

# Electroweak $SU(2)_L \times U(1)_Y$ model with strong spontaneously fermion-mass-generating gauge dynamics

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**ABSTRACT:** Higgs sector of the Standard model (SM) is replaced by quantum flavor dynamics (QFD), the gauged flavor  $SU(3)_f$  symmetry with scale  $\Lambda$ . Anomaly freedom requires addition of three  $\nu_R$ . The approximate QFD Schwinger-Dyson equation for the Euclidean infrared fermion self-energies  $\Sigma_f(p^2)$  has the spontaneous-chiral-symmetry-breaking solutions ideal for seesaw: (1)  $\Sigma_f(p^2) = M_{fR}^2/p$  where three Majorana masses  $M_{fR}$  of  $\nu_{fR}$  are of order  $\Lambda$ . (2)  $\Sigma_f(p^2) = m_f^2/p$  where three Dirac masses  $m_f = m_{(0)}1 + m_{(3)}\lambda_3 + m_{(8)}\lambda_8$  of SM fermions are *exponentially suppressed w.r.t.*  $\Lambda$ , and *degenerate for all SM fermions in  $f$* . (1)  $M_{fR}$  break  $SU(3)_f$  symmetry completely;  $m_{(3)}, m_{(8)}$  superimpose the tiny breaking to  $U(1) \times U(1)$ . All flavor gluons thus acquire self-consistently the masses  $\sim \Lambda$ . (2) All  $m_f$  break the electroweak  $SU(2)_L \times U(1)_Y$  to  $U(1)_{em}$ . Symmetry partners of the composite Nambu-Goldstone bosons are the genuine Higgs particles: (1) three  $\nu_R$ -composed Higgses  $\chi_i$  with masses  $\sim \Lambda$ . (2) Two new SM-fermion-composed Higgses  $h_3, h_8$  with masses  $\sim m_{(3)}, m_{(8)}$ , respectively. (3) The SM-like SM-fermion-composed Higgs  $h$  with mass  $\sim m_{(0)}$ , the effective Fermi scale.  $\Sigma_f(p^2)$ -dependent vertices in the electroweak Ward-Takahashi identities imply: the axial-vector ones give rise to the  $W$  and  $Z$  masses at Fermi scale. The polar-vector ones give rise to the fermion mass splitting in  $f$ . At the present exploratory stage the splitting comes out unrealistic.

**KEYWORDS:** Gauge Symmetry, Nonperturbative Effects, Spontaneous Symmetry Breaking

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## 1 Introduction

The Principle of spontaneous symmetry breaking, ingeniously realized in the SM of electroweak interactions [1–3] in the form of the Higgs mechanism [4, 5] turned out to be extremely successful. First, three ‘would-be’ Nambu-Goldstone (NG) bosons, pre-prepared in the spontaneously condensing elementary scalar electroweak complex doublet Higgs field give rise to the longitudinal polarization states of massive  $W^\pm$  and  $Z$  bosons. Second, the condensate of the Higgs field, which defines the Fermi (electroweak) scale gives rise to the fermion masses in terms of *theoretically arbitrary* (infinitely renormalized) Yukawa coupling constants. Third, the observable remnant of the Higgs mechanism is a real scalar above the condensate which remains in the Higgs field. It describes the massive spinless Higgs boson  $H$  with uniquely prescribed interactions. In 2012 such a boson was discovered at CERN LHC [6, 7]. The resulting picture confirmed experimentally with steadily increasing accuracy is theoretically consistent all the way up to the Planck scale.

In this way, however, the Higgs mechanism as a source of particle masses is not complete; it is blind to three important facts: first, the neutrinos are massive. Second, there is the dark matter. Third, if the CERN Higgs were indeed the Higgs boson of the Standard model, the lepton and quark masses would be forever the phenomenological, theoretically arbitrary parameters. The spectra of quantum systems are, however, as a rule, calculable.

Fortunately, the principles are more general than their particular realizations. Already the masters of the Higgs mechanism, knowing that the same principle of spontaneously broken symmetry is realized also in superconductors, pointed out (P. Higgs in [4, 5], F. Englert and R. Brout in [8]) that the Higgs boson could be a composite fermion-anti-fermion particle. Literature on this subject is vast [9]. The very description of bound states in relativistic quantum field theory requires strong coupling and is therefore exceedingly difficult

to handle. Consequently, no model of this sort, which would be comparably convincing as the weakly coupled Higgs mechanism, is available. Our suggestion presented here is no exception.

We refer with admiration to the papers of Heinz Pagels and co-workers [10–12]: these papers analyze some *consequences* of the dynamical Higgs mechanism which they call a quantum flavor dynamics (QFD) [10]. The main points of their QFD, dealing with the SM fermions and new gauge dynamics, without specifying the Lagrangian, are common with the approach suggested here: first, the dynamically generated fermion masses are finite and calculable [10]. Second, their dynamical, spontaneous generation implies the masses of  $W$  and  $Z$  bosons [11]. Third, there is a composite Higgs particle [11]. We find the name QFD very appropriate and take the liberty of using it for our gauge quantum flavor dynamics (QFD) defined by gauging properly the flavor  $SU(3)$  symmetry index.

It is obvious that if the SM fermion masses are generated (somehow) dynamically the electroweak gauge boson masses come out as the necessary consequence of the existence theorem of Goldstone. The composite Higgs is then the plausible symmetry partner of the composite ‘would-be’ NG bosons. Its existence is not, however, guaranteed by an existence theorem. This idea was elaborated heuristically in several less known papers [13–15] as well as in several famous ones [16, 17]. Majority of them of course quote the basic ref. [11].

In the present model the SM fermion self-energies  $\Sigma_f(p^2)$  generated by the strong-coupling QFD are by symmetry the same in each flavor  $f$ . This does not at all correspond to reality. Fortunately, there are the electroweak interactions. The necessary appearance of  $\Sigma_f(p^2)$  in the electroweak Ward-Takahashi identities seems to supplement the QFD naturally.

The paper is organized as follows: in section 2 we define the QFD and argue that it generates spontaneously the fixed fermion mass pattern. In section 3 we describe the consequences of this pattern dictated by the powerful Goldstone theorem supplemented with the Nambu-Jona-Lasinio-like assumption [18] on the existence of the NG symmetry partners. In section 4 we discuss the effects of  $\Sigma_f(p^2)$  in the electroweak WT identities: the axial-vector terms with NG poles have the robust effect of generating the  $W$  and  $Z$  boson masses with observed ratio given by the Weinberg angle. The polar-vector terms which we suggest for computing the fermion mass splitting in  $f$  are strongly model-dependent. The present computation has therefore only illustrative character. Section 5 contains our brief general conclusions.

## 2 Properties of QFD

Gauging the flavor or family (or horizontal, or generation) symmetry is so natural that it cannot be new [19–22]. Being completely and badly broken it has to be broken spontaneously. The safe way is to use the weakly coupled Higgs sector. There is the arbitrariness in choosing the gauge group, the assignment of the chiral fermion fields to the representations of the gauge group, and the choice of the Higgs fields.

Here we consider the real world with three SM fermion chiral families, i.e., the gauge flavor dynamics is  $SU(3)_f$ . We put all chiral SM fermion multiplets into triplets of the

flavor group. The sacred requirement of an anomaly freedom then uniquely implies adding one triplet of the right-handed neutrino fields  $\nu_R^f$ . This is the starting point of the model of Tsutomu Yanagida [23]. Its fermion and flavor gluon Lagrangian has the form

$$\mathcal{L}_f = -\frac{1}{4}F_{a\mu\nu}F_a^{\mu\nu} + \bar{q}_L i \not{D} q_L + \bar{u}_R i \not{D} u_R + \bar{d}_R i \not{D} d_R + \bar{l}_L i \not{D} l_L + \bar{e}_R i \not{D} e_R + \bar{\nu}_R i \not{D} \nu_R. \quad (2.1)$$

Yanagida himself [23] describes the observed broken gauge flavor and electroweak gauge symmetries with two vastly different mass scales put by hands by an extended sector of weakly interacting Higgs fields. They are uniquely defined by the available gauge-invariant Yukawa couplings. Be it as it may, this slightly extended Standard model is an elegant quantum field theory realization of the seesaw [24–30], and also the basis of understanding the baryogenesis via leptogenesis [31].

In conclusion of his paper Yanagida suggests that the model is “a possible candidate for the spontaneous mass generation by dynamical symmetry breaking”, having in mind the famous model of Nambu and Jona-Lasinio [18]. The present model wants to be a honest attempt in realizing this suggestion: we pretend to obtain two vastly different mass scales spontaneously as two distinct solutions in two different channels of one matrix QFD Schwinger-Dyson equation (gap equation) for the chirality-changing fermion self-energies  $\Sigma(p^2)$ .

The field tensor  $F$  describes the kinetic term of eight flavor gluons  $C$  and their self-interactions, the covariant derivatives  $D$  describe their interactions with chiral fermions, both right- and left-handed. The  $q_L$  and  $l_L$  are the quark and lepton electroweak doublets, respectively,  $u_R, d_R, e_R, \nu_R$  are the electroweak singlets. Their weak hypercharges are uniquely fixed by the corresponding electric charges.

The gauge  $SU(3)_f$  interaction, characterized by one dimensionless coupling constant  $h$ , is *asymptotically free* at high momenta and hence *strongly interacting in the infrared*. This means that by dimensional transmutation the dimensionless  $h$  can turn into the theoretically arbitrary mass scale  $\Lambda$ .

The Lagrangian  $\mathcal{L}_f$  is always considered together with the standard electroweak gauge  $SU(2)_L \times U(1)_Y$  forces known to remain weakly coupled all the way up to the Planck scale: in the first stage, when considering the effects of the strong-coupling QFD we take the electroweak interactions only as a weak external perturbation; basically, QFD should only know what are the electric charges of the fermions present. In the second stage we take the electroweak interactions into account perturbatively. We are interested in the effects of  $\Sigma_f(p^2)$  which appear in the electroweak Ward-Takahashi identities. The QCD does not influence the phenomena described here.

Like in the QCD Lagrangian, also in  $\mathcal{L}_f$  both the left- and the right-handed fermion fields interact with the octet of the corresponding massless gauge bosons as  $SU(3)$  triplets. This similarity is highly suspicious. At the first sight it seems that what we suggest contradicts the Vafa-Witten no-go theorem [32]. Since in QCD we trust we are obliged to provide a truly good reason why in the infrared the suggested QFD should self-break, generating spontaneously the masses of its elementary excitations (leptons, quarks and flavor gluons), whereas the QCD confines.

We believe there is such good reason: while the QCD deals with the electrically charged quarks, the QFD deals also with the electrically neutral sterile right-handed neutrinos *which can be the massive Majorana particles*. It is utmost important that their hard mass term

$$\mathcal{L}_{\text{Majorana}} = -\frac{1}{2}\bar{\nu}_R M_R (\nu_R)^c + \text{h.c.} \quad (2.2)$$

unlike the Dirac mass term, is strictly prohibited by the  $SU(3)_f$  gauge symmetry: it transforms as  $3^* \times 3^* = 3 + 6^*$  which does not contain unity. It can, however, be generated dynamically à la Nambu-Jona-Lasinio [18] provided this option is energetically favorable (which we assume). It is easy to see that the relevant part of the Lagrangian

$$\mathcal{L}_{\text{int}} = \frac{1}{2}h \left( \bar{\nu}_R \gamma_\mu \frac{1}{2} \lambda_a \nu_R + \overline{(\nu_R)^c} \gamma_\mu \left[ -\frac{1}{2} \lambda_a^T \right] (\nu_R)^c \right) C_a^\mu \quad (2.3)$$

in contrast with the vector-like QCD Lagrangian, is effectively chiral: the charge conjugate neutrino field  $(\nu_R)^c = C(\bar{\nu}_R)^T$  is of course a *left-handed field*,  $(\nu_R)^c = (\nu^c)_L$ . Unlike the other left-handed fields in the Lagrangian  $\mathcal{L}_f$  it transforms, however, as the *antitriplet* of  $SU(3)_f$ :  $T_a(L) = -\frac{1}{2}\lambda_a^T$ . An answer to the mandatory question why the Majorana masses of the left-handed neutrinos are not dynamically generated is suggested in [33].

The strategy is in principle crystal-clear: first, in the Lagrangian dealing only with chiral fermions and with gauge fields *all hard fermion mass terms* are strictly prohibited by the underlying gauge chiral  $SU(3)_f \times SU(2)_L \times U(1)_Y$  invariance.<sup>1</sup> Second, if the appropriate chirality-changing fermion self-energies  $\Sigma(p^2)$  are spontaneously generated by the QFD strongly coupled in the infrared, they fix the pattern of spontaneously broken symmetries uniquely. The powerful Goldstone theorem then yields a number of strong conclusions. We supplement the Goldstone theorem with a plausible assumption of the existence of the genuine symmetry partners of the composite ‘would-be’ NG bosons. It is gratifying that the generic properties of the SM are reproduced. Harsh support or invalidation of our suggestion can apparently be given only by the non-perturbative lattice computations. For gauge theories with chiral fermions they are, however, at present not available [34, 35].

Our primary aim therefore is to find the non-perturbative solutions of the Schwinger-Dyson (SD) equation for the chiral-symmetry-breaking  $\Sigma(p^2)$  in the full fermion propagators  $S^{-1}(p) = \not{p} - \Sigma(p)$  for all fermions of the  $SU(3)_f \times SU(2)_L \times U(1)_Y$  gauge-invariant Lagrangian. The matrix SD equation of QFD (in Landau gauge) is [12]

$$\Sigma(p) = 3 \int \frac{d^4 k}{(2\pi)^4} \frac{\bar{h}_{ab}^2 ((p-k)^2)}{(p-k)^2} T_a(R) \Sigma(k) \left[ k^2 + \Sigma^+(k) \Sigma(k) \right]^{-1} T_b(L). \quad (2.4)$$

According to the NJL self-consistent reasoning [18] we first *assume* that the gauge flavor  $SU(3)_f$  is completely self-broken, and subsequently *find* the corresponding symmetry-breaking solutions.

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<sup>1</sup>The fermion mass terms themselves are super-renormalizable but, being non-invariant they break down the gauge symmetry in loops. This implies the necessity of introducing the non-invariant gauge boson mass counter-terms, i.e., the unitarity-violating hard gauge boson masses.

It is important to realize that the SD equation is universal: (i) the Majorana mass is the R-L bridge between  $\nu_R$  which transforms as a flavor triplet, and between the left-handed  $(\nu_R)^c$  which transforms as flavor anti-triplet ( $3^* \times 3^* = 3 + 6^*$ ). The corresponding  $\Sigma(p^2)$  which gives rise to the three Majorana masses  $M_{fR}$  is the complex  $3 \times 3$  matrix, the symmetric sextet by the Pauli principle, and  $T_a(R) = \frac{1}{2}\lambda_a$ ,  $T_a(L) = -\frac{1}{2}\lambda_a^T$ . (ii) The Dirac mass is the R-L bridge between the right- and the left-handed fermion fields both transforming as flavor triplets ( $3^* \times 3 = 1 + 8$ ): the corresponding  $\Sigma(p^2)$  which gives rise to three Dirac masses  $m_f$  is a general complex  $3 \times 3$  matrix, and  $T_a(R) = T_a(L) = \frac{1}{2}\lambda_a$ .

*Because there is nothing in QFD which would distinguish between the neutrino, the charged lepton, the charge  $Q = 2/3$  quark, and the charge  $Q = -1/3$  quark in given family the masses  $m_f$  must come out degenerate for all fermion species in family  $f$ . The difference between the Majorana and Dirac mass matrices turns out, however, substantial.*

We have demonstrated elsewhere [36] that in a separable approximation for the kernel of the SD equation (2.4) there are explicit solutions for  $\Sigma(p^2)$  with the following properties:

- (1) There are three Euclidean Majorana self-energies  $\Sigma_f(p^2) = M_{fR}^2/p$  where the three Majorana masses  $M_{fR}$  are

$$M_{fR} \sim \Lambda \tag{2.5}$$

- (2) There are three Euclidean Dirac self-energies  $\Sigma_f(p^2) = m_f^2/p$  where the three Dirac masses  $m_f$  degenerate for  $\nu_f, e_f, u_f, d_f$  in family  $f$  are exponentially small with respect to  $\Lambda$ :

$$m_f = \Lambda \exp(-1/4\alpha_f) \tag{2.6}$$

Here  $\alpha_f$  are three dimensionless effective coupling constants of separable approximation.

We note that the paradigm-changing superconducting gap of BCS [37] was also the result of a weird separable approximation: in the gap equation it simply ignored the vast majority of interactions of electrons in superconductors. Only many years later this issue was clarified by Polchinski [38].

Another important note is this [36]: the obtained functional behavior of  $\Sigma(p^2) \sim 1/p$  is apparently good only in the infrared (IR), i.e., at low momenta where the momentum-dependent coupling is large. There it defines the fermion masses [39–42]. In solving the SD equation in the separable approximation the integration over momenta runs only up to  $\Lambda$ . At high momenta, i.e., in the ultraviolet (UV) the behavior of  $\Sigma(p^2)$  is dictated by asymptotic freedom of QFD [12]:  $\Sigma(p^2) \sim 1/p^2$ . In our approximation the QFD is not asymptotically, but entirely free. As the fermion mass is a low-momentum phenomenon the high-momentum regime is not essential for our purposes.

How many free parameters are *ultimately* necessary for computing the fermion masses in QFD? The strong non-Abelian  $SU(3)_f$  dynamics is characterized by one theoretically arbitrary parameter, the scale  $\Lambda$ . Consequently, *if our basic strong assumption of the complete self-breaking is warranted* both the Majorana masses  $M_{fR}$  and the Dirac masses  $m_f$  should ultimately be the *calculable* multiples of  $\Lambda$  [10]. In any case, in the sterile neutrino sector the sextet is characterized by three vacuum expectation values [43] as is

the singlet plus octet in the SM fermion sector. Clearly, the flavor gluon dynamics uniquely relates them.

Our belief here is entirely analogous to the belief in understanding the hadron mass spectrum of the confining QCD in the chiral limit: with one theoretically arbitrary scale  $\Lambda_{\text{QCD}}$  there are the massless NG pions, whereas the masses of all other hadrons of the first family ( $m_u, m_d \ll \Lambda_{\text{QCD}}$ ) are ultimately the calculable multiples of  $\Lambda_{\text{QCD}}$  (so far only approximately by a computer). The case of QFD is even more complex because besides the masses of its elementary excitations (leptons, quarks and flavor gluons) there are also the masses of its expected unconfined but strongly coupled collective excitations or bound states. Putting the effectively chiral QFD on the lattice will be hard [34, 35], because the hard Majorana masses of sterile neutrinos are strictly prohibited by symmetry: their flavor group representation  $3 \times 3 = 3^* + 6$  does not contain unity.

### 3 Goldstone theorem implies

We will assume in the following that the fermion mass pattern  $M_{fR} \gg m_f$  obtained in a crude approximation is the generic property of QFD at strong coupling. It then follows that it fixes the spontaneous symmetry-breaking pattern of the underlying gauge  $SU(3)_f \times SU(2)_L \times U(1)_Y$  symmetry down to  $U(1)_{\text{em}}$  uniquely, and the Goldstone theorem implies several strong reliable conclusions. The Goldstone theorem is supplemented with a plausible assumption of the existence of the genuine symmetry partners of the composite ‘would-be’ NG bosons. For transparency we rewrite  $m_f$  as ( $\lambda_0 = \sqrt{\frac{2}{3}}\mathbf{1}$ )

$$m_f \equiv m_{(0)}\lambda_0 + m_{(3)}\frac{1}{2}\lambda_3 + m_{(8)}\frac{1}{2}\lambda_8, \tag{3.1}$$

where

$$m_{(0)} = \frac{1}{\sqrt{6}}(m_1 + m_2 + m_3), \tag{3.2a}$$

$$m_{(3)} = m_1 - m_2, \tag{3.2b}$$

$$m_{(8)} = \frac{1}{\sqrt{3}}(m_1 + m_2 - 2m_3). \tag{3.2c}$$

- I. (1) The Majorana masses  $M_{fR}$  break down the gauge symmetry  $SU(3)_f$  spontaneously and completely [23, 36, 43]. Consequently, eight ‘would-be’ NG bosons composed of sterile neutrinos give rise to masses  $m_{iC}$  of all flavor gluons  $C_i^\mu$  of order  $\Lambda$  [44–46]. In the  $SU(3)_f$  WT identities for the sterile neutrino sector [36] the NG bosons are convincingly identified as massless poles together with their quantum numbers: some are scalars, some are pseudo-scalars.
- (2) Eight ‘would-be’ NG bosons and one genuine pseudo NG boson resulting from spontaneous breakdown of global anomalous  $U(1)$  symmetry of the right-handed neutrino sector belong to the complex symmetric composite sextet [36]

$$\Phi_{\{fg\}} = \frac{1}{\Lambda^2}(\bar{\nu}_{fR}(\nu_{gR})^c). \tag{3.3}$$

Consequently, as a remnant of the symmetry ( $8 + 1 + 3 = 12$ ), there should be existing three genuine Higgs-like composite bosons  $\chi_i$  with masses of order  $\Lambda$  [36].

- II. (1) The Dirac mass  $\bar{\psi}_{fR} m_{(0)} \psi_{fL} + \text{h.c.}$  is  $SU(3)_f$  invariant, i.e., the QFD symmetry in isolation would allow the hard Dirac fermion mass common to all fermions of three families. Such a term breaks, however, the electroweak  $SU(2)_L \times U(1)_Y$  chiral symmetry spontaneously down to  $U(1)_{\text{em}}$  even if that is considered as a weak external perturbation. Consequently, three multi-component NG bosons composed of all electroweakly interacting leptons and quarks are dynamically generated. Only in the second stage, when the gauge electroweak interactions are switched on these NG bosons become ‘would-be’, giving incoherently rise to masses  $m_W$  and  $m_Z$  of  $W$  and  $Z$  bosons, respectively, in terms of  $\sum m_f$ , the induced Fermi (electroweak) scale. In the  $SU(2)_L \times U(1)_Y$  WT identities for the SM fermions [36], see eq. (4.7), (4.8) the NG bosons are convincingly identified as massless poles together with their quantum numbers: as the fermion masses in families are degenerate they are the pure pseudo-scalars. For the same reason the canonical Weinberg relation  $m_W/m_Z = \cos \theta_W$  is exact. We note a difference between the composite ‘would-be’ NG bosons of spontaneously broken symmetries  $SU(3)_f$  and  $SU(2)_L \times U(1)_Y$ : in both cases they are composed by the strong-coupling QFD, but in the former case they are always ‘would-be’.
- (2) Three ‘would-be’ NG bosons belong to the complex multi-component composite doublet (index  $a$  in the following formula)

$$\phi^a = \frac{1}{\Lambda^2} \left[ (\bar{e}_{fR} l_L^{af}) + (\bar{d}_{fR} q_L^{af}) + ((\nu_{fR})^c (l_L^a)^c) + ((u_{fR})^c (q_L^a)^c) \right]. \quad (3.4)$$

For the  $SU(2)$  spinors  $\phi$  the definition of charge conjugation includes the definition  $\phi^c = i\tau_2 \phi^*$ . Consequently, as a remnant of the symmetry (as in the Standard model) ( $3 + 1 = 4$ ), there is the mandatory retro-diction of one genuine multi-component composite SM-like Higgs boson  $h$  with mass at the induced electroweak scale.

- III. (1) The Dirac masses  $m_{(3)}$  and  $m_{(8)}$  break down spontaneously the flavor  $SU(3)_f$  gauge symmetry in the SM sector down to  $U(1) \times U(1)$  [4, 5, 36], and the electroweak  $SU(2)_L \times U(1)_Y$  chiral symmetry spontaneously down to  $U(1)_{\text{em}}$ . Consequently, six multi-component ‘would-be’ NG bosons composed of the electroweakly interacting leptons and quarks contribute a tiny amounts to the huge masses of six flavor gluons [4, 5], and three multi-component ‘would-be’ NG bosons, also composed of the electroweakly interacting leptons and quarks provide extra contributions to  $m_W$  and  $m_Z$ . The canonical Weinberg relation between them remains intact.
- (2) All multi-component ‘would-be’ NG bosons discussed above are contained in the composite operator

$$\phi_i^a = \frac{1}{\Lambda^2} \left[ \left( \bar{e}_R \frac{1}{2} \lambda_i l_L^a \right) + \left( \bar{d}_R \frac{1}{2} \lambda_i q_L^a \right) + \left( (\nu_{fR})^c \frac{1}{2} \lambda_i (l_L^a)^c \right) + \left( (u_{fR})^c \frac{1}{2} \lambda_i (q_L^a)^c \right) \right]. \quad (3.5)$$



It is important to consider those fermion bilinear combinations of the chiral fermion fields  $\psi_{L,R}$  with the same electric charge, i.e., those which are responsible for the fermion masses  $m_{(3)}$  and  $m_{(8)}$ , i.e.,  $\bar{\psi}_R \lambda_{3,8} \psi_L$ . They belong to the SU(3) octet. To have it real we have in mind that its redundant component becomes the ‘would-be’ NG boson of otherwise neglected SU(2)<sub>L</sub> × U(1)<sub>Y</sub>. Consequently, as a remnant of the symmetry (6 + 2 = 8) there should be existing two additional multi-component composite Higgs-like bosons  $h_3$  and  $h_8$  with characteristic Yukawa couplings and masses at the electroweak scale. It is rather remarkable that namely such a possibility was explicitly mentioned as an example of the non-Abelian Higgs mechanism by Peter Higgs in his seminal ‘Abelian’ paper [4, 5].

- IV. There are interesting phenomena associated with spontaneous breakdown of global chiral Abelian symmetries of the model [36]. The anomalous ones result in observable new axion-like particles, the anomaly-free one can be gauged, resulting in new massive  $Z'$  gauge boson.

#### 4 Electroweak WT identities imply

So far we have analyzed the consequences of spontaneous emergence of  $\Sigma_f(p^2)$  generated by the strongly coupled QFD. The chiral-symmetry-breaking  $\Sigma_f(p^2)$ s manifest operationally by new terms also in the *electroweak interactions by virtue of the Ward-Takahashi identities*. The WT identities have to remain valid regardless of whether the underlying symmetry were broken spontaneously or not. Consequently, the new  $\Sigma_f(p^2)$ -dependent symmetry-breaking terms in them should have the important and theoretically reliable implications. The EW WT identities are (we temporarily omit the flavor index  $f$ ) [10, 47, 48]

$$(p' - p)_\mu \Gamma_A^\mu(p', p) = eQ_i [S^{-1}(p') - S^{-1}(p)], \tag{4.1}$$

$$(p' - p)_\mu \Gamma_W^\mu(p', p) = \frac{e}{2\sqrt{2} \sin \theta_W} \left\{ [S^{-1}(p')T^+ - T^+S^{-1}(p)] - [S^{-1}(p')T^+\gamma_5 + T^+\gamma_5S^{-1}(p)] \right\}, \tag{4.2}$$

$$(p' - p)_\mu \Gamma_Z^\mu(p', p) = \frac{e}{\sin 2\theta_W} \left\{ [S^{-1}(p')T_Z^i - T_Z^iS^{-1}(p)] - [S^{-1}(p')T_{3L}^i\gamma_5 + T_{3L}^i\gamma_5S^{-1}(p)] \right\}, \tag{4.3}$$

where by  $T_Z^i$  we denote

$$T_Z^i \equiv (T_{3L}^i - 2Q_i \sin^2 \theta_W). \tag{4.4}$$

The index  $i$ , ( $i = \nu, l, u, d$ ) distinguishes different SM fermion species. The proper vertices themselves, which satisfy the WT identities and have no unwanted kinematic singularities,

have the form

$$\Gamma_A^\mu(p', p) = eQ_i[\gamma^\mu - (p' + p)^\mu \Sigma'(p', p)], \quad (4.5)$$

$$\Gamma_W^{+\mu}(p', p) = \frac{e}{2\sqrt{2}\sin\theta_W} \left\{ [\gamma^\mu - (p' + p)^\mu \Sigma'(p', p)] T^+ - \left[ \gamma^\mu \gamma_5 T^+ - \frac{(p' - p)^\mu}{(p' - p)^2} (\Sigma(p') + \Sigma(p)) \gamma_5 T^+ \right] \right\}, \quad (4.6)$$

$$\Gamma_Z^\mu(p', p) = \frac{e}{\sin 2\theta_W} \left\{ [\gamma^\mu - (p' + p)^\mu \Sigma'(p', p)] T_Z^i - \left[ \gamma^\mu \gamma_5 - \frac{(p' - p)^\mu}{(p' - p)^2} (\Sigma(p') + \Sigma(p)) \gamma_5 \right] T_{3L}^i \right\}. \quad (4.7)$$

The ‘derivative’ is defined as

$$\Sigma'(p', p) \equiv \frac{\Sigma(p') - \Sigma(p)}{p'^2 - p^2}. \quad (4.8)$$

It in fact points to the difference between the  $\Sigma(p^2)$ -dependent terms in the axial-vector and polar-vector vertices. In the former ones they mark the famous massless ‘would-be’ Nambu-Goldstone poles and persist even for constant  $\Sigma$ s, i.e., for hard fermion masses. This is how the NG pole manifests in the NJL model. This robust effect is supported by the existence theorem.

In polar-vector vertices the appearance of chiral-symmetry breaking  $\Sigma$ s is much more subtle. They appear only provided they are momentum-dependent. Consequently, their consequences depend crucially upon their functional form and in evaluating their credibility we should be very humble.

#### 4.1 Masses of $W$ and $Z$ bosons

How the composite ‘would-be’ NG bosons become the longitudinal polarization states of massive  $W$  and  $Z$  bosons is well known. In the present model the mechanism was described in detail in [36] and here we merely quote the result. For the Euclidean  $\Sigma_f(p^2) = m_f^2/p$  the new axial-vector vertices contribute to the gauge-boson polarization tensor loop and the electroweak gauge-boson masses are given by the famous Pagels-Stokar formula in the form of the sum rules. The technique is identical to that used in technicolor [49, 50].

$$m_W^2 = \frac{1}{4} g^2 \frac{5}{4\pi} \sum_f m_f^2 = \frac{5\alpha}{4\sin^2\theta_W} \sum_f m_f^2 \quad (4.9)$$

$$m_Z^2 = \frac{1}{4} (g^2 + g'^2) \frac{5}{4\pi} \sum_f m_f^2 = \frac{5\alpha}{4\sin^2\theta_W \cos^2\theta_W} \sum_f m_f^2. \quad (4.10)$$

The Weinberg relation  $m_W = m_Z \cos\theta_W$  is the consequence of the degeneracy of the fermion masses in electroweak doublets. It is due to the fact that all chiral fermion fields are in flavor triplets. If some strong dynamics would produce spontaneously the fermion masses with large splitting, the relation would be badly broken [11].

## 4.2 SM-fermion mass splitting

Inspired by prescient Heinz Pagels who attempted to compute the  $u - d$  quark mass difference due to the electromagnetic interaction [10] with new  $\Sigma(p^2)$ -dependent vertex we suggest here that the SM fermion mass splitting in families is entirely due to the electroweak interactions with peculiar vectorial  $\Sigma(p^2)$ -dependent vertices enforced by the WT identities [10, 47, 48, 51].

The new  $\Sigma_f(p^2)$ -dependent polar-vector electroweak vertices are

$$\Gamma'_A{}^\mu(p', p) = -eQ_i(p' + p)^\mu \Sigma'(p', p), \quad (4.11)$$

$$\Gamma'_{W^+}{}^\mu(p', p) = -\frac{e}{2\sqrt{2}\sin\theta_W}(p' + p)^\mu \Sigma'(p', p)T^+, \quad (4.12)$$

$$\Gamma'_Z{}^\mu(p', p) = -\frac{e}{\sin 2\theta_W}(p' + p)^\mu \Sigma'(p', p)T_Z^i. \quad (4.13)$$

Being *chirality-changing* they are perfectly suited for computing different fermion masses in a given family in terms of the *known parameters*. An obvious objection is of course how it can be that the weakly coupled electroweak interaction describes the observed *enormous* top-bottom quark mass splitting. There is hope. The computation of the fermion pole-mass splitting amounts to solving a complicated algebraic equation (4.18) which crucially depends upon the functional form of the resulting  $\Sigma_f^i(p^2)$ . It of course critically depends upon the functional form of  $\Sigma_f(p^2)$ . Another obvious objection is that the observed ordering of fermion masses in different families is *different* whereas the electroweak interactions are *identical* for all three families: in the first family the  $u$  quark is lighter than the  $d$  quark, whereas in other two families the corresponding ordering is dramatically reversed. In the second family the muon is heavier than the  $s$  quark, whereas in other two families the corresponding ordering is reversed. There is hope. The ‘known’ parameters  $m_Z/m_f$  and  $m_W/m_f$  which come from QFD and from the axial-vector terms in the WT identities could account for these properties.

It is utmost important that the fermion mass splitting due to the *weakly coupled* electroweak interactions is not spontaneous: it is not associated with any additional spontaneous electroweak gauge symmetry breaking. The very formation of both the composite ‘would-be’ NG bosons and of the composite genuine Higgs bosons, exhibiting spontaneous breakdown of  $SU(3)_f \times SU(2)_L \times U(1)_Y$  down to  $U(1)_{\text{em}}$  can solely be due to the strong coupling, i.e., to QFD.

Important consequence of this reasoning is the neutrino mass spectrum. As described below the Dirac neutrino mass matrix  $m_f^\nu$  results from characteristic contributions from  $W$  and  $Z$  exchanges and, because of the Majorana mass matrix  $M_{fR}$  of sterile right-handed neutrinos *the neutrino mass spectrum of the model is given by the famous seesaw mass matrix* [24–30].

We treat the contributions of the photon,  $W$  and  $Z$  boson on the same footing and take the massive propagators in the ‘soft’ form of a massive vector boson coupled to a conserved current:

$$D_{W,Z}^{\mu\nu}(q) = -i \frac{g^{\mu\nu} - q^\mu q^\nu / q^2}{q^2 - m_{W,Z}^2}, \quad (4.14)$$

i.e., in parallel with the massless photon propagator in a Landau (transverse) gauge. Namely this form is obtained when summing the contributions of the massless ‘would-be’ NG poles in the  $W, Z$  boson polarisation tensors in order to obtain the massive  $W, Z$  boson poles [8]. It also enables immediate cross-check with a photon contribution by setting  $m_{W,Z}$  to zero.

The  $A$  and the  $Z, W$ -boson corrections to the Dirac fermion self-energies  $\delta_{A,Z,W}^i \Sigma_f(p^2)$ ,  $i = \nu, l, u, d$  are characterized by the strengths  $-e^2 Q_i^2$  and  $-e^2 P_{iZ,W}^2$ , respectively, where (for  $\sin^2 \theta_W \sim 0.23$ )

$$\begin{aligned} Q_\nu^2 &= 0, & P_{\nu Z}^2 &= 1/4 \sin^2 2\theta_W \cong 0.35, \\ Q_l^2 &= 1, & P_{lZ}^2 &= (1 - 4 \sin^2 \theta_W)^2 / 4 \sin^2 2\theta_W \cong 0.00, \\ Q_u^2 &= \frac{4}{9}, & P_{uZ}^2 &= \left(1 - \frac{8}{3} \sin^2 \theta_W\right)^2 / 4 \sin^2 2\theta_W \cong 0.05, \\ Q_d^2 &= \frac{1}{9}, & P_{dZ}^2 &= \left(1 - \frac{4}{3} \sin^2 \theta_W\right)^2 / 4 \sin^2 2\theta_W \cong 0.17. \end{aligned}$$

The  $W$ -boson correction to the Dirac self-energies  $\delta_W \Sigma_f(p^2)$  is characterized by the strength which does not depend upon the fermion species distinguishing index  $i$ :  $-e^2 P_W^2$  where  $P_W^2 = 1/8 \sin^2 \theta_W \cong 0.54$ .

For the SM fermion  $i = \nu, l, u, d$  in a family  $f$  the contributions  $\delta_{A,W,Z}^i \Sigma_f(p^2)$  to the common  $\Sigma_f(p^2) = m_f^2/p$  due to the new  $\Sigma_f(p^2)$ -dependent vertices are given in the lowest order by the Feynman diagrams of figure 1. After the Wick rotation  $k_0 \rightarrow -ik_0$  they have the form

$$\delta_A^i \Sigma_f(p^2) = -e^2 Q_i^2 \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(p-k)^4} \frac{\Sigma_f(p^2) - \Sigma_f(k^2)}{k^2 + \Sigma_f(k^2)} \frac{p^2 k^2 - (p \cdot k)^2}{p^2 - k^2}, \quad (4.15)$$

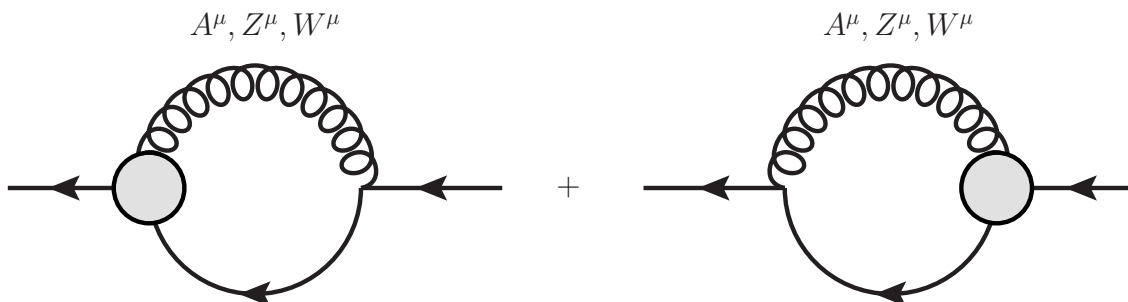
$$\begin{aligned} \delta_{Z,W}^i \Sigma_f(p^2) &= -e^2 P_{iZ,W}^2 \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(p-k)^2} \frac{1}{(p-k)^2 + m_{Z,W}^2} \frac{\Sigma_f(p^2) - \Sigma_f(k^2)}{k^2 + \Sigma_f(k^2)} \\ &\quad \times \frac{p^2 k^2 - (p \cdot k)^2}{p^2 - k^2}. \end{aligned} \quad (4.16)$$

It is rather remarkable that these integrals can be computed exactly. In order to obtain a comprehensible, illustrative fermion mass formula we make some elementary simplifying approximations. All necessary details are given in the appendix. The result is the *Euclidean*  $\Sigma_f^i(p^2)$ :

$$\Sigma_f^i(p^2) = \frac{m_f^2}{p} + \delta_A^{i(\text{IR})} \Sigma_f(p^2) + \delta_Z^{i(\text{IR})} \Sigma_f(p^2) + \delta_W^{(\text{IR})} \Sigma_f(p^2). \quad (4.17)$$

The fermion masses  $m_f^i$  are given as the poles of the full fermion propagators with the momentum-dependent chirality-changing self-energies  $\Sigma_f^i(p^2)$  in the *Minkowski* space. For this reason we replace in (4.17) the Euclidean  $p = \sqrt{p^2}$  by  $p = \sqrt{-p^2}$  where the Minkowski  $p^2$  can be both positive and negative. The resulting pole equation to be solved for  $p^2 = m_f^{i2}$  with fixed  $i, f$  has the form

$$p^2 = \Sigma_f^{i+}(p^2) \Sigma_f^i(p^2). \quad (4.18)$$



**Figure 1.** The lowest-order contributions of new  $\Sigma$ -dependent vertices to the fermion mass splitting in  $f$ .

Here

$$\Sigma_f^i(p^2) = -i \frac{m_f^2}{\sqrt{p^2}} A_f^i(p^2) + i \sqrt{p^2} B_f^i(p^2). \quad (4.19)$$

The real dimensionless functions  $A$ ,  $B$  are

$$\begin{aligned} A_f^i(p^2) = & 1 + \frac{3\alpha}{8\pi} Q_i^2 \left[ 1 - \frac{m_f^2}{p^2} \operatorname{arctg} \frac{p^2}{m_f^2} \right] \\ & + \frac{3\alpha}{8\pi} P_{iZ}^2 \left\{ -\frac{1}{2} \left( 1 - \frac{m_f^2}{m_Z^2} \right) \operatorname{arctg} \frac{p^2}{m_f^2} \right. \\ & \quad \left. - \frac{m_f^2}{p^2} \left[ \operatorname{arctg} \frac{p^2}{m_f^2} + \frac{3m_Z^4 + m_f^4}{6m_f^2 m_Z^2} \ln \left( 1 + \frac{p^4}{m_f^4} \right) \right] \right\} \\ & + \frac{3\alpha}{8\pi} P_W^2 \left\{ -\frac{1}{2} \left( 1 - \frac{m_f^2}{m_W^2} \right) \operatorname{arctg} \frac{p^2}{m_f^2} \right. \\ & \quad \left. - \frac{m_f^2}{p^2} \left[ \operatorname{arctg} \frac{p^2}{m_f^2} + \frac{3m_W^4 + m_f^4}{6m_f^2 m_W^2} \ln \left( 1 + \frac{p^4}{m_f^4} \right) \right] \right\}, \\ B_f^i(p^2) = & \frac{3\alpha}{32\pi} \left( P_{iZ}^2 \frac{m_f^2}{m_Z^2} + P_W^2 \frac{m_f^2}{m_W^2} \right) \left[ \frac{5}{3} - \ln \left( 1 + \frac{p^4}{m_f^4} \right) \right]. \end{aligned} \quad (4.21)$$

Our ignorance about  $M_{fR}$  does not allow to specify the neutrino propagator in which the neutrino mass is given by the seesaw mass formula. As such a propagator is needed in computing the  $W$  contribution to the charged lepton masses, we have to restrict ourselves at present merely to the computing of the quark mass splitting.

For an illustration we present the approximate explicit solution of the pole equation (4.18) assuming  $m_f^2/m_f^{i2} \ll 1$ . This amounts to solving the simplified equation  $B_f^i(m_f^{i2}) = \pm 1$ . It is interesting that in this approximation the dependence upon  $\alpha$  disappears. The consistency with the assumption demands the sign plus, and the result is

$$m_f^i = m_f \exp \left\{ \frac{50\pi \sum m_f^2}{9m_f^2 \left[ (T_{3L}^i - 2Q_i \sin^2 \theta_W)^2 + \frac{1}{2} \right]} \right\}. \quad (4.22)$$

Although aesthetically appealing this mass formula does not approximate the quark world. This, however, suggests that the all-important infrared part of  $\Sigma_f^i(p^2)$  could be guessed.

In general, with the Euclidean  $\Sigma_f(p^2) = m_f^2/p$  resulting from a crude separable approximation the hopes remain unfulfilled: the numerical solutions of the equation (4.18) exhibit only the small, nonrealistic fermion mass splitting.

## 5 Conclusion

Replacement of the weakly coupled Higgs sector of the SM with many parameters by the strong-coupling gauge QFD with one parameter is an immodest challenge. We believe it is in the spirit of the old good traditions of theoretical physics. As a quantum field theory it makes sense if and only if the fermion sector of the Standard model is uniquely extended, for purely theoretical reason of anomaly freedom, by three right-handed neutrinos. Spontaneously, i.e., in solutions the matrix SD equation of QFD, so far in a separable approximation, generates from one scale  $\Lambda$  *in two different channels two different sets of solutions with uniquely fixed and vastly different fermion masses*: first, three Majorana masses  $M_{fR} \sim \Lambda$  of  $\nu_R^f$ . Second, three exponentially small Dirac masses  $m_f \sim \exp(-1/\alpha_f)\Lambda$  ( $\alpha_f$  are the effective dimensionless couplings of separable approximation) degenerate for all SM fermions in  $f$ . The soft Majorana masses  $\Sigma_{fR}(p^2) = M_{fR}^2/p$  spontaneously emerge despite the fact that the hard ones are strictly prohibited by the hidden chiral symmetry of the  $\nu_R^f$  sector of QFD; the soft SM Dirac masses  $\Sigma_f(p^2) = m_f^2/p$  spontaneously emerge despite the fact that the hard ones are strictly prohibited by the chiral SM.

We know no way of knowing whether this highly desirable property is inherent to QFD at strong coupling or not. In any case the same property  $M_{fR} \gg m_f$  was obtained by Yanagida [23] easily in a very useful model with fermion content identical to ours, but with the rich Higgs sector with elementary Higgs fields in representations dictated by symmetry of allowed Yukawa couplings. Not surprisingly our fermion-composite Higgs-type operators have the same quantum numbers.

If our main conjecture is warranted there is no fundamental electroweak mass scale. The only genuine scale in the game is the huge QFD scale  $\Lambda$ . Although theoretically arbitrary the phenomenology of the emerging picture (e.g. seesaw or FCNC) forces  $\Lambda$  to be fixed by one experimental datum as huge. The Dirac masses  $m_f$  of the SM fermions come out spontaneously exponentially small with respect to  $\Lambda$ . The Goldstone theorem implies the  $W, Z$  masses of the order of  $\sum_f m_f$ , the *induced* or *effective* electroweak mass scale. The Higgs boson is light because it is intrinsically related with this scale as the genuine partner of the longitudinal polarization states of massive  $W$  and  $Z$  bosons. There is no fine tuning. In the resulting ‘improved’ Standard model the facts ‘ignored’ by the Standard one mentioned in the Introduction are addressed and bona fide explained:

- (1) By seesaw the present model describes its three active neutrinos as extremely light *Majorana* fermions. In comparison with the general effective field theory prediction of Steven Weinberg [52] the present ordinary quantum field theory is uniquely defined. It simply needs three right-handed neutrinos for anomaly freedom in much the same way the famous Weinberg’s model of leptons [1–3] needs quarks [53].
- (2) By the same logic the super-heavy right-handed neutrinos should be phenomenologically equally important as ordinary SM fermions. In close parallel with QCD

nucleons  $N \sim \epsilon_{abc} q^a q^b q^c$  which make the luminous matter of the Universe the model offers the stable heavy-neutrino composites  $S \sim \eta_{abc} \nu_R^a \nu_R^b \nu_R^c$  ( $\eta$  is an appropriate SU(3) Clebsch-Gordan coefficient) strongly coupled by QFD as a natural possibly stable-enough candidate for the dark matter of the Universe. It is conceivable that such a composite-fermion-made dark Universe might even look not dissimilar to our luminous one. More generally, the heavy sector with the fundamental scale  $\Lambda$ , which is absolutely necessary for electroweak physics at Fermi scale provides the contact with astro-particle physics and cosmology. First of all, the heavy Majorana neutrinos are the necessary ingredient in Fukugida-Yanagida's baryogenesis via leptogenesis [31]. Furthermore, the classically scale-invariant matter sector with a huge scale  $\Lambda$  due to the dimensional transmutation is indispensable for conservative understanding of the induced quantum gravity suggested by Sakharov [54] and advocated at present by Donoghue [55]. Also, the heavy composite higgses  $\chi$  potentially could be identified with inflatons [56].

- (3) QFD apparently computes the fermion masses  $m_f$  and  $M_{fR}$ , but that is not sufficient: it does not distinguish between different SM fermion species within one family. Fortunately, there are the electroweak gauge interactions. We have computed the mass splitting of each calculable  $m_f$  into  $m_f^i$ ,  $i = \nu, e, u, d$  unequivocally in terms of the known parameters entering the photon,  $W$  and  $Z$  boson loops with  $\Sigma_f(p^2)$  dependent vectorial vertices. The computation is, unfortunately, hampered by serious uncertainties. First, with the aim of obtaining the explicit fermion mass formula the perturbative computation was slightly simplified. Second and more important, the very form of the original SD equation and the separable approximation to its kernel result from approximations at strong coupling which are not under theoretical control. Hence, the fermion mass formula (4.18) based on the explicit form  $\Sigma_f(p^2) = m_f^2/p$  should be understood merely as an illustration of the general idea. Would we know  $M_{fR}$  we could predict, using the seesaw mass formula, the highly needed neutrino mass spectrum.

The present approach touches the Higgs paradigm of the origin of particle masses. In the first step the strongly coupled QFD spontaneously and self-consistently generates three Majorana masses  $M_{fR}$  of the right-handed neutrinos and the masses of all flavor gluons, both of order  $\Lambda$ . It is important that this step is a spontaneous breakdown of chiral symmetry. At the same time the QFD spontaneously generates also three Dirac masses  $m_f \ll M_{fR}$  of the SM fermions. By symmetry they are the same for all SM fermion species in  $f$ . The Goldstone theorem implies two types of the SM-fermion-composite 'would-be' NG excitations: (1) six of them correspond to the additional breakdown of  $SU(3)_f$  by  $m_{(3)}$  and  $m_{(8)}$  down to  $U(1) \times U(1)$ . They belong to the composite flavor octet and disappear from the spectrum as additional longitudinal components of massive flavor gluons. Hence, the massive  $h_3$  and  $h_8$  remain. (2) All  $m_f$  contribute to the spontaneous breakdown of the electroweak  $SU(2)_L \times U(1)_Y$  down to  $U(1)_{em}$ . Consequently, there are three SM-fermion-composite NG bosons and one massive  $h$ , all belonging to the complex composite doublet.

In the following step the dynamical role of the electroweak interactions, although weakly coupled, becomes profound: first, the  $\Sigma_f(p^2)$ -dependent terms in the *axial-vector*

parts of the electroweak WT identities are responsible for the  $W$  and  $Z$  boson masses. Second, the  $\Sigma_f(p^2)$ -dependent terms in the *polar-vector* parts of the electroweak WT identities are suggested to be responsible for the SM fermion mass splitting of  $m_f$  into  $m_f^i, i = \nu, l, u, d$ .

In evaluating the consequences of the model it is important to distinguish between those which crucially depend upon the functional form of  $\Sigma_f(p^2)$  and those which rely essentially only upon their very existence and their gross features. We believe that the unfulfilled expectation in computing the fermion mass splitting in  $f$  does not disqualify the whole idea.

IF the basic assumption of the spontaneous, strong-coupling generation of  $M_{fR} \gg m_f$  by QFD is justified, even without the necessity of knowing the details of  $\Sigma_f(p^2)$  there are the firm, and currently observable consequences: they are due to the powerful existence theorem of Goldstone supplemented with the natural NJL assumption of the existence of the NG symmetry partners:

First of all, after July 4, 2012 the model is obliged to contain the CERN Higgs boson with properties similar, though not necessarily identical with those of the elementary SM Higgs  $H$ . We believe our  $h$  is such a boson. We interpret it as an a posteriori confirmation of the assumption (in fact due to Nambu and Jona-Lasinio) of the existence of the NG symmetry partners. Derivation of its fermion-loop-generated couplings with the electroweak gauge bosons  $A, W, Z$  is in progress [48]. Similarity with the tree-level couplings of  $H$  with  $W$  and  $Z$  of the Standard model proportional to the masses  $m_W$  and  $m_Z$ , respectively is to be expected due to the sum rules (4.9), (4.10).

Above all, there is a robust prediction of two new composite electroweakly interacting Higgs bosons  $h_3$  and  $h_8$  at Fermi scale. The model stands and falls with these Higgs-like particles: they are the remnants of the flavor octet [4, 5], the composite fermion-antifermion scalar strongly bound by QFD and responsible for the spontaneous emergence of  $m_{(3)}$  and  $m_{(8)}$ . These scalars are distinguished by the effective flavor-sensitive Yukawa couplings proportional to the flavor matrices  $\lambda_3$  and  $\lambda_8$ , respectively. This signals that not all three families are alike.

The rigid predictive electroweak model presented above represents so far merely a possible framework, or scenario. The most difficult step is to provide the convincing argument for  $M_{fR} \gg m_f$ . Further, it remains to compute the quantum electroweak corrections to all quantities in which the Dirac fermion masses  $m_f$ , generated by QFD, enter degenerate:

- (i) The Yukawa couplings of composite Higgses  $h, h_3, h_8$  with fermions.
- (ii) The effective loop-generated couplings of composite Higgses  $h, h_3, h_8$  with the electroweak gauge fields  $A, W, Z$ .
- (iii) The electroweak corrections to the Pagels-Stokar mass formula for  $m_W, m_Z$ .
- (iv) We believe that the general idea of the electroweak fermion mass splitting remains alive. How to look for the convincing functional form of  $\Sigma_f(p^2)$  is, however, completely outside our imagination.



Reliable computation of masses of  $h$ ,  $h_3$  and  $h_8$  as well as the computation of other properties of the expected strongly bound QFD bound states (e.g. the dark matter neutrino composites) which are not of the NG nature requires the generically strong-coupling tools; also a formidable task.

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## A Evaluation of the electroweak fermion mass splitting

In the Euclidean integrals for  $\delta_{A,W,Z}^i \Sigma_f(p^2)$  we fix without loss of generality the external four-momentum  $p$  as  $p = (p, \vec{0})$ , integrate over the angles, and for  $\Sigma_f(p^2) = m_f^2/p$  we get

$$\delta_A^i \Sigma_f(p^2) = \frac{2\alpha}{\pi^2} Q_i^2 m_f^2 p \int_0^\infty I(p, k) \frac{k^6 dk}{(p+k)(k^4+m_f^4)}, \quad (\text{A.1})$$

$$\delta_{Z,W}^i \Sigma_f(p^2) = \frac{2\alpha}{\pi^2} P_{iZ,W}^2 m_f^2 p \int_0^\infty I_{Z,W}(p, k) \frac{k^6 dk}{(p+k)(k^4+m_f^4)}, \quad (\text{A.2})$$

where

$$\begin{aligned} I(p, k) &= \int_0^\pi \frac{\sin^4 \Theta d\Theta}{(p^2 + k^2 - 2pk \cos \Theta)^2} \\ &= \frac{3\pi}{16p^4 k^4} [p^4 + k^4 - (p^2 + k^2)|p^2 - k^2|], \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} I_{Z,W}(p, k) &= \int_0^\pi \frac{\sin^4 \Theta d\Theta}{(p^2 + k^2 - 2pk \cos \Theta)(p^2 + k^2 + m_{Z,W}^2 - 2pk \cos \Theta)} \\ &= \frac{\pi}{16p^4 k^4} \left\{ 3 \left[ p^4 + k^4 + (p^2 + k^2)m_{Z,W}^2 + \frac{1}{3}m_{Z,W}^4 \right] \right. \\ &\quad \left. + \frac{1}{m_{Z,W}^2} \left[ (p^2 - k^2)^2 |p^2 - k^2| \right. \right. \\ &\quad \left. \left. - ((p-k)^2 + m_{Z,W}^2)^{3/2} ((p+k)^2 + m_{Z,W}^2)^{3/2} \right] \right\}. \end{aligned} \quad (\text{A.4})$$

The absolute value  $|p-k|$  in both  $I(p, k)$  and  $I_{Z,W}(p, k)$  naturally leads to splitting the integrals into: (a)  $\int_0^p$  which we call infrared (IR) and (b)  $\int_p^\infty$  which we call ultraviolet (UV). We consider  $p \gg m_W, m_Z$ .

In the integral for  $\delta_A^i \Sigma_f(p^2)$  we merely replace the term  $(p+k)$  in the integrand by  $k$  and obtain

$$\delta_A^i \Sigma_f(p^2) = \delta_A^{i(\text{IR})} \Sigma_f(p^2) + \delta_A^{i(\text{UV})} \Sigma_f(p^2), \quad (\text{A.5})$$

where

$$\delta_A^{i(\text{IR})}\Sigma_f(p^2) = \frac{3\alpha}{8\pi}Q_i^2 \left( \frac{m_f^2}{p} - \frac{m_f^4}{p^3} \arctg \frac{p^2}{m_f^2} \right), \quad (\text{A.6})$$

$$\delta_A^{i(\text{UV})}\Sigma_f(p^2) = \frac{3\alpha}{8\pi}Q_i^2 p \left( \frac{\pi}{2} - \arctg \frac{p^2}{m_f^2} \right). \quad (\text{A.7})$$

In the integral for  $\delta_{Z,W}^i \Sigma_f(p^2)$  we make similar simplifications: (i) replace the term  $(p+k)$  in the integrand by  $k$ ; (ii) replace the term  $((p-k)^2 + m_{Z,W}^2)^{3/2}((p+k)^2 + m_{Z,W}^2)^{3/2}$  in  $I_{Z,W}(p,k)$  by  $(p^2 + m_{Z,W}^2)^3$ ; (iii) in (UV) we neglect the gauge boson masses:

$$\delta_{Z,W}^i \Sigma_f(p^2) = \delta_{Z,W}^{i(\text{IR})} \Sigma_f(p^2) + \delta_{Z,W}^{i(\text{UV})} \Sigma_f(p^2), \quad (\text{A.8})$$

where

$$\begin{aligned} \delta_{Z,W}^{i(\text{IR})}\Sigma_f(p^2) = \frac{3\alpha}{8\pi}P_{iZ,W}^2 \left\{ \frac{1}{2} \frac{m_f^2}{p} \left( 1 - \frac{m_f^2}{m_{Z,W}^2} \right) \arctg \frac{p^2}{m_f^2} \right. \\ \left. + \frac{m_f^4}{p^3} \left[ -\arctg \frac{p^2}{m_f^2} + \frac{1}{2} \left( \frac{m_{Z,W}^2}{m_f^2} + \frac{m_f^2}{3m_{Z,W}^2} \right) \ln \left( 1 + \frac{p^4}{m_f^4} \right) \right] \right. \\ \left. + \frac{1}{4} p \frac{m_f^2}{m_{Z,W}^2} \left[ \frac{5}{3} - \ln \left( 1 + \frac{p^4}{m_f^4} \right) \right] \right\}, \end{aligned} \quad (\text{A.9})$$

$$\delta_{Z,W}^{i(\text{UV})}\Sigma_f(p^2) = \frac{3\alpha}{8\pi}P_{iZ,W}^2 p \left( \frac{\pi}{2} - \arctg \frac{p^2}{m_f^2} \right). \quad (\text{A.10})$$

In accordance with approximations used in solving the SD equation for  $\Sigma_f(p^2)$  the ultraviolet pieces of  $\Sigma_f^i(p^2)$  can be ignored. Consequently, the resulting  $\Sigma_f^i(p^2)$  which defines the SM fermion mass splitting is given by the formula (4.17).

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