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Two body non-leptonic D^0 decays from LCSR and implications for $\Delta a_{\mathrm{CP}}^{\mathrm{dir}}$

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Abstract: Motivated by the recent measurements of CP violating effects in singly Cabibbo suppressed D^0 decays, we revisit the theoretical predictions of these channels. Using up-to-date values for the decay constants and form factors, we find already within naive QCD factorisation, surprisingly good agreement between the central values of the branching ratios and the corresponding experimental data. We further extend the study of these modes by employing the method of light-cone sum rules (LCSR) with light-meson light-cone distribution amplitudes. Using for the first time this framework to compute the leading contribution to the decay amplitude, we can again describe well the experimental branching ratios for the modes $D^0 \to \pi^+ K^-$, $D^0 \to K^+ K^-$, $D^0 \to \pi^+ \pi^-$ and $D^0 \to K^+ \pi^-$. The combination of our results with known predictions for the penguin contributions obtained with LCSR, leads to an upper bound for the value of direct CP violation expected in the Standard Model of $|\Delta a_{\mathrm{CP}}^{\mathrm{dir}}| \leq 2.4 \times 10^{-4}$, which is approximately a factor six smaller than the current measurement.

Keywords: CP Violation, Specific QCD Phenomenology

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Contents

1 Introduction

Studies of charmed hadrons decays provide complementary information with respect to the b-system on the structure of the Standard Model (SM), see e.g. ref. [1] for a review. From a theoretical point of view, however, a robust description of the charm sector is currently more challenging due to the smaller value of the charm quark mass and, correspondingly, the larger size of the strong coupling at this scale. In spite of this, it was recently found that lifetimes of charmed mesons and baryons can be consistently computed within the heavy quark expansion (HQE), yielding results in agreement [2–4] with the experimental data [5] within uncertainties. This framework is currently quite advanced, with NLO-QCD corrections to the spectator quark contributions [6–8], as well as the coefficient at LO-QCD of the Darwin operator [9–11] known by now and, in the case of charmed mesons, also three-loop sum rule estimates for the matrix element of the arising four-quark dimension-six operators available [12, 13] — in addition, first steps towards a determination of these non-perturbative inputs using Lattice QCD are being taken [14]. On the other hand, the applicability of the HQE to mixing observables is plagued by the presence of an extreme GIM [15] suppression, making the theoretical interpretation of these quantities still not fully understood. Specifically, analyses based on simplified investigations of exclusive decays [16, 17], i.e. taking only phase space effects into account but no dynamical QCD contributions, lead to a range of values for the mixing parameters that is consistent with the experimental data [5], whereas studies computed within the HQE yield extremely suppressed results, see e.g. ref. [18]. Despite recent progress made in assessing the uncertainty of the HQE prediction [19], it is still unclear how to deal with such a strong GIM suppression from a theoretical point of view.

Another peculiarity of the charm system is that CP violation effects are expected to be tiny, see e.g. ref. [1]. Therefore, it came as a big surprise when in 2011 [20] the LHCb

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Collaboration found evidence of sizable CP violating effects in singly Cabibbo suppressed decays of neutral D mesons into two pions or two kaons. The quantity considered is the difference of two time-integrated CP asymmetries, i.e.

$$\Delta A_{\rm CP} \equiv A_{\rm CP}(K^+K^-) - A_{\rm CP}(\pi^+\pi^-)\,, \tag{1.1}$$

with the time dependent asymmetry defined as

$$A_{\rm CP}(f;t) = \frac{\Gamma(D^0(t)\to f) - \Gamma(\overline{D}^0(t)\to f)}{\Gamma(D^0(t)\to f) + \Gamma(\overline{D}^0(t)\to f)}\,, \tag{1.2}$$

for an arbitrary final state f. The experimental discovery was finally made by the LHCb Collaboration in 2019 [21]. As the dominant contribution to eq. (1.2) comes from direct CP violation, in the following we will consider only $\Delta a_{\rm CP}^{\rm dir}$. The current experimental average reads [21]

$$\Delta a_{\rm CP}^{\rm dir}|_{\rm exp} = (-15.7 \pm 2.9) \times 10^{-4}\,, \tag{1.3}$$

while a comprehensive summary of the evolution of both the experimental and theoretical determinations for this observable can be found in ref. [22] (from 2013) and in ref. [23] (from 2019). Recently, the LHCb Collaboration also presented a new measurement of the CP asymmetry in the $D^0 \to K^+K^-$ channel [24]. The latter, combined with the result for $\Delta a_{\rm CP}^{\rm dir}$ in eq. (1.3), leads to the following values for the size of CP asymmetry in the two individual modes, namely

$$a_{\rm CP}^{\rm dir}(K^+K^-)|_{\rm exp} = (7.7 \pm 5.7) \times 10^{-4}\,, \tag{1.4}$$

$$a_{\rm CP}^{\rm dir}(\pi^+\pi^-)|_{\rm exp} = (23.2 \pm 6.1) \times 10^{-4}\,, \tag{1.5}$$

where eq. (1.5) provides the first evidence of CP violation in a specific D-meson decay.

Exclusive hadronic decays of charmed hadrons pose further challenges to robust theoretical predictions and a wide range of theory results can be found in the literature. From naive estimates, see e.g. ref. [25], the value of $\Delta a_{\rm CP}^{\rm dir}$ is expected to be approximately one order of magnitude smaller than the one in eq. (1.3). This result was also confirmed in ref. [26] using the framework of light-cone sum rule (LCSR) [27–29] with pion and kaon light-cone distribution amplitudes (LCDAs), and similar conclusions were obtained in a recent analysis of final state interactions employing dispersion relations [30]. Consequently, the large experimental value in eq. (1.3) has also triggered several investigations of physics beyond the SM (BSM) [23, 31–33].

At the same time, in the literature there are also theory estimates that point towards a SM origin of the experimental result for $\Delta A_{\rm CP}$. These include analyses based on U-spin relations, see e.g. ref. [34], studies of rescattering contributions with potential large enhancements due to nearby resonances like the $f_0(1710)$ or $f_0(1790)$ [35], as well as investigations of final state interactions [36]. On the other hand, the effect of nearby resonances is in principle also included in the approach of ref. [30], where no sign of large enhancement was found and, furthermore, ref. [30] points out some inconsistencies e.g. in ref. [36]. In addition, also approaches based on topological diagram analyses have been employed [37–39]. In particular,

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in ref. [38] the experimental data were combined with certain theory assumptions, including ad-hoc guesses on the relative size of the QCD-penguin exchange graphs, in order to estimate the different topological contributions, clearly not constituting a first principle determination. Finally, it is also worth commenting that the new experimental results for the individual CP asymmetries shown in eqs. (1.4), (1.5), would imply a huge U-spin symmetry breaking [40, 41]. Currently then, a clear theory interpretation of the data is still missing.

In ref. [26], Khodjamirian and Petrov used LCSR to estimate the size of penguin contributions in $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$, whereas the value of the leading decay amplitude, needed to predict $\Delta a_{\rm CP}^{\rm dir}$, see section 2, was extracted from experimental data on the corresponding branching fractions. The latter are known quite precisely, and the PDG quotes [42]

$$\mathcal{B}(D^0 \to K^+K^-)\big|_{\rm exp} = (4.08 \pm 0.06) \times 10^{-3}\,, \tag{1.6}$$

$$\mathcal{B}(D^0 \to \pi^+\pi^-)\big|_{\rm exp} = (1.454 \pm 0.024) \times 10^{-3}\,. \tag{1.7}$$

The result obtained for the penguin amplitude was approximately one order of magnitude smaller than what would be required to explain the value of $\Delta a_{\rm CP}^{\rm dir}$ in eq. (1.3). To gain further insight into the question whether this difference is an artefact of the method used or a signal of tension between the SM prediction and the data, in this paper we present a new determination of the leading contribution to the decay amplitude of two-body singly Cabibbo suppressed D^0 decays, employing the same method as used in ref. [26] for the penguin contribution, i.e. LCSRs. Specifically, we compute for the first time within this framework the tree-level amplitudes for the modes $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$, and thus the corresponding branching fractions, in order to compare with the experimental values in eqs. (1.6), (1.7). In addition, we also extend our analysis to the Cabibbo favoured and doubly Cabibbo suppressed decays $D^0 \to \pi^+K^-$ and $D^0 \to K^+\pi^-$, for which the experimental results of the branching fractions read [42]

$$\mathcal{B}(D^0 \to \pi^+K^-)\big|_{\rm exp} = (3.947 \pm 0.030) \times 10^{-2}\,, \tag{1.8}$$

$$\mathcal{B}(D^0 \to K^+\pi^-)\big|_{\rm exp} = (1.50 \pm 0.07) \times 10^{-4}\,. \tag{1.9}$$

Obtaining a good agreement would considerably strengthen our confidence in the applicability of LCSR for two-body non-leptonic D^0 meson decays, and represent a first crucial step towards a more robust understanding of the strong dynamics in these channels.

Our paper is organised as follows: in section 2 we introduce the theoretical framework needed to describe branching ratios and CP asymmetries, starting from the effective Hamiltonian. In section 3 we revisit the naive QCD factorisation estimates for the branching fractions, using updated theoretical and experimental inputs. The main result of our paper, the calculation of the tree-level amplitude from LCSR, is described in section 4. Section 5 is devoted to the description of the numerical analysis and the discussion of our results, including predictions for the branching fractions and their implications on the bound on $|\Delta a_{\rm CP}^{\rm dir}|$. Finally, in section 6 we conclude and discuss future potential improvements to our calculation.

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2 The general framework

2.1 Effective Hamiltonian and decay amplitudes

The singly Cabibbo suppressed decays[1] $D^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$, can be described by introducing the $\Delta C = 1$ effective Hamiltonian governing the flavour-changing charm-quark transitions $c \to q\bar{q}u$, with $q = d, s$, namely [43]

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} \lambda_q \Big(C_1 O_1^q + C_2 O_2^q \Big) + \text{h.c.}\,, \tag{2.1}$$

where $\lambda_q \equiv V_{cq}^* V_{uq}$ are the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and the current-current operators $O_{1,2}^q$ read respectively

$$O_1^q = (\bar{q}^i \Gamma_\mu c^i)(\bar{u}^j \Gamma^\mu q^j)\,, \qquad O_2^q = (\bar{q}^i \Gamma_\mu c^j)(\bar{u}^j \Gamma^\mu q^i)\,. \tag{2.2}$$

Here $\Gamma_\mu \equiv \gamma_\mu(1-\gamma_5)$, i, j, are colour indices, and the corresponding Wilson coefficients $C_i(\mu_1)$ are evaluated at the renormalisation scale $\mu_1 \sim m_c$, with their numerical values shown in table 1. Note that in eq. (2.1) the contribution of the penguin and chromomagnetic operators has been neglected due to the smallness of their Wilson coefficients compared to the accuracy of our study.

Similarly to ref. [26], we also introduce a compact notation for the combination of the effective operators $O_{1,2}^q$ and of their Wilson coefficients in eq. (2.1), i.e.

$$\mathcal{O}^q \equiv -\frac{G_F}{\sqrt{2}} \sum_{i=1,2} C_i O_i^q\,, \quad \text{with } q = d, s\,, \tag{2.3}$$

so that each decay amplitude can be expressed in terms of the corresponding CKM structure respectively as [26]

$$\mathcal{A}(D^0 \to \pi^+\pi^-) = \lambda_d \langle \pi^+\pi^- | \mathcal{O}^d | D^0 \rangle + \lambda_s \langle \pi^+\pi^- | \mathcal{O}^s | D^0 \rangle\,, \tag{2.4}$$

$$\mathcal{A}(D^0 \to K^+K^-) = \lambda_s \langle K^+K^- | \mathcal{O}^s | D^0 \rangle + \lambda_d \langle K^+K^- | \mathcal{O}^d | D^0 \rangle\,. \tag{2.5}$$

Using the unitarity of the CKM matrix $\lambda_d + \lambda_s + \lambda_b = 0$, eqs. (2.4), (2.5), are then recast in the following form

$$\mathcal{A}(D^0 \to \pi^+\pi^-) = \lambda_d\, \mathcal{A}_{\pi\pi} \left[1 - \frac{\lambda_b}{\lambda_d} \frac{\mathcal{P}_{\pi\pi}}{\mathcal{A}_{\pi\pi}} \right]\,, \tag{2.6}$$

$$\mathcal{A}(D^0 \to K^+K^-) = \lambda_s\, \mathcal{A}_{KK} \left[1 - \frac{\lambda_b}{\lambda_s} \frac{\mathcal{P}_{KK}}{\mathcal{A}_{KK}} \right]\,, \tag{2.7}$$

where we have defined, respectively [26]

$$\mathcal{A}_{\pi\pi} = \langle \pi^+\pi^- | \mathcal{O}^d | D^0 \rangle - \langle \pi^+\pi^- | \mathcal{O}^s | D^0 \rangle\,, \tag{2.8}$$

$$\mathcal{A}_{KK} = \langle K^+K^- | \mathcal{O}^s | D^0 \rangle - \langle K^+K^- | \mathcal{O}^d | D^0 \rangle\,, \tag{2.9}$$

[1] Note that the description can be easily generalised to the Cabibbo favoured and doubly Cabibbo suppressed decays $D^0 \to \pi^+K^-$ and $D^0 \to K^+\pi^-$. We omit this for brevity.

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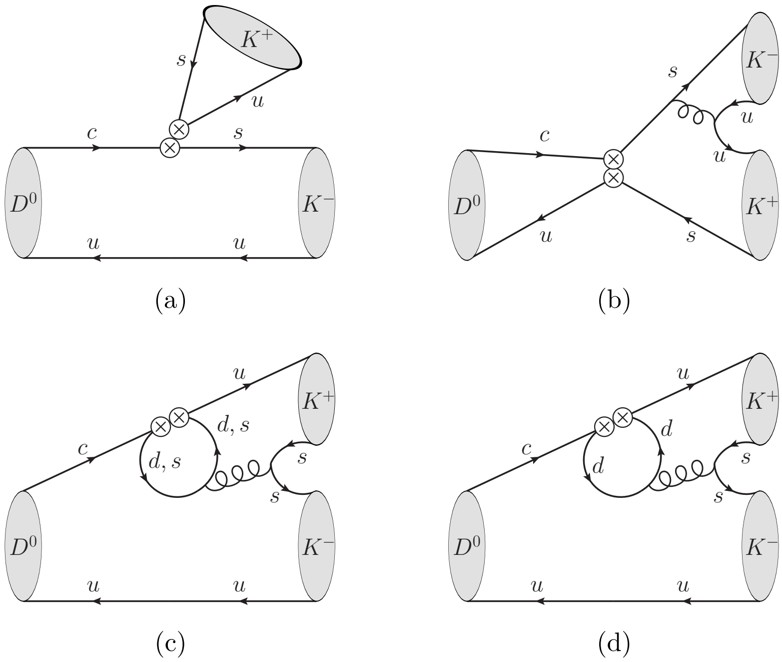

Figure 1. Examples of tree-level (a), exchange (b) and penguin (c) topologies contributing to $\mathcal{A}_{KK}$. Example of penguin topology contributing to $\mathcal{P}_{KK}$ (d). The corresponding diagrams for $\mathcal{A}_{\pi\pi}$ and $\mathcal{P}_{\pi\pi}$ can be obtained replacing $K \to \pi$, $s \leftrightarrow d$.

μ_1[GeV]	1	1.27	1.5	2
$C_1(\mu_1)$	1.34 (1.25)	1.27 (1.20)	1.24 (1.18)	1.19 (1.14)
$C_2(\mu_1)$	-0.62 (-0.48)	-0.53 (-0.40)	-0.47 (-0.36)	-0.40 (-0.30)

Table 1. Comparison of the Wilson coefficients at LO-QCD (NLO-QCD) for different values of the renormalisation scale μ_1.

and

$$\mathcal{P}_{\pi\pi} = \langle \pi^+\pi^- | \mathcal{O}^s | D^0 \rangle \,, \qquad \mathcal{P}_{KK} = \langle K^+K^- | \mathcal{O}^d | D^0 \rangle \,. \tag{2.10}$$

The leading CKM amplitudes $\mathcal{A}_{\pi\pi}$, $\mathcal{A}_{KK}$, in eqs. (2.8), (2.9), receive contributions from color-allowed tree-level, exchange and penguin topologies, on the other hand, only the penguin topology can contribute to $\mathcal{P}_{\pi\pi}$, $\mathcal{P}_{KK}$, in eq. (2.10), cf. figure 1.

2.2 Branching fractions and CP asymmetries

The branching ratio for the decay $D^0 \to K^+K^-$ can be then expressed as

$$\mathcal{B}(D^0 \to K^+K^-) = \mathcal{N}_{KK} |\lambda_s|^2 |\mathcal{A}_{KK}|^2 \left| 1 - \frac{\lambda_b}{\lambda_s} \frac{\mathcal{P}_{KK}}{\mathcal{A}_{KK}} \right|^2 \,, \tag{2.11}$$

where the prefactor $\mathcal{N}_{KK}$ is

$$\mathcal{N}_{KK} \equiv \frac{\sqrt{\lambda(m_D^2, m_K^2, m_K^2)}}{16\pi m_D^3}\, \tau(D^0)\,, \tag{2.12}$$

with $\lambda(a,b,c) \equiv (a-b-c)^2 - 4bc$ being the Källen function and $\tau(D^0)$ the total lifetime of the D^0 meson. Similarly, the direct CP asymmetry, defined as

$$a_{\mathrm{CP}}^{\mathrm{dir}}(f) \equiv \frac{\Gamma(D^0 \to f) - \Gamma(\overline{D}^0 \to \bar{f})}{\Gamma(D^0 \to f) + \Gamma(\overline{D}^0 \to \bar{f})}\,, \tag{2.13}$$

reads

$$a_{\mathrm{CP}}^{\mathrm{dir}}(K^+K^-) = -\frac{2\left|\frac{\lambda_b}{\lambda_s}\right| \sin\gamma \left|\frac{\mathcal{P}_{KK}}{\mathcal{A}_{KK}}\right| \sin\phi_{KK}}{1 - 2\left|\frac{\lambda_b}{\lambda_s}\right| \cos\gamma \left|\frac{\mathcal{P}_{KK}}{\mathcal{A}_{KK}}\right| \cos\phi_{KK} + \left|\frac{\lambda_b}{\lambda_s}\right|^2 \left|\frac{\mathcal{P}_{KK}}{\mathcal{A}_{KK}}\right|^2}\,, \tag{2.14}$$

where we have defined the strong phase difference $\phi_{KK} \equiv \arg\left(\mathcal{P}_{KK}/\mathcal{A}_{KK}\right)$, and introduced the angle $\gamma \equiv -\arg(\lambda_b/\lambda_s)$. Note that the corresponding expressions for the mode $D^0 \to \pi^+\pi^-$ can be obtained by replacing $KK \to \pi\pi$, $\lambda_s \to \lambda_d$ and $\sin\gamma \to -\sin\gamma$ in eqs. (2.11), (2.12), and (2.14).

Taking into account that $\lambda_b/\lambda_{d,s} \ll 1$, it thus follows that while the amplitudes $\mathcal{A}_{\pi\pi}$, $\mathcal{A}_{KK}$, give the dominant contribution to the branching fractions, i.e.

$$\mathcal{B}(D^0 \to \pi^+\pi^-) \simeq \mathcal{N}_{\pi\pi}|\lambda_d|^2|\mathcal{A}_{\pi\pi}|^2\,, \tag{2.15}$$

$$\mathcal{B}(D^0 \to K^+K^-) \simeq \mathcal{N}_{KK}|\lambda_s|^2|\mathcal{A}_{KK}|^2\,, \tag{2.16}$$

the direct CP asymmetries are driven by the ratio of the penguin over the CKM leading amplitudes, that is

$$a_{\mathrm{CP}}^{\mathrm{dir}}(\pi^+\pi^-) \simeq 2\left|\frac{\lambda_b}{\lambda_d}\right| \sin\gamma \left|\frac{\mathcal{P}_{\pi\pi}}{\mathcal{A}_{\pi\pi}}\right| \sin\phi_{\pi\pi}\,, \tag{2.17}$$

$$a_{\mathrm{CP}}^{\mathrm{dir}}(K^+K^-) \simeq -2\left|\frac{\lambda_b}{\lambda_s}\right| \sin\gamma \left|\frac{\mathcal{P}_{KK}}{\mathcal{A}_{KK}}\right| \sin\phi_{KK}\,. \tag{2.18}$$

From the above results, together with $|\lambda_d| \simeq |\lambda_s|$, we arrive at the following expression for the difference of direct CP asymmetries $\Delta a_{\mathrm{CP}}^{\mathrm{dir}}$, that is

$$\Delta a_{\mathrm{CP}}^{\mathrm{dir}} \simeq -2\left|\frac{\lambda_b}{\lambda_s}\right| \sin\gamma \left(\left|\frac{\mathcal{P}_{KK}}{\mathcal{A}_{KK}}\right| \sin\phi_{KK} + \left|\frac{\mathcal{P}_{\pi\pi}}{\mathcal{A}_{\pi\pi}}\right| \sin\phi_{\pi\pi}\right)\,. \tag{2.19}$$

The penguin amplitudes $\mathcal{P}_{\pi\pi}$, $\mathcal{P}_{KK}$, were estimated in ref. [26] using the framework of LCSR with respectively pion and kaon LCDAs, and following previous studies of the $B \to \pi\pi$ decay [44, 45]. The values of $|\mathcal{A}_{\pi\pi}|$ and $|\mathcal{A}_{KK}|$, required to determine $a_{\mathrm{CP}}^{\mathrm{dir}}$, were instead extracted from the precise experimental data available on the branching ratios, taking into account the relations in eqs. (2.15), (2.16). At the same time, assuming naive power counting, we can express $\mathcal{A}_{\pi\pi}$ and $\mathcal{A}_{KK}$ in eqs. (2.8), (2.9), as

$$\mathcal{A}_{\pi\pi} = \langle \pi^+\pi^- | \mathcal{O}^d | D^0 \rangle \Big|_{\mathrm{tree}} + \mathcal{O}(\alpha_s) + \mathcal{O}(1/m_c)\,, \tag{2.20}$$

$$\mathcal{A}_{KK} = \langle K^+K^- | \mathcal{O}^s | D^0 \rangle \Big|_{\mathrm{tree}} + \mathcal{O}(\alpha_s) + \mathcal{O}(1/m_c)\,, \tag{2.21}$$

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retaining the dominant contribution due to the tree-level amplitude and neglecting sub-leading diagrams due to both hard and soft QCD corrections. In this approximation it thus follows that

$$\mathcal{A}_{\pi\pi} \simeq -\frac{G_F}{\sqrt{2}}\left(C_1+\frac{C_2}{3}\right)\langle\pi^+\pi^-|O_1^d|D^0\rangle\Big|_{\text{tree}}\,, \tag{2.22}$$

$$\mathcal{A}_{KK} \simeq -\frac{G_F}{\sqrt{2}}\left(C_1+\frac{C_2}{3}\right)\langle K^+K^-|O_1^s|D^0\rangle\Big|_{\text{tree}}\,. \tag{2.23}$$

The tree-level matrix elements in eqs. (2.22), (2.23), can be then estimated using naive QCD factorisation (nQCDf), see section 3, or determined from a LCSR computation, see section 4. Importantly, in the latter case, the combination with the results for $\mathcal{P}_{\pi\pi}$, $\mathcal{P}_{KK}$, obtained in ref. [26], allows us to consistently estimate the ratios in eqs. (2.17), (2.18), and ultimately to obtain a constraint on $|\Delta a_{\rm CP}^{\rm dir}|$ entirely within the same theoretical framework. Note that for brevity, the suffix "tree" will be omitted in the following, although this should always be understood.

The computation of the tree-level hadronic matrix elements for the singly Cabibbo suppressed D^0 decays can be easily extended to the Cabibbo favourite and doubly Cabibbo suppressed decays $D^0\to\pi^+K^-$ and $D^0\to K^+\pi^-$, which are also driven by color-allowed tree-level topologies. Therefore, we include these two additional channels in our analysis. As for the color-suppressed tree-level decays like $D^0\to\pi^0\pi^0$, the prefactor $(C_1+C_2/3)$ is replaced by $(C_2+C_1/3)$, where the latter combination shows an almost perfect cancellation. This makes the analysis of these decays extremely sensitive to the accuracy of the computation, and hence we leave the inclusion of these channels to future studies that will also take higher order perturbative contributions into account.

3 The decay amplitudes in naive QCDf

A first estimate of the tree-level hadronic matrix elements in eqs. (2.22), (2.23), can be obtained within nQCDf. Considering, for instance, the decay $D^0\to K^+K^-$, the nQCDf approximation leads to

$$\langle K^+K^-|O_1^s|D^0\rangle\Big|_{\rm nQCDf} = i f_K(m_D^2-m_K^2)f_0^{DK}(m_K^2)\,, \tag{3.1}$$

where f_K is the kaon decay constant and $f_0^{DK}(m_K^2)$ the scalar form factor evaluated at $q^2=m_K^2$. Hence, the amplitude $\mathcal{A}_{KK}$ becomes

$$\mathcal{A}_{KK}\Big|_{\rm nQCDf} = -i\frac{G_F}{\sqrt{2}}\left(C_1+\frac{C_2}{3}\right)f_K(m_D^2-m_K^2)f_0^{DK}(m_K^2)\,. \tag{3.2}$$

Similar expressions for the remaining modes can be easily obtained by properly replacing $f_K\to f_\pi$, $f_0^{DK}(m_K^2)\to f_0^{D\pi}(m_\pi^2)\simeq f_0^{D\pi}(0)$, etc.

Using the values for the Wilson coefficients shown in table 1, and for parameters like, masses, decay constants, etc., those presented in table 2, as well as Lattice QCD determinations for the form factors [46],[2] namely

$$f_0^{DK}(0) = 0.765 \pm 0.031\,, \qquad f_0^{DK}(m_K^2) = 0.789 \pm 0.028\,, \tag{3.3}$$

$$f_0^{D\pi}(0) = 0.612 \pm 0.035\,, \qquad f_0^{D\pi}(m_K^2) = 0.639 \pm 0.032\,, \tag{3.4}$$

we arrive at the following estimates for the branching fractions

$$\mathcal{B}(D^0 \to K^+K^-)\Big|_{\rm nQCDf} = \left(3.40^{+0.40}_{-0.35}\right) \times 10^{-3}\,, \tag{3.5}$$

$$\mathcal{B}(D^0 \to \pi^+\pi^-)\Big|_{\rm nQCDf} = \left(1.90^{+0.28}_{-0.26}\right) \times 10^{-3}\,, \tag{3.6}$$

$$\mathcal{B}(D^0 \to \pi^+K^-)\Big|_{\rm nQCDf} = \left(4.55^{+0.56}_{-0.50}\right) \times 10^{-2}\,, \tag{3.7}$$

$$\mathcal{B}(D^0 \to K^+\pi^-)\Big|_{\rm nQCDf} = \left(1.48^{+0.20}_{-0.19}\right) \times 10^{-4}\,, \tag{3.8}$$

where the quoted uncertainties include the variation of the input parameters, as well as of the renormalisation scale in the interval $1\,\text{GeV} \leq \mu_1 \leq 2\,\text{GeV}$, all combined in quadrature. We stress however, that these should not be understood as the final theory errors, since uncertainties due to contributions that cannot be captured by the nQCDf approximation are not included and could potentially be sizable.

Given the non-trivial hadronic structure of these decays and the challenges posed by the charm sector, the agreement between the nQCDf estimates and the corresponding data in eqs. (1.6)–(1.9) appears, surprisingly, excellent. In particular, using updated input values for the form factors, we are able to reproduce the large experimental result for the $\mathrm{SU}(3)_f$ breaking in the $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$ modes.

To get a first idea of the possible size of sub-leading contributions to the decay amplitude, we consider the deviation of the experimental branching ratios from the nQCDf results in eqs. (3.5)–(3.8), as well as the corresponding deviation at the amplitude level in order to subtract the effect of simple phase space and CKM factors. Specifically, we define

$$\delta\mathcal{B}^{\rm nQCDf} \equiv \frac{(\mathcal{B}^{\rm exp} - \mathcal{B}^{\rm nQCDf})}{\mathcal{B}^{\rm exp}}\,, \qquad \delta\mathcal{A}^{\rm nQCDf} \equiv \frac{(\mathcal{A}^{\rm exp} - \mathcal{A}^{\rm nQCDf})}{\mathcal{A}^{\rm exp}}\,, \tag{3.9}$$

obtaining the values summarised in the table below.

	$D^0 \to K^+K^-$	$D^0 \to \pi^+\pi^-$	$D^0 \to \pi^+K^-$	$D^0 \to K^+\pi^-$
$\delta\mathcal{B}^{\rm nQCDf}$	0.17	-0.31	-0.15	0.02
$\delta\mathcal{A}^{\rm nQCDf}$	0.09	-0.14	-0.07	0.01

These results demonstrate that the decay amplitudes for the channels considered are indeed dominated by the contribution of the tree-level topology, and that therefore there is no

[2] We use the most recent and so far only published Lattice QCD results using $N_f = 2+1+1$ ensembles [46], There are, in fact, also older Lattice QCD determinations based on using $N_f = 2+1$ ensembles [47, 48], which indicate slightly smaller $\mathrm{SU}(3)_f$ breaking effects in the form factors compared to ref. [46]. For more details, see the FLAG review [49].

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indication for a large enhancement of sub-leading diagrams. It is important to stress that compared to previous analyses, like e.g. ref. [25], several inputs have changed considerably. For instance, the $D \to \pi$ and $D \to K$ form factors show now a significant $\mathrm{SU}(3)_f$ breaking effect, cf. eqs. (3.3), (3.4), whereas the values from 2005 used in ref. [25] were almost $\mathrm{SU}(3)_f$ symmetric. In addition, the experimental data for the branching ratios have also changed, e.g. the value of $\Gamma(D^0 \to K^0 \bar{K}^0)$, which is expected to vanish in nQCDf, went down by more than a factor of five, while it seemed to be sizable in 2006, thus potentially contradicting the nQCDf estimate.

4 The decay amplitudes from LCSR

In this section, we outline the computation of the tree-level matrix elements in eqs. (2.22), (2.23), using the framework of LCSR. For definiteness, we consider the decay $D^0 \to K^+K^-$, as it is more general. The calculation of the remaining channels is in fact analogous and the corresponding results can be obtained from those presented here implementing the proper replacements i.e. $K \to \pi$, $m_s \to m_d \to 0$, $m_K \to m_\pi \to 0$, etc. Note that the analysis largely follows the studies of the $B \to \pi\pi$ and $B \to \pi K$ decays performed in refs. [44, 50].

We start by introducing the three-point correlation function

$$F_\mu(p,q) = i^2 \int d^4x\, e^{-ip\cdot x} \int d^4y\, e^{iq\cdot y}\, \langle K^-(p-q)|\mathrm{T}\Big\{ j_5^D(x), O_1^s(0), j_\mu^K(y)\Big\}\,|0\rangle\,, \qquad (4.1)$$

where $j_\mu^K = \bar{s}\gamma_\mu\gamma_5 u$ and $j_5^{D^0} = im_c\, \bar{c}\gamma_5 u$ are interpolating currents of the K^+ and D^0 mesons, respectively, with the corresponding off-shell momenta p^μ and q^μ. The correlation function $F_\mu(p,q)$ admits the Lorentz decomposition

$$F_\mu(p,q) = F_q(p^2,q^2)\, q_\mu + F_p(p^2,q^2)\, p_\mu\,, \qquad (4.2)$$

in terms of the scalar functions $F_q(p^2,q^2)$ and $F_p(p^2,q^2)$, depending on the two independent Lorentz invariant variables p^2 and q^2. We stress that only the coefficient of q_μ contributes to the dispersion relations and is thus relevant for our analysis.

For sufficiently large space-like values of the momenta squared $P^2 \equiv -p^2 \gg \Lambda^2$, $Q^2 \equiv -q^2 \gg \Lambda^2$, with Λ denoting a hadronic scale of the order of few hundreds MeV, one can show that the dominant contribution to the integrals in eq. (4.1) comes from the light-cone region $x^2 \sim 0$ and $y^2 \sim 0$, see e.g. ref. [44]. Therefore, with the kinematics fixed above, the so-called light-cone operator-product expansion (LC-OPE) is applicable, allowing us to represent the correlation function in the form of convolution of hard scattering kernels with the corresponding kaon LCDAs of growing twist.

The general expression for the non-local two-particle kaon-to-vacuum matrix element expanded near the light-cone and up to twist-4, can be found e.g. in the appendix of ref. [51],

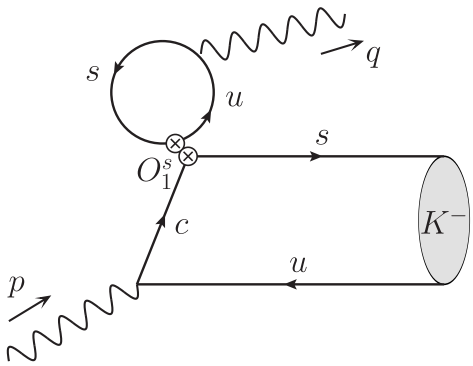

Figure 2. Diagram relevant for the computation of the LC-OPE for the correlation function in eq. (4.1).

and reads

$$\langle K^-(k)|\bar{s}^i_\alpha(x_1)u^j_\beta(x_2)|0\rangle = \frac{i\delta^{ij}}{12} f_K \int_0^1 e^{iu(k\cdot x_1)+i\bar{u}(k\cdot x_2)}\Big([\not{p}\gamma_5]_{\beta\alpha}\phi_{2K}(u) \tag{4.3}$$
$$- [\gamma_5]_{\beta\alpha}\mu_K \phi^p_{3K}(u) + \frac{1}{6}[\sigma_{\mu\nu}\gamma_5]_{\beta\alpha} k^\mu (x_1-x_2)^\nu \mu_K \phi^\sigma_{3K}(u)$$
$$+ \frac{1}{16}[\not{p}\gamma_5]_{\beta\alpha}(x_1-x_2)^2\phi_{4K}(u) - \frac{i}{2}[(\not{x}_1-\not{x}_2)\gamma_5]_{\beta\alpha}\int_0^u dv\psi_{4K}(v)\Big),$$

where i, j, denote the quark colour indices, α, β, are spinor indices, f_K is the kaon decay constant, ϕ_{2K}, ..., ψ_{4K}, are the kaon LCDAs of twist-2, 3, and 4, $\bar{u} = 1-u$, and $\mu_K = m_K^2/(m_u+m_s)$ denotes the chirally enhanced parameter. The diagram relevant for the derivation of the LC-OPE is shown in figure 2,[3] and its computation leads to the following result

$$F_q(p^2,q^2)\Big|_{\text{OPE}} = m_c f_K \int_0^1 du \sum_\phi \phi(u) \sum_{n=1}^{3} \frac{c^\phi_n(u,q^2)}{[\tilde{s}(u,q^2)-p^2]^n} \ln\left(\frac{m_s^2-q^2}{\mu^2}\right), \tag{4.4}$$

where $\phi = \{\phi_{2K}(u), \ldots, \psi_{4K}(u)\}$, and we have only retained the logarithmic term arising from the loop calculation, as this is the only relevant for the derivation of the dispersion relations. Moreover, the function $\tilde{s}(u,q^2)$ in eq. (4.4) reads

$$\tilde{s}(u,q^2) = \frac{m_c^2 - \bar{u}q^2 + u\bar{u}m_K^2}{u}, \tag{4.5}$$

and note that the coefficients $c^n_\phi(u,q^2)$ have been suitably manipulated so that the dependence on p^2 is contained only in the denominators. Their explicit expressions can be found in appendix A.

[3] The computation of the correlator in eq. (4.1) actually leads also to a second diagram corresponding to an annihilation topology. However, its contribution to the dispersion relations vanishes, as expected, given the LO-QCD accuracy of our analysis.

The OPE result in eq. (4.4) can be then linked to the desired matrix element via the derivation of hadronic dispersion relations, see e.g. refs. [44, 52] for details. In particular, after isolating the ground D^0- and K^+-meson states in the p^2- and q^2-channels, respectively, the contribution of the excited states and of the continuum is approximated by means of the quark hadron duality, the latter introducing two effective threshold parameters s_0^D and s_0^K. In addition, a Borel transformation in both channels i.e. $p^2 \to M^2$, $q^2 \to M'^2$, is performed in order to suppress the contribution of the continuum. The final sum-rule takes then the form

$$i\langle K^+K^-|O_1^s|D^0\rangle = \frac{e^{m_D^2/M^2} e^{m_K^2/M'^2}}{\pi^2 f_K f_D\, m_D^2} \int\limits_{m_s^2}^{s_0^K} ds' \int\limits_{m_c^2}^{s_0^D} ds\, e^{-s/M^2} e^{-s'/M'^2}\, \mathrm{Im}_{s'} \mathrm{Im}_s [F_q(s,s')]_{\mathrm{OPE}}\,, \tag{4.6}$$

where the expression for the imaginary part of $F_q(s,s')|_{\mathrm{OPE}}$ can be easily derived using the results given e.g. in the appendix of ref. [52].

It is worth emphasising that up to power corrections of the order of s_0^K/m_D^2, the result in eq. (4.6) can be factorized into the product of the two-point sum rule for the decay constant f_K and of the LCSR for the $D \to K$ form factor, both at LO-QCD, see e.g. ref. [44] for more details.

5 Numerical analysis and results

In this section we discuss the choice of the input parameters and present our results for the leading contribution to the amplitudes $\mathcal{A}_{\pi\pi}$, $\mathcal{A}_{KK}$, in eqs. (2.22), (2.23). For convenience, all the values used in the numerical analysis are also summarised in table 2.

For the pion and kaon LCDAs up to twist-4, we use the same expressions as in the corresponding LCSR studies of the $D \to \pi$ and $D \to K$ form factors [56]. In particular, the twist-2 LCDAs are obtained in the form of a truncated expansion in the Gegenbauer polynomials $C_n^{(3/2)}(u - \bar{u})$ with corresponding coefficients a_n^π and a_n^K, respectively. For the pion LCDAs, the odd Gegenbauer coefficients vanish, the values of a_2^π and a_4^π are taken from the recent study [54], while we neglect $a_{n>4}^\pi$. For the kaon LCDAs, we take the value of a_1^K from ref. [55], of a_2^K from ref. [56] (based on refs. [60, 61]), and neglect $a_{n>2}^K$.[4] The value of the chirally-enhanced parameters μ_π and μ_K, entering the twist-3 LCDAs, are obtained by employing well-known relations in chiral perturbation theory [65], and can be found e.g. in ref. [53]. The remaining parameters entering the twist-3 and twist-4 LCDAs are taken to be the same as in the study [56], corresponding to the values obtained in ref. [57]. As for the renormalisation scale μ, following ref. [56], we fix the central value to $\mu = 1.5\,\mathrm{GeV} \sim \sqrt{m_D^2 - m_c^2}$, and vary this in the interval $1\,\mathrm{GeV} \le \mu \le 2\,\mathrm{GeV}$. The scale dependence at one-loop accuracy for the LCDAs parameters is taken from ref. [57]. The running of the strong coupling, as well as of the quark masses in the $\overline{\mathrm{MS}}$-scheme, is implemented using the Mathematica package `RunDec` [66], fixing the values of $\alpha_s(m_Z)$, $\overline{m}_c(\overline{m}_c)$, and $\overline{m}_s(2\,\mathrm{GeV})$ as given in ref. [42].

[4] Other determinations of the Gegenbauer coefficients $a_n^{\pi,K}$ are available in the literature, see e.g. refs. [60, 62–64]. However, we have explicitly checked that the effect of using different values for these parameters is negligible compared to the accuracy of our study and amounts to at most a few percent, as expected, since these parametrise subleading corrections in the conformal expansion of the LCDAs.

Parameters of the LCDAs					
μ_π	(2.50 ± 0.30) GeV	[53]	μ_K	(2.49 ± 0.26) GeV	[53]
a_2^π	0.275 ± 0.055	[54]	a_1^K	0.10 ± 0.04	[55]
a_4^π	0.185 ± 0.065	[54]	a_2^K	0.25 ± 0.15	[56]
f_3^π	$(0.0045 \pm 0.0015)\,\mathrm{GeV}^2$	[57]	f_3^K	$(0.0045 \pm 0.0020)\,\mathrm{GeV}^2$	[57]
ω_3^π	-1.5 ± 0.7	[57]	ω_3^K	-1.2 ± 0.7	[57]
λ_3^π	0	—	λ_3^K	1.6 ± 0.4	[57]
δ_π^2	$(0.18 \pm 0.06)\,\mathrm{GeV}^2$	[57]	δ_K^2	$(0.20 \pm 0.06)\,\mathrm{GeV}^2$	[57]
ω_4^π	0.2 ± 0.1	[57]	ω_4^K	0.2 ± 0.1	[57]
$\kappa_{4\pi}$	0	—	κ_{4K}	-0.12 ± 0.01	[57]
Sum rule parameters					
s_0^π	$(0.7 \pm 0.1)\,\mathrm{GeV}^2$	[26]	s_0^K	$(1.2 \pm 0.1)\,\mathrm{GeV}^2$	[26]
M_π^2	$(1.0 \pm 0.5)\,\mathrm{GeV}^2$	[58]	M_K^2	$(1.0 \pm 0.5)\,\mathrm{GeV}^2$	[58]
s_0^D	$8.2^{+1.4}_{-0.6}\,\mathrm{GeV}^2$	(5.1)	M_D^2	$(4.5 \pm 1.0)\,\mathrm{GeV}^2$	[56]
CKM parameters					
$\lvert V_{us}\rvert$	$0.22500^{+0.00024}_{-0.00021}$	[59]	$\frac{\lvert V_{ub}\rvert}{\lvert V_{cb}\rvert}$	$0.08848^{+0.00224}_{-0.00219}$	[59]
$\lvert V_{cb}\rvert$	$0.04145^{+0.00035}_{-0.00061}$	[59]	δ	$\left(65.5^{+1.3}_{-1.2}\right)^\circ$	[59]
Other parameters					
$m_{\pi^\pm}$	0.13957 GeV	[42]	$m_{K^\pm}$	0.493677 GeV	[42]
f_π	(0.1302 ± 0.0008) GeV	[49]	f_K	(0.1557 ± 0.0003) GeV	[49]
$\overline{m}_c$	(1.27 ± 0.02) GeV	[42]	$\overline{m}_s$	$(0.0934^{+0.0086}_{-0.0034})$ GeV	[42]
$\alpha_s(m_Z)$	0.1179 ± 0.0009	[42]	m_{D^0}	1.86484 GeV	[42]
f_D	(0.2120 ± 0.0007) GeV	[49]	$\tau(D^0)$	(0.4013 ± 0.0010) ps	[42]

Table 2. Values of the parameters used in the numerical analysis. All numbers quoted correspond to the non-perturbative parameters evaluated at the renormalistation scale $\mu = 1\,\mathrm{GeV}$, apart from μ_π and μ_K, whose values are given at $\mu = 2\,\mathrm{GeV}$.

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Meson decay constants are determined precisely from Lattice QCD calculations, and we take their values from ref. [49]. Meson masses, also known very precisely, are taken from the PDG [42]. Finally, to predict branching fractions and $\Delta a_{\rm CP}^{\rm dir}$, we need in addition to fix the value of the Wilson coefficients and the CKM parameters. Leading-order results for $C_1(\mu_1)$ and $C_2(\mu_1)$ are implemented using the expressions given in ref. [43], see also table 3 of ref. [2], and we vary the renormalisation scale in the interval $\mu_1 = (1.5 \pm 0.5)\,\text{GeV}$. As for the CKM matrix elements, we use the standard parametrisation, taking the values of $|V_{us}|$, $|V_{cb}|$, $|V_{ub}/V_{cb}|$, and δ, from the CKMfitter [59] (similar values can be obtained from the UTFit [67]).

Concerning the choice of the sum rule parameters, we mostly follow ref. [26]. Thus, for the Borel parameters we adopt the same interval $M'^2 = (1\pm 0.5)\,\text{GeV}^2$ for both the pion and kaon channels, while for the respective threshold continuum parameters we use $s_0^\pi = (0.7\pm 0.1)\,\text{GeV}^2$ and $s_0^K = (1.2 \pm 0.1)\,\text{GeV}^2$ [68, 69]. The Borel parameter for the D-meson channel is chosen to be in the interval $M^2 = (4.5 \pm 1.0)\,\text{GeV}^2$, following ref. [56], while we fix the threshold continuum parameter s_0^D by differentiating with respect to $1/M^2$ both sides of the sum-rule in the case of pion final state, cf. eq. (4.6), obtaining the following relation

$$[m_{D^0}^2]_{\rm LCSR} = \frac{\displaystyle\int_{m_c^2}^{s_0^D} ds\, s\, e^{-s/M^2} \int_0^{s_0^\pi} ds' e^{-s'/M'^2}\, \text{Im}_{s'}\text{Im}_s\big[F_q(s,s')\big]_{\rm OPE}}{\displaystyle\int_{m_c^2}^{s_0^D} ds\, e^{-s/M^2} \int_0^{s_0^\pi} ds' e^{-s'/M'^2}\, \text{Im}_{s'}\text{Im}_s\big[F_q(s,s')\big]_{\rm OPE}}\,. \tag{5.1}$$

The sum-rule result for the D^0-meson mass squared is then fitted to its experimental value, leading to $s_0^D \approx 8.2\,\text{GeV}^2$ in correspondence of the central values of the remaining input parameters. The uncertainty on s_0^D, shown in table 2, is derived varying M^2 within its error range.

Using the values of the input parameters as described above, we obtain the following LCSR predictions for the tree-level matrix elements of the four modes analysed[5]

$$\langle K^+K^-|O_1^s|D^0\rangle\Big|_{\rm LCSR} = i\left(0.413^{+0.069}_{-0.111} \pm 0.165\right)\,\text{GeV}^3 = i\,0.413^{+0.179}_{-0.199}\,\text{GeV}^3, \tag{5.2}$$

$$\langle \pi^+\pi^-|O_1^d|D^0\rangle\Big|_{\rm LCSR} = i\left(0.236^{+0.047}_{-0.073} \pm 0.094\right)\,\text{GeV}^3 = i\,0.236^{+0.105}_{-0.119}\,\text{GeV}^3\,, \tag{5.3}$$

$$\langle \pi^+K^-|O_1^{sd}|D^0\rangle\Big|_{\rm LCSR} = i\left(0.261^{+0.049}_{-0.079} \pm 0.105\right)\,\text{GeV}^3 = i\,0.261^{+0.116}_{-0.131}\,\text{GeV}^3\,, \tag{5.4}$$

$$\langle K^+\pi^-|O_1^{ds}|D^0\rangle\Big|_{\rm LCSR} = i\left(0.380^{+0.067}_{-0.106} \pm 0.152\right)\,\text{GeV}^3 = i\,0.380^{+0.166}_{-0.185}\,\text{GeV}^3\,, \tag{5.5}$$

where the first uncertainties are due to variation of all the input parameters and of the renormalisation scale, and the second account for missing higher-order QCD effects, which we conservatively estimate within the 40% range.

With the LCSR results in eq. (5.2)–(5.5), and the expressions derived in section 2 for the amplitude and branching fraction, cf. eqs. (2.15), (2.22), we arrive at the following values

[5] The operators introduced in eqs. (5.4), (5.5), read $O_1^{q_1q_2} = (\bar q_1^i \Gamma_\mu c^i)(\bar u^j \Gamma^\mu q_2^j)$, with $q_i = d, s$.

for the decays considered

$$\mathcal{B}(D^0 \to K^+K^-)\big|_{\rm LCSR} = \left(3.67^{+3.90}_{-2.69}\right) \times 10^{-3}\,, \tag{5.6}$$

$$\mathcal{B}(D^0 \to \pi^+\pi^-)\big|_{\rm LCSR} = \left(1.40^{+1.53}_{-1.06}\right) \times 10^{-3}\,, \tag{5.7}$$

$$\mathcal{B}(D^0 \to \pi^+K^-)\big|_{\rm LCSR} = \left(2.99^{+3.26}_{-2.26}\right) \times 10^{-2}\,, \tag{5.8}$$

$$\mathcal{B}(D^0 \to K^+\pi^-)\big|_{\rm LCSR} = \left(1.80^{+1.93}_{-1.33}\right) \times 10^{-4}\,. \tag{5.9}$$

The central values of the LCSR predictions are in surprisingly good agreement with the data in eqs. (1.6)–(1.9); on the other hand, the uncertainties are large, which mainly follows from our very conservative treatment of missing corrections. Again, these results do not indicate the presence of unexpectedly big sub-leading effects.

Next, we discuss our estimate for $|\Delta a_{\rm CP}^{\rm dir}|$. With the results for $\mathcal{A}_{KK}$, $\mathcal{A}_{\pi\pi}$, obtained using eqs. (5.2), (5.3), and implementing the expressions for the penguin matrix elements $\mathcal{P}_{KK}$, $\mathcal{P}_{\pi\pi}$, from ref. [26], in order to account for correlations originating from the use of the same theoretical framework and input parameters, we obtain the following values for the ratios entering eqs. (2.17), (2.18), that is

$$\left|\frac{\mathcal{P}_{KK}}{\mathcal{A}_{KK}}\right|_{\rm LCSR} = 0.066^{+0.031}_{-0.029}\,, \qquad \left|\frac{\mathcal{P}_{\pi\pi}}{\mathcal{A}_{\pi\pi}}\right|_{\rm LCSR} = 0.089^{+0.042}_{-0.037}\,, \tag{5.10}$$

in perfect agreement with the results of ref. [26]. Note that we have again added a conservative 40% uncertainty, to account for missing higher-order QCD contributions, higher-twist effects, and corrections of the order $O(s_0^{K,\pi}/m_D^2)$ not included in ref. [26]. Using the results in eq. (5.10), and allowing for arbitrary strong phase differences, that is varying both $\sin\phi_{\pi\pi}$ and $\sin\phi_{KK}$ from -1 to 1 in eq. (2.19), we obtain the following upper bound for $|\Delta a_{\rm CP}^{\rm dir}|$, namely

$$|\Delta a_{\rm CP}^{\rm dir}|_{\rm LCSR} \leq 2.4 \times 10^{-4}\,, \tag{5.11}$$

which is about 6 times smaller than the current central experimental value in eq. (1.3).

Before concluding, it is worth emphasising that we also computed the tree-level matrix elements in eqs. (2.22), (2.23), using LCSR with D-meson LCDAs. In the latter case, which largely follows the calculation performed in the recent study of non-leptonic B-meson decays [52], the parameters of the D-meson LCDAs were taken from the corresponding ones for the B-meson assuming heavy quark flavour symmetry. Interestingly, including contributions with two-particle D-meson LCDAs up to twist-3, we obtained central values in agreement with the results in eqs. (5.6), (5.7). On the other hand, the respective uncertainties were also found to be huge, reflecting the poor precision with which the D-meson LCDAs are known.

6 Conclusion and outlook

In this paper we have employed the framework of LCSR to determine the SM predictions for several non-leptonic D^0 decays. Focusing on the computation of the leading contribution to the decay amplitude, i.e. considering only the color-allowed tree-level topology at leading order in QCD and including two-particle pion/kaon LCDAs up to twist-4, we find a surprisingly good agreement with the experimental values of the branching ratios for the modes $D^0 \to \pi^+ K^-$, $D^0 \to K^+ K^-$, $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+\pi^-$. Our results for the decay amplitudes, combined with the expressions for the penguin diagrams obtained within LCSR in ref. [26], lead to a bound for $|\Delta a_{\rm CP}^{\rm dir}|$ which is considerably lower than the current experimental determination.

To obtain a more profound statement, we plan to extend our theoretical study to include the following improvements:

1. Calculate the contribution of condensate and soft-gluon effects, i.e. three-particle LCDAs. Furthermore, higher-twist corrections could be investigated.

2. Extend our study to decays that are governed by color-suppressed tree-level topologies.

3. Calculate higher order perturbative QCD corrections to the sum-rule result. This will also allow to extend our approach to other topologies than the color-allowed tree-level one, like annihilation and penguin diagrams, and will have a crucial impact on the theoretical determination of the strong phases.

Finally, in light of the surprising agreement of our results — and in particular of the nQCDf estimates — with the experimental branching ratios, we consider it to be valuable to contemplate a scenario in which the above listed future improvements to the sum-rule predictions will not change considerably the current picture. In such a scenario we could conclude:

◇ The framework of LCSR can be successfully employed to predict the branching ratios for the decays $D^0 \to \pi^+\pi^-$, $K^+\pi^-$, π^+K^-, K^+K^-. The color-allowed tree-level diagrams give the dominant contribution to the branching fractions and the remaining topologies only lead to smaller corrections. The current large uncertainties in the theoretical predictions presented in eqs. (5.6)–(5.9) can be systematically reduced including the improvements listed above.

◇ The size of $\mathrm{SU}(3)_f$ breaking effects turns out to be very large and it is well accommodated by the values of the decay constants and form factors. Comparing the central values of eq. (5.2) and eq. (5.3) we find $\mathrm{SU}(3)_f$ breaking effects of the order of 75% at the amplitude level, thus questioning the applicability of this symmetry for D-meson decays. In this respect, we would like to note that studies like refs. [70–72] only set a lower limit of 30% on the possible size of the $\mathrm{SU}(3)_f$ breaking, while allowing also much larger values, in consistency with our finding.

◇ The upper bound on $|\Delta a_{\rm CP}^{\rm dir}|$, given in eq. (5.11), is found to be about a factor of 6 smaller than the current experimental average given in eq. (1.3). If the experimental numbers will stay, in particular the central values for the individual CP asymmetries given in eqs. (1.4), (1.5), then this could be a first glimpse of physics beyond the SM.

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Note added. While this work was being completed, ref. [73] appeared on the arXiv. Assuming the validity of the SM for the description of D^0 decays, the authors find experimental evidence for large penguin/rescattering effects. Despite being a very interesting study, this cannot provide a decisive statement on the actual nature of these large effects, namely if they could be accommodated within the SM or not. Therefore, the latter work does not contradict the conclusions of our paper, where first steps towards a fully QCD based calculation indicate that the current experimental value for $\Delta a_{\rm CP}^{\rm dir}$ cannot be reproduced in the SM.

A Expressions of the OPE coefficients

Here, we list the non-vanishing coefficients $c_n^{\phi}(u, q^2)$, introduced in eq. (4.4).

$$c_1^{\phi_{2K}} = \frac{m_c\,(m_s^2-q^2)^2}{8\pi^2 q^6 u^2}\Big[m_c^2\left(2m_s^2+q^2\right)+q^2\left(q^2(2u-1)-u^2m_K^2\right) + m_s^2\left(q^2(u-2)-2u^2m_K^2\right)\Big]\,, \tag{A.1}$$

$$c_1^{\phi_{3K}^p} = \frac{\mu_K\,(m_s^2-q^2)^2}{8\pi^2 q^6 u}\Big[m_c^2\left(2m_s^2+q^2\right)+q^2\left(-u^2m_K^2+2q^2u-q^2\right) + m_s^2\left(-2u^2m_K^2+q^2u+q^2\right)\Big]\,, \tag{A.2}$$

$$c_1^{\phi_{3K}^\sigma} = \frac{\mu_K\,(m_s^2-q^2)^2}{48\pi^2 q^6 u^2}\Big[m_c^2\left(2m_s^2+q^2\right)+q^2\left(-u^2m_K^2+4q^2u-3q^2\right) + m_s^2\left(-2u^2m_K^2+2q^2u-3q^2\right)\Big]\,, \tag{A.3}$$

$$\begin{aligned} c_2^{\phi_{3K}^\sigma} = \frac{\mu_K\,(m_s^2-q^2)^2}{48\pi^2 q^6 u^3}\Big[&m_c^4\left(2m_s^2+q^2\right) \\ &+ m_c^2\left(m_s^2\left(q^2(u-3)-4u^2m_K^2\right)+2q^2u\left(q^2-um_K^2\right)\right) \\ &+\left(q^2-u^2m_K^2\right)\left(m_s^2\left(-2u^2m_K^2+(u+1)q^2\right)+q^2\left(-u^2m_K^2+(2u-1)q^2\right)\right)\Big]\,, \end{aligned} \tag{A.4}$$

$$c_2^{\phi_{4K}} = \frac{m_c^3\,(m_s^2-q^2)^2\,(2m_s^2+q^2)}{16\pi^2 q^6 u^3}\,, \tag{A.5}$$

$$c_3^{\phi_{4K}} = -\frac{m_c^3\,(m_s^2-q^2)^2}{16\pi^2 q^6 u^4}\Big[m_c^2\left(2m_s^2+q^2\right)+q^2\left(q^2(2u-1)-u^2m_K^2\right) + m_s^2\left(q^2(u-2)-2u^2m_K^2\right)\Big]\,, \tag{A.6}$$

$$c_1^{\psi 4K} = \frac{m_c \left(m_s^2 - q^2\right)^2 \left(2m_s^2 + q^2\right)}{8\pi^2 q^6 u}, \tag{A.7}$$

$$c_2^{\psi 4K} = -\frac{m_c \left(m_s^2 - q^2\right)^2}{8\pi^2 q^6 u^2} \Big[m_c^2 \left(2m_s^2 + q^2\right) + q^2 \left(-u^2 m_K^2 + 2q^2 u - q^2\right) + m_s^2 \left(-2u^2 m_K^2 + q^2 u + q^2\right) \Big]. \tag{A.8}$$

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