Extreme black hole anabasis

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ABSTRACT: We study the $\mathsf{SL}(2)$ transformation properties of spherically symmetric perturbations of the Bertotti-Robinson universe and identify an invariant μ that characterizes the backreaction of these linear solutions. The only backreaction allowed by Birkhoff's theorem is one that destroys the $AdS_2 \times S^2$ boundary and builds the exterior of an asymptotically flat Reissner-Nordström black hole with $Q = M\sqrt{1 - \mu/4}$. We call such backreaction with boundary condition change an *anabasis*. We show that the addition of linear anabasis perturbations to Bertotti-Robinson may be thought of as a boundary condition that defines a *connected* $AdS_2 \times S^2$. The connected AdS_2 is a nearly-AdS_2 with its $\mathsf{SL}(2)$ broken appropriately for it to maintain connection to the asymptotically flat region of Reissner-Nordström. We perform a backreaction calculation with matter in the connected $AdS_2 \times S^2$ and show that it correctly captures the dynamics of the asymptotically flat black hole.

KEYWORDS: Black Holes, Space-Time Symmetries

Contents

1 Introduction

Birkhoff's theorem in four dimensions tells us that all spherically symmetric spacetimes with vanishing Ricci tensor are static and therefore described by the Schwarzschild metric. The theorem extends to the Einstein-Maxwell equations with the Schwarzschild solution replaced by the Reissner-Nordström one. On the other hand, in the Einstein-Maxwell theory another spherically symmetric solution of importance is the Bertotti-Robinson universe with metric given by the direct product of AdS_2 with a two-sphere. This is consistent with Birkhoff's theorem because Bertotti-Robinson agrees with the near-horizon of extreme and near-extreme Reissner-Nordström, and the theorem is only a local statement.[1] Reissner-Nordström is asymptotically flat while Bertotti-Robinson is asymptotically $AdS_2 \times S^2$.

If one considers the linearized Einstein-Maxwell equations around Reissner-Nordström, then within spherical symmetry one finds a two-parameter family of solutions parametrized by the change in mass δM and charge δQ relative to the background. In other words, such linearized perturbations are moving towards other Reissner-Nordström solutions. On the other hand, if one considers the linearized Einstein-Maxwell equations around Bertotti-Robinson then one finds a four-parameter family of solutions. Why is that? One may identify two of the four parameters with $\delta M \pm \delta Q$ and derive the corresponding solutions from appropriate near-horizon limits of the linearized solutions around Reissner-Nordström. When $\delta M = \delta Q$ this results in a linearized solution around Bertotti-Robinson that respects

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[1]The precise statement of Birkhoff's theorem is that a C^2 solution of the Einstein (resp. Einstein-Maxwell) equations which is spherically symmetric in an open set $\mathcal{V}$ is locally equivalent to part of the maximally extended Schwarzschild (resp. Reissner-Nordström) solution in $\mathcal{V}$ (see, e.g., [1]).

the $\mathsf{SL}(2)$ symmetry associated with the AdS_2 factor of the background. This $\mathsf{SL}(2)$-preserving linear solution is asymptotically $AdS_2 \times S^2$ and moves towards another Bertotti-Robinson. When $\delta M \neq \delta Q$ the near-horizon limit produces a linear solution around Bertotti-Robinson that breaks its $\mathsf{SL}(2)$ symmetry. Therefore acting with the background's $\mathsf{SL}(2)$ isometries we may obtain two additional linear solutions. The $\mathsf{SL}(2)$-breaking triplet of solutions are not asymptotically $AdS_2 \times S^2$ and it is well known that the backreaction of these solutions destroys the AdS_2 boundary [2]. In the past, this has led to the slogans that "AdS_2 has no dynamics" or that "AdS_2 admits no finite energy excitations." While not incorrect, these slogans are true only as long as one insists on asymptotically AdS_2 boundary conditions.

In this paper, we study the $\mathsf{SL}(2)$-breaking triplet solutions with boundary conditions that allow the near-horizon $AdS_2 \times S^2$ throat to maintain its connection with the exterior asymptotically flat Reissner-Nordström. In this context, we first identify an $\mathsf{SL}(2)$-invariant quantity μ associated with any solution in the triplet. Then we show that when $\mu = 0$ the corresponding solution may be thought of as beginning to build the asymptotically flat region of extreme Reissner-Nordström starting from its $AdS_2 \times S^2$ throat. When $\mu > 0$ the corresponding solution is beginning to build the asymptotically flat region of near-extreme Reissner-Nordström with $Q = M\sqrt{1 - \mu/4}$. In other words, when $\mu \geq 0$ the backreaction of these $\mathsf{SL}(2)$-breaking linear solutions makes sense provided we allow the boundary condition change that leads to an asymptotically flat nonlinear solution. We call such backreaction with boundary condition change an *anabasis*—an adventure of climbing out of the black hole throat into the weak gravity regime.

Next, consider perturbing an extreme Reissner-Nordström black hole using a spherically symmetric infalling matter source with energy-momentum tensor of order $\epsilon \ll 1$. Generically, one expects the fully backreacted nonlinear endpoint of this perturbation to be a near-extreme Reissner-Nordström with $Q = M\sqrt{1 - \mathcal{O}(\epsilon)}$ [3]. To leading order in ϵ, the initial and final states only differ in their near-horizon region. More precisely, the near-horizon throat geometry remains locally $AdS_2 \times S^2$ before as well as after the perturbation but the associated $\mathsf{SL}(2)$ symmetry breaking induced by the gluing to the exterior region is different in the extreme and near-extreme cases. We define the *connected $AdS_2 \times S^2$* throat as the geometry obtained by the addition of anabasis perturbations. This may also be thought of as a boundary condition for the backreaction calculation. We show that this leads to a consistent backreaction calculation in $AdS_2 \times S^2$ that captures the dynamics of the asymptotically flat black hole.

In section 2, we derive the spherically symmetric perturbations of the Bertotti-Robinson universe and study their $\mathsf{SL}(2)$ transformation properties. In particular, we identify the invariant quantity μ associated with each $\mathsf{SL}(2)$-breaking perturbation. Section 3 singles out standard Poincaré and Rindler anabasis perturbations responsible for building the exterior in extreme and near-extreme Reissner-Nordström, respectively. Section 4 presents the general transformation from Poincaré to Rindler $AdS_2 \times S^2$. In section 5, we do a backreaction calculation for a pulse of energy ϵ in the connected $AdS_2 \times S^2$ throat of a Reissner-Nordström black hole. Section 6 contains further discussion of our work, especially in relation to the AdS/CFT correspondence and models of two-dimensional dilaton gravity in AdS_2.

2 Perturbations of Bertotti-Robinson

The Einstein-Maxwell equations in four dimensions read ($G = c = 1$)

$$R_{\mu\nu} = 8\pi T_{\mu\nu}\,, \quad \nabla^\mu F_{\mu\nu} = 0\,, \tag{2.1}$$

with $4\pi T_{\mu\nu} = F_{\mu\rho}F_\nu{}^\rho - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}$, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The Bertotti-Robinson universe

$$\frac{1}{M^2}ds^2 = -r^2 dt^2 + \frac{dr^2}{r^2} + d\Omega^2\,, \quad A_t = Mr\,, \tag{2.2}$$

is a spherically symmetric conformally flat exact solution with uniform electromagnetic field $F_{rt} = M$. Clearly the Bertotti-Robinson metric is a direct product $AdS_2 \times S^2$. From now on, we set $M = 1$ and restore it only when beneficial for clarity.

Consider the most general spherically symmetric perturbation

$$h_{\mu\nu} = \begin{pmatrix} h_{tt}(t,r) & h_{tr}(t,r) & 0 & 0 \\ & h_{rr}(t,r) & 0 & 0 \\ & & h_{\theta\theta}(t,r) & 0 \\ & & & h_{\theta\theta}\sin^2\theta \end{pmatrix}\,, \tag{2.3}$$

$$a_\mu = \begin{pmatrix} a_t(t,r) & a_r(t,r) & 0 & 0 \end{pmatrix}\,.$$

The linearized Einstein-Maxwell equations are invariant under the gauge transformations

$$h_{\mu\nu} \to h_{\mu\nu} + \mathcal{L}_\xi g_{\mu\nu}\,, \quad a_\mu \to a_\mu + \mathcal{L}_\xi A_\mu + \nabla_\mu \Lambda\,, \tag{2.4}$$

for any vector field ξ and scalar function Λ. Within the spherically symmetric ansatz, we may use this gauge freedom, with appropriate $\xi = \xi^t(t,r)\partial_t + \xi^r(t,r)\partial_r\,, \Lambda = \Lambda(t,r)$, to set

$$h_{tt} = h_{rr} = a_t = 0\,, \tag{2.5}$$

and remove from h_{tr} any addition of the form $h_{tr} = c_1(r) + c_2(t)/r$ for arbitrary c_1, c_2. Note, however, that for perturbations around the Bertotti-Robinson solution $h_{\theta\theta}$ is gauge invariant. Therefore, all physical information for perturbations of Bertotti-Robinson is contained in $h_{\theta\theta}$.

We find that the most general solution to the linearized Einstein-Maxwell equations around Bertotti-Robinson is given by

$$h_{\theta\theta} = \Phi_0 + ar + brt + cr\left(t^2 - 1/r^2\right)\,, \tag{2.6}$$

$$h_{tr} = -\frac{1}{2}rt\left[\Phi_0 + 2ar + brt + \frac{2}{3}cr\left(t^2 - 9/r^2\right)\right]\,, \tag{2.7}$$

$$f_{tr} = \partial_t a_r = h_{\theta\theta} - \Phi_0/2\,. \tag{2.8}$$

Thus, the spherically symmetric perturbations of Bertotti-Robinson are a four-parameter family of solutions parametrized by the constants Φ_0, a, b, c.

2.1 SL(2) transformations and invariants

The background (2.2) is invariant under the SL(2) transformations associated with the AdS_2 factor:

$$H(\alpha): \qquad t \to t + \alpha \,, \tag{2.9}$$

$$D(\beta): \qquad t \to t/\beta \,, \quad r \to \beta r \,, \tag{2.10}$$

$$K(\gamma): \qquad t \to \frac{t - \gamma \left(t^2 - 1/r^2\right)}{1 - 2\gamma t + \gamma^2 \left(t^2 - 1/r^2\right)} \,, \quad r \to r \left[1 - 2\gamma t + \gamma^2 \left(t^2 - 1/r^2\right)\right] \,. \tag{2.11}$$

Here $H(\alpha)$, $D(\beta)$, $K(\gamma)$ are the time translations, dilations, and special conformal transformations for real parameters α, β, γ with $\beta > 0$. The special conformal coordinate transformation must be followed by a gauge field transformation $A \to A + d \ln \frac{r(t-1/\gamma)+1}{r(t-1/\gamma)-1}$.

The SL(2) invariance of the background implies that if we act with an SL(2) transformation on any of the solutions (2.6)–(2.8) we will obtain another solution to the linearized Einstein-Maxwell equations around the same background. Note, however, that the SL(2) transformations do not necessarily preserve the gauge (2.5). Fortunately, as we have previously emphasized, $h_{\theta\theta}$ is gauge invariant and therefore uniquely labels each physically distinct solution in every gauge.

The four-parameter solution (2.6) consists of an SL(2)-invariant solution Φ_0 together with the SL(2)-breaking triplet

$$\Phi = ar + brt + cr \left(t^2 - 1/r^2\right) \,. \tag{2.12}$$

Clearly, the SL(2)-invariant solution Φ_0 corresponds to a rescaling of (2.2) by $M \to M + \delta M$ with $\Phi_0 = 2M \, \delta M$. In the remainder of the paper, we will focus on the SL(2)-breaking solutions Φ.

The action of the SL(2) transformations on Φ is given by

$$H(\alpha): a \to a + b\alpha + c\alpha^2 \,, \qquad b \to b + 2c\alpha \,, \qquad c \to c \,, \tag{2.13}$$

$$D(\beta): a \to a\beta \,, \qquad\qquad b \to b \,, \qquad\qquad c \to c/\beta \,, \tag{2.14}$$

$$K(\gamma): a \to a \,, \qquad\qquad b \to b - 2a\gamma \,, \qquad c \to c - b\gamma + a\gamma^2 \,. \tag{2.15}$$

Using the above, we identify the following SL(2) invariant

$$\mu = b^2 - 4ac \,. \tag{2.16}$$

Moreover, we note that for $\mu < 0$ we have $\operatorname{sgn} a = \operatorname{sgn} c \neq 0$ being an additional SL(2) invariant, while for $\mu = 0$ it is $\operatorname{sgn}(a + c)$ that is also SL(2)-invariant.

From (2.13)–(2.15), we see that each of the SL(2) transformations preserves one and only one of the three solutions in (2.12). If we think of (a, b, c) as charges associated with the corresponding SL(2) transformations, then μ may be identified with the quadratic SL(2) Casimir, which is invariant under the group action. In fact, under a general SL(2) isometry, (a, b, c) transforms in the coadjoint representation, thereby generating a coadjoint orbit. The invariant μ labels the coadjoint orbits of SL(2), which come in three types (elliptic, parabolic, and hyperbolic) according to the sign of μ.

Before we end this section, let us fix two standard choices for the general solution (2.12). For $\mu > 0$ one may always find an $\mathsf{SL}(2)$ transformation that will set

$$\Phi = -\sqrt{\mu}\, rt\,, \quad \mu > 0\,. \tag{2.17}$$

Similarly, for $\mu = 0$ and $\mathrm{sgn}(a + c) = 1$ one may set

$$\Phi = 2r\,, \quad \mu = 0\,, \mathrm{sgn}(a + c) = 1\,. \tag{2.18}$$

Specifically, when $\mu > 0$ we may get to $\Phi = -\sqrt{\mu}\, rt$ by acting on (2.12) with the following series of $\mathsf{SL}(2)$ transformations:

$$H\left(\frac{a}{\sqrt{b^2 - 4ac}}\right) \circ K\left(\frac{b + \sqrt{b^2 - 4ac}}{2a}\right)\,, \qquad \text{for} \quad a \neq 0\,, \tag{2.19}$$

$$K(\gamma) \circ H\left(1/\gamma\right) \circ K(\gamma) \circ K\left(\frac{c}{b}\right)\,, \qquad \text{for} \quad a = 0\,, b > 0\,, \tag{2.20}$$

$$K\left(\frac{c}{b}\right)\,, \qquad \text{for} \quad a = 0\,, b < 0\,. \tag{2.21}$$

Likewise, when $\mu = 0$ we may get to $\Phi = \mathrm{sgn}(a+c)\, 2r$ by acting on (2.12) with the following series of $\mathsf{SL}(2)$ transformations:

$$D\left(\frac{2}{|a|}\right) \circ K\left(\frac{b}{2a}\right)\,, \qquad \text{for} \quad abc \neq 0\,, \tag{2.22}$$

$$D\left(\frac{2}{\gamma^2 |c|}\right) \circ K\left(1/\gamma\right) \circ H(\gamma)\,, \qquad \text{for} \quad b = a = 0\,, \tag{2.23}$$

$$D\left(\frac{2}{|a|}\right)\,, \qquad \text{for} \quad b = c = 0\,. \tag{2.24}$$

3 Anabasis and the connected throat

The Bertotti-Robinson solution (2.2) may be derived from near-horizon near-extremality scalings of the Reissner-Nordström black hole solution of mass M and charge Q, with outer/inner horizons at $r_\pm = M \pm \sqrt{M^2 - Q^2}$,

$$ds^2 = -\left(1 - \frac{2M}{\hat{r}} + \frac{Q^2}{\hat{r}^2}\right) d\hat{t}^2 + \left(1 - \frac{2M}{\hat{r}} + \frac{Q^2}{\hat{r}^2}\right)^{-1} d\hat{r}^2 + \hat{r}^2 d\Omega^2\,, \quad \hat{A}_{\hat{t}} = -\frac{Q}{\hat{r}}\,. \tag{3.1}$$

This implies that the leading corrections in these scaling limits are, by construction, solutions of the linearized Einstein-Maxwell equations around Bertotti-Robinson. There are two essentially distinct scaling limits of the black hole exterior that yield the Bertotti-Robinson solution.

The first scaling limit is most simply described by setting $Q = M$, making the coordinate and gauge transformation,

$$r = \frac{\hat{r} - M}{\lambda M}\,, \quad t = \frac{\lambda \hat{t}}{M}\,, \quad A = \hat{A} + d\hat{t}\,, \tag{3.2}$$

to obtain

$$\frac{1}{M^2}ds^2 = -\left(\frac{r}{1+\lambda r}\right)^2 dt^2 + \left(\frac{r}{1+\lambda r}\right)^{-2} dr^2 + (1+\lambda r)^2 d\Omega^2\,, \quad A_t = M\frac{r}{1+\lambda r}\,, \quad (3.3)$$

and then taking the limit $\lambda \to 0$. At order $\mathcal{O}(1)$ this produces exactly (2.2). The leading correction is of order $\mathcal{O}(\lambda)$ and it is given by

$$h_{tt} = 2r^3\,, \quad h_{rr} = 2/r\,, \quad h_{\theta\theta} = 2r\,, \quad f_{rt} = -2r\,. \quad (3.4)$$

By construction, this solves the linearized Einstein-Maxwell equations around (2.2).

Comparing the gauge invariant $h_{\theta\theta}$ in the above with (2.6) we see that this is the $\Phi = 2r$ solution.[2] Hence the $\mathsf{SL}(2)$-breaking $\mu = 0$ solution $\Phi = 2r$ may be thought of as beginning to build the asymptotically flat region of an extreme Reissner-Nordström starting from its near-horizon Bertotti-Robinson throat. In other words, the nonlinear solution obtained from the $\mu = 0$ perturbation of $AdS_2 \times S^2$, when backreaction is fully taken into account in the Einstein-Maxwell theory, is the extreme Reissner-Nordström black hole.

The second scaling limit is described by setting $Q = M\sqrt{1 - \lambda^2\kappa^2}$, making the coordinate and gauge transformation,

$$\rho = \frac{\hat{r} - r_+}{\lambda r_+}\,, \quad \tau = \frac{\lambda\hat{t}}{M}\,, \quad A = \hat{A} + d\hat{t}\,, \quad (3.5)$$

to obtain

$$\frac{1}{M^2}ds^2 = -\frac{\rho(\rho + 2\kappa + \lambda\kappa\rho)}{(1+\lambda\kappa)(1+\lambda\rho)^2}d\tau^2 + \frac{(1+\lambda\kappa)^3(1+\lambda\rho)^2}{\rho(\rho + 2\kappa + \lambda\kappa\rho)}d\rho^2 + (1+\lambda\kappa)^2(1+\lambda\rho)^2 d\Omega^2\,,$$

$$A_\tau = \frac{M}{\lambda}\left(1 - \sqrt{\frac{1-\lambda\kappa}{1+\lambda\kappa}}\frac{1}{1+\lambda\rho}\right)\,, \quad (3.6)$$

and then taking the limit $\lambda \to 0$. At order $\mathcal{O}(1)$ this produces

$$\frac{1}{M^2}ds^2 = -\rho(\rho + 2\kappa)d\tau^2 + \frac{d\rho^2}{\rho(\rho + 2\kappa)} + d\Omega^2\,, \quad A_\tau = M(\rho + \kappa)\,. \quad (3.7)$$

The leading correction is of order $\mathcal{O}(\lambda)$ and it is given by

$$h_{\tau\tau} = 2\rho(\rho + \kappa)^2\,, \quad h_{\rho\rho} = \frac{2\left(\rho^2 + 3\kappa^2 + 3\kappa\rho\right)}{\rho(\rho + 2\kappa)^2}\,, \quad h_{\theta\theta} = 2(\rho + \kappa)\,, \quad f_{\rho\tau} = -2\rho - \kappa\,. \quad (3.8)$$

By construction, this solves the linearized Einstein-Maxwell equations around (3.7).

Locally, the $\mathcal{O}(1)$ results of the two scaling limits we have considered [eqs. (2.2) and (3.7)] are diffeomorphic — they are both the Bertotti-Robinson universe. Indeed, the coordinate transformation [4]

$$\tau = -\frac{1}{2\kappa}\ln\left(t^2 - 1/r^2\right)\,, \quad \rho = -\kappa(1 + rt)\,, \quad (3.9)$$

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[2]Indeed, one may align (3.4) with the $a = 2$, $b = c = \Phi_0 = 0$ solution (2.6)–(2.8) by adjusting the gauge via (2.4) with $\xi = 2rt\partial_t - r^2\partial_r\,, \Lambda = 0$.
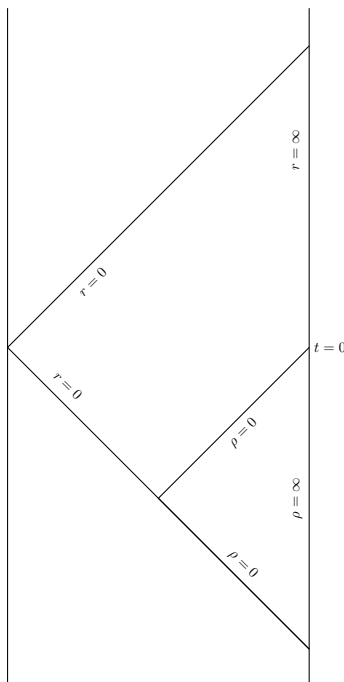
Figure 1. Penrose diagram of $AdS_2 \times S^2$ with relative placement of the Poincaré patch (2.2) and the Rindler patch (3.7) according to the transformation (3.9).

together with $A \to A - d\Lambda$, $\Lambda = \frac{1}{2} \ln \frac{rt-1}{rt+1} = -\frac{1}{2} \ln \frac{\rho}{\rho+2\kappa}$ maps (3.7) to (2.2). Globally, on the Penrose diagram of $AdS_2 \times S^2$, the coordinates in (2.2) cover a Poincaré patch while the coordinates in (3.7) cover a Rindler patch. The transformation (3.9) situates the two patches relative to each other as shown in figure 1. It follows that under this transformation the leading $\mathcal{O}(\lambda)$ correction (3.8) transforms to a solution of the linearized Einstein-Maxwell equations around (2.2). Comparing the gauge invariant $h_{\theta\theta} = 2(\rho + \kappa) = -2\kappa rt$ with (2.6), we see that this is the $\Phi = -2\kappa rt$ solution. Hence the $\mathsf{SL}(2)$-breaking $\sqrt{\mu} = 2\kappa$ solution $\Phi = -2\kappa rt$ may be thought of as beginning to build the asymptotically flat region of a near-extreme Reissner-Nordström starting from its near-horizon Bertotti-Robinson throat. In other words, the nonlinear solution obtained from the $\mu > 0$ perturbation of $AdS_2 \times S^2$, when backreaction is fully taken into account in the Einstein-Maxwell theory, is the near-extreme Reissner-Nordström black hole with $Q = M\sqrt{1 - \mu/4}$.

We end this section by bringing attention to the fact that the anabasis solutions $\Phi = 2r$ and $\Phi = 2(\rho + \kappa)$ that begin to build the asymptotically flat black hole exteriors are positive

$$\Phi > 0 \,. \tag{3.10}$$

Intuitively, this is because Φ measures the increase in the size of the S^2 as one climbs out of a black hole's throat towards its asymptotically flat region. In particular, notice that when the Rindler anabasis solution $\Phi = 2(\rho + \kappa)$ is mapped to $\Phi = -2\kappa rt$ via (3.9), this leads to the range $rt \leq -1$ shown in figure 1.

4 General Poincaré to Rindler transformation

The transformation between the Poincaré and Rindler backgrounds used in the previous section [eq. (3.9)] maps the Rindler anabasis solution $h_{\theta\theta} = 2(\rho + \kappa)$ to the $\mu > 0$ perturbation $\Phi = -\sqrt{\mu}\, rt$ in Poincaré coordinates (2.17) with $\sqrt{\mu} = 2\kappa$. As explained in section 2.1, the standard $\Phi = -\sqrt{\mu}\, rt$ solution is related by $\mathsf{SL}(2)$ transformations to any $\mathsf{SL}(2)$-breaking solution Φ (2.12) with the same $\mu > 0$. As a result, there is a two-parameter generalization of the standard Poincaré to Rindler transformation (3.9) that can map the $h_{\theta\theta} = 2(\rho + \kappa)$ Rindler anabasis solution to a general Poincaré solution Φ (2.12) with $\sqrt{\mu} = 2\kappa$.

On the Penrose diagram of $AdS_2 \times S^2$, the two-parameter generalization of figure 1 allows for the Rindler patch to have arbitrary vertical location and size with respect to the Poincaré one. This is shown in figure 2. The general coordinate transformation is

$$
\begin{aligned}
t &= \left(1 + \nu^2\right) \frac{e^{\kappa\tau}\sqrt{\rho(\rho + 2\kappa)}\left(\rho + \kappa + \psi e^{\kappa\tau}\sqrt{\rho(\rho + 2\kappa)}\right)}{\left(\rho + \kappa + \psi e^{\kappa\tau}\sqrt{\rho(\rho + 2\kappa)}\right)^2 - \kappa^2} - \nu\,, \\
r &= \frac{1}{(1 + \nu^2)\,\kappa}\left(e^{-\kappa\tau}\sqrt{\rho(\rho + 2\kappa)}\left(1 + \psi^2 e^{2\kappa\tau}\right) + 2\psi(\rho + \kappa)\right)\,,
\end{aligned}
\tag{4.1}
$$

accompanied by $A \to A + d\Lambda$,

$$
\begin{aligned}
\Lambda &= -\frac{1}{2}\ln\frac{\rho}{\rho + 2\kappa} + \ln\frac{\rho + \psi e^{\kappa\tau}\sqrt{\rho(\rho + 2\kappa)}}{\rho + 2\kappa + \psi e^{\kappa\tau}\sqrt{\rho(\rho + 2\kappa)}} \\
&= \frac{1}{2}\ln\frac{[r(t + \nu) - 1]\left[\psi r(t + \nu) - (1 + \nu^2)\, r - \psi\right]}{[r(t + \nu) + 1]\left[\psi r(t + \nu) - (1 + \nu^2)\, r + \psi\right]}\,,
\end{aligned}
\tag{4.2}
$$

with $\psi = \nu - \chi \geq 0$. The derivation of this general transformation is in appendix A.

Using the above general transformation we may ask again: when is it possible to map a Poincaré solution $\Phi = ar + brt + cr\left(t^2 - 1/r^2\right)$ to the Rindler anabasis solution $\Phi = 2(\rho + \kappa)$? We find that the answer is again: when and only when $\mu = b^2 - 4ac > 0$. For $\mu > 0$ we find that the parameter identification is

$$
\kappa = \sqrt{b^2 - 4ac}/2 = \sqrt{\mu}/2\,,
\tag{4.3}
$$

and

$$
\nu = \frac{b - \sqrt{b^2 - 4ac}}{2c}\,, \qquad \chi = \frac{a + c}{\sqrt{b^2 - 4ac}}\,, \qquad \text{for} \quad c < 0\,,
\tag{4.4}
$$

$$
\nu = a/b\,, \qquad\qquad \chi = a/b\,, \qquad\qquad \text{for} \quad c = 0\,, b > 0\,,
\tag{4.5}
$$

$$
\nu = +\infty\,, \qquad\qquad \chi = -a/b\,, \qquad\quad \text{for} \quad c = 0\,, b < 0\,.
\tag{4.6}
$$

Notice that the above does not include any solutions with $c > 0$. The reason is the following. As noted in (3.10), only solutions with $\Phi > 0$ may be used for anabasis to an asymptotically flat black hole region. For $c > 0$, near the boundary $r \to \infty$, the general Poincaré solution is given by $\Phi \approx r\left(a + bt + ct^2\right)$, which is positive in a portion
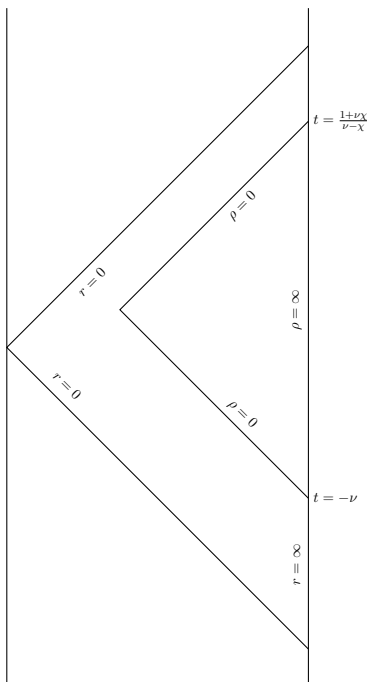
Figure 2. Penrose diagram of $AdS_2 \times S^2$ with relative placement of the Poincaré patch (2.2) and the Rindler patch (3.7) according to the transformation (4.1).

of the boundary that has the form $(-\infty, t_1) \cup (t_2, +\infty)$. As a result, no $\mu > 0$ solution Φ with $c > 0$ may be transformed to the Rindler anabasis solution $\Phi = 2(\rho + \kappa)$ by a single transformation (4.1)–(4.2) everywhere near the boundary. That said, if we allow for topology change when $c > 0$, it may be possible to perform anabasis to two separate asymptotically flat regions.

5   Backreaction calculation in the connected throat

In the previous sections, we have seen that the AdS_2 backreaction problem for an $\mathsf{SL}(2)$-breaking electrovacuum solution Φ in four-dimensional Einstein-Maxwell theory is consistent as long as one does not insist on maintaining AdS_2 boundary conditions but rather considers such linear solutions as black hole anabasis solutions that build the asymptotically flat regions of extreme (for $\mu = 0$) and near-extreme (for $\mu > 0$) Reissner-Nordström. In this section, we show that the addition of anabasis perturbations to AdS_2 may also be thought of as a boundary condition for a *connected AdS_2*. The connected AdS_2 is a nearly-AdS_2 with its $\mathsf{SL}(2)$ broken appropriately for it to maintain connection to the asymptotically flat region of a Reissner-Nordström black hole.

Consider, for example, throwing a matter pulse of energy $\epsilon > 0$ into the connected AdS_2 throat of an extreme Reissner-Nordström. That is, add to the right hand side of the Einstein equation in (2.1) the matter energy-momentum tensor

$$8\pi T_{vv}^{\mathrm{matter}} = \epsilon \delta(v - v_0), \quad v = t - 1/r. \tag{5.1}$$

For $\epsilon \ll 1$ we may find an $\mathcal{O}(\epsilon)$ metric and gauge field perturbation around the Bertotti-Robinson universe that generalizes the solution (2.6)–(2.8) according to

$$h_{\theta\theta} = \Phi_0 + ar + brt + cr\left(t^2 - 1/r^2\right) - \frac{\epsilon}{2}\frac{r^2(t-v_0)^2 - 1}{r}\Theta(v - v_0)\,, \qquad (5.2)$$

$$h_{tr} = -\frac{1}{2}rt\left[\Phi_0 + 2ar + brt + \frac{2}{3}cr\left(t^2 - 9/r^2\right)\right] \qquad (5.3)$$

$$+ \frac{\epsilon}{2}\frac{r^3(t-v_0)^3 - 9r(t-v_0) + 6\ln(r(t-v_0)) + 8}{3r}\Theta(v - v_0)\,,$$

$$f_{tr} = \partial_t a_r = h_{\theta\theta} - \Phi_0/2\,. \qquad (5.4)$$

Before the pulse, for $v < v_0$, we impose the causal boundary condition for a connected AdS_2 throat given by the $\mu = 0$ Poincaré anabasis solution $\Phi = 2r$,

$$h_{\theta\theta} = 2r\,, \quad \text{for} \quad v < v_0\,. \qquad (5.5)$$

That is to say, we set $a = 2$ and $b = c = \Phi_0 = 0$. Then after the pulse, for $v > v_0$, we get

$$h_{\theta\theta} = 2r - \frac{\epsilon}{2}\frac{r^2(t-v_0)^2 - 1}{r}\,, \quad \text{for} \quad v > v_0\,. \qquad (5.6)$$

This solution after the pulse is a $\mu = 4\epsilon$ solution which, using the results from section 4, maps to the Rindler anabasis solution $h_{\theta\theta} = 2(\rho + \kappa)$ via (4.1)–(4.2) with

$$\kappa = \sqrt{\epsilon}\,, \quad \nu = \frac{2}{\sqrt{\epsilon}} - v_0\,, \quad \chi = \frac{1}{\sqrt{\epsilon}} - \frac{1+v_0^2}{4}\sqrt{\epsilon}\,. \qquad (5.7)$$

We thus see that we have a backreaction calculation in the connected AdS_2 throat that is consistent with the expectation from the physics of Reissner-Nordström: throwing a pulse of energy $\epsilon \ll 1$ into the extreme black hole with $Q = M$ "shifts the horizon" and the black hole becomes near extreme with $Q = M\sqrt{1-\epsilon}$. This is shown in figure 3.

6 Discussion

In this paper, we have studied backreaction in the context of $AdS_2 \times S^2$ connected to an asymptotically flat region in four-dimensional Einstein-Maxwell theory. We imposed spherical symmetry but considered both electrovacuum solutions as well as a matter source in the form of a null ingoing pulse. We have seen that backreaction with boundary condition change, which we call anabasis, is consistent with Reissner-Nordström physics.

In AdS/CFT, anabasis is dual to following the inverse renormalization group (RG) flow, from IR to UV, for an appropriate irrelevant deformation of the boundary field theory that does not respect AdS boundary conditions. This is not something discussed very often in the AdS/CFT literature for at least two reasons. First, it is a difficult question to study systematically because it is hard to identify appropriate solvable irrelevant deformations of CFTs. Second, it runs somewhat contrary to the spirit of AdS/CFT which is a complete self-contained theory in itself — a theory in which even when one studies irrelevant deformations, one may wish to restrict oneself to deformations which do not destroy
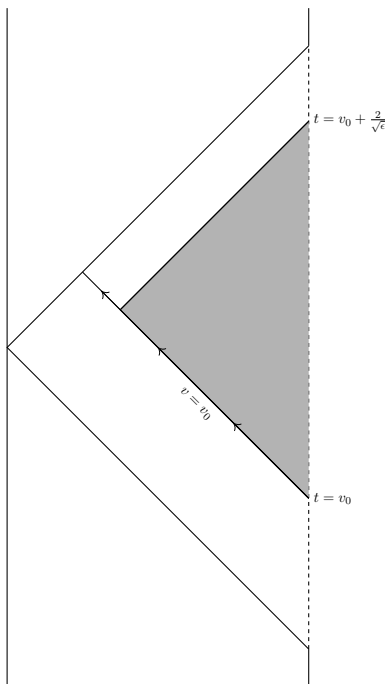
Figure 3. Penrose diagram of $AdS_2 \times S^2$. The dashed boundary signifies that this AdS_2 is maintaining connection with the exterior of Reissner-Nordström. An ingoing pulse of energy ϵ enters the extreme throat (Poincaré patch) at $t = v_0$. The black hole horizon shifts and the throat becomes near-extreme (shaded Rindler patch).

the boundary of AdS. Historically, of course, AdS/CFT was discovered by a low-energy near-horizon limit from string theory in asymptotically flat spacetime. A recent body of work that carries out an anabasis by following a flow for a single-trace irrelevant deformation of a CFT$_2$, which goes under the name $T\overline{T}$ and changes AdS_3 asymptotics to flat with a linear dilaton, may be found in [5–8].[3] For AdS_2, it is not clear to us how an inverse RG flow would be implemented microscopically due to the apparent lack of irrelevant deformations of quantum mechanical models with conformal symmetry. For an RG flow along a relevant deformation, useful for flowing away from AdS_2 but not for an anabasis from it, see, e.g., [12].

The gravitational aspects of AdS_2 anabasis studied in this paper do not rely on the existence of a holographic dual and are expected to be readily generalizable to a wide class of theories with (near-)extreme black holes which universally exhibit AdS_2-like near-horizon geometries [13]. This includes rotating black holes such as Kerr which near extremality has a throat geometry, the Near-Horizon-Extreme-Kerr (NHEK) solution [14], with backreaction properties similar to $AdS_2 \times S^2$ [15, 16]. In appendix B, we give the $\mathsf{SL}(2)$-breaking triplet of linear perturbations of NHEK that generalizes (2.6)–(2.8). Beyond near-horizon approximations, AdS_2 makes an appearance in other contexts where approximate space-

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[3]A different double-trace $T\overline{T}$ deformation of CFT$_2$ has been holographically interpreted as a gravitational theory in an AdS_3 that is cut off at a finite interior surface [9] (see also [10]). This is not related to anabasis as it may be obtained from mixed boundary conditions that respect the AdS_3 boundary [11].

time decoupling occurs, such as the interaction region of colliding shock electromagnetic plane waves [17], or near certain highly localized matter distributions [18]. The ideas in this paper may also be relevant in such contexts.

A model of two-dimensional dilaton gravity in AdS_2 that is solvable with backreaction, as well as with the addition of matter, is the Jackiw-Teitelboim (JT) theory [19, 20]. This model captures many of the universal aspects in the spherically symmetric sector of higher dimensional gravity near extreme black hole horizons, and it has been studied extensively from the holographic perspective beginning with [21–24]. In JT theory, the geometry is fixed to being locally AdS_2 but the $\mathsf{SL}(2)$ is broken by a dilaton Φ_{JT}. Comparing with our gravitational perturbations, we may identify $\Phi = \Phi_{JT}$, noting that (2.12) solves the JT equation of motion $\nabla_\mu \nabla_\nu \Phi_{JT} - g_{\mu\nu} \nabla^2 \Phi_{JT} + g_{\mu\nu} \Phi_{JT} = 0$ for AdS_2 in Poincaré coordinates. This is because in the ansatz (2.3) we have Φ measuring the variation in the size of the S^2 and, in this ansatz, dimensional reduction of higher dimensional gravity down to two dimensions is known to lead to JT theory with the dilaton Φ_{JT} measuring precisely this variation (see e.g [25–28]). Continuing the comparison, the $\mathsf{SL}(2)$-invariant μ defined in (2.16) may be identified with the ADM mass of the 2D black holes in JT theory [21, 29]. It follows that the mass of the 2D AdS_2 black hole in JT is the deviation from extremality of the 4D Reissner-Nordström in Einstein-Maxwell. A comment is in order here. It is often said in the literature that JT is a nearly-AdS_2 theory with the "nearly" part, which is due to the dilaton's breaking of the $\mathsf{SL}(2)$ symmetry of AdS_2, associated with a departure from extremality. As we have seen in this paper, however, this is not necessarily so because for $\mu = 0$ the $\mathsf{SL}(2)$ may be broken only in order to build the exterior of an exactly extreme Reissner-Nordström.

The connected $AdS_2 \times S^2$, which we defined in section 5 in order to perform a consistent Reissner-Nordström backreaction calculation, is nearly-AdS_2 in the sense that its $\mathsf{SL}(2)$ has been broken by the addition of anabasis perturbations that make this AdS_2 an approximate one. We also saw that this $\mathsf{SL}(2)$ breaking may be thought of as a choice of boundary condition for the backreaction calculation. A comprehensive study of various boundary conditions for the JT theory has been carried out in [30]. However, it appears that none of the boundary conditions contained therein would yield an anabasis as they at best correspond to mixed boundary conditions that do not destroy the AdS_2 boundary (of the double-trace type in AdS/CFT terms). On the other hand, the boundary term used in [31] for what is called therein "permeable boundary conditions" appears to be a better candidate for defining a connected AdS_2 in JT theory. Indeed, matching fields across the $AdS_2 \times S^2$ boundary in Reissner-Nordström, as in the calculations of [32, 33], necessitates boundary conditions that are "leaky" from the AdS_2 point of view.

Broadening the pulse used in section 5 and replacing it with a finite-width wavepacket, one may arrange to have such a wavepacket enter the $AdS_2 \times S^2$ region by sending in low energy waves from past null infinity in Reissner-Nordström. The calculation may be set up using matched asymptotic expansions and features leaky boundary conditions [34].

Generically, the backreaction of an extreme Reissner-Nordström black hole results, as in section 5, in a near-extreme one. However, there is a notable exception. In [3] it was found that there exist fine-tuned initial data for a massless scalar perturbing extreme

Reissner-Nordström, for which an instability of the scalar field at the event horizon — the Aretakis instability — persists for arbitrarily long evolution and leads to a spacetime that may be thought of as a dynamical extreme black hole. It was observed that, at late times, this dynamical extreme black hole has the same exterior as extreme Reissner-Nordström but differs from it at the horizon. In [35], the Aretakis instability of the perturbing massless scalar on extreme Reissner-Nordström was analyzed using the symmetries of its $AdS_2 \times S^2$ throat. It would be interesting to study the dynamical extreme black hole of [3] using a connected $AdS_2 \times S^2$ as defined in this paper.

Acknowledgments

A   Derivation of the transformation in section 4

In this appendix, we give some details about the derivation of the general Poincaré to Rindler transformation (4.1)–(4.2) in section 4. In particular, we show how this general transformation may be derived by composing a global AdS_2 time translation with a Poincaré time translation of the basic transformation (3.9).

The global AdS_2 coordinates, with $ds_2^2 = (-d\eta^2 + d\sigma^2)/\sin^2\sigma$, are related to the Poincaré coordinates via $t \pm \frac{1}{r} = \tan\left(\frac{\eta \pm \sigma}{2}\right)$. The translation $\eta \to \eta + \eta_0$ then moves the Poincaré patch vertically on the Penrose diagram of AdS_2 as shown in the left panel of figure 4. In the overlapping region, with $ds_2^2 = -r^2 dt^2 + dr^2/r^2 = -\tilde{r}^2 d\tilde{t}^2 + d\tilde{r}^2/\tilde{r}^2$, the relation between the two different sets of Poincaré coordinates reads

$$
\begin{aligned}
t &= -\left(1 + \nu^2\right) \frac{\tilde{r}^2\left(\tilde{t} - \nu\right)}{\tilde{r}^2\left(\tilde{t} - \nu\right)^2 - 1} - \nu \,, \\
r &= \frac{1}{1 + \nu^2} \frac{\tilde{r}^2\left(\tilde{t} - \nu\right)^2 - 1}{\tilde{r}} \,,
\end{aligned}
\tag{A.1}
$$

with $\nu = \cot(\eta_0/2)$. For the Bertotti-Robinson solution, the above coordinate transformation must be supplemented by the gauge field transformation $A \to A + d\Lambda \,, \Lambda = \ln \frac{r(t+\nu)-1}{r(t+\nu)+1}$.

The resizing of the Rindler patch situated as in figure 1 is achieved by a translation of the Poincaré time coordinate in the transformation (3.9). Specifically, the transformation

$$
\begin{aligned}
\tilde{t} &= -e^{-\kappa\tau} \frac{\rho + \kappa}{\sqrt{\rho(\rho + 2\kappa)}} + \chi \,, \\
\tilde{r} &= \frac{1}{\kappa} e^{\kappa\tau} \sqrt{\rho(\rho + 2\kappa)} \,,
\end{aligned}
\tag{A.2}
$$

together with $A \to A + d\Lambda \,, \Lambda = \frac{1}{2} \ln \frac{\tilde{r}(\tilde{t}-\chi)-1}{\tilde{r}(\tilde{t}-\chi)+1}$ implements the mapping shown in the right panel of figure 4.
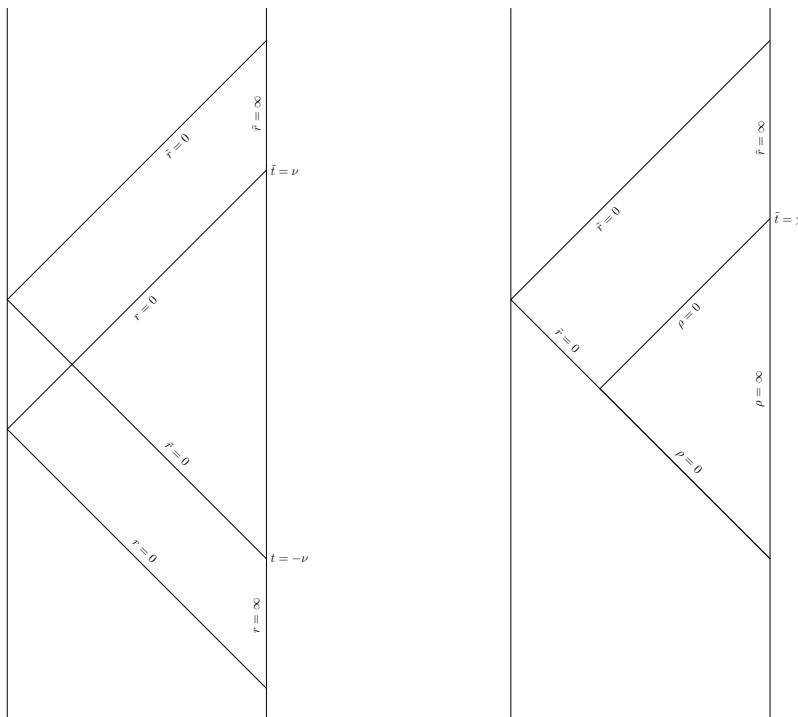
Figure 4. Penrose diagrams of $AdS_2 \times S^2$. Left: the two Poincaré patches are related by a global time translation according to the transformation (A.1). Right: the Poincaré and Rindler patches are situated according to the transformation (A.2).

The transformation (4.1)–(4.2) is a composition of (A.1) with (A.2). Note that one must have $\nu \geq \chi$ in order for the Rindler patch to be contained entirely inside the Poincaré one covered by the (t, r) coordinates.

B SL(2)-breaking NHEK perturbations

The Near-Horizon of Extreme-Kerr (NHEK) was obtained in [14] by a scaling limit $\lambda \to 0$ applied to extreme Kerr, analogous to the one that produces Bertotti-Robinson from extreme Reissner-Nordström. The $\mathcal{O}(1)$ NHEK metric is given by

$$ds^2 = 2M^2\Gamma \left[-r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda^2 \left(d\phi + r dt \right)^2 \right], \tag{B.1}$$

$$\Gamma(\theta) = \frac{1 + \cos^2\theta}{2}, \quad \Lambda(\theta) = \frac{2\sin\theta}{1 + \cos^2\theta}. \tag{B.2}$$

NHEK has an isometry group given by $\mathsf{SL}(2) \times \mathsf{U}(1)$, where the $\mathsf{U}(1)$ is due to axial symmetry and the $\mathsf{SL}(2)$ is again associated with the AdS_2 part in the above.

An $\mathsf{SL}(2)$-breaking triplet of axially symmetric linear perturbations of NHEK that generalizes (2.6)–(2.8) is given by

$$h_{tt} = 4\Gamma\partial_t^2\Phi + r^2\left(2\Gamma - 1\right)\left(1 + \Lambda^2\right)\Phi + 4rA_t\Gamma\Lambda^2, \qquad h_{tr} = 2rA_r\Gamma\Lambda^2, \qquad \text{(B.3)}$$

$$h_{t\phi} = \Phi r\left(2\Gamma - 1\right)\Lambda^2 + 2A_t\Gamma\Lambda^2, \qquad h_{rr} = \frac{\Phi}{r^2}, \qquad h_{r\phi} = 2A_r\Gamma\Lambda^2, \qquad \text{(B.4)}$$

$$h_{\theta\theta} = \Phi, \quad h_{\phi\phi} = \Phi\left(2\Gamma - 1\right)\Lambda^2, \qquad \text{(B.5)}$$

with

$$A_t = -r^2\frac{\Gamma^2\Lambda^2}{8}\partial_r\Phi, \quad A_r = \frac{1}{r^2}\left(1 - \frac{\Gamma^2\Lambda^2}{8}\right)\partial_t\Phi, \qquad \text{(B.6)}$$

and Φ as in (2.12). We derived the above as follows. First, we obtained the anabasis solution, corresponding to $\Phi = 2r$, from the $\mathcal{O}(\lambda)$ term in the expansion of [14]. Then, we applied $\mathsf{SL}(2)$ transformations to generate the triplet while adjusting the gauge for clarity. We note that for NHEK perturbations, even within axial symmetry, $h_{\theta\theta}$ is not gauge-invariant.

We expect the anabasis of these three perturbations towards Kerr to proceed in a similar fashion to the analysis carried out above for Reissner-Nordström. However, it is worth emphasizing that there is no analog of Birkhoff's theorem for axisymmetric spacetimes and that there exist axisymmetric propagating gravitational wave perturbations of Kerr and NHEK. These are typically studied in the Newman-Penrose formalism as in [15, 16]. In [36] (see also [37]) an attempt was made to find NHEK perturbations using a metric ansatz judiciously picked to accommodate the anabasis perturbation to (near-)extreme Kerr. Unfortunately, the solutions found in [36] that go beyond the above triplet are singular at the poles $\theta = 0, \pi$.[4]

Finally, note that in another gauge, our solution triplet takes the simple form

$$h_{\mu\nu} = 2M^2\Gamma\begin{pmatrix} r^2\Phi\Gamma & h_{tr} & 0 & 0 \\ & \frac{\Phi(2-\Gamma)}{r^2} & 0 & 0 \\ & & \Phi\Gamma^2\Lambda^2 & 0 \\ & & & 0 \end{pmatrix}, \qquad \text{(B.7)}$$

$$h_{tr} = \frac{1}{3}rt\left[2ar + brt + \frac{2}{3}cr\left(t^2 + 9/r^2\right)\right]. \qquad \text{(B.8)}$$

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[4]In their notation, Ψ diverges at the poles unless $\chi = \Phi$, in which case their $\mathsf{SL}(2)$-breaking solution reduces to the above triplet.

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