Scotogenic neutrino masses with gauged matter parity and gauge coupling unification

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ABSTRACT: Building up on previous work we propose a Dark Matter (DM) model with gauged matter parity and dynamical gauge coupling unification, driven by the same physics responsible for scotogenic neutrino mass generation. Our construction is based on the extended gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N$, whose spontaneous breaking leaves a residual conserved matter parity, M_P , stabilizing the DM particle candidates of the model. The key role is played by Majorana $SU(3)_L$ -octet leptons, allowing the successful gauge coupling unification and a one-loop scotogenic neutrino mass generation. Theoretical consistency allows for a *plethora* of new particles at the $\lesssim \mathcal{O}(10)$ TeV scale, hence accessible to future collider and low-energy experiments.

KEYWORDS: Beyond Standard Model, Gauge Symmetry, Neutrino Physics

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1 Introduction

The supersymmetric approach to gauge coupling unification has so far not been vindicated experimentally, neither at colliders, nor through the observation of proton decay [1]. However, the historical discovery of neutrino oscillations and the growing evidence for a weakly interacting massive particle (WIMP) as, perhaps, the most viable candidate for cosmological Dark Matter motivate us to seek new roads to unification. We start from the phenomenologically safe foundations provided by the Standard Model (SM) with the gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ extended to $SU(3)_c \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N$. The use of $SU(3)_L$ as an extended electroweak symmetry has been advocated by its ability to explain the observed number of families through the anomaly cancellation requirement [2, 3]. Moreover, a suggestion was made within the original $SU(3)_c \otimes SU(3)_L \otimes U(1)$ framework proposed in [2] of how neutrino masses and gauge coupling unification could emerge together [4], so that the physics responsible for small neutrino masses could also drive the unification of the gauge couplings. This model provides a radiative seesaw mechanism for calculable neutrino masses, arising from quantum corrections mediated by new $SU(3)_c \otimes SU(3)_L \otimes U(1)$ gauge bosons. Apart from its somewhat *ad hoc* nature, the model lacked an explanation for cosmological dark matter, another major drawback of particle physics. The key property of any dark matter candidate is its stability on cosmological time scales, suggesting the existence of a (nearly) preserved stabilizing symmetry. Recently it has been proposed that the latter could be a discrete residual matter parity symmetry, surviving the spontaneous breaking of the extended gauge symmetry [5–7]. This happens

within a simple $U(1)$ -extension of the gauge symmetry, allowing the implementation of a conserved matter parity

$$M_P = (-1)^{3(B-L)+2s}. \quad (1.1)$$

analogous to the R-parity in supersymmetric theories.

In this paper we put together all these attractive features, proposing a theory of calculable scotogenic Majorana neutrino masses [8] in the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N$ framework, or 3-3-1-1, for short. Dark matter is a weakly interacting massive particle (WIMP) that mediates neutrino mass generation. Gauge couplings unify for a 3-3-1 scale just above the TeV range, making the model directly testable at the LHC. For that we introduce an $SU(3)_L$ Majorana octet, responsible for both neutrino mass generation and gauge coupling unification.

The paper is organized as follows. In section 2 we present the model setup, while in section 3 we discuss the scalar sector and symmetry breaking. In section 4 we analyse the neutrino mass matrix and the scotogenic mechanism. Dark matter and gauge coupling unification are discussed in sections 5 and 6, respectively. A brief outlook is given in section 7.

2 The model setup

2.1 Field content

We study a 3-3-1-1 model based on the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N$ gauge symmetry introduced in [7] and the implementation of a conserved matter parity according to eq. (1.1). This is very much analogous to the R-parity symmetry imposed on the supersymmetric theories. Except that the M_P discrete symmetry is not imposed *ad hoc* as a global symmetry, but rather arises as a remnant of the spontaneously broken $B - L$ gauge group [5–7]. It leads to the stability of the lightest M_P -odd particle, and hence to a potentially viable WIMP dark matter candidate. In our model, electric charge Q and the $B - L$ generators are embedded into the gauge symmetry as

$$Q = T_3 - \frac{T_8}{\sqrt{3}} + X, \quad (2.1)$$

$$B - L = -\frac{2}{\sqrt{3}}T_8 + N, \quad (2.2)$$

with T_i ($i = 1, 2, 3, \dots, 8$), X and N being the respective generators of $SU(3)_L$, $U(1)_X$ and $U(1)_N$. The lepton sector of the model includes three leptonic triplets¹

$$l_{aL} = \begin{pmatrix} \nu_a \\ e_a \\ N_a \end{pmatrix}_L, \quad (2.3)$$

¹Notice that in the original 3-3-1 formulation [2], left-handed leptons transform as anti-triplets of $SU(3)_L$. This configuration can be recovered by exchanging all triplets and anti-triplets in the present work.

Field	SU(3) _c	SU(3) _L	U(1) _X	U(1) _N	Q	$M_P = (-1)^{3(B-L)+2s}$
q_{iL}	3	$\bar{\mathbf{3}}$	0	0	$(-\frac{1}{3}, \frac{2}{3}, -\frac{1}{3})^T$	$(++-)^T$
q_{3L}	3	3	$\frac{1}{3}$	$\frac{2}{3}$	$(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3})^T$	$(++-)^T$
u_{aR}	3	1	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	+
d_{aR}	3	1	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	+
U_{3R}	3	1	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{2}{3}$	-
D_{iR}	3	1	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	-
l_{aL}	1	3	$-\frac{1}{3}$	$-\frac{2}{3}$	$(0, -1, 0)^T$	$(++-)^T$
e_{aR}	1	1	-1	-1	-1	+
ν_{iR}	1	1	0	-4	0	-
ν_{3R}	1	1	0	5	0	+
Ω_{aL}	1	8	0	0	$\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$	$\begin{pmatrix} - & - & + \\ - & - & + \\ + & + & - \end{pmatrix}$
η	1	3	$-\frac{1}{3}$	$\frac{1}{3}$	$(0, -1, 0)^T$	$(++-)^T$
ρ	1	3	$\frac{2}{3}$	$\frac{1}{3}$	$(1, 0, 1)^T$	$(++-)^T$
χ	1	3	$-\frac{1}{3}$	$-\frac{2}{3}$	$(0, -1, 0)^T$	$(--+)^T$
ϕ	1	1	0	2	0	+
σ	1	1	0	1	0	-

Table 1. 3311 model field content ($a = 1, 2, 3$ and $i = 1, 2$ are family indices). Note the non-standard charges of the ν_R .

each with its new neutral lepton N_{aL} , $a = 1, 2, 3$. The quark fields are arranged as

$$q_{iL} = \begin{pmatrix} d_i \\ -u_i \\ D_i \end{pmatrix}_L \quad q_{3L} = \begin{pmatrix} u_3 \\ d_3 \\ U_3 \end{pmatrix}_L. \quad (2.4)$$

Anomaly cancellation requires that two families of quarks q_{iL} ; $i = 1, 2$ transform as anti-triplets and one q_{3L} as a triplet, as in the original Singer-Valle-Schechter (SVS) scheme [2]. This way, the number of the fermion families is equal to the number of colors, thus allowing a natural explanation of the number of generations of the SM fermions. The full field content is shown in table 1.

Notice that, in addition to the field content of the model of ref. [7], our present setup includes three Majorana octets Ω_{aL} ($a = 1, 2, 3$). They are crucial for achieving a successful gauge coupling unification scenario, as well as for providing the tiny masses of the light active neutrinos via a one-loop scotogenic mechanism. The latter is possible thanks to the inclusion of a scalar singlet σ . This electrically neutral field has a nontrivial charge under the preserved remnant matter-parity symmetry, providing a viable scalar WIMP

dark matter candidate. Our model also includes three right-handed neutrinos with non-standard $U(1)_N$ charges, which have been introduced in [9] to ensure an anomaly free gauge symmetry. These fields do not take part in the neutrino mass generation mechanism, and they do not mix with the other neutral fermions of the model. The two ν_{iR} fermion fields can acquire a Majorana mass after spontaneous symmetry breaking, by the inclusion of a scalar field transforming as $(\mathbf{1}, \mathbf{1}, 0, 8)$, while a mass term for ν_{3R} requires a scalar with quantum numbers $(\mathbf{1}, \mathbf{1}, 0, -10)$. In order to keep the analysis of the scalar sector as simple as possible here we do not include those extra scalar fields.

The gauged $B - L$ symmetry is spontaneously broken by two units as the singlet scalar ϕ develops a vacuum expectation value (VEV). As seen from the assignments in table 1 this leaves a discrete remnant symmetry M_P specified in eq. (1.1). The most general VEV alignment for the scalars consistent with a preserved M_P symmetry is [10]

$$\langle \eta \rangle = \frac{1}{\sqrt{2}}(v_1, 0, 0)^T, \quad \langle \rho \rangle = \frac{1}{\sqrt{2}}(0, v_2, 0)^T, \quad \langle \chi \rangle = \frac{1}{\sqrt{2}}(0, 0, w)^T, \quad \langle \phi \rangle = \frac{1}{\sqrt{2}}\Lambda, \quad \langle \sigma \rangle = 0. \quad (2.5)$$

Here we will assume the hierarchy $w, \Lambda, \gg v_1, v_2$, leading to the following spontaneous symmetry breaking (SSB) pattern

$$\begin{aligned} & SU(3)_C \times SU(3)_L \times U(1)_X \times U(1)_N \\ & \quad \downarrow w, \Lambda \\ & SU(3)_C \times SU(2)_L \times U(1)_Y \times M_P \\ & \quad \downarrow v_1, v_2 \\ & SU(3)_C \times U(1)_Q \times M_P. \end{aligned} \quad (2.6)$$

2.2 Majorana octet lepton fields

The presence of the $SU(3)_L$ octet fermions, Ω , is a key ingredient of the model in order to ensure gauge coupling unification and to generate radiative masses for the active neutrinos. In this subsection we will show the explicit derivation of the electric charge and matter parity assignments of the components of the Majorana leptonic octets. Our starting point is the $SU(3)_L$ algebra, which is described by

$$[T_a, T_b] = if_{abc}T_c, \quad T_a = \frac{\lambda_a}{2}, \quad (2.7)$$

where λ_a , $a = 1, \dots, 8$ are the Gell-Mann matrices. The corresponding Cartan subalgebra is spanned by $H_1 = T_3$ and $H_2 = T_8$. The $SU(3)_L$ algebra in the Cartan basis reads

$$[H_i, E_{\alpha_a}] = (\alpha_a)_i E_{\alpha_a}, \quad E_{\alpha_a}^\dagger = E_{-\alpha_a}, \quad i = 1, 2, \quad a = 1, 2, 3, \quad (2.8)$$

where the ladder operators E_{α_a} are given by

$$E_{\alpha_1} = \frac{1}{\sqrt{2}} (T_4 + iT_5) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (2.9)$$

$$E_{\alpha_2} = \frac{1}{\sqrt{2}} (T_6 - iT_7) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad (2.10)$$

$$E_{\alpha_3} = \frac{1}{\sqrt{2}} (T_1 + iT_2) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (2.11)$$

and the coefficients $(\alpha_a)_i$ are the components of the roots

$$\vec{\alpha}_1 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2} \right), \quad \vec{\alpha}_2 = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2} \right), \quad \vec{\alpha}_3 = (1, 0). \quad (2.12)$$

The $SU(3)_L$ leptonic octet, Ω , can be decomposed in the Cartan basis of the $SU(3)_L$ generators as follows

$$\Omega = \Omega^{(3)} T_3 + \Omega^{(8)} T_8 + \Omega_{\pm}^{(1)} E_{\pm\alpha_1} + \Omega_{\pm}^{(2)} E_{\pm\alpha_2} + \Omega_{\pm}^{(3)} E_{\pm\alpha_3}. \quad (2.13)$$

Using eqs. (1.1), (2.1), (2.2) and the charge assignments $Q_X(\Omega) = Q_N(\Omega) = 0$ given in table 1, we find the Q and $B - L$ charge assignments as well as matter parity M_P of the component fields in (2.13), namely

$$\Omega(T_3, T_8): \quad \Omega^{(3)}(0,0), \quad \Omega^{(8)}(0,0), \quad \Omega_{\pm}^{(1)}\left(\pm\frac{1}{2}, \pm\frac{\sqrt{3}}{2}\right), \quad \Omega_{\pm}^{(2)}\left(\pm\frac{1}{2}, \mp\frac{\sqrt{3}}{2}\right), \quad \Omega_{\pm}^{(3)}(\pm 1, 0) \quad (2.14)$$

$$\Omega(Q, B-L; M_P): \quad \Omega^{(3)}(0,0;-1), \quad \Omega^{(8)}(0,0;-1), \quad \Omega_{\pm}^{(1)}(0, \mp 1; +1), \quad \Omega_{\pm}^{(2)}(\pm 1, \pm 1; +1), \quad \Omega_{\pm}^{(3)}(\pm 1, 0; -1). \quad (2.15)$$

Then, the leptonic Octet can be expressed in matrix form as

$$\Omega_L = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} \Omega^{(3)} + \frac{1}{\sqrt{6}} \Omega^{(8)} & \Omega_+^{(3)} & \Omega_+^{(1)} \\ \Omega_-^{(3)} & -\frac{1}{\sqrt{2}} \Omega^{(3)} + \frac{1}{\sqrt{6}} \Omega^{(8)} & \Omega_-^{(2)} \\ \Omega_-^{(1)} & \Omega_+^{(2)} & -\frac{2}{\sqrt{6}} \Omega^{(8)} \end{pmatrix}_L, \quad (2.16)$$

Note that $(\Omega^c)_{ij} = (\Omega_{ji})^c$. Therefore, $Q(\Omega_{ij}) = Q(\Omega_{ij}^c)$ and $M_P(\Omega_{ij}) = M_P(\Omega_{ij}^c)$, as should be for a Majorana field $\Omega_M = \Omega_L \oplus (\Omega_L)^c$. From eqs. (2.15) and (2.16) we find the charge Q and M_P assignments of the leptonic octet, Ω , shown in table 1.

In the following discussion, we will denote the components of Ω in the canonical normalization as $\Omega^{(3)} = \sqrt{2}\Psi^0$, $\Omega^{(8)} = \sqrt{2}\tilde{N}$, $\Omega_{\pm}^{(3)} = \sqrt{2}E^{\pm}$, $\Omega_{\pm}^{(2)} = \sqrt{2}\tilde{E}^{\pm}$, $\Omega_{+}^{(1)} = \sqrt{2}\Delta$, $\Omega_{-}^{(1)} = \sqrt{2}\tilde{\Delta}$. In this notation, the octet takes the form

$$\Omega_L = \begin{pmatrix} \frac{1}{\sqrt{2}}\Psi + \frac{1}{\sqrt{6}}\tilde{N} & E^{+} & \Delta \\ E^{-} & -\frac{1}{\sqrt{2}}\Psi + \frac{1}{\sqrt{6}}\tilde{N} & \tilde{E}^{-} \\ \tilde{\Delta} & \tilde{E}^{+} & -\frac{2}{\sqrt{6}}\tilde{N} \end{pmatrix}_L, \quad (2.17)$$

$$(\Omega_L)^c = \begin{pmatrix} \frac{1}{\sqrt{2}}(\Psi_L)^c + \frac{1}{\sqrt{6}}(\tilde{N}_L)^c & (E_L^{-})^c & (\tilde{\Delta}_L)^c \\ (E_L^{+})^c & -\frac{1}{\sqrt{2}}(\Psi_L)^c + \frac{1}{\sqrt{6}}(\tilde{N}_L)^c & (\tilde{E}_L^{+})^c \\ (\Delta_L)^c & (\tilde{E}_L^{-})^c & -\frac{2}{\sqrt{6}}(\tilde{N}_L)^c \end{pmatrix}, \quad (2.18)$$

$$\bar{\Omega}_L = \begin{pmatrix} \frac{1}{\sqrt{2}}\bar{\Psi}_L + \frac{1}{\sqrt{6}}\bar{\tilde{N}}_L & \bar{E}_L^{-} & \bar{\Delta}_L \\ \bar{E}_L^{+} & -\frac{1}{\sqrt{2}}\bar{\Psi}_L + \frac{1}{\sqrt{6}}\bar{\tilde{N}}_L & \bar{\tilde{E}}_L^{+} \\ \bar{\Delta}_L & \bar{\tilde{E}}_L^{-} & -\frac{2}{\sqrt{6}}\bar{\tilde{N}}_L \end{pmatrix}. \quad (2.19)$$

3 Symmetry breaking

The most general 3-3-1-1 gauge-invariant scalar potential of the model is given by

$$\begin{aligned} V = & \mu_1^2 \rho^\dagger \rho + \mu_2^2 \chi^\dagger \chi + \mu_3^2 \eta^\dagger \eta + \mu_4^2 \phi^\dagger \phi + \mu_5^2 \sigma^\dagger \sigma \\ & + \lambda_1 (\rho^\dagger \rho)^2 + \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 (\eta^\dagger \eta)^2 \\ & + \lambda_4 (\rho^\dagger \rho)(\chi^\dagger \chi) + \lambda_5 (\rho^\dagger \rho)(\eta^\dagger \eta) + \lambda_6 (\chi^\dagger \chi)(\eta^\dagger \eta) \\ & + \lambda_7 (\rho^\dagger \chi)(\chi^\dagger \rho) + \lambda_8 (\rho^\dagger \eta)(\eta^\dagger \rho) + \lambda_9 (\chi^\dagger \eta)(\eta^\dagger \chi) \\ & + \lambda_{10} (\phi^\dagger \phi)(\rho^\dagger \rho) + \lambda_{11} (\phi^\dagger \phi)(\chi^\dagger \chi) + \lambda_{12} (\phi^\dagger \phi)(\eta^\dagger \eta) \\ & + \lambda_{13} (\sigma^\dagger \sigma)(\rho^\dagger \rho) + \lambda_{14} (\sigma^\dagger \sigma)(\chi^\dagger \chi) + \lambda_{15} (\sigma^\dagger \sigma)(\eta^\dagger \eta) \\ & + \lambda_{16} (\phi^\dagger \phi)^2 + \lambda_{17} (\sigma^\dagger \sigma)^2 + \lambda_{18} (\phi^\dagger \phi)(\sigma^\dagger \sigma) + \lambda_{19} [(\sigma^\dagger \phi)(\eta^\dagger \chi) + \text{h.c.}] \\ & + \left[\frac{\mu_t}{2} \rho \eta \chi + \frac{\mu_s}{2} \phi^\dagger \sigma \sigma + \frac{\mu_u}{2} (\eta^\dagger \chi) \sigma + \text{h.c.} \right], \end{aligned} \quad (3.1)$$

where the λ_k ($k = 1, 2, \dots, 19$) are dimensionless parameters, whereas the μ_r ($r = 1, 2, \dots, 5$), μ_t , μ_s , μ_u have dimension of mass. We emphasize that in our model we have no need of imposing any global symmetries, all the ingredients for the scotogenic neutrino mass generation are already contained in the gauge symmetry group. To ensure that the

M_P symmetry remains conserved we require $\mu_5^2 > 0$, implying that the M_P -odd scalar σ does not develop a nonzero VEV.

The minimization conditions of the scalar potential yield the following relations:

$$\begin{aligned}\mu_1^2 &= \frac{v_1 w \mu_t - v_2 (\lambda_{10} \Lambda^2 + 2\lambda_1 v_2^2 + \lambda_5 v_1^2 + \lambda_4 w^2)}{2v_2}, \\ \mu_2^2 &= \frac{v_1 v_2 \mu_t - w (\lambda_{11} \Lambda^2 + \lambda_4 v_2^2 + \lambda_6 v_1^2 + 2\lambda_2 w^2)}{2w}, \\ \mu_3^2 &= \frac{v_2 w \mu_t - v_1 (\lambda_{12} \Lambda^2 + 2\lambda_3 v_1^2 + \lambda_5 v_2^2 + \lambda_6 w^2)}{2v_1}, \\ \mu_4^2 &= -\frac{1}{2} (2\lambda_{16} \Lambda^2 + \lambda_{10} v_2^2 + \lambda_{12} v_1^2 + \lambda_{11} w^2).\end{aligned}\quad (3.2)$$

The scalar potential of the model has been previously analyzed in ref. [9]. Here we just quote the results relevant for the modified neutrino mass generation mechanism of the present model. Decomposing the scalar multiplets in components as

$$\eta = \begin{pmatrix} \frac{v_1 + s_1 + ia_1}{\sqrt{2}} \\ \eta_2^- \\ \frac{s'_3 + ia'_3}{\sqrt{2}} \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho_1^+ \\ \frac{v_2 + s_2 + ia_2}{\sqrt{2}} \\ \rho_3^+ \end{pmatrix}, \quad \chi = \begin{pmatrix} \frac{s'_1 + ia'_1}{\sqrt{2}} \\ \chi_2^- \\ \frac{w + s_3 + ia_3}{\sqrt{2}} \end{pmatrix}, \quad \phi = \frac{\Lambda + s_\phi + ia_\phi}{\sqrt{2}}, \quad \sigma = \frac{s_\sigma + ia_\sigma}{\sqrt{2}}. \quad (3.3)$$

The CP-even neutral scalar sector of the model was studied in [9]. Besides the 125 GeV SM-like Higgs identified with

$$h \approx \frac{v_1 s_1 + v_2 s_2}{\sqrt{v_1^2 + v_2^2}}, \quad (3.4)$$

and having a mass

$$m_h^2 \approx \frac{\Lambda^2 v_1 v_2 w \mu_t ((\lambda_4 \lambda_{11} - 2\lambda_2 \lambda_{10}) \lambda_{12} - \lambda_5 (\lambda_{11}^2 - 4\lambda_2 \lambda_{16}) + \lambda_6 (\lambda_{10} \lambda_{11} - 2\lambda_4 \lambda_{16})) w^2 - \lambda_{16} \mu_t^2}{m_{H_1}^2 m_{H_2}^2 m_{H_3}^2}. \quad (3.5)$$

There are three additional heavy Higgs bosons, given by the approximate expressions

$$H_1 \approx \frac{v_2 s_1 - v_1 s_2}{\sqrt{v_1^2 + v_2^2}}, \quad m_{H_1}^2 \approx \frac{(v_1^2 + v_2^2) w \mu_t}{2v_1 v_2}, \quad (3.6)$$

$$H_2 \approx \cos \xi s_3 - \sin \xi s_4, \quad m_{H_2}^2 \approx \lambda_{16} \Lambda^2 + \lambda_2 w^2 - \sqrt{\lambda_{16}^2 \Lambda^4 + \lambda_2^2 w^4 + \lambda_{11}^2 \Lambda^2 w^2 - 2\lambda_2 \lambda_{16} \Lambda^2 w^2},$$

$$H_3 \approx \sin \xi s_3 + \cos \xi s_4, \quad m_{H_3}^2 \approx \lambda_{16} \Lambda^2 + \lambda_2 w^2 + \sqrt{\lambda_{16}^2 \Lambda^4 + \lambda_2^2 w^4 + \lambda_{11}^2 \Lambda^2 w^2 - 2\lambda_2 \lambda_{16} \Lambda^2 w^2},$$

valid under the assumption $\Lambda, w, \mu_t \gg v_1, v_2$. There are also two physical real scalars φ_1, φ_2 and one Nambu-Goldstone boson G_1 , defined as

$$\begin{pmatrix} \varphi_1 \\ \varphi_2 \\ G_1 \end{pmatrix} = U^s \begin{pmatrix} s'_1 \\ s'_3 \\ s_\sigma \end{pmatrix} = \begin{pmatrix} \frac{v_1 \cos \theta_s}{\sqrt{w^2 + v_1^2}} & \frac{w \cos \theta_s}{\sqrt{w^2 + v_1^2}} & \sin \theta_s \\ -\frac{v_1 \sin \theta_s}{\sqrt{w^2 + v_1^2}} & -\frac{w \sin \theta_s}{\sqrt{w^2 + v_1^2}} & \cos \theta_s \\ \frac{w}{\sqrt{w^2 + v_1^2}} & -\frac{v_1}{\sqrt{w^2 + v_1^2}} & 0 \end{pmatrix} \begin{pmatrix} s'_1 \\ s'_3 \\ s_\sigma \end{pmatrix}, \quad (3.7)$$

where the mixing angle θ_s satisfies the relation

$$\tan 2\theta_s = \frac{2v_1 w \sqrt{v_1^2 + w^2} (\lambda_{19}\Lambda + \mu_u)}{v_1 w (-2\mu_5^2 - \Lambda(\lambda_{18}\Lambda + 2\mu_s) - \lambda_{13}v_2^2 - \lambda_{15}v_1^2 + \lambda_9(v_1^2 + w^2) - \lambda_{14}w^2) + v_2\mu_t(v_1^2 + w^2)}. \quad (3.8)$$

The emergence of a Nambu-Goldstone boson in the CP-even scalar sector follows from of the existence of a non-hermitian gauge boson X^0 , whose real part must absorb G_1 after the SSB, so as to acquire a consistent mass, while its imaginary part absorbs an analogous CP-odd Goldstone boson, as we discuss below.

The CP-odd neutral sector consists of four Nambu-Goldstone bosons, $G_{2,3,4,5}$, and three massive states, denoted as A_1 , $\tilde{\varphi}_1$ and $\tilde{\varphi}_2$. Three of these four Nambu-Goldstone bosons are given by

$$G_2 = \frac{v_1 a_1 - v_2 a_2}{\sqrt{v_1^2 + v_2^2}}, \quad G_3 = \frac{v_1 a_1 - w a_2}{\sqrt{v_1^2 + w^2}}, \quad G_4 = a_\phi, \quad (3.9)$$

and correspond to the longitudinal components of the physical gauge bosons, Z , Z' , Z'' , respectively. On the other hand, the massive state A_1 is

$$A_1 = \frac{v_2 w a_1 + v_1 w a_2 + v_1 v_2 a_3}{\sqrt{(v_2 w)^2 + (v_1 w)^2 + (v_1 v_2)^2}}, \quad m_{A_1}^2 = \frac{\mu_t (v_1^2 w^2 + v_2^2 w^2 + v_2^2 v_1^2)}{2v_1 v_2 w}, \quad (3.10)$$

whereas the two physical states $\tilde{\varphi}_1$, $\tilde{\varphi}_2$ and the fifth Goldstone G_5 are defined as

$$\begin{pmatrix} \tilde{\varphi}_1 \\ \tilde{\varphi}_2 \\ G_5 \end{pmatrix} = U^a \begin{pmatrix} a'_1 \\ a'_3 \\ a_\sigma \end{pmatrix} = \begin{pmatrix} -\frac{v_1 \cos \theta_a}{\sqrt{w^2 + v_1^2}} & \frac{w \cos \theta_a}{\sqrt{w^2 + v_1^2}} & \sin \theta_a \\ \frac{v_1 \sin \theta_a}{\sqrt{w^2 + v_1^2}} & -\frac{w \sin \theta_a}{\sqrt{w^2 + v_1^2}} & \cos \theta_a \\ \frac{w}{\sqrt{w^2 + v_1^2}} & \frac{v_1}{\sqrt{w^2 + v_1^2}} & 0 \end{pmatrix} \begin{pmatrix} a'_1 \\ a'_3 \\ a_\sigma \end{pmatrix}, \quad (3.11)$$

with mixing angle

$$\tan 2\theta_a = \frac{2v_1 w \sqrt{v_1^2 + w^2} (\mu_u - \lambda_{19}\Lambda)}{v_1 w (-\lambda_{18}\Lambda^2 - 2\mu_5^2 + 2\Lambda\mu_s - \lambda_{13}v_2^2 - \lambda_{15}v_1^2 + \lambda_9(v_1^2 + w^2) - \lambda_{14}w^2) + v_2\mu_t(v_1^2 + w^2)}. \quad (3.12)$$

Notice that the Goldstone bosons G_5 and G_1 combine into a single complex neutral would-be Goldstone, absorbed by the longitudinal component of the non-Hermitian neutral gauge boson X^0 .

The real scalars φ_1 , φ_2 , $\tilde{\varphi}_1$ and $\tilde{\varphi}_2$ acquire squared masses given by

$$m_{\varphi_{1,2}}^2 = \frac{1}{4v_1 w} \left\{ v_1 w (\lambda_{18}\Lambda^2 + 2\mu_5^2 + 2\Lambda\mu_s + \lambda_{13}v_2^2 + \lambda_{15}v_1^2 + \lambda_9(v_1^2 + w^2) + \lambda_{14}w^2) + v_2\mu_t(v_1^2 + w^2) \right. \\ \left. \mp \mathcal{F}_s \left\{ (v_1 w (\lambda_{18}\Lambda^2 + 2\mu_5^2 + 2\Lambda\mu_s + \lambda_{13}v_2^2 + \lambda_{15}v_1^2 + \lambda_9(v_1^2 + w^2) + \lambda_{14}w^2) + v_2\mu_t(v_1^2 + w^2))^2 \right. \right. \\ \left. \left. - 4v_1 w (v_1^2 + w^2) (v_2\mu_t(\lambda_{18}\Lambda^2 + 2\mu_5^2 + 2\Lambda\mu_s + \lambda_{13}v_2^2 + \lambda_{14}w^2) + v_1 w (\lambda_9(\lambda_{18}\Lambda^2 + 2\mu_5^2 + 2\Lambda\mu_s + \lambda_{14}w^2) \right. \right. \\ \left. \left. - (\lambda_{19}\Lambda + \mu_u)^2 + \lambda_9\lambda_{13}v_2^2) + \lambda_{15}v_2v_1\mu_t + \lambda_9\lambda_{15}v_1^3w) \right\}^{1/2} \right\},$$

$$\begin{aligned}
 \mathcal{F}_s = & \text{sgn} \left\{ v_1 w \left(-2\mu_5^2 - \Lambda(\lambda_{18}\Lambda + 2\mu_s) - \lambda_{13}v_2^2 - \lambda_{15}v_1^2 + \lambda_9(v_1^2 + w^2) - \lambda_{14}w^2 \right) + v_2\mu_t(v_1^2 + w^2) \right\}, \\
 m_{\varphi_{1,2}}^2 = & \frac{1}{4v_1w} \left\{ v_1w \left(\lambda_{18}\Lambda^2 + 2\mu_5^2 - 2\Lambda\mu_s + \lambda_{13}v_2^2 + \lambda_{15}v_1^2 + \lambda_9(v_1^2 + w^2) + \lambda_{14}w^2 \right) + v_2\mu_t(v_1^2 + w^2) \right. \\
 & \mp \mathcal{F}_a \left\{ \left(v_1w \left(\lambda_{18}\Lambda^2 + 2\mu_5^2 - 2\Lambda\mu_s + \lambda_{13}v_2^2 + \lambda_{15}v_1^2 + \lambda_9(v_1^2 + w^2) + \lambda_{14}w^2 \right) + v_2\mu_t(v_1^2 + w^2) \right)^2 \right. \\
 & - 4v_1w(v_1^2 + w^2) \left(v_2\mu_t(\lambda_{18}\Lambda^2 + 2\mu_5^2 - 2\Lambda\mu_s + \lambda_{13}v_2^2 + \lambda_{14}w^2) + v_1w(\lambda_9(\lambda_{18}\Lambda^2 + 2\mu_5^2 - 2\Lambda\mu_s + \lambda_{14}w^2) \right. \\
 & \left. \left. - (\lambda_{19}\Lambda - \mu_u)^2 + \lambda_9\lambda_{13}v_2^2 \right) + \lambda_{15}v_2v_1^2\mu_t + \lambda_9\lambda_{15}v_1^3w \right) \left. \right\}^{1/2} \left. \right\}, \quad (3.13) \\
 \mathcal{F}_a = & \text{sgn} \left\{ v_1w \left(-\lambda_{18}\Lambda^2 - 2\mu_5^2 + 2\Lambda\mu_s - \lambda_{13}v_2^2 - \lambda_{15}v_1^2 + \lambda_9(v_1^2 + w^2) - \lambda_{14}w^2 \right) + v_2\mu_t(v_1^2 + w^2) \right\},
 \end{aligned}$$

As shown in detail in ref. [9], in the limit $\mu_s, \mu_u \rightarrow 0$, one obtains a degenerate physical scalar spectrum $m_{\varphi_{1,2}}^2 = m_{\varphi_{1,2}}^2$. This degeneracy is lifted in our present model by the inclusion of the scalar singlet σ , a feature which is crucial in order to implement our scotogenic neutrino mass generation mechanism, as described in the next section.

4 Yukawa sector

The 3-3-1-1 gauge-invariant Lagrangian of the model responsible for the fermion mass terms is given by

$$\begin{aligned}
 -\mathcal{L}_{\text{Yukawa}} = & y_{3a}^u \bar{q}_{3L} \eta u_{aR} + y_{ia}^u \bar{q}_{iL} \rho^* u_{aR} + y^U \bar{q}_{3L} \chi U_{3R} \\
 & + y_{3a}^d \bar{q}_{3L} \rho d_{aR} + y_{ia}^d \bar{q}_{iL} \eta^* d_{aR} + y_{ij}^D \bar{q}_{iL} \chi^* D_{jR} \\
 & + y_{ab}^e \bar{l}_{aL} \rho e_{bR} + \frac{M_{8ab}}{2} \text{Tr}[\bar{\Omega}_{aL}(\Omega_{bL})^c] + y_{ab}^\Omega \bar{l}_{aL}(\Omega_{bL})^c \chi + \text{h.c.} \quad (4.1)
 \end{aligned}$$

It includes Yukawa interaction terms as well as the bare octet mass term. First note that the quark spectrum of our model coincides with the one presented in [9].

We now focus on the tree-level masses of the neutral fermions. After spontaneous symmetry breaking, the last term in eq. (4.1) becomes

$$y_{ab}^\Omega \bar{l}_{aL}(\Omega_{bL})^c \chi = y_{ab}^\Omega \left(\bar{\nu} \quad \bar{e} \quad \bar{N} \right)_{aL} \begin{pmatrix} \frac{1}{\sqrt{2}}(\Psi_L)^c + \frac{1}{\sqrt{6}}(\tilde{N}_L)^c & (E_L^-)^c & (\tilde{\Delta}_L)^c \\ (E_L^+)^c & -\frac{1}{\sqrt{2}}(\Psi_L)^c + \frac{1}{\sqrt{6}}(\tilde{N}_L)^c & (\tilde{E}_L^+)^c \\ (\Delta_L)^c & (\tilde{E}_L^-)^c & -\frac{2}{\sqrt{6}}(\tilde{N}_L)^c \end{pmatrix}_b \begin{pmatrix} \frac{s'_1 + ia'_1}{\sqrt{2}} \\ \chi_2^- \\ \frac{w + s_3 + ia_3}{\sqrt{2}} \end{pmatrix}. \quad (4.2)$$

This product contains the following bilinear terms

$$y_{ab}^\Omega \bar{l}_{aL}(\Omega_{bL})^c \chi \supset \frac{y_{ab}^\Omega w}{\sqrt{2}} \left(\overline{\nu_{aL}}(\tilde{\Delta}_{bL})^c + \overline{e_{aL}}(\tilde{E}_{bL}^+)^c - \frac{2}{\sqrt{6}} \overline{N_{aL}}(\tilde{N}_{bL})^c \right), \quad (4.3)$$

with $a, b = 1, 2, 3$. In turn, the lepton octet mass term in eq. (4.1) includes the following bilinears

$$\frac{M_{8ab}}{2} \text{Tr}[\bar{\Omega}_{aL}(\Omega_{bL})^c] \supset \frac{M_{8ab}}{2} \left[\left(\overline{\Psi_{aL}} \right) (\Psi_{bL})^c + \overline{\tilde{N}_{aL}}(\tilde{N}_{bL})^c + \overline{\tilde{\Delta}_{aL}}(\Delta_{bL})^c + \overline{\Delta_{aL}}(\tilde{\Delta}_{bL})^c \right], \quad (4.4)$$

From these expressions we can identify the mass and mixing of the neutral fermions in the model, namely six states Ψ_{aL} , N_{aL} , $\tilde{N}_{aL}$, ν_{aL} , Δ_{aL} , and $\tilde{\Delta}_{aL}$ in each family.

First of all note that, from eqs. (4.4), (4.3), it follows that the three neutral singlet states Ψ_a remain unmixed with other fermions after spontaneous symmetry breaking, their mass matrix being just the bare octet mass M_{8ab} in eq. (4.4). The two “dark” or M_P -odd fermions N_{aL} and $\tilde{N}_{aL}$ mix through the mass matrix

$$M_N = \begin{pmatrix} 0 & M_{N\tilde{N}} \\ M_{N\tilde{N}}^T & M_8 \end{pmatrix}, \quad (4.5)$$

written in the basis $(N_L, \tilde{N}_L^c)$ and with $(M_{N\tilde{N}})_{ab} = -y_{ab}^\Omega w/\sqrt{3}$. Note that this matrix structure resembles the seesaw mechanism though it involves different fields, i.e. heavy electrically neutral Majorana fermions. Using the general expansion method in [11] this matrix can be diagonalized perturbatively by a unitary transformation, defining six physical heavy Majorana states denoted by $S_{\alpha L}$ through

$$\begin{pmatrix} N_L^c \\ \tilde{N}_L^c \end{pmatrix} = U S_L^c, \quad (4.6)$$

such that $M'_N = U^T M_N U = \text{diag}(M'_\alpha)$, $\alpha = 1, \dots, 6$. In the following analysis, only the lower blocks of the unitary matrix U will be relevant. We adopt the following notation for the relation between N_L^c and S_L^c :

$$\tilde{N}_{aL}^c = U_{a\alpha} S_{\alpha L}^c. \quad (4.7)$$

For simplicity, we will assume that the entries of the mass matrix $M_{N\tilde{N}}$ are all real, and consequently that the matrix U is orthogonal.

Finally, eqs. (4.4) and (4.3) provide a tree-level mass matrix for the remaining neutral fermions, including the active neutrino fields, which, in the basis $(\nu_L, \tilde{\Delta}_L, \Delta_L)$, has the form

$$M_\nu^{\text{tree}} = \begin{pmatrix} 0 & y^\Omega w/\sqrt{2} & 0 \\ (y^\Omega)^T w/\sqrt{2} & 0 & M_8 \\ 0 & M_8 & 0 \end{pmatrix}. \quad (4.8)$$

It can be shown that this matrix has rank 6, implying that the three light active neutrinos remain massless at tree-level. This is a very important consistency check of our construction, i.e. neutrinos remain massless at tree-level. Their non-zero masses arise radiatively at one-loop via the scotogenic mechanism, which we explain in the next subsection.

4.1 Scotogenic neutrino masses

We first notice that the key ingredient for a scotogenic mechanism for neutrino masses is already present in the basic $\text{SU}(3)_c \otimes \text{SU}(3)_L \otimes \text{U}(1)_X \otimes \text{U}(1)_N$ framework of ref. [9]. Indeed, non-zero neutrino masses arise in our model at one-loop level, from the diagram in figure 1. Moreover, two *ad hoc* features of the original scotogenic model in ref. [8] become automatic by the 3-3-1-1 embedding:

Figure 1. Scotogenic loop for Majorana neutrino mass.

1. the discrete symmetry responsible for WIMP dark matter stability is simply matter-parity M_P in eq. (1.1), which survives as a residual discrete symmetry after the complete spontaneous symmetry breaking chain of eq. (2.6).
2. the dark $SU(2)$ scalar doublet needed in the scotogenic loop in figure 1 is readily identified with the first two components of the χ triplet, whose third component is responsible for $SU(3)_L$ breakdown.

Finally, the scotogenic loop is closed in our model by the inclusion of the fermion octet, which also plays a double role, as mediator of neutrino masses, as well as the dominant field responsible for our proposed gauge coupling unification mechanism, see section 6.

Following from the analysis of the scalar potential in eq. (3.1), the Yukawa interaction terms contained in eq. (4.2) relevant for the neutrino masses are

$$\begin{aligned}
 y_{ab}^{\Omega} \bar{l}_{aL} (\Omega_{bL})^c \chi &\supset y_{ab}^{\Omega} \bar{\nu}_{aL} \left(\frac{1}{2} \Psi_{bL}^c + \frac{1}{2\sqrt{3}} \tilde{N}_{bL}^c \right) (s'_1 + ia'_1) \\
 &= y_{ab}^{\Omega} \bar{\nu}_{aL} \left(\frac{1}{2} \Psi_{bL}^c + \frac{1}{2\sqrt{3}} U_{b\alpha} S_{\alpha L}^c \right) (U_{1i}^s \varphi_i + iU_{1i}^a \tilde{\varphi}_i).
 \end{aligned} \tag{4.9}$$

Then, according to figure 1 the one loop level light active neutrino mass matrix is given by

$$\begin{aligned}
 (M_{\nu})_{ab} &= \sum_{c=1}^3 \sum_{i=1}^2 \frac{y_{ac}^{\Omega} y_{bc}^{\Omega} M_{8c}}{32\pi^2} \left[(U_{i1}^s)^2 \frac{m_{\varphi_i}^2}{m_{\varphi_i}^2 - M_{8c}^2} \ln \left(\frac{m_{\varphi_i}^2}{M_{8c}^2} \right) - (U_{i1}^a)^2 \frac{m_{\varphi_i}^2}{m_{\varphi_i}^2 - M_{8c}^2} \ln \left(\frac{m_{\varphi_i}^2}{M_{8c}^2} \right) \right] \\
 &+ \sum_{\alpha=1}^6 \sum_{i=1}^2 \frac{(y^{\Omega} U)_{a\alpha} (y^{\Omega} U)_{b\alpha} M'_{\alpha}}{96\pi^2} \left[(U_{i1}^s)^2 \frac{m_{\varphi_i}^2}{m_{\varphi_i}^2 - M_{\alpha}^{\prime 2}} \ln \left(\frac{m_{\varphi_i}^2}{M_{\alpha}^{\prime 2}} \right) - (U_{i1}^a)^2 \frac{m_{\varphi_i}^2}{m_{\varphi_i}^2 - M_{\alpha}^{\prime 2}} \ln \left(\frac{m_{\varphi_i}^2}{M_{\alpha}^{\prime 2}} \right) \right].
 \end{aligned} \tag{4.10}$$

In the above expression, the mass splitting between the CP even and CP odd scalars running in the internal lines of the loop is induced by the scalar singlet σ through the trilinear interactions $\frac{\mu_s}{\sqrt{2}} \phi^{\dagger} \sigma \sigma$ and $\frac{\mu_u}{\sqrt{2}} (\eta^{\dagger} \chi) \sigma$. As pointed out before, a degenerate physical scalar spectrum $m_{\varphi_{1,2}}^2 = m_{\varphi_{1,2}}^2$ is obtained in the limit $\mu_s, \mu_u \rightarrow 0$. In such limit one therefore obtains vanishing neutrino masses. Natural tiny values of the light active neutrino masses are achieved by invoking non-zero couplings μ_s and μ_u , which produce a small mass splitting between the virtual CP-even and CP-odd scalars that take part as mediators in the scotogenic loop.

Finally, to close this section it is worth mentioning that in the limit $\mu_s, \mu_u \rightarrow 0$, the Lagrangian of the model acquires an accidental $U(1)$ symmetry, under which the scalar fields σ and ϕ transform with the same charge, whereas the rest of the fields remain invariant. This accidental $U(1)$ symmetry is explicitly broken by the trilinear scalar couplings μ_s and μ_u , thus implying that in our model the light neutrino masses are symmetry-protected. Note that the neutrino oscillation data do not determine the allowed Yukawa couplings in a unique manner. Indeed, the diagonalised M_ν and the lepton mixing matrix in the left side of eq. (43) are given by the oscillation data. The Yukawa coupling y^Ω can be determined once the other mass scales $m_\phi, m_{\tilde{\phi}}, M'_\alpha, M_{8c}$ are chosen. But this procedure is not unique and involves a free general complex rotation matrix. These Yukawa couplings can then be related and constrained from other observables. A dedicated phenomenological analysis lies beyond the scope of the current work and will be presented elsewhere.

5 WIMP scotogenic dark matter

Due to the presence of a conserved discrete matter-parity surviving spontaneous symmetry breaking, our model contains a potentially stable dark matter candidate, namely the lightest electrically neutral M_P -odd particle. The phenomenology of a scalar scotogenic dark matter in our model was previously studied in [9]. There, the viability of the real scalar field φ_2 as a WIMP dark matter candidate was analyzed within a simplified scenario where all the non-SM fields were assumed to be heavy, with $|\theta_s| \ll 1$, and therefore decoupled. In such limit, φ_2 is mostly composed of the electroweak singlet scalar σ , and thus it mainly annihilates into a pair of Higgs fields, through the simplified Higgs portal quartic scalar interaction $\lambda_{\text{eff}} h^2 (\varphi_2)^2$. In this regime, the dark matter candidate φ_2 has a small coupling with the Z -boson, evading direct detection Higgs-portal experimental bounds [12].

Even in such constrained scenario, φ_2 yields viable relic densities [9], while evading direct detection bounds. This happens in two viable mass regions: near half of the Higgs mass, where resonant annihilation of dark matter into the Higgs boson takes place, and also for masses m_{φ_2} above 1 TeV, where the direct detection constraints on the effective Higgs portal coupling are weak [13]. These are shown in the left and right panels in figure (2). Notice that the allowed regions for scalar dark matter can be considerably larger than those of the simplified scenario described above [9], due to the presence of re-scattering effects [15] coming from the other scalar dark fields present in the model.

Concerning fermionic scotogenic dark matter, there are several M_P -odd Majorana fermions that may play this role in the present model, including the previously discussed fields $\Psi_{aL}, N_{aL}, \tilde{N}_{aL}, \Delta_{aL}, \tilde{\Delta}_{aL}$. As shown above, most of these fields have masses dictated by the octet mass M_8 , which plays an important role in gauge coupling unification, as discussed in the next section. Since we will assume that the scale M_8 is higher than the 3-3-1-1 symmetry breaking scale, most of the Majorana fermions will be heavy. In fact, their masses are restricted by the gauge coupling unification hypothesis, leading to a limited region accommodating viable relic densities.

There are, however, two scenarios for lighter WIMP dark matter fermions. The first one emerges from the observation that the fields N_{aL} and $\tilde{N}_{aL}$ undergo a seesaw mechanism

Figure 2. Viable mass regions where the field φ_2 of the simplified model described in the text behaves as a dark matter candidate. The red regions correspond to the current direct detection limits [12, 13]. The blue regions represent values of the effective coupling λ_{eff} where the corresponding relic density is incompatible with the Planck measurement [14].

when $M_8 > M_{N\tilde{N}}$, see eq. (4.5). This can naturally drive the mass of the lightest physical fermion S_{1L} below the few TeV scale. In this case, the fermionic dark matter candidates can annihilate into a SM fermion pair and neutral lepton pairs via the s -channel exchange of Z' and Z'' gauge bosons and t -channel exchange of neutral scalars, respectively. Moreover, a pair of heavy neutral gauge bosons can be produced by the t -exchange of Majorana fermions, and a scalar pair can be produced via the t -exchange of neutral leptons. Finally, a neutral scalar and a heavy neutral gauge boson can in turn be created from the s channel exchange of Z' and Z'' of the same neutral fermions. Detailed studies of the dark matter relic abundance in the scenario of a Majorana dark matter candidate in radiative neutrino mass models have been carried out in [16]. It was shown that a fermionic dark matter candidate should be heavier than about 3 TeV in order to successfully comply with the available experimental constraints. Other studies of fermionic dark matter constraints in models with extended gauge symmetry were performed in [17, 18]. This ~ 3 TeV limit can be qualitatively understood by noticing that the dominant contribution to the annihilation cross section into SM states arises from the s channel exchange of heavy neutral gauge bosons Z' and Z'' , as pointed out in [16, 17]. Within the simplified scenario where the Z' exchange dominates over the exchange of Z'' (which corresponds to taking $\Lambda \gg w$), the thermally-averaged annihilation cross section of fermionic dark matter candidate into SM states can be estimated as [17]:

$$\langle\sigma v\rangle \approx \left(\frac{\alpha}{150 \text{ GeV}}\right)^2 \left(\frac{M_\Omega}{3 \text{ TeV}}\right)^2 \approx \left(\frac{M_\Omega}{3 \text{ TeV}}\right)^2 \text{ pb}, \quad (5.1)$$

which shows that the measured value of the dark matter relic abundance

$$\Omega_{DM} h^2 = \frac{0.1 \text{ pb}}{\langle\sigma v\rangle}, \quad (5.2)$$

can be naturally explained by having a fermionic dark matter candidate with a mass of the order of 3 TeV.

Finally, one can achieve light WIMP dark matter fermions in the present model by giving masses to the M_P -odd right-handed neutrinos ν_{iR} through spontaneous symmetry

breaking with the inclusion of adequate scalar fields. In such scenario, one can easily accomodate a viable relic density compatible with direct detection bounds. A dedicated analysis of the WIMP dark matter phenomenology is beyond the scope of this paper.

6 Gauge coupling unification

One of the primary goals of the grand unification programme consists in embedding the SM gauge group $\mathcal{G}_{\text{SM}} \equiv \text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y$ into a bigger gauge group with only one coupling constant. At some very high energy scale M_U , the unified group $\mathcal{G}_U$ breaks down to the SM gauge group (or some gauge extension thereof) followed by different evolution of the gauge couplings leading to their SM values at the electroweak symmetry breaking scale. Interestingly, one can have several intermediate scales between the unification scale M_U and the Z -pole, m_Z , corresponding to multi-stage breaking of $\mathcal{G}_U$ to $\mathcal{G}_{\text{SM}}$. The most straightforward route to unification was proposed by Georgi and Glashow [19], who pointed out that the SM can be embedded into the rank-4 simple Lie group $\text{SU}(5)$, implying the unification of all the SM coupling constants.

In what follows of this section, instead of specifying a unified gauge group upfront, we will take a phenomenological approach to find the possibility for a dynamical gauge coupling unification. This exploratory approach not only allows us to unravel the phenomenological possibilities for the unification of the electroweak interaction with the new interactions of our scotogenic framework, but also provides important hints on the possible interesting UV-completions on which we comment towards the end of this section.

Having already discussed how the SM gauge group and particle content can be embedded into our 3-3-1-1 theory, in this section we proceed to perform a comprehensive study of the Renormalisation Group Equations (RGE) in order to determine the conditions under which the SM gauge couplings, embedded into the extended gauge group $\text{SU}(3)_C \otimes \text{SU}(3)_L \otimes \text{U}(1)_X \otimes \text{U}(1)_N$, lead to a successful gauge coupling unification at some high scale M_U . *A priori* we do not make any assumptions about the ultimate unified gauge group. Instead we work with the 3-3-1-1 model, in order to explore various possibilities. At the one-loop level the evolution of the gauge couplings g_i is governed by the renormalisation group equations (RGEs)

$$\mu \frac{\partial g_i}{\partial \mu} = \beta_i(g_i) \equiv \frac{b_i}{16\pi^2} g_i^3, \quad (6.1)$$

where μ is the renormalization scale. The behaviour of the gauge couplings with the energy can also be expressed in the form

$$\frac{1}{\alpha_i(\mu_2)} = \frac{1}{\alpha_i(\mu_1)} - \frac{b_i}{2\pi} \ln \left(\frac{\mu_2}{\mu_1} \right), \quad (6.2)$$

where $\alpha_i = g_i^2/4\pi$ and the one-loop beta-coefficients b_i are given by

$$b_i = -\frac{11}{3} \mathcal{C}_2(G_i) + \frac{2}{3} \sum_{R_f} T(R_f) \prod_{j \neq i} d_j(R_f) + \frac{1}{3} \sum_{R_s} T(R_s) \prod_{j \neq i} d_j(R_s). \quad (6.3)$$

Here, $\mathcal{C}_2(G_i)$ is the quadratic Casimir invariant corresponding to the adjoint representations,

$$\mathcal{C}_2(G) \equiv \begin{cases} N & \text{if } \text{SU}(N), \\ 0 & \text{if } \text{U}(1). \end{cases} \quad (6.4)$$

whereas $T(R_f)$ and $T(R_s)$ correspond to the Dynkin indices of the irreducible representation $R_{f,s}$ for a given fermion and scalar, respectively. For the case of $\text{SU}(N)$ they are

$$T(R_{f,s}) \equiv \begin{cases} 1/2 & \text{if } R_{f,s} \text{ is fundamental,} \\ N & \text{if } R_{f,s} \text{ is adjoint,} \\ 0 & \text{if } R_{f,s} \text{ is singlet.} \end{cases} \quad (6.5)$$

The quantity $d(R_{f,s})$ in (6.3) is the dimension of a given representation $R_{f,s}$ under all gauge groups except for the i -th gauge group under consideration.

As shown above, in our 3-3-1-1 model the electric charge operator is defined as

$$Q = T_3 - \frac{1}{\sqrt{3}}T_8 + X, \quad (6.6)$$

where the $\text{SU}(3)_L$ generators are normalised as $\text{Tr}(T_i T_j) = \frac{1}{2}\delta_{ij}$. Note that the $\text{U}(1)_X$ charge, X , enters in the definition of electric charge Q and hence can be related to the hypercharge after the breaking of $\text{SU}(3)_C \otimes \text{SU}(3)_L \otimes \text{U}(1)_X \otimes \text{U}(1)_N$ to the SM gauge group $\text{SU}(3)_C \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y$ as

$$Y = -\frac{1}{\sqrt{3}}T_8 + X. \quad (6.7)$$

Therefore the initial value for α_X at the 3-3-1-1 symmetry breaking scale M_X can be obtained using the hypercharge α_Y evolution from the Z -pole to M_X . In addition, we can define the normalized charge operators X_N and Y_N , which satisfy the relations

$$X = n_X X_N, \quad Y = n_Y Y_N, \quad (6.8)$$

with the normalizations of X and the hypercharge Y being related by

$$n_Y^2 = \frac{1}{3} + n_X^2. \quad (6.9)$$

We recall that in an embedding of the SM gauge group into some unified simple group the hypercharge normalization is usually chosen so that it matches with the normalization for the $\text{SU}(N)$ counterparts in the SM gauge group,

$$\text{Tr}[T_i T_j] = \frac{1}{2}\delta_{ij}. \quad (6.10)$$

For instance, in a $\text{SU}(5)$ theory by fixing the normalisation of the fundamental representation one can fix the $\text{U}(1)_Y$ normalisation to $n_Y^2 = 5/3$. Here, in the absence of any specific unification group we will treat n_Y , *a priori*, as a free parameter.

Relevant Gauge group	Scale of running	Gauge group G_i	Notation for b_i	Value of b_i
$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$	$M_Z < \mu < M_X$	$SU(3)_C$	b_{3C}	-7
		$SU(2)_L$	b_{2L}	$-\frac{19}{6}$
		$U(1)_Y$	b_Y^{UN}	$\frac{41}{6}$
$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N$	$M_X < \mu < M_8$	$SU(3)_C$	b_{3C}^X	-5
		$SU(3)_L$	b_{3L}^X	$-\frac{13}{2}$
		$U(1)_X$	b_X^{UN}	$\frac{26}{3}$
		$U(1)_N$	b_N^{UN}	$\frac{163}{3}$
$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N$	$M_8 < \mu < M_U$	$SU(3)_C$	$b_{3C}^\Omega = b_{3C}^X$	-5
		$SU(3)_L$	b_{3L}^Ω	$-\frac{1}{2}$
		$U(1)_X$	$b_X^{\Omega;\text{UN}} = b_X^{\text{UN}}$	$\frac{26}{3}$
		$U(1)_N$	$b_N^{\Omega;\text{UN}} = b_N^{\text{UN}}$	$\frac{163}{3}$

Table 2. Values of b_i for different gauge groups (G_i) relevant for RG running of gauge couplings at different energy scales.

We also notice that the $U(1)_N$ charge does not contribute to the electric charge, and therefore can be considered as an “electrically neutral new charge” (ENNC). As a result, the initial value for α_N at the 3-3-1-1 symmetry breaking scale M_X and the normalisation of $U(1)_N$,

$$N = n_N N_N, \quad (6.11)$$

remain free parameters which cannot be fixed by electroweak gauge coupling input values or hypercharge normalization.

Furthermore, for the sake of generality, we take the 3-3-1-1 symmetry breaking scale (M_X) and the fermionic octet mass scale $M_8 > M_X$ as independent scales. In table 2, we summarize the one-loop RGE beta coefficients governing the evolution of the relevant gauge couplings at different scales. Before addressing the evolution of the $U(1)_N$ gauge coupling, it is straightforward to find the unification scale M_U for $SU(3)_c$, $SU(2)_L$ and $U(1)_X$ and the hypercharge normalization n_Y as a function of the intermediate symmetry breaking scales. First we note that the normalized couplings are related by

$$n_Y^2 (\alpha_Y^N)^{-1} = \frac{1}{3} \alpha_{3L}^{-1} + \left(n_Y^2 - \frac{1}{3} \right) (\alpha_X^N)^{-1}. \quad (6.12)$$

Taking the 3-3-1-1 symmetry breaking scale (M_X) and the mass scale for the fermionic octets Ω_{aL} , $a = 1, 2, 3$, $M_8 > M_X$ as independent parameters, and using eq. (6.2) we then obtain

$$\alpha_U^{-1} = \frac{1}{n_Y^2 - \frac{1}{3}} \left\{ \alpha_{\text{em}}^{-1}(M_Z) \cos^2 \theta_w(M_Z) - \frac{1}{3} \alpha_{2L}^{-1}(M_Z) - \frac{b_Y^{\text{UN}} - \frac{1}{3} b_{2L}}{2\pi} \ln \left(\frac{M_X}{M_Z} \right) - \frac{b_X^{\text{UN}}}{2\pi} \ln \left(\frac{M_U}{M_X} \right) \right\}, \quad (6.13)$$

$$\alpha_U^{-1} = \alpha_{2L}^{-1}(M_Z) - \frac{b_{2L}}{2\pi} \ln \left(\frac{M_X}{M_Z} \right) - \frac{b_{3L}^X}{2\pi} \ln \left(\frac{M_8}{M_X} \right) - \frac{b_{3L}^\Omega}{2\pi} \ln \left(\frac{M_U}{M_8} \right), \quad (6.14)$$

$$\alpha_U^{-1} = \alpha_{3C}^{-1}(M_Z) - \frac{b_{3C}}{2\pi} \ln \left(\frac{M_X}{M_Z} \right) - \frac{b_{3C}^X}{2\pi} \ln \left(\frac{M_U}{M_X} \right), \quad (6.15)$$

where we note that the fermionic octets only affect the evolution of α_{3L} from M_8 to M_U . Hence in the above we denote the beta coefficients for the running of $SU(3)_L$ from M_X to

M_8 by b_{3L}^X and M_8 to M_U by b_{3L}^Ω , respectively. From eqs. (6.14) and (6.15) one obtains the unification scale M_U as a function of M_X and M_8 as

$$M_U(M_X, M_8) = \frac{M_X^{\frac{b_{3C}^X}{b_{3C}^X - b_{3L}^\Omega}}}{M_8^{\frac{b_{3L}^\Omega}{b_{3C}^X - b_{3L}^\Omega}}} \left(\frac{M_8}{M_X} \right)^{\frac{b_{3L}^X}{b_{3C}^X - b_{3L}^\Omega}} \left(\frac{M_X}{M_Z} \right)^{\frac{b_{2L} - b_{3C}}{b_{3C}^X - b_{3L}^\Omega}} \exp \left[2\pi \frac{\alpha_{3C}^{-1}(M_Z) - \alpha_{2L}^{-1}(M_Z)}{b_{3C}^X - b_{3L}} \right]. \quad (6.16)$$

Using eqs. (6.12) and (6.13) the normalization n_Y^2 can be obtained as

$$n_Y^2 = \frac{1}{3} + \left[\alpha_{\text{em}}^{-1}(M_Z) \cos^2 \theta_w(M_Z) - \frac{1}{3} \alpha_{2L}^{-1}(M_Z) - \frac{b_Y^{\text{UN}} - \frac{1}{3} b_{2L}}{2\pi} \ln \left(\frac{M_X}{M_Z} \right) + \frac{b_X^{\text{UN}}}{2\pi} \ln \left(\frac{M_U(M_X, M_8)}{M_X} \right) \right] \\ \times \left[\alpha_{2L}^{-1}(M_Z) - \frac{b_{2L}}{2\pi} \ln \left(\frac{M_X}{M_Z} \right) - \frac{b_{3L}^X}{2\pi} \ln \left(\frac{M_8}{M_X} \right) - \frac{b_{3L}^\Omega}{2\pi} \ln \left(\frac{M_U(M_X, M_8)}{M_8} \right) \right]^{-1}, \quad (6.17)$$

where $M_U(M_X, M_8)$ is given by eq. (6.16). The one-loop beta coefficients relevant for the running between different scales are collected in table 2.

In figure 3 left plot, we show the unification scale as a function of the 3-3-3-1 symmetry breaking scale M_X (cf. eq. (6.16)), for three different benchmark choices $M_8 = M_X$ (solid curve), $M_8 = 3M_X$ (dashed curve) and $M_8 = 10M_X$ (dot-dashed curve). The blue band indicates the range for the unification scale consistent with the current experimental limit on proton decay lifetime if the 3-3-1-1 gauge group is embedded in a unified gauge group. However, we note that a dynamical gauge coupling unification achieved with an anomaly-free set (under the 3-3-1-1 gauge group) of fields [20–26], is not subject to such a constraint. The tilted red line corresponds to the asymptotic limit $M_X = M_U$. In figure 3 right plot, we show the corresponding hypercharge normalization as a function of the 3-3-1-1 symmetry breaking scale M_X , for the benchmark choices described above. The red line shows the standard hypercharge normalization for a reference SU(5) unified theory. Note that for a given benchmark M_8 value, the relevant (solid, dashed or dot-dashed) curves in the left and right panels of figure 3 correspond to the $\text{SU}(3)_c \times \text{SU}(3)_L \times \text{U}(1)_X$ unification scale M_U and the relevant hypercharge normalization required for a successful unification of gauge couplings. Therefore, such a curve represents a family of gauge coupling unification possibilities, with each point corresponding to a particular choice of the 3-3-1-1 symmetry breaking scale M_X .

In figure 4 we show such an example point in the dashed curves in figure 3 with the 3-3-1-1 symmetry breaking scale $M_X = 10 \text{ TeV}$ and $M_8 = 3M_X = 30 \text{ TeV}$, demonstrating a successful $\text{SU}(3)_c \times \text{SU}(3)_L \times \text{U}(1)_X$ unification.

Having discussed the $\text{SU}(3)_c \times \text{SU}(3)_L \times \text{U}(1)_X$ unification, we now explore the possibility of $\text{U}(1)_N$ unification with $\text{SU}(3)_c \times \text{SU}(3)_L \times \text{U}(1)_X$. First we note that, *a priori*, it is a valid theoretical possibility that $\text{SU}(3)_c \times \text{SU}(3)_L \times \text{U}(1)_X$ can first unify into a larger gauge group G_{3-3-1} , independent of $\text{U}(1)_N$, and at some higher energy scale the unification of $\text{U}(1)_N$ and G_{3-3-1} takes place. However, for the sake of simplicity, we will only consider the case of $\text{U}(1)_N$ unifying at the same scale of G_{3-3-1} unification.

We recall that $\text{U}(1)_N$ does not contribute to the electric charge and therefore the initial value for α_N at M_X and its normalization n_N defined in eq. (6.8) are not fixed by the hypercharge and electroweak input parameters. Therefore, in order to derive the viable

Figure 3. (Left) unification scale M_U as a function of the 3-3-1-1 symmetry breaking scale M_X , for three benchmark choices $M_8 = M_X$ (solid curve), $M_8 = 3M_X$ (dashed curve) and $M_8 = 10M_X$ (dot-dashed curve). (Right) hypercharge normalization n_Y^2 as a function of M_X , for the same benchmark choices as the left panel.

Figure 4. An example of $SU(3)_c \times SU(3)_L \times U(1)_X$ unification for the 3-3-1-1 symmetry breaking scale $M_X = 10 \text{ TeV}$ and $M_8 = 3M_X = 30 \text{ TeV}$, corresponding to the dashed curves in figure 3.

Figure 5. The left panel shows the α_N^{-1} contours in the $U(1)_N$ normalisation (n_N^2) vs the 3-3-1-1 symmetry breaking scale M_X plane, for a benchmark choice $M_8 = M_X$, while the right one is the same as the left, but with $M_8 = 10M_X$.

Figure 6. An example of $SU(3)_c \times SU(3)_L \times U(1)_X \times U(1)_N$ unification for a phenomenologically accessible 3-3-1-1 symmetry breaking scale $M_X = 10$ TeV and $M_8 = 3M_X = 30$ TeV, corresponding to the dashed curves in figure 3.

initial value for α_N at the 3-3-1-1 symmetry breaking scale M_X and the normalisation n_N , we require that the $U(1)_N$ coupling must unify with the remaining gauge groups associated to $SU(3)_C$, $SU(3)_L$, $U(1)_X$ at the same scale M_U , leading to the relation

$$\alpha_U^{-1} = \alpha_{3C}^{-1}(M_U) = \alpha_{3L}^{-1}(M_U) = \left(\alpha_X^N(M_U)\right)^{-1} = \left(\alpha_N^N(M_U)\right)^{-1}, \quad (6.18)$$

subject to the b coefficients shown in table 2. This in turn yields

$$\alpha_N^{-1}(M_X) = n_N^2 \alpha_U^{-1} + \frac{b_N^{UN}}{2\pi} \ln \left(\frac{M_U}{M_X} \right), \quad (6.19)$$

where α_U^{-1} is obtained using any of the eqs. (6.10), (6.11), or (6.12), and $M_U(M_X, M_8)$ is given by eq. (6.14). In figure 5 we show the contours for $\alpha_N^{-1}(M_X)$ in the $U(1)_N$ normalisation(n_N^2) vs 3-3-1-1 symmetry breaking scale M_X plane (predicted by the requirement that the $U(1)_N$ coupling must unify with the remaining gauge groups associated to $SU(3)_C$, $SU(3)_L$, $U(1)_X$ at the same scale M_U), for two benchmark choices: in the left plot $M_8 = M_X$ and in the right one $M_8 = 10M_X$.

In order to explicitly give an example of a unification scenario, in figure 6 we show $SU(3)_c \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N$ unification for a 3-3-1-1 symmetry breaking scale $M_X = 10$ TeV and $M_8 = 3M_X = 30$ TeV, with the corresponding input for $\alpha_N^{-1}(M_X)$ computed using eq. (6.16).

Intriguingly, we notice from figures 3 and 5 that successful gauge coupling unification can occur for a 3-3-1-1 symmetry breaking scale M_X and fermionic octet mass scale M_8 around $\mathcal{O}(10)$ TeV, accessible at the current and future collider experiments. Therefore, our 3-3-1-1 model provides a very exciting phenomenological alternative for having new physics at an energy scale around $\mathcal{O}(10)$ TeV in the form of the gauge bosons associated with 3-3-1-1 symmetry breaking, as well as the fermionic octet. Such mass scales can not only be explored at collider experiments [27], but also lead to interesting charged lepton flavour violation signals [28–31]. Moreover they may also be probed in low-energy neutrino experiments, e.g. neutrinoless double beta decay searches [32].

To conclude this section we comment on the possible embeddings of the 3-3-1-1 gauge group. One of the minimal possibilities is to unify the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N$

gauge group inside $SU(6) \otimes U(1)_N$. In this case the $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ part of the 3-3-1-1 gauge group unifies into $SU(6)$ [33, 34]. The fermion content of the model (except for the octets) can be embedded into the anomaly-free combination of $SU(6)$ representations $\bar{6} + \bar{6} + 15 + 20$, while the octets can be embedded in the 35 multiplet of $SU(6)$ which does not contribute to the anomaly. Given that the specific multiplicity of the fermionic triplets in the 3-3-1-1 model is dictated by the number of families, the required combination of anomaly-free $SU(6)$ multiplets to accomodate such structure requires additional fields. In order to identify the required multiplets in the unified theory it may be useful to make use of flux breaking tools implemented through the Hosotani mechanism [35]. As seen above, by normalizing the fundamental representation of $SU(6)$ the hypercharge and $U(1)_X$ normalizations can be fixed to $n_Y = \sqrt{5/3}$ and $n_X = 2/\sqrt{3}$ respectively. Moreover, the $SU(6)$ multiplets required for 3-3-1-1 unification can be further embedded in a $E(6)$ theory with one of its maximal subgroups being $SU(6) \otimes SU(2)$. The 27 representation of $E(6)$ can break into $\bar{6}$ and 15 representations of $SU(6)$ and the 78 can break into 35, 20 and 1 representations of $SU(6)$. The $E(6)$ embedding can be particularly interesting from the perspective of $E(6)$ F -theories [36–40]. Finally, note that a 3-3-1-1 embedding into a unified $SU(6) \otimes U(1)_N$ group would lead to further constraints on the unification scale, due to the fact that the $SU(6)$ gauge bosons can mediate a proton decay mode such as $p \rightarrow e^+ \pi^0$. Experimental searches for the latter lead to a stringent limit $M_U \gtrsim 10^{15.5} \text{ GeV}$ [1]. We would like to note that there are different variants of the standard minimal 3-3-1 model. These have different assignments of the quarks and leptons, e.g. the SVS [9], the flipped (e.g. [41]) and the sequential (e.g. [33]) variants of the 3-3-1 model. While all of them are in principle consistent with gauge coupling unification [33] only those with large leptonic multiplets (e.g. leptonic octets) allow for a 3-3-1 symmetry breaking scale within reach of upcoming collider experiments.

7 Summary and outlook

As a follow-up of our previous work we have now proposed a scotogenic scheme where dark matter stability is ensured by a gauged matter parity symmetry, eq. (1.1). The same physics responsible for neutrino mass generation drives the unification of the fundamental gauge couplings. A crucial role is played by the leptonic octets in the model: they are responsible for generating the light active neutrino masses through a Scotogenic mechanism (see figure 1), while driving gauge coupling unification (see figures 3–6). Their masses can be accessible to experiments at $\mathcal{O}(10)$ TeV scale. Taking such dynamical unification approach as the guiding principle, we have used the unification of the electrically neutral $U(1)_N$ symmetry with $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ to predict the initial value and normalization for the coupling strength of the new interaction associated with $U(1)_N$ (see figure 5). Indeed, while not exclusive, this approach is very attractive, as it naturally predicts the free parameters associated with the new interaction. The construction is suggestive of a *plethora* of new physics associated to the new gauge bosons and to the exotic states dictated by the 3-3-1-1 gauge symmetry, which can be probed at future collider and low-energy experiments. In addition to the possibility of observing the new exotic fermions at future

collider searches, this model can also be probed at the upcoming experiments searching for charged Lepton Flavour Violation (cLFV). In particular, there are two distinct types of cLFV contributions that can arise in this model. One is the standard “scotogenic” diagram for $\mu \rightarrow e\gamma$, with dark particles running in the loop, as noted in [8]. Such a contribution is strongly suppressed, since it goes as the fourth power of the mixing angle between the electrically charged components of the $SU(3)_L$ scalar triplets χ and ρ . This mixing angle is of the order of the ratio between the electroweak scale and the 3-3-1-1 symmetry breaking scale, i.e. $\lesssim \mathcal{O}(10^{-2})$. As a result, apart from the loop suppression, the scotogenic contribution to the $\mu \rightarrow e\gamma$ decay has another suppression factor of $\mathcal{O}(10^{-8})$. Taking reasonable values for the neutrino Yukawa couplings, the mass of the neutral heavy lepton and the electrically charged scalar mediators, we find that such scotogenic contribution is several orders of magnitude below the current experimental upper limit of MEG. Another type of contribution in this model is the inverse-seesaw-type contribution [42], which in this case involves the leptonic octets and W boson in the loop. Such a contribution is suppressed by the mixing between the light-neutrino and neutral-octet. As expected, taking $M_8 \sim w \sim 10^4$ GeV and $\text{Tr}[y^\Omega y^{\Omega\dagger}] \sim 2 \times 10^{-5}$, one finds that the inverse seesaw contribution gives $\text{Br}[\mu \rightarrow e\gamma] \sim 8.7 \times 10^{-14}$, which will be probed at MEG-II.

To conclude, we stress once again that our model does not require supersymmetry, though it can be made supersymmetric, should one desire that route to address the gauge hierarchy problem instead of, say, the warping of extra spacetime dimensions. In contrast, our suggestion provides a potentially testable approach where other drawbacks of the SM are addressed in an interconnected manner, such as

- number of the fermion families equals the number of colors,
- WIMP dark matter mediates neutrino mass generation,
- dark matter stability results from a residual gauge matter-parity,
- dynamical unification of gauge couplings.

In short, we have illustrated an idea which seems worth of a dedicated scrutiny of its potential implications.

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