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Unruh-DeWitt detector responses for complex scalar fields in de Sitter spacetime

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Abstract: We derive the response function for a comoving, pointlike Unruh-DeWitt particle detector coupled to a complex scalar field ϕ, in the $(3+1)$-dimensional cosmological de Sitter spacetime. The field-detector coupling is taken to be proportional to $\phi^\dagger\phi$. We address both conformally invariant and massless minimally coupled scalar field theories, respectively in the conformal and the Bunch-Davies vacuum. The response function integral for the massless minimal complex scalar, not surprisingly, shows divergences and accordingly we use suitable regularisation scheme to find out well behaved results. The regularised result also contains a de Sitter symmetry breaking logarithm, growing with the cosmological time. Possibility of extension of these results with the so called de Sitter α-vacua is discussed. While we find no apparent problem in computing the response function for a real scalar in these vacua, a complex scalar field is shown to contain some possible ambiguities in the detector response. The case of the minimal and nearly massless scalar field theory is also briefly discussed.

Keywords: Renormalization Regularization and Renormalons, Nonperturbative Effects

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Contents

1 Introduction

An Unruh-DeWitt detector is conventionally a point particle (like an atom) that can couple to a quantum field. The detector has internal discrete energy levels which, along with the field may be excited/de-excited to higher/lower levels. Such (de-)excitation depends upon the trajectory of the detector, the field-detector coupling and also the particular initial and final states we are looking into. One particularly interesting quantity is the response function of the detector, representing the rate of quantum transitions occurring per unit proper time along detector's trajectory. The associated quanta are not necessarily actual created particles which may give rise to flow of energy and momentum, but instead they may be an outcome of application of the external energy required to maintain detector's particular trajectory. We refer our reader to [1] and references therein for a discussion. The response functions for an Unruh-DeWitt detector have been investigated in various contexts in the flat spacetime. This includes various non-inertial trajectories, e.g. [2–9] (also references therein). See also [10–12] and references therein for dynamics of entangled detectors interacting with quantum fields.

The de Sitter spacetime is physically very well motivated in the context of the early inflationary as well as the current universe. It has gained considerable attention in the context of the Unruh-DeWitt detector model. The earliest of such discussions can be seen in [1] and references therein, where it was shown that the response function for a comoving detector in the cosmological de Sitter spacetime for a conformal scalar in a conformal

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vacuum is thermal, although there is no actual particle creation in this scenario. This analysis was later extended in various directions, including scalar fields without conformal symmetry, in the static de Sitter coordinate and also quite extensively in the context of quantum entanglement and decoherence, e.g. [13–32] and references therein.

In this work we compute at the leading order of the perturbation theory, the response functions of a comoving Unruh-DeWitt detector for complex scalar fields, for both conformally symmetric and massless minimal cases. It is well known that a massless minimal scalar can be a very good candidate for the inflaton. We also wish to discuss the possibility of extending these results in the context of the de Sitter α-vacua [33–43]. Since the field-detector coupling must be hermitian, for a complex scalar it must be at the simplest non-trivial form proportional to $\phi^\dagger\phi$,[1] unlike the case of a real scalar [1]. Apart from considering this as a theoretical model of handling Dirac fermionic fields, such quadratic couplings can also be expected emerging in low energy effective theories of some interacting theories where a scalar describes composite particles at low energies [44, 45]. The same is true for any complex field like a Dirac fermion. Due to such coupling, one obtains product of two Wightman functions in the integral of the response function and thus perhaps not unexpectedly, one encounters divergences needing suitable regularization. We refer our reader for discussions on the Unruh-DeWitt detector models for complex fields in the Rindler space including entanglement dynamics to [44–52]. Expectedly, in curved spacetime, the issue of the non-linear interaction between the detector and the field as well as the corresponding divergences may become much more relevant receiving curvature contributions. In order to analyse the curvature effects, we, therefore, consider the response of the Unruh-DeWitt detector in a maximally symmetric spacetime with constant curvature. This study can be considered as a theoretical approach of handling the divergences appearing in more realistic non-linear couplings in various kind of fields in de Sitter spacetime. We refer our reader to [53] and references therein for discussions on loop effects with a massless minimal complex scalar in the context of scalar quantum electrodynamics in de Sitter spacetime.

A quantum field in the de Sitter spacetime may inherit many inequivalent vacua, depending upon its symmetry structure. The Bunch-Davies vacuum, for example, is suited for describing early time inflationary modes [54] according to comoving observers. The one parameter family of de Sitter invariant α-vacua [33] on the other hand, may be interesting in the context of the trans-Planckian physics. For example, discussion on such vacua in the context of the so called trans-Planckian censorship conjecture can be seen in [55] and references therein. Since an α-vacuum can be expressed as a squeezed state over the Bunch-Davies states, one can expect nontrivial feature in the detector response function with respect to such a vacuum. Another curious point to note is, unlike the Bunch-Davies case, the timelike parameter that defines a positive frequency α-mode will not be the cosmological time, which is the proper time along a comoving detector's trajectory. Also, it was reported earlier in certain contexts that such vacua may not be good candidates

[1]One can also envisage a Hermitian linear coupling with either $\phi_R = \phi + \phi^\dagger$ or $\phi_I = i(\phi^\dagger - \phi)$, which technically will be no different from the monopole coupling of a real scalar field.

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to do perturbation theory, owing to their inherent non-local characteristics [35–43]. While we wish to reinforce this argument in this paper, one possibly cannot completely rule these vacua out based upon the limitations offered by the perturbative approaches, for it is possible that they should be treated via some (hitherto unknown) non-perturbative techniques. Not surprisingly, the analysis of α-vacua has invited enough attention and there has been a significant effort devoted towards isolating their signatures in inflationary paradigm [37, 56, 57].

The paper is summarised as follows. In the next section we briefly review the basic setup for the Unruh-DeWitt detector model. Using this, we discuss in section 3 the response functions for a *real* conformal and massless minimal scalar. In section 4, we discuss complex conformal and massless minimal scalars respectively in the conformal and the Bunch-Davies vacua. For the latter in particular, suitable regularization scheme is employed in order to find out well behaved result. This regularization involves in particular, adding a fictitious conformal scalar field with divergent detector-field interaction, in order to cancel a divergence appearing in the response function. This result also possesses a de Sitter symmetry breaking logarithm growing with time analogous to the infrared secular growth, reported earlier in e.g. [58–62]. However, such term is absent for the case of a real scalar [13]. In section 5, we discuss generalisation of complex fields' results to the α-vacua and we point out some possible ambiguities. The case of the nearly massless minimal scalar is briefly mentioned in section 6. Finally we conclude in section 7. Even though we stick to the first order perturbation theory throughout, we argue in section 7 that the response function for real scalars in the α-vacua can be defined and computed at any arbitrary order of the perturbation theory.

2 The setup

Following [1, 33, 34], we briefly review below the basic setup we use in this paper, for the sake of completeness.

The de Sitter metric in the spatially flat cosmological coordinates in $(3 + 1)$-dimensions reads

$$ds^2 = -dt^2 + e^{2Ht}\left(dx^2 + dy^2 + dz^2\right), \tag{2.1}$$

where $H = \sqrt{\Lambda/3}$ is the Hubble constant. Defining the conformal time, $\eta = -e^{-Ht}/H$, the metric takes a conformally flat form,

$$ds^2 = \frac{1}{H^2\eta^2}\left[-d\eta^2 + dx^2 + dy^2 + dz^2\right]. \tag{2.2}$$

The generic free action for a real scalar field reads,

$$S = -\frac{1}{2}\int \sqrt{-g}\, d^4x \left[(\nabla_\mu\phi)(\nabla^\mu\phi) + m^2\phi^2 + \xi R\phi^2\right],$$

whereas for a complex scalar field it reads,

$$S = -\int \sqrt{-g}\, d^4x \left[(\nabla_\mu\phi^\dagger)(\nabla^\mu\phi) + m^2|\phi|^2 + \xi R|\phi|^2\right].$$

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We are chiefly interested in two cases here: a) a conformal scalar ($m^2 + \xi R = R/6$) and b) a massless minimal scalar ($m^2 + \xi R = 0$). The case of a nearly massless and minimal scalar will be briefly discussed in section 6. We shall set below $\hbar = 1 = c$.

We shall use the formalism of particle detectors in curved spacetime discussed in e.g. [1] and references therein. Let us first discuss a real scalar field theory. The simplest coupling of this field with a pointlike detector (e.g. an atom) is taken as,

$$\mathcal{L}_{\rm int} = g\mu(\tau)\phi(x(\tau)),$$

where g is a coupling constant and μ is the monopole moment operator of the detector. In the Heisenberg picture, $\mu(\tau) = e^{iH_0\tau}\,\mu\, e^{-iH_0\tau}$, where H_0 is the free Hamiltonian of the detector, and τ is the proper time along detector's trajectory. We shall specialise to a comoving trajectory and hence will take detector's spatial points to be fixed.

Thus the first order matrix element for the field-detector combined system to make a transition from an initial state $|i\rangle$ to a final state $|f\rangle$ is given by

$$\mathcal{M}_{fi} = ig\,\langle E|\mu|E_0\rangle \int_{\tau_i}^{\tau_f} d\tau e^{-i(E-E_0)\tau} \langle\phi_f|\phi(x(\tau))|\phi_i\rangle, \tag{2.3}$$

where we have taken $|i\rangle = |E_0\rangle \otimes |\phi_i\rangle$ and $|f\rangle = |E\rangle \otimes |\phi_f\rangle$, and E_0 and E are respectively the energy eigenvalues of the detector in these states. The transition probability is given by

$$|\mathcal{M}_{fi}|^2 = g^2\,|\langle E|\mu|E_0\rangle|^2 \int_{\tau_i}^{\tau_f} d\tau_1\, d\tau_2\, e^{-i(E-E_0)(\tau_1-\tau_2)}\, \langle\phi_i|\phi(x_2(\tau_2))|\phi_f\rangle\langle\phi_f|\phi(x_1(\tau_1))|\phi_i\rangle.$$

However, it is more interesting to sum over all possible final states of the quantum field $|\phi_f\rangle$. Thus if we take the initial state $|\phi_i\rangle$ of the field to be its vacuum, using the completeness relation, we have an effective probability of a definitive transition only in the detector state

$$\begin{aligned}\overline{\mathcal{F}}(\Delta E) &= \int \mathcal{D}\phi_f |\mathcal{M}_{fi}|^2 \\ &= g^2\,|\langle E|\mu|E_0\rangle|^2 \int_{\tau_i}^{\tau_f} d\tau_1\, d\tau_2\, e^{-i(E-E_0)(\tau_1-\tau_2)}\, \langle\phi_i|\phi(x_2(\tau_2))\phi(x_1(\tau_1))|\phi_i\rangle,\end{aligned} \tag{2.4}$$

where

$$\langle\phi_i|\phi(x_2(\tau_2))\,\phi(x_1(\tau_1))|\phi_i\rangle \equiv iG^+(x_2(\tau_2) - x_1(\tau_1)),$$

is the Wightman function. It is then convenient to define two new temporal variables,

$$\tau_+ = \frac{\tau_1+\tau_2}{2} \qquad \Delta\tau = \tau_1 - \tau_2.$$

Assuming further adiabatic turn on and off of the detector-field coupling, taking $\tau_i = 0$ and $\tau_f \to \infty$, the response function of the detector per unit τ_+ is defined as (in units of $g^2\,|\langle E|\mu|E_0\rangle|^2$ which we shall stick to in the remaining of the paper)

$$\frac{d\mathcal{F}(\Delta E)}{d\tau_+} = \int_{-\infty}^{\infty} d(\Delta\tau)\, e^{-i\Delta E\Delta\tau}\, iG^+(\Delta\tau), \tag{2.5}$$

where $\mathcal{F} := \overline{\mathcal{F}}/(g|\langle E|\mu|E_0\rangle|)^2$ and we have written $\Delta E = E - E_0$. $\Delta E > 0$ ($\Delta E < 0$) denotes excitation (de-excitation) of the detector interacting with the quantum field. The

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integral (2.5) was computed in [13] in the Bunch-Davies vacuum for a conformal, a massless minimally coupled as well as for a minimally coupled and nearly massless scalar field (see also [1]).

Eq. (2.5) can be extended to the de Sitter α-vacua in a straightforward manner as follows [34]. The mode function u_α corresponding to the choice of the α-vacua are related to the mode function $u(x)$ corresponding to the Bunch-Davies vacuum[2] through a Bogoliubov tranformation [33],

$$u_\alpha(x) = \cosh\alpha\, u(x) + \sinh\alpha\, u^\star(x), \tag{2.6}$$

where α is a spacetime independent real parameter. Then the de Sitter invariant Wightman function in these vacua reads [33],

$$G^+_\alpha(x,x') = \cosh^2\alpha\, G^+(x,x') + \sinh^2\alpha\, G^+(\overline{x},\overline{x}') + \frac{1}{2}\sinh 2\alpha \left(G^+(x,\overline{x}') + G^+(\overline{x},x')\right), \tag{2.7}$$

where a bar over the spacetime points denotes the antipodal position, which corresponds to $\eta \to -\eta$ in eq. (2.2). All the G^+'s on the right hand side of the above equation stand for the Bunch-Davies vacuum. For a scalar field of mass m and non-minimal coupling ξ, $G^+(x,x')$ reads [33],

$$iG^+(x,x') = \frac{H^2}{16\pi^2}\Gamma\left(\frac{3}{2}-\nu\right)\Gamma\left(\frac{3}{2}+\nu\right)\, {}_2F_1\left(\frac{3}{2}-\nu, \frac{3}{2}+\nu, 2; 1-\frac{y}{4}\right), \tag{2.8}$$

where

$$\nu = \left(\frac{9}{4} - 12\xi - \frac{m^2}{H^2}\right)^{1/2},$$

and the de Sitter invariant interval y written in terms of the conformal time reads,

$$y(x,x') = \frac{-(\eta-\eta'-i\epsilon)^2 + |\vec{x}-\vec{x}'|^2}{\eta\eta'},$$

where $\epsilon = 0^+$. Rewriting things now in the cosmological time t and setting $\vec{x} = \vec{x}'$ for a comoving detector, we have

$$y(t,t') = -4\left(\sinh\frac{H\Delta t}{2} - i\epsilon\right)^2. \tag{2.9}$$

Likewise we have for the antipodal transformations,

$$\begin{aligned} y(t,\overline{t}') &= 4\left(\cosh\frac{H\Delta t}{2} + i\epsilon\right)^2 \qquad y(\overline{t},t') = 4\left(\cosh\frac{H\Delta t}{2} - i\epsilon\right)^2 \\ y(\overline{t},\overline{t}') &= -4\left(\sinh\frac{H\Delta t}{2} + i\epsilon\right)^2. \end{aligned} \tag{2.10}$$

Since we have set the comoving spatial separation to zero, the cosmological time t becomes the proper time along detector's trajectory, $\tau = t$. Putting these all in together, the

[2]Bunch-Davies mode function $u(x)$ assumes a positive frequency appearance at early time i.e. $u(x) \to e^{-ik\eta + i\vec{k}\cdot\vec{x}}$ for $k\eta \to -\infty$ and hence have been the preferred choice of initial state for the quantum fields in de Sitter background.

response function for the de Sitter α-vacua is then obtained once we replace the Wightman function appearing in eq. (2.5) by eq. (2.7), [34]

$$\frac{d\mathcal{F}_\alpha(\Delta E)}{dt_+} = \int_{-\infty}^{\infty} d(\Delta t)\, e^{-i\Delta E \Delta t}\, iG^+_\alpha(\Delta t), \tag{2.11}$$

and further use eqs. (2.8), (2.9) and (2.10) into it.

A few comments pertaining eq. (2.11) are in order here. First, we note that we have not changed the $e^{-i\Delta E\Delta t}$ term according to the antipodal transformation at all. This is because this term originates purely from the detector, eq. (2.3), and not from the field. Since the detector is a pointlike and localised object, the antipodal transformation simply should not act on this term. We also note that the nonlocal characteristic of the α-vacua is manifest from eq. (2.7), where we have added Wightman functions corresponding to antipodal points. Thus there seems to be an apparent interpretational problem of coupling a pointlike particle detector to the field in this case. However, we may get rid of this issue by recalling that the Bogoliubov rotation made from the Bunch-Davies modes, eq. (2.6), is purely local. Since on the other hand the Wightman function does not represent propagation of the field, the same appearing in eq. (2.11) can be interpreted as just an expectation value of the operator $\phi(x)\phi(x')$ with respect to the vacuum corresponding to the α modes. With this interpretation, we shall see below that at least for a real scalar field we can compute the response function in the α-vacua without any apparent ambiguity.

Finally as discussed before, for a complex scalar field the simplest non-trivial detector-field coupling is quadratic in the field,

$$\mathcal{L}_{\rm int} = g\mu(t)\phi^\dagger(x(t))\phi(x(t)). \tag{2.12}$$

Alike the fermions, e.g. [50], the quadratic coupling is necessary in order to make the field-detector interaction Hamiltonian hermitian. Accordingly, the integral for the response function will contain a product of two Wightman functions. We address this issue in detail in section 4.

3 Response functions of a real scalar field in α-vacua

3.1 The conformal scalar

We start with the simplest case of a conformally invariant scalar field theory. Even though such a field does not create particles in a conformally flat spacetime such as the de Sitter in the conformal vacuum, it is well known that the Unruh-DeWitt detector records a thermal response function in the conformal or the Bunch-Davies vacuum [1]. This result was later extended to the de Sitter α-vacua in [34]. We briefly reproduce this result below chiefly to fix the notations we shall be using in the rest of this paper.

In the conformal vacuum ($\nu = 1/2$) eq. (2.8) in the comoving frame of the detector becomes

$$iG^+(x,x') = \frac{H^2}{4\pi^2 y} = -\frac{H^2}{16\pi^2(\sinh\frac{H\Delta t}{2} - i\epsilon)^2}, \tag{3.1}$$

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and hence eq. (2.11) gives, when written in terms of a dimensionless temporal coordinate, $u = H\Delta t/2$,

$$\frac{d\mathcal{F}_\alpha(\Delta E)}{dt_+} = -\frac{H}{8\pi^2}\int_{-\infty}^{\infty} du\, e^{-2i\Delta E u/H}\left[\frac{\cosh^2\alpha}{(\sinh u - i\epsilon)^2} + \frac{\sinh^2\alpha}{(\sinh u + i\epsilon)^2}\right.$$
$$\left. -\frac{1}{2}\sinh 2\alpha\left(\frac{1}{(\cosh u - i\epsilon)^2} + \frac{1}{(\cosh u + i\epsilon)^2}\right)\right]. \tag{3.2}$$

The above integrals can easily be evaluated e.g. by using a semicircular contour closing in the lower half plane. Note that since the function $\cosh u$ is never vanishing on the real line, from hereafter we shall not retain the $\pm i\epsilon$ terms for them. We next introduce a dimensionless energy difference for convenience,

$$p := \frac{2\Delta E}{H},$$

in terms of which the response function is found to be [34],

$$\left.\frac{d\mathcal{F}_\alpha(p)}{dt_+}\right|_{\text{conf.}} = \frac{pH}{4\pi}\,\frac{\left(\cosh\alpha - e^{\pi p/2}\sinh\alpha\right)^2}{e^{\pi p} - 1}. \tag{3.3}$$

Setting $\alpha = 0$ above recovers the thermal spectra associated with the usual conformal vacuum [1]. We have plotted eq. (3.3) scaled by the $\alpha = 0$ result in figure 1. A couple of things are perhaps worth noting: a) the response becomes gradually stronger with the increasing α values b) taking $\alpha \geq 0$, the numerator of eq. (3.3) forces the response function to be vanishing at some p-value, for a given α and c) unlike the $\alpha = 0$ case [1], there is no unique meaning of eq. (3.3) in terms of pure absorption or pure emission. This should correspond to the fact that firstly, for $\alpha \neq 0$ the timelike parameter corresponding to the Bogoliubov rotated modes, eq. (2.6), is not the proper time along the detector, which dictates its time evolution. Second, such mode mixing makes an α-vacuum to be a squeezed state over all the Bunch-Davies states. These two facts force the detector to undergo both excitations and de-excitations, as is manifest in eq. (3.3). In different de Sitter vacua the non-thermal response have been obtained from field content analysis as well [16], though an Unruh-DeWitt detector does not always measure the field content.

3.2 The minimally coupled massless scalar

As is evident, one cannot simply set $\nu = 3/2$ corresponding to the massless minimal coupling in eq. (2.8), owing to the fact that there exists no de Sitter invariant Wightman function for a massless minimal scalar. One thus needs to find it independently [33],

$$iG^+(y) = \frac{H^2}{4\pi^2}\left(\frac{1}{y} - \frac{1}{2}\ln y + \frac{1}{2}\ln\left(a(\eta)a(\eta')\right) + \ln 2 - \frac{1}{4}\right). \tag{3.4}$$

Compared to eq. (3.1), the above thus contains additional terms including one that breaks the de Sitter symmetry. Eq. (2.11) in this case reads,

$$\frac{d\mathcal{F}_\alpha(p)}{dt_+} = \frac{H}{2\pi^2}\int_{-\infty}^{\infty} du\, e^{-ipu}\left[-\cosh^2\alpha\left(\frac{1}{4(\sinh u - i\epsilon)^2} + \frac{1}{2}\ln\left(-4(\sinh u - i\epsilon)^2\right)\right)\right.$$
$$-\sinh^2\alpha\left(\frac{1}{4(\sinh u + i\epsilon)^2} + \frac{1}{2}\ln\left(-4(\sinh u + i\epsilon)^2\right)\right)$$

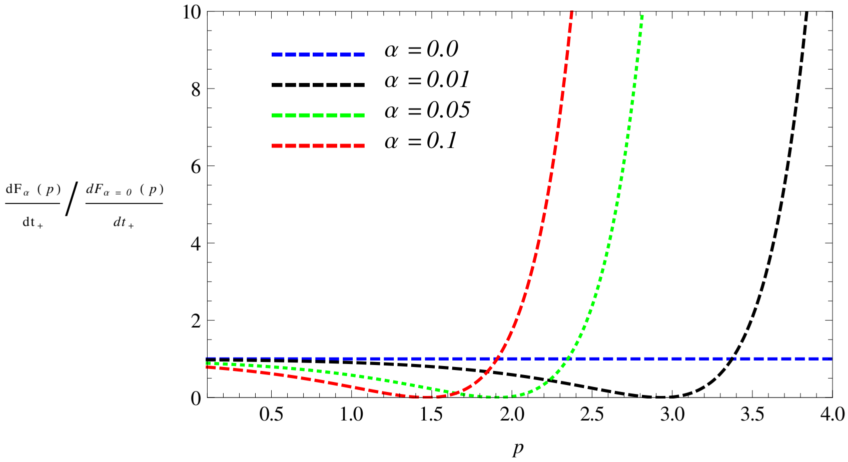

Figure 1. Variation of the response function (3.3) [34], when scaled by the conformal vacuum ($\alpha = 0$) result, with respect to the dimensionless energy difference p for different α-values.

$$+\sinh 2\alpha\left(\frac{1}{2\cosh^2 u}-\ln\left(4\cosh^2 u\right)\right)$$
$$+\left(\frac{1}{2}\ln(a(t)a(t'))+\ln 2-\frac{1}{4}\right)e^{2\alpha}\Bigg], \tag{3.5}$$

where $p = 2\Delta E/H$ as earlier. Using eq. (3.3), we rewrite the above equation as

$$\begin{aligned}\frac{d\mathcal{F}_\alpha(p)}{dt_+} &= \frac{d\mathcal{F}_\alpha(p)}{dt_+}\Big|_{\text{conf.}} + \frac{H}{2\pi^2}\left(\frac{1}{2}\ln(a(t)a(t'))+\ln 2-\frac{1}{4}\right)e^{2\alpha}\delta(p)\\ &-\frac{H}{4\pi^2}\int_{-\infty}^{\infty}du\, e^{-ipu}\Big[\cosh^2\alpha\,\ln\left(-4(\sinh u-i\epsilon)^2\right)\\ &+\sinh^2\alpha\,\ln\left(-4(\sinh u+i\epsilon)^2\right)-\sinh 2\alpha\ln\left(4\cosh^2 u\right)\Big].\end{aligned} \tag{3.6}$$

We note that as $\epsilon \to 0$,

$$\mathrm{Arg}\left(-4(\sinh u \mp i\epsilon)^2\right) = \pm\pi\,\mathrm{sgn}(u), \tag{3.7}$$

where sgn stands for the 'sign' function.

Following [13], then the regularised form of the logarithmic integrals of eq. (3.6) can be found by introducing an infinitesimal positive imaginary part in p and then by integrating them by parts. Some calculations after using eq. (3.3) yields,

$$\begin{aligned}\frac{d\mathcal{F}_\alpha(p)}{dt_+}\Big|_{\text{MM}} &= \frac{pH}{4\pi}\frac{\left(\cosh\alpha-e^{\pi p/2}\sinh\alpha\right)^2+(4/p)^2\left(\cosh\alpha+e^{\pi p/2}\sinh\alpha\right)^2}{e^{\pi p}-1}\\ &+\frac{H}{2\pi^2}\left(\frac{1}{2}\ln(e^{H(t+t')})+\ln 2-\frac{1}{4}\right)e^{2\alpha}\,\delta(p),\end{aligned} \tag{3.8}$$

where the suffix "MM" stands for massless and minimal scalar field. We shall argue in section 7 that the above results for a real conformal or massless minimal scalar goes through arbitrary order of the perturbation theory, without encountering any difficulty. Putting $\alpha = 0$ recovers the result of [13].

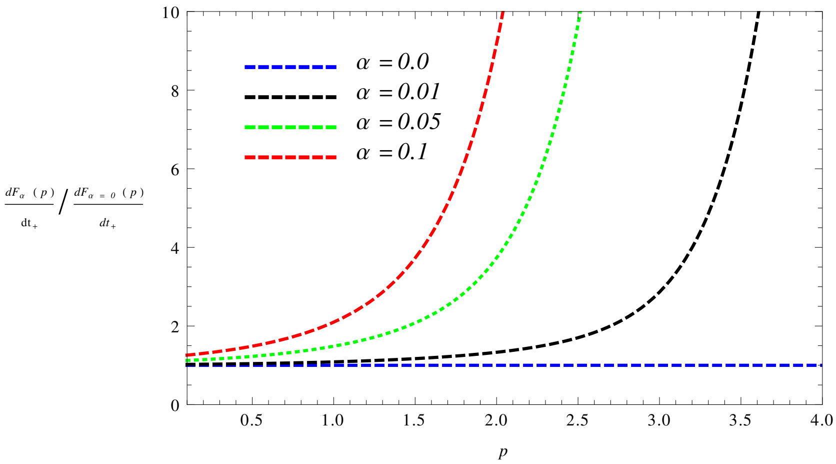

Figure 2. Variation of the response function (3.8) with $p \neq 0$, when scaled by the Bunch-Davies ($\alpha = 0$) result, with respect to p for different α-values. Unlike figure 1, the response function is monotonic here.

Note that the term proportional to $H(t + t')$ in the above equation is not de Sitter invariant. The physical origin of this is related to the non-existence of a vacuum state for a massless minimal scalar that corresponds to the full isometry of the de Sitter background. This non-existence leads to the de Sitter breaking logarithm in the Wightman function in eq. (3.4), eventually being reflected in eq. (3.8). In fact such terms are always expected to originate from a state that does not obey the full isometry of the spacetime. For example, even in the flat spacetime one can construct analogous symmetry breaking two point functions for non-vacuum states, leading to symmetry breaking terms in the response function [63].

The first term on the right hand side of eq. (3.8) diverges as $p \to 0$ whereas the second term diverges at $p = 0$. However, for a detector *undergoing* transitions with *discrete* internal energy levels (i.e., $p = 2\Delta E/H \neq 0$), the δ-function term drops out. On the other hand, for a detector with continuum energy levels, it seems that possibly we need to integrate eq. (3.8) over all p values with an appropriate density of states in order to give the response function a well defined meaning. The behaviour of the first term on the right hand side of eq. (3.8) as $p \to 0$ dictates the density of state must vanish at least as $\mathcal{O}(p^2)$ as $p \to 0$. However, this makes the integral related to the second term vanishing.

We shall focus in this work *only* on detectors with $p \neq 0$, so that we may ignore the $\delta(p)$-term anyway, whenever it appears. The simplest physical example of such a detector is of course, a two level system undergoing level transition. Even though this makes the de Sitter breaking term in eq. (3.8) disappear, for a complex, massless minimal scalar such logarithm will indeed appear from the integration of the Wightman functions, as we shall see below.

We have plotted in figure 2 the characteristics of the response function ($p \neq 0$), by scaling it with the Bunch-Davies result ($\alpha = 0$). The behaviour is monotonous, as compared to the conformal scalar, figure 1.

4 Complex scalar in the Bunch-Davies vacuum

4.1 Conformal complex scalar in conformal vacuum

Using eq. (2.12), the first order response function for a complex scalar field is given by,

$$\frac{d\mathcal{F}(\Delta E)}{dt_+} = \int_{-\infty}^{\infty} d(\Delta t) e^{-i\Delta E \Delta t} \left[(iG^+(\Delta t))^2 + (iG^+(0))^2 \right]. \tag{4.1}$$

The second integrand on the right hand side is divergent and is present for any spacetime. It was suggested earlier in [44, 46] to replace the argument of $iG^+(0)$ by an infinitesimal cut-off and obtain a term proportional to $\delta(p)$, so that one can ignore it for transitions with $p \neq 0$.

A more systematic and satisfactory prescription to deal with this divergence was suggested recently in [45], by replacing the field-detector interaction Lagrangian density, eq. (2.12), with the normal ordered one,

$$\mathcal{L}_{\text{int}} = g\mu(t)\phi^\dagger(x(t))\phi(x(t)) \to g\mu(t) : \phi^\dagger(x(t))\phi(x(t)) :$$

Since the $iG^+(0)$ term appearing in eq. (4.1) essentially represents the vacuum to vacuum transition in the coincidence limit, the normal ordering prescription just gets rid of any such divergence beforehand. Thus whenever we have a quadratic field-detector coupling such as eq. (2.12), we must normal order it. Having dealt with this, however, we shall encounter additional divergences (both ultraviolet and infrared) for a massless minimally coupled complex scalar from the first integral of eq. (4.1), which needs further suitable regularization.

However, the first integral does not show any divergence for a conformal scalar in the conformal vacuum. Using eq. (3.1), eq. (4.1) becomes,

$$\frac{d\mathcal{F}(p)}{dt_+} = \frac{H^3}{128\pi^4} \int_{-\infty}^{\infty} du \, \frac{e^{-ipu}}{(\sinh u - i\epsilon)^4}. \tag{4.2}$$

We can perform the above integration just like the real scalar field by choosing the integration contour to be a semicircle in the lower half plane and taking the poles lying on the negative imaginary axis. We find

$$\left.\frac{d\mathcal{F}(p)}{dt_+}\right|_{\text{complex, conf.}} = \frac{p^3 H^3}{384\pi^3} \frac{1}{e^{\pi p} - 1}. \tag{4.3}$$

Note that the thermal factor remains unchanged compared to the real scalar field.

4.2 The minimally coupled massless complex scalar

Massless complex scalar fields with interactions have previously been studied in context of cosmology [53] and also in BEC systems mimicking gravity systems [64], where study of detector response may be more feasible. Also, as we discussed previously, the quadratic coupling with complex scalars will give us good theoretical exposure of handling the resulting divergences in more physical, e.g. fermionic systems. Therefore, in order to study curvature effects and non-linear couplings in physically realisable systems, we consider complex scalar field interactions as our first step. Further, since we are interested mainly in studying the curvature effects in the detector response, we consider the massless limit first before going to a more realistic small mass limit in section 6.

However, as we shall see this case will not be as simple as that of the conformal one. In fact there will be finite as well as divergent terms in the expression for the rate of the response function, originating at the coincidence limit of the Wightman functions. We shall not be able to compute the response function in a closed form and will eventually resort to numerical analysis. However, before we do so, we first need to regularise the integral and also need to cast it into a form which can be handled numerically without any ambiguity.

We have the rate of the response function,

$$\begin{aligned}\frac{d\mathcal{F}(p)}{dt_+} = \frac{H^3}{8\pi^4}\int_{-\infty}^{\infty} du\, e^{-ipu}\Bigg[&\frac{(y\ln y-2)^2}{4y^2} + 2\left(\frac{1}{2}\ln(a(t)a(t')) + \ln 2 - \frac{1}{4}\right)\left(\frac{1}{y} - \frac{1}{2}\ln y\right)\\ &+\left(\frac{1}{2}\ln(a(t)a(t')) + \ln 2 - \frac{1}{4}\right)^2\Bigg], \end{aligned} \tag{4.4}$$

which can be rewritten as (after excluding a δ-function term, cf., the discussion at the end of section 3.2),

$$\begin{aligned}\frac{d\mathcal{F}(p)}{dt_+} = \frac{H^3}{8\pi^4}\int_{-\infty}^{\infty} du\, e^{-ipu}\Bigg[&\frac{\left[2(\sinh u - i\epsilon)^2\ln(-4(\sinh u - i\epsilon)^2) + 1\right]^2}{16(\sinh u - i\epsilon)^4}\\ &-\frac{\left(\frac{1}{2}\ln(a(t)a(t')) + \ln 2 - \frac{1}{4}\right)}{2(\sinh u - i\epsilon)^2}\\ &-\left(\frac{1}{2}\ln(a(t)a(t')) + \ln 2 - \frac{1}{4}\right)\ln(-4(\sinh u - i\epsilon)^2)\Bigg]. \end{aligned} \tag{4.5}$$

Let us evaluate the first integral of eq. (4.5) first, which is the most non-trivial. It reads,

$$\int_{-\infty}^{\infty} du\, e^{-ipu}\left[\frac{1}{16(\sinh u - i\epsilon)^4} + \frac{\ln\left(-4(\sinh u - i\epsilon)^2\right)}{4(\sinh u - i\epsilon)^2} + \frac{\left(\ln\left(-4(\sinh u - i\epsilon)^2\right)\right)^2}{4}\right]. \tag{4.6}$$

The first integral in eq. (4.6) is the same as that of eq. (4.2),

$$\frac{\pi p^3}{48}\frac{1}{e^{\pi p} - 1}. \tag{4.7}$$

Let us now evaluate the second integral of eq. (4.6), which is problematic due to the branch cut of the logarithm. To tackle this, after using eq. (3.7) and expanding the logarithm in powers of e^{-2u}, we rewrite it as

$$-\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}\int_0^{\infty} du\left(\frac{e^{-(ip+2n)u}}{(\sinh u - i\epsilon)^2} + \text{c.c.}\right) + \frac{i}{2}\left(\partial_p + \frac{\pi}{2}\right)\int_0^{\infty} du\left(\frac{e^{-ipu}}{(\sinh u - i\epsilon)^2} - \text{c.c.}\right), \tag{4.8}$$

where "c.c." denotes complex conjugation and we have used

$$\int_{-\infty}^{\infty} du\,\frac{e^{-ipu}\,\mathrm{sgn}(u)}{(\sinh u - i\epsilon)^2} = \int_0^{\infty} du\left(\frac{e^{-ipu}}{(\sinh u - i\epsilon)^2} - \text{c.c.}\right). \tag{4.9}$$

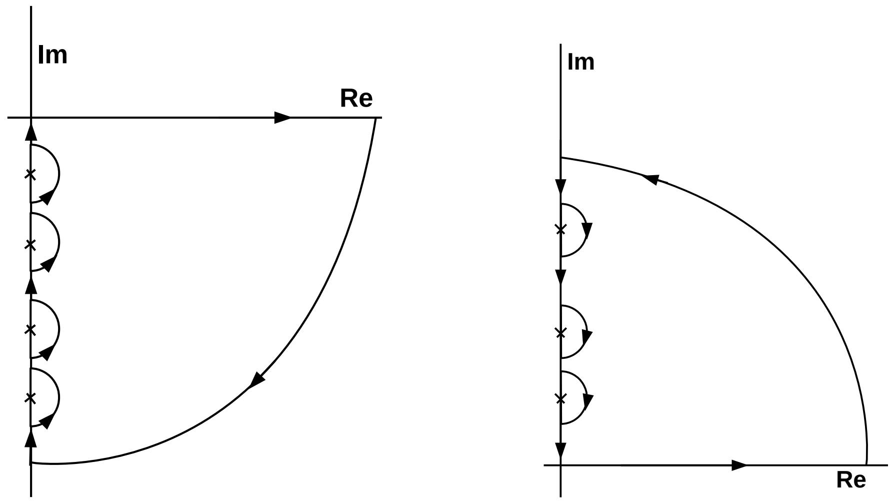

Figure 3. Contours to evaluate (4.8).

We shall evaluate the above integral first. We write,

$$\int_0^\infty du \frac{e^{-ipu}}{(\sinh u - i\epsilon)^2} = -i\frac{\partial}{\partial\epsilon}\int_0^\infty du \frac{e^{-ipu}}{(\sinh u - i\epsilon)}. \tag{4.10}$$

The poles of the integrand are located at

$$u_n = i(n\pi + (-1)^n\epsilon) \qquad n = 0,\, \pm1,\, \pm2, \cdots$$

We use a quarter-circular contour in the fourth quadrant to evaluate this integral, as shown in the first of figure 3 and let the radius of the quarter-circle go to infinity. The poles are avoided using infinitesimal semicircular deformations,

$$z_n = u_n + \bar{\epsilon}e^{i\theta} \quad (\bar{\epsilon} = 0^+) \qquad -\pi/2 \le \theta \le \pi/2\,.$$

The arc of the quarter-circle does not contribute to the integration. Computing the effect of the deformations and then performing the derivative with respect to ϵ, eq. (4.10), we have

$$\int_0^\infty du \frac{e^{-ipu}}{(\sinh u - i\epsilon)^2} = -\frac{\pi p}{e^{\pi p}-1} + i\left(\int_0^\infty du_I \frac{e^{-pu_I}}{(\sin u_I + \epsilon)^2}\right)_{\text{poles excluded}} + \mathcal{O}(\bar{\epsilon}), \tag{4.11}$$

where $u_I = -\mathrm{Im}(u)$ along the negative imaginary axis. Note that the poles of the integral on the right hand side of eq. (4.11) are located at $n\pi - (-1)^n\epsilon$, $n = 1, 2, \cdots$, which are excluded via the first contour of figure 3.

As a check of consistency, we have

$$\int_{-\infty}^\infty du \frac{e^{-ipu}}{(\sinh u - i\epsilon)^2} = \int_0^\infty du \left(\frac{e^{-ipu}}{(\sinh u - i\epsilon)^2} + \text{c.c.}\right) = -\frac{2\pi p}{e^{\pi p}-1}, \tag{4.12}$$

where the complex conjugate of the first integral within parenthesis can either be found by using the second contour of figure 3, or by simply complex conjugating eq. (4.11), since the integral on the right hand side of this equation is real. We can see that eq. (4.12) recovers the result of eq. (3.3) with $\alpha = 0$.

Note that unless we converted the second order pole of the integral to first order via eq. (4.10), the computation of the above infinitesimal deformations of the contour would not have been possible, for such deformations always yield diverging results for poles beyond first order.

We now have from eqs. (4.9), (4.11),

$$\int_{-\infty}^{\infty} du \, \frac{e^{-ipu} \, \mathrm{sgn}(u)}{(\sinh u - i\epsilon)^2} = 2i \left(\int_0^{\infty} du_I \frac{e^{-pu_I}}{(\sin u_I + \epsilon)^2} \right)_{\text{poles excluded}} . \tag{4.13}$$

The integral on the right hand side is written as,

$$\int_0^{\pi} du_I \frac{e^{-pu_I}}{(\sin u_I + \epsilon)^2} + \sum_{n=1}^{\infty} \int_{n\pi+\epsilon}^{(n+1)\pi-\epsilon} du_I \, \csc^2 u_I \, e^{-pu_I} , \tag{4.14}$$

where $\epsilon = 0^+$ and hence the integration limits exclude the poles. Note that we have got rid of the ϵ term for the integrals in the summation. Each of the integrals give divergent contribution as we approach the poles. We tackle with them by treating all the integrals in an equal footing as follows. We note from eq. (4.13) that the coincidence limit of the Wightman function $u \to 0$ on its left hand side corresponds to the points where $\sin u_I = 0$ on the right hand side, achieved via the contour of figure 3. Thus we shall regularise the integrals of eq. (4.14) by using suitable regularization near each pole, effectively regularising the very short distance divergent correlation as $u \to 0$ and thereby giving the response function a physical meaning. This task can be largely simplified if we first set all ϵ terms to zero in eq. (4.14) and simply rewrite it as

$$\sum_{n=0}^{\infty} \int_{n\pi}^{(n+1)\pi} du_I \, \csc^2 u_I \, e^{-pu_I} ,$$

which can be rewritten after a change of variable as,

$$\sum_{n=0}^{\infty} e^{-p\pi n} \int_0^{\pi} dx \, \csc^2 x \, e^{-px} .$$

Note that the only poles in the above integral are now located at $x = 0, \pi$. Performing the sum and further breaking the integration limits, the above can be rewritten as,

$$\mathrm{csch} \, \frac{\pi p}{2} \int_0^{\pi/2} dx \csc^2 x \, \cosh p \left(\frac{\pi}{2} - x \right) ,$$

so that the only pole of the above integration is located at $x = 0$. Accordingly, we break the above integration into three pieces and write after using eq. (4.13),

$$\begin{aligned} &\int_{-\infty}^{\infty} du \, \frac{e^{-ipu} \, \mathrm{sgn}(u)}{(\sinh u - i\epsilon)^2} \\ &\quad = 2i \left(\coth \frac{\pi p}{2} \int_0^{\pi/2} dx \, \csc^2 x \, - \, p \int_0^{\pi/2} dx \, x \, \csc^2 x \right) \\ &\qquad + 2i \, \mathrm{csch} \, \frac{\pi p}{2} \int_0^{\pi/2} dx \, \csc^2 x \, \left[\cosh \left(\frac{\pi}{2} - x \right) p - \cosh \frac{\pi p}{2} + px \sinh \frac{\pi p}{2} \right] . \end{aligned} \tag{4.15}$$

The first two integrals in the above expression diverge as $x \to 0$. It is easy to see from eq. (4.4) that there is no flat space limit of this divergence as the integrals vanish in the $H \to 0$ limit. Such short distance divergence structure is expected to be present in all Hadamard states. Before we deal with this divergence, let us also evaluate the first integral of eq. (4.8), using the contour of figure 3. After computing the effect of deformations around the infinitesimal semicircles, we can express it as,

$$-\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n}\int_0^{\infty} du\left(\frac{e^{-(ip+2n)u}}{(\sinh u - i\epsilon)^2}+\text{c.c.}\right) \\ = \frac{\pi p}{e^{p\pi}-1}\sum_{n=1}^{\infty}\frac{1}{n}+\frac{1}{\sinh\frac{\pi p}{2}}\sum_{n=1}^{\infty}\frac{1}{n}\int_0^{\pi/2} dx\,\csc^2 x\,\sin 2nx\,\sinh\left(\frac{\pi}{2}-x\right)p\,. \tag{4.16}$$

The first term on the right hand side corresponds to infinitesimal deformation of the contour in figure 3, whereas the second integral corresponds to the 'poles excluded' part as earlier. For the second integral we slightly lift its lower limit, so that we can use the formula 1.441 of [65],

$$\sum_{n=1}^{\infty}\frac{\sin 2nx}{n}=\frac{\pi-2x}{2}\qquad (0<x<\pi), \tag{4.17}$$

to rewrite eq. (4.16) as

$$\frac{\pi p}{e^{p\pi}-1}\sum_{n=1}^{\infty}\frac{1}{n}+\frac{\pi}{2\sinh\frac{\pi p}{2}}\int_{\epsilon}^{\pi/2} dx\,\csc^2 x\,\sinh\left(\frac{\pi}{2}-x\right)p \\ -\frac{1}{\sinh\frac{\pi p}{2}}\int_{\epsilon}^{\pi/2} dx\,x\,\csc^2 x\,\sinh\left(\frac{\pi}{2}-x\right)p\,. \tag{4.18}$$

The first term diverges as $\zeta(1)$ whereas the integrals diverge as $x \to \epsilon$. Accordingly, we now separate the above into non-divergent and divergent pieces,

$$\frac{\pi p}{e^{p\pi}-1}\sum_{n=1}^{\infty}\frac{1}{n}+\frac{\pi}{2}\int_0^{\pi/2} dx\,\csc^2 x-\left(1+\frac{\pi p}{2}\coth\frac{\pi p}{2}\right)\int_0^{\pi/2} dx\,x\csc^2 x \\ +\frac{\pi}{2\sinh\frac{\pi p}{2}}\int_0^{\pi/2} dx\,\csc^2 x\left[\sinh\left(\frac{\pi}{2}-x\right)p-\sinh\frac{\pi p}{2}+px\cosh\frac{\pi p}{2}\right] \\ -\frac{1}{\sinh\frac{\pi p}{2}}\int_0^{\pi/2} dx\,x\,\csc^2 x\left[\sinh\left(\frac{\pi}{2}-x\right)p-\sinh\frac{\pi p}{2}\right]. \tag{4.19}$$

Combining the above expression and eq. (4.15), we have the divergent part of eq. (4.8) (i.e., the second integral of eq. (4.6)),

$$\int_{-\infty}^{\infty} du\, e^{-ipu}\frac{\ln\left(-4(\sinh u-i\epsilon)^2\right)}{4(\sinh u-i\epsilon)^2}\bigg|_{\text{div.}}=\frac{\pi}{2}\left(\operatorname{csch}^2\frac{\pi p}{2}-\coth\frac{\pi p}{2}+1\right)\int_0^{\pi/2} dx\,\csc^2 x \\ -\frac{\pi p}{e^{\pi p}-1}\int_0^{\pi/2} dx\,x\csc^2 x+\frac{\pi p}{e^{\pi p}-1}\sum_{n=1}^{\infty}\frac{1}{n}\,. \tag{4.20}$$

After inserting suitable regulators, we rewrite the second and third of the above divergences as,

$$
\begin{aligned}
&-\frac{\pi p}{e^{\pi p}-1}\left(\int_0^{\pi/2} dx\, x\csc^2 x-\sum_{n=1}^{\infty}\frac{1}{n}\right) \qquad (4.21)\\
&\quad \to -\frac{\pi p}{e^{\pi p}-1}\left(\int_0^{\pi/2} dx\, x\csc^2 (x+\epsilon)-\sum_{n=1}^{\infty}\frac{(1-\epsilon')^n}{n}\right)=\frac{\pi p}{e^{\pi p}-1}\ln\frac{\epsilon}{\epsilon'}\,.
\end{aligned}
$$

Note that we could have set $\epsilon = \epsilon'$ above to get rid of the term as a whole. However, since these cut-offs are used to regularise respectively, an integration and a series, for the sake of generality we have kept them distinct.

Likewise the first divergence of eq. (4.20) can be regularised by setting $\csc x \to \csc(x+\epsilon)$, and we have

$$\int_{-\infty}^{\infty} du\, e^{-ipu}\frac{\ln\left(-4(\sinh u - i\epsilon)^2\right)}{4(\sinh u-i\epsilon)^2}\Bigg|_{\text{div.}} = \frac{\pi\coth\frac{\pi p}{2}}{e^{\pi p}-1}\frac{1}{\epsilon}+\frac{\pi p}{e^{\pi p}-1}\ln\frac{\epsilon}{\epsilon'}\,. \qquad (4.22)$$

The right hand side is divergent in the absence of regulators introduced and therefore depends upon the fashion in which the regularisation is carried out. Interestingly these divergences can easily be tackled through the renormalisation of the off-diagonal matrix elements of the detector's monopole operator, μ, in its energy eigenbasis, as follows.[3] We modify the field-detector interaction coupling by adding another monopole operator that does *not* couple to the field,

$$\mathcal{L}_{\text{int}} = g\mu(t) : \phi^\dagger(x(t))\phi(x(t)) : + \mu'(t)\,.$$

Analogous modification for a real scalar field yields the shift of the detector energy level, as has been discussed recently in [30]. The response function with the above modification becomes (after ignoring a δ-function as earlier),

$$\frac{d\mathcal{F}(p)}{dt_+} = \frac{2}{H}\int_{-\infty}^{\infty} du\, e^{-ipu}\left(iG^+(u)\right)^2 + \frac{2}{gH}\left(\frac{\langle E|\mu'|E_0\rangle}{\langle E|\mu|E_0\rangle}+\text{c.c.}\right)\int_{-\infty}^{\infty} du\, e^{-ipu}\, iG^+(u)\,. \qquad (4.23)$$

The first term on the right hand side gives the usual response function integral for a massless minimal complex scalar, eq. (4.5), whereas the second term yields the response function for a massless minimal *real* scalar, eq. (3.8) (with $\alpha = 0$ and $p \neq 0$). Our objective is to exploit this new term to cancel the contribution eq. (4.22). We find the desired condition after a little calculation,

$$\langle E|\mu'|E_0\rangle = -\frac{gH^3}{8\pi^2 p\,(1+16/p^2)}\left(\frac{\coth\frac{\pi p}{2}}{\epsilon}+p\ln\frac{\epsilon}{\epsilon'}\right)\langle E|\mu|E_0\rangle\,. \qquad (4.24)$$

[3] This result can also be obtained through the identification $\epsilon = \epsilon'$, followed by a principal value computation of the first integral on the right hand side of eq. (4.20). There might exist some other viable regularisation or renormalisation schemes as well to tackle this divergence. Of course, like any other divergent observable in QFT, the physically meaningful finite result may be different in different regularisation schemes and only experiments can ascertain which scheme is correct.

This corresponds to an operator relationship,

$$\mu' = -\frac{gH^3}{8\pi^2} \sum_{i,j,\, i\neq j} C_{ij} |i\rangle\langle i|\mu|j\rangle\langle j| \tag{4.25}$$

where the states represent the complete orthonormal energy eigenkets of the detector, and C_{ij}'s are real numbers depend upon the energy levels. Denoting then $|E_0\rangle$ and $|E\rangle$ respectively by, for example $|i=0\rangle$ and $|i=1\rangle$, we obtain from eqs. (4.24), (4.25), the requirement

$$C_{10}(p = 2(E-E_0)/H) = \frac{1}{p(1+16/p^2)} \left(\frac{\coth \frac{\pi p}{2}}{\epsilon} + p \ln \frac{\epsilon}{\epsilon'} \right). \tag{4.26}$$

It is clear that by construction, μ' will cancel the divergence for any level transition of the detector. Note also that C_{10} is even in p, i.e. $C_{01} = C_{10}$. Likewise $C_{ij} = C_{ji}$ for all i, j.

Note that μ' is non-diagonal in the energy eigenbasis of the detector, eq. (4.25). Thus it is clear from eq. (4.24) that here we are basically renormalising the off-diagonal matrix elements of the operator μ via the operator μ', which serves as a counterterm. However in doing so, we have the appearance of a detector-curvature interaction, gH^3, in eq. (4.25). This seems to be curious, for such non-minimal interaction of the detector and the spacetime curvature is introduced at the level of the Lagrangian density itself. It will be interesting to see whether this term can produce any finite or observable effects at higher order of the perturbation theory. We hope to pursue this issue in a future work.

We also refer our reader to [45] for a discussion on the renormalisation of the Unruh-DeWitt detector response in the flat spacetime with different interacting field theories like QED.

Collecting now the finite pieces from eqs. (4.15), (4.19) and using eq. (4.8), we obtain the regularised expression of the second integral of eq. (4.6),

$$\begin{aligned}
&\int_{-\infty}^{\infty} du\, e^{-ipu} \frac{\ln\left(-4(\sinh u - i\epsilon)^2\right)}{4(\sinh u - i\epsilon)^2}\Bigg|_{\text{Regularised}} \\
&= \frac{\pi}{2} \operatorname{csch} \frac{\pi p}{2} \coth \frac{\pi p}{2} \int_0^{\pi/2} dx\, \csc^2 x \left[\cosh\left(\frac{\pi}{2} - x\right) p - \cosh \frac{\pi p}{2} + px \sinh \frac{\pi p}{2} \right] \\
&\quad - \operatorname{csch} \frac{\pi p}{2} \int_0^{\pi/2} dx\, \csc^2 x \left[\left(\frac{\pi}{2} - x\right) \sinh\left(\frac{\pi}{2} - x\right) p \right. \\
&\qquad\qquad\qquad\qquad \left. - \left(\frac{\pi}{2} - x\right) \sinh \frac{\pi p}{2} + \frac{\pi p x}{2} \cosh \frac{\pi p}{2} \right] \\
&\quad - \frac{\pi}{e^{\pi p} - 1} \int_0^{\pi/2} dx\, \csc^2 x \left[e^{px} - 1 - px \right] \\
&\quad - \operatorname{csch} \frac{\pi p}{2} \int_0^{\pi/2} dx\, x \csc^2 x \left[\sinh\left(\frac{\pi}{2} - x\right) p - \sinh \frac{\pi p}{2} \right].
\end{aligned} \tag{4.27}$$

Finally, we come to the third integral of eq. (4.6). After using eq. (3.7), it takes the form (after ignoring a term containing $\delta(p)$),

$$\frac{1}{2} \int_0^{\infty} du\, \cos pu \left(\ln(4 \sinh^2 u) \right)^2 + \pi \int_0^{\infty} du\, \sin pu\, \ln(4 \sinh^2 u). \tag{4.28}$$

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After expanding the logarithms, the above integral can be rewritten as,

$$-2\left(\partial_p^2+\pi\partial_p\right)\int_0^\infty du\,\cos pu-4\sum_{n=1}^\infty\frac{1}{n}\partial_p\int_0^\infty du\,\sin pu\,e^{-2nu}$$
$$-2\pi\sum_{n=1}^\infty\frac{1}{n}\int_0^\infty du\,\sin pu\,e^{-2nu}+2\sum_{m,n=1}^\infty\frac{1}{mn}\int_0^\infty du\,\cos pu\,e^{-2(m+n)u} \tag{4.29}$$

The first two integrals diverge as $u\to\infty$. Such infrared divergence can be regularised by introducing an infinitesimal positive imaginary part in p. Accordingly, we get

$$\int_0^\infty du\,\cos pu=0. \tag{4.30}$$

Using this and also integrating by parts, eq. (4.29) can be put into a regularised form

$$4\sum_{n=1}^\infty\frac{p^2-4n^2}{n(p^2+4n^2)^2}-\sum_{n=1}^\infty\frac{2\pi p}{n(p^2+4n^2)}+8\sum_{m,n=1}^\infty\frac{1}{n\,(p^2+4(m+n)^2)}. \tag{4.31}$$

The various summations appearing above, as is evident, are all convergent. Thus eqs. (4.7), (4.27) and (4.31), when added together, gives a regularised expression for the first integral of eq. (4.5).

The remaining integrals of eq. (4.5), i.e. the second and the third ones were already evaluated respectively in section 3.1 and section 3.2. Thus, combing them with eqs. (4.7), (4.27) and (4.31), we finally obtain a fully regularised expression of the detector response function of eq. (4.5),

$$\begin{aligned}
&\frac{d\mathcal{F}(p)}{dt_+}\bigg|_{\text{complex, MM}}\\
&=\frac{p^3H^3}{384\pi^3}\frac{1}{e^{\pi p}-1}+\left(\frac{1}{2}\ln(a(t)a(t'))+\ln 2-\frac{1}{4}\right)\frac{pH^3}{8\pi^3}\frac{1}{e^{\pi p}-1}\left(1+\frac{8}{p^2}\right)\\
&\quad+\frac{H^3}{2\pi^4}\left[\sum_{n=1}^\infty\frac{p^2-4n^2}{n(p^2+4n^2)^2}-\sum_{n=1}^\infty\frac{\pi p}{2n(p^2+4n^2)}+2\sum_{m,n=1}^\infty\frac{1}{n\,(p^2+4(m+n)^2)}\right]\\
&\quad+\frac{H^3}{16\pi^3}\operatorname{csch}\frac{\pi p}{2}\coth\frac{\pi p}{2}\int_0^{\pi/2}dx\,\csc^2 x\left[\cosh\left(\frac{\pi}{2}-x\right)p-\cosh\frac{\pi p}{2}+px\sinh\frac{\pi p}{2}\right]\\
&\quad-\frac{H^3}{8\pi^4}\operatorname{csch}\frac{\pi p}{2}\int_0^{\pi/2}dx\,\csc^2 x\left[\left(\frac{\pi}{2}-x\right)\sinh\left(\frac{\pi}{2}-x\right)p\right.\\
&\qquad\qquad\left.-\left(\frac{\pi}{2}-x\right)\sinh\frac{\pi p}{2}+\frac{\pi p x}{2}\cosh\frac{\pi p}{2}\right]\\
&\quad-\frac{H^3}{8\pi^4}\operatorname{csch}\frac{\pi p}{2}\int_0^{\pi/2}dx\,x\,\csc^2 x\left[\sinh\left(\frac{\pi}{2}-x\right)p-\sinh\frac{\pi p}{2}\right]\\
&\quad-\frac{H^3}{8\pi^3}\frac{\pi}{e^{\pi p}-1}\int_0^{\pi/2}dx\,\csc^2 x\,\left[e^{px}-1-px\right].
\end{aligned} \tag{4.32}$$

Before we proceed, we note the regularization procedures we adopted to derive the above expression. First, we discussed a couple of possible regularization cum renormalization schemes to tackle the short distance ultraviolet divergence of eq. (4.20). Second, we

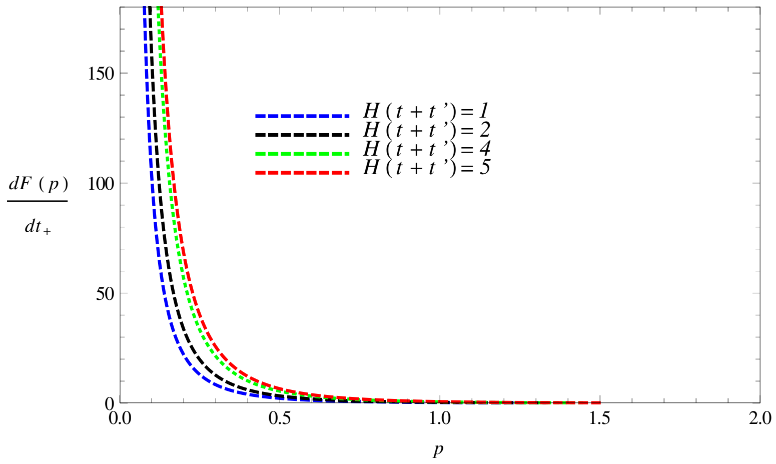

Figure 4. Plot of eq. (4.32) (after scaling it by $H^3/8\pi^3$) with respect to p for different values of $H(t+t') = 2Ht_+$.

also needed to introduce an infinitesimal positive imaginary part in p, in order to regularise the infrared divergence of some of the integrals of eq. (4.29).

If we let $p \to \infty$ in eq. (4.32), each of the terms of the first two lines as well as the last integral vanish. The remaining three integrals do not vanish individually in this limit, but they cancel with each other to yield a vanishing contribution. This is expected, as $p \to \infty$ corresponds to energy level separation of the detector much larger than the spacetime energy scale, H.

Note that there is a logarithm in eq. (4.32), increasing monotonically with $2Ht_+ = H(t+t')$, indicating breakdown of the perturbation theory as $Ht, Ht' \gg 1$. This seems to be analogous to the secular growth reported earlier in the context of perturbative quantum field theory in de Sitter space, e.g. [58–62] (also references therein). Such secular growth is absent for a real massless minimal scalar, eq. (3.8), for in that case the de Sitter breaking term is accompanied by a δ-function. We also note that in this limit the p-dependence of eq. (4.32) becomes qualitatively similar to that of the real scalar, eq. (3.8) (with $\alpha = 0$ and $p \neq 0$). Finally, we note that eq. (4.32) diverges for small p-values, as of the real scalar, eq. (3.8).

Since the response function eq. (4.32) is now regularised, we investigate its behaviour numerically without any trouble, as a function of the dimensionless energy p, figure 4.

5 The ambiguity of complex scalar field with α-vacua

We finally come to the case of complex scalar fields with α-vacua. However, we argue below that the detector response function is not well defined in this case. The response function for a conformal scalar in this case is given as,

$$\frac{d\mathcal{F}_\alpha(p)}{dt_+} = \frac{H^3}{128\pi^4} \int_{-\infty}^{\infty} du\, e^{-ipu} \left[\frac{\cosh^2\alpha}{(\sinh u - i\epsilon)^2} + \frac{\sinh^2\alpha}{(\sinh u + i\epsilon)^2} - \frac{\sinh 2\alpha}{\cosh^2 u} \right]^2 . \tag{5.1}$$

Expanding the square, making some rearrangements of terms and also redefining ϵ in some of the integrals, we find

$$\begin{aligned}\frac{d\mathcal{F}_\alpha(p)}{dt_+} = \frac{H^3}{128\pi^4}\int_{-\infty}^{\infty} du\, e^{-ipu}\Bigg[&\frac{\cosh^4\alpha}{(\sinh u - i\epsilon)^4} + \frac{\sinh^4\alpha}{(\sinh u + i\epsilon)^4} \\ &+\frac{\sinh^2 2\alpha}{2}\left(\frac{1}{\sinh^4 u} + \frac{2}{\cosh^4 u}\right) \\ &-8\sinh 2\alpha\left(\frac{\cosh^2\alpha}{(\sinh 2u - i\epsilon)^2} + \frac{\sinh^2\alpha}{(\sinh 2u + i\epsilon)^2}\right)\Bigg]. \end{aligned} \tag{5.2}$$

Note that there is an integral containing $\sinh^{-4} u$ without any $i\epsilon$. This term arises due to the multiplication $(\sinh u - i\epsilon)^2(\sinh u + i\epsilon)^2$, while squaring. All but the integral containing this term can be straightforwardly evaluated. The integral containing $\sinh^{-4} u$ is problematic because it does not converge on the real line, nor we can attempt to compute any principal value, for it diverges in the presence of poles beyond the first order. We cannot re-insert any $i\epsilon$ term now, for the answer will depend upon the sign of that term. Also, the contour of figure 3 cannot be used here, for we can compute the effect of the infinitesimal semicircular deformations to avoid the poles only if the poles are of first order.

Similar cancellation of the $i\epsilon$ regulator and some other ambiguities in the context of the perturbation theory in de Sitter α-vacua were reported earlier in [35–43]. Such ambiguity seems to originate from the inherent non-local characteristic of the α-vacua, coming from the antipodal transformations discussed in section 2. Thus even though we may give the detector response function for a real scalar in the α-vacua a meaning in the sense of just an expectation value (cf. the discussions at the end of section 2), apparently it fails for a complex scalar. In [41] (also references therein), it was suggested to modify the Feynman propagator by adding two sources, in order to tackle the non-locality. However, for a pointlike, localised particle detector it is not clear to us how much appropriate will be any such analogous modification. Since such a detector model is very generic and well motivated and also keeping in mind the existing results on the de Sitter α-vacua mentioned above, to the best of our knowledge and understanding, it seems that the above problem should not be attributed to the model of the detector we are working in, but it should be regarded as a generic problem of the perturbation theory with the α-vacua.

Nevertheless, we may still try to obtain a regularised version of the problematic integral as follows. However the caveat is, this will *not* yield a unique result, as described below. Let us first rewrite the integral as

$$\int_{-\infty}^{\infty} du\,\frac{e^{-ipu}}{\sinh^4 u} = \frac{1}{12}\lim_{\epsilon'\to 0}\partial^3_{\epsilon'}\left(\int_{-\infty}^{\infty} du\,\frac{e^{-ipu}}{(\sinh u - \epsilon')} + \text{c.c.}\right),$$

where ϵ' is real, no matter positive or negative. Since the pole on the real axis of the integrand on the right hand side is of first order, its principal value is well defined. This allows us to thus *define* a regularised value of our original integration as,

$$\begin{aligned}&\int_{-\infty}^{\infty} du\,\frac{e^{-ipu}}{\sinh^4 u}\Bigg|_{\text{Regularised}} \\ &:= \frac{1}{12}\lim_{\epsilon'\to 0}\partial^3_{\epsilon'}\left(\text{PV}\int_{-\infty}^{\infty} du\,\frac{e^{-ipu}}{(\sinh u - \epsilon')} + \text{PV}\int_{-\infty}^{\infty} du\,\frac{e^{ipu}}{(\sinh u - \epsilon')}\right).\end{aligned} \tag{5.3}$$

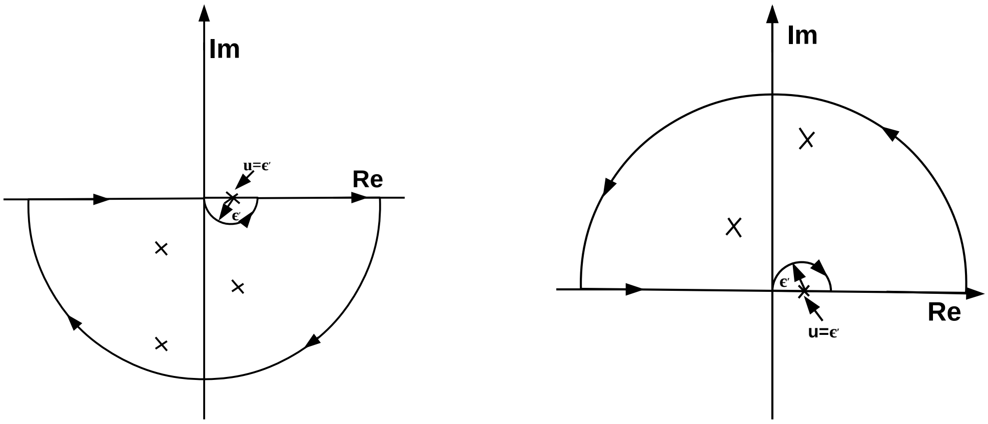

Figure 5. Possible contours to evaluate eq. (5.4).

Accepting this definition, the contour for the first integration on the right hand side is taken to be a semicircle with an infinitesimal semicircular deformation of radius ϵ' centred at $u = \epsilon'$, in the lower half plane. Thus the poles we pick up are located at $u_n = in\pi + (-1)^n\epsilon'$ $(n = -1, -2, -3, \dots)$. For the second, we use similar contour in the upper half plane, picking up the poles at $u_n = in\pi + (-1)^n\epsilon'$ $(n = 1, 2, 3, \dots)$, figure 5. Splitting the integral into two parts on the right hand side in the definition of eq. (5.4) will ensures its real valuedness. We find

$$\int_{-\infty}^{\infty} du\, \frac{e^{-ipu}}{\sinh^4 u}\bigg|_{\text{Regularised}} = -\frac{\pi p^3}{3}\left[\frac{1}{e^{\pi p}+1} - \frac{1}{2} + \frac{2}{p^2}\right]. \tag{5.4}$$

Evaluating the rest of the integrals in eq. (5.2), we obtain a regularised detector response function for a conformal complex scalar in the α-vacua,

$$\begin{aligned} \frac{d\mathcal{F}_\alpha(p)}{dt_+}\bigg|_{\text{complex conf.}} = \frac{p^3H^3}{384\pi^3}\Bigg[&\frac{\cosh^4\alpha + \sinh^4\alpha\; e^{\pi p}}{e^{\pi p}-1} \\ &+\frac{12\sinh 2\alpha\left(\cosh^2\alpha + \sinh^2\alpha\, e^{\pi p/2}\right)}{p^2(e^{\pi p/2}-1)} \\ &-\frac{1}{2}\sinh^2 2\alpha\left(\frac{e^{\pi p}-1-2e^{\pi p/2}(e^{\pi p}+1)}{e^{2\pi p}-1} - \frac{1}{2} + \frac{2}{p^2}\right)\Bigg]. \end{aligned} \tag{5.5}$$

Setting $\alpha = 0$ recovers the result of eq. (4.3).

As evident, the above result is not unique, for eq. (5.4) would change if we change the integration contour. For example, in the first of figures 5, we could have made the infinitesimal semicircular deformation in the upper half plane as well (and the opposite in the second of figure 5), which will lead to a change of sign of the second and third terms appearing on the right hand side of eq. (5.5). We face similar ambiguity for the case of a massless minimal scalar field as well. Again, such ambiguities can be attributed towards regularisation scheme adopted for the divergent integrals, and in the absence of any limiting consistency condition for the α-vacua, perhaps only experiments should be able to verify or rule-out the correct regularisation for obtaining physical finite results.

6 The case of a nearly minimally coupled massless scalar

We shall end with a comment on the case of a nearly massless minimally coupled scalar. The Wightman function in this case reads,

$$iG^+(y) = \frac{H^2}{4\pi^2}\left(\frac{1}{y} - \frac{1}{2}\ln y + \frac{1}{2s} + \ln 2 - 1\right), \tag{6.1}$$

where $s = 3/2 - \nu$ with $|s| \ll 1$ is a small parameter and y is given by eq. (2.9). Note that the de Sitter symmetry breaking logarithm is absent here compared to eq. (3.6). Comparing eq. (6.1) and eq. (3.6) it is clear that we can compute the detector response for this scalar by just making the replacement,

$$\left(\frac{1}{2}\ln(a(t)a(t')) + \ln 2 - \frac{1}{4}\right) \to \left(\frac{1}{2s} + \ln 2 - 1\right),$$

appearing in any of the expressions for the response function for the massless minimal scalar field (such as eq. (4.32)). Thus despite s is large, there will be no term growing with time in this case, as compared to the exactly massless one. On the other hand, if we take a complex scalar field in the α-vacua, problems exactly similar to section 5 will prevail.

7 Discussions

We have computed, in this work, the response function for the Unruh-DeWitt detector coupled to a complex scalar field at the first order perturbation theory, for both conformal and a massless minimal couplings. The latter requires certain regularization procedure in order to give the response function a physical meaning. We have discussed extension of these results to the de Sitter α-vacua and have pointed out some possible ambiguities for a complex scalar.

We have also shown that for a real scalar field theory with a field-detector coupling linear in the field operator, with the interpretation discussed at the end of section 2, we can compute the response function for the α-vacua (see also [34]). It is easy to argue that such computation extends to any arbitrary order of the perturbation theory. This is because at the n-th order, we have a term like $\prod_n \int d\tau_n \phi(\tau_n)$ from the S-matrix expansion. Since there are as many integrations as the number of field operators, we shall never have two Wightman functions appearing in a single integral. Accordingly, the cancellation of the $i\epsilon$ regulators as of section 5 does not occur in this case. For example, the second order correction in the response function can be evaluated (up to some numerical factors) as

$$\frac{d^2\mathcal{F}_\alpha(\Delta E)}{dt_+\, dt'_+} \sim \left(\int_{-\infty}^{\infty} d(\Delta t)\, e^{-i\Delta E \Delta t}\, iG^+_\alpha(\Delta t)\right)^2 + \int_{-\infty}^{\infty} d(\Delta t)\, e^{-i\Delta E \Delta t}\, iG^+_\alpha(\Delta t) \times \int_{-\infty}^{\infty} d(\Delta t')\, e^{i\Delta E \Delta t'}\, iG^+_\alpha(\Delta t'),$$

which can indeed be computed without any ambiguity. Similarly, the corrections to the response function can be obtained for higher orders as well.

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For the massless and minimal complex scalar in particular (section 4.2), we have discussed a viable regularisation cum renormalisation scheme. It will be interesting to check how this can consistently tackle the divergences at higher order of the perturbation theory as well.

Computation of the response function for a fermionic field will be more realistic. Since massless fermion is conformally invariant, we may expect in this case the spectrum to be qualitatively similar to that of a complex conformal scalar. Also, we do not expect any de Sitter breaking logarithms for fermions. It will also be interesting to investigate the massless minimal complex scalar field theory from various perspective of quantum entanglement, e.g. entanglement harvesting. The effect of background primordial electromagnetic fields on a charged scalar will also be interesting, for in this case we expect de Sitter breaking terms indicating instability at late times analogous to that of the growing logarithm of eq. (4.32), even in the massive case. We shall come back to these issues in future works.

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