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Contributions of $K_0^*(1430)$ and $K_0^*(1950)$ in the three-body decays $B \to K\pi h$

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Abstract: We study the contributions of the resonant states $K_0^*(1430)$ and $K_0^*(1950)$ in the three-body decays $B \to K\pi h$ (with $h = \pi, K$) in the perturbative QCD approach. The crucial nonperturbative input $F_{K\pi}(s)$ in the distribution amplitudes of the S-wave $K\pi$ system is derived from the matrix element of vacuum to $K\pi$ pair. The CP averaged branching fraction of the quasi-two-body decay process $B \to K_0^*(1950)h \to K\pi h$ is about one order smaller than that of the corresponding decay $B \to K_0^*(1430)h \to K\pi h$. In view of the important contribution from the S-wave $K\pi$ system for the $B \to K\pi h$ decays, it is not appropriate to neglect the $K_0^*(1950)$ in the theoretical or experimental studies for the relevant three-body B meson decays. The predictions in this work for the relevant decays are consistent with the existing experimental data.

Keywords: B physics, Branching fraction, CP violation, Flavor physics, Hadron-Hadron scattering (experiments)

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1 Introduction

The charmless three-body hadronic B meson decay processes provide us a field to appraise different dynamical models of strong interaction, to investigate hadronic final-state interactions and analyze hadron spectroscopy, to determine the fundamental quark mixing parameters and understand CP asymmetries. In order to extract the significative information from experimental results and present the effective and accurate predictions for the three-body B decays, some methods have been adopted in abundant works, such as the U-spin, isospin and flavor SU(3) symmetries in [1–10], the QCD factorization (QCDF) in [11–25] and the perturbative QCD (PQCD) approach in [26–28]. The three-body decays $B \to K\pi h$, with h is the pion or kaon, have been studied by Belle [29–35], BaBar [36–43] and LHCb [44–51] Collaborations in recent years. These decays especially the $B \to K\pi\pi$ were found to be a clean source for the extraction of the Cabibbo-Kobayashi-Maskawa (CKM) [52, 53] angle γ [54–61]. The relevant processes also provide new possibilities for the measurements of the CP violation in the B decays [30, 45–47].

The total decay amplitude for the B meson decays into three light mesons K, π and h as the final state can be described as the coherent sum of the nonresonant and resonant contributions in the isobar formalism [62–64]. The nonresonant contributions are spread all over the phase space and play an important role in the corresponding decay processes [65–67]. The resonant contributions from low energy scalar, vector and tensor resonances are known experimentally, in most cases, to be the dominated proportion of the related decays and could be studied in the quasi-two-body framework [68–70] when the rescattering effects [71] and three-body effects [72, 73] are neglected. For the three-body decays $B \to K\pi h$, one has the resonant contributions from the $K\pi$, πh and Kh pairs which are originated from different intermediate states and as well containing the two-body final state interactions. And the $J^P = 0^+$ component of the $K\pi$ spectrum, denoted as $(K\pi)^*_0$, is always found very important for the relevant physical observables.

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The kaon-pion scattering has been extensively studied in refs. [74–80] in recent years. While the primary source of the information on $I = 1/2$ S-wave $K\pi$ system comes from the LASS experiment for the reaction $K^-p \to K^-\pi^+n$ [81]. The $K\pi$ S-wave amplitude has also been studied in detail in the decays $D^+ \to K^-\pi^+\pi^+$ by E791 [82], FOCUS [83, 84] and CLEO [85], $\eta_c \to K\bar{K}\pi$ by BaBar [86] and $\tau^- \to K_S\pi^-\nu_\tau$ by Belle [87] with the methods of Breit-Wigner functions [88], K-matrix formalism [89–91] or model-independent partial-wave analysis. To describe the slowly increasing phase as a function of the $K\pi$ invariant mass, the scalar $K\pi$ scattering amplitude was written as the relativistic Breit-Wigner term [88] for the resonance $K_0^*(1430)$ in the LASS parametrization together with an effective range nonresonant component in [81], and the effective range term has been applied a cutoff to the slowly varying part close to the charm hadron mass at about 1.8 GeV for the three-body B decays in the experimental studies [38, 40, 42, 43]. At about 1.95 GeV one will find the presence of the resonance $K_0^*(1950)$ in [81] and also in the η_c decays in [86, 92]. This state was assigned as a radial excitation of the 0^+ member of the $L = 1$ triplet in the LASS analysis [81]. The lowest-lying broad component of the S-wave $K\pi$ system is the $K_0^*(700)$ [93], also named as κ or $K_0^*(800)$ in literature [82, 84, 85, 94–99], which has commonly been placed together with the resonant states σ, $f_0(980)$ and $a_0(980)$ into an SU(3) flavor nonet, and they have been suspected to be exotics [100–106].

In this work, we will focus on the contributions of the resonant state $K_0^*(1430)$ in the $B \to K\pi h$ decay processes in the PQCD approach based on the $k_{\rm T}$ factorization theorem [107–110]. The contributions of the resonant state $K_0^*(1950)$ in the hadronic three-body B meson decays involving S-wave $K\pi$ pair have been ignored in the relevant theoretical studies and be noticed only by LHCb Collaboration very recently in the works [111, 112]. We will systematically estimate, for the first time, the contributions from the state $K_0^*(1950)$ for the $B \to K\pi h$ decays in this work. As for the resonance $K_0^*(700)$, we shall leave to the future studies in view of its ambiguous internal structure and the accompanying complicated results for the three-body B decays [113], in addition, the corresponding contributions have been covered up by the effective range part of LASS line shape for the experimental results [38–40, 42, 43].

For the quasi-two-body decays $B \to K_0^*(1430, 1950)h \to K\pi h$, the subprocesses of the $B \to K\pi h$ decays, the intermediate state K_0^*, as demonstrated in the figure 1, is generated in the hadronization of quark-antiquark pair including one s or $\bar{s}$-quark. The process $K_0^* \to K\pi$, which can not be calculated in the PQCD approach, is always shrunken as the decay constants in the twist-2 and twist-3 light-cone distribution amplitudes of the scalar mesons [113–115] in the studies of the two-body B meson decays involving the scalar mesons $K_0^*(700)$ and $K_0^*(1430)$, see ref. [116] and the references therein for examples. While in the quasi-two-body framework based on PQCD, one can easily introduce the nonperturbative subprocess $K_0^* \to K\pi$ into a time-like form factor in the distribution amplitudes of the $K\pi$ pair. The quasi-two-body framework based on PQCD has been discussed in detail in [68] and has been adopted in some studies on the quasi-two-body B meson decay processes recently [117–126].

This work is organized as follows. In section II, we give a brief introduction for the theoretical framework. In section III, we show the numerical results and give some discus-

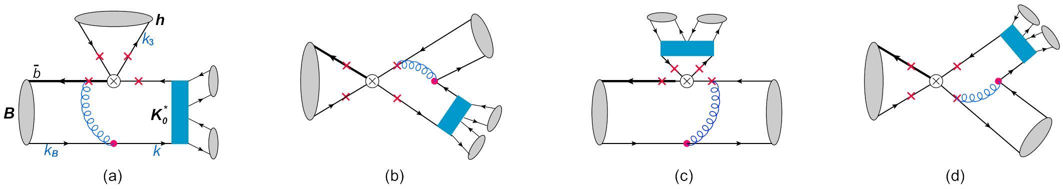

Figure 1. Typical Feynman diagrams for the decay processes $B \to K_0^* h \to K\pi h$, $h = (\pi, K)$. The symbol $\otimes$ is the weak vertex, $\times$ denotes possible attachments of hard gluons and the rectangle represents the scalar resonances K_0^*.

sions. Conclusions are presented in section IV. The factorization formulas and functions for the related quasi-two-body decay amplitudes are collected in the appendix.

2 Framework

In the rest frame of B meson, we define its momentum p_B and light spectator quark momentum k_B as

$$p_B = \frac{m_B}{\sqrt{2}}(1, 1, 0_{\mathrm{T}}), \quad k_B = \left(\frac{m_B}{\sqrt{2}} x_B, 0, k_{B\mathrm{T}}\right), \tag{2.1}$$

in the light-cone coordinates, where x_B is the momentum fraction and m_B is the mass. For the resonant states K_0^* and the $K\pi$ pair generated from it by the strong interaction as revealed in the figure 1, we define their momentum $p = \frac{m_B}{\sqrt{2}}(\zeta, 1, 0)$. Its easy to validate $\zeta = s/m_B^2$, where the invariant mass square $s = p^2 = m_{K\pi}^2$ for the $K\pi$ pair. The light spectator quark comes from B meson and goes into intermediate state in the hadronization of K_0^* as shown in figure 1 (a) has the momentum $k = (0, \frac{m_B}{\sqrt{2}} z, k_{\mathrm{T}})$. For the bachelor final state h and its spectator quark, their momenta p_3 and k_3 have the definitions as

$$p_3 = \frac{m_B}{\sqrt{2}}(1 - \zeta, 0, 0_{\mathrm{T}}), \quad k_3 = \left(\frac{m_B}{\sqrt{2}}(1 - \zeta) x_3, 0, k_{3\mathrm{T}}\right). \tag{2.2}$$

Where x_3 and z, which run from 0 to 1, are the corresponding momentum fractions.

The matrix element from the vacuum to the $K^+\pi^-$ final state is given by [127]

$$\langle K^+(p_1)\pi^-(p_2)|\bar{d}\gamma_\mu(1-\gamma_5)s|0\rangle = \left[(p_1 - p_2)_\mu - \frac{\Delta_{K\pi}}{p^2} p_\mu\right] F_+^{K\pi}(s) + \frac{\Delta_{K\pi}}{p^2} p_\mu F_0^{K\pi}(s), \tag{2.3}$$

with the $p_1(p_2)$ is the momentum for kaon(pion) in the $K\pi$ system, $\Delta_{K\pi} = (m_K^2 - m_\pi^2)$ and $m_K(m_\pi)$ is the mass of $K(\pi)$ meson. The $F_+^{K\pi}(s)$ is the vector form factor which has been discussed in detail in the refs. [87, 128–135]. The the scalar form factor $F_0^{K\pi}(s)$ is defined as [136–138]

$$\langle K\pi|\bar{q}s|0\rangle = C_X \frac{\Delta_{K\pi}}{m_s - m_q} F_0^{K\pi}(s) = B_0 C_X F_0^{K\pi}(s)\,, \tag{2.4}$$

where q is the light quark u or d, the isospin factor $C_X = 1$ for $X = \{K^+\pi^-, K^0\pi^+\}$ and $C_X = 1/\sqrt{2}$ for $X = \{K^+\pi^0, K^0\pi^0\}$. The constant B_0 equals to $\Delta_{K\pi}/(m_s - m_q)$. The

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form factor $F_0^{K\pi}(s)$ above is suppose to be one when s goes to zero. When the $K^+\pi^-$ pair originated from the resonant state $K_0^*(1430)^0$, we have [137]

$$\langle K^+\pi^-|\bar{d}s|0\rangle \approx \langle K^+\pi^-|K_0^{*0}\rangle \frac{1}{\mathcal{D}_{K_0^*}} \langle K_0^{*0}|\bar{d}s|0\rangle = \Pi_{K_0^* K\pi}\langle K_0^{*0}|\bar{d}s|0\rangle \ , \tag{2.5}$$

and

$$\Pi_{K_0^* K\pi} = \frac{g_{K_0^* K\pi}}{\mathcal{D}_{K_0^*}} \approx \frac{B_0}{\bar{f}_{K_0^*} m_{K_0^*}} F_0^{K\pi}(s) \ , \tag{2.6}$$

with $\bar{f}_{K_0^*} = \frac{m_{K_0^*}}{m_s(\mu)-m_d(\mu)} \cdot f_{K_0^*}$, the decay constants defined by $\langle K_0^{*0}|\bar{d}s|0\rangle = m_{K_0^*}\bar{f}_{K_0^*}$ and $\langle K_0^{*0}(p)|\bar{d}\gamma_\mu s|0\rangle = f_{K_0^*}p_\mu$ [113], and the mass $m_{K_0^*}$ could be replaced by the invariant mass $\sqrt{s}$ for the off-shell K_0^*. One can find different values of $f_{K_0^*}$ for $K_0^*(1430)$ in [139], we employ $f_{K_0^*(1430)} m^2_{K_0^*(1430)} = 0.0842 \pm 0.0045\,\mathrm{GeV}^3$ [140] and $f_{K_0^*(1950)} m^2_{K_0^*(1950)} = 0.0414$ GeV^3 [141] in this work. The Breit-Wigner formula for the denominator $\mathcal{D}_{K_0^*} = m^2_{K_0^*} - s - i m_{K_0^*}\Gamma(s)$, with the mass-dependent decay width $\Gamma(s) = \Gamma_0 \frac{q}{q_0}\frac{m_{K_0^*}}{\sqrt{s}}$ and Γ_0 is the full width for resonant state K_0^*. In the rest frame of the resonance K_0^*, its daughter kaon or pion has the magnitude of the momentum as

$$q = \frac{1}{2}\sqrt{\left[s-(m_K+m_\pi)^2\right]\left[s-(m_K-m_\pi)^2\right]/s} \ . \tag{2.7}$$

The q_0 in $\Gamma(s)$ is the value for q at $s = m^2_{K_0^*}$. The coupling constant $g_{K_0^* K\pi} = \langle K^+\pi^-|K_0^{*0}\rangle$, one has [18]

$$g_{K_0^* K\pi} = \sqrt{\frac{8\pi m^2_{K_0^*}\Gamma_{K_0^*\to K\pi}}{q_0}} \ , \tag{2.8}$$

where the $\Gamma_{K_0^*\to K\pi}$ is the partial width for $K_0^* \to K\pi$.

The S-wave $K\pi$ system distribution amplitudes are collected into [113, 138, 142, 143]

$$\Phi_{K\pi}(z,s) = \frac{1}{\sqrt{2N_c}}\left[\not{p}\phi(z,s) + \sqrt{s}\phi^s(z,s) + \sqrt{s}(\not{v}\not{n}-1)\phi^t(z,s)\right] , \tag{2.9}$$

with the $v = (0, 1, 0_{\mathrm{T}})$ and $n = (1, 0, 0_{\mathrm{T}})$ being the dimensionless vectors. The twist-2 light-cone distribution amplitude has the form [113, 138, 142]

$$\phi(z,s) = \frac{F_{K\pi}(s)}{2\sqrt{2N_c}}\left\{6z(1-z)\left[a_0(\mu) + \sum_{m=1}^{\infty} a_m(\mu) C_m^{3/2}(2z-1)\right]\right\} \ , \tag{2.10}$$

with $C_m^{3/2}$ the Gegenbauer polynomials, $a_0 = (m_s(\mu) - m_q(\mu))/\sqrt{s}$ for $(K_0^{*-}, \bar{K}_0^{*0})$ and $a_0 = (m_q(\mu) - m_s(\mu))/\sqrt{s}$ for (K_0^{*+}, K_0^{*0}) according to ref. [142]. The a_m are scale-dependent Gegenbauer moments, with $a_1 = -0.57 \pm 0.13$ and $a_3 = -0.42 \pm 0.22$ at the scale $\mu = 1\,\mathrm{GeV}$ for the resonance $K_0^*(1430)$, and the contributions from the even terms could be neglected [113]. There is no available Gegenbauer moments for the state $K_0^*(1950)$, we employ the scale-dependent a_1 and a_3 of $K_0^*(1430)$ for the entire S-wave $K\pi$ system in

$m_{B^0} = 5.280$	$m_{B^\pm} = 5.279$	$m_{B^0_s} = 5.367$	$m_{\pi^\pm} = 0.140$	$m_{\pi^0} = 0.135$
$m_{K^\pm} = 0.494$	$m_{K^0} = 0.498$	$f_K = 0.156$	$f_\pi = 0.130$	
$m_{K_0^*(1430)} = 1.425 \pm 0.050$		$\Gamma_{K_0^*(1430)} = 0.270 \pm 0.080$		
$m_{K_0^*(1950)} = 1.945 \pm 0.010 \pm 0.020$		$\Gamma_{K_0^*(1950)} = 0.201 \pm 0.034 \pm 0.079$		
$\lambda = 0.22453 \pm 0.00044$	$A = 0.836 \pm 0.015$	$\bar{\rho} = 0.122^{+0.018}_{-0.017}$	$\bar{\eta} = 0.355^{+0.012}_{-0.011}$	

Table 1. Masses, decay constants, full widths of $K_0^*(1430)$ and $K_0^*(1950)$ (in units of GeV) and Wolfenstein parameters [93].

the numerical calculation. For the twist-3 light-cone distribution amplitudes in this work, we take the asymptotic forms as

$$\phi^s(z,s) = \frac{F_{K\pi}(s)}{2\sqrt{2N_c}}\,, \qquad \phi^t(z,s) = \frac{F_{K\pi}(s)}{2\sqrt{2N_c}}(1-2z)\,. \tag{2.11}$$

The factor $F_{K\pi}(s)$ is related to scalar form factor $F_0^{K\pi}(s)$ by $F_{K\pi}(s) = \frac{B_0}{m_{K_0^*}} F_0^{K\pi}(s)$.

The distribution amplitudes for B meson and the bachelor final state h in this work are the same as those widely employed in the studies of the hadronic B meson decays in the PQCD approach, one can find their expressions and parameters in the appendix.

3 Results and discussions

In the numerical calculation, we adopt the decay constants $f_B = 0.189\,\text{GeV}$, $f_{B_s} = 0.231\,\text{GeV}$ [144], the mean lifetimes $\tau_{B^0} = (1.520 \pm 0.004) \times 10^{-12}$ s, $\tau_{B^+} = (1.638 \pm 0.004) \times 10^{-12}$ s and $\tau_{B^0_s} = (1.509 \pm 0.004) \times 10^{-12}$ s [93] for the B^0, B^+ and B^0_s mesons, respectively. The masses and the decay constants for the relevant particles in the numerical calculation in this work, the full widths for $K_0^*(1430)$ and $K_0^*(1950)$, and the Wolfenstein parameters of the CKM matrix are presented in table 1.

Utilizing the differential branching fraction eq. (A.8) and the decay amplitudes collected in appendix A, we obtain the CP averaged branching fractions ($\mathcal{B}$) and the direct CP asymmetries ($\mathcal{A}_{\rm CP}$) in table 2 and table 3 for the concerned quasi-two-body decay processes involving the resonances $K_0^*(1430)$ and $K_0^*(1950)$ as the intermediate states, respectively. The results for those quasi-two-body decays with one daughter of the K_0^* is the neutral pion are omitted. One will get a half value of the $\mathcal{B}$ and the same value of the $\mathcal{A}_{\rm CP}$ of the corresponding result in tables 2, 3 for a decay with the subprocesses $K_0^* \to K\pi^0$ considering the isospin relation. For example, we have

$$\mathcal{B}(B^+ \to K_0^*(1430)^+\pi^0 \to K^+\pi^0\pi^0) = \frac{1}{2}\mathcal{B}(B^+ \to K_0^*(1430)^+\pi^0 \to K^0\pi^+\pi^0), \tag{3.1}$$

while these two processes have the same direct CP asymmetry.

For the PQCD predictions in tables 2, 3, the shape parameters $\omega_B = 0.40 \pm 0.04$ or $\omega_{B_s} = 0.50 \pm 0.05$ in eq. (A.3) for the $B^{+,0}$ or B^0_s contribute the first error. The second error for each PQCD result comes from the Gegenbauer moments a_1 and a_3 in the eq. (2.10). The

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Decay modes	Quasi-two-body results	
$B^+ \to K_0^*(1430)^0\pi^+ \to K^+\pi^-\pi^+$	$\mathcal{B}(10^{-5})$	$2.27 \pm 0.59(\omega_B) \pm 0.17(a_{3+1}) \pm 0.34(m_0^\pi{+}a_2^\pi)$
	$\mathcal{A}_{\rm CP}(\%)$	$-1.3 \pm 0.2(\omega_B) \pm 0.4(a_{3+1}) \pm 0.2(m_0^\pi{+}a_2^\pi)$
$B^+ \to K_0^*(1430)^+\pi^0 \to K^0\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$7.86 \pm 2.16(\omega_B) \pm 0.55(a_{3+1}) \pm 1.36(m_0^\pi{+}a_2^\pi)$
	$\mathcal{A}_{\rm CP}(\%)$	$1.5 \pm 0.4(\omega_B) \pm 0.8(a_{3+1}) \pm 0.4(m_0^\pi{+}a_2^\pi)$
$B^+ \to K_0^*(1430)^+\bar{K}^0 \to K^0\pi^+\bar{K}^0$	$\mathcal{B}(10^{-7})$	$2.33 \pm 0.04(\omega_B) \pm 1.29(a_{3+1}) \pm 0.34(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	$-18.4 \pm 5.8(\omega_B) \pm 2.7(a_{3+1}) \pm 5.4(m_0^K{+}a_2^K)$
$B^+ \to \bar{K}_0^*(1430)^0K^+ \to K^-\pi^+K^+$	$\mathcal{B}(10^{-6})$	$2.86 \pm 0.54(\omega_B) \pm 0.51(a_{3+1}) \pm 0.42(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	$17.9 \pm 0.4(\omega_B) \pm 8.0(a_{3+1}) \pm 0.9(m_0^K{+}a_2^K)$
$B^0 \to K_0^*(1430)^+\pi^- \to K^0\pi^+\pi^-$	$\mathcal{B}(10^{-5})$	$2.07 \pm 0.54(\omega_B) \pm 0.14(a_{3+1}) \pm 0.30(m_0^\pi{+}a_2^\pi)$
	$\mathcal{A}_{\rm CP}(\%)$	$0.3 \pm 0.5(\omega_B) \pm 0.8(a_{3+1}) \pm 0.1(m_0^\pi{+}a_2^\pi)$
$B^0 \to K_0^*(1430)^0\pi^0 \to K^+\pi^-\pi^0$	$\mathcal{B}(10^{-5})$	$1.39 \pm 0.35(\omega_B) \pm 0.11(a_{3+1}) \pm 0.18(m_0^\pi{+}a_2^\pi)$
	$\mathcal{A}_{\rm CP}(\%)$	$-1.8 \pm 0.4(\omega_B) \pm 0.2(a_{3+1}) \pm 0.1(m_0^\pi{+}a_2^\pi)$
$B^0 \to K_0^*(1430)^+K^- \to K^0\pi^+K^-$	$\mathcal{B}(10^{-8})$	$5.77 \pm 2.38(\omega_B) \pm 2.92(a_{3+1}) \pm 0.62(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	$4.9 \pm 6.4(\omega_B) \pm 3.7(a_{3+1}) \pm 3.6(m_0^K{+}a_2^K)$
$B^0 \to K_0^*(1430)^-K^+ \to \bar{K}^0\pi^-K^+$	$\mathcal{B}(10^{-7})$	$3.84 \pm 1.48(\omega_B) \pm 1.95(a_{3+1}) \pm 0.09(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	$-5.0 \pm 2.6(\omega_B) \pm 6.7(a_{3+1}) \pm 3.0(m_0^K{+}a_2^K)$
$B^0 \to K_0^*(1430)^0\bar{K}^0 \to K^+\pi^-\bar{K}^0$	$\mathcal{B}(10^{-7})$	$3.04 \pm 0.15(\omega_B) \pm 2.04(a_{3+1}) \pm 0.36(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	—
$B^0 \to \bar{K}_0^*(1430)^0K^0 \to K^-\pi^+K^0$	$\mathcal{B}(10^{-6})$	$2.89 \pm 0.53(\omega_B) \pm 0.65(a_{3+1}) \pm 0.41(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	—
$B_s^0 \to K_0^*(1430)^-\pi^+ \to \bar{K}^0\pi^-\pi^+$	$\mathcal{B}(10^{-5})$	$3.77 \pm 0.78(\omega_B) \pm 0.51(a_{3+1}) \pm 0.01(m_0^\pi{+}a_2^\pi)$
	$\mathcal{A}_{\rm CP}(\%)$	$15.5 \pm 1.6(\omega_B) \pm 3.2(a_{3+1}) \pm 1.0(m_0^\pi{+}a_2^\pi)$
$B_s^0 \to \bar{K}_0^*(1430)^0\pi^0 \to K^-\pi^+\pi^0$	$\mathcal{B}(10^{-7})$	$5.03 \pm 0.38(\omega_B) \pm 1.52(a_{3+1}) \pm 0.80(m_0^\pi{+}a_2^\pi)$
	$\mathcal{A}_{\rm CP}(\%)$	$59.2 \pm 3.2(\omega_B) \pm 7.1(a_{3+1}) \pm 2.5(m_0^\pi{+}a_2^\pi)$
$B_s^0 \to K_0^*(1430)^+K^- \to K^0\pi^+K^-$	$\mathcal{B}(10^{-5})$	$1.44 \pm 0.20(\omega_B) \pm 0.26(a_{3+1}) \pm 0.25(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	$0.4 \pm 0.3(\omega_B) \pm 1.8(a_{3+1}) \pm 1.2(m_0^K{+}a_2^K)$
$B_s^0 \to K_0^*(1430)^-K^+ \to \bar{K}^0\pi^-K^+$	$\mathcal{B}(10^{-5})$	$1.74 \pm 0.16(\omega_B) \pm 0.84(a_{3+1}) \pm 0.24(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	$-51.1 \pm 1.1(\omega_B) \pm 6.7(a_{3+1}) \pm 5.5(m_0^K{+}a_2^K)$
$B_s^0 \to K_0^*(1430)^0\bar{K}^0 \to K^+\pi^-\bar{K}^0$	$\mathcal{B}(10^{-5})$	$1.47 \pm 0.22(\omega_B) \pm 0.24(a_{3+1}) \pm 0.25(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	—
$B_s^0 \to \bar{K}_0^*(1430)^0K^0 \to K^-\pi^+K^0$	$\mathcal{B}(10^{-5})$	$1.19 \pm 0.06(\omega_B) \pm 0.71(a_{3+1}) \pm 0.17(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	—

Table 2. PQCD predictions of the CP averaged branching fractions and the direct CP asymmetries for the quasi-two-body $B \to K_0^*(1430)h \to K\pi h$ decays.

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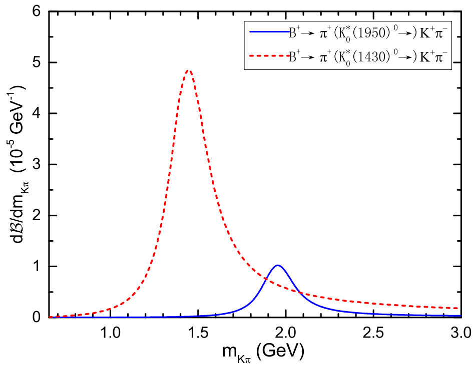

Figure 2. Differential branching fractions from threshold of $K\pi$ pair to 3 GeV for the $B^+ \to K_0^*(1430)^0\pi^+ \to K^+\pi^-\pi^+$ and $B^+ \to K_0^*(1950)^0\pi^+ \to K^+\pi^-\pi^+$ decays.

third one is induced by the chiral masses m_0^h and the Gegenbauer moment $a_2^h = 0.25 \pm 0.15$ of the bachelor final state pion or kaon. The large uncertainties of the decay widths of the states $K_0^*(1430)$ and $K_0^*(1950)$ in the table 1 result in quite small errors, which have been neglected, for these quasi-two-body predictions in the tables 2 and 3. The reason is that the variation effect of decay width Γ in the denominator $\mathcal{D}_{K_0^*}$ of eq. (2.6) will be mainly canceled out by the uncertainty of $\Gamma_{K_0^* \to K\pi}$ (equals to $\Gamma \cdot \mathcal{B}(K_0^* \to K\pi)$) in the numerator $g_{K_0^* K\pi}$. For instance, the corresponding errors for the decay process $B^+ \to K_0^*(1430)^0\pi^+ \to K^+\pi^-\pi^+$ are 0.04×10^{-5} and 0.2% for its branching fraction and direct CP asymmetry, respectively, while for $B^+ \to K_0^*(1950)^0\pi^+ \to K^+\pi^-\pi^+$, the two errors are 0.01×10^{-6} and 0.1%. There are other errors, which come from the uncertainties of the Wolfenstein parameters of the CKM matrix, the parameters in the distribution amplitudes for bachelor pion or kaon, the masses and the decay constants of the initial and final states, etc. are small and have been neglected. One can find that for those decay modes with the main contributions come from the annihilation diagrams of figure 1, their branching fraction errors generated from the variations of the a_1 and a_3 could be larger than the corresponding errors from ω_B or ω_{B_s}, because there is no shape parameter for B meson in the factorizable annihilation diagrams.

In this work, the branching fraction of a quasi-two-body decay process involving the resonant state $K_0^*(1950)$ is predicted to be roughly one order smaller than the corresponding decay mode with the resonance $K_0^*(1430)$. Or rather, the ratios between the CP averaged branching fractions for the decays in table 3 and table 2 with (without) the factorizable emission diagrams of figure 1 (c) are about 12%-15% (6%-9%). The difference mainly originated from the $(S-P)(S+P)$ amplitude the eq. (A.43), which has the intermediate state invariant mass factor $m_B\sqrt{\zeta}$ ($\equiv \sqrt{s}$). This factor makes the proportion originated from eq. (A.43) in the total branching ratio for a quasi-two-body decay mode invloving $K_0^*(1950)$ larger than that of the corresponding decay process including $K_0^*(1430)$ because of the larger pole mass of the resonance $K_0^*(1950)$. Take the decays $B^+ \to K_0^*(1430, 1950)^0\pi^+ \to K^+\pi^-\pi^+$ as the examples, when we neglect the contribution from the $(S-P)(S+P)$ amplitude of the eq. (A.43), the ratio between two branching fractions of the decays $B^+ \to K_0^*(1950)^0\pi^+ \to K^+\pi^-\pi^+$ and $B^+ \to K_0^*(1430)^0\pi^+ \to K^+\pi^-\pi^+$

will drop to 8% from about 15%. From the lines of the differential branching fractions for $B^+ \to K_0^*(1950)^0\pi^+ \to K^+\pi^-\pi^+$ and $B^+ \to K_0^*(1430)^0\pi^+ \to K^+\pi^-\pi^+$ in figure 2, one can find that the main portion of the branching fractions lies in the region around the corresponding pole mass of the intermediate states.

We must stress that the ratios between the corresponding branching fractions in table 3 and table 2, and also the branching fractions in table 3 for the quasi-two-body decays involving $K_0^*(1950)$ are squared dependent on the result $f_{K_0^*(1950)} m^2_{K_0^*(1950)} = 0.0414\,\mathrm{GeV}^3$ [141]. If the value 0.0414 becomes two times larger, the ratios and the branching fractions in table 3 will become four times larger than their current values. In ref. [112], there are two branching fractions measured by LHCb to be

$$\mathcal{B}(B^0 \to \eta_c K_0^*(1950)^0 \to \eta_c K^+\pi^-) = (2.18 \pm 1.04 \pm 0.04^{+0.80}_{-1.43} \pm 0.25) \times 10^{-5}, \quad (3.2)$$

$$\mathcal{B}(B^0 \to \eta_c K_0^*(1430)^0 \to \eta_c K^+\pi^-) = (14.50 \pm 2.10 \pm 0.28^{+2.01}_{-1.60} \pm 1.67) \times 10^{-5}. \quad (3.3)$$

The two central values above give us the ratio about 0.15 between these two branching factions, but there is no diagrams like figure 1 (c) for $B^0 \to \eta_c K_0^{*0}$ decays. Because of the large errors for $B^0 \to \eta_c K_0^*(1950)^0$, we can not extract the decay constant $f_{K_0^*(1950)}$ from this measurement. While from the data of the fit fractions for $\eta_c \to K_S^0 K^\pm \pi^\mp$ in [86] and $\eta_c \to K^+K^-\pi^0$ in [92] both from BaBar, one can expect a larger value than $0.0414\,\mathrm{GeV}^3$ for the $f_{K_0^*(1950)} m^2_{K_0^*(1950)}$.

The two-body branching fractions for $B \to K_0^* h$ can be extracted from the quasi-two-body predictions of this work with the relation

$$\Gamma(B \to K_0^* h \to K\pi h) = \Gamma(B \to K_0^* h) \times \mathcal{B}(K_0^* \to K\pi). \quad (3.4)$$

In ref. [145], a parameter η was defined to measure the violation of the factorization relation the eq. (3.4) in the D meson decays. For the $B \to K_0^*(1430)h$ and $B \to K_0^*(1430)h \to K\pi h$ decays, we have

$$\begin{aligned}\eta &= \frac{\Gamma(B \to K_0^*(1430)h \to K\pi h)}{\Gamma(B \to K_0^*(1430)h) \times \mathcal{B}(K_0^* \to K\pi)} \\ &\approx \frac{m^2_{K_0^*(1430)}}{4\pi m_B} \frac{\Gamma_{K_0^*(1430)}}{\hat{q}_h q_0} \int_{(m_K+m_\pi)^2}^{(m_B-m_h)^2} \frac{ds}{s} \frac{\lambda^{1/2}(m_B^2, s, m_h^2)\lambda^{1/2}(s, m_K^2, m_\pi^2)}{(s - m^2_{K_0^*(1430)})^2 + (m_{K_0^*(1430)}\Gamma_{K_0^*}(s))^2}, \end{aligned} \quad (3.5)$$

where $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$, the $\hat{q}_h$ is the expression of eq. (A.9) in the rest frame of B meson and fixed at $s = m^2_{K_0^*(1430)}$. With eq. (3.5), we have $\eta = 0.90$ for the decays $B^+ \to K_0^*(1430)^0\pi^+$, which means the violation of the factorization relation is not large when neglecting the effect of the invariant mass s in the decay amplitudes of the quasi-two-body decays. In order to check this conclusion, we calculate the decay $B^+ \to K_0^*(1430)^0\pi^+$ in the two-body framework of the PQCD approach, and we have $\mathcal{B}(B^+ \to K_0^*(1430)^0\pi^+) = 35.2 \times 10^{-6}$, which is about 96.2% of the result in table 4 extracted with eq. (3.4), and $\mathcal{A}_{\rm CP}(B^+ \to K_0^*(1430)^0\pi^+) = -1.0\%$ is consistent with the -1.3% in table 2.

The comparison of the PQCD branching fractions with the experimental measurements for the two-body decays $B^+ \to K_0^*(1430)^0\pi^+$, $B^+ \to K_0^*(1430)^+\pi^0$ and $B^0 \to$

Decay modes	Quasi-two-body results	
$B^+ \to K_0^*(1950)^0\pi^+ \to K^+\pi^-\pi^+$	$\mathcal{B}(10^{-6})$	$3.36 \pm 0.86(\omega_B) \pm 0.24(a_{3+1}) \pm 0.51(m_0^\pi{+}a_2^\pi)$
	$\mathcal{A}_{\rm CP}(\%)$	$1.5 \pm 0.3(\omega_B) \pm 0.2(a_{3+1}) \pm 0.3(m_0^\pi{+}a_2^\pi)$
$B^+ \to K_0^*(1950)^+\pi^0 \to K^0\pi^+\pi^0$	$\mathcal{B}(10^{-6})$	$1.19 \pm 0.32(\omega_B) \pm 0.08(a_{3+1}) \pm 0.21(m_0^\pi{+}a_2^\pi)$
	$\mathcal{A}_{\rm CP}(\%)$	$3.5 \pm 0.1(\omega_B) \pm 0.4(a_{3+1}) \pm 0.2(m_0^\pi{+}a_2^\pi)$
$B^+ \to K_0^*(1950)^+\bar{K}^0 \to K^0\pi^+\bar{K}^0$	$\mathcal{B}(10^{-8})$	$1.86 \pm 0.04(\omega_B) \pm 0.60(a_{3+1}) \pm 0.38(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	$-9.2 \pm 5.3(\omega_B) \pm 4.0(a_{3+1}) \pm 2.8(m_0^K{+}a_2^K)$
$B^+ \to \bar{K}_0^*(1950)^0K^+ \to K^-\pi^+K^+$	$\mathcal{B}(10^{-7})$	$3.59 \pm 0.66(\omega_B) \pm 0.54(a_{3+1}) \pm 0.54(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	$19.2 \pm 0.1(\omega_B) \pm 7.4(a_{3+1}) \pm 1.4(m_0^K{+}a_2^K)$
$B^0 \to K_0^*(1950)^+\pi^- \to K^0\pi^+\pi^-$	$\mathcal{B}(10^{-6})$	$2.99 \pm 0.77(\omega_B) \pm 0.20(a_{3+1}) \pm 0.45(m_0^\pi{+}a_2^\pi)$
	$\mathcal{A}_{\rm CP}(\%)$	$1.9 \pm 0.5(\omega_B) \pm 0.5(a_{3+1}) \pm 0.1(m_0^\pi{+}a_2^\pi)$
$B^0 \to K_0^*(1950)^0\pi^0 \to K^+\pi^-\pi^0$	$\mathcal{B}(10^{-6})$	$2.01 \pm 0.50(\omega_B) \pm 0.15(a_{3+1}) \pm 0.26(m_0^\pi{+}a_2^\pi)$
	$\mathcal{A}_{\rm CP}(\%)$	$0.4 \pm 0.6(\omega_B) \pm 0.3(a_{3+1}) \pm 0.3(m_0^\pi{+}a_2^\pi)$
$B^0 \to K_0^*(1950)^+K^- \to K^0\pi^+K^-$	$\mathcal{B}(10^{-9})$	$5.14 \pm 1.90(\omega_B) \pm 1.66(a_{3+1}) \pm 0.29(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	$-2.8 \pm 10(\omega_B) \pm 10.6(a_{3+1}) \pm 3.3(m_0^K{+}a_2^K)$
$B^0 \to K_0^*(1950)^-K^+ \to \bar{K}^0\pi^-K^+$	$\mathcal{B}(10^{-8})$	$2.36 \pm 0.95(\omega_B) \pm 1.10(a_{3+1}) \pm 0.06(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	$-1.0 \pm 2.4(\omega_B) \pm 8.5(a_{3+1}) \pm 1.3(m_0^K{+}a_2^K)$
$B^0 \to K_0^*(1950)^0\bar{K}^0 \to K^+\pi^-\bar{K}^0$	$\mathcal{B}(10^{-8})$	$2.22 \pm 0.08(\omega_B) \pm 1.05(a_{3+1}) \pm 0.35(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	—
$B^0 \to \bar{K}_0^*(1950)^0K^0 \to K^-\pi^+K^0$	$\mathcal{B}(10^{-7})$	$3.36 \pm 0.64(\omega_B) \pm 0.58(a_{3+1}) \pm 0.48(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	—
$B_s^0 \to K_0^*(1950)^-\pi^+ \to \bar{K}^0\pi^-\pi^+$	$\mathcal{B}(10^{-6})$	$3.35 \pm 0.59(\omega_B) \pm 0.37(a_{3+1}) \pm 0.01(m_0^\pi{+}a_2^\pi)$
	$\mathcal{A}_{\rm CP}(\%)$	$12.9 \pm 7.0(\omega_B) \pm 3.1(a_{3+1}) \pm 0.8(m_0^\pi{+}a_2^\pi)$
$B_s^0 \to \bar{K}_0^*(1950)^0\pi^0 \to K^-\pi^+\pi^0$	$\mathcal{B}(10^{-8})$	$3.74 \pm 0.35(\omega_B) \pm 1.01(a_{3+1}) \pm 0.48(m_0^\pi{+}a_2^\pi)$
	$\mathcal{A}_{\rm CP}(\%)$	$57.1 \pm 4.0(\omega_B) \pm 8.1(a_{3+1}) \pm 5.5(m_0^\pi{+}a_2^\pi)$
$B_s^0 \to K_0^*(1950)^+K^- \to K^0\pi^+K^-$	$\mathcal{B}(10^{-6})$	$2.03 \pm 0.31(\omega_B) \pm 0.19(a_{3+1}) \pm 0.32(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	$0.6 \pm 0.2(\omega_B) \pm 1.0(a_{3+1}) \pm 0.9(m_0^K{+}a_2^K)$
$B_s^0 \to K_0^*(1950)^-K^+ \to \bar{K}^0\pi^-K^+$	$\mathcal{B}(10^{-6})$	$1.26 \pm 0.15(\omega_B) \pm 0.54(a_{3+1}) \pm 0.20(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	$-45.1 \pm 1.3(\omega_B) \pm 4.6(a_{3+1}) \pm 5.7(m_0^K{+}a_2^K)$
$B_s^0 \to K_0^*(1950)^0\bar{K}^0 \to K^+\pi^-\bar{K}^0$	$\mathcal{B}(10^{-6})$	$2.13 \pm 0.33(\omega_B) \pm 0.19(a_{3+1}) \pm 0.33(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	—
$B_s^0 \to \bar{K}_0^*(1950)^0K^0 \to K^-\pi^+K^0$	$\mathcal{B}(10^{-7})$	$7.65 \pm 0.54(\omega_B) \pm 4.56(a_{3+1}) \pm 1.48(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	—

Table 3. PQCD predictions of the CP averaged branching fractions and the direct CP asymmetries for the quasi-two-body $B \to K_0^*(1950)h \to K\pi h$ decays.

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Two-body decays	This work	Data	Ref.
$B^+ \to K_0^*(1430)^0\pi^+$	$36.6 \pm 11.3 \pm 3.9$	$34.6 \pm 3.3 \pm 4.2^{+1.9}_{-1.8}$	BaBar [43]
		$32.0 \pm 1.2 \pm 2.7^{+9.1}_{-1.4} \pm 5.2$	BaBar [38]
		$51.6 \pm 1.7 \pm 6.8^{+1.8}_{-3.1}$	Belle [30]
$B^+ \to K_0^*(1430)^+\pi^0$	$12.7 \pm 4.2 \pm 1.4$	$11.9 \pm 1.7 \pm 1.0^{+0.0}_{-1.3}$	BaBar [43]
$B^0 \to K_0^*(1430)^+\pi^-$	$33.4 \pm 10.2 \pm 3.6$	$29.9^{+2.3}_{-1.7} \pm 1.6 \pm 0.6 \pm 3.2$	BaBar [40]
		$49.7 \pm 3.8 \pm 6.7^{+1.2}_{-4.8}$	Belle [31]

Table 4. Comparison of the extracted predictions with the experimental measurements for the relevant two-body branching fractions (in units of 10^{-6}). The first error for the theoretical results is added in quadrature from the errors in table 2, the second error comes from the uncertainty of $\mathcal{B}(K_0^*(1430) \to K\pi) = 0.93 \pm 0.10$ [93].

$K_0^*(1430)^+\pi^-$ are shown in the table 4, with the first error added in quadrature from the errors in table 2 and the second error comes from the uncertainty of $\mathcal{B}(K_0^*(1430) \to K\pi) = 0.93 \pm 0.10$ [93] for these theoretical results. The branching fraction and direct CP asymmetry for $B^+ \to K_0^*(1430)^0\pi^+$ in *Review of Particle Physics* [93] averaged from the results in [30, 38, 43] are $39^{+6}_{-5} \times 10^{-6}$ and 0.061 ± 0.032, respectively, which are consistent with the predictions $(36.6 \pm 11.3 \pm 3.9) \times 10^{-6}$ in table 4 and $(-1.3 \pm 0.5)\%$ in table 2. Because of the large uncertainty of the $\mathcal{A}_{\rm CP} = 0.26^{+0.18}_{-0.14}$ for $B^+ \to K_0^*(1430)^+\pi^0$ in [93], we can not evaluate the significance of the prediction $(1.5 \pm 1.0)\%$, but our branching fraction agrees very well with BaBar's result in [43] for this decay mode. For the decay $B^0 \to K_0^*(1430)^+\pi^-$, one has two results as listed in table 4 from BaBar and Belle Collaborations, its average $\mathcal{B}$ is presented to be $(33 \pm 7) \times 10^{-6}$ in *Review of Particle Physics* [93], this value agrees well with the PQCD prediction $(33.4 \pm 10.2 \pm 3.6) \times 10^{-6}$. There is an upper limit of 2.2×10^{-6} for the decay $B^+ \to \bar{K}_0^*(1430)^0 K^+$, which is below our expectation. Our predictions in this work will be tested by future experiments. In the very recent work, LHCb Collaboration presented the branching fractions for the combined decays $B_s^0 \to \overset{(-)}{K}{}^0\pi^\pm K^\mp$ as [51]

$$\mathcal{B}(B_s^0 \to K_0^*(1430)^\pm K^\mp \to \overset{(-)}{K}{}^0\pi^\pm K^\mp)$$
$$= (19.4 \pm 1.4 \pm 0.4 \pm 15.6 \pm 2.0 \pm 0.3) \times 10^{-6}, \quad (3.6)$$
$$\mathcal{B}(B_s^0 \to \overset{(-)}{K}{}^*_0(1430)^0 \overset{(-)}{K}{}^0 \to K^\mp\pi^\pm \overset{(-)}{K}{}^0)$$
$$= (20.5 \pm 1.6 \pm 0.6 \pm 5.7 \pm 2.2 \pm 0.3) \times 10^{-6}, \quad (3.7)$$

which are in agreement with the PQCD predictions in table 5.

On the experimental side, the LASS parametrization [36, 81]

$$R(s) = \frac{\sqrt{s}}{q\cot\delta_B - iq} + e^{2i\delta_B}\frac{m_0\Gamma_0\frac{m_0}{q_0}}{m_0^2 - s - im_0\Gamma_0\frac{q}{m}\frac{m_0}{q_0}}\,, \quad (3.8)$$

are employed in most cases to describe the S-wave $K\pi$ system, where m_0 and Γ_0 are now the pole mass and full width for $K_0^*(1430)$, and $\cot\delta_B = \frac{1}{aq} + \frac{1}{2}rq$ with the parameters $a = 2.07 \pm 0.10\,\text{GeV}^{-1}$ and $r = 3.32 \pm 0.34\,\text{GeV}^{-1}$ [36]. The relativistic Breit-Wigner term

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Decay modes	Quasi-two-body results	
$B_s^0 \to K_0^*(1430)^\pm K^\mp \to \overset{(-)}{K}{}^0\pi^\pm K^\mp$	$\mathcal{B}(10^{-5})$	$1.97 \pm 0.45(\omega_B) \pm 0.10(a_{3+1}) \pm 0.43(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	$-7.7 \pm 1.5(\omega_B) \pm 1.3(a_{3+1}) \pm 4.3(m_0^K{+}a_2^K)$
$B_s^0 \to \overset{(-)}{K}{}^*_0(1430)^0\overset{(-)}{K}{}^0 \to K^\mp\pi^\pm\overset{(-)}{K}{}^0$	$\mathcal{B}(10^{-5})$	$1.50 \pm 0.36(\omega_B) \pm 0.09(a_{3+1}) \pm 0.40(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	—
$B_s^0 \to K_0^*(1950)^\pm K^\mp \to \overset{(-)}{K}{}^0\pi^\pm K^\mp$	$\mathcal{B}(10^{-6})$	$3.20 \pm 0.69(\omega_B) \pm 0.21(a_{3+1}) \pm 0.62(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	$3.1 \pm 0.2(\omega_B) \pm 2.9(a_{3+1}) \pm 1.6(m_0^K{+}a_2^K)$
$B_s^0 \to \overset{(-)}{K}{}^*_0(1950)^0\overset{(-)}{K}{}^0 \to K^\mp\pi^\pm\overset{(-)}{K}{}^0$	$\mathcal{B}(10^{-6})$	$2.67 \pm 0.59(\omega_B) \pm 0.25(a_{3+1}) \pm 0.61(m_0^K{+}a_2^K)$
	$\mathcal{A}_{\rm CP}(\%)$	—

Table 5. PQCD predictions of the CP averaged branching fractions and the direct CP asymmetries for the combined decays $B_s^0 \to K\pi K$, with the resonances $K_0^*(1430)$ and $K_0^*(1950)$ as the intermediate states.

of eq. (3.8) is different from eq. (2.6). Before the $F_{K\pi}(s)$ in eqs. (2.10)–(2.11) be replaced by the LASS expression, a coefficient is needed for $R(s)$. We have the replacement

$$F_{K\pi}(s) \to \hat{R}(s) = \frac{q_0}{m_0^2\Gamma_0} g_{K_0^*(1430)K\pi} \bar{f}_{K_0^*(1430)} R(s) \tag{3.9}$$

on the theoretical side. With $\hat{R}(s)$ in the concerned quasi-two-body decay amplitudes, one could in principle have the predictions for the decays $B \to (K\pi)_0^* h$, including the results as same as the values in the table 2 for the resonance $K_0^*(1430)$ and the contributions from the nonresonant effective range term. But we argue that, considering the nonresonant term of a three-body decay amplitude should not be included in the resonance distribution amplitudes the eq. (2.9), it's improper for the effective range term of the eq. (3.8) to be studied in the quasi-two-body framework with the same expressions of the decay amplitudes in appendix A.

The two-body decays $B^+ \to K_0^*(1430)^0\pi^+$, $B^+ \to K_0^*(1430)^+\pi^0$, $B^0 \to K_0^*(1430)^+\pi^-$ and $B^0 \to K_0^*(1430)^0\pi^0$ have been studied in ref. [113] and updated in [142] in the QCDF with $K_0^*(1430)$ being the first excited states of $K_0^*(700)$ (scenario 1) or the lowest lying scalar state (scenario 2), and in the scenario 2 $K_0^*(700)$ is treated as a four-quark state. In view of the discussions for $K_0^*(700)$ in [100–106], we will consider only the results for the $K_0^*(1430)$ in the scenario 2 in this work. The branching fractions in [113, 142] for the four decays invloving the $K_0^*(1430)$ are all smaller when comparing with the measurements and our results but with quite large errors as shown in table 6. The difference between our predictions and the concerned results in [113, 142] may be partly due to the dynamical enhancement of penguin contributions in PQCD approach as discussed in detail in [107, 108, 146], and also due to the small values for the decay constant $f_{K_0^*(1430)}$ and the $B \to K_0^*(1430)$ and $B \to \pi$ transition form factors $F_{0,1}^{BK_0^*}$ and $F_{0,1}^{B\pi}$ in [113, 142]. The value $f_{K_0^*(1430)} = 34\,\mathrm{MeV}$ [113] will make our branching ratios involving $K_0^*(1430)$ 1.5 times smaller than the results in table 6. With the parameters in this work, the form

Two-body decays	This work	Theory	ref.
$B^+ \to K_0^*(1430)^0\pi^+$	$36.6 \pm 11.3 \pm 3.9$	$11.0^{+10.3+7.5+49.9}_{-6.0-3.5-10.1}$	[113]
		$12.9^{+4.6+4.1+38.5}_{-3.7-3.4-9.1}$	[142]
		$47.6^{+11.3+3.7+6.9}_{-10.1-3.6-5.1}$	[147]
$B^+ \to K_0^*(1430)^+\pi^0$	$12.7 \pm 4.2 \pm 1.4$	$5.3^{+4.7+1.6+22.3}_{-2.8-1.7-4.7}$	[113]
		$7.4^{+2.4+2.1+20.1}_{-1.9-1.8-5.0}$	[142]
		$28.8^{+6.8+1.9+3.2}_{-6.1-1.9-3.5}$	[147]
$B^0 \to K_0^*(1430)^+\pi^-$	$33.4 \pm 10.2 \pm 3.6$	$11.3^{+9.4+3.7+45.8}_{-5.8-3.7-9.9}$	[113]
		$13.8^{+4.5+4.1+38.3}_{-3.6-3.5-9.5}$	[142]
		$43.0^{+10.2+3.1+7.0}_{-9.1-2.9-5.2}$	[147]
$B^0 \to K_0^*(1430)^0\pi^0$	$22.4 \pm 6.6 \pm 2.4$	$6.4^{+5.4+2.2+26.1}_{-3.3-2.1-5.7}$	[113]
		$5.6^{+2.6+2.4+18.8}_{-1.3-1.2-3.9}$	[142]
		$18.4^{+4.4+1.5+4.0}_{-3.9-1.4-2.9}$	[147]

Table 6. Comparison of the extracted predictions with the results in literature for the relevant two-body branching fractions (in units of 10^{-6}). The sources of the errors of our results are the same as in table 4.

factors $F_{0,1}^{B\pi}(0) = 0.26$ and $F_{0,1}^{BK_0^*(1430)}(0) = 0.42$ could be induced in the PQCD approach. The value 0.26 is close to 0.25 for $F_{0,1}^{B\pi}(0)$ in [113, 142], but the 0.42 for $F_{0,1}^{BK_0^*}(0)$ is two times larger than the value 0.21 in refs. [113, 142]. In the PQCD approach, the two-body decays $B \to K_0^*(1430)\pi$ were studied in [147], with the branching fractions larger than the corresponding results of this work except the decay $B^0 \to K_0^*(1430)^0\pi^0$ which is $18.4^{+4.4+1.5+4.0}_{-3.9-1.4-2.9} \times 10^{-6}$ in [147] as listed in table 6. The result $28.8^{+6.8+1.9+3.2}_{-6.1-1.9-3.5} \times 10^{-6}$ in [147] is about double of our prediction and BaBar's measurement [43] for the decay $B^+ \to K_0^*(1430)^+\pi^0$. The difference between the results in [147] and our predictions could be be explained as the different input parameters. The decays $B \to K_0^*(1430)K$ have been studied in the QCDF in [148]. One can find the comparison of relevant branching fractions in table 7. The $\mathcal{A}_{\rm CP} = -22.51^{+4.90+5.63+19.61}_{-7.57-9.36-22.86}\%$ for the decay $B^+ \to K_0^*(1430)^+\bar{K}^0$ in [148] is consistent with the result $(-18.4 \pm 5.8 \pm 2.7 \pm 5.4)\%$ in table 2, while the $\mathcal{A}_{\rm CP} = -2.60^{+1.61+0.59+3.52}_{-1.76-0.59-5.47}\%$ for $B^+ \to \bar{K}_0^*(1430)^0K^+$ in [148] is smaller than the PQCD prediction $(17.9 \pm 0.4 \pm 8.0 \pm 0.9)\%$ in this work and with an opposite sign.

With $m_{K\pi}$ in the region $(0.64\sim1.76)$ GeV, the branching ratios of the decay processes $B^- \to [\bar{K}_0^*(1430)^0 \to K^-\pi^+]\pi^-$ and $\bar{B}^0 \to [K_0^*(1430)^- \to \bar{K}^0\pi^-]\pi^+$ were calculated in QCDF in ref. [13] with the predictions $(11.6\pm0.6)\times10^{-6}$ and $(11.1\pm0.5)\times10^{-6}$, respectively. These two decays have also been studied in ref. [14] in the $m_{K\pi}$ region $(1.0 \sim 1.76)$ GeV and the branching ratios are $(12.11 \pm 0.32) \times 10^{-6}$ and $(11.05 \pm 0.25) \times 10^{-6}$, respectively. In the PQCD approach we have $(16.6 \pm 5.3) \times 10^{-6}$ and $(15.2 \pm 4.7) \times 10^{-6}$ in the region $m_{K\pi} \in (0.64 \sim 1.76)$ GeV, $(16.4 \pm 5.1) \times 10^{-6}$ and $(15.0 \pm 4.6) \times 10^{-6}$ in the region $m_{K\pi} \in (1.0 \sim 1.76)$ GeV for the branching ratios of the decays $B^+ \to K_0^*(1430)^0\pi^+ \to K^+\pi^-\pi^+$ and $B^0 \to K_0^*(1430)^+\pi^- \to K^0\pi^+\pi^-$, respectively, which are consistent with the results

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Two-body decays	This work	QCDF [148]
$B^+ \to K_0^*(1430)^+ \bar{K}^0$	$3.76 \pm 2.16 \pm 0.40$	$1.14^{+0.54+1.40+1.17}_{-0.38-0.56-0.92}$
$B^+ \to \bar{K}_0^*(1430)^0 K^+$	$39.9 \pm 13.8 \pm 4.3$	$33.70^{+10.33+5.52+3.37}_{-8.47-4.82-3.94}$
$B^0 \to K_0^*(1430)^+ K^-$	$0.93 \pm 0.61 \pm 0.10$	$1.07^{+0.72+0.03+2.27}_{-0.47-0.04-0.97}$
$B^0 \to K_0^*(1430)^- K^+$	$6.19 \pm 3.95 \pm 0.67$	$0.58^{+0.45+0.02+0.14}_{-0.29-0.03-0.05}$
$B^0 \to K_0^*(1430)^0 \bar{K}^0$	$4.90 \pm 3.34 \pm 0.53$	$2.39^{+1.20+1.95+2.67}_{-0.85-0.90-2.00}$
$B^0 \to \bar{K}_0^*(1430)^0 K^0$	$46.1 \pm 15.0 \pm 5.0$	$40.47^{+13.36+6.09+6.06}_{-10.77-5.38-6.16}$

Table 7. Comparison of the extracted predictions with the QCDF results in [148] for the relevant two-body branching fractions (in units of 10^{-7}). The sources of the errors of our results are the same as in table 4.

Decay modes	This work	Theory	ref.
$B^+ \to K_0^*(1430)^0 \pi^+ \to K^+\pi^-\pi^+$	22.7 ± 7.0	$11.3^{+0.0+3.3+0.1}_{-0.0-2.8-0.1}$	[18]
		$11.5^{+0.0+3.3+0.0}_{-0.0-2.8-0.0}$	[21]
$B^+ \to K_0^*(1430)^+ \pi^0 \to K^0\pi^+\pi^0$	7.86 ± 2.61	$5.4^{+0.0+1.6+0.1}_{-0.0-1.4-0.1}$	[18]
		$5.6^{+0.0+1.6+0.0}_{-0.0-1.4-0.0}$	[21]
$B^+ \to \bar{K}_0^*(1430)^0 K^+ \to K^-\pi^+K^+$	2.86 ± 0.85	$1.0^{+0.0+0.2+0.0}_{-0.0-0.2-0.0}$	[18]
		$1.0^{+0.0+0.2+0.0}_{-0.0-0.2-0.0}$	[21]
$B^0 \to K_0^*(1430)^+ \pi^- \to K^0\pi^+\pi^-$	20.7 ± 6.3	$10.3^{+0.0+2.9+0.0}_{-0.0-2.5-0.0}$	[18]
		$10.6^{+0.0+3.0+0.0}_{-0.0-2.6-0.0}$	[21]
$B^0 \to K_0^*(1430)^0 \pi^0 \to K^+\pi^-\pi^0$	13.9 ± 4.1	$4.1^{+0.0+1.4+0.0}_{-0.0-1.2-0.0}$	[18]
		$4.2^{+0.0+1.4+0.0}_{-0.0-1.2-0.0}$	[21]

Table 8. Comparison of the PQCD predictions with the theoretical results for the relevant quasi-two-body branching fractions (in units of 10^{-6}). The errors of this work have been added in quadrature.

in refs. [13, 14] within errors. The three-body decays $B \to K\pi h$ have been discussed in detail in refs. [18, 21] in QCDF. The comparison of PQCD predictions in this work with the related results in [18, 21] are listed in table 8. From table 4 and table 8, one can find that the PQCD predictions are totally larger than the QCDF results [18, 21] but closer to the available data.

There is no direct CP asymmetries for $B^0_{(s)} \to K_0^{*0}\bar{K}^0$ and $B^0_{(s)} \to \bar{K}_0^{*0}K^0$ in tables 2, 3, because these decays have contributions only from the penguin operators in their decay amplitudes. For the decays $B^0 \to K_0^*(1430)^+\pi^- \to K^0\pi^+\pi^-$ and $B^0 \to K_0^*(1430)^0\pi^0 \to K^+\pi^-\pi^0$ via the $b \to sq\bar{q}$ transition at quark level, the very small proportion of the total branching ratio from the current-current operators led to the small direct CP asymmetries for these two decays as shown in table 2. The same pattern will appear again for the decays $B^+ \to K_0^*(1430)^0\pi^+ \to K^+\pi^-\pi^+$ and $B^+ \to K_0^*(1430)^+\pi^0 \to K^0\pi^+\pi^0$, and also for the corresponding decays with the $K_0^*(1430)$ be replaced by the $K_0^*(1950)$

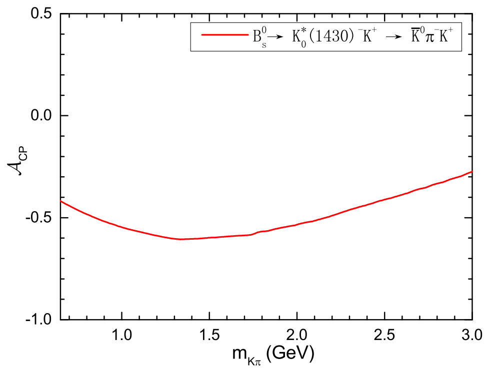

Figure 3. Differential direct CP asymmetry for the decay $B_s^0 \to K_0^*(1430)^- K^+ \to \bar{K}^0\pi^- K^+$.

as the intermediate, but not for the decays $B_s^0 \to K_0^*(1430)^-\pi^+ \to \bar{K}^0\pi^-\pi^+$ and $B_s^0 \to \bar{K}_0^*(1430)^0\pi^0 \to K^-\pi^+\pi^0$ via the $b \to dq\bar{q}$ transition. The interference between the weak and the strong phases of the decay amplitudes from current-current and penguin operators results in the large direct CP asymmetries for the $B_s^0 \to K_0^*(1430)^-\pi^+ \to \bar{K}^0\pi^-\pi^+$ and $B_s^0 \to \bar{K}_0^*(1430)^0\pi^0 \to K^-\pi^+\pi^0$ decays. As an example, we display the differential distribution curve of the $\mathcal{A}_{\rm CP}$ in $m_{K\pi}$ for the decay process $B_s^0 \to K_0^*(1430)^- K^+ \to \bar{K}^0\pi^- K^+$ in figure 3.

For the decays $B^+ \to K_0^*(1430)^0\pi^+$ and $B^0 \to K_0^*(1430)^+\pi^-$, with the isospin limit, one has the ratio [147]

$$R = \frac{\tau_{B^0}}{\tau_{B^+}} \frac{\mathcal{B}(B^+ \to K_0^*(1430)^0\pi^+)}{\mathcal{B}(B^0 \to K_0^*(1430)^+\pi^-)} \approx 1\,. \tag{3.10}$$

With the predictions in table 6, we have the ratio $R = 1.017 \pm 0.003$ in this work. The small error for R is because the cancellation between the errors of two branching ratios, which means the increase or the decrease of the parameters that caused the errors will result in nearly identical change of the weight for the numerator and denominator of R. For the decays $B^+ \to K_0^*(1430)^+\pi^0$ and $B^0 \to K_0^*(1430)^0\pi^0$, the diagrams of figure 1 (a), (c), (d) will contribute to the branching fractions, the decay amplitudes from figure 1 (a) are same for both $B^+ \to K_0^*(1430)^+\pi^0$ and $B^0 \to K_0^*(1430)^0\pi^0$, but the decay amplitudes from figure 1 (c), (d) have the opposite sign considering the difference for $\bar{u}u$ and $\bar{d}d$ to form a neutral pion. It is not strange for the ratio between the branching fractions of $B^+ \to K_0^*(1430)^+\pi^0$ and $B^0 \to K_0^*(1430)^0\pi^0$ away from unity.

A relation for the direct CP asymmetries of the two-body decays $B^+ \to K^+\pi^0$, $B^+ \to K^0\pi^+$, $B^0 \to K^+\pi^-$ and $B^0 \to K^0\pi^0$ was suggested in ref. [149] as

$$\begin{aligned} \mathcal{A}_{\rm CP}(B^+ \to K^+\pi^0)\frac{2\mathcal{B}(B^+ \to K^+\pi^0)}{\mathcal{B}(B^0 \to K^+\pi^-)}\frac{\tau_{B^0}}{\tau_{B^+}} &+ \mathcal{A}_{\rm CP}(B^0 \to K^0\pi^0)\frac{2\mathcal{B}(B^0 \to K^0\pi^0)}{\mathcal{B}(B^0 \to K^+\pi^-)} \\ = \mathcal{A}_{\rm CP}(B^0 \to K^+\pi^-) &+ \mathcal{A}_{\rm CP}(B^+ \to K^0\pi^+)\frac{\mathcal{B}(B^+ \to K^0\pi^+)}{\mathcal{B}(B^0 \to K^+\pi^-)}\frac{\tau_{B^0}}{\tau_{B^+}}. \end{aligned} \tag{3.11}$$

Considering the same transitions at quark level, one could extend the eq. (3.11) to the $B \to K_0^*(1430)\pi$ decays with the replacement $K \to K_0^*(1430)$. This relation is satisfied within

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errors with the $\mathcal{A}_{\rm CP}(B^0 \to K_0^*(1430)^+\pi^-) = (0.3 \pm 0.9)\%$, $\mathcal{A}_{\rm CP}(B^+ \to K_0^*(1430)^0\pi^+) = (-1.3 \pm 0.5)\%$, $\mathcal{A}_{\rm CP}(B^+ \to K_0^*(1430)^+\pi^0) = (1.5 \pm 1.0)\%$ and $\mathcal{A}_{\rm CP}(B^0 \to K_0^*(1430)^0\pi^0) = (-1.8 \pm 0.5)\%$, and relevant branching fractions in table 2. One can find that the relation eq. (3.11) will also hold for $B \to K_0^*(1950)\pi$ decays with the values in table 3.

4 Conclusion

In this work, we studied the contributions from the resonant state $K_0^*(1430)$ and, for the first time, from the resonance $K_0^*(1950)$ in the three-body decays $B \to K\pi h$ in the PQCD approach. The crucial nonperturbative input factor $F_{K\pi}(s)$ in the distribution amplitudes of the S-wave $K\pi$ system was derived from the matrix element of the vacuum to $K\pi$ final state and was related to the scalar time-like form factor $F_0^{K\pi}(s)$ by the relation $F_{K\pi}(s) = B_0/m_{K_0^*} F_0^{K\pi}(s)$. This relation also means that the LASS parametrization for the $(K\pi)_0^*$ system which frequently appeared in the experimental works cannot be adopted directly for the $K\pi$ system distribution amplitudes in the PQCD approach.

With $f_{K_0^*(1430)} m^2_{K_0^*(1430)} = 0.0842 \pm 0.0045\,\text{GeV}^3$ and $f_{K_0^*(1950)} m^2_{K_0^*(1950)} = 0.0414\,\text{GeV}^3$, the branching fractions and the direct CP asymmetries for the concerned quasi-two-body decays $B \to K_0^*(1430, 1950)h \to K\pi h$ were calculated. An important conclusion is that the CP averaged branching fraction of a quasi-two-body process with $K_0^*(1950)$ as the intermediate state is about one order smaller than the corresponding decay mode involving the resonance $K_0^*(1430)$. In view of the important contribution from the S-wave $K\pi$ system for the $B \to K\pi h$ decays, it is not appropriate to neglect the $K_0^*(1950)$ in the theoretical or experimental studies for the relevant three-body B meson decays. We compared our predictions with the related results in literature and found the predictions in this work for the relevant decays agree well with the existing experimental results from BaBar, Belle and LHCb Collaborations.

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A Decay amplitudes

The Lorentz invariant decay amplitude $\mathcal{A}$ for the quasi-two-body decay $B \to K_0^* h \to K\pi h$ in the PQCD approach, according to figure 1, is given by [26, 68]

$$\mathcal{A} = \Phi_B \otimes H \otimes \Phi_h \otimes \Phi_{K\pi} \,. \tag{A.1}$$

The symbol $\otimes$ here means convolutions in parton momenta, the hard kernel H contains one hard gluon exchange at the leading order in strong coupling α_s as in the two-body formalism. The distribution amplitudes Φ_B, Φ_h and $\Phi_{K\pi}$ absorb the nonperturbative dynamics in the relevant decay processes.

The B meson light-cone matrix element can be decomposed as [150–152]

$$\Phi_B = \frac{i}{\sqrt{2N_c}}(\not{p}_B + m_B)\gamma_5\phi_B(k_B), \tag{A.2}$$

where the distribution amplitude ϕ_B is of the form

$$\phi_B(x_B, b_B) = N_B x_B^2(1-x_B)^2 \exp\left[-\frac{(x_B m_B)^2}{2\omega_B^2} - \frac{1}{2}(\omega_B b_B)^2\right], \tag{A.3}$$

with N_B the normalization factor. The shape parameters $\omega_B = 0.40 \pm 0.04\,\mathrm{GeV}$ for B^0 and $B^\pm$, $\omega_{B_s} = 0.50 \pm 0.05$ for B_s^0, respectively.

The light-cone wave functions for pion and kaon are written as [153–156]

$$\Phi_h = \frac{i}{\sqrt{2N_c}}\gamma_5\left[\not{p}_3\phi^A(x_3) + m_0^h\phi^P(x_3) + m_0^h(\not{n}\not{v} - 1)\phi^T(x_3)\right]. \tag{A.4}$$

The distribution amplitudes of $\phi^A(x_3), \phi^P(x_3)$ and $\phi^T(x_3)$ are

$$\phi^A(x_3) = \frac{f_h}{2\sqrt{2N_c}}6x_3(1-x_3)\left[1 + a_1^h C_1^{3/2}(t) + a_2^h C_2^{3/2}(t) + a_4^h C_4^{3/2}(t)\right], \tag{A.5}$$

$$\phi^P(x_3) = \frac{f_h}{2\sqrt{2N_c}}\left[1 + \left(30\eta_3 - \frac{5}{2}\rho_h^2\right)C_2^{1/2}(t) - 3\left[\eta_3\omega_3 + \frac{9}{20}\rho_h^2(1+6a_2^h)\right]C_4^{1/2}(t)\right], \tag{A.6}$$

$$\phi^T(x_3) = \frac{f_h}{2\sqrt{2N_c}}(-t)\left[1 + 6\left(5\eta_3 - \frac{1}{2}\eta_3\omega_3 - \frac{7}{20}\rho_h^2 - \frac{3}{5}\rho_h^2 a_2^h\right)(1 - 10x_3 + 10x_3^2)\right], \tag{A.7}$$

with $t = 2x_3 - 1$, $C_{2,4}^{1/2}(t)$ and $C_{1,2,4}^{3/2}(t)$ are Gegenbauer polynomials. The chiral masses m_0^h for pion and kaon are $m_0^\pi = (1.4 \pm 0.1)\,\mathrm{GeV}$ and $m_0^K = (1.6 \pm 0.1)\,\mathrm{GeV}$ as they in ref. [157]. The Gegenbauer moments $a_1^\pi = 0, a_1^K = 0.06, a_2^h = 0.25, a_4^h = -0.015$ and the parameters $\rho_h = m_h/m_0^h, \eta_3 = 0.015, \omega_3 = -3$ are adopted in the numerical calculation.

For the the differential branching fraction, we have [93]

$$\frac{d\mathcal{B}}{d\zeta} = \tau_B \frac{q_h q}{64\pi^3 m_B}\overline{|\mathcal{A}|^2}, \tag{A.8}$$

The magnitude momentum for the bachelor h is

$$q_h = \frac{1}{2}\sqrt{\left[\left(m_B^2 - m_h^2\right)^2 - 2\left(m_B^2 + m_h^2\right)s + s^2\right]/s} \tag{A.9}$$

in the center-of-mass frame of the K_0^*, where m_h is the mass of the bachelor state. The direct CP asymmetry $\mathcal{A}_{\mathrm{CP}}$ is defined as

$$\mathcal{A}_{\mathrm{CP}} = \frac{\mathcal{B}(\bar{B} \to \bar{f}) - \mathcal{B}(B \to f)}{\mathcal{B}(\bar{B} \to \bar{f}) + \mathcal{B}(B \to f)} \tag{A.10}$$

For errors of the $\mathcal{B}$ and $\mathcal{A}_{\mathrm{CP}}$ induced by the parameter $\mathcal{P} \pm \Delta\mathcal{P}$ in this work, we employ the formulas

$$\Delta\mathcal{B} = \left|\frac{\partial\mathcal{B}}{\partial\mathcal{P}}\right|\Delta\mathcal{P}, \qquad \Delta\mathcal{A}_{\mathrm{CP}} = \left|\frac{\partial\mathcal{A}_{\mathrm{CP}}}{\partial\mathcal{P}}\right|\Delta\mathcal{P} = \frac{2(\mathcal{B}\Delta\overline{\mathcal{B}} - \overline{\mathcal{B}}\Delta\mathcal{B})}{(\overline{\mathcal{B}} + \mathcal{B})^2}. \tag{A.11}$$

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With the subprocesses $K_0^{*+} \to \{K^0\pi^+, \sqrt{2}K^+\pi^0\}$, $K_0^{*0} \to \{K^+\pi^-, \sqrt{2}K^0\pi^0\}$, $K_0^{*-} \to \{\bar{K}^0\pi^-, \sqrt{2}K^-\pi^0\}$ and $\bar{K}_0^{*0} \to \{K^-\pi^+, \sqrt{2}\bar{K}^0\pi^0\}$, and the K_0^* is $K_0^*(1430)$ or $K_0^*(1950)$, the concerned quasi-two-body decay amplitudes are given as follows:

$$\begin{aligned}
\mathcal{A}\left(B^+ \to K_0^{*0}\pi^+\right) = &\frac{G_F}{\sqrt{2}}\Bigg\{V_{ub}^*V_{us}[a_1F_{Ah}^{LL} + C_1M_{Ah}^{LL}] - V_{tb}^*V_{ts}\Bigg[\left(a_4 - \frac{a_{10}}{2}\right)F_{Th}^{LL} \\
&+\left(a_6 - \frac{a_8}{2}\right)F_{Th}^{SP} + \left(C_3 - \frac{C_9}{2}\right)M_{Th}^{LL} + \left(C_5 - \frac{C_7}{2}\right)M_{Th}^{LR} \\
&+(a_4 + a_{10})F_{Ah}^{LL} + (a_6 + a_8)F_{Ah}^{SP} + (C_3 + C_9)M_{Ah}^{LL} \\
&+(C_5 + C_7)M_{Ah}^{LR}\Bigg]\Bigg\}, \qquad (A.12)
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}\left(B^+ \to K_0^{*+}\pi^0\right) = &\frac{G_F}{2}\Bigg\{V_{ub}^*V_{us}[a_2F_{TK_0^*}^{LL} + C_2M_{TK_0^*}^{LL} + a_1(F_{Th}^{LL} + F_{Ah}^{LL}) \\
&+C_1(M_{Th}^{LL} + M_{Ah}^{LL})] - V_{tb}^*V_{ts}\Bigg[\frac{3}{2}(a_9 - a_7)F_{TK_0^*}^{LL} + \frac{3C_{10}}{2}M_{TK_0^*}^{LL} \\
&+\frac{3C_8}{2}M_{TK_0^*}^{SP} + (a_4 + a_{10})(F_{Th}^{LL} + F_{Ah}^{LL}) + (a_6 + a_8)(F_{Th}^{SP} + F_{Ah}^{SP}) \\
&+(C_3 + C_9)(M_{Th}^{LL} + M_{Ah}^{LL}) + (C_5 + C_7)(M_{Th}^{LR} + M_{Ah}^{LR})\Bigg]\Bigg\}, \qquad (A.13)
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}\left(B^+ \to K_0^{*+}\bar{K}^0\right) = &\frac{G_F}{\sqrt{2}}\Bigg\{V_{ub}^*V_{ud}[a_1F_{AK_0^*}^{LL} + C_1M_{AK_0^*}^{LL}] - V_{tb}^*V_{td}\Bigg[\left(a_4 - \frac{a_{10}}{2}\right)F_{TK_0^*}^{LL} \\
&+\left(a_6 - \frac{a_8}{2}\right)F_{TK_0^*}^{SP} + \left(C_3 - \frac{C_9}{2}\right)M_{TK_0^*}^{LL} + \left(C_5 - \frac{C_7}{2}\right)M_{TK_0^*}^{LR} \\
&+(a_4 + a_{10})F_{AK_0^*}^{LL} + (a_6 + a_8)F_{AK_0^*}^{SP} + (C_3 + C_9)M_{AK_0^*}^{LL} \\
&+(C_5 + C_7)M_{AK_0^*}^{LR}\Bigg]\Bigg\}, \qquad (A.14)
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}\left(B^+ \to \bar{K}_0^{*0}K^+\right) = &\frac{G_F}{\sqrt{2}}\Bigg\{V_{ub}^*V_{ud}[a_1F_{Ah}^{LL} + C_1M_{Ah}^{LL}] - V_{tb}^*V_{td}\Bigg[\left(a_4 - \frac{a_{10}}{2}\right)F_{Th}^{LL} \\
&+\left(a_6 - \frac{a_8}{2}\right)F_{Th}^{SP} + \left(C_3 - \frac{C_9}{2}\right)M_{Th}^{LL} + \left(C_5 - \frac{C_7}{2}\right)M_{Th}^{LR} \\
&+(a_4 + a_{10})F_{Ah}^{LL} + (a_6 + a_8)F_{Ah}^{SP} + (C_3 + C_9)M_{Ah}^{LL} \\
&+(C_5 + C_7)M_{Ah}^{LR}\Bigg]\Bigg\}, \qquad (A.15)
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}\left(B^0 \to K_0^{*+}\pi^-\right) = &\frac{G_F}{\sqrt{2}}\Bigg\{V_{ub}^*V_{us}[a_1F_{Th}^{LL} + C_1M_{Th}^{LL}] - V_{tb}^*V_{ts}\Bigg[(a_4 + a_{10})F_{Th}^{LL} \\
&+(a_6 + a_8)F_{Th}^{SP} + (C_3 + C_9)M_{Th}^{LL} + (C_5 + C_7)M_{Th}^{LR} \\
&+\left(a_4 - \frac{a_{10}}{2}\right)F_{Ah}^{LL} + \left(a_6 - \frac{a_8}{2}\right)F_{Ah}^{SP} + \left(C_3 - \frac{C_9}{2}\right)M_{Ah}^{LL} \\
&+\left(C_5 - \frac{C_7}{2}\right)M_{Ah}^{LR}\Bigg]\Bigg\}, \qquad (A.16)
\end{aligned}$$

$$
\begin{aligned}
\mathcal{A}\left(B^0 \to K_0^{*0}\pi^0\right) =& \frac{G_F}{2}\bigg\{V_{ub}^* V_{us}[a_2 F_{TK_0^*}^{LL} + C_2 M_{TK_0^*}^{LL}] - V_{tb}^* V_{ts}\bigg[\bigg(\frac{3}{2}(a_9 - a_7) F_{TK_0^*}^{LL} \\
&+ \frac{3C_{10}}{2} M_{TK_0^*}^{LL} + \frac{3C_8}{2} M_{TK_0^*}^{SP} - \left(a_4 - \frac{a_{10}}{2}\right)(F_{Th}^{LL} + F_{Ah}^{LL}) \\
&- \left(a_6 - \frac{a_8}{2}\right)(F_{Th}^{SP} + F_{Ah}^{SP}) - \left(C_3 - \frac{C_9}{2}\right)(M_{Th}^{LL} + M_{Ah}^{LL}) \\
&- \left(C_5 - \frac{C_7}{2}\right)(M_{Th}^{LR} + M_{Ah}^{LR})\bigg]\bigg\}, \qquad \text{(A.17)} \\
\mathcal{A}\left(B^0 \to K_0^{*+}K^-\right) =& \frac{G_F}{\sqrt{2}}\bigg\{V_{ub}^* V_{ud}[a_2 F_{AK_0^*}^{LL} + C_2 M_{AK_0^*}^{LL}] - V_{tb}^* V_{td}\bigg[(a_3 + a_9 - a_5 - a_7) F_{AK_0^*}^{LL} \\
&+ (C_4 + C_{10}) M_{AK_0^*}^{LL} + (C_6 + C_8) M_{AK_0^*}^{SP} + \left(a_3 - \frac{a_9}{2} - a_5 + \frac{a_7}{2}\right) F_{Ah}^{LL} \\
&+ \left(C_4 - \frac{C_{10}}{2}\right) M_{Ah}^{LL} + \left(C_6 - \frac{C_8}{2}\right) M_{Ah}^{SP}\bigg]\bigg\}, \qquad \text{(A.18)} \\
\mathcal{A}\left(B^0 \to K_0^{*-}K^+\right) =& \frac{G_F}{\sqrt{2}}\bigg\{V_{ub}^* V_{ud}[a_2 F_{Ah}^{LL} + C_2 M_{Ah}^{LL}] - V_{tb}^* V_{td}\bigg[(a_3 + a_9 - a_5 - a_7) F_{Ah}^{LL} \\
&+ (C_4 + C_{10}) M_{Ah}^{LL} + (C_6 + C_8) M_{Ah}^{SP} + \left(a_3 - \frac{a_9}{2} - a_5 + \frac{a_7}{2}\right) F_{AK_0^*}^{LL} \\
&+ \left(C_4 - \frac{C_{10}}{2}\right) M_{AK_0^*}^{LL} + \left(C_6 - \frac{C_8}{2}\right) M_{AK_0^*}^{SP}\bigg]\bigg\}, \qquad \text{(A.19)} \\
\mathcal{A}\left(B^0 \to K_0^{*0}\bar{K}^0\right) =& -\frac{G_F}{\sqrt{2}}\bigg\{V_{tb}^* V_{td}\bigg[\left(a_4 - \frac{a_{10}}{2}\right) F_{TK_0^*}^{LL} + \left(a_6 - \frac{a_8}{2}\right)(F_{TK_0^*}^{SP} + F_{AK_0^*}^{SP}) \\
&+ \left(C_3 - \frac{C_9}{2}\right) M_{TK_0^*}^{LL} + \left(C_5 - \frac{C_7}{2}\right)(M_{TK_0^*}^{LR} + M_{AK_0^*}^{LR}) + \bigg(\frac{4}{3}\bigg(C_3 + C_4 \\
&- \frac{C_9 + C_{10}}{2}\bigg) - a_5 + \frac{a_7}{2}\bigg) F_{AK_0^*}^{LL} + \left(C_3 + C_4 - \frac{C_9 + C_{10}}{2}\right) M_{AK_0^*}^{LL} \\
&+ \left(C_6 - \frac{C_8}{2}\right)(M_{AK_0^*}^{SP} + M_{Ah}^{SP}) + \left(a_3 - \frac{a_9}{2} - a_5 + \frac{a_7}{2}\right) F_{Ah}^{LL} \\
&+ \left(C_4 - \frac{C_{10}}{2}\right) M_{Ah}^{LL}\bigg]\bigg\}, \qquad \text{(A.20)} \\
\mathcal{A}\left(B^0 \to \bar{K}_0^{*0}K^0\right) =& -\frac{G_F}{\sqrt{2}}\bigg\{V_{tb}^* V_{td}\bigg[\left(a_4 - \frac{a_{10}}{2}\right) F_{Th}^{LL} + \left(a_6 - \frac{a_8}{2}\right)(F_{Th}^{SP} + F_{Ah}^{SP}) \\
&+ \left(C_3 - \frac{C_9}{2}\right) M_{Th}^{LL} + \left(C_5 - \frac{C_7}{2}\right)(M_{Th}^{LR} + M_{Ah}^{LR}) + \bigg(\frac{4}{3}\bigg(C_3 + C_4 \\
&- \frac{C_9 + C_{10}}{2}\bigg) - a_5 + \frac{a_7}{2}\bigg) F_{Ah}^{LL} + (C_3 + C_4 - \frac{C_9 + C_{10}}{2}) M_{Ah}^{LL} \\
&+ \left(C_6 - \frac{C_8}{2}\right)(M_{Ah}^{SP} + M_{AK_0^*}^{SP}) + \left(a_3 - \frac{a_9}{2} - a_5 + \frac{a_7}{2}\right) F_{AK_0^*}^{LL} \\
&+ \left(C_4 - \frac{C_{10}}{2}\right) M_{AK_0^*}^{LL}\bigg]\bigg\}, \qquad \text{(A.21)} \\
\mathcal{A}\left(B_s^0 \to K_0^{*-}\pi^+\right) =& \frac{G_F}{\sqrt{2}}\bigg\{V_{ub}^* V_{ud}[a_1 F_{TK_0^*}^{LL} + C_1 M_{TK_0^*}^{LL}] - V_{tb}^* V_{td}\bigg[(a_4 + a_{10}) F_{TK_0^*}^{LL}
\end{aligned}
$$

$$+(a_6+a_8)F^{SP}_{TK_0^*}+(C_3+C_9)M^{LL}_{TK_0^*}+(C_5+C_7)M^{LR}_{TK_0^*}$$
$$+\left(a_4-\frac{a_{10}}{2}\right)F^{LL}_{AK_0^*}+\left(a_6-\frac{a_8}{2}\right)F^{SP}_{AK_0^*}+\left(C_3-\frac{C_9}{2}\right)M^{LL}_{AK_0^*}$$
$$+\left(C_5-\frac{C_7}{2}\right)M^{LR}_{AK_0^*}\Big]\Big\}, \tag{A.22}$$

$$\mathcal{A}\left(B_s^0\to\bar{K}_0^{*0}\pi^0\right)=\frac{G_F}{2}\Big\{V_{ub}^*V_{ud}[a_2F^{LL}_{TK_0^*}+C_2M^{LL}_{TK_0^*}]-V_{tb}^*V_{td}\Big[\Big(-a_4-\frac{3a_7}{2}$$
$$+\frac{5C_9}{3}+C_{10}\Big)F^{LL}_{TK_0^*}-\left(a_6-\frac{a_8}{2}\right)F^{SP}_{TK_0^*}+\left(-C_3+\frac{3a_{10}}{2}\right)M^{LL}_{TK_0^*}$$
$$-\left(C_5-\frac{C_7}{2}\right)M^{LR}_{TK_0^*}+\frac{3C_8}{2}M^{SP}_{TK_0^*}-\left(a_4-\frac{a_{10}}{2}\right)F^{LL}_{AK_0^*}-\Big(a_6$$
$$-\frac{a_8}{2}\Big)F^{SP}_{AK_0^*}-\left(C_3-\frac{C_9}{2}\right)M^{LL}_{AK_0^*}-\left(C_5-\frac{C_7}{2}\right)M^{LR}_{AK_0^*}\Big]\Big\}, \tag{A.23}$$

$$\mathcal{A}\left(B_s^0\to K_0^{*+}K^-\right)=\frac{G_F}{\sqrt{2}}\Big\{V_{ub}^*V_{us}[a_1F^{LL}_{Th}+C_1M^{LL}_{Th}+a_2F^{LL}_{AK_0^*}+C_2M^{LL}_{AK_0^*}]$$
$$-V_{tb}^*V_{ts}\Big[(a_4+a_{10})F^{LL}_{Th}+(a_6+a_8)F^{SP}_{Th}+(C_3+C_9)M^{LL}_{Th}$$
$$+(C_5+C_7)M^{LR}_{Th}+\left(\frac{4}{3}\left(C_3+C_4-\frac{C_9+C_{10}}{2}\right)-a_5+\frac{a_7}{2}\right)F^{LL}_{Ah}$$
$$+\left(a_6-\frac{a_8}{2}\right)F^{SP}_{Ah}+\left(C_3+C_4-\frac{C_9+C_{10}}{2}\right)M^{LL}_{Ah}$$
$$+\left(C_5-\frac{C_7}{2}\right)M^{LR}_{Ah}+\left(C_6-\frac{C_8}{2}\right)M^{SP}_{Ah}+(a_3+a_9-a_5-a_7)F^{LL}_{AK_0^*}$$
$$+(C_4+C_{10})M^{LL}_{AK_0^*}+(C_6+C_8)M^{SP}_{AK_0^*}\Big]\Big\}, \tag{A.24}$$

$$\mathcal{A}\left(B_s^0\to K_0^{*-}K^+\right)=\frac{G_F}{\sqrt{2}}\Big\{V_{ub}^*V_{us}[a_1F^{LL}_{TK_0^*}+C_1M^{LL}_{TK_0^*}+a_2F^{LL}_{Ah}+C_2M^{LL}_{Ah}]$$
$$-V_{tb}^*V_{ts}\Big[(a_4+a_{10})F^{LL}_{TK_0^*}+(a_6+a_8)F^{SP}_{TK_0^*}+(C_3+C_9)M^{LL}_{TK_0^*}$$
$$+(C_5+C_7)M^{LR}_{TK_0^*}+\left(\frac{4}{3}\left(C_3+C_4-\frac{C_9+C_{10}}{2}\right)-a_5+\frac{a_7}{2}\right)F^{LL}_{AK_0^*}$$
$$+\left(a_6-\frac{a_8}{2}\right)F^{SP}_{AK_0^*}+\left(C_3+C_4-\frac{C_9+C_{10}}{2}\right)M^{LL}_{AK_0^*}$$
$$+\left(C_5-\frac{C_7}{2}\right)M^{LR}_{AK_0^*}+\left(C_6-\frac{C_8}{2}\right)M^{SP}_{AK_0^*}+(a_3+a_9-a_5-a_7)F^{LL}_{Ah}$$
$$+(C_4+C_{10})M^{LL}_{Ah}+(C_6+C_8)M^{SP}_{Ah}\Big]\Big\}, \tag{A.25}$$

$$\mathcal{A}\left(B_s^0\to K_0^{*0}\bar{K}^0\right)=-\frac{G_F}{\sqrt{2}}\Big\{V_{tb}^*V_{ts}\Big[\left(a_4-\frac{a_{10}}{2}\right)F^{LL}_{Th}+\left(a_6-\frac{a_8}{2}\right)(F^{SP}_{Th}+F^{SP}_{Ah})$$
$$+\left(C_3-\frac{C_9}{2}\right)M^{LL}_{Th}+\left(C_5-\frac{C_7}{2}\right)(M^{LR}_{Th}+M^{LR}_{Ah})+\Big(\frac{4}{3}\Big(C_3+C_4$$

$$
\begin{aligned}
&-\frac{C_9+C_{10}}{2}\bigg)-a_5+\frac{a_7}{2}\bigg)F^{LL}_{Ah}+\bigg(C_3+C_4-\frac{C_9+C_{10}}{2}\bigg)M^{LL}_{Ah}\\
&+\bigg(C_6-\frac{C_8}{2}\bigg)(M^{SP}_{Ah}+M^{SP}_{AK_0^*})+\bigg(a_3-\frac{a_9}{2}-a_5+\frac{a_7}{2}\bigg)F^{LL}_{AK_0^*}\\
&+\bigg(C_4-\frac{C_{10}}{2}\bigg)M^{LL}_{AK_0^*}\bigg]\bigg\},
\end{aligned}
\tag{A.26}
$$

$$
\begin{aligned}
\mathcal{A}\left(B_s^0\to \bar{K}_0^{*0}K^0\right) = -\frac{G_F}{\sqrt{2}}\bigg\{V_{tb}^*V_{ts}\bigg[&\bigg(a_4-\frac{a_{10}}{2}\bigg)F^{LL}_{TK_0^*}+\bigg(a_6-\frac{a_8}{2}\bigg)(F^{SP}_{TK_0^*}+F^{SP}_{AK_0^*})\\
&+\bigg(C_3-\frac{C_9}{2}\bigg)M^{LL}_{TK_0^*}+\bigg(C_5-\frac{C_7}{2}\bigg)(M^{LR}_{TK_0^*}+M^{LR}_{AK_0^*})+\bigg(\frac{4}{3}\bigg(C_3+C_4\\
&-\frac{C_9+C_{10}}{2}\bigg)-a_5+\frac{a_7}{2}\bigg)F^{LL}_{AK_0^*}+\bigg(C_3+C_4-\frac{C_9+C_{10}}{2}\bigg)M^{LL}_{AK_0^*}\\
&+(C_6-\frac{C_8}{2}\bigg)(M^{SP}_{AK_0^*}+M^{SP}_{Ah})+\bigg(a_3-\frac{a_9}{2}-a_5+\frac{a_7}{2}\bigg)F^{LL}_{Ah}\\
&+\bigg(C_4-\frac{C_{10}}{2}\bigg)M^{LL}_{Ah}\bigg]\bigg\},
\end{aligned}
\tag{A.27}
$$

in which G_F is the Fermi coupling constant, V's are the CKM matrix elements. The combinations a_i of the Wilson coefficients are defined as

$$
a_1=C_2+\frac{C_1}{3},\quad a_2=C_1+\frac{C_2}{3},\quad a_3=C_3+\frac{C_4}{3},\quad a_4=C_4+\frac{C_3}{3},\qquad a_5=C_5+\frac{C_6}{3},
\tag{A.28}
$$

$$
a_6=C_6+\frac{C_5}{3},\quad a_7=C_7+\frac{C_8}{3},\quad a_8=C_8+\frac{C_7}{3},\quad a_9=C_9+\frac{C_{10}}{3},\qquad a_{10}=C_{10}+\frac{C_9}{3}.
\tag{A.29}
$$

It should be understood that the Wilson coefficients C and the amplitudes F and M for the factorizable and nonfactorizable contributions, respectively, appear in convolutions in momentum fractions and impact parameters b.

The general amplitudes for the decays $B\to K_0^*h\to K\pi h$ in the decay amplitudes eq. (A.12)-eq. (A.27) are given according to the figure 1, the typical Feynman diagrams in the PQCD approach. In the following expressions, we will employ LL and LR to denote the contributions from $(V-A)(V-A)$ and $(V-A)(V+A)$ operators, respectively. For the contribution from $(S-P)(S+P)$ operators which come from the Fierz transformation of the $(V-A)(V+A)$ operators, we will use SP to denote it. The emission diagrams are depicted in figure 1 (a) and (c) with $B\to K_0^*$ and $B\to h$ transitions, and described as the subscripts TK_0^* and Th in their amplitudes, respectively. The factorizable and nonfactorizable diagrams have been merged in figure 1, which could be distinguished easily from the attachments of the hard gluons. Those diagrams with two attachments of the hard gluon passed the weak vertex are nonfactorizable diagrams, we name their expressions with M, while the others are factorizable, and we name their expressions with F. There are two similar merged annihilation diagrams, the figure 1 (b) and (d), with the subscripts AK_0^* and Ah in their amplitudes, respectively, which demonstrate the W annihilation and W-exchange, space-like penguin and time-like penguin annihilation-type diagrams.

With the ratio $r_0 = m_0^h/m_B$, the amplitudes from figure 1(a) are written as

$$\begin{aligned}
F_{TK_0^*}^{LL} &= 8\pi C_F m_B^4 f_{K(\pi)}(\zeta-1)\int dx_B dz \int b_B db_B b db \phi_B(x_B,b_B) \\
&\quad\times\Big\{\big[\sqrt{\zeta}(2z-1)(\phi^s+\phi^t)-(z+1)\phi\big]E_{a12}(t_{a1})h_{a1}(x_B,z,b_B,b) \\
&\quad+\Big(\zeta\phi-2\sqrt{\zeta}\phi^s\Big)E_{a12}(t_{a2})h_{a2}(x_B,z,b_B,b)\Big\}, \qquad \text{(A.30)}\\
F_{TK_0^*}^{LR} &= -F_{TK_0^*}^{LL}, \qquad \text{(A.31)}\\
F_{TK_0^*}^{SP} &= 16\pi C_F m_B^4 r_0 f_{K(\pi)}\int dx_B dz \int b_B db_B b db \phi_B(x_B,b_B) \\
&\quad\times\Big\{\big[\phi[\zeta(2z-1)-1]+\sqrt{\zeta}[z\phi^t-(z+2)\phi^s]\big]E_{a12}(t_{a1})h_{a1}(x_B,z,b_B,b) \\
&\quad+\Big[\phi(2\zeta-x_B)-2\sqrt{\zeta}\phi^s(\zeta-x_B+1)\Big]E_{a12}(t_{a2})h_{a2}(x_B,z,b_B,b)\Big\}, \qquad \text{(A.32)}\\
M_{TK_0^*}^{LL} &= 32\pi C_F m_B^4/\sqrt{2N_c}(\zeta-1)\int dx_B dz dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B,b_B)\phi^A \\
&\quad\times\Big\{\big[[\zeta(1-x_3-z)+x_B+x_3-1]\phi+\sqrt{\zeta}z(\phi^s-\phi^t)\big] \\
&\quad\times E_{a34}(t_{a3})h_{a3}(x_B,z,x_3,b_B,b_3)+\big[[x_3(1-\zeta)-x_B]\phi+z[\phi-\sqrt{\zeta}(\phi^s+\phi^t)]\big] \\
&\quad\times E_{a34}(t_{a4})h_{a4}(x_B,z,x_3,b_B,b_3)\Big\}, \qquad \text{(A.33)}\\
M_{TK_0^*}^{LR} &= 32\pi C_F m_B^4 r_0/\sqrt{2N_c}\int dx_B dz dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B,b_B) \\
&\quad\times\Big\{\big[[\zeta(1-x_3)+x_B+x_3-1][\phi+\sqrt{\zeta}(\phi^s-\phi^t)](\phi^P+\phi^T) \\
&\quad+\sqrt{\zeta}z(\sqrt{\zeta}\phi+\phi^s+\phi^t)(\phi^T-\phi^P)\big]E_{a34}(t_{a3})h_{a3}(x_B,z,x_3,b_B,b_3) \\
&\quad+\big[[(1-\zeta)x_3-x_B][\phi+\sqrt{\zeta}(\phi^s-\phi^t)](\phi^P-\phi^T) \\
&\quad+\sqrt{\zeta}z(\sqrt{\zeta}\phi+\phi^s+\phi^t)(\phi^P+\phi^T)\big]E_{a34}(t_{a4})h_{a4}(x_B,z,x_3,b_B,b_3)\Big\}, \qquad \text{(A.34)}\\
M_{TK_0^*}^{SP} &= 32\pi C_F m_B^4/\sqrt{2N_c}(\zeta-1)\int dx_B dz dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B,b_B)\phi^A \\
&\quad\times\Big\{\big[[(x_3-1)\zeta-x_B+z-x_3+1]\phi-\sqrt{\zeta}z(\phi^s+\phi^t)\big] \\
&\quad\times E_{a34}(t_{a3})h_{a3}(x_B,z,x_3,b_B,b_3)+\big[[x_B+x_3(\zeta-1)]\phi-z\sqrt{\zeta}(\sqrt{\zeta}\phi-\phi^s+\phi^t)\big] \\
&\quad\times E_{a34}(t_{a4})h_{a4}(x_B,z,x_3,b_B,b_3)\Big\}, \qquad \text{(A.35)}
\end{aligned}$$

with the color factor $C_F = 4/3$. The amplitudes from figure 1(b) are written as

$$\begin{aligned}
F_{AK_0^*}^{LL} &= 8\pi C_F m_B^4 f_B \int dz dx_3 \int b db b_3 db_3 \\
&\quad\times\Big\{\big[(1-\zeta)(z-1)\phi\phi^A+2\sqrt{\zeta}r_0[(2-z)\phi^s+z\phi^t]\phi^P\big]E_{b12}(t_{b1})h_{b1}(z,x_3,b,b_3) \\
&\quad+\big[(1-\zeta)[x_3(1-\zeta)+\zeta]\phi\phi^A+2\sqrt{\zeta}r_0\phi^s[\big(\zeta(x_3-1)-x_3\big)(\phi^P+\phi^T) \\
&\quad-(\phi^P-\phi^T)]\big]E_{b12}(t_{b2})h_{b2}(z,x_3,b,b_3)\Big\}, \qquad \text{(A.36)}\\
F_{AK_0^*}^{LR} &= -F_{AK_0^*}^{LL}, \qquad \text{(A.37)}
\end{aligned}$$

$$
\begin{aligned}
F^{SP}_{AK_0^*} &= 16\pi C_F m_B^4 f_B \int dz dx_3 \int b db b_3 db_3 \\
&\quad \times \Big\{ \big[(\zeta-1)\sqrt{\zeta}(z-1)(\phi^s+\phi^t)\phi^A + 2r_0\phi\phi^P[\zeta(z-1)-1]\big] \\
&\quad \times E_{b12}(t_{b1})h_{b1}(z,x_3,b,b_3) + \big[2\sqrt{\zeta}(1-\zeta)\phi^s\phi^A + x_3 r_0\phi(\zeta-1)(\phi^P-\phi^T) \\
&\quad -2\zeta r_0\phi\phi^P\big] E_{b12}(t_{b2})h_{b2}(z,x_3,b,b_3)\Big\}, \qquad \text{(A.38)}
\end{aligned}
$$

$$
\begin{aligned}
M^{LL}_{AK_0^*} &= 32\pi C_F m_B^4/\sqrt{2N_c} \int dx_B dz dx_3 \int b_B db_B b db \phi_B(x_B,b_B) \\
&\quad \times \Big\{ \big[[\zeta^2(1-z-x_3) + \zeta(x_B+2x_3+z-1) - (x_B+x_3)]\phi\phi^A \\
&\quad +\sqrt{\zeta} r_0[(x_B+(1-\zeta)(x_3-1))(\phi^s-\phi^t)(\phi^P+\phi^T) \\
&\quad +z(\phi^s+\phi^t)(\phi^T-\phi^P) + 4\phi^s\phi^P]\big] E_{b34}(t_{b3})h_{b3}(x_B,z,x_3,b_B,b) \\
&\quad +\big[(\zeta^2-1)(z-1)\phi\phi^A + \sqrt{\zeta} r_0[(\zeta(x_3-1)-x_3+x_B)(\phi^s+\phi^t)(\phi^P-\phi^T) \\
&\quad +(z-1)(\phi^s-\phi^t)(\phi^P+\phi^T)]\big] E_{b34}(t_{b4})h_{b4}(x_B,z,x_3,b_B,b)\Big\}, \qquad \text{(A.39)}
\end{aligned}
$$

$$
\begin{aligned}
M^{LR}_{AK_0^*} &= 32\pi C_F m_B^4/\sqrt{2N_c} \int dx_B dz dx_3 \int b_B db_B b db \phi_B(x_B,b_B) \\
&\quad \times \Big\{ \big[(\zeta-1)(z+1)\sqrt{\zeta}(\phi^s-\phi^t)\phi^A + r_0\phi[(x_3(1-\zeta)+x_B-2)(\phi^P+\phi^T) \\
&\quad +\zeta z(\phi^T-\phi^P) + 2\zeta\phi^T]\big] E_{b34}(t_{b3})h_{b3}(x_B,z,x_3,b_B,b) \\
&\quad +\big[(1-z)\sqrt{\zeta}(\zeta-1)(\phi^s-\phi^t)\phi^A + r_0\phi[(\zeta x_3-(x_3-x_B))(\phi^P+\phi^T) \\
&\quad +\zeta z(\phi^P-\phi^T) - 2\zeta\phi^P]\big] E_{b34}(t_{b4})h_{b4}(x_B,z,x_3,b_B,b)\Big\}, \qquad \text{(A.40)}
\end{aligned}
$$

$$
\begin{aligned}
M^{SP}_{AK_0^*} &= 32\pi C_F m_B^4/\sqrt{2N_c} \int dx_B dz dx_3 \int b_B db_B b db \phi_B(x_B,b_B) \\
&\quad \times \Big\{ \big[(z\zeta+z-1)(\zeta-1)\phi\phi^A + \sqrt{\zeta} r_0[(\zeta(1-x_3)+x_B+x_3-1)(\phi^s+\phi^t)(\phi^T-\phi^P) \\
&\quad +z(\phi^s-\phi^t)(\phi^P+\phi^T) - 4\phi^s\phi^P]\big] E_{b34}(t_{b3})h_{b3}(x_B,z,x_3,b_B,b) \\
&\quad +\big[(1-\zeta)[\zeta(z-2)+x_3(\zeta-1)+x_B]\phi\phi^A + r_0\sqrt{\zeta}[(1-z)(\phi^s+\phi^t)(\phi^P-\phi^T) \\
&\quad +((1-\zeta)x_3+\zeta-x_B)(\phi^s-\phi^t)(\phi^P+\phi^T)]\big] E_{b34}(t_{b4})h_{b4}(x_B,z,x_3,b_B,b)\Big\}, \qquad \text{(A.41)}
\end{aligned}
$$

The amplitudes from figure 1(c) are

$$
\begin{aligned}
F^{LL}_{Th} &= 8\pi C_F m_B^4 F_{K\pi}(s)/\mu_s \int dx_B dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B,b_B) \\
&\quad \times \Big\{ \big[(1-\zeta)((\zeta-1)x_3-1)\phi^A - r_0[\big(2x_3(\zeta-1)+\zeta+1\big)\phi^P + (\zeta-1)(2x_3-1)\phi^T]\big] \\
&\quad \times E_{c12}(t_{c1})h_{c1}(x_B,x_3,b_B,b_3) + \big[\zeta(\zeta-1)x_B\phi^A + 2r_0(\zeta x_B+\zeta-1)\phi^P\big] \\
&\quad \times E_{c12}(t_{c2})h_{c2}(x_B,x_3,b_B,b_3)\Big\}, \qquad \text{(A.42)}
\end{aligned}
$$

$$
\begin{aligned}
F^{SP}_{Th} &= 16\pi C_F m_B^4 \sqrt{\zeta} F_{K\pi}(s) \int dx_B dx_3 \int b_B db_B b_3 db_3 \phi_B(x_B,b_B) \\
&\quad \times \Big\{ \big[(\zeta-1)\phi^A + r_0[x_3(\zeta-1)(\phi^P-\phi^T) - 2\phi^P]\big] E_{c12}(t_{c1})h_{c1}(x_B,x_3,b_B,b_3) \\
&\quad + \big[(\zeta-1)x_B\phi^A + 2r_0(\zeta+x_B-1)\phi^P\big] E_{c12}(t_{c2})h_{c2}(x_B,x_3,b_B,b_3)\Big\}, \qquad \text{(A.43)}
\end{aligned}
$$

$$
\begin{aligned}
M_{Th}^{LL} &= 32\pi C_F m_B^4/\sqrt{2N_c}\int dx_B dz dx_3 \int b_B db_B b db \phi_B(x_B,b_B)\phi \\
&\times\Big\{\big[(\zeta^2-1)(x_B+z-1)\phi^A + r_0[\zeta(x_B+z)(\phi^P+\phi^T)+x_3(\zeta-1)(\phi^P-\phi^T)-2\zeta\phi^P]\big] \\
&\times E_{c34}(t_{c3})h_{c3}(x_B,z,x_3,b_B,b)+\big[(1-\zeta)x_3[(\zeta-1)\phi^A+r_0(\phi^P+\phi^T)] \\
&-(x_B-z)[(\zeta-1)\phi^A+\zeta r_0(\phi^P-\phi^T)]\big]E_{c34}(t_{c4})h_{c4}(x_B,z,x_3,b_B,b)\Big\}, \qquad \text{(A.44)}\\
M_{Th}^{LR} &= 32\pi C_F m_B^4\sqrt{\zeta}/\sqrt{2N_c}\int dx_B dz dx_3 \int b_B db_B b db \phi_B(x_B,b_B) \\
&\times\Big\{\big[(\zeta-1)(x_B+z-1)(\phi^s+\phi^t)\phi^A + r_0[(\zeta(1-x_3)+x_3)(\phi^s-\phi^t)(\phi^P+\phi^T) \\
&+(x_B+z-1)(\phi^s+\phi^t)(\phi^T-\phi^P)]\big]E_{c34}(t_{c3})h_{c3}(x_B,z,x_3,b_B,b) \\
&+\big[(z-x_B)(\phi^s-\phi^t)[(\zeta-1)\phi^A+r_0(\phi^T-\phi^P)]+(\zeta-1)r_0x_3(\phi^s+\phi^t) \\
&\times(\phi^P+\phi^T)]\big]E_{c34}(t_{c4})h_{c4}(x_B,z,x_3,b_B,b)\Big\}, \qquad \text{(A.45)}
\end{aligned}
$$

The amplitudes from figure 1(d) are

$$
\begin{aligned}
F_{Ah}^{LL} &= 8\pi C_F m_B^4 f_B \int dz dx_3 \int b db b_3 db_3 \\
&\times\Big\{\big[[(\zeta-1)[(\zeta-1)x_3+1]\phi\phi^A - 2\sqrt{\zeta}r_0\phi^s[(\zeta-1)x_3(\phi^P-\phi^T)+2\phi^P]\big] \\
&\times E_{d12}(t_{d1})h_{d1}(z,x_3,b,b_3)+\big[z[2\sqrt{\zeta}r_0(\phi^s+\phi^t)\phi^P+(1-\zeta)\phi\phi^A] \\
&-2(\zeta-1)\sqrt{\zeta}r_0(\phi^s-\phi^t)\phi^P\big]E_{d12}(t_{d2})h_{d2}(z,x_3,b,b_3)\Big\}, \qquad \text{(A.46)}\\
F_{Ah}^{LR} &= -F_{Ah}^{LL}, \qquad \text{(A.47)}\\
F_{Ah}^{SP} &= 16\pi C_F m_B^4 f_B \int dz dx_3 \int b db b_3 db_3 \\
&\times\Big\{\big[2(1-\zeta)\sqrt{\zeta}\phi^s\phi^A + r_0\phi[((\zeta-1)x_3+1)(\phi^P+\phi^T)+\zeta(\phi^P-\phi^T)]\big] \\
&\times E_{d12}(t_{d1})h_{d1}(z,x_3,b,b_3)+\Big[(1-\zeta)\sqrt{\zeta}z(\phi^s-\phi^t)\phi^A+2r_0(\zeta(z-1)+1)\phi\phi^P\Big] \\
&\times E_{d12}(t_{d2})h_{d2}(z,x_3,b,b_3)\Big\}, \qquad \text{(A.48)}\\
M_{Ah}^{LL} &= 32\pi C_F m_B^4/\sqrt{2N_c}\int dx_B dz dx_3 \int b_B db_B b db \phi_B(x_B,b_B) \\
&\times\Big\{\big[[(x_B+z-1)\zeta^2+\zeta-(x_B+z)]\phi\phi^A+\sqrt{\zeta}r_0[(x_3-\zeta(x_3-1))(\phi^s-\phi^t) \\
&\times(\phi^P+\phi^T)+(x_B+z-1)(\phi^s+\phi^t)(\phi^T-\phi^P)-4\phi^s\phi^P]\big]E_{d34}(t_{d3})h_{d3}(x_B,z,x_3,b_B,b) \\
&+\big[(1-\zeta)[\zeta(x_3-x_B+z-1)-x_3+1]\phi\phi^A-\sqrt{\zeta}r_0[(x_B-z)(\phi^s-\phi^t)(\phi^P+\phi^T) \\
&+(1-x_3)(\zeta-1)(\phi^s+\phi^t)(\phi^P-\phi^T)]\big]E_{d34}(t_{d4})h_{d4}(x_B,z,x_3,b_B,b)\Big\}, \qquad \text{(A.49)}\\
M_{Ah}^{LR} &= 32\pi C_F m_B^4/\sqrt{2N_c}\int dx_B dz dx_3 \int b_B db_B b db \phi_B(x_B,b_B) \\
&\times\Big\{\big[(\zeta-1)\sqrt{\zeta}(x_B+z-2)(\phi^s+\phi^t)\phi^A+r_0\phi[\zeta(x_B+z-1)(\phi^P+\phi^T) \\
&+(\zeta x_3-x_3-1)(\phi^P-\phi^T)-2\zeta\phi^P]\big]E_{d34}(t_{d3})h_{d3}(x_B,z,x_3,b_B,b)
\end{aligned}
$$

$$+\big[\sqrt{\zeta}(\zeta-1)(x_B-z)(\phi^s+\phi^t)\phi^A+r_0\phi[\zeta(x_B-z)(\phi^P+\phi^T)$$
$$+(1-x_3)(\zeta-1)(\phi^P-\phi^T)]\big]E_{d34}(t_{d4})h_{d4}(x_B,z,x_3,b_B,b)\Big\}, \tag{A.50}$$

$$M_{Ah}^{SP}=32\pi C_F m_B^4/\sqrt{2N_c}\int dx_B dz dx_3\int b_B db_B b db \phi_B(x_B,b_B)$$
$$\times\Big\{\big[(1-\zeta)[\zeta(x_B+z+x_3-2)-x_3+1]\phi\phi^A+\sqrt{\zeta}r_0[(x_B+z-1)(\phi^s-\phi^t)$$
$$(\phi^P+\phi^T)+(x_3\zeta-\zeta-x_3)(\phi^s+\phi^t)(\phi^P-\phi^T)+4\phi^s\phi^P]\big]E_{d34}(t_{d3})h_{d3}(x_B,z,x_3,b_B,b)$$
$$+\big[(1-\zeta^2)(x_B-z)\phi\phi^A+r_0\sqrt{\zeta}[(1-x_3)(\zeta-1)(\phi^s-\phi^t)(\phi^P+\phi^T)$$
$$+(x_B-z)(\phi^s+\phi^t)(\phi^P-\phi^T)]\big]E_{d34}(t_{d4})h_{d4}(x_B,z,x_3,b_B,b)\Big\}, \tag{A.51}$$

B PQCD functions

In this section, we group the functions which appear in the factorization formulas of this work.

With $\bar{\zeta}=(1-\zeta)$, $\bar{x}_3=(1-x_3)$ and $\bar{z}=(1-z)$, the involved hard scales are chosen as

$$t_{a1}=\max\Big\{m_B\sqrt{z},\ 1/b_B,\ 1/b\Big\}, \tag{B.1}$$
$$t_{a2}=\max\Big\{m_B\sqrt{|x_B-\zeta|},\ 1/b_B,\ 1/b\Big\}, \tag{B.2}$$
$$t_{a3}=\max\Big\{m_B\sqrt{x_Bz},\ m_B\sqrt{z|\bar{\zeta}\bar{x}_3-x_B|},\ 1/b_B,\ 1/b_3\Big\}, \tag{B.3}$$
$$t_{a4}=\max\Big\{m_B\sqrt{x_Bz},\ m_B\sqrt{z|x_B-x_3\bar{\zeta}|},\ 1/b_B,\ 1/b_3\Big\}, \tag{B.4}$$
$$t_{b1}=\max\Big\{m_B\sqrt{1-z},\ 1/b,\ 1/b_3\Big\}, \tag{B.5}$$
$$t_{b2}=\max\Big\{m_B\sqrt{\zeta+x_3\bar{\zeta}},\ 1/b,\ 1/b_3\Big\}, \tag{B.6}$$
$$t_{b3}=\max\Big\{m_B\sqrt{\bar{z}(\zeta+x_3\bar{\zeta})},\ m_B\sqrt{1-z(\bar{x}_3\bar{\zeta}-x_B)},\ 1/b_B,\ 1/b\Big\}, \tag{B.7}$$
$$t_{b4}=\max\Big\{m_B\sqrt{\bar{z}(\zeta+x_3\bar{\zeta})},\ m_B\sqrt{\bar{z}|x_B-\zeta-x_3\bar{\zeta}|},\ 1/b_B,\ 1/b\Big\}, \tag{B.8}$$
$$t_{c1}=\max\Big\{m_B\sqrt{x_3\bar{\zeta}},\ 1/b_B,\ 1/b_3\Big\}, \tag{B.9}$$
$$t_{c2}=\max\Big\{m_B\sqrt{x_B\bar{\zeta}},\ 1/b_B,\ 1/b_3\Big\}, \tag{B.10}$$
$$t_{c3}=\max\Big\{m_B\sqrt{x_Bx_3\bar{\zeta}},\ m_B\sqrt{|1-x_B-z|[x_3\bar{\zeta}+\zeta]},\ 1/b_B,\ 1/b\Big\}, \tag{B.11}$$
$$t_{c4}=\max\Big\{m_B\sqrt{x_Bx_3\bar{\zeta}},\ m_B\sqrt{|x_B-z|x_3\bar{\zeta}},\ 1/b_B,\ 1/b\Big\}, \tag{B.12}$$
$$t_{d1}=\max\Big\{m_B\sqrt{1-x_3\bar{\zeta}},\ 1/b,\ 1/b_3\Big\}, \tag{B.13}$$
$$t_{d2}=\max\Big\{m_B\sqrt{z\bar{\zeta}},\ 1/b,\ 1/b_3\Big\}, \tag{B.14}$$

$$t_{d3} = \max\left\{m_B\sqrt{\bar{x}_3 z\bar{\zeta}},\ m_B\sqrt{1-(x_3\bar{\zeta}+\zeta)(1-x_B-z)},\ 1/b_B,\ 1/b\right\}, \tag{B.15}$$

$$t_{d4} = \max\left\{m_B\sqrt{\bar{x}_3 z\bar{\zeta}},\ m_B\sqrt{|x_B-z|\bar{x}_3\bar{\zeta}},\ 1/b_B,\ 1/b\right\}. \tag{B.16}$$

The hard functions are written as

$$\begin{aligned} h_{a1}(x_B,z,b_B,b) &= K_0(m_B\sqrt{x_B z}b_B)\Big[\theta(b_B-b)K_0\Big(m_B\sqrt{z}b_B\Big)I_0\Big(m_B\sqrt{z}b\Big) \\ &\quad +(b\leftrightarrow b_B)\Big]S_t(z), \end{aligned} \tag{B.17}$$

$$\begin{aligned} h_{a2}(x_B,z,b_B,b) &= K_0\Big(m_B\sqrt{x_B z}b\Big)S_t(x_B) \\ &\quad\times\begin{cases} \frac{i\pi}{2}\big[\theta(b-b_B)H_0^{(1)}(m_B\sqrt{\zeta-x_B}b)J_0(m_B\sqrt{\zeta-x_B}b_B) \\ \qquad +(b\leftrightarrow b_B)\big], \qquad x_B<\zeta, \\ \big[\theta(b-b_B)K_0(m_B\sqrt{x_B-\zeta}b)I_0(m_B\sqrt{x_B-\zeta}b_B) \\ \qquad +(b\leftrightarrow b_B)\big], \qquad x_B\geq\zeta, \end{cases} \end{aligned} \tag{B.18}$$

$$\begin{aligned} h_{a3}(x_B,z,x_3,b_B,b_3) &= \Big[\theta(b_B-b_3)K_0\Big(m_B\sqrt{x_B z}b_B\Big)I_0\Big(m_B\sqrt{x_B z}b_3\Big)+(b_B\leftrightarrow b_3)\Big] \\ &\quad\times\begin{cases} \frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{z[\bar{\zeta}\bar{x}_3-x_B]}b_3), & \bar{\zeta}\bar{x}_3>x_B, \\ K_0(m_B\sqrt{z[x_B-\bar{\zeta}\bar{x}_3]}b_3), & \bar{\zeta}\bar{x}_3\leq x_B, \end{cases} \end{aligned} \tag{B.19}$$

$$\begin{aligned} h_{a4}(x_B,z,x_3,b_B,b_3) &= \Big[\theta(b_B-b_3)K_0\Big(m_B\sqrt{x_B z}b_B\Big)I_0\Big(m_B\sqrt{x_B z}b_3\Big)+(b_B\leftrightarrow b_3)\Big] \\ &\quad\times\begin{cases} \frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{z[x_3\bar{\zeta}-x_B]}b_3), & x_3\bar{\zeta}>x_B, \\ K_0(m_B\sqrt{z[x_B-x_3\bar{\zeta}]}b_3), & x_3\bar{\zeta}\leq x_B, \end{cases} \end{aligned} \tag{B.20}$$

$$\begin{aligned} h_{b1}(z,x_3,b,b_3) &= \left(\frac{i\pi}{2}\right)^2 H_0^{(1)}\Big(m_B\sqrt{\bar{z}(\zeta+x_3\bar{\zeta})}b_3\Big)S_t(z) \\ &\quad\times\Big[\theta(b-b_3)H_0^{(1)}\Big(m_B\sqrt{1-z}b\Big)J_0\Big(m_B\sqrt{1-z}b_3\Big)+(b\leftrightarrow b_3)\Big], \end{aligned} \tag{B.21}$$

$$\begin{aligned} h_{b2}(z,x_3,b,b_3) &= \left(\frac{i\pi}{2}\right)^2 H_0^{(1)}\Big(m_B\sqrt{\bar{z}(\zeta+x_3\bar{\zeta})}b\Big)S_t(x_3)\Big[\theta(b-b_3) \\ &\quad\times H_0^{(1)}\Big(m_B\sqrt{\zeta+x_3\bar{\zeta}}b\Big)J_0\Big(m_B\sqrt{\zeta+x_3\bar{\zeta}}b_3\Big)+(b\leftrightarrow b_3)\Big], \end{aligned} \tag{B.22}$$

$$\begin{aligned} h_{b3}(x_B,z,x_3,b_B,b) &= \frac{i\pi}{2}K_0\Big(m_B\sqrt{1-z(\bar{x}_3\bar{\zeta}-x_B)}b_B\Big)\Big[\theta(b_B-b) \\ &\quad\times H_0^{(1)}\Big(m_B\sqrt{\bar{z}(\zeta+x_3\bar{\zeta})}b_B\Big)J_0\Big(m_B\sqrt{\bar{z}(\zeta+x_3\bar{\zeta})}b\Big)+(b_B\leftrightarrow b)\Big], \end{aligned} \tag{B.23}$$

$$\begin{aligned} h_{b4}(x_B,z,x_3,b_B,b) &= \frac{i\pi}{2}\Big[\theta(b_B-b)H_0^{(1)}\Big(m_B\sqrt{\bar{z}(\zeta+x_3\bar{\zeta})}b_B\Big) \\ &\quad\times J_0\Big(m_B\sqrt{\bar{z}(\zeta+x_3\bar{\zeta})}b\Big)+(b_B\leftrightarrow b)\Big] \\ &\quad\times\begin{cases} \frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{\bar{z}(\zeta+x_3\bar{\zeta}-x_B)}b_B), & x_B<\zeta+x_3\bar{\zeta}, \\ K_0(m_B\sqrt{\bar{z}(x_B-\zeta-x_3\bar{\zeta})}b_B), & x_B\geq\zeta+x_3\bar{\zeta}, \end{cases} \end{aligned} \tag{B.24}$$

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$$\begin{aligned} h_{c1}(x_B,x_3,b_B,b_3) &= K_0(m_B\sqrt{x_Bx_3\bar\zeta}b_B)\Big[\theta(b_B-b_3)K_0\Big(m_B\sqrt{x_3\bar\zeta}b_B\Big) \\ &\quad\times I_0\Big(m_B\sqrt{x_3\bar\zeta}b_3\Big)+(b_3\leftrightarrow b_B)\Big]S_t(x_3), \end{aligned} \tag{B.25}$$

$$h_{c2}(x_B,x_3,b_B,b_3) = h_{c1}(x_3,x_B,b_3,b_B), \tag{B.26}$$

$$\begin{aligned} h_{c3}(x_B,z,x_3,b_B,b) &= \Big[\theta(b_B-b)K_0\Big(m_B\sqrt{x_Bx_3\bar\zeta}b_B\Big)I_0\Big(m_B\sqrt{x_Bx_3\bar\zeta}b\Big)+(b_B\leftrightarrow b)\Big] \\ &\quad\times\begin{cases} \frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{(1-x_B-z)[x_3\bar\zeta+\zeta]}b), & x_B+z<1, \\ K_0(m_B\sqrt{(x_B+z-1)[x_3\bar\zeta+\zeta]}b), & x_B+z\geq 1, \end{cases} \end{aligned} \tag{B.27}$$

$$\begin{aligned} h_{c4}(x_B,z,x_3,b_B,b) &= \Big[\theta(b_B-b)K_0\Big(m_B\sqrt{x_Bx_3\bar\zeta}b_B\Big)I_0\Big(m_B\sqrt{x_Bx_3\bar\zeta}b\Big)+(b_B\leftrightarrow b)\Big] \\ &\quad\times\begin{cases} \frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{x_3(z-x_B)\bar\zeta}b), & x_B<z, \\ K_0(m_B\sqrt{x_3(x_B-z)\bar\zeta}b), & x_B\geq z, \end{cases} \end{aligned} \tag{B.28}$$

$$\begin{aligned} h_{d1}(z,x_3,b,b_3) &= \left(\frac{i\pi}{2}\right)^2 H_0^{(1)}\Big(m_B\sqrt{\bar x_3z\bar\zeta}b\Big)S_t(x_3)\Big[\theta(b-b_3) \\ &\quad\times H_0^{(1)}\Big(m_B\sqrt{1-x_3\bar\zeta}b\Big)J_0\Big(m_B\sqrt{1-x_3\bar\zeta}b_3\Big)+(b\leftrightarrow b_3)\Big], \end{aligned} \tag{B.29}$$

$$\begin{aligned} h_{d2}(z,x_3,b,b_3) &= \left(\frac{i\pi}{2}\right)^2 H_0^{(1)}(m_B\sqrt{\bar x_3z\bar\zeta}b_3)S_t(z) \\ &\quad\times\Big[\theta(b-b_3)H_0^{(1)}\Big(m_B\sqrt{z\bar\zeta}b\Big)J_0\Big(m_B\sqrt{z\bar\zeta}b_3\Big)+(b\leftrightarrow b_3)\Big], \end{aligned} \tag{B.30}$$

$$\begin{aligned} h_{d3}(x_B,z,x_3,b_B,b) &= \frac{i\pi}{2}K_0(m_B\sqrt{1-x_3(1-x_B-z)\bar\zeta+(x_B+z-1)\zeta}b_B) \\ &\quad\times\Big[\theta(b_B-b)H_0^{(1)}\Big(m_B\sqrt{\bar x_3z\bar\zeta}b_B\Big)J_0\Big(m_B\sqrt{\bar x_3z\bar\zeta}b\Big)+(b_B\leftrightarrow b)\Big], \end{aligned} \tag{B.31}$$

$$\begin{aligned} h_{d4}(x_B,z,x_3,b_B,b) &= \frac{i\pi}{2}\Big[\theta(b_B-b)H_0^{(1)}\Big(m_B\sqrt{\bar x_3z\bar\zeta}b_B\Big)J_0\Big(m_B\sqrt{\bar x_3z\bar\zeta}b\Big)+(b_B\leftrightarrow b)\Big] \\ &\quad\times\begin{cases} \frac{i\pi}{2}H_0^{(1)}(m_B\sqrt{\bar x_3(z-x_B)\bar\zeta}b_B), & x_B<z, \\ K_0(m_B\sqrt{\bar x_3(x_B-z)\bar\zeta}b_B), & x_B\geq z, \end{cases} \end{aligned} \tag{B.32}$$

where $H_0^{(1)}(\chi)=J_0(\chi)+iY_0(\chi)$. The factor $S_t(\chi)$ with the expression [158]

$$S_t(\chi)=\frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[\chi(1-\chi)]^c, \tag{B.33}$$

resums the threshold logarithms $\ln^2\chi$ appearing in the hard kernels to all orders, and the parameter c has its expression as $c=0.04Q^2-0.51Q+1.87$ with Q^2 the invariant mass square of the final state f in the $B\to f$ transition [143, 159].

The evolution factors in the factorization expressions are given by

$$E_{a12}(t) = \alpha_s(t)\exp[-S_B(t)-S_{K_0^*}(t)], \tag{B.34}$$

$$E_{a34}(t) = \alpha_s(t)\exp[-S_B(t)-S_{K_0^*}(t)-S_h]|_{b=b_B}, \tag{B.35}$$

$$E_{b12}(t) = \alpha_s(t)\exp[-S_{K_0^*} - S_h(t)], \tag{B.36}$$

$$E_{b34}(t) = \alpha_s(t)\exp[-S_B(t) - S_{K_0^*}(t) - S_h]|_{b_3=b}, \tag{B.37}$$

$$E_{c12}(t) = \alpha_s(t)\exp[-S_B(t) - S_{K_0^*}(t)], \tag{B.38}$$

$$E_{c34}(t) = \alpha_s(t)\exp[-S_B(t) - S_{K_0^*}(t) - S_h]|_{b_3=b_B}, \tag{B.39}$$

$$E_{d12}(t) = E_{b12}(t), \tag{B.40}$$

$$E_{d34}(t) = E_{b34}(t), \tag{B.41}$$

in which the Sudakov exponents are defined as

$$S_B = s\left(x_B\frac{m_B}{\sqrt{2}}, b_B\right) + \frac{5}{3}\int_{1/b_B}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \tag{B.42}$$

$$S_{K_0^*} = s\left(z\frac{m_B}{\sqrt{2}}, b\right) + s\left((1-z)\frac{m_B}{\sqrt{2}}, b\right) + 2\int_{1/b}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \tag{B.43}$$

$$S_h = s\left(x_3\frac{m_B}{\sqrt{2}}, b_3\right) + s\left((1-x_3)\frac{m_B}{\sqrt{2}}, b_3\right) + 2\int_{1/b_3}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \tag{B.44}$$

with the quark anomalous dimension $\gamma_q = -\alpha_s/\pi$. The explicit form for the function $s(Q,b)$ is [152]

$$\begin{aligned} s(Q,b) =& \frac{A^{(1)}}{2\beta_1}\hat{q}\ln\left(\frac{\hat{q}}{\hat{b}}\right) - \frac{A^{(1)}}{2\beta_1}\left(\hat{q}-\hat{b}\right) + \frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{\hat{b}}-1\right) \\ & - \left[\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1}\ln\left(\frac{e^{2\gamma_E-1}}{2}\right)\right]\ln\left(\frac{\hat{q}}{\hat{b}}\right) + \frac{A^{(1)}\beta_2}{4\beta_1^3}\hat{q}\left[\frac{\ln(2\hat{q})+1}{\hat{q}} - \frac{\ln(2\hat{b})+1}{\hat{b}}\right] \\ & + \frac{A^{(1)}\beta_2}{8\beta_1^3}\left[\ln^2(2\hat{q}) - \ln^2(2\hat{b})\right], \end{aligned} \tag{B.45}$$

with the variables are

$$\hat{q} \equiv \ln[Q/(\sqrt{2}\Lambda)], \quad \hat{b} \equiv \ln[1/(b\Lambda)], \tag{B.46}$$

and the coefficients $A^{(i)}$ and β_i are

$$\begin{aligned} \beta_1 =& \frac{33-2n_f}{12}, \quad \beta_2 = \frac{153-19n_f}{24}, \quad A^{(1)} = \frac{4}{3}, \\ A^{(2)} =& \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3}\beta_1\ln\left(\frac{1}{2}e^{\gamma_E}\right), \end{aligned} \tag{B.47}$$

where n_f is the number of the quark flavors and γ_E is the Euler constant.

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