1 + \omega^2\bar{\psi}_{2L}\phi_2 + \omega\bar{\psi}_{3L}\phi_3) l_{2R} \\ &\quad + h_3(\bar{\psi}_{1L}\phi_1 + \omega\bar{\psi}_{2L}\phi_2 + \omega^2\bar{\psi}_{3L}\phi_3) l_{3R} + \text{H.c.} \end{aligned} \tag{3.3}$$

The mass Lagrangian for the charged leptons is then given by

$$\begin{aligned} -\mathcal{L}_l^{\text{mass}} &= h_1 v \bar{l}_{1L} l_{1R} + h_2 v \bar{l}_{1L} l_{2R} + h_3 v \bar{l}_{1L} l_{3R} \\ &\quad + h_1 v \bar{l}_{2L} l_{1R} + h_2 v\omega^2 \bar{l}_{2L} l_{2R} + h_3 v\omega \bar{l}_{2L} l_{3R} \\ &\quad + h_1 v \bar{l}_{3L} l_{1R} + h_2 v\omega \bar{l}_{3L} l_{2R} + h_3 v\omega^2 \bar{l}_{3L} l_{3R} + \text{H.c.} \end{aligned} \tag{3.4}$$

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The mass Lagrangian for the charged leptons reads

$$-\mathcal{L}_l^{\text{mass}} = (\bar{l}_{1L}, \bar{l}_{2L}, \bar{l}_{3L}) M_l (l_{1R}, l_{2R}, l_{3R})^T + H.c, \tag{3.5}$$

where

$$M_l = \begin{pmatrix} h_1 v & h_2 v & h_3 v \\ h_1 v & h_2 v\omega^2 & h_3 v\omega \\ h_1 v & h_2 v\omega & h_3 v\omega^2 \end{pmatrix}. \tag{3.6}$$

This matrix can be diagonalized as,

$$U_L^\dagger M_l U_R = \begin{pmatrix} \sqrt{3}h_1 v & 0 & 0 \\ 0 & \sqrt{3}h_2 v & 0 \\ 0 & 0 & \sqrt{3}h_3 v \end{pmatrix} \equiv \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \tag{3.7}$$

where

$$U_L = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \qquad U_R = 1. \tag{3.8}$$

As will see in section 5, in this case, the exact tribimaximal mixing form is obtained, by choosing the right vev's in the neutrino sector.

The experimental values for masses of the charged leptons at the weak scale are given as [8]:

$$m_e = 0.511\,\text{MeV}, \qquad m_\mu = 105.658\ \text{MeV}, \qquad m_\tau = 1776.82\,\text{MeV} \tag{3.9}$$

from which it follows that $h_1 \ll h_2 \ll h_3$. On the other hand, if we choose the VEV $v = 100\,GeV$ then $h_1 \sim 10^{-6}$, $h_2 \sim 10^{-4}$, $h_3 \sim 10^{-2}$.

4 Quark masses

To generate masses for quarks with a minimal Higgs content, we additionally introduce the following Higgs triplets:

$$\eta_i = \begin{pmatrix} \eta_{i1}^0 \\ \eta_{i2}^- \\ \eta_{i3}^0 \end{pmatrix} \sim [3, -1/3, -1/3, \underline{3}] \qquad (i = 1, 2, 3), \tag{4.1}$$

$$\chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \chi_3^0 \end{pmatrix} \sim [3, -1/3, 2/3, \underline{1}]\,. \tag{4.2}$$

The Higgs content and Yukawa couplings in the quark sector are summarized in table 1.

The Yukawa interactions are

$$\begin{aligned} -\mathcal{L}_q = {} & h_3^d \bar{Q}_{3L}(\phi d_R)_1 + h_1^u \bar{Q}_{1L}(\phi^* u_R)_{1''} + h_2^u \bar{Q}_{2L}(\phi^* u_R)_{1'} \\ & + h_3^u \bar{Q}_{3L}(\eta u_R)_1 + h_1^d \bar{Q}_{1L}(\eta^* d_R)_{1''} + h_2^d \bar{Q}_{2L}(\eta^* d_R)_{1'} \\ & + f_3 \bar{Q}_{3L}\chi U_R + f_1 \bar{Q}_{1L}\chi^* D_{1R} + f_2 \bar{Q}_{2L}\chi^* D_{2R} + \text{H.c.} \end{aligned}$$

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Couplings	Higgs multiplets
$\bar{Q}_{3L}U_R \sim \left(3^*, \frac{1}{3}, -\frac{2}{3}, \underline{1}\right)$	$\chi \sim \left(3, -\frac{1}{3}, \frac{2}{3}, \underline{1}\right)$
$\bar{Q}_{2L}D_{2R} \sim \left(3, -\frac{1}{3}, \frac{2}{3}, \underline{1}\right)$	$\chi^* \sim \left(3^*, \frac{1}{3}, -\frac{2}{3}, \underline{1}\right)$
$\bar{Q}_{1L}D_{1R} \sim \left(3, -\frac{1}{3}, \frac{2}{3}, \underline{1}\right)$	$\chi^* \sim \left(3^*, \frac{1}{3}, -\frac{2}{3}, \underline{1}\right)$
$\bar{Q}_{3L}d_R \sim \left(3^*, -\frac{2}{3}, \frac{1}{3}, \underline{3}^*\right)$	$\phi \sim \left(3, \frac{2}{3}, -\frac{1}{3}, \underline{3}\right)$
$\bar{Q}_{1L}u_R \sim \left(3, \frac{2}{3}, -\frac{1}{3}, \underline{3}\right)$	$\phi^* \sim \left(3^*, -\frac{2}{3}, \frac{1}{3}, \underline{3}^*\right)$
$\bar{Q}_{2L}u_R \sim \left(3, \frac{2}{3}, -\frac{1}{3}, \underline{3}\right)$	$\phi^* \sim \left(3^*, -\frac{2}{3}, \frac{1}{3}, \underline{3}^*\right)$
$\bar{Q}_{3L}u_R \sim \left(3^*, \frac{1}{3}, \frac{1}{3}, \underline{3}\right)$	$\eta \sim \left(3, -\frac{1}{3}, -\frac{1}{3}, \underline{3}^*\right)$
$\bar{Q}_{1L}d_R \sim \left(3, -\frac{1}{3}, -\frac{1}{3}, \underline{3}^*\right)$	$\eta^* \sim \left(3^*, \frac{1}{3}, \frac{1}{3}, \underline{3}\right)$
$\bar{Q}_{2L}d_R \sim \left(3, -\frac{1}{3}, -\frac{1}{3}, \underline{3}^*\right)$	$\eta^* \sim \left(3^*, \frac{1}{3}, \frac{1}{3}, \underline{3}\right)$

Table 1. List of couplings which form a singlet from the invariance under the T_7.

$$\begin{aligned}
&= h_3^d\bar{Q}_{3L}(\phi_1 d_{1R} + \phi_2 d_{2R} + \phi_3 d_{3R}) \\
&\quad + h_1^u\bar{Q}_{1L}(\phi_1^* u_R + \omega^2\phi_2^* u_{2R} + \omega\phi_3^* u_{3R}) \\
&\quad + h_2^u\bar{Q}_{2L}(\phi_1^* u_R + \omega\phi_2^* u_{2R} + \omega^2\phi_3^* u_{3R}) \\
&\quad + h_3^u\bar{Q}_{3L}(\eta_1 u_{1R} + \eta_2 u_{2R} + \eta_3 u_{3R}) \\
&\quad + h_1^d\bar{Q}_{1L}(\eta_1^* d_{1R} + \omega^2\eta_2^* d_{2R} + \omega\eta_3^* d_{3R}) \\
&\quad + h_2^d\bar{Q}_{2L}(\eta_1^* d_{1R} + \omega\eta_2^* d_{2R} + \omega^2\eta_3^* d_{3R}) \\
&\quad + f_3\bar{Q}_{3L}\chi U_R + f_1\bar{Q}_{1L}\chi^* D_{1R} + f_2\bar{Q}_{2L}\chi^* D_{2R} + \text{H.c.}
\end{aligned} \tag{4.3}$$

We suppose that T_7 is broken into Z_3 like the case of the charged lepton sector, i,e, the VEVs of η and χ are given as $\langle\eta\rangle = (\langle\eta_1\rangle, \langle\eta_1\rangle, \langle\eta_1\rangle)$ with

$$\langle\eta_1\rangle = (u \quad 0 \quad 0)^T, \tag{4.4}$$

and

$$\langle\chi\rangle = (0 \quad 0 \quad v_\chi)^T. \tag{4.5}$$

The mass Lagrangian for quarks is given by

$$\begin{aligned}
-\mathcal{L}_q^{\text{mass}} &= -h_1^u v_1\bar{u}_{1L}u_{1R} - h_1^u v_2\omega^2\bar{u}_{1L}u_{2R} - h_1^u v_3\omega\bar{u}_{1L}u_{3R} \\
&\quad - h_2^u v_1\bar{u}_{2L}u_{1R} - h_2^u v_2\omega\bar{u}_{2L}u_{2R} - h_2^u v_3\omega^2\bar{u}_{2L}u_{3R} \\
&\quad + h_3^u u_1\bar{u}_{3L}u_{1R} + h_3^u u_2\bar{u}_{3L}u_{2R} + h_3^u u_3\bar{u}_{3L}u_{3R} \\
&\quad + h_1^d u_1\bar{d}_{1L}d_{1R} + \omega^2 h_1^d u_2\bar{d}_{1L}d_{2R} + \omega h_1^d u_3\bar{d}_{1L}d_{3R} \\
&\quad + h_2^d u_1\bar{d}_{2L}d_{1R} + \omega h_2^d u_2\bar{d}_{2L}d_{2R} + \omega^2 h_2^d u_3\bar{d}_{2L}d_{3R} \\
&\quad + h_3^d v_1\bar{d}_{3L}d_{1R} + h_3^d v_2\bar{d}_{3L}d_{2R} + h_3^d v_3\bar{d}_{3L}d_{3R} \\
&\quad + f_3 v_\chi\bar{U}_L U_R + f_1 v_\chi\bar{D}_{1L}D_{1R} + f_2 v_\chi\bar{D}_{2L}D_{2R} + \text{H.c.}
\end{aligned} \tag{4.6}$$

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The exotic quarks get masses

$$m_U = f_3 v_\chi, \quad m_{D_{1,2}} = f_{1,2} v_\chi. \tag{4.7}$$

The mass matrices for ordinary up-quarks and down-quarks are, respectively, obtained as follows:

$$M_u = \begin{pmatrix} -h_1^u v & -h_1^u v\omega^2 & -h_1^u v\omega \\ -h_2^u v & -h_2^u v\omega & -h_2^u v\omega^2 \\ h_3^u u & h_3^u u & h_3^u u \end{pmatrix}, \quad M_d = \begin{pmatrix} h_1^d u & h_1^d u\omega^2 & h_1^d u\omega \\ h_2^d u & h_2^d u\omega & h_2^d u\omega^2 \\ h_3^d v & h_3^d v & h_3^d v \end{pmatrix}. \tag{4.8}$$

The structure of the up- and down-quark mass matrices in (4.8) is similar to those in [33], i.e, in the model under consideration there is no CP violation in the quark mixing matrix. The mass matrices M_u, M_d in (4.8) are diagonalized as follows

$$\begin{aligned} U_L^{u+} M_u U_R^u &= \begin{pmatrix} -\sqrt{3}h_1^u v & 0 & 0 \\ 0 & -\sqrt{3}h_2^u v & 0 \\ 0 & 0 & \sqrt{3}h_3^u u \end{pmatrix} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \\ U_L^{d+} M_d U_R^d &= \begin{pmatrix} \sqrt{3}h_1^d u & 0 & 0 \\ 0 & \sqrt{3}h_2^d u & 0 \\ 0 & 0 & \sqrt{3}h_3^d v \end{pmatrix} = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}. \end{aligned} \tag{4.9}$$

where

$$U_R^u = U_R^d = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{pmatrix}, \quad U_L^u = U_L^d = 1 \tag{4.10}$$

The unitary matrices, which couple the left-handed up- and down-quarks to those in the mass bases are unit matrices. Therefore we get the Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$U_{\mathrm{CKM}} = U_L^{d\dagger} U_L^u = 1. \tag{4.11}$$

Note that the property in (4.11) is common for some models based on the discrete symmetry groups [164, 165].

The up and down quark masses are

$$\begin{aligned} m_u &= -\sqrt{3}h_1^u v, & m_c &= -\sqrt{3}h_2^u v, & m_t &= \sqrt{3}h_3^u u, \\ m_d &= \sqrt{3}h_1^d u, & m_s &= \sqrt{3}h_2^d u, & m_d &= \sqrt{3}h_3^d v. \end{aligned} \tag{4.12}$$

The current mass values for the quarks are given by [8]:

$$\begin{aligned} m_u &= 2.3^{+0.7}_{-0.5}\ \text{MeV}, & m_c &= 1.275 \pm 0.025\ \text{GeV}, & m_t &= 173.5 \pm 0.6 \pm 0.8,\ \text{GeV} \\ m_d &= 4.8^{+0.7}_{-0.3}\ \text{MeV}, & m_s &= 95 \pm 5\ \text{MeV}, & m_b &= 4.18 \pm 0.03\ \text{GeV}. \end{aligned} \tag{4.13}$$

It is obvious that if $|u| \sim |v|$, the Yukawa coupling hierarchies are $|h_1^u| \ll h_2^u \ll h_3^u$, $h_1^d \ll h_2^d \ll h_3^d$ and the couplings between up-quarks (h_2^u, h_3^u) and Higgs scalar multiplets are slightly heavier than those of down-quarks $(h_2^d \ll h_3^d)$, respectively.

5 Neutrino mass and mixing

The neutrino masses arise from the couplings of $\bar{\psi}^c_L \psi_L$ to scalars, where $\bar{\psi}^c_L \psi_L$ transforms as $3^* \oplus 6$ under $\mathrm{SU}(3)_L$ and $\underline{3} \oplus \underline{3}^* \oplus \underline{3}^*$ under T_7. It is worth mentioning that, with the T_7 group, $\underline{3} \times \underline{3} \times \underline{3}$ has two invariants and $\underline{3} \times \underline{3} \times \underline{3}^*$ has one invariant. For the known scalar triplets (ϕ, χ, η), there is no available interaction because of the $\mathcal{L}$-symmetry. We will therefore propose new $\mathrm{SU}(3)_L$ anti-sextets instead coupling to $\bar{\psi}^c_L \psi_L$ responsible for the neutrino masses. To obtain a realistic neutrino spectrum, the antisextets transform as follows[2]

$$\sigma_i = \begin{pmatrix} \sigma^0_{11} & \sigma^+_{12} & \sigma^0_{13} \\ \sigma^+_{12} & \sigma^{++}_{22} & \sigma^+_{23} \\ \sigma^0_{13} & \sigma^+_{23} & \sigma^0_{33} \end{pmatrix}_i \sim [6^*, 2/3, -4/3, \underline{3}^*] \quad (i = 1, 2, 3), \tag{5.1}$$

Following the potential minimization conditions, we have the followings alignments:

(1) The first alignment: $\langle \sigma_1 \rangle = \langle \sigma_2 \rangle = \langle \sigma_3 \rangle$ then T_7 is broken into Z_3 consisting of the elements $\{e, b, b^2\}$.

(2) The second alignment: $\langle \sigma_1 \rangle \neq \langle \sigma_2 \rangle \neq \langle \sigma_3 \rangle$ or $\langle \sigma_1 \rangle \neq \langle \sigma_2 \rangle = \langle \sigma_3 \rangle$ or $\langle \sigma_2 \rangle \neq \langle \sigma_1 \rangle \neq \langle \sigma_3 \rangle$ or $\langle \sigma_3 \rangle \neq \langle \sigma_1 \rangle \neq \langle \sigma_2 \rangle$ then T_7 is broken into {Identity}.

(3) The third alignment: $0 = \langle \sigma_1 \rangle \neq \langle \sigma_2 \rangle = \langle \sigma_3 \rangle \neq 0$ or $0 = \langle \sigma_2 \rangle \neq \langle \sigma_3 \rangle = \langle \sigma_1 \rangle \neq 0$ or $0 = \langle \sigma_3 \rangle \neq \langle \sigma_1 \rangle = \langle \sigma_2 \rangle \neq 0$ then T_7 is broken into {Identity}.

(4) The fourth alignment: $0 = \langle \sigma_1 \rangle \neq \langle \sigma_2 \rangle \neq \langle \sigma_3 \rangle \neq 0$ or $0 = \langle \sigma_2 \rangle \neq \langle \sigma_1 \rangle \neq \langle \sigma_3 \rangle \neq 0$ or $0 = \langle \sigma_3 \rangle \neq \langle \sigma_1 \rangle \neq \langle \sigma_2 \rangle \neq 0$ then T' is broken into {Identity}.

(5) The fifth alignment: $0 = \langle \sigma_1 \rangle = \langle \sigma_2 \rangle \neq \langle \sigma_3 \rangle \neq 0$ or $0 = \langle \sigma_1 \rangle = \langle \sigma_3 \rangle \neq \langle \sigma_2 \rangle \neq 0$ or $0 = \langle \sigma_2 \rangle = \langle \sigma_3 \rangle \neq \langle \sigma_1 \rangle \neq 0$ then T_7 is broken into {Identity}.

To obtain a realistic neutrino spectrum, in this work we argue that both the breakings $T_7 \to Z_3$ and $T_7 \to \{\text{identity}\}$ (Instead of $Z_3 \to \{\text{identity}\}$) must be taken place in neutrino sector. However, the VEVs of σ does only one of these tasks. The $T_7 \to Z_3$ can be achieved by a $\mathrm{SU}(3)_L$ anti-sextet σ given in (5.1) with the VEVs is set as $\langle \sigma \rangle = (\langle \sigma_1 \rangle, \langle \sigma_1 \rangle, \langle \sigma_1 \rangle)$ under T_7, where

$$\langle \sigma_1 \rangle = \begin{pmatrix} \lambda_\sigma & 0 & v_\sigma \\ 0 & 0 & 0 \\ v_\sigma & 0 & \Lambda_\sigma \end{pmatrix}. \tag{5.2}$$

To achieve the second direction of the breakings $T_7 \to \{\text{Identity}\}$ (equivalently to $Z_3 \to \{\text{Identity}\}$), we additionally introduce another $\mathrm{SU}(3)_L$ anti-sextet Higgs scalar which is either put in $\underline{3}$ or $\underline{3}^*$ under T_7. This is equivalent to breaking the subgroup Z_3 of the first direction into {Identity}, and it can be achieved within each case below.

[2] Note that in the model under consideration, if the choice is a $\mathrm{SU}(3)_L$ triplet [for example $\rho \sim (3, 2/3, -4/3, \underline{3}^*)$] instead of $\mathrm{SU}(3)_L$ anti-sextet σ as in (5.1), there will be a contribution from term of $(\bar{\psi}^c_L \psi_L)_{\underline{3}^*} \rho$ added to the elements $(M_D)_{11}, (M_D)_{22}, (M_D)_{33}$ of the matrix M_D which is the same order. The lepton mixing matrix therefore can only reach to U_{HPS} but not $\theta_{13} \neq 0$.

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1. A new $\mathrm{SU}(3)_L$ anti-sextet s which is put in the $\underline{3}$ under T_7,

$$s_i = \begin{pmatrix} s^0_{11} & s^+_{12} & s^0_{13} \\ s^+_{12} & s^{++}_{22} & s^+_{23} \\ s^0_{13} & s^+_{23} & s^0_{33} \end{pmatrix}_i \sim [6^*, 2/3, -4/3, \underline{3}], \quad (i = 1, 2, 3) \tag{5.3}$$

with the VEVs given by $\langle s \rangle = (\langle s_1 \rangle, 0, 0)^T$, where

$$\langle s_1 \rangle = \begin{pmatrix} \lambda_s & 0 & v_s \\ 0 & 0 & 0 \\ v_s & 0 & \Lambda_s \end{pmatrix}. \tag{5.4}$$

2. Another $\mathrm{SU}(3)_L$ anti-sextet σ' is put in the $\underline{3}^*$ under T_7, with the VEVs chosen by

$$\sigma'_i = \begin{pmatrix} \sigma'^0_{11} & \sigma'^+_{12} & \sigma'^0_{13} \\ \sigma'^+_{12} & \sigma'^{++}_{22} & \sigma'^+_{23} \\ \sigma'^0_{13} & \sigma'^+_{23} & \sigma'^0_{33} \end{pmatrix}_i \sim [6^*, 2/3, -4/3, \underline{3}^*], \quad (i = 1, 2, 3)$$

$$\langle \sigma'_1 \rangle = \begin{pmatrix} \lambda'_\sigma & 0 & v'_\sigma \\ 0 & 0 & 0 \\ v'_\sigma & 0 & \Lambda'_\sigma \end{pmatrix}, \ \langle \sigma'_2 \rangle = \langle \sigma'_3 \rangle = 0. \tag{5.5}$$

Note that σ' differs from σ only in the VEVs alignment. Combining both cases, after calculation, we obtain the Yukawa interactions:

$$\begin{aligned} -\mathcal{L}_\nu &= \frac{1}{2}x(\bar{\psi}^c_L \sigma)_{\underline{3}^*}\psi_L + y(\bar{\psi}^c_L s)_{\underline{3}^*}\psi_L + \frac{z}{2}(\bar{\psi}^c_L \sigma')_{\underline{3}^*}\psi_L + \mathrm{H.c.} \\ &= \frac{1}{2}x(\bar{\psi}^c_{1L}\sigma_2\psi_{1L} + \bar{\psi}^c_{2L}\sigma_3\psi_{2L} + \bar{\psi}^c_{3L}\sigma_1\psi_{3L}) \\ &\quad + y(\bar{\psi}^c_{2L}s_3\psi_{1L} + \bar{\psi}^c_{3L}s_1\psi_{2L} + \bar{\psi}^c_{1L}s_2\psi_{3L}) \\ &\quad + \frac{z}{2}(\bar{\psi}^c_{1L}\sigma'_2\psi_{1L} + \bar{\psi}^c_{2L}\sigma'_3\psi_{2L} + \bar{\psi}^c_{3L}\sigma'_1\psi_{3L}) + \mathrm{H.c.} \end{aligned} \tag{5.6}$$

The mass Lagrangian for the neutrinos is given by

$$\begin{aligned} -\mathcal{L}^{\mathrm{mass}}_\nu &= \frac{1}{2}x(\lambda_\sigma\bar{\nu}^c_{1L}\nu_{1L} + v_\sigma\bar{\nu}^c_{1L}N^c_{1R} + v_\sigma\bar{N}_{1R}\nu_{1L} + \Lambda_\sigma\bar{N}_{1R}N^c_{1R}) \\ &\quad + \frac{1}{2}x(\lambda_\sigma\bar{\nu}^c_{2L}\nu_{2L} + v_\sigma\bar{\nu}^c_{2L}N^c_{2R} + v_\sigma\bar{N}_{2R}\nu_{2L} + \Lambda_\sigma\bar{N}_{2R}N^c_{2R}) \\ &\quad + \frac{1}{2}x(\lambda_\sigma\bar{\nu}^c_{3L}\nu_{3L} + v_\sigma\bar{\nu}^c_{3L}N^c_{3R} + v_\sigma\bar{N}_{3R}\nu_{3L} + \Lambda_\sigma\bar{N}_{3R}N^c_{3R}) \\ &\quad + y(\lambda_s\bar{\nu}^c_{3L}\nu_{2L} + v_s\bar{\nu}^c_{3L}N^c_{2R} + v_s\bar{N}_{3R}\nu_{2L} + \Lambda_s\bar{N}_{3R}N^c_{2R}) \\ &\quad + \frac{1}{2}z(\lambda'_\sigma\bar{\nu}^c_{3L}\nu_{3L} + v'_\sigma\bar{\nu}^c_{3L}N^c_{3R} + v'_\sigma\bar{N}_{3R}\nu_{3L} + \Lambda'_\sigma\bar{N}_{3R}N^c_{3R}) + \mathrm{H.c.} \end{aligned} \tag{5.7}$$

The neutrino mass Lagrangian can be written in matrix form as follows

$$-\mathcal{L}^{\mathrm{mass}}_\nu = \frac{1}{2}\bar{\chi}^c_L M_\nu \chi_L + \mathrm{h.c.}, \tag{5.8}$$

where

$$\chi_L \equiv (\nu_L \quad N_R^c)^T, \qquad\qquad M_\nu \equiv \begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix},$$

$$\nu_L = (\nu_{1L}, \nu_{2L}, \nu_{3L})^T, \qquad\qquad N_R = (N_{1R}, N_{2R}, N_{3R})^T, \tag{5.9}$$

and the mass matrices are then obtained by

$$M_{L,R,D} = \begin{pmatrix} a_{L,R,D} & 0 & 0 \\ 0 & a_{L,R,D} & b_{L,R,D} \\ 0 & b_{L,R,D} & a_{L,R,D} + c_{L,R,D} \end{pmatrix}, \tag{5.10}$$

with

$$\begin{aligned} a_L &= \lambda_\sigma x, & a_D &= v_\sigma x, & a_R &= \Lambda_\sigma x, \\ b_L &= \lambda_s y, & b_D &= v_s y, & b_R &= \Lambda_s y, \\ c_L &= \lambda'_\sigma z, & c_D &= v'_\sigma z, & c_R &= \Lambda'_\sigma z. \end{aligned} \tag{5.11}$$

Three observed neutrinos gain masses via a combination of type I and type II seesaw mechanisms derived from (5.8) and (5.10) as

$$M_{\rm eff} = M_L - M_D^T M_R^{-1} M_D = \begin{pmatrix} A & 0 & 0 \\ 0 & B_1 & C \\ 0 & C & B_2 \end{pmatrix}, \tag{5.12}$$

where

$$\begin{aligned} A &= a_L - \frac{a_D^2}{a_R}, \\ B_1 &= a_L - \frac{a_R b_D^2 - 2a_D b_D b_R + a_D^2(a_R + d_R)}{a_R^2 - b_R^2 + a_R d_R} \\ B_2 &= B_1 + d_L + \frac{2(b_D b_R - a_D a_R)d_D + (a_D^2 - b_D^2)d_R - a_R d_D^2}{a_R^2 - b_R^2 + a_R d_R}, \\ C &= b_L - \frac{(a_D^2 + b_D^2)b_R - (2a_D a_R + a_D d_R)b_D + (a_D b_R - a_R b_D)d_D}{a_R^2 - b_R^2 + a_R d_R}. \end{aligned} \tag{5.13}$$

We can diagonalize the mass matrix (5.12) as follows $U_\nu^T M_{\rm eff} U_\nu = \mathrm{diag}(m_1, m_2, m_3)$, with

$$\begin{aligned} m_1 &= \frac{1}{2}\left(B_1 + B_2 + \sqrt{(B_1+B_2)^2 + 4C^2}\right), \\ m_2 &= A, \\ m_3 &= \frac{1}{2}\left(B_1 + B_2 - \sqrt{(B_1+B_2)^2 + 4C^2}\right), \end{aligned} \tag{5.14}$$

and the corresponding neutrino mixing matrix:

$$U_\nu = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{K^2+1}} & 0 & \frac{K}{\sqrt{K^2+1}} \\ -\frac{K}{\sqrt{K^2+1}} & 0 & \frac{1}{\sqrt{K^2+1}} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix}, \tag{5.15}$$

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where

$$K = \frac{B_1 - B_2 - \sqrt{4C^2 + (B_1 - B_2)^2}}{2C}. \tag{5.16}$$

Combining (3.8) and (5.15), we get the lepton mixing matrix:

$$U_L^\dagger U_\nu = \frac{1}{\sqrt{3}} \begin{pmatrix} \frac{1-K}{\sqrt{K^2+1}} & 1 & \frac{1+K}{\sqrt{K^2+1}} \\ \frac{\omega(1-K\omega)}{\sqrt{K^2+1}} & 1 & \frac{\omega(\omega+K)}{\sqrt{K^2+1}} \\ \frac{\omega(\omega-K)}{\sqrt{K^2+1}} & 1 & \frac{\omega(K\omega+1)}{\sqrt{K^2+1}} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & i \end{pmatrix}. \tag{5.17}$$

It is worth noting that in our model, K given in (5.16) is an arbitrary number. Hence in general the lepton mixing matrix given in (5.17) is different to U_{HPS} in (1.1), but similar to the original version of trimaximal mixing considered in ref. [170] which is based on the $\Delta(27)$ group extension of the Standard Model.Although there are some phenomenological predictions of the model are similar to those of in ref. [170] but the fundamental difference between our model with the well known one is the prediction of CP violation (our model predicts maximal CP violation $\delta = \pi/2, 3\pi/2$ with $\theta_{23} \neq \pi/4$, but in ref. [170], the maximal CP violation $\delta = \pi/2, 3\pi/2$ achieved with $\theta_{23} = \pi/4$). In the case where T_7 is broken into Identity (Instead of $Z_3 \to$ Identity) only by s, i.e, without contribution of σ' (or $\lambda'_\sigma = v'_\sigma = \Lambda'_\sigma = 0$), the lepton mixing matrix (5.17) being equal to U_{HPS} as given in (1.1). This is a good features of T_7 with tensor product $\underline{3} \otimes \underline{3}$ given in (A.3).

In the standard Particle Data Group (PDG) parametrization, the lepton mixing matrix (U_{PMNS}) can be parametrized as

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & -s_{12}c_{13} & -s_{13}e^{-i\delta} \\ s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & -s_{23}c_{13} \\ s_{12}s_{23} + c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} + s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times P. \tag{5.18}$$

where $P = \mathrm{diag}(1, e^{i\alpha}, e^{i\beta})$, and $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ with θ_{12}, θ_{23} and θ_{13} being the solar angle, atmospheric angle and the reactor angle respectively. δ is the Dirac CP violating phase while α and β are the two Majorana CP violating phases.

From the (5.17) and (5.18) we rule out $\alpha = 0, \beta = \frac{\pi}{2}$, and the lepton mixing matrix in (5.17) can be parameterized in three Euler's angles θ_{ij} as follows:

$$s_{13}e^{-i\delta} = \frac{-1-K}{\sqrt{3}\sqrt{K^2+1}}, \tag{5.19}$$

$$t_{12} = \frac{\sqrt{K^2+1}}{K-1}, \tag{5.20}$$

$$t_{23} = -\frac{\omega + K}{1 + K\omega}. \tag{5.21}$$

Substituting $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ into (5.21) yields:

$$K = k_1 + ik_2,$$

$$k_1 = \frac{t_{23}^2 - 4t_{23} + 1}{2(t_{23}^2 - t_{23} + 1)}, \quad k_2 = \frac{\sqrt{3}(t_{23}^2 - 1)}{2(t_{23}^2 - t_{23} + 1)}. \tag{5.22}$$

The expression (5.22) tells us that $k_1^2+k_2^2 \equiv |K|^2 = 1$. Combining (5.19) and (5.20) yields:

$$\begin{aligned} e^{-i\delta} &= \frac{1}{\sqrt{3}s_{13}t_{12}}\frac{1+K}{1-K} = \frac{1}{\sqrt{3}s_{13}t_{12}}\left[\frac{1-k_1^2-k_2^2}{[(1-k_1)^2+k_2^2]}+\frac{2k_2}{[(1-k_1)^2+k_2^2]}i\right] \\ &= -\frac{i(1-t_{23})}{s_{13}t_{12}(1+t_{23})} \equiv \cos\delta - i\sin\delta \end{aligned}$$

or

$$\cos\delta = 0, \qquad \sin\delta = \frac{1-t_{23}}{s_{13}t_{12}(t_{23}+1)}. \tag{5.23}$$

Since $\cos\delta = 0$ so that $\sin\delta$ must be equal to ± 1, it is then $\delta = \frac{\pi}{2}$ or $\delta = \frac{3\pi}{2}$. Thus, our model predicts the maximal Dirac CP violating phase which is the same as in refs. [170, 171] the maximal CP violation $\delta = \frac{\pi}{2}, \frac{3\pi}{2}$ is achieved with $\theta_{23} = \pi/4$,[3] and this is one of the most striking prediction of the model under consideration.

Up to now the precise evaluation of θ_{23} is still an open problem while θ_{12} and θ_{13} are now very constrained [8]. From (5.23), our model can provide constraints on θ_{23} from θ_{12} and θ_{13} which satisfy [8] as follows.

(i) In the case $\delta = \frac{\pi}{2}$, from (5.23) we have the relation among three Euler's angles as follows:

$$\begin{aligned} t_{23} &= \frac{1-s_{13}t_{12}}{1+s_{13}t_{12}} \\ &= \frac{2-\sqrt{2}\sqrt{\frac{2-2\sqrt{1-\sin^2(2\theta_{12})}+\sin^2(2\theta_{12})}{\sin^2(2\theta_{12})}}\sqrt{1-\sqrt{1-\sin^2(2\theta_{13})}}}{2+\sqrt{2}\sqrt{\frac{2-2\sqrt{1-\sin^2(2\theta_{12})}+\sin^2(2\theta_{12})}{\sin^2(2\theta_{12})}}\sqrt{1-\sqrt{1-\sin^2(2\theta_{13})}}}. \end{aligned} \tag{5.25}$$

In figure 1, we have plotted the values of t_{23} as functions of $\sin^2(2\theta_{13})$ and $\sin^2(2\theta_{12})$ with $\sin^2(2\theta_{12}) \in (0.833, 0.881)$, $\sin^2(2\theta_{13}) \in (0.085, 0.111)$ given in (1.2). If $\sin^2(2\theta_{13}) = 0.098\,(\theta_{13} = 9.11^o)$ we have the relation between t_{23} and $\sin^2(2\theta_{12})$ as shown in figure 2.

For the best fit values of θ_{12} and θ_{13} given in [8], $\sin^2(2\theta_{12}) = 0.857$, $\sin^2(2\theta_{13}) = 0.098$ we obtain $t_{23} = 0.8075\,(\theta_{23} = 38.92^o)$, and

$$K = -0.930528 - 0.366221i, \qquad (|K| = 1). \tag{5.26}$$

The lepton mixing matrix in (5.17) then takes the form:

$$U \simeq \begin{pmatrix} 0.831597 & 0.57735 & 0.157754 \\ -0.552417 & 0.57735 & -0.799061 \\ -0.27918 & 0.57735 & 0.641307 \end{pmatrix}. \tag{5.27}$$

[3] It would be interesting to note that the above conclusion is superficial. From eq. (16) in [170], it follows

$$\cos\delta\frac{2\tan\theta_{23}}{1-\tan^2\theta_{23}} = \frac{2\cos_{13}\cot 2\theta_{13}}{\sqrt{2-3\sin^2\theta_{13}}}. \tag{5.24}$$

If $\delta = \frac{\pi}{2}, \frac{3\pi}{2}$ and $\theta_{13} \neq 0$ then $\cos\delta = 0$, $\sin\theta_{13} \neq 0$, with $\theta_{23} = \frac{\pi}{4}$ (maximal), denominator in the left-hand side (5.24) vanishes. Thus, the left-hand side of (5.24) gets the form $\frac{0}{0}$, which does not have meaning. (as will see bellow in our model $\theta_{23} \neq \pi/4$).

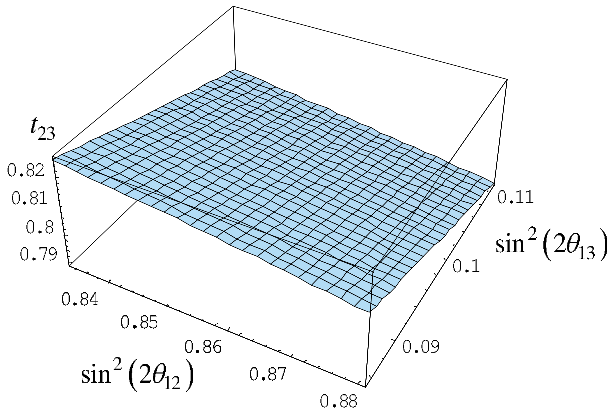

Figure 1. t_{23} as a function of $\sin^2(2\theta_{12})$ and $\sin^2(2\theta_{13})$ with $\sin^2(2\theta_{12}) \in (0.833, 0.881)$ and $\sin^2(2\theta_{13}) \in (0.085, 0.111)$ in the case $\delta = \frac{\pi}{2}$.

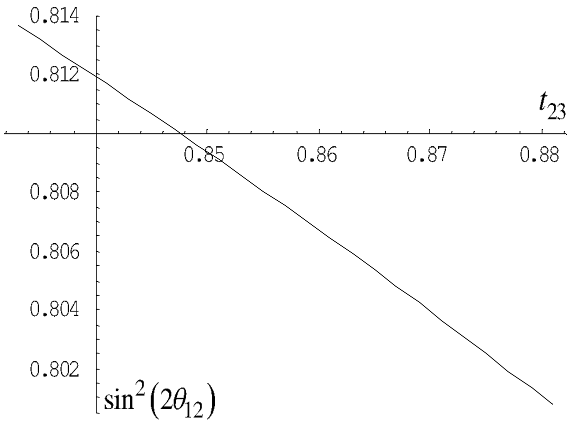

Figure 2. t_{23} as a function of $\sin^2(2\theta_{12})$ with $\sin^2(2\theta_{12}) \in (0.833, 0.881)$ and $\sin^2(2\theta_{13}) = 0.098$ in the case $\delta = \frac{\pi}{2}$.

These results also implies that in the model under consideration, the value of the Jarlskog invariant J_{CP} which determines the magnitude of CP violation in neutrino oscillations is determined [172]:

$$J_{CP} = \frac{1}{8}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\sin\delta = 0.03527. \tag{5.28}$$

(ii) In the case $\delta = \frac{3\pi}{2}$, we have the relation among three Euler's angles as follows:

$$\begin{aligned} t_{23} &= \frac{1+s_{13}t_{12}}{1-s_{13}t_{12}} \\ &= \frac{2+\sqrt{2}\sqrt{\frac{2-2\sqrt{1-\sin^2(2\theta_{12})}+\sin^2(2\theta_{12})}{\sin^2(2\theta_{12})}}\sqrt{1-\sqrt{1-\sin^2(2\theta_{13})}}}{2-\sqrt{2}\sqrt{\frac{2-2\sqrt{1-\sin^2(2\theta_{12})}+\sin^2(2\theta_{12})}{\sin^2(2\theta_{12})}}\sqrt{1-\sqrt{1-\sin^2(2\theta_{13})}}}. \end{aligned} \tag{5.29}$$

In figure 3, we have plotted the values of t_{23} as a function of $\sin^2(2\theta_{12})$ and $\sin^2(2\theta_{13})$ with $\sin^2(2\theta_{12}) \in (0.833, 0.881)$ and $\sin^2(2\theta_{13}) \in (0.085, 0.111)$ in the case $\delta = \frac{3\pi}{2}$.

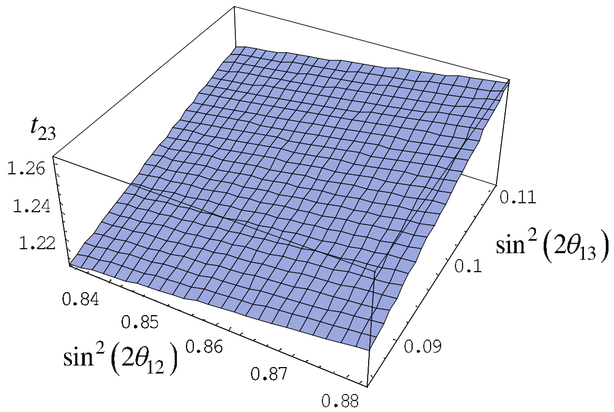

Figure 3. t_{23} as a function of $\sin^2(2\theta_{12})$ and $\sin^2(2\theta_{13})$ with $\sin^2(2\theta_{12}) \in (0.833, 0.881)$ and $\sin^2(2\theta_{13}) \in (0.085, 0.111)$ in the case $\delta = \frac{3\pi}{2}$.

For the best fit values of θ_{12} and θ_{13} given in [8], $\sin^2(2\theta_{12}) = 0.857$, $\sin^2(2\theta_{13}) = 0.098$ we obtain $t_{23} = 1.2384\,(\theta_{23} = 51.079^o)$, and

$$K = -0.930527 + 0.366223i, \qquad (|K| = 1). \tag{5.30}$$

The lepton mixing matrix in (5.17) in this case takes the form:

$$U \simeq \begin{pmatrix} 0.831597 & 0.57735 & -0.157755 \\ -0.279179 & 0.57735 & -0.641306 \\ -0.552418 & 0.57735 & 0.799061 \end{pmatrix}, \tag{5.31}$$

and the value of the Jarlskog invariant J_{CP} is determined [172]:

$$J_{CP} = \frac{1}{8}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\sin\delta \simeq -0.03527. \tag{5.32}$$

Until now values of neutrino masses (or the absolute neutrino masses) as well as the mass ordering of neutrinos is unknown. The tritium experiment [173, 174] provides an upper bound on the absolute value of neutrino mass

$$m_i \leq 2.2\,\mathrm{eV} \tag{5.33}$$

A more stringent bound was found from the analysis of the latest cosmological data [175]

$$m_i \leq 0.6\,\mathrm{eV}, \tag{5.34}$$

while arguments from the growth of large-scale structure in the early Universe yield the upper bound [176]

$$\sum_{i=1}^{3} m_i \leq 0.5\,\mathrm{eV}. \tag{5.35}$$

The neutrino mass spectrum can be the normal mass hierarchy ($|m_1| \simeq |m_2| < |m_3|$), the inverted hierarchy ($|m_3| < |m_1| \simeq |m_2|$) or nearly degenerate ($|m_1| \simeq |m_2| \simeq |m_3|$). The mass ordering of neutrino depends on the sign of Δm^2_{23} which is currently unknown. In the case of 3-neutrino mixing, in the model under consideration, the two possible signs of Δm^2_{23} correspond to two types of neutrino mass spectrum can be provided. Combining (5.14) and the two experimental constraints on squared mass differences of neutrinos as shown in (1.2) and the values of K in (5.26) or in (5.30), we have the solutions as shown bellows.

5.1 Normal case ($\Delta m^2_{23} > 0$)

5.1.1 The case $\delta = \frac{\pi}{2}$

In this case, combining (5.16) and the values of K in (5.26), we obtain

$$B_1 = B_2 - (1.67146 \times 10^{-7} + 0.732442i)C. \tag{5.36}$$

Substituting B_1 from (5.36) into (5.14) and combining with the two experimental constraints on squared mass differences of neutrinos as shown in (1.2), we get the solutions (in [eV]) given in appendix C.

From (5.36), (C.1) and (C.5) we see that A must be a real number in order to make the light neutrino masses $m_{1,2,3}$ to be real. In general, $B_{1,2}, C$ are complex numbers, α is also a complex number but $\mathrm{Im}(\alpha) \ll \mathrm{Re}(\alpha)$; $m_{1,2}$ are real numbers, m_3 is a complex number with $\mathrm{Im}(m_3) \ll \mathrm{Re}(m_3)$ but as will see below in the regions of the model parameters m_3 is real number, too.

The solutions in equations from (C.1) to (C.4) have the same absolute values of $m_{1,2,3}$, the unique difference is the sign of $m_{1,3}$. So, here we only consider in detail the case in (C.1)[4] Using the upper bound on the absolute value of neutrino mass (5.34) we can restrict the values of A: $A \leq 0.6\,\mathrm{eV}$. However, in the case in (C.1), $A \in (0.0087, 0.01)\,\mathrm{eV}$ or $A \in (-0.01, -0.0087)\,\mathrm{eV}$ are good regions of A that can reach the realistic neutrino mass hierarchy. In this region of A, $B_{1,2}$ and C are complex numbers. The real part and the imaginary part of $B_{1,2}$ and C as functions of A (or m_2) are plotted in figures 4a and 4b, respectively. In figure 5, we have plotted the absolute value $|m_{1,3}|$ as functions of m_2 with $m_2 \in (0.00867, 0.05)\,\mathrm{eV}$. This figure shows that there exist allowed regions for values m_2 (or A) where either normal or quasi-degenerate neutrino masses spectrum is achieved. The quasi-degenerate mass hierarchy is obtained when A lies in a region $[0.05\,\mathrm{eV}, +\infty]$ (A increases but must be small enough because of the scale of $m_{1,2,3}$). The normal mass hierarchy will be obtained if A takes the values around $(0.0087, 0.01)\,\mathrm{eV}$ or $(-0.01, -0.0087)\,\mathrm{eV}$. The figures 6a and 6b give the sum $\sum_{i=1}^{3} m_i$ and $\sum_{i=1}^{3} |m_i|$ with $m_2 \in (0.0087, 0.05)\,\mathrm{eV}$, respectively.

[4] The expressions from (C.1) to (C.4) show that $m_i \quad (i = 1, 2, 3)$ depends only on a parameter $A = m_2$ so we consider $m_{1,3}$ as functions of m_2. However, to have an explicit hierarchy on neutrino masses, in the following figures, m_2 should be included.

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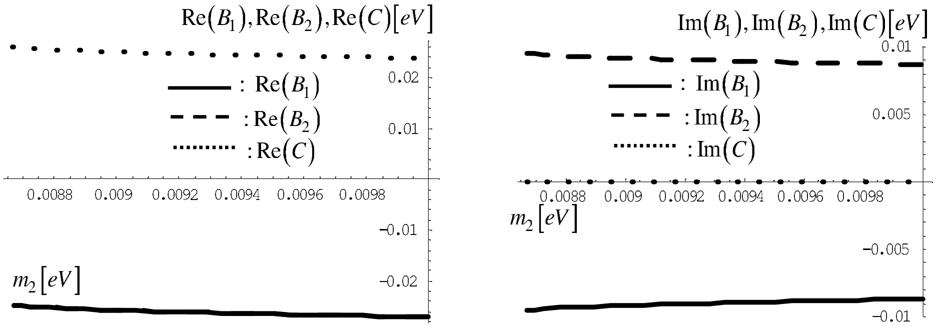


(a) (b)

Figure 4. (a) The real part of $B_{1,2}$ and C as functions of m_2, (b) The imaginary part of $B_{1,2}$ and C as functions of m_2 in the case of $\Delta m_{23}^2 > 0$.

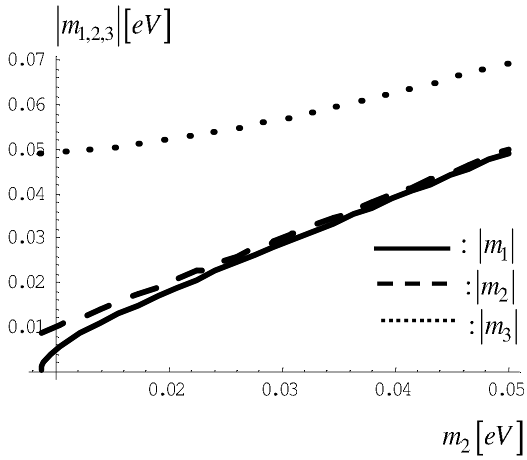
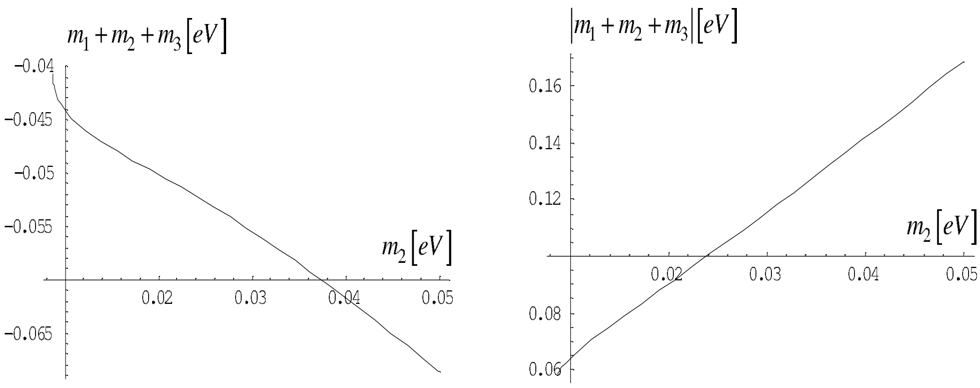


Figure 5. $|m_{1,3}|$ as functions of m_2 with $m_2 \in (0.00867, 0.05)\,\mathrm{eV}$ in the case of $\Delta m_{23}^2 > 0$.



(a) (b)

Figure 6. (a) The sum $\sum_{i=1}^{3} m_i$ as a function of A with $A \in (0.00867, 0.05)\,\mathrm{eV}$; (b) The sum $\sum_{i=1}^{3} |m_i|$ as a function of A with $A \in (0.00867, 0.05)\,\mathrm{eV}$ in the case of $\Delta m_{23}^2 > 0$.

From the expressions (5.17) , (5.30) and (C.1), it is easily to obtain the effective mass $\langle m_{ee}\rangle$ governing neutrinoless double beta decay [177–182],

$$\begin{aligned}\langle m_{ee}\rangle &= \left|\sum_{i=1}^{3} U_{ei}^2 m_i\right| \\ &= \frac{A}{3} - (0.333333 - 8.90616\times 10^{-18} i)\sqrt{4A^2 - 0.0003} \\ &\quad +(0.0248863 - 1.26619\times 10^{-8} i)\sqrt{\Gamma}, \end{aligned} \tag{5.37}$$

$$\Gamma = 0.002245 + (2 - 2.6469\times 10^{-23} i)A^2 - (1.73176 + 1.22425\times 10^{-7} i)\sqrt{\gamma}, \tag{5.38}$$

$$\begin{aligned}\gamma &= (1.88579\times 10^{-7} + 1.33377 i)(2.44973\times 10^{-19} + 0.00866025 i - Ai) \\ &\quad \times(4.79026\times 10^{-19} + 0.0481664 i + A)(1.84151\times 10^{-18} - 0.0481664 i + A) \\ &\quad \times(0.00866025 - 6.69766\times 10^{-26} i + A). \end{aligned} \tag{5.39}$$

$$m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 m_i^2}\,, \tag{5.40}$$

where

$$\begin{aligned}\sum_{i=1}^{3} |U_{ei}|^2 m_i^2 &= 2.13672\times 10^{-6} + 1.09955 A^2 \\ &\quad -(0.0430971 + 3.0467\times 10^{-9} i)\sqrt{\gamma'} + 0.0248863\sqrt{4A^2 - 0.0003}\sqrt{\Gamma'}, \end{aligned} \tag{5.41}$$

with

$$\begin{aligned}\gamma' &= -2.32077\times 10^{-7} + 3.28127\times 10^{-14} i + (0.00299432 - 4.23359\times 10^{-10} i)A^2 \\ &\quad +(1.33377 - 1.88579\times 10^{-7} i)A^4, \\ \Gamma' &= 0.002245 + (2 - 2.64698\times 10^{-23} i)A^2 - (1.73176 + 1.22425\times 10^{-7} i\sqrt{\gamma'}). \end{aligned} \tag{5.42}$$

We also note that in the normal spectrum, $|m_1| \approx |m_2| < |m_3|$, so m_1 given in (C.1) is the lightest neutrino mass. Hence, it is denoted as $m_1 \equiv m_{\rm light}$. In figures 7a and 7b, we have plotted the value $|m_{ee}|$, $|m_\beta|$ and $|m_{\rm light}|$ as functions of m_2 with $m_2 \in (-0.05, -0.0087)$ eV and $m_2 \in (0.0087, 0.05)$ eV, respectively.

To get explicit values of the model parameters, we assume $m_2 = 10^{-2}$ eV, which is safely small. Then the other neutrino masses are explicitly given as $m_1 \simeq -5.298\times 10^{-3}$ eV, $m_2 \simeq 10^{-2}$ eV, $m_3 \simeq -4.95\times 10^{-2}$ eV and $|m_{ee}| \simeq 1.09\times 10^{-3}$ eV, $|m_\beta| \simeq 1.178\times 10^{-2}$ eV. This solution means a normal neutrino mass spectrum as mentioned above and consistent with the recent experimental data [8, 183, 184]. It follows that

$$\begin{aligned} C &\simeq 0.0237465 - 8.39362\times 10^{-10} i \simeq 0.0237465\,\text{eV}, \\ B_1 &= -0.0270968 - 0.00869645 i, \qquad B_2 = -0.0276928 + 0.0232392 i. \end{aligned} \tag{5.43}$$

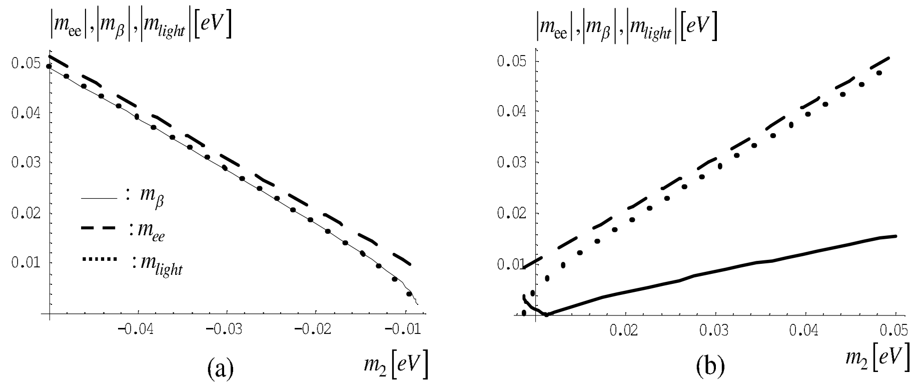


Figure 7. The $|m_{ee}|$, $|m_\beta|$ and $|m_{\text{light}}|$ as functions of m_2 from (C.1) in the case of $\Delta m^2_{23} > 0$. (a) $m_2 \in (0.00867, 0.05)\,\text{eV}$, (b) $m_2 \in (-0.05, -0.00867)\,\text{eV}$.

Furthermore, by assuming that

$$\lambda_s = -\lambda_\sigma = -\lambda_{\sigma'} = -1\,\text{eV},\ v_s = v_\sigma = -v'_\sigma,\ \Lambda_s = -\Lambda_\sigma = -\Lambda'_\sigma = -v^2, \tag{5.44}$$

we obtain

$$\begin{aligned} A &= 2x, & C &= y\left(-2 + \frac{4x^2}{x^2 - y^2 + xz}\right), \\ B_1 &= x\left(2 + \frac{4y^2}{x^2 - y^2 + xz}\right), & B_2 &= 2(z - x) + \frac{4x^3}{x^2 - y^2 + xz}. \end{aligned} \tag{5.45}$$

Combining (5.43) and (5.45) yields: $x \simeq 5 \times 10^{-3}$, $y \simeq (-4.52717 - 7.71265i) \times 10^{-3}$, $z \simeq (-10.4861 + 5.89481i) \times 10^{-3}$.

5.1.2 The case $\delta = \frac{3\pi}{2}2$

In this case, combining (5.16) and the values of K in (5.30), we obtain

$$B_1 = B_2 + (2.01498 \times 10^{-7} + 0.732446i)C. \tag{5.46}$$

Substituting B_1 from (5.46) into (5.14) and combining with the two experimental constraints on squared mass differences of neutrinos as shown in (1.2), we obtain four solutions (in [eV]) given in appendix D.

Similar to the case $\delta = \frac{\pi}{2}$ in subsection 5.1.1, in this case we also have four solutions in which $m_{1,3}$ have the same absolute values, the unique difference is the sign of $m_{1,3}$. So, we only consider one solution in the case (D.1). The values $|m_{1,3}|$ as functions of m_2 are shown in figures 8a, 8b. Furthermore, if we assume $m_2 = 10^{-2}\,\text{eV}$, which is safely small. Then the other neutrino masses are explicitly given as $m_1 \simeq 8.997 \times 10^{-3}\,\text{eV}$, $m_3 \simeq 5.00 \times 10^{-3}\,\text{eV}$. It follows that $C \simeq 2.37465 - 1.01188 \times 10^{-7} i\,\text{eV}$, $B_1 \simeq -2.70967 + 0.869651i$, $B_2 \simeq -2.76928 - 2.32392i$. Furthermore, with the assumption (5.44), we obtain $x \simeq 5 \times 10^{-3}$, $y \simeq (-4.52717 + 7.71265i) \times 10^{-3}$, $z \simeq (-10.4861 - 5.8948i) \times 10^{-2}$.

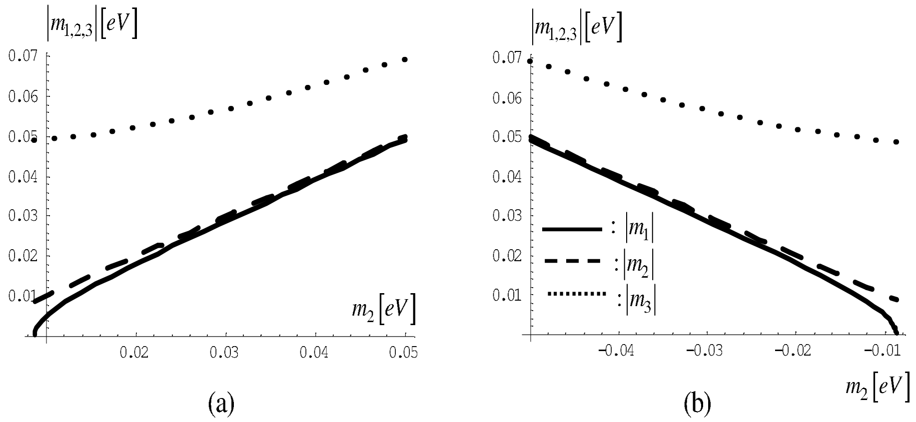


Figure 8. $|m_{1,3}|$ as functions of m_2 from (D.1) in the case of $\Delta m_{23}^2 > 0$. (a) The $|m_{1,3}|$ as functions of m_2 with $m_2 \in (0.00867, 0.05)\,\mathrm{eV}$, (b) The $|m_{1,3}|$ as functions of m_2 with $m_2 \in (-0.05, -0.00867)\,\mathrm{eV}$.

5.2 Inverted case ($\Delta m_{23}^2 < 0$)

Similar to the normal case, in this case we also have four solutions in which $m_{1,3}$ have the same absolute values, the unique difference is the sign of $m_{1,3}$. So, in the case $K = -0.930528 - 0.366221i$, we only consider one solution in the form:

$$\begin{aligned}
B_1 &= B_2 - (1.67146 \times 10^{-7} + 0.732442i)C, \\
C &= 0.5\sqrt{\alpha_1 - 2\sqrt{\beta_1}}, \\
B_2 &= 0.5\sqrt{4A^2 - 0.0003} + (8.3573 \times 10^{-8} + 0.366221i)C \\
&\qquad\qquad\qquad\qquad -0.5\sqrt{(3.46353 + 2.4485 \times 10^{-7})C^2}, \\
m_1 &= 0.5\sqrt{4A^2 - 0.0003}, \qquad m_2 = A, \\
m_3 &= 0.5\sqrt{4A^2 - 0.0003} - \sqrt{\Gamma}.
\end{aligned} \tag{5.47}$$

where

$$\begin{aligned}
\Gamma &= -0.002395 + 2.58494 \times 10^{-26}i + (2 - 2.64698 \times 10^{-23}i)A^2 \\
&\quad -(1.73176 + 1.22425 \times 10^{-7}i)\sqrt{\beta_1}, \\
\alpha_1 &= -0.00276597 + 1.95536 \times 10^{-10}i + (2.30978 - 1.63287 \times 10^{-7}i)A^2, \\
\beta_1 &= (2.32077 \times 10^{-7} - 3.28127 \times 10^{-14}i) - (0.00319439 - 4.51646 \times 10^{-10}i)A^2 \\
&\quad +(1.33377 - 1.88579 \times 10^{-7}i)A^4.
\end{aligned} \tag{5.48}$$

In figures 9a and 9b, we have plotted the real and the imaginary part of $B_{1,2}$ and C in (5.47) as functions of m_2 with $m_2 \in (0.0482, 0.05)\,\mathrm{eV}$, respectively, in the case of $\Delta m_{23}^2 < 0$ and $\delta = \frac{\pi}{2}$. The absolute value $|m_{1,3}|$ as functions of m_2 with $m_2 \in (-0.1, -0.0482)\,\mathrm{eV}$ and $m_2 \in (0.0482, 0.1)\,\mathrm{eV}$ are plotted in figures 10a and 10b, respectively. These figures show that there exist allowed regions for value of m_2 (or A) where either inverted or quasi-degenerate neutrino mass hierarchy achieved. The quasi-degenerate mass hierarchy obtained when A lies in a region $[0.1\,\mathrm{eV}, +\infty]$ 0r $[-\infty, -0.1\,\mathrm{eV}]$ ($|A|$ increases but must be

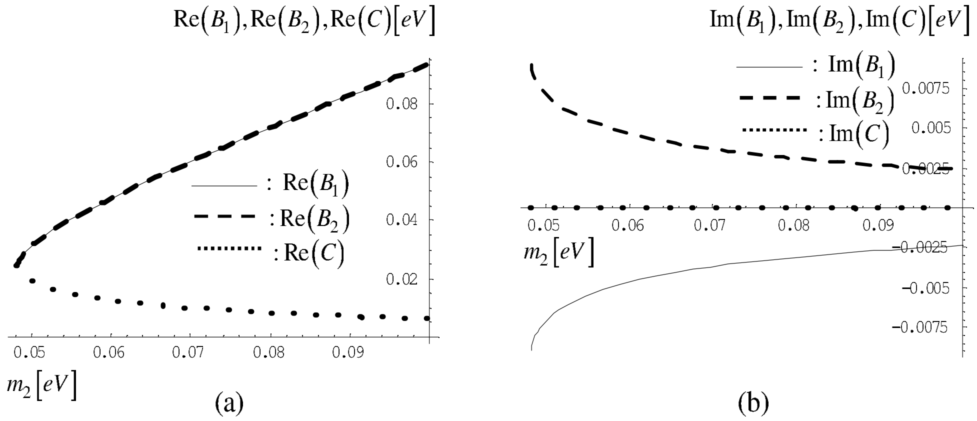


Figure 9. (a) The real part of $B_{1,2}$ and C as functions of m_2, (b) The imaginary part of $B_{1,2}$ and C as functions of m_2 in the case of $\Delta m_{23}^2 < 0$ and $\delta = \frac{\pi}{2}$.

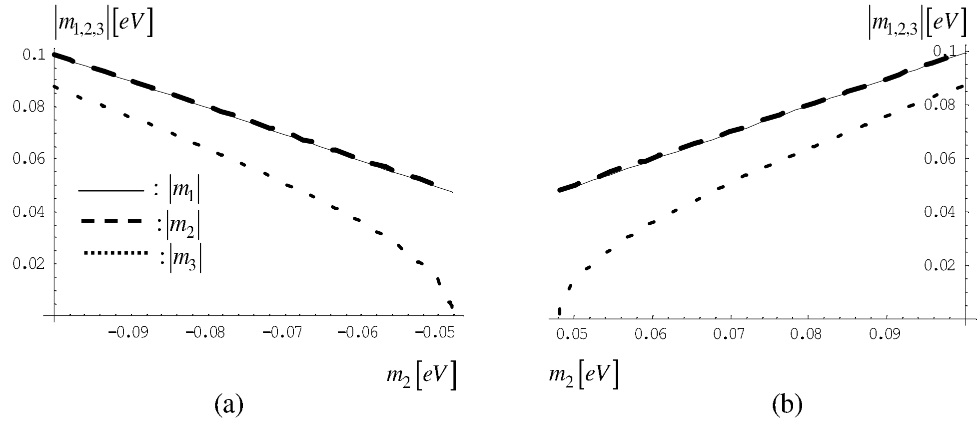


Figure 10. (a) The $|m_{1,3}|$ as functions of m_2 in the case of $\Delta m_{23}^2 < 0$ and $\delta = \frac{\pi}{2}$ with (a) $m_2 \in (-0.0482, -0.1)$ eV, (b) $m_2 \in (0.0482, 0.1)$ eV.

small enough because of the scale of $m_{1,2,3}$). The inverted mass hierarchy will be obtained if A takes the values around $(-0.1, -0.0482)$ eV or $(0.0482, 0.1)$ eV. Figures 11a and 11b give the sum $\sum_{i=1}^{3} |m_i|$ with $m_2 \in (-0.1, -0.0482)$ eV and $m_2 \in (0.0482, 0.1)$ eV, respectively.

In the inverted spectrum, $|m_3| \approx |m_1| \simeq |m_2|$, and $m_3 \equiv m_{\rm light}$ given in (5.47) is the lightest neutrino mass. $|m_{ee}|$, $|m_\beta|$ and $|m_{\rm light}|$ as functions of m_2 with $m_2 \in (-0.1, -0.0482)$ eV and $m_2 \in (0.0482, 0.1)$ eV are plotted in figures 12a and 12b, respectively.

In similarity to the normal case, to get explicit values of the model parameters, we assume $m_2 = 5 \times 10^{-2}$ eV, which is safely small. Then the other neutrino masses are explicitly given as $m_1 \simeq 4.925 \times 10^{-2}$ eV and $m_3 \simeq 1.342 \times 10^{-2}$ eV. It follows that $C \simeq 0.0192514 - 6.80475 \times 10^{-10} i$ eV, $B_1 \simeq 0.0313303 - 0.00705026i$ eV, $B_2 \simeq 0.0313303 + 0.00705026i$ eV. Furthermore, with the assumption (5.44), we obtain $x \simeq 2.50 \times 10^{-2}$, $y \simeq (-0.778994 + 1.4801i) \times 10^{-2}$, $z \simeq (0.0146926 + 0.0228004i) \times 10^{-2}$.

The relations between m_{ee}, m_β and $m_{\rm light}$ in both normal and inverted hierarchy are shown in figures 13, 14 and 15, respectively.

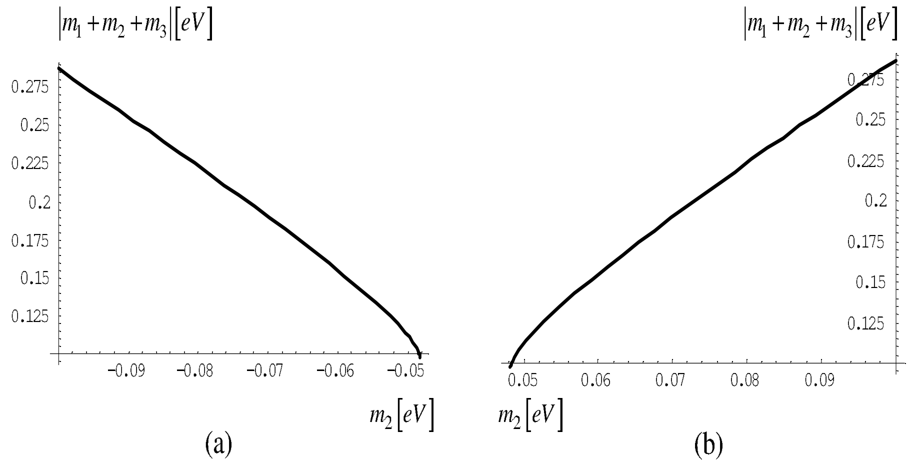


Figure 11. (a) The sum $\sum_{i=1}^{3} m_i$ as a function of m_2 with $m_2 \in (0.0482, 0.05)$ eV in the case of $\Delta m_{23}^2 < 0$ and $K = -0.930528 - 0.366221i$, (b) The sum $\sum_{i=1}^{3} |m_i|$ as a function of m_2 with $m_2 \in (0.0482, 0.05)$ eV in the case of $\Delta m_{23}^2 < 0$ and $K = -0.930528 - 0.366221i$.

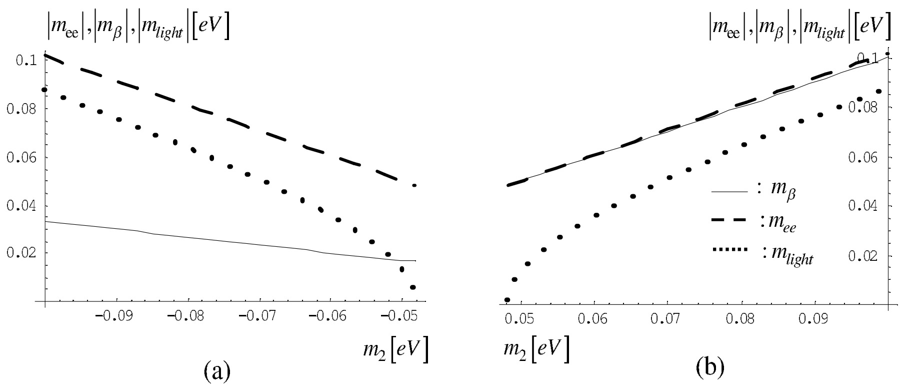


Figure 12. The $|m_{ee}|$, $|m_\beta|$ and $|m_{\rm light}|$ as functions of m_2 from (5.47) in the case of $\Delta m_{23}^2 < 0$ and $\delta = \frac{\pi}{2}$. (a) $m_2 \in (-0.1, -0.0482)$ eV, (b) $m_2 \in (0.0482, 0.1)$ eV.

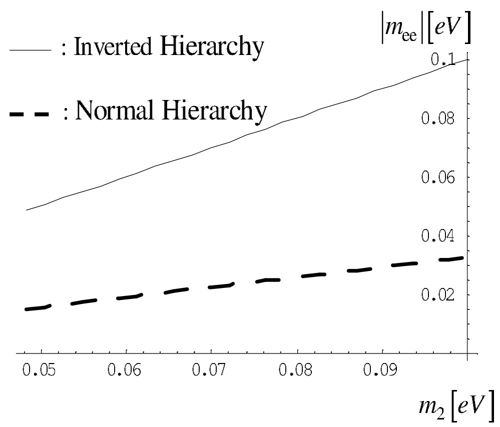


Figure 13. $|m_{ee}|$ as functions of m_2 in normal and inverted hierarchy with $\delta = \frac{\pi}{2}$.

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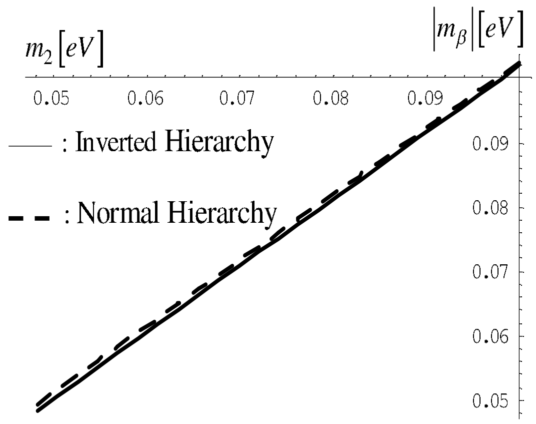

Figure 14. $|m_\beta|$ as functions of m_2 in normal and inverted hierarchy with $\delta = \frac{\pi}{2}$.

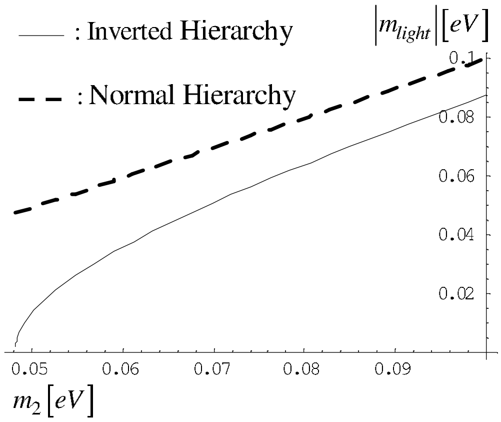

Figure 15. $|m_{\text{light}}|$ as functions of m_2 in normal and inverted hierarchy with $\delta = \frac{\pi}{2}$.

6 Remark on breaking, VEVs and rho parameter

To obtain a realistic neutrino spectrum, in this work we argue that both the breakings $T_7 \to Z_3$ and $T_7 \to$ {identity} must be taken place in neutrino sector while only the breaking $T_7 \to Z_3$ is taken place in charged lepton and quark sectors. The quark masses at the tree-level can be fitted but then the CKM matrix is diagonal.

A breaking of the lepton parity due to the odd VEVs $\langle \eta_3^0 \rangle$, $\langle \chi_1^0 \rangle$, or a violation of $\mathcal{L}$ and/nor S_3 symmetry in terms of Yukawa interactions will disturb the tree level matrix resulting in mixing between SM and exotic quarks and/or possibly providing the desirable quark mixing pattern [165–167]. To get a realistic pattern of SM quarks mixing, we should add radiative correction or use the effective six dimensional operators (for details, see ref. [187]). However, detailed study on this problem is out of the scope of this work.

Note that $\Lambda_\sigma, \Lambda_s, \Lambda_{\sigma'}$ are needed to the same order and not to be so large that can naturally be taken at TeV scale as the VEV v_χ of χ. This is because v_σ, v_s and $v_{\sigma'}$ carry lepton number, simultaneously breaking the lepton parity which is naturally constrained to be much smaller than the electroweak scale [160, 161, 164–166]. This is also behind a theoretical fact that v_χ, Λ_σ are scales for the gauge symmetry breaking in the first stage from $SU(3)_L \otimes U(1)_X \to SU(2)_L \otimes U(1)_Y$ in the original form of 3-3-1 models [160, 161, 169]. They will provide masses for the new gauge bosons: Z', X and Y. Also, the exotic quarks gain masses from v_χ while the neutral fermions masses arise from $\Lambda_\sigma, \Lambda_s, \Lambda_{\sigma'}$. The second stage of the gauge symmetry breaking from $SU(2)_L \otimes U(1)_Y \to U(1)_Q$ is achieved by the electroweak scale VEVs such as u, v responsible for ordinary particle masses. In combination with those of type II seesaw as determined, in our model, the following limit is often taken into account [160, 161, 164–166]:

$$(\mathrm{eV})^2 \sim \lambda_\sigma^2, \lambda_s^2, \lambda_{\sigma'}^2 \ll v_\sigma^2, v_s^2, v_{\sigma'}^2 \ll u^2, v^2 \ll v_\chi^2 \sim \Lambda_\sigma^2 \sim \Lambda_s^2 \sim \Lambda_{\sigma'}^2 \sim (\mathrm{TeV})^2. \tag{6.1}$$

Our model contains a lot of $SU(3)_L$ scalar triplets that may modify the precision electroweak parameter such as S, T, U [188, 189] and ρ parameters. The most serious one can result from the tree-level contributions to the ρ parameter. To see this let us approximate W, Z mass and ρ parameter:[5]

$$\begin{aligned} M_W^2 &\simeq \frac{g^2}{2} v_W^2, \qquad M_Z^2 \simeq \frac{g^2 v_W^2}{2c_W^2}, \\ M_Y^2 &\simeq \frac{g^2}{2}\left(6\Lambda_\sigma^2 + 2\Lambda_s^2 + 2\Lambda_{\sigma'}^2 + v_\chi^2\right), \end{aligned} \tag{6.2}$$

and

$$\rho = \frac{M_W^2}{c_W^2 M_Z^2} \simeq 1 + \frac{\lambda_s^2}{v_W^2}, \tag{6.3}$$

where $v_W^2 \simeq (3u^2 + 3v^2) = (246\,\mathrm{GeV})^2$ is naturally given according to (6.1) with $u \sim v \sim 100\,\mathrm{GeV}$. Since $\lambda_s = 6v_\sigma^2 + 2v_s^2 + 2v_{\sigma'}^2$ is in eV scale responsible for the observed neutrino masses, the ρ in (6.3) is absolutely close to the unity and in agreement with the data [8].

The mixings between the charged gauge bosons $W - Y$ and the neutral ones $Z' - W_4$ are in the same order since they are proportional to $\frac{v_\sigma}{\Lambda_\sigma}$, and in the limit $\lambda_s, \lambda_\sigma, v_s, v_\sigma \to 0$ these mixing angles tend to zero. In addition, from (6.1) and (6.2), it follows that M_W^2 is much smaller than M_Y^2.

7 Conclusions

In this paper, we have constructed the T_7 model based on $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge symmetry responsible for fermion masses and mixing. Neutrinos get masses from only anti-sextets which are in triplets $\underline{3}$ and $\underline{3}^*$ under T_7. The flavor mixing patterns and mass splitting are obtained without perturbation. The number of Higgs multiplets needed in order to allow the fermions to gain masses are less than those of S_3, S_4 and D_4 [165–167]. The tribimaximal form obtained with the breaking $T_7 \to Z_3$ in charged lepton sector and both $T_7 \to Z_3$ and $Z_3 \to \{\mathrm{Identity}\}$ must be taken place in neutrino sector but only apart in

[5] We have used the notation $s_W = \sin\theta_W, c_W = \cos\theta_W, t_W = \tan\theta_W$, and the continuation of the gauge coupling constant g of the $SU(3)_L$ at the spontaneous symmetry breaking point [167, 185, 186], $t = \frac{3\sqrt{2}s_W}{\sqrt{3-4s_W^2}}$ was used.

class	n	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3	χ_{3^*}
C_1	1	1	1	1	1	3	3
C_2	7	3	1	ω	ω^2	0	0
C_3	7	3	1	ω^2	ω	0	0
C_4	3	7	1	1	1	ξ	ξ^*
C_5	3	7	1	1	1	ξ^*	ξ

Table 2. Character table of T_7 group.

breakings $Z_3 \to \{\text{Identity}\}$ (without contribution of σ'), and the upper bound on neutrino mass $\sum_{i=1}^{3} m_i$ at the level is presented. From the Dirac CP violation phase we obtain the relation between Euler's angles which is consistent with the experimental in PDG 2012. On the other hand, the realistic lepton mixing can be obtained if both the direction for breakings $T_7 \to Z_3$ and $Z_3 \to \{\text{Identity}\}$ are taken place in neutrino sectors. The CKM matrix is the identity matrix at the tree-level. The Dirac CP violation phase δ is predicted to either $\frac{\pi}{2}$ or $\frac{3\pi}{2}$ which is maximal CP violation.

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A T_7 group and Clebsch-Gordan coefficients

The tetrahedral group A_4 has 12 elements and four equivalence classes with three inequivalent one-dimensional representations and one three-dimensional one, which is the smallest group with only a real $\underline{3}$ representation. The Frobenius group T_7 has 21 elements and five equivalence classes with three inequivalent one-dimensional representations and two three-dimensional once, which is the smallest group with a pair of complex $\underline{3}$ and $\underline{3}^*$ representations. It is generated by

$$a = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix}, \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \tag{A.1}$$

where $\rho = \exp(2\pi i/7)$, so that $a^7 = 1$, $b^3 = 1$, and $ab = ba^4$. The character table of T_7 is given in table 2, where $\xi = -1/2 + i\sqrt{7}/2$.

The conjugacy classes generated from a and b are

$$\begin{aligned} &C_1 : \{e\}, \\ &C_2 : \{b\,,\ ba\,,\ ba^2\,,\ ba^3\,,\ ba^4\,,\ ba^5\,,\ ba^6\}, \\ &C_3 : \{b^2\,,\ b^2a\,,\ b^2a^2\,,\ b^2a^3\,,\ b^2a^4\,,\ b^2a^5\,,\ b^2a^6, \end{aligned}$$

$$C_4 : \{a\,,\ a^2\,,\ a^4\},$$
$$C_5 : \{a^3\,,\ a^5\,,\ a^6\}. \tag{A.2}$$

Let us put $\underline{3}(1,2,3)$ which means some $\underline{3}$ multiplet such as $x = (x_1, x_2, x_3) \sim \underline{3}$ or $y = (y_1, y_2, y_3) \sim \underline{3}$ and so on, and similarly for the other representations. Moreover, the numbered multiplets such as $(\ldots, ij, \ldots)$ mean $(\ldots, x_i y_j, \ldots)$ where x_i and y_j are the multiplet components of different representations x and y, respectively. In the following the components of representations in l.h.s. will be omitted and should be understood, but they always exist in order in the components of decompositions in r.h.s. All the group multiplication rules of T_7 as given below.

$$\begin{aligned}
&\underline{1}\otimes\underline{1}=\underline{1}(11), && \underline{1}\otimes\underline{1}'=\underline{1}'(11), && \underline{1}\otimes\underline{1}''=\underline{1}''(11),\\
&\underline{1}'\otimes\underline{1}''=\underline{1}(11), && \underline{1}'\otimes\underline{1}'=\underline{1}''(11), && \underline{1}''\otimes\underline{1}''=\underline{1}'(11),\\
&\underline{1}\otimes\underline{3}=\underline{3}(11,12,13), && \underline{1}'\otimes\underline{3}=\underline{3}(11,\omega 12,\omega^2 13), && \underline{1}''\otimes\underline{3}=\underline{3}(11,\omega^2 12,\omega 13),\\
&\underline{1}\otimes\underline{3}^*=\underline{3}^*(11,12,13), && \underline{1}'\otimes\underline{3}^*=\underline{3}^*(11,\omega 12,\omega^2 13), && \underline{1}''\otimes\underline{3}^*=\underline{3}^*(11,\omega^2 12,\omega 13),\\
&\underline{3}\otimes\underline{3}=\underline{3}(33,11,22)\oplus\underline{3}^*(23,31,12)\oplus\underline{3}^*(32,13,21),\\
&\underline{3}^*\otimes\underline{3}^*=\underline{3}^*(33,11,22)\oplus\underline{3}(23,31,12)\oplus\underline{3}(32,13,21),\\
&\underline{3}\otimes\underline{3}^*=\underline{1}(11+22+33)\oplus\underline{1}'(11+\omega 22+\omega^2 33)\\
&\qquad\oplus\underline{1}''(11+\omega^2 22+\omega 33)\oplus\underline{3}(21,32,13)\oplus\underline{3}^*(12,23,31).
\end{aligned} \tag{A.3}$$

Note that $\underline{3}\times\underline{3}\times\underline{3}$ has two invariants and $\underline{3}\times\underline{3}\times\underline{3}^*$ has one invariant.

B The numbers

In the following we will explicitly point out the lepton number (L) and lepton parity (P_l) of the model particles (notice that the family indices are suppressed):

Particles	L	P_l
N_R, u, d, ϕ_1^+,$\phi_1'^+$, ϕ_2^0,$\phi_2'^0$, η_1^0,$\eta_1'^0$, η_2^-,$\eta_2'^-$ χ_3^0, σ_{33}^0, s_{33}^0	0	1
ν_L, l, U, D^*, ϕ_3^+,$\phi_3'^+$, η_3^0, $\eta_3'^0$, χ_1^{0*}, χ_2^+, σ_{13}^0, σ_{23}^+, s_{13}^0, s_{23}^+	-1	-1
σ_{11}^0, σ_{12}^+, σ_{22}^{++}, s_{11}^0, s_{12}^+, s_{22}^{++}	-2	1

C The solutions with $\delta = \frac{\pi}{2}$ in the normal case

- The first case:

$$\begin{aligned}
C &= 0.5\sqrt{\alpha - 2\sqrt{\beta}},\\
B_2 &= -0.5\sqrt{4A^2 - 0.0003} + (8.3573\times10^{-8} + 0.366221i)C\\
&\quad -0.5\sqrt{(3.46353 + 2.4485\times10^{-7})C^2},\\
m_1 &= -0.5\sqrt{4A^2 - 0.0003}, \qquad m_2 = A,\\
m_3 &= -0.5\sqrt{4A^2 - 0.0003}\\
&\quad -\sqrt{0.002245 + (2 - 2.64698\times10^{-23}i)A^2 - (1.73176 + 1.22425\times10^{-7}i)\sqrt{\beta}}.
\end{aligned} \tag{C.1}$$

- The second case:

$$\begin{aligned}
C &= 0.5\sqrt{\alpha + 2\sqrt{\beta}}, \\
B_2 &= -0.5\sqrt{4A^2 - 0.0003} + (8.3573 \times 10^{-8} + 0.366221i)C \\
&\quad -0.5\sqrt{(3.46353 + 2.4485 \times 10^{-7})C^2}, \\
m_1 &= -0.5\sqrt{4A^2 - 0.0003}, \qquad m_2 = A, \\
m_3 &= -0.5\sqrt{4A^2 - 0.0003} \\
&\quad -\sqrt{0.002245 + (2 - 2.64698 \times 10^{-23}i)A^2 + (1.73176 + 1.22425 \times 10^{-7}i)\sqrt{\beta}},
\end{aligned} \tag{C.2}$$

- The third case:

$$\begin{aligned}
C &= -0.5\sqrt{\alpha - 2\sqrt{\beta}}, \\
B_2 &= 0.5\sqrt{4A^2 - 0.0003} + (8.3573 \times 10^{-8} + 0.366221i)C \\
&\quad -0.5\sqrt{(3.46353 + 2.4485 \times 10^{-7})C^2}, \\
m_1 &= 0.5\sqrt{4A^2 - 0.0003}, \qquad m_2 = A, \\
m_3 &= 0.5\sqrt{4A^2 - 0.0003} \\
&\quad -\sqrt{0.002245 + (2 - 2.64698 \times 10^{-23}i)A^2 + (1.73176 + 1.22425 \times 10^{-7}i)\sqrt{\beta}},
\end{aligned} \tag{C.3}$$

- The fourth case:

$$\begin{aligned}
C &= 0.5\sqrt{\alpha + 2\sqrt{\beta}}, \\
B_2 &= 0.5\sqrt{4A^2 - 0.0003} + (8.3573 \times 10^{-8} + 0.366221i)C \\
&\quad -0.5\sqrt{(3.46353 + 2.4485 \times 10^{-7})C^2}, \\
m_1 &= 0.5\sqrt{4A^2 - 0.0003}, \qquad m_2 = A, \\
m_3 &= 0.5\sqrt{4A^2 - 0.0003} \\
&\quad -\sqrt{0.002245 + (2 - 2.64698 \times 10^{-23}i)A^2 + (1.73176 + 1.22425 \times 10^{-7}i)\sqrt{\beta}},
\end{aligned} \tag{C.4}$$

where

$$\begin{aligned}
\alpha &= (0.00259273 - 1.8329 \times 10^{-10}i) + (2.30978 - 1.63287 \times 10^{-7}i)A^2, \\
\beta &= -2.32077 \times 10^{-7} + 3.28127 \times 10^{-14}i \\
&\quad +(0.00299432 - 4.23359 \times 10^{-10}i)A^2 + (1.33377 - 1.88579 \times 10^{-7}i)A^4.
\end{aligned} \tag{C.5}$$

D The solutions with $\delta = \frac{3\pi}{2}$ in the normal case

- The first case:

$$\begin{aligned}
C &= 0.5\sqrt{\alpha' - 2\sqrt{\beta'}}, \\
B_2 &= -0.5\sqrt{4A^2 - 0.0003} + (1.00749 \times 10^{-7} + 0.366223i)C \\
&\quad -0.5\sqrt{(3.46352 + 2.95173 \times 10^{-7})C^2},
\end{aligned}$$

$$
\begin{aligned}
m_1 &= -0.5\sqrt{4A^2 - 0.0003}, \quad m_2 = A, \\
m_3 &= -0.5\sqrt{4A^2 - 0.0003} \\
&\quad -\sqrt{0.002245 + 2.58494\times 10^{-26}i + 2A^2 - (1.73176 + 1.47587\times 10^{-7}i)\sqrt{\beta'}}.
\end{aligned} \tag{D.1}
$$

- The second case:

$$
\begin{aligned}
C &= 0.5\sqrt{\alpha + 2\sqrt{\beta}}, \\
B_2 &= -0.5\sqrt{4A^2 - 0.0003} + (1.00749\times 10^{-7} + 0.366223i)C \\
&\quad -0.5\sqrt{(3.46352 + 2.95173\times 10^{-7})C^2}, \\
m_1 &= -0.5\sqrt{4A^2 - 0.0003}, \quad m_2 = A, \\
m_3 &= -0.5\sqrt{4A^2 - 0.0003} \\
&\quad -\sqrt{0.002245 + 2.58494\times 10^{-26}i + 2A^2 + (1.73176 + 1.47587\times 10^{-7}i)\sqrt{\beta'}}.
\end{aligned} \tag{D.2}
$$

- The third case:

$$
\begin{aligned}
C &= 0.5\sqrt{\alpha' - 2\sqrt{\beta'}}, \\
B_2 &= 0.5\sqrt{4A^2 - 0.0003} - (1.00749\times 10^{-7} + 0.366223i)C \\
&\quad -0.5\sqrt{(3.46352 + 2.95173\times 10^{-7})C^2}, \\
m_1 &= 0.5\sqrt{4A^2 - 0.0003}, \quad m_2 = A, \\
m_3 &= 0.5\sqrt{4A^2 - 0.0003} \\
&\quad -\sqrt{0.002245 + 2.58494\times 10^{-26}i + 2A^2 - (1.73176 + 1.47587\times 10^{-7}i)\sqrt{\beta'}},
\end{aligned} \tag{D.3}
$$

- The fourth case:

$$
\begin{aligned}
C &= 0.5\sqrt{\alpha' + 2\sqrt{\beta'}}, \\
B_2 &= 0.5\sqrt{4A^2 - 0.0003} - (1.00749\times 10^{-7} + 0.366223i)C \\
&\quad -0.5\sqrt{(3.46352 + 2.95173\times 10^{-7})C^2}, \\
m_1 &= 0.5\sqrt{4A^2 - 0.0003}, \quad m_2 = A, \\
m_3 &= 0.5\sqrt{4A^2 - 0.0003} \\
&\quad -\sqrt{0.002245 + (2 - 2.64698\times 10^{-23}i)A^2 + (1.73176 + 1.47587\times 10^{-7}i)\sqrt{\beta'}},
\end{aligned} \tag{D.4}
$$

where

$$
\begin{aligned}
\alpha' &= (0.00259274 - 2.20962\times 10^{-10}i) + (2.30979 - 1.96848\times 10^{-7}i)A^2, \\
\beta' &= -2.32078\times 10^{-7} + 3.95569\times 10^{-14}i \\
&\quad +(0.00299433 - 5.10375\times 10^{-10}i)A^2 + (1.33378 - 2.27338\times 10^{-7}i)A^4.
\end{aligned} \tag{D.5}
$$

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