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N-point tree-level scattering amplitude in the new Berkovits' string

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Abstract: We give a proof that the pure spinor superstring theory in a novel infinite tension limit, as was discussed recently by Berkovits, reproduces the tree-level scattering amplitudes of the ten-dimensional $\mathcal{N} = 1$ super Yang-Mills in its heterotic version and type II supergravity in its type II version. The Yang-Mills case agrees with the result obtained by Mafra, Schlotterer, Stieberger and Tsimpis.

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Contents

1 Introduction

Recently a new formula was proposed by Cachazo, He and Yuan (CHY) to compute the tree-level scattering amplitudes of massless bosons (doubly-colored scalar with cubic self-interaction, pure gluon and pure graviton) in any dimensions [1, 2], which is constructed upon scattering equations that govern the relation between scattering data and an underlying punctured Riemann sphere in the connected prescription [3–8]. This formula has been proven by Britto-Cachazo-Feng-Witten (BCFW) recursion relations [9]. Given the twistor string origin of such construction, Mason and Skinner found a new ambitwistor string theory whose tree-level scattering produces this formula [10]. Moreover, they pointed out that this new version of twistor string can be obtained by taking the chiral infinite tension limit of the ordinary string theory and they gave an explicit example in the bosonic case. This was extended by Berkovits very recently to the superstring in the pure spinor formalism [11]. By investigating its connection with the RNS formalism in Mason and Skinner's discussion, this infinite tension pure spinor string theory was also claimed to give rise to the scattering-equation-based formula. A particularly interesting aspect of this extension is that, since the pure spinor formalism naturally encodes space-time supersymmetry, this has the potential of extending the original CHY formula to the supersymmetric case, at least in ten dimensions where this string theory sits.

In this short paper, we give a proof that at tree-level Berkovits' infinite tension limit of heterotic string and type II string computes the scattering amplitudes from ten-dimensional $\mathcal{N}=1$ super Yang-Mills (SYM) and type II supergraivty (SUGRA), respectively. The proof uses the results of Mafra, Schlotterer and Stieberger (and also later on with Broedel) on the disk amplitudes of ordinary superstring in the pure spinor formalism [12–14]. This is expected since the constructions of vertex operators are very similar between the two theories. The main difference comes with the moduli, which are now holomorphic coordinates on a Riemann sphere instead of ordered coordinates on the real axis that by conformal symmetry describe points on the boundary of a disk. With this, the scattering equations directly make an appearance in the amplitude [10, 11]. As a result of Kawai-Lewellen-Tye (KLT) orthogonality pointed out in [2], in the heterotic version this just reduces to the SYM tree level amplitude as given in [15], and in the type II version it leads to SUGRA as the KLT of two copies of SYM. The paper is organized as follows. We first give a detailed proof for the case of heterotic string in section 2. Since the proof for the type II string shares a lot in common, we only discuss in detail the differences in section 3. A quick review of Berkovits' theory in each case is summarized at the beginning of each section.

2 Tree-level SYM amplitude

The new action proposed by Berkovits for heterotic superstring, which is expected to describe the $\mathcal{N}=1$ SYM scattering amplitudes in ten dimensions, is given by [11]

$$S=\int d^2z\,(P_m\bar\partial X^m+p_\alpha\bar\partial\theta^\alpha+w_\alpha\bar\partial\lambda^\alpha+b\bar\partial c)+S_c, \tag{2.1}$$

where λ^α is a ten dimensional pure spinor (this means $\lambda\gamma^m\lambda=0$, $m=0,\ldots,9$) and S_C is the worldsheet action for the current algebra. The BRST operator is defined as

$$Q=\int dz\,(\lambda^\alpha d_\alpha+c(P_m\partial X^m+p_\alpha\partial\theta^\alpha+w_\alpha\partial\lambda^\alpha+T_c)+bc\partial c), \tag{2.2}$$

where d_α is the Green-Schwarz constraint

$$d_\alpha=p_\alpha-\frac{1}{2}P_m(\gamma^m\theta)_\alpha. \tag{2.3}$$

The massless vertex operator describing the $\mathcal{N}=1$ SYM multiplet are

$$\begin{aligned}V&=c\,\tilde V^I\,J_I, &&\text{Unintegrated,}\\ U&=\tilde U^I\,J_I, &&\text{Integrated,}\end{aligned} \tag{2.4}$$

where

$$\begin{aligned}\tilde V^I&=e^{ik\cdot X}\lambda^\alpha A^I_\alpha(\theta),\\ \tilde U^I&=e^{ik\cdot X}\bar\delta(k\cdot P)[P^m A^I_m+d_\alpha W^{\alpha I}+\frac{1}{2}N_{mn}\mathcal{F}^{mnI}].\end{aligned} \tag{2.5}$$

In the above, $N_{mn}:=\frac{1}{2}(\lambda\gamma_{mn}w)$, $\{A_\alpha(\theta),A_m(\theta),W^\alpha(\theta),\mathcal{F}^{mn}(\theta)\}$ are the $\mathcal{N}=1$ SYM superfields

$$\begin{aligned}A_\alpha(\theta)&=\frac{1}{2}a_m(\gamma^m\theta)_\alpha-\frac{1}{3}(\xi\gamma_m\theta)(\gamma^m\theta)_\alpha-\frac{1}{32}F_{mn}(\gamma_p\theta)_\alpha(\theta\gamma^{mnp}\theta)+\cdots\\ A_m(\theta)&=a_m-(\xi\gamma_m\theta)-\frac{1}{8}(\theta\gamma_m\gamma^{pq}\theta)F_{pq}+\frac{1}{12}(\theta\gamma_m\gamma^{pq}\theta)(\partial_p\xi\gamma_q\theta)+\cdots\\ W^\alpha(\theta)&=\xi^\alpha-\frac{1}{4}(\gamma^{mn}\theta)^\alpha F_{mn}+\frac{1}{4}(\gamma^{mn}\theta)^\alpha(\partial_m\xi\gamma_n\theta)+\frac{1}{48}(\gamma^{mn}\theta)^\alpha(\theta\gamma_n\gamma^{pq}\theta)\partial_m F_{pq}+\cdots\\ \mathcal{F}_{mn}(\theta)&=F_{mn}-2(\partial_{[m}\xi\gamma_{n]}\theta)+\frac{1}{4}(\theta\gamma_{[m}\gamma^{pq}\theta)\partial_{n]}F_{pq}+\frac{1}{6}\partial_{[m}(\theta\gamma_{n]}{}^{pq}\theta)(\xi\gamma_q\theta)\partial_p+\cdots\end{aligned} \tag{2.6}$$

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and the current $J_I(\sigma)$ satisfies

$$J_I(\sigma_i)J_J(\sigma_j) \sim \frac{k\delta_{IJ}}{\sigma_{ij}^2} + \frac{f_{IJ}^K J_K(\sigma_j)}{\sigma_{ij}}. \tag{2.7}$$

From the action (2.1) it is simple to read the OPE's

$$\begin{aligned}
&d_\alpha(\sigma_i)d_\beta(\sigma_j) \sim -\frac{\gamma^m_{\alpha\beta}P_m}{\sigma_{ij}}, \quad d_\alpha(\sigma_i)\theta^\beta(\sigma_j) \sim \frac{\delta^\beta_\alpha}{\sigma_{ij}}, \quad d_\alpha(\sigma_i)f(X(\sigma_j),\theta(\sigma_j)) \sim \frac{D_\alpha f(X(\sigma_j)}{\sigma_{ij}},\\
&N^{mn}(\sigma_i)N_{pq}(\sigma_j) \sim \frac{4N^{[m}{}_{[p}\delta^{n]}_{q]}}{\sigma_{ij}} - \frac{6\delta^m_{[p}\delta^n_{q]}}{\sigma_{ij}^2}, \quad N^{mn}(\sigma_i)\lambda^\alpha(\sigma_j) \sim -\frac{1}{2}\frac{(\lambda\gamma^{mn})^\alpha}{\sigma_{ij}}, \quad P^m(\sigma_i)P_n(\sigma_j) \sim 0,\\
&P^m(\sigma_i)f(X(\sigma_j),\theta(\sigma_j)) \sim -\frac{(k^m)f(X(\sigma_j),\theta(\sigma_j))}{\sigma_{ij}},
\end{aligned} \tag{2.8}$$

where $D_\alpha := \partial_\alpha + \frac{1}{2}(\gamma^m\theta)_\alpha\partial_m$ is the covariant derivative. Note that these are the same OPE's as found in the pure spinor superstring formalism [16], except for the OPE of $P^m(\sigma_i)P_n(\sigma_j)$, which in the pure spinor formalism of ordinary string has a double pole.[1]

The tree-level amplitude prescription is given by the correlation function

$$\mathcal{A}_N = \int \prod_{i=1}^{N-2} d\sigma_i \, \langle V_1(\sigma_1 = 0)U_2 \cdots U_{N-2}V_{N-1}(\sigma_{N-1} = 1)V_N(\sigma_N = \infty)\rangle, \tag{2.9}$$

where the three unintegrated vertex operators $\{V_1(\sigma_1 = 0), V_{N-1}(\sigma_{N-1} = 1), V_N(\sigma_N = \infty)\}$ fix the $\mathrm{SL}(2,\mathbb{C})$ gauge symmetry on the sphere. Since there is no correlation between c, J_I and the vertices $\{\tilde{V}^I, \tilde{U}^I\}$, the above formula can be decomposed as

$$\mathcal{A}_N = \int \prod_{i=1}^{N-2} d\sigma_i \langle c(\sigma_1)c(\sigma_{N-1})c(\sigma_N)\rangle \; \langle \tilde{V}_1^{I_1}\tilde{U}_2^{I_2}\ldots\tilde{U}_{N-2}^{I_{N-2}}\tilde{V}_{N-1}^{I_{N-1}}\tilde{V}_N^{I_N}\rangle \; \langle J_{I_1}J_{I_2}\ldots J_{I_N}\rangle, \tag{2.10}$$

where the c-ghost correlator just produces a Vandermonde factor

$$\langle c(\sigma_1)c(\sigma_{N-1})c(\sigma_N)\rangle = \sigma_{1,N-1}\sigma_{N-1,N}\sigma_{N,1}. \tag{2.11}$$

2.1 X^m and P^m integration

We first perform the phase space integration. In the path integral prescription (2.10) the X^m effective action contribution, obtained by absorbing the plane waves factors from the vertices, is given by

$$S[X,P] = -\int d^2\sigma \left(\frac{1}{2\pi}P_m\bar{\partial}X^m - i\sum_{i=1}^N k_i\cdot X\delta^{(2)}(\sigma-\sigma_i)\right). \tag{2.12}$$

Integrating the zero modes of the X^m field leads to the usual momentum conservation $\delta^{(10)}(\sum_i k_i^m)$. The non-zero modes integration implies the constraint

$$\bar{\partial}P^m = -2\pi i\sum_{i=1}^N k_i^m\delta^{(2)}(\sigma-\sigma_i), \tag{2.13}$$

[1] Note that the OPE between P^m and any superfield has the same convention as in [12]. The idea is to apply the results obtained in that paper.

which, on the sphere, has the unique solution

$$P^m(\sigma) = \sum_{i=1}^{N} \frac{-(ik_i^m)}{\sigma - \sigma_i}. \tag{2.14}$$

This solution must be replaced in the integrated vertex operators, i.e., at the vertex U_i we have

$$P^m \longrightarrow P^m(\sigma_i) = \sum_{j\neq i}^{N} \frac{-(ik_j^m)}{\sigma_i - \sigma_j} \tag{2.15}$$

$$\bar{\delta}(k_i \cdot P) \longrightarrow \bar{\delta}(k_i \cdot P(\sigma_i)) = \delta\left(i\sum_{j\neq i}^{N} \frac{(ik_j)\cdot(ik_j)}{\sigma_i - \sigma_j}\right) = \delta\left(i\sum_{j\neq i}^{N} \frac{s_{ij}}{\sigma_{ij}}\right), \tag{2.16}$$

where[2] $s_{ij} := (ik_i)\cdot(ik_j)$. The solution (2.15) is equivalent to consider the OPE given in (2.8) and we can write the Dirac delta as

$$\bar{\delta}(k_i \cdot P(\sigma_i)) = \delta\left(\sum_{j\neq i}^{N} \frac{s_{ij}}{\sigma_{ij}}\right), \tag{2.17}$$

since the overall i factor does not affect the final answer.[3] Hence, we can conclude that the integration by the X^m and P^m fields imply the OPE (2.8), the $N-3$ independent scattering equations (2.17) and the momentum conservation.

2.2 N-point pure spinor amplitude

In order to compute the pure spinor correlator we must note that every single pole contraction is the same as those given in [12]. This is simple to see, since the only difference between the operators in (2.5) and those used in [12] is the missing term

$$\partial\theta^\alpha A_\alpha(X,\theta) \tag{2.18}$$

in the definition of the integrated vertices, whose OPE's involve only double poles (all possible simple poles from this term cancel away in the end). In addition, the operator Π^m in the ordinary string, which is replaced by P^m in the new vertex operator (2.5), has the OPE's

$$\Pi^m(\sigma_i)\Pi_n(\sigma_j) \sim -\frac{\delta^m_n}{\sigma_{ij}^2}, \qquad \Pi^m(\sigma_i) f(X(\sigma_j),\theta(\sigma_j)) \sim -\frac{k^m f(X(\sigma_j),\theta(\sigma_j))}{\sigma_{ij}} \tag{2.19}$$

where $f(X(\sigma_j),\theta(\sigma_j))$ is any superfield. Note that the only difference between (2.8) and (2.19) is the double pole. These indicate that all differences enter into the terms with double poles, and so we must have a careful look at these terms before moving on.

[2] The definition of s_{ij} matches with one given in [12].

[3] Integrating out the phase space $\{P^m, X^m\}$ implies that it is not necessary to consider the OPE between the Dirac delta $\bar{\delta}(k\cdot P)$ and the superfields.

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In [12] it was argued that terms involving double poles always combine to produce a prefactor of the form

$$\frac{(1+s_{ij})}{\sigma_{ij}^2}, \tag{2.20}$$

whose numerator plays the role of canceling the tachyon pole $1/(1+s_{ij})$ produced by integration of the Koba-Nielsen (KN) factor $\prod_{i<j}|\sigma_i-\sigma_j|^{-s_{ij}}$ (since such a pole is expected to be spurious). At the integrand level this means that the double poles are actually spurious as well, and hence the aim is to remove the appearance of the double poles. In the treatment of [12] for ordinary string, this is done by integration by parts in the presence of the KN factor, e.g.,

$$\int d\sigma_a \frac{d}{d\sigma_a}\left(\frac{1}{\sigma_{a,b}}\prod_{i<j}|\sigma_i-\sigma_j|^{-s_{ij}}\right)=-\int d\sigma_a\left(\frac{1+s_{ab}}{\sigma_{ab}^2}+\sum_{i\neq a,b}\frac{s_{ai}}{\sigma_{ab}\sigma_{ai}}\right)\prod_{i<j}|\sigma_i-\sigma_j|^{-s_{ij}}=0, \tag{2.21}$$

and so the effect of this operation is equivalent to the substitution

$$\frac{(1+s_{ab})}{\sigma_{ab}^2}\longrightarrow -\sum_{i\neq a,b}\frac{s_{ai}}{\sigma_{ab}\sigma_{ai}}. \tag{2.22}$$

It is important to point out that, in the calculation of ordinary string, the presence of the term (2.18) in the integrated vertex and the double pole in (2.19) contribute and only contribute to the term "1" in the numerator of (2.20), and this "1" term receives no contribution from anything else. Here we just show this explicitly in the simplest example at five points. According to the calculation in [17], when fixing $\{\sigma_1,\sigma_4,\sigma_5\}$, the terms with double poles reads

$$\frac{(1+s_{23})}{\sigma_{23}^2}\,\langle\tilde{V}(\sigma_1)\,[A_\alpha(\sigma_2)W^\alpha(\sigma_3)+A_\alpha(\sigma_3)W^\alpha(\sigma_2)-A_m(\sigma_2)A^m(\sigma_3)]\,\tilde{V}(\sigma_4)\,\tilde{V}(\sigma_5)\rangle, \tag{2.23}$$

where with a slight abuse of notation we denote $\tilde{V}=\lambda_\alpha A^\alpha$. If we study the contribution from the term (2.18), from (2.8) it is easy to see that the only non-trivial OPE's are[4]

$$(\partial\theta_\alpha A^\alpha)(\sigma_2)\,(d_\beta W^\beta)(\sigma_3)\sim\frac{A_\alpha(\sigma_2)W^\alpha(\sigma_3)}{\sigma_{23}^2},\quad (d_\beta W^\beta)(\sigma_2)\,(\partial\theta_\alpha A^\alpha)(\sigma_3)\sim\frac{A_\alpha(\sigma_3)W^\alpha(\sigma_2)}{\sigma_{23}^2}. \tag{2.24}$$

Moreover, the non-trivial OPE among Π_m's given in (2.19) produces an additional term

$$(\Pi^m A_m)(\sigma_2)\,(\Pi^n A_n)(\sigma_3)\sim -\frac{A_m(\sigma_2)A^m(\sigma_3)}{\sigma_{23}^2}. \tag{2.25}$$

When we switch from ordinary string to the twistor string constructed by Berkovits, one can check that (2.24) and (2.25) are the only OPE's that cease to contribute to the vertices correlator, and so the change to the result (2.23) is only to delete the "1" from the prefactor

[4]Here we only write out the double-pole terms. As stated before, the simple-pole terms from these additional OPE's eventually cancel each other, and thus are of no interests.

$(1+s_{23})$. In general, in the computation of Berkovits' twistor string, we just need to switch the prefactors $(1+s_{ij})$ to the corresponding s_{ij}.[5]

Now, in the context of twistor string, there is no longer any KN factor. Instead, since the σ variables are evaluated under the delta constraints (2.17), the way to get rid of the presence of double poles is to apply substitution on the support of the corresponding scattering equations, e.g.,

$$\frac{s_{ab}}{\sigma_{ab}^2} \longrightarrow -\sum_{i\neq a,b} \frac{s_{ai}}{\sigma_{ab}\sigma_{ai}}. \tag{2.26}$$

From (2.22) and (2.26), we see that although the differences in OPE's between ordinary string and Berkovits' twistor string lead to different appearances of double-pole terms, after canceling these spurious poles they actually give the same result for the vertices correlator.

Due to this fact, we are justified to directly apply the results obtained in [12]

$$\begin{aligned}\langle \tilde{V}_1^{I_1}\tilde{U}_2^{I_2}\ldots\tilde{U}_{N-2}^{I_{N-2}}\tilde{V}_{N-1}^{I_{N-1}}\tilde{V}_N^{I_N}\rangle = \delta^{(10)}\left(\sum_i k_i^m\right)\prod_{i=2}^{N-2}\delta\left(\sum_{j\neq i}\frac{s_{ij}}{\sigma_{ij}}\right)\cdot \\ \cdot\sum_{\beta\in S_{N-3}} A_{YM}(1,\beta,N-1,N)\prod_{k=2}^{N-2}\sum_{m=1}^{k-1}\frac{s_{\beta(m)\beta(k)}}{\sigma_{\beta(m)\beta(k)}},\end{aligned} \tag{2.27}$$

where $A_{YM}(1,\beta,N-1,N) = A_{YM}(1,\beta(2),\ldots,\beta(N-3),N-1,N)$ is the SYM scattering amplitude which is given in terms of the BRST building blocks [15]. Furthermore, from [14] we know that by manipulations with partial fraction relations the last factor above can be rewritten as[6]

$$\prod_{k=2}^{N-2}\sum_{m=1}^{k-1}\frac{s_{\beta(m)\beta(k)}}{\sigma_{\beta(m)\beta(k)}} = \sigma_{1,N}\sigma_{N,N-1}\sigma_{N-1,1}\sum_{\gamma\in S_{N-3}}\mathcal{S}[\beta|\gamma]\frac{1}{(1,\gamma,N,N-1)}, \tag{2.28}$$

where

$$(1,\gamma,N,N-1) := \sigma_{1\gamma(2)}\sigma_{\gamma(2)\gamma(3)}\cdots\sigma_{\gamma(N-2)N}\sigma_{N,N-1}\sigma_{N-1,1}$$

denotes the Parker-Taylor factor, and

$$\mathcal{S}[\beta|\gamma] := \prod_{a=2}^{N-2}\left(s_{1,\beta(a)} + \sum_{b=2}^{a-1}\theta(\beta(b),\beta(a))_\gamma\, s_{\beta(b),\beta(a)}\right)$$

is the infinite tension limit of the $(N-3)!\times(N-3)!$ momentum kernel, with $\theta(a,b)_\beta = 1$ if the ordering of the labels a,b is the same in both sets β and γ, and zero otherwise [18].

On the other hand, the current algebra correlator gives[7]

$$\langle J_{I_1}J_{I_2}\cdots J_{I_N}\rangle = \sum_{\Pi\in S_{N-1}}\frac{\mathrm{Tr}(T^{I_1}T^{\Pi(I_2)}\cdots T^{\Pi(I_N)})}{(1,\Pi(2),\ldots,\Pi(N))}. \tag{2.29}$$

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[5] The authors are grateful to Carlos Mafra for discussions over this issue.

[6] Rigorously speaking, this identity holds only when σ_N is gauge-fixed at infinity. However, for the general gauge where σ_N is finite, the requirement of $\mathrm{SL}(2,\mathbb{C})$ invariance of $\mathcal{A}_N$ guarantees that the r.h.s. below is the correct answer. This will become obvious later in (2.32).

[7] In addition to the single-trace terms, the current algebra correlator also produces multi-trace terms [10, 19]. As stated in [10], the multi-trace terms are associated to coupling Yang-Mills to gravity. Here we care about pure Yang-Mills and so we only focus on the single-trace terms.

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Due to the delta constraints in (2.27), the formula actually reduces to a rational function with the $\{\sigma\}$ variables evaluated on the solutions to the scattering equations. On the support of these equations, it is known from [20] that the Parke-Taylor factors in (2.29) can be linearly decomposed onto a $(n-3)!$ basis due to the validity of Bern-Carrasco-Johansson relations [21]

$$\frac{1}{(1,\Pi(2),\ldots,\Pi(N))} = \sum_{\alpha\in S_{N-3}} \mathcal{K}[\Pi,\alpha]\frac{1}{(1,\alpha,N-1,N)} \tag{2.30}$$

in the same way as

$$A_{YM}(1,\Pi(2),\ldots,\Pi(N)) = \sum_{\alpha\in S_{N-3}} \mathcal{K}[\Pi,\alpha]A_{YM}(1,\alpha,N-1,N), \tag{2.31}$$

with $\mathcal{K}[\Pi,\alpha]$ some function only depending on the kinematic invariants $\{s_{ij}\}$ and the two orderings Π,α (which is not relevant to our discussion).[8]

To this end, we see that the two copies of Vandermonde factor $(\sigma_{1,N-1},\sigma_{N-1,1},\sigma_{N,1})$ from the c-ghost correlation and (2.28) combine with the measure and the delta constraints in (2.27) to form fully permutation invariant and SL$(2,\mathbb{C})$ covariant objects

$$\begin{aligned}\int \frac{d^N\sigma}{\text{vol SL}(2,\mathbb{C})} &:= \sigma_{1,N-1}\sigma_{N-1,N}\sigma_{N,1}\int\sum_{i-2}^{N-2} d\sigma_i,\\ \prod{}'\left(\sum\frac{s_{ij}}{\sigma_{ij}}\right) &:= \sigma_{1,N}\sigma_{N,N-1}\sigma_{N-1,1}\prod_{i=2}^{N-2}\delta\left(\sum_{j\neq i}^{N}\frac{s_{ij}}{\sigma_{ij}}\right).\end{aligned} \tag{2.32}$$

Hence by assembling different pieces in (2.10), the whole amplitude can be expressed as

$$\begin{aligned}\mathcal{A}_N = &\sum_{\Pi\in S_{N-1}}\sum_{\alpha\in S_{N-3}} \text{Tr}(T^{I_1}T^{\Pi(I_2)}\cdots T^{\Pi(I_N)})\mathcal{K}[\Pi,\alpha]\int\frac{d^N\sigma}{\text{vol SL}(2,\mathbb{C})}\prod{}'\left(\sum\frac{s_{ij}}{\sigma_{ij}}\right)\frac{1}{(1,\alpha,N-1,N)}\\ &\times\sum_{\beta\in S_{N-3}} A_{YM}(1,\beta,N-1,N)\sum_{\gamma\in S_{N-3}}\mathcal{S}[\beta|\gamma]\frac{1}{(1,\gamma,N,N-1)}\delta^{(10)}\left(\sum_i k_i^m\right).\end{aligned} \tag{2.33}$$

It is easy to see that the part

$$m[\gamma|\alpha] := \int\frac{d^N\sigma}{\text{vol SL}(2,\mathbb{C})}\prod{}'\left(\sum\frac{s_{ij}}{\sigma_{ij}}\right)\frac{1}{(1,\gamma,N,N-1)}\frac{1}{(1,\alpha,N-1,N)} \tag{2.34}$$

is exactly the double partial amplitude in the doubly colored ϕ^3 theory computed by CHY formula in [2]. Since from there we know that as the result of KLT orthogonality

$$m[\gamma|\alpha] = (\mathcal{S}[\gamma|\alpha])^{-1}, \tag{2.35}$$

(2.33) reduces to

$$\mathcal{A}_N = \delta^{(10)}\left(\sum_i k_i^m\right)\sum_{\Pi\in S_{N-1}}\sum_{\alpha\in S_{N-3}}\text{Tr}(T^{I_1}T^{\Pi(I_2)}\cdots T^{\Pi(I_N)})\mathcal{K}[\Pi,\alpha]A_{YM}(1,\alpha,N-1,N), \tag{2.36}$$

[8]The $\mathcal{K}[\Pi,\alpha]$ matrix is the field theory limit of the string theory integrals $F[\Pi,\alpha]$ [12–14].

which by (2.31) is indeed the full tree-level amplitude of ten-dimensional $\mathcal{N}=1$ SYM as originally computed in [15].

In (2.36), the polarization vectors and spinors are solely encoded into the $(N-3)!$ basis $A_{YM}(1,\alpha,N-1,N)$. Note that the $A_{YM}(1,\alpha,N-1,N)$ amplitude only depends on the pure spinor variable and the superfields $\{A_\alpha(\theta), A_m(\theta), W^\alpha(\theta), \mathcal{F}_{mn}(\theta)\}$. Therefore, in order to compute the scattering between gluons and gluinos one must take into account the pure spinor measure

$$\langle(\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta)\rangle = 1. \tag{2.37}$$

Using the theta expansion (2.6) for the component amplitude involving gluons only, and the measure (2.37), we can compare (2.33) with the CHY formula

$$\mathcal{A}_{YM}^{\text{CHY}}(1,\beta,N-1,N) = \int \frac{d^N\sigma}{\text{vol SL}(2,\mathbb{C})}\prod{}' \left(\sum \frac{s_{ij}}{\sigma_{ij}}\right)\frac{1}{(1,\alpha,N-1,N)}\,\text{Pf}'\Psi. \tag{2.38}$$

and conclude that the factor

$$\sum_{\beta\in S_{N-3}} A_{YM}(1,\beta,N-1,N)\Big|_{\text{gluons}} \sum_{\gamma\in S_{N-3}} \mathcal{S}[\beta|\gamma]\frac{1}{(1,\gamma,N,N-1)} \tag{2.39}$$

becomes a Pfaffian.[9] In this way (2.33) is related to the original CHY formula (apart from the momentum conservation).

3 Tree-level SUGRA amplitude

In the version of Berkovits' theory for type II superstring, which is expected to describe type II supergravity scattering amplitudes in ten dimensions, the action reads [11]

$$S = \int d^2z(P_m\bar\partial X^m + p_\alpha\bar\partial\theta^\alpha + w_\alpha\bar\partial\lambda^\alpha + \hat p_{\hat\alpha}\bar\partial\hat\theta^{\hat\alpha} + \hat w_{\hat\alpha}\bar\partial\hat\lambda^{\hat\alpha}), \tag{3.1}$$

where λ^α and $\hat\lambda^{\hat\alpha}$ are pure spinors. The BRST charge is defined as

$$Q = \int dz(\lambda^\alpha d_\alpha + \hat\lambda^{\hat\alpha}\hat d_{\hat\alpha}) \tag{3.2}$$

where $d_\alpha(\hat d_{\hat\alpha})$ is the Green-Schwarz constraint given in (2.3).

The massless vertex operators are the double copy of the vertices defined previously in (2.4), but now without c-ghost and J^I current. With a little change of notation for later convenience, these are given by

$$\begin{aligned} V &= e^{ik\cdot X}\tilde V\,\tilde{\hat V}, && \text{Unintegrated},\\ U &= e^{ik\cdot X}\bar\delta(k\cdot P)\,\tilde U\,\tilde{\hat U}, && \text{Integrated}, \end{aligned} \tag{3.3}$$

[9] The 4-point amplitude can be checked straightforwardly since that $\mathcal{S}[2|2] = s_{12}$ and that $A_{YM}(1,2,3,4)\Big|_{\text{gluons}}$ is given in [12, 22].

where

$$\begin{aligned}\tilde{V} &= \lambda^\alpha A_\alpha(\theta),\\ \tilde{U} &= P^m A_m + d_\alpha W^\alpha + \frac{1}{2} N_{mn}\mathcal{F}^{mn},\end{aligned} \tag{3.4}$$

and the $\{\tilde{\hat{V}}, \tilde{\hat{U}}\}$ are defined in a similar way (with the hatted version of the fields).

3.1 N-point correlator and KLT formula

The computation in this case greatly resembles that for the heterotic string, and so here we only summarize the differences. Since there is no correlation between the hatted and non-hatted fields, the tree-level amplitude prescription reads

$$\begin{aligned}\mathcal{M}_N &= \int \prod_{i=1}^{N-2} d\sigma_i \, \langle V_1(\sigma_1=0)U_2\cdots U_{N-2}V_{N-1}(\sigma_{N-1}=1)V_N(\sigma_N=\infty)\rangle\\ &= \delta^{(10)}\left(\sum_i k_i^m\right)\int \prod_{i=1}^{N-2} d\sigma_i\ \delta\left(\sum_{j\neq i}^{N}\frac{s_{ij}}{\sigma_{ij}}\right)\ \langle \tilde{V}_1\tilde{U}_2\ldots\tilde{U}_{N-2}\tilde{V}_{N-1}\tilde{V}_N\rangle\ \langle \tilde{\hat{V}}_1\tilde{\hat{U}}_2\ldots\tilde{\hat{U}}_{N-2}\tilde{\hat{V}}_{N-1}\tilde{\hat{V}}_N\rangle \end{aligned}\tag{3.5}$$

where we have already performed the phase space integration, which is the same as that discussed in the SYM case. Each of the remaining correlators above is computed in the same way as that in (2.27) and (2.28). Hence one can check that $\mathcal{M}_N$ can be expressed as

$$\mathcal{M}_N = \delta^{(10)}\left(\sum_i k_i^m\right)\sum_{\beta\in S_{N-3}}\sum_{\hat{\beta}\in S_{N-3}} A_{YM}(1,\beta,N-1,N)\ H[\beta|\hat{\beta}]\ \hat{A}_{YM}(1,\hat{\beta},N,N-1), \tag{3.6}$$

where

$$\begin{aligned}H[\beta|\hat{\beta}] &= \int \frac{d^N\sigma}{\text{vol SL}(2,\mathbb{C})}\prod{}'\left(\sum\frac{s_{ij}}{\sigma_{ij}}\right)\sum_{\gamma\in S_{N-3}}\mathcal{S}[\beta|\gamma]\frac{1}{(1,\gamma,N,N-1)}\sum_{\hat{\gamma}\in S_{N-3}}\mathcal{S}[\hat{\gamma}|\hat{\beta}]\frac{1}{(1,\hat{\gamma},N-1,N)}\\ &= \sum_{\gamma,\hat{\gamma}\in S_{N-3}}\mathcal{S}[\beta|\gamma]m[\gamma|\hat{\gamma}]\mathcal{S}[\hat{\gamma}|\hat{\beta}].\end{aligned}\tag{3.7}$$

Then by the relation (2.35) it is clear that

$$\mathcal{M}_N = \delta^{(10)}\left(\sum_i k_i^m\right)\sum_{\beta\in S_{N-3}}\sum_{\hat{\beta}\in S_{N-3}} A_{YM}(1,\beta,N-1,N)\ \mathcal{S}[\beta|\hat{\beta}]\ \hat{A}_{YM}(1,\hat{\beta},N,N-1), \tag{3.8}$$

which is just the KLT relation in constructing SUGRA amplitude from the corresponding SYM amplitude. So we have also confirmed that this theory indeed produces the amplitudes of the type II SUGRA at tree level.

Analogous to subsection 2.2 , when (3.6) is restricted to graviton scattering, and using the pure spinor measure (2.37), we can connect (3.8) to the CHY formula for pure gravitons, similar to (2.39) for gluons.

4 Conclusion and discussion

We conclude that at tree level the Berkovits' infinite tension limit theory of the heterotic string (type II string) produces the ten-dimensional $\mathcal{N}=1$ SYM ($\mathcal{N}=2$ SUGRA) scattering amplitudes. The computation straightforwardly leads to the scattering-equation-based

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structure in CHY formula, in similar way as discussed by Mason and Skinner in the RNS formalism. Note that the result is manifestly supersymmetric, since $A_{YM}(1, \beta, N, N-1)$ only depends on the superfields wrote in (2.6). This result is in perfect agreement with the structure of the superstring amplitude given in the paper [12], which was one of the most important reference to our proof.

At the time when this paper was being prepared, Adamo, Casali and Skinner published a new work studying Mason and Skinner's ambitwistor string at one loop [23]. In particular, the extention of scattering equations to loop levels was proposed, and one-loop amplitudes for NS-NS external states in the type II ambitwistor string were calculated. It is interesting to see how Berkovits' theory works at loop levels. The main drawback to compute loop level in the new Berkovits' string is to obtain a well defined b-ghost. However, since a lot of progress have been done on the pure spinor formalism, for example [24, 25], it should not be hard to find a b-ghost and so to perform the loop-level scattering amplitude computation.

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