Adjoint SU(5) GUT model with modular S_4 symmetry

Ya Zhao and Hong-Hao Zhang

*School of Physics, Sun Yat-Sen University,
GuangZhou, GuangDong 510275, P.R. China*

E-mail: zhaoya@mail.ustc.edu.cn, zh98@mail.sysu.edu.cn

ABSTRACT: We study the textures of SM fermion mass matrices and their mixings in a supersymmetric adjoint SU(5) Grand Unified Theory with modular S_4 being the horizontal symmetry. The Yukawa entries of both quarks and leptons are expressed by modular forms with lower weights. Neutrino sector has an adjoint SU(5) representation 24 as matter superfield, which is a triplet of S_4 . The effective light neutrino masses is generated through Type-III and Type-I seesaw mechanism. The only common complex parameter in both charged fermion and neutrino sectors is modulus τ . Down-type quarks and charged leptons have the same joint effective operators with adjoint scalar in them, and their mass discrepancy in the same generation depends on Clebsch-Gordan factor. Especially for the first two generations the respective Clebsch-Gordan factors made the double Yukawa ratio $y_d y_\mu / y_e y_s = 12$, in excellent agreement with the experimental result. We reproduce proper CKM mixing parameters and all nine Yukawa eigenvalues of quarks and charged leptons. Neutrino masses and MNS parameters are also produced properly with normal ordering is preferred.

KEYWORDS: Discrete Symmetries, GUT, Neutrino Physics

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1 Introduction

Despite of great success, especially the discovery of Higgs boson [1, 2], the standard model (SM) still has some problems unsolved in particle physics. One of the problems is the origin of flavor structure for the SM fermions, which mainly refers to the enormous mass difference among different generations, and distinct mixing patterns between lepton and quark sector.

The masses of charged fermions, from lightest electron of MeV level to heaviest top quark being 173 GeV, span almost 5 orders of magnitude. The situation becomes even worse when the neutrino sector is contained, since the neutrino masses of sub-eV are nearly 7 orders of magnitude smaller than that of electron. Besides the absolute mass scale and mass ordering are still yet to be determined by future high-precision neutrino experiments. Apart from the large mass hierarchy, quark CKM parameters manifest a mixing pattern of small angles [3]. However the situation is completely different in lepton MNS mixing matrix. The precise neutrino oscillation data has provided us a picture of two large angles θ_{12} and θ_{23} , and one small angle θ_{13} , which is comparable with the quark Cabibbo angle θ_C .

It is an unsolved puzzle to interpret the observed flavor structure in quark and lepton sector. The flavor parameters, include all the fermion masses, mixing angles as well as CP violating phases arise from the dependence of Yukawa on flavor. Since symmetry plays

an important role in physics, it is worth to constrain the Yukawa interactions with the supervision of flavor symmetry, hence the mass hierarchies and mixing patterns of SM fermions could be interpreted by the additional symmetry beyond the gauge symmetry of the SM. But note that flavor symmetry is not the unique top-down scenario to understand the flavor structure. The other possible top-down approaches include, such as anarchy [4], extra dimension [5, 6] and string theory [7–9].

For the last two decades, the precise measurement of flavor parameters, especially that of lepton mixing angles, has motivated the model building by using discrete symmetry group to elucidate the different flavor structures between quarks and leptons. Non-Abelian discrete groups, such as lower order groups A_4 , A_5 , S_4 and other ones with higher order, are wildly used in such kind of works. And it is indeed easy to reproduce at least the two large leptonic mixing angles. For reviews see refs. [10–15]. In the models apart from the essential Higgs, some extra scalar fields called flavons are introduced. They are singlets under SM gauge group but have nontrivial representations under flavor group, whose vacuum orientation in flavor space can induce specific Yukawa textures for fermions. In neutrino sector the famous Tri-Bimaximal mixing [16, 17] is ubiquitous in many realistic models.

Non-Abelian discrete groups as flavor symmetries are success in explaining the leptonic mixing pattern with large angles, with the price of introducing a number of gauge singlets called flavon fields in conventional studies. The flavor symmetry is broken when the scalar flavons acquiring vacuum expectation values (VEV) by non-trivial dynamical way. Such VEVs with specific configurations control the flavor textures of fermions and thus a few of free coupling parameters appear in the Yukawa entries. Besides the values of VEVs themselves are determined by some other parameters. The more parameters in the traditional flavor models, the less predictive power they have. And the fermion masses are still often acquired by tuning the coupling parameters, even though the VEVs of the flavons can be nearly fixed such that all the couplings can be naturally of the same order, e.g., of order one.

Recently the modular symmetry provides another possibility to interpret the flavor issues [18]. The simplest modular symmetry implementation only demands one complex field, called modulus τ , as the unique source of modular symmetry breaking when it develops a VEV, hence the vacuum configuration problem is greatly simplified. In such a simplest framework the Yukawa couplings are just modular forms which are the holomorphic functions of modulus τ , then the flavon fields are not indispensable ingredient for model building, and thus tremendously reduce the amount of particle content and the free parameters of the theory.

The modular invariance models are based on the level N finite modular group Γ_N , e.g., from $N = 2$ to 5, $\Gamma_2 \simeq S_3$ [19–22], $\Gamma_3 \simeq A_4$ [18, 23–34], $\Gamma_4 \simeq S_4$ [35–43] and $\Gamma_5 \simeq A_5$ [41, 44, 45], which are all inhomogeneous modular groups. In such models the Yukawa couplings are weight k modular forms with k being even numbers. Most of the studies focus on the lepton flavor issues, while few of them include quark sector [20, 24, 25, 31]. The unified quark and lepton models can be implemented in the context of $SU(5)$ grand unified theories (GUT) combined with the modular symmetries [21, 32]. Besides the topics on the fermion flavor structures, the related phenomenological issues has been discussed

in those works, such as the dark matter models [27], baryon number violation [26] and leptogenesis [22, 33, 34]. On the other side, the double covering of finite modular group which is homogeneous, has been used as flavor symmetry as well [46–51].

Motivated by the phenomenological viable mass ratios between quarks and leptons, and the idea of Yukawa couplings can be modular forms, in the study we combine the modular flavor symmetry $\Gamma_4 \simeq S_4$ with the supersymmetric adjoint SU(5) GUT [52] to forge the flavor textures of quarks and leptons simultaneously. In refs. [21, 32] the neutrino masses are generated via type-I [53–56] seesaw by adding at least two gauge singlets, i.e., right-handed heavy neutrinos. However there exist another two possibilities in SU(5) GUT to produce light neutrino masses: first is type-II seesaw mechanism [57–59] by adding an extra Higgs $\mathbf{15}_H$, and second is Type-I plus Type-III [60] seesaw by adding fermionic field in the $\mathbf{24}$ dimensional representation. The two cases in SU(5) GUT have not been explored yet in the recent modular flavor models. In this paper we explore for the first time the second scheme to produce effective light neutrino masses. Meanwhile we would like to give rise to a realistic Yukawa ratios between charged leptons and down-quarks. The Yukawa ratio in each generation is directly derived from the novel Clebsch-Gordan (CG) factor, which can be comparable to the phenomenological values at GUT scale. For the modular flavor model built on SU(5) GUT, the flavon-free model is such that the double Yukawa eigenvalue ratio $y_\mu y_d / y_e y_s$ equal to 12 for the first and second families of charged leptons and down quarks. The other viable cases which can generate the acceptable ratios, however, still require the flavon or so called weighton to compensate the loss of mass dimension.

There is no flavon but a modulus τ in the modular symmetry breaking sector, meanwhile the gauge symmetry is broken by one adjoint scalar H_{24} . We aim at minimizing the amount of free coupling parameters in the unified model. For higher weight modular forms, it will bring more free coupling parameters, and thus decrease the predictive power of the model. Thus we use lower weight modular forms to give the modular invariant operators.

In order to achieve the above goal, we should strictly constrain the representations and weights under S_4 for the fields. To be specific, all the $\mathbf{10}$ -dimensional matter superfields are S_4 singlets and have distinct weights. The three families of $\bar{\mathbf{5}}_s$ are divided into an doublet and a singlet but have the same weight. At last the $\mathbf{24}$ fermionic superfields are sealed in a triplet. For the scalar sector, the $\mathbf{5}_H$, $\mathbf{45}_H$ and $\mathbf{24}_H$ are just singlets with distinct weights. Please see table. 1 for details.

In quark and charged lepton sector the Yukawa matrices are very sparse with several texture zeros. The adjoint scalar H_{24} couples to the matter superfields in down quark sector, then it induces the novel ratio of CG factors 1/2 and 6 for the first two families of leptons and down quarks, and 3/2 for the third one. In neutrino sector we introduce the $\mathbf{24}$ dimensional matter field rather than a gauge singlet to produce the neutrino masses through Type-I and Type-III seesaw mechanism. Since we introduce the adjoint matter fields rather than gauge singlets to produce the effective masses of light neutrinos, the Yukawa matrices are slightly different for the heavy ρ_3 and ρ_0 in the $\mathbf{24}$. And the same for Majorana mass matrices, since the mass terms include two nontrivial couplings: the pure mass term which is the same for ρ_3 and ρ_0 , and the new interaction between $\mathbf{24}$ and H_{24} , which splits the masses of ρ_3 and ρ_0 . Such new interaction is of course absent in the models which gauge singlets are responsible for seesaw mechanism.

The layout of the paper is arranged as follows. In section 2 we introduce the framework for the model building work, including the brief review on modular group, especially the one with level $N = 4$, $\Gamma_4 \simeq S_4$ and the modular forms of weight 2 and 4. Then we give the basic aspects of adjoint SU(5) GUT. In section 3 we present the flavor model based on adjoint SU(5) GUT combined with modular S_4 flavor symmetry. We show that the GUT flavor model can be built without introducing a gauge singlet scalar. The entries of Yukawa matrices of quarks and charged leptons have only modular forms in them. In section 4, we first give the convention for Yukawa matrices and the SUSY threshold corrections, then we present the data to be used and perform the numerical fit to the Yukawa matrices with threshold effects included. Section 5 devotes to the summary of the study.

2 The framework

In the section we shall briefly discuss the framework and environment used for the construction of model. First we give a brief review on the basic concepts of modular symmetry with lower level N and modular forms of weight k . The finite modular group Γ_4 is used for our model building work, so we give the modular forms with weight $k = 2, 4$. Secondly we introduce the basic aspects of adjoint SU(5) grand unified theory, including the matter multiplets and scalar Higgs together with the vacuum configurations of the scalars.

2.1 Modular group and modular forms

The modular group $\bar{\Gamma}$ implies the linear fractional transformations γ that act on the complex τ in the upper-half complex plane

$$\tau \rightarrow \gamma\tau = \frac{a\tau + b}{c\tau + d}, \quad a, b, c, d \in \mathbb{Z}, \quad ad - bc = 1, \quad \text{Im}\tau > 0, \quad (2.1)$$

The generators S and T are the two transformations satisfying

$$S^2 = (ST)^3 = \mathbb{I}, \quad (2.2)$$

with the representation matrices as

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad (2.3)$$

lead to

$$S : \tau \rightarrow -\frac{1}{\tau}, \quad T : \tau \rightarrow \tau + 1. \quad (2.4)$$

We introduce the series of infinite normal subgroups $\Gamma(N)$, $N = 1, 2, 3, \dots$ of $\text{SL}(2, \mathbb{Z})$

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{SL}(2, \mathbb{Z}), \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}. \quad (2.5)$$

For $N = 1, 2$ we define $\bar{\Gamma}(N) = \Gamma(N)/\{\mathbb{I}, -\mathbb{I}\}$ and for $N > 2$ we have $\bar{\Gamma}(N) = \Gamma(N)$. Taking the quotient $\Gamma_N \equiv \bar{\Gamma}/\bar{\Gamma}(N)$, one can obtain a finite subgroup called finite modular

group. Especially for $N \leq 5$ the groups Γ_N are isomorphic to the permutation groups $\Gamma_2 \simeq S_3, \Gamma_3 \simeq A_4, \Gamma_4 \simeq S_4$ and $\Gamma_5 \simeq A_5$, which are ubiquitous in the construction of flavor models.

Modular forms $f(\tau)$ of weight k and level N are holomorphic functions transforming under the groups $\Gamma(N)$ in the way as

$$f(\gamma\tau) = (c\tau + d)^k f(\tau), \quad \gamma \in \Gamma(N). \quad (2.6)$$

Here k is even number, and N is natural. Given the weight k and level N , the modular forms span a linear space of dimension equals to $k+1$. The basis in the linear space can be choosed such that the modular form $f_i(\tau)$ in a multiplet transforms according to a unitary representation ρ of the group Γ_N :

$$f_i(\gamma\tau) = (c\tau + d)^k \rho(\gamma)_{ij} f_j(\tau), \quad \gamma \in \Gamma_N. \quad (2.7)$$

In the study we take $N = 4$ as the case of interest and construct an explicit grand unified flavor model to elucidate the fermion masses and mixings. In the case of lowest weight 2, there are 5 linear independent modular forms. These modular forms are explicitly expressed in terms of Dedekind eta-function $\eta(\tau)$:

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \quad q = e^{i2\pi\tau}. \quad (2.8)$$

To be specific, the modular forms are defined as the functions of $\eta(\tau)$ and its derivatives of the form

$$Y(c_1, \dots, c_6 | \tau) \equiv c_1 \frac{\eta'(\tau + \frac{1}{2})}{\eta(\tau + \frac{1}{2})} + c_2 \frac{\eta'(4\tau)}{\eta(4\tau)} + c_3 \frac{\eta'(\frac{\tau}{4})}{\eta(\frac{\tau}{4})} + c_4 \frac{\eta'(\frac{\tau+1}{4})}{\eta(\frac{\tau+1}{4})} + c_5 \frac{\eta'(\frac{\tau+2}{4})}{\eta(\frac{\tau+2}{4})} + c_6 \frac{\eta'(\frac{\tau+3}{4})}{\eta(\frac{\tau+3}{4})}, \quad (2.9)$$

with the coefficients $c_1 + \dots + c_6 = 0$. For the case of weight 2, the basis is comprised of five modular forms as follow:

$$Y_1(\tau) \equiv Y(1, 1, \omega, \omega^2, \omega, \omega^2 | \tau), \quad (2.10)$$

$$Y_2(\tau) \equiv Y(1, 1, \omega^2, \omega, \omega^2, \omega | \tau), \quad (2.11)$$

$$Y_3(\tau) \equiv Y(1, -1, -1, -1, 1, 1 | \tau), \quad (2.12)$$

$$Y_4(\tau) \equiv Y(1, -1, -\omega^2, -\omega, \omega^2, \omega | \tau), \quad (2.13)$$

$$Y_5(\tau) \equiv Y(1, -1, -\omega, -\omega^2, \omega, \omega^2 | \tau), \quad (2.14)$$

with $\omega \equiv e^{2\pi i/3}$. The modular forms $(Y_1, Y_2)^T$ and $(Y_3, Y_4, Y_5)^T$ transform as a doublet and triplet of S_4 respectively, i.e., they are denoted as

$$Y_2(\tau) \equiv \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}, \quad Y_{3'}(\tau) \equiv \begin{pmatrix} Y_3(\tau) \\ Y_4(\tau) \\ Y_5(\tau) \end{pmatrix}. \quad (2.15)$$

One can get modular forms of higher weights from the tensor productions of weight 2 modular forms, and thus obtain different irreps of S_4 . Taking weight $k = 4$ for example, we

then have 9 independent modular forms, and they are arranged into the following singlets and multiplets irreps of S_4

$$\begin{aligned} Y_1^{(4)} &= Y_1 Y_2, & Y_2^{(4)} &= \begin{pmatrix} Y_2^2 \\ Y_1^2 \end{pmatrix}, \\ Y_3^{(4)} &= \begin{pmatrix} Y_1 Y_4 - Y_2 Y_5 \\ Y_1 Y_5 - Y_2 Y_3 \\ Y_1 Y_3 - Y_2 Y_4 \end{pmatrix}, & Y_{3'}^{(4)} &= \begin{pmatrix} Y_1 Y_4 + Y_2 Y_5 \\ Y_1 Y_5 + Y_2 Y_3 \\ Y_1 Y_3 + Y_2 Y_4 \end{pmatrix}. \end{aligned} \quad (2.16)$$

We use the above modular forms of weight 2 and 4 to construct our modular invariant model in section 3. For higher weight modular forms, one may refer to ref. [36] for further information.

2.2 Basic aspects of adjoint SU(5)

In this part we shall give the fermion sector and the scalars used for the model setup. According to the distinct representations under the gauge symmetry, the matter fields are divided into the following three parts: anti-fundamental $\bar{\mathbf{5}}$, anti-symmetrical tensor $\mathbf{10}$ and the adjoint $\mathbf{24}$. We assume the usual three generations of $\bar{\mathbf{5}}$ and $\mathbf{10}$, which have the decomposition under the SM gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. The theory of renormalizable adjoint SU(5) can be seen in ref. [61]. Concerning our model, we promote the matter fields to superfields in Minimal Supersymmetry Standard Model (MSSM) and embed them into SU(5) representations $\bar{\mathbf{5}}$ and $\mathbf{10}$. For $\bar{\mathbf{5}}$ we denote

$$F_i = (d_R^c \quad d_B^c \quad d_G^c \quad e \quad -\nu)_i = d_i^c \oplus \ell_i \quad (2.17)$$

and for $\mathbf{10}$

$$T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -u_G^c & u_B^c & -u_R & -d_R \\ u_G^c & 0 & -u_R^c & -u_B & -d_B \\ -u_B^c & u_R^c & 0 & -u_G & -d_G \\ u_R & u_B & u_G & 0 & -e^c \\ d_R & d_B & d_G & e^c & 0 \end{pmatrix}_i = u_i^c \oplus q_i \oplus e_i^c, \quad (2.18)$$

where $i = 1, 2, 3$ indicates the family indices of standard model (SM), and R, B, G stand for the color indices. The adjoint matter field $\mathbf{24}$ reads

$$\mathbf{24} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} \boldsymbol{\lambda} \cdot \boldsymbol{\rho}_8 - \sqrt{\frac{2}{15}} \rho_0 & \rho_{(3,2)} \\ \rho_{(\bar{3},2)} & \frac{1}{\sqrt{2}} \boldsymbol{\sigma} \cdot \boldsymbol{\rho}_3 + \sqrt{\frac{3}{10}} \rho_0 \end{pmatrix}, \quad (2.19)$$

in which λ^i are the Gell-Mann matrices. The scalar fields contain the following Higgs fields

$$\begin{aligned} \mathbf{5}_H &= T_{(3,1)} \oplus H_1, \\ \mathbf{45}_H &= S_8 \oplus S_{(\bar{6},1)} \oplus S_{(3,3)} \oplus S_{(\bar{3},2)} \oplus S_{(3,1)} \oplus S_{(\bar{3},1)} \oplus H_2, \\ \mathbf{24}_H &= \Sigma_8 \oplus \Sigma_3 \oplus \Sigma_{(3,2)} \oplus \Sigma_{(\bar{3},2)} \oplus \Sigma_{24}. \end{aligned} \quad (2.20)$$

Fields	T_1	T_2	T_3	$F = (F_1, F_2)$	F_3	A	H_5	$H_{\bar{5}}$	H_{45}	$H_{\bar{45}}$	H_{24}
SU(5)	10	10	10	$\bar{\mathbf{5}}$	$\bar{\mathbf{5}}$	24	5	$\bar{\mathbf{5}}$	45	$\bar{\mathbf{45}}$	24
$\Gamma_4 \equiv S_4$	1	1	1	2	1	3'	1	1'	1	1	1'
k_I	0	-1	1	1	1	-1	-2	1	0	-1	-2

Table 1. Field content and their representation assignments under the gauge group SU(5), modular S_4 and their weights k_I in the model.

The VEVs of the scalars are listed as the following form

$$\langle \mathbf{5}_H \rangle = \frac{v_5}{\sqrt{2}}(0, 0, 0, 0, 1)^T, \quad (2.21)$$

$$\langle \mathbf{24}_H \rangle = v_{24} \text{diag}(1, 1, 1, -3/2, -3/2), \quad (2.22)$$

$$\langle \mathbf{45}_H \rangle_j^{i5} = \frac{v_{45}}{\sqrt{2}} [\text{diag}(1, 1, 1, -3, 0)]_j^i, \quad \langle \mathbf{45}_H \rangle_j^{in} = 0, \quad (2.23)$$

where $i, j = 1, \dots, 5$, $n = 1, \dots, 4$ and $v = \sqrt{|v_5|^2 + 24|v_{45}|^2} = 246 \text{ GeV}$.

3 Adjoint SU(5) GUT flavor model

Now let us present the supersymmetric adjoint SU(5) GUT flavor model in details. First we assign the representations and weights of chiral supermultiplets. For sake of simplicity we shall impose certain constraints on the assignments. For matter fields, three generations of **10** dimensional representations, denoted by $T_{1,2,3}$, are all S_4 singlets, i.e., $\rho_{T_i} \sim \mathbf{1}$, ($i = 1, 2, 3$), but have distinct weights for each family. The first two generations of $\bar{\mathbf{5}}$ dimensional representations, named as $F_{1,2}$, are assigned to be a doublet of S_4 , i.e., $F = (F_1, F_2)$, while the third generation F_3 is a S_4 singlet. Nevertheless the three $\bar{\mathbf{5}}$ s have the same weight. In addition to the **10**- and $\bar{\mathbf{5}}$ -dimensional representations in SU(5) GUTs, the adjoint **24** dimensional matter superfields A have also three families, and they are assigned to be an triplet of S_4 , denoted by $A = (A_1, A_2, A_3)$.

For the Higgs sector H_5 , H_{45} , $H_{\bar{5}}$ and $H_{\bar{45}}$ are responsible for the electroweak symmetry broken, while the GUT gauge group is broken by an adjoint scalar field H_{24} . We shall not intend to explain the mass hierarchy of charged fermions, which is beyond the power of modular symmetry. And in order to made the coupling terms minimality, we do not introduce any flavon fields like conventional models did. Instead all the Yukawa couplings are modular forms with specific weights to ensure the modular invariance. The only modular symmetry breaking source arises from the modulus τ developing its VEV. For simplicity the modular weights are assigned such that the Yukawa couplings have as lower weights as possible. The representations and weights of the chiral supermultiplets are listed in the table 1.

3.1 SU(5) breaking

For the purpose of our model construction, we start with SU(5) breaking superpotential. Since we have only one adjoint scalar field H_{24} in the model, the gauge symmetry broken

into SM gauge group is realized by the adjoint scalar acquiring the vacuum expectation value of H_{24} . The SU(5) breaking superpotential is simply as

$$\mathcal{W}_{24} = M_{24} Y_1^{(4)} \text{Tr} H_{24}^2 + \lambda Y_{1'}^{(6)} \text{Tr} H_{24}^3, \quad (3.1)$$

then one can get the VEV of H_{24} , i.e., $\langle H_{24} \rangle$ in eq. (2.22) with

$$v_{24} = \frac{4M_{24}Y_1^{(4)}}{3\lambda_1 Y_{1'}^{(6)}}. \quad (3.2)$$

Besides causing the GUT breaking, the adjoint scalar field H_{24} would also couple to the matter fields leading to novel Clebsch-Gordan factors, and especially the exotic Yukawa coupling ratios between down quark sector and charged lepton sector at GUT scale. See the next section for details.

3.2 Charged fermions

In this section we will present the Yukawa couplings for charged fermions. Since the flavons are not an essential part of the model, the relevant Yukawa coupling terms in up-type quark sector manifest in renormalisable level, while those in down-type quarks and charged lepton sector show in effective operators. Since the quarks have enormous mass hierarchies but small mixing angles, the resulting Yukawa matrices should be controllable in the entries. A practical viable scheme is to induce texture zeros for Yukawa matrices. And another keypoint is the GUT-scale Yukawa ratios between leptons and down-quarks have to fulfil certain phenomenological constraints from experiments as well.

The Yukawa coupling terms for up quarks in superpotential involve two Higgs fields, i.e., H_5 and H_{45} . Because of the symmetry constraints only the following couplings are allowed

$$\mathcal{W}_u = \alpha_u T_1 T_1 H_{45} + \beta_u T_2 T_2 H_5 Y_1^{(4)} + \gamma_u T_3 T_3 H_5 + g_u T_2 T_3 H_{45}, \quad (3.3)$$

which leads to a block diagonal mass matrix

$$M_u = \begin{bmatrix} \alpha_u v_{45} & 0 & 0 \\ 0 & \beta_u Y_1 Y_2 v_5 & g_u v_{45} \\ 0 & -g_u v_{45} & \gamma_u v_5 \end{bmatrix}. \quad (3.4)$$

The operators of dimension 4, such as those in eq. (3.3), are expressed by the diagram in figure 1,¹ in which the dashed line stands for scalar Higgs and solid lines are matter multiplets. In table 2 we give the list of dimension 4 operators corresponding to figure 1 including those in the next sections.

In SU(5) GUTs, the left and right handed components of up-type quarks reside in the same representations T_i , that is why the Yukawa couplings are of the form $T_i T_j H_s$ ($s = 5, 45$) which made the Yukawa elements either symmetric or antisymmetric. However those of down-type quarks live in different representations: F_i includes the right handed component and T_i has left handed quark doublet. And vice verse for charged leptons. Then

¹The supergraphs were drawn with JaxoDraw [62, 63].

Figure 1. The supergraph for dimension 4 operators.

Field	1	2	3	4	5	6	7	8
ϕ_1	T_1	T_2	T_3	T_2	T_1	F	F_3	A
ϕ_2	T_1	T_2	T_3	T_3	F_3	A	A	A
ϕ_3	H_{45}	H_5	H_5	H_{45}	$H_{\overline{45}}$	H_5	H_5	H_{24}

Table 2. The list for dimension 4 operators corresponding to figure 1.

the interactions of both down quarks and charged leptons are written by the same joint operators of the form $T_i F_j H_{\bar{s}}$. Accordingly the corresponding Yukawa couplings, Y_d and Y_e , are just mutual transposed relation up to $\mathcal{O}(1)$ CG factors. Therefore the eigenvalues have to satisfy certain phenomenological GUT relations. For the first two families, the following double Yukawa ratio [64]

$$\frac{y_\mu}{y_s} \left(\frac{y_e}{y_d} \right)^{-1} = 10.7^{+1.8}_{-0.8}, \quad (3.5)$$

is a strong restriction on model building. Besides the Yukawa eigenvalues in both up and down quarks sectors, the quark mixing CKM parameters have to be fulfilled as well. All the requirements imply that only certain CG factors can be realistic in GUT flavor models. To be specific, in our model the effective superpotential in down quarks sector and charged leptons sector is written by the operators

$$\mathcal{W}_d = \frac{\alpha_d}{\Lambda} [T_1 H_{\overline{45}}]_{45} [F H_{24}]_{\overline{45}} Y_{\mathbf{2}} + \frac{\beta_d}{\Lambda} [T_2 H_{24}]_{10} [F H_{\bar{5}}]_{\overline{10}} Y_{\mathbf{2}} + \frac{\gamma_d}{\Lambda} [T_3 H_{\bar{5}}]_{\bar{5}} [F_3 H_{24}]_{\bar{5}} + g_d T_1 F_3 H_{\overline{45}}, \quad (3.6)$$

where the SU(5) contraction $[XY]_R$ of fields X and Y denotes a tensor in the representation R . Here the adjoint scalar H_{24} is crucial to form the dimension five operators which result in the new Yukawa ratios between leptons and quarks. After the GUT gauge symmetry is broken when H_{24} develops its VEV along the hypercharge direction, the Clebsch-Gordan factors emerge in the entries of lepton and quark Yukawa matrices. These SU(5) tensor contractions are realized by integrating out heavy messengers. The effective operators in the superpotential are then generated, see figure 2. For the model to work, we list the messenger fields and their representations as well as weights in table 3.

Figure 2. The supergraph for dimension 5 operators in the superpotential of down quarks.

Field	ϕ_1	ϕ_2	ϕ_3	ϕ_4	Γ	Field	Υ_1	Υ_2	Υ_3
1	T_1	$H_{\overline{45}}$	F	H_{24}	Υ_1	SU(5)	$\overline{45}$	$\overline{10}$	$\overline{5}$
2	T_2	H_{24}	F	$H_{\overline{5}}$	Υ_2	$\Gamma_4 \equiv S_4$	1	1'	1'
3	T_3	$H_{\overline{5}}$	F	H_{24}	Υ_3	k_I	-1	3	-2

Table 3. Left panel: the dimension 5 operators corresponding to figure 2, Right panel: the charges of messenger fields appear in the dimension 5 operators.

We define the quantity $\epsilon = \langle H_{24} \rangle / \Lambda$, then the mass matrix of down-type quarks reads

$$M_d = \begin{bmatrix} \alpha_d v_{\overline{45}} \epsilon Y_2 & -\alpha_d v_{\overline{45}} \epsilon Y_1 & g_d v_{\overline{45}} \\ \beta_d v_{\overline{5}} \epsilon Y_2 & \beta_d v_{\overline{5}} \epsilon Y_1 & 0 \\ 0 & 0 & \gamma_d v_{\overline{5}} \epsilon \end{bmatrix}, \quad (3.7)$$

and that of charged leptons is simply the transposed M_d up to the CG coefficients of Yukawa couplings

$$M_e = \begin{bmatrix} C_1 \alpha_d v_{\overline{45}} \epsilon Y_2 & C_2 \beta_d v_{\overline{5}} \epsilon Y_2 & 0 \\ -C_1 \alpha_d v_{\overline{45}} \epsilon Y_1 & C_2 \beta_d v_{\overline{5}} \epsilon Y_1 & 0 \\ C_4 g_d v_{\overline{45}} & 0 & C_3 \gamma_d v_{\overline{5}} \epsilon \end{bmatrix}, \quad (3.8)$$

The CG coefficients in the model are $C_1 = -1/2$, $C_2 = 6$, $C_3 = -3/2$ and $C_4 = -3$. Note that the fourth CG factor is the famous Georgi-Jarlskog relation [65]. The first three ones are the main predictions for the mass relations between quarks and leptons, which can be realized in conventional models, such as [66, 67]. We can check that the double ratio in eq. (3.5) is satisfied for the present choice. The set of the CG coefficients, $C_1 = -1/2$ and $C_2 = 6$, is in fact the only one that can be realized in GUT without flavons. The other two possibilities in realistic flavor GUT models, e.g., (A) $C_1 = 4/9, C_2 = 9/2$ [68] and (B) $C_1 = -8/27, C_2 = -3$ [69], however, at least one scalar field has to be added to compensate the loss of mass dimension. In the two cases the mass of messenger pair Γ and $\bar{\Gamma}$ is generated by a heavy scalar field Λ_{24} , who lives in the SU(5) adjoint representation. Then the leptonic and down-type quark-like components of Γ and $\bar{\Gamma}$ obtain different masses split by CG factors. After integrating out the heavy messenger fields, the CG factors inversely enter in the Yukawa matrix entries of charged leptons and down-quarks [70]. We will briefly build two toy models to elucidate the two cases. For sake of simplicity we assume only one flavon field ϕ (or weighton [31]) which is a singlet under a modular symmetry (not necessary Γ_4), and the appropriate weight is assigned. And we just show the operators

Figure 3. The supergraphs for the operators in eq. (3.9) (left) and eq. (3.10) (right).

which result in the diagonal entries of the mass matrix. For case (A) we may write the superpotential for the first two families as

$$\mathcal{W}_d^A = y_{11}^d T_1 F_1 H_{\overline{5}} \frac{\phi^2}{H_{24}^2} Y_{\mathbf{r},a}^{(k)} + \frac{y_{22}^d}{\Lambda} [T_2 H_{45}]_{\overline{5}} [F_2 H_{24}]_5 Y_{\mathbf{r}',a'}^{(k')}, \quad (3.9)$$

in which $Y_{\mathbf{r},a}^{(k)}$ and $Y_{\mathbf{r}',a'}^{(k')}$ denote the components of modular form multiplets. For case (B) the superpotential is

$$\mathcal{W}_d^B = y_{11}^d T_1 F_1 H_{\overline{5}} \frac{\phi^3}{H_{24}^3} Y_{\mathbf{r},b}^{(k)} + y_{22}^d T_2 H_{45} F_2 Y_{\mathbf{r}',b'}^{(k')}. \quad (3.10)$$

The effective operators of the superpotentials are generated after integrate out the heavy messenger fields. We show in figure 3(a) and figure 3(b) the supergraphs corresponding to case (A) and case (B) respectively. The details for building such models are beyond the scope of this work.

3.3 Neutrino

In this section we shall give the neutrino interactions which induce light neutrino masses. We assume neutrinos to be Majorana type, and the light masses are generated by seesaw mechanism. In most of typical flavor models, including the flavor GUT models, the light neutrino masses can be produced through type-I seesaw mechanism which demands at least two superheavy right-handed Majorana neutrinos to suppress the Yukawa couplings. In the implementation ways of seesaw mechanisms in SU(5) GUTs, however, there are another two ways to do the same thing, the first scheme is type-II seesaw [57–59] by adding an extra Higgs $\mathbf{15}_H$, and the second one is Type-I plus Type-III seesaw [60] by introducing the fermionic fields in the $\mathbf{24}$ dimensional representation. In our model we assume the second scheme as the unique origin of neutrino masses and no more extra matter fields are involved. The matter chiral superfield A lives in SU(5) adjoint representation $\mathbf{24}$ and is also an S_4 triplet. According to the symmetry constraints in, the neutrino Yukawa interactions reads

$$\mathcal{W}_{Y_A} = \frac{y_{\nu_1}}{\Lambda} F A H_5 Y_{\mathbf{3}'} + \frac{y_{\nu_2}}{\Lambda} F_3 A H_5 Y_{\mathbf{3}'}. \quad (3.11)$$

Note that the second term would vanish if $(\rho_{F_3}, \rho_A) = (\mathbf{1}, \mathbf{3})$ or $(\mathbf{1}', \mathbf{3}')$. If we made the choice, the third column of Yukawa texture will be vanishing which features a zero

determinant of neutrino mass matrix, no matter what structure of Majorana mass matrix is. We drop the case at present. The Yukawa matrix is then

$$\mathcal{Y}_{\rho_3} = \frac{y_{\nu_1}}{2} \begin{bmatrix} Y_4 & Y_5 & 0 \\ Y_3 & Y_4 & 0 \\ Y_5 & Y_3 & 0 \end{bmatrix} + \frac{y_{\nu_2}}{2} \begin{bmatrix} 0 & 0 & Y_3 \\ 0 & 0 & Y_5 \\ 0 & 0 & Y_4 \end{bmatrix}$$

$$\mathcal{Y}_{\rho_0} = \sqrt{\frac{3}{5}} \mathcal{Y}_{\rho_3}. \quad (3.12)$$

If we introduce SU(5) singlet **1**, e.g., an N^c , as right-handed Majorana neutrinos, the pure mass term of the form MN^cN^c is the only interaction. However in adjoint SU(5) there is an extra interaction form besides pure mass term. Since we have **24** dimensional matter superfield responsible for the generation of neutrino masses, the new interactions between **24** and **24_H** have to be considered. Now we give all the mass terms of **24** as

$$\mathcal{W}_A = M A A Y_2 + M' A A Y_{\mathbf{3}'} + \lambda_1 A A H_{24} Y_2^{(4)} + \lambda_2 A A H_{24} Y_{\mathbf{3}'}^{(4)} + \lambda_3 A A H_{24} Y_{\mathbf{3}}^{(4)}. \quad (3.13)$$

The second and the fifth terms vanish because the tensor productions of $\mathbf{3}' \otimes \mathbf{3}'$ have to be antisymmetric (see appendix A) to form an invariant with modular forms. Applying the decomposition to eq. (3.13), the fermionic singlet ρ_0 and triplet ρ_3 have the mass matrices of the form

$$M_{\rho_3} = \frac{M}{4} \begin{bmatrix} 0 & Y_1 & Y_2 \\ Y_1 & Y_2 & 0 \\ Y_2 & 0 & Y_1 \end{bmatrix} - \frac{3v'_{24}}{4\sqrt{30}} \left\{ \lambda_1 \begin{bmatrix} 0 & -Y_2^2 & Y_1^2 \\ -Y_2^2 & Y_1^2 & 0 \\ Y_1^2 & 0 & -Y_2^2 \end{bmatrix} + \lambda_2 \begin{bmatrix} 2Y_{\mathbf{3}',1}^{(4)} & -Y_{\mathbf{3}',3}^{(4)} & -Y_{\mathbf{3}',2}^{(4)} \\ -Y_{\mathbf{3}',3}^{(4)} & 2Y_{\mathbf{3}',2}^{(4)} & -Y_{\mathbf{3}',1}^{(4)} \\ -Y_{\mathbf{3}',2}^{(4)} & -Y_{\mathbf{3}',1}^{(4)} & 2Y_{\mathbf{3}',3}^{(4)} \end{bmatrix} \right\},$$

$$M_{\rho_0} = \frac{M}{4} \begin{bmatrix} 0 & Y_1 & Y_2 \\ Y_1 & Y_2 & 0 \\ Y_2 & 0 & Y_1 \end{bmatrix} - \frac{v'_{24}}{4\sqrt{30}} \left\{ \lambda_1 \begin{bmatrix} 0 & -Y_2^2 & Y_1^2 \\ -Y_2^2 & Y_1^2 & 0 \\ Y_1^2 & 0 & -Y_2^2 \end{bmatrix} + \lambda_2 \begin{bmatrix} 2Y_{\mathbf{3}',1}^{(4)} & -Y_{\mathbf{3}',3}^{(4)} & -Y_{\mathbf{3}',2}^{(4)} \\ -Y_{\mathbf{3}',3}^{(4)} & 2Y_{\mathbf{3}',2}^{(4)} & -Y_{\mathbf{3}',1}^{(4)} \\ -Y_{\mathbf{3}',2}^{(4)} & -Y_{\mathbf{3}',1}^{(4)} & 2Y_{\mathbf{3}',3}^{(4)} \end{bmatrix} \right\}, \quad (3.14)$$

where the abbreviation $Y_{\mathbf{3}',i}^{(4)} (i = 1, 2, 3)$ denote the components of weight 4 modular form $Y_{\mathbf{3}'}^{(4)}$ for the expression more compact, e.g., $Y_{\mathbf{3}',1}^{(4)} = Y_1 Y_4 + Y_2 Y_5$ and else, see eq. (2.16).

The neutrino masses are generated by Type-I and Type-III seesaw, which are realized by integrating out ρ_0 and ρ_3 , respectively. We write the effective light neutrino mass matrix as the sum of the contributions from Type-I and Type-III seesaw

$$M_\nu^{SS} = -(\mathcal{Y}_{\rho_0}^T M_{\rho_0}^{-1} \mathcal{Y}_{\rho_0} + \mathcal{Y}_{\rho_3}^T M_{\rho_3}^{-1} \mathcal{Y}_{\rho_3}) v_u^2, \quad (3.15)$$

where $v_u = v \sin \beta$ with $v = \sqrt{v_u^2 + v_d^2} = 174 \text{ GeV}$ and $\tan \beta = v_u/v_d$, as usual defined in MSSM.

4 Phenomenology

The superpotential in the matter sector is simply given by

$$\mathcal{W}_{\text{matt}} = \mathcal{W}_u + \mathcal{W}_d + \mathcal{W}_{Y_A} + \mathcal{W}_A, \quad (4.1)$$

in which each term is given by eqs. (3.3), (3.6), (3.11) and (3.13), respectively. As the GUT and modular symmetry breaking we write the Yukawa matrices of fermions in the following convention

$$\begin{aligned} W_Y = & (Y_u)_{ij} q_i * H_u u_j^c + (Y_d)_{ij} q_i * H_d d_j^c + (Y_e)_{ij} \ell_i * H_d e_j^c + (\mathcal{Y}_{\rho_3})_{ij} \ell_i^T i \sigma_2 (\rho_3)_j H_u \\ & + (\mathcal{Y}_{\rho_0})_{ij} \ell_i^T i \sigma_2 (\rho_0)_j H_u + (M_{\rho_3})_{ij} \text{Tr}((\rho_3)_i (\rho_3)_j) + (M_{\rho_0})_{ij} (\rho_0)_i (\rho_0)_j, \end{aligned} \quad (4.2)$$

where H_u and H_d are the two Higgs doublets in MSSM. The neutrino Yukawa matrices and mass matrices have been given by eqs. (3.12) and (3.14) in section 3.3. The Yukawa matrices of charged fermions are then read as

$$Y_u = \begin{bmatrix} \alpha_u & 0 & 0 \\ 0 & \beta_u Y_1 Y_2 & g_u \\ 0 & -g_u & \gamma_u \end{bmatrix} \equiv \begin{bmatrix} u_{11} & 0 & 0 \\ 0 & u_{22} & u_{23} \\ 0 & -u_{23} & u_{33} \end{bmatrix}, \quad (4.3)$$

for up quarks, and

$$Y_d = \begin{bmatrix} \alpha_d Y_2 & -\alpha_d Y_1 & g_d \\ \beta_d Y_2 & \beta_d Y_1 & 0 \\ 0 & 0 & \gamma_d \end{bmatrix} \equiv \begin{bmatrix} d_{11} & -d_{11} Y_1 / Y_2 & d_{13} \\ d_{22} Y_2 / Y_1 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix}, \quad (4.4)$$

for down quarks. Observing the Yukawa matrices Y_u and Y_d are very sparse with some texture zeros, one can conclude the CKM Cabibbo angle θ_{12}^q is totally generated by the mixing in the down sector, and of course the same for the θ_{13}^q as well as CP phase δ^q . The mixing in up sector completely determines the angle θ_{23}^q .

In SU(5) GUT the Yukawa matrix of charged leptons is the transpose of that of down quarks, i.e., $Y_e \sim Y_d^T$, up to the order one CG coefficients,

$$Y_e = \begin{bmatrix} -\frac{1}{2} d_{11} & 6 d_{22} Y_2 / Y_1 & 0 \\ -\frac{1}{2} d_{11} Y_1 / Y_2 & 6 d_{22} & 0 \\ -3 d_{13} & 0 & -\frac{3}{2} d_{33} \end{bmatrix}. \quad (4.5)$$

It is obvious that the down quarks and charged leptons follow the following Yukawa ratios

$$\frac{y_\tau}{y_b} \approx \frac{3}{2}, \quad \frac{y_\mu}{y_s} \approx 6, \quad \frac{y_e}{y_d} \approx \frac{1}{2}, \quad (4.6)$$

in which y_ℓ ($\ell = e, \mu, \tau$) and y_{d_i} ($i = d, s, b$) are the eigenvalues of Y_e and Y_d , respectively. Thus the CG factors $C_1 = 1/2$ and $C_2 = 6$ made the double Yukawa ratio $y_d y_\mu / y_e y_s = 12$ which is in good agreement with the data in eq. (3.5).

4.1 SUSY threshold corrections

The Yukawa couplings and mixing observables are defined at superhigh GUT scale in our model, therefore the values from low energy experiments must run up to the ones at GUT scale. Moreover the SUSY radiative threshold corrections are the requisite factor for matching the MSSM at the SUSY scale M_{SUSY} to the SM [71–74]. The running of MSSM Yukawa parameters from M_Z to M_{GUT} has been analysed in [64], where the $\tan\beta$ enhanced 1-loop SUSY threshold effects are discussed in detail. The matching relations between the eigenvalues of the MSSM and the SM Yukawa coupling matrices are parameterized as

$$y_{u,c,t}^{\text{MSSM}} = \frac{y_{u,c,t}^{\text{SM}}}{\sin\bar{\beta}}, \quad (4.7)$$

$$y_{d,s}^{\text{MSSM}} = \frac{y_{d,s}^{\text{SM}}}{(1 + \bar{\eta}_q) \cos\bar{\beta}}, \quad y_b^{\text{MSSM}} = \frac{y_b^{\text{SM}}}{(1 + \bar{\eta}_b) \cos\bar{\beta}}, \quad (4.8)$$

$$y_{e,\mu}^{\text{MSSM}} = \frac{y_{e,\mu}^{\text{SM}}}{(1 + \bar{\eta}_\ell) \cos\bar{\beta}}, \quad y_\tau^{\text{MSSM}} = \frac{y_\tau^{\text{SM}}}{\cos\bar{\beta}}, \quad (4.9)$$

and the quark CKM parameters are also corrected by

$$\theta_{i3}^{q,\text{MSSM}} = \frac{1 + \bar{\eta}_b}{1 + \bar{\eta}_q} \theta_{i3}^{q,\text{SM}}, \quad \theta_{12}^{q,\text{MSSM}} = \theta_{12}^{q,\text{SM}}, \quad \delta^{q,\text{MSSM}} = \delta^{q,\text{SM}}. \quad (4.10)$$

One can notice, to a good approximation, the threshold corrections have no impact on θ_{12}^q and δ^q and the running of Yukawa couplings Y_f^{MSSM} depends only on $\bar{\eta}_b$ and $\tan\bar{\beta}$. Especially in the limit that the threshold corrections to charged leptons are neglected, i.e., $\bar{\eta}_\ell = 0$, then $\tan\bar{\beta}$ reduces to the usual $\tan\beta$. We will adopt the scenario in the model. Nevertheless the $\bar{\eta}_q$ can not be dropped. The reason is that the ratio y_μ/y_s at GUT scale is approximately 4.5 without SUSY threshold corrections, but the large CG factor 6 in (4.5) needs a compensation from $\bar{\eta}_q$ which is approximately +0.33 [75]. The two CG factors $C_2 = 6$ and $C_3 = -3/2$ appeared in the Yukawa matrix Y_e in eq. (4.5) require a relative large $\tan\beta$ to generate substantial threshold corrections. In fact for the large Yukawa coupling ratios in (4.6), both large $\tan\beta$ and large threshold corrections are required (cf. [76]). Accordingly we set $\tan\beta = 35$, $\bar{\eta}_b = 0.13125$ and $\bar{\eta}_q = 0.3$. We notice that the threshold parameters can be free, but the fixed values are enough to reproduce correct observables.

4.2 Numerical analysis

The Yukawa matrices for neutrinos, up- and down- quarks and leptons are presented in eqs. (3.15), (4.3), (4.4) and (4.5), respectively. The only common parameter τ among them has been bounded in the upper half complex plane. Modular symmetry itself, however, is enable to give rise to the hierarchical fermion masses, and mixing parameters. The free parameters appear in the mass matrices are

$$P_i = \{\tau, \alpha_u, \beta_u, \gamma_u, g_u, \alpha_d, \beta_d, \gamma_d, g_d, y_{\nu_1}, y_{\nu_2}, M, \lambda_1, \lambda_2, \}, \quad (4.11)$$

and the physical observables Q^{obs} in the GUT model include

$$\begin{aligned} Q_q &= \{y_u, y_c, y_t, y_d, y_s, y_b, \theta_{12}^q, \theta_{13}^q, \theta_{23}^q, \delta^q\}, \\ Q_\ell &= \{y_e, y_\mu, y_\tau, \Delta\mathcal{M}_{21}^2, \Delta\mathcal{M}_{31}^2, \theta_{12}^\ell, \theta_{13}^\ell, \theta_{23}^\ell, \delta^\ell\}. \end{aligned} \quad (4.12)$$

We shall construct a global χ^2 function for all the observable quantities when fitting the coupling matrices in eqs. (4.3), (4.4) and (4.5) for charged fermions and eqs. (3.12) and (3.14) for neutrinos. The χ^2 function to be minimized is defined as

$$\chi^2 = \sum_i \left(\frac{Q_i(P_j) - Q_i^{\text{obs}}}{\sigma_i} \right)^2, \quad (4.13)$$

where $Q_i(P_j)$ denote the model predicted values for observables and $Q^{\text{obs}} = Q_q + Q_\ell$ are the central values with σ_i the 1σ errors.

Before performing the fit, we would like to elucidate the data used in the minimization. As stressed in section 4.1, the model is defined at the high energy scale, the observable quantities should be set at the GUT scale. The quantities at GUT energy scale can be achieved from the low energy scale where the experimental values are determined by the renormalization group equations (RGEs). Here in our work we adopt the values of Yukawa couplings at the GUT scale $M_X = 2 \times 10^{16}$ GeV, assuming minimal SUSY breaking $M_{\text{SUSY}} = 1$ TeV with a large $\tan\beta = 35$ [64]. The Yukawa values y_f give rise to the fermion masses as $m_f = y_f v_H$ with $v_H = 174$ GeV. Meanwhile the CKM mixing angles and CP phase are also taken as the values at the GUT scale. All the Yukawa values and CKM observables as well as their 1σ errors are listed in the left panel of table 4. For the neutrino sector, the lepton mixing angles, CP phase and neutrino mass squared differences we adopted are taken from NuFit 5.0 [77]. We show their center values and 1σ errors in the right panel of table 4. The above input data are used for the estimation of our χ^2 .

Since the quark sector and neutrino sector have the modulus τ as the common parameter which determines the modular forms, the remaining free parameters are just Yukawa coupling coefficients. It is equivalent and more convenient to fit the entries of Yukawa matrices rather than Yukawa coefficients themselves. Therefore we just fit the entries of quark Yukawa matrices, u_{ij} s in eq. (4.3) and d_{ij} s in eq. (4.4). In neutrino sector, we define the coupling parameter ratios

$$r_{21} = \frac{y_{\nu_2}}{y_{\nu_1}}, \quad r_1 = \frac{\lambda_1 v_{24}}{M}, \quad r_2 = \frac{\lambda_2 v_{24}}{M}, \quad (4.14)$$

and an overall mass scale $y_{\nu_1}^2 v_u^2 / M$ are the parameters to be fitted. So, instead of fitting the primitive free parameters showed in eq. (4.11), we take the following equivalent parameter set

$$\overline{P}_i = \{\tau, u_{11}, u_{22}, u_{33}, u_{23}, d_{11}, d_{22}, d_{33}, d_{13}, r_{21}, r_1, r_2, y_{\nu_1}^2 v_u^2 / M\}. \quad (4.15)$$

In table 5 we show the model best fit input parameters in quark Yukawa and neutrino mass matrices which minimizes the χ_q^2 and χ_ℓ^2 , respectively. We present our fit results for all

Observable	μ_i	σ_i
$\theta_{12}^q/^\circ$	13.027	0.041
$\theta_{13}^q/^\circ$	0.166	0.006
$\theta_{23}^q/^\circ$	1.924	0.031
$\delta^q/^\circ$	69.213	3.094
y_u	2.889×10^{-6}	8.954×10^{-7}
y_c	1.413×10^{-3}	4.946×10^{-5}
y_t	0.529	0.016
y_d	1.3879×10^{-4}	1.5267×10^{-5}
y_s	2.7467×10^{-3}	1.4832×10^{-4}
y_b	0.1867	2.766×10^{-3}
y_e	7.3834×10^{-5}	4.430×10^{-7}
y_μ	1.5590×10^{-2}	9.354×10^{-5}
y_τ	0.2819	1.751×10^{-3}

Observable	NO	IO
$\theta_{12}/^\circ$	$33.44^{+0.78}_{-0.75}$	$33.45^{+0.78}_{-0.75}$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	8.61 ± 0.12
$\theta_{23}/^\circ$	$49.0^{+1.0}_{-1.4}$	$49.3^{+1.0}_{-1.2}$
$\delta/^\circ$	195^{+51}_{-25}	286^{+27}_{-32}
$\Delta\mathcal{M}_{21}^2/10^{-5}$	$7.42^{+0.21}_{-0.20}$	$7.42^{+0.21}_{-0.20}$
$\Delta\mathcal{M}_{31}^2/10^{-3}$	$2.514^{+0.028}_{-0.027}$	$-2.497^{+0.028}_{-0.028}$

Table 4. Left panel: the Observables of charged fermions at the GUT scale for $\tan\beta = 35$ are taken from [64]. The SUSY breaking scale are set at $M_{SUSY} = 1$ TeV and the threshold correction parameters $\bar{\eta}_b = 0.13125$ and $\bar{\eta}_q = 0.3$. Right panel: the values of neutrino Observables are taken from NuFit5.0 [77] without the atmospheric data from SuperKamiokande. NO (IO) denotes the Normal (Inverted) Ordering of neutrino masses.

the Yukawas and mass matrices in table 6. In the left panel of table 6 we give the resulting best fit values and pulls to the ten quark observables: three quark CKM mixing angles θ_{ij}^q ($ij = 12, 13, 23$) and one CP violating phase δ^q , six Yukawas y_q ($q = u, c, t, d, s, b$). Also the minimum $\chi_{\min,q}^2 \sim 0.45$ is given at the last row. We also list in table 6 (right panel) the best fit values and pulls to six neutrino observables and three charged lepton Yukawas. The minimum $\chi_{\min,\ell}^2$ is just $\mathcal{O}(1)$. The best fit point has the total $\chi_{\min}^2 = \chi_{\min,q}^2 + \chi_{\min,\ell}^2 \simeq 1.6$. One can see that the model favours normal ordering neutrino masses, and we found the minimum $\chi_{\min,\ell}^2 = 8.478$ for inverted ordering. Besides the values of absolute neutrino masses m_i , the Majorana phases φ_{21} and φ_{31} are pure theoretical predictions. The mass sum, the β -decay effective mass m_β as well as the neutrinoless double beta ($0\nu\beta\beta$) decay amplitude parameter m_{ee} are also given as predictions in the table. Specifically the bounds on the above mass related quantities are given by

$$\begin{aligned}
 \sum_i m_i &\leq 120 \text{ meV}, \\
 m_\beta &= \left[\sum_i m_i^2 |U_{ei}|^2 \right]^{1/2} < (61 \sim 165) \text{ meV}, \\
 m_{ee} &= \left| \sum_i U_{ei}^2 m_i \right| < 1.1 \text{ eV} (90\% \text{ C.L.}),
 \end{aligned}
 \tag{4.16}$$

which are taken from PLANCK [78], KamLAND-ZEN [79] and KATRIN [80], respectively.

Quark Input	Value	Neutrino Input	Value
u_{11}	2.887×10^{-6}	$r_1 = \lambda_1 v_{24}/M$	$-0.06587 + 2.09536i$
u_{22}	8.476×10^{-4}	$r_2 = \lambda_2 v_{24}/M$	$0.36449 + 1.72602i$
u_{33}	0.5304	$r_{21} = y_{\nu_2}/y_{\nu_1}$	2.26385
u_{23}	-1.775×10^{-2}	$y_{\nu_1}^2 v_u^2/M$ (eV)	0.00362
d_{11}	2.667×10^{-6}		
d_{22}	1.020×10^{-4}	Common Input	Value
d_{33}	6.032×10^{-3}	$\text{Re}\tau$	0.4768
d_{13}	$2.031 \times 10^{-5} e^{-0.657i\pi}$	$\text{Im}\tau$	0.9288

Table 5. Left panel: the best fit input parameters in quark sector. The Yukawa entries u_{ij} and d_{ij} include the original Yukawa coupling coefficients and the modular forms. Upper Right panel: the best fit input parameters in neutrino sector. We list the ratios of neutrino couplings in eqs. (3.11) and (3.13). Lower Right panel: the only common input parameter to both sectors is the modulus τ .

The leptonic mixing matrix elements U_{ei} are taken from the standard parametrization [3]

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}c_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\varphi_{21}/2} & 0 \\ 0 & 0 & e^{\varphi_{31}/2} \end{pmatrix}, \quad (4.17)$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$, δ is lepton Dirac CP violating phase and φ_{21} , φ_{31} are two Majorana CP phases. Comparing the model predictions in table 6 to these bounds, we see that the predicted values are well below the corresponding upper bounds. For the sum of neutrino masses, our result is still below the tightest and most robust upper limit $M_\nu < 0.118 \text{ eV}$ [81].

There are 17 real input parameters in table 5, to fit 19 measured data points in table 4. Hence the number of the degree of freedom (d.o.f) is naively 2, which is the difference of the number of observables and inputs. This translates to a reduced χ^2 , i.e., $\chi_{\text{red}}^2 = \chi_{\text{min}}^2/\text{d.o.f} = 0.808$. We take this value as a good fit. The fit has been performed by using the Mathematica package Mixing Parameter Tools (MPT) [82].

We can see that in quark and lepton sectors the only common input parameter is modulus τ , which is close to the boundary (right cusp) of the fundamental region. The Yukawas are in general complex, however we can always absorb most of the phases by redefinition of the fields. Specifically for the quark Yukawa inputs, the only complex parameter is d_{13} which mainly controls the magnitude of the CKM matrix element V_{ub} and part of the CP phase δ^q . Meanwhile the other inputs, all u_{ij} s and the rest of d_{ij} s ($ij \neq 13$) are real. In neutrino sector we also have reduced the total number of real parameters to be six, in which only r_1 and r_2 are complex. The Yukawa ratio r_{21} and the overall mass scale $y_{\nu_1}^2 v_u^2/M$ are all real parameters.

Quark output	Value	pull
$\theta_{12}^q/^\circ$	13.0233	-0.0839
$\theta_{13}^q/^\circ$	0.16639	0.0002
$\theta_{23}^q/^\circ$	1.9238	0
$\delta^q/^\circ$	69.0969	0.0376
y_u	2.889×10^{-6}	0.0
y_c	1.413×10^{-3}	0.0
y_t	0.529	0.0
y_d	1.4378×10^{-4}	0.327
y_s	2.6685×10^{-3}	-0.527
y_b	0.1873	0.2312
$\chi_{\min,q}^2$		0.4467

Lepton output	Value	pull
$\theta_{12}/^\circ$	33.4916	0.0675
$\theta_{13}/^\circ$	8.5344	-0.285
$\theta_{23}/^\circ$	49.7665	0.613
$\delta/^\circ$	208.2733	0.349
$\Delta\mathcal{M}_{21}^2/10^{-5}$	7.524	0.4966
$\Delta\mathcal{M}_{31}^2/10^{-3}$	2.514	0
y_e	7.377×10^{-5}	-0.1432
y_μ	1.560×10^{-2}	0.1433
y_τ	0.281	-0.5478
$\chi_{\min,\ell}^2$		1.1694
Predictions	Value	
Mass Ordering	NO	
m_1/meV	11.728	
m_2/meV	14.585	
m_3/meV	51.493	
$\sum_i m_i/\text{meV}$	77.809	
m_β/meV	14.674	
m_{ee}/meV	11.040	
φ_{21}/π	0.105	
φ_{31}/π	1.323	

Table 6. Left panel: the quark fit output results. All the quark sector is fitted to the 1σ interval. Right panel: the output results of lepton sector. The absolute values of three light neutrino masses and the ordering as well as the two Majorana phases are pure model predictions. We have found a minimum $\chi_{\min,\ell}^2 = 8.478$ for inverted ordering.

5 Summary

In the study we explored an supersymmetric adjoint $SU(5)$ GUT flavor model based on modular $\Gamma_4 \simeq S_4$ symmetry. We have shown the model can produce correct masses and mixing parameters of both quarks and leptons simultaneously. No flavons are introduced to the model, only the complex modulus τ is responsible for the breaking of modular symmetry. By assigning suitable representations and weights to chiral superfields, only finite coupling terms are presented in the effective operators in quark sector and lepton sector. We obtained very sparse Yukawa matrices of up- and down-quarks (and of course charged leptons) with some texture zeros. Also in neutrino sector we have only two Yukawa coupling terms, and three terms in Majorana mass terms. The effective light neutrino masses are generated through Type-I plus Type-III seesaw mechanism. The modulus τ is the only common field appeared in both quark and lepton sector as spurion.

For simplicity we have used modular forms with lower weight in the model, since higher weight would bring more free parameters. The assignments of representations under S_4 and weights for the chiral superfields are also highly constrained such that the Yukawa coupling terms are uniquely fixed by the modular forms. The **10** dimensional matter fields are all S_4 singlets, while $\bar{\mathbf{5}}_s$ are divided into a S_4 doublet for the first two generations and a singlet for third one. Meanwhile the adjoint superfields **24** are collected in the triplet of S_4 . All the scalars of course transform as singlets of S_4 . We also assigned distinct weights for the superfields such that the number of free Yukawa parameters is as little as possible.

With the delicate representation assignments of field content, the superpotentials of the model is relatively simple. Unlike the models in [21, 32] which have more parameters than observables, our model has less free parameters and thus more predictive than the above two works. The resulting operators in each sector have limit coupling parameters. The model predicts that down quarks and charged leptons have a rigid Yukawa coupling ratios generated by CG factors which arise from the SU(5) contractions of the effective 5D operators. The double Yukawa ratio $y_\mu y_d / y_e y_s$ equal to 12 for the first two families of charged leptons and down quarks. The model has only 17 real parameters in total, and 19 observables to be fit, thus the degree of freedom (d.o.f) is 2. We obtained the reduced chi-square $\chi^2/\text{d.o.f} \simeq 0.81$ for quark and lepton sectors combined, which is a good fit. The model favours an normal mass ordering over inverted ordering for light neutrinos, with a $\Delta\chi^2 \sim 7$. Moreover we obtained the absolute values of three light neutrino masses, the effective masses of β -decay and $0\nu\beta\beta$ decay as well as the Majorana CP phases as model predictions.

At last we give a outlook for the model. Since the masses of the fields live in the adjoint representation are splited by the adjoint scalar H_{24} , the lightest one is ρ_3 . It is crucial to realize the baryogenesis via leptogenesis [83, 84] in the context. In the case the net asymmetry of $B - L$ can be generated in the out of equilibrium decays of ρ_3 and ρ_0 as well as their superpartners in the adjoint representation. So it is worth studying further the phenomenology according to the model.

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A S_4 group

The discrete group S_4 who has 24 elements is the permutation group of four objects. The two generators S and T in different irreducible presentations are given as follows

$$\mathbf{1} : \quad S = 1, \quad T = 1 \quad (\text{A.1})$$

$$\mathbf{1}' : \quad S = -1, \quad T = 1 \quad (\text{A.2})$$

$$\mathbf{2}: \quad S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix} \quad (\text{A.3})$$

$$\mathbf{3}: \quad S = \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega & 2\omega^2 & -1 \\ 2\omega^2 & -1 & 2\omega \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \quad (\text{A.4})$$

$$\mathbf{3}': \quad S = \frac{1}{3} \begin{pmatrix} 1 & -2\omega & -2\omega^2 \\ -2\omega & -2\omega^2 & 1 \\ -2\omega^2 & 1 & -2\omega \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \quad (\text{A.5})$$

where $\omega = e^{2\pi i/3} = (i\sqrt{3} - 1)/2$. In the basis we can obtain the decomposition of the product representations and the Clebsch-Gordan factors. The product rules of S_4 group, with a_i, b_i as the elements of multiplet in the product, are given by following

$$\star \quad \mathbf{1} \otimes r = r \sim ab_i \quad (\text{A.6})$$

$$\star \quad \mathbf{1}' \otimes \mathbf{1}' = 1 \sim ab \quad (\text{A.7})$$

$$\star \quad \mathbf{1}' \otimes \mathbf{2} = 2 \sim \begin{pmatrix} ab_1 \\ -ab_2 \end{pmatrix} \quad (\text{A.8})$$

$$\star \quad \mathbf{1}' \otimes \mathbf{3} = 3' \sim \begin{pmatrix} ab_1 \\ ab_2 \\ ab_3 \end{pmatrix} \quad (\text{A.9})$$

$$\star \quad \mathbf{1}' \otimes \mathbf{3}' = 3 \sim \begin{pmatrix} ab_1 \\ ab_2 \\ ab_3 \end{pmatrix} \quad (\text{A.10})$$

The product rules with two-dimensional representation are given by:

$$\begin{aligned} \star \quad \mathbf{2} \otimes \mathbf{2} &= \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{2} \\ \mathbf{1} &\sim a_1 b_2 + a_2 b_1, & \mathbf{1}' &\sim a_1 b_2 - a_2 b_1 \\ \mathbf{2} &\sim \begin{pmatrix} a_2 b_2 \\ a_1 b_1 \end{pmatrix} \end{aligned} \quad (\text{A.11})$$

$$\begin{aligned} \star \quad \mathbf{2} \otimes \mathbf{3} &= \mathbf{3} \oplus \mathbf{3}' \\ \mathbf{3} &\sim \begin{pmatrix} a_1 b_2 + a_2 b_3 \\ a_1 b_3 + a_2 b_1 \\ a_1 b_1 + a_2 b_2 \end{pmatrix}, & \mathbf{3}' &\sim \begin{pmatrix} a_1 b_2 - a_2 b_3 \\ a_1 b_3 - a_2 b_1 \\ a_1 b_1 - a_2 b_2 \end{pmatrix} \end{aligned} \quad (\text{A.12})$$

$$\begin{aligned} \star \quad \mathbf{2} \otimes \mathbf{3}' &= \mathbf{3} \oplus \mathbf{3}' \\ \mathbf{3} &\sim \begin{pmatrix} a_1 b_2 - a_2 b_3 \\ a_1 b_3 - a_2 b_1 \\ a_1 b_1 - a_2 b_2 \end{pmatrix}, & \mathbf{3}' &\sim \begin{pmatrix} a_1 b_2 + a_2 b_3 \\ a_1 b_3 + a_2 b_1 \\ a_1 b_1 + a_2 b_2 \end{pmatrix} \end{aligned} \quad (\text{A.13})$$

and the three-dimensional representations have the following product rules

$$\begin{aligned}
 \star \quad \mathbf{3} \otimes \mathbf{3} &= \mathbf{3}' \otimes \mathbf{3}' = 1 \oplus 2 \oplus 3 \oplus 3' \\
 \mathbf{1} &\sim a_1 b_1 + a_2 b_3 + a_3 b_2 \\
 \mathbf{2} &\sim \begin{pmatrix} a_2 b_2 + a_3 b_1 + a_1 b_3 \\ a_3 b_3 + a_1 b_2 + a_2 b_1 \end{pmatrix} \\
 \mathbf{3} &\sim \begin{pmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_3 b_1 - a_1 b_3 \end{pmatrix}, \quad \mathbf{3}' \sim \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_3 b_1 - a_1 b_3 \end{pmatrix} \quad (\text{A.14})
 \end{aligned}$$

$$\begin{aligned}
 \star \quad \mathbf{3} \otimes \mathbf{3}' &= \mathbf{1}' \oplus 2 \oplus 3 \oplus 3' \\
 \mathbf{1}' &\sim a_1 b_1 + a_2 b_3 + a_3 b_2 \\
 \mathbf{2} &\sim \begin{pmatrix} a_2 b_2 + a_3 b_1 + a_1 b_3 \\ -a_3 b_3 - a_1 b_2 - a_2 b_1 \end{pmatrix} \\
 \mathbf{3} &\sim \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_3 b_1 - a_1 b_3 \end{pmatrix}, \quad \mathbf{3}' \sim \begin{pmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_3 b_1 - a_1 b_3 \end{pmatrix} \quad (\text{A.15})
 \end{aligned}$$

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