

# Generalised $\mu$ - $\tau$ symmetries and calculable gauge kinetic and mass mixing in $U(1)_{L_\mu-L_\tau}$ models

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**ABSTRACT:** Extensions of the standard model with a  $U(1)$  gauge symmetry contain gauge invariant kinetic mixing,  $\sin\chi$ , and gauge non-invariant mass mixing,  $\delta M^2$ , between the hypercharge and the new gauge boson  $Z'$ . These represent a priori incalculable but phenomenologically important parameters of the theory. They become calculable if there exist spontaneously or softly broken symmetries which forbid them at tree level but allow their generation at the loop level. We discuss various symmetries falling in this category in the context of the gauged  $L_\mu - L_\tau$  models and their interplay with lepton mixing. It is shown that one gets phenomenologically inconsistent lepton mixing parameters if these symmetries are exact. Spontaneous breaking of these symmetries can lead to consistent lepton mixing and also generates finite and calculable values of these parameters at one or two loop order depending on the underlying symmetry. We calculate these parameters in two specific cases: (i) the standard seesaw model with  $\mu$ - $\tau$  symmetry broken by the masses of the right-handed neutrinos and (ii) in a model containing a pair of vectorlike charged leptons which break  $\mu$ - $\tau$  symmetry. In case (i), the right-handed neutrinos are the only source of gauge mixing. The kinetic mixing parameters are suppressed and vanish if the right-handed neutrinos decouple from the theory. In contrast, there exists a finite gauge mixing in case (ii) which survives even when the masses of vectorlike leptons are taken to infinity, exhibiting non-decoupling behaviour. The seesaw model discussed here represents a complete framework with practically no kinetic mixing and hence can survive a large number of experimental probes used to rule out specific ranges in the coupling  $g'$  and mass  $M_{Z'}$ . The model can generate non-universality in tau decays, which can be tested in future experiments.

**KEYWORDS:** Beyond Standard Model, Discrete Symmetries, Gauge Symmetry

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**1 Introduction**

The observed deviations from the Standard Model (SM) predictions in the anomalous magnetic moment of muon,  $B$  meson decays, and the need to explain the origin of the dark matter in the universe has led to various extensions of the SM gauge symmetry. One of the simplest extensions corresponds to the addition of a U(1) gauge group and the most economical among them are the ones which do not require extension in the fermionic content of the SM. Three such possibilities corresponding to differences in the leptonic flavour indices  $L_\alpha - L_\beta$  have been identified long ago [1–3]. Most conspicuous of these three choices is the U(1) gauge group corresponding to  $L_\mu - L_\tau$ . In the simplest form, the gauge boson of  $L_\mu - L_\tau$  does not couple to quarks and the first generation leptons, thereby avoiding many constraints coming from these sectors. Phenomenology of  $L_\mu - L_\tau$  gauge symmetry has been extensively discussed in a number of papers (see for incomplete list of references [4, 5, 5–34]) in various contexts.

The additional U(1) symmetry may be broken at a scale smaller than the electroweak scale in which case the new interactions are constrained by a variety of low energy processes. The new gauge boson having mass in the range 100-400 MeV is advocated (see, for example, [35]) as an interesting possibility in case of the  $L_\mu - L_\tau$  symmetry. Such a gauge boson is consistent with various constraints from the laboratory experiments and may also explain the possible discrepancy between the measured value of the muon ( $g - 2$ ) and that predicted in the SM [5]. Alternatively, if  $U(1)_{L_\mu - L_\tau}$  is broken at a scale significantly larger than the electroweak scale, then all the effects associated with the new gauge boson would be suppressed by its mass. Such effects would appear as non-renormalizable operators in

the effective theory below the  $L_\mu - L_\tau$  breaking scale. An exception to this, in case of all the U(1) gauge groups, is a dimension four operator allowed by gauge symmetries. It is conveniently parameterized as [36–38]

$$-\frac{\sin \chi}{2} F_{\mu\nu}^B F_{Z'}^{\mu\nu} \tag{1.1}$$

where  $F_{\mu\nu}^B, F_{Z'}^{\mu\nu}$  are the field strength tensors for the hypercharge and the  $Z'$  gauge bosons respectively. An associated operator which can result after breaking of hypercharge and U(1) symmetry is the mass mixing among two gauge bosons  $B$  and  $Z'$ :

$$\delta M^2 B^\mu Z'_\mu . \tag{1.2}$$

These two parameters provide a window into new gauge symmetry if it is broken at a very large scale.

The Kinetic Mixing (KM) parameters,  $\sin \chi$  and  $\delta M^2$ , are arbitrary a priori but can be constrained from various experiments. The main effect of these parameters is to mix the additional gauge boson with  $Z$  thereby coupling quarks and electron to  $Z'$  and changing the existing couplings of  $Z$  to fermions. This leads to observable effects in precision electroweak tests, atomic parity violation [38], neutrino trident production [39], the low energy elastic scattering of the solar neutrino with electrons observed in Borexino [40] and coherent elastic  $\nu N$  scattering [41]. One obtains significant bounds on  $\sin \chi$  from these processes. These are model dependent. A summary of various constraints can be found, for example, in [42]. In models with a diagonal charged lepton mass matrix, one obtains [35, 42]:  $\sin \chi \sim 10^{-5} - 10^{-6}$  for  $M_{Z'} \sim 100 - 200$  MeV. One can obtain some meaningful predictions for  $\sin \chi$  and  $\delta M^2$  by invoking discrete symmetry which forbids them at the tree level. If this symmetry remains unbroken, then the  $L_\mu - L_\tau$  symmetry broken at a very high scale would remain completely hidden. On the other hand, the spontaneous breaking of such discrete symmetry would lead to calculable values for the said parameters. One possible symmetry forbidding eqs. (1.1), (1.2) was considered first in [4]. This corresponds to interchanging  $\mu$  and  $\tau$  degrees accompanied by a change in sign of  $Z'$ . One could consider various generalizations of this symmetry any of which can be used to forbid interactions in eqs. (1.1), (1.2) at the tree level. One of the aims of this paper is to discuss possible classes of symmetries which forbid eqs. (1.1), (1.2) and their implications for the leptonic mixing. The symmetries invoked to forbid KM parameters directly influence the neutrino mixing pattern since they also constrain the leptonic Yukawa couplings. We show that none of the symmetries used here to forbid the KM parameters at tree level can remain unbroken if one is to obtain consistent mixing angles and CP violation in the leptonic sector. The breaking of these symmetries then generates the KM parameters at the 1-loop or at the 2-loop level as we discuss.

KM parameter  $\sin \chi$  gets generated at the 1-loop level by the charged leptons in the standard scenario adopted in many works [20, 22, 35, 36, 43]. This happens however for a specific case in which the charged lepton mass matrix is  $L_\mu - L_\tau$  invariant and hence diagonal and break  $\mu$ - $\tau$  symmetry. The neutrino mass matrix in this case cannot also be invariant under the  $\mu$ - $\tau$  symmetry if it is to reproduce the observed leptonic mixing angles. One gets an additional contribution to KM from this mass matrix. The neutrino

contribution to  $\sin \chi$  is suppressed by the neutrino masses, i.e.  $\sin \chi \sim \mathcal{O}(\frac{m^2}{q^2})$ , but the contribution to  $\delta M^2$  contains a divergent piece if the charged leptons and three light Majorana neutrinos are the only sources which generate eqs. (1.1), (1.2). This makes the resulting  $Z$ - $Z'$  mixing incalculable. The divergent contribution can be removed only in a complete theory with spontaneously or softly broken  $\mu$ - $\tau$  symmetry. The other aim of the paper is to provide explicit models within which all the infinities which would arise through ad-hoc breaking of  $\mu$ - $\tau$  symmetry vanish. We present two specific examples, one which leads to unobservably small and the other with a fairly large value of  $\sin \chi$ . The first example is the standard seesaw model in which the source of  $\mu$ - $\tau$  breaking is confined in the Right-Handed (RH) neutrino mass matrix. The standard contribution from the charged leptons considered in the literature actually vanish in this case. We present detailed calculations showing that one gets finite  $\sin \chi$  and  $\delta M^2$  at the 1-loop from the neutrino sector. Both these parameters are suppressed in the model and vanish in the limit of very large RH neutrino masses. However,  $\delta M^2$  can get finite non-decoupling contribution, independent of the RH neutrino masses, in the presence of  $\mu$ - $\tau$  symmetry breaking in the Dirac neutrino mass matrix. The second example we discuss corresponds to adding the charged vectorlike fermions to the SM. Their mass terms, allowed by gauge symmetry, provide the only source of the  $\mu$ - $\tau$  (or other analogous symmetry) breaking and are responsible for generation of finite contribution to mixing parameters. This model shows the non-decoupling effects and contains a direct contribution from the vectorlike fermions, which does not vanish when vectorlike masses are taken to infinity. This contribution thus could be large.

The paper is organized as follows. We discuss symmetries which lead to vanishing KM in section 2 and their consequences on the leptonic mixing in section 3. Evaluation of KM at 1-loop in a general framework is given in section 4 and two specific examples are discussed in details in section 5. We summarize our results in 6 and give a short discussion of the already existing literature of the phenomenology of the type of models discussed here.

## 2 Symmetries for vanishing kinetic mixing in $L_\mu - L_\tau$ model

The minimal version of the  $L_\mu - L_\tau$  model is obtained by assigning equal and opposite U(1) charges to the leptonic doublets  $L'_\mu$  and  $L'_\tau$ , where  $L'_\alpha = (\nu'_{\alpha L}, l'_{\alpha L})^T$ ,  $\alpha = e, \mu, \tau$ . The right handed charged leptons  $l'_{\mu R}$  and  $l'_{\tau R}$  carry the same  $L_\mu - L_\tau$  charges as  $L'_\mu$  and  $L'_\tau$ , respectively. Rest of the SM fermions are neutral under the additional U(1). The neutral current interactions of leptons are then given by

$$-\mathcal{L}_{\text{NC}} = g_Y B_\mu \left( -\frac{1}{2} \bar{L}'_\alpha \gamma^\mu L'_\alpha - \bar{l}'_{\alpha R} \gamma^\mu l'_{\alpha R} \right) + g' Z'_\mu \left( \bar{l}'_\mu \gamma^\mu l'_\mu - \bar{l}'_\tau \gamma^\mu l'_\tau + \bar{\nu}'_{\mu L} \gamma^\mu \nu'_{\mu L} - \bar{\nu}'_{\tau L} \gamma^\mu \nu'_{\tau L} \right), \tag{2.1}$$

$\mathcal{L}_{\text{NC}}$  is invariant under the following transformations [4]:

$$L'_\mu \leftrightarrow L'_\tau, \quad l'_{\mu R} \leftrightarrow l'_{\tau R}, \quad B_\mu \rightarrow B_\mu, \quad Z'_\mu \rightarrow -Z'_\mu. \tag{2.2}$$

This symmetry acts as the standard  $\mu$ - $\tau$  interchange symmetry on the leptons. It also changes the sign of the new gauge boson in addition. The  $\mu$ - $\tau$  interchanges symmetry

forbids the kinetic and mass mixing terms of eqs. (1.1), (1.2) at tree level. If this symmetry is also respected by the Higgs sector and the Yukawa couplings of leptons then the entire Lagrangian is invariant under it, and the KM remains absent to all orders. If this is not the case, the KM will get generated at the loop level. It is easy to derive conditions under which the leptonic contribution to KM remains zero at 1-loop.

We collectively represent  $l'_{\alpha L}$ ,  $l'_{\alpha R}$ ,  $\nu'_{\alpha L}$  as  $f'_\alpha$ . The mixing matrix  $U_f$  connecting the mass eigenstates  $f_i$  of fermions to the weak eigenstates  $f'_\alpha$  is defined as

$$f'_\alpha = (U_f)_{\alpha i} f_i. \quad (2.3)$$

In the mass basis, the couplings to  $Z'$  boson given in eq. (2.1) change to

$$g' Z'_\mu F_{ij}^f \bar{f}_i \gamma^\mu f_j, \quad (2.4)$$

with

$$F_{ij}^f = (U_f)_{\mu i}^* (U_f)_{\mu j} - (U_f)_{\tau i}^* (U_f)_{\tau j}. \quad (2.5)$$

The diagonal couplings,  $F_{ii}^f$ , vanish if

$$|(U_f)_{\mu i}|^2 = |(U_f)_{\tau i}|^2. \quad (2.6)$$

This equation, termed as the  $\mu$ - $\tau$  reflection symmetry, has useful phenomenological consequences [44–46] when applied to the leptonic mixing matrix  $U_{\text{PMNS}}$ . It has implication for the KM as well. Since  $B$  has only flavour diagonal couplings, and  $Z'$  has only off-diagonal couplings when eq. (2.6) is satisfied, the fermion loop connecting them in vacuum polarization diagram is absent and the KM cannot arise at the 1-loop level. This however requires that eq. (2.6) holds individually for all the fermion mixing matrices  $U_{l_L}$ ,  $U_{l_R}$  and  $U_{\nu_L}$ . Eq. (2.6) in this case represents sufficient conditions for vanishing of the leptonic contribution to KM parameters at the 1-loop level.

The  $\mu$ - $\tau$  interchange symmetry is a special case leading to eq. (2.6). If the Yukawa couplings of leptons and the Higgs sector respect this symmetry then the Majorana neutrino and the charged lepton mass matrices  $M_\nu$  and  $M_l$  respectively satisfy [47]

$$S^T M_\nu S = M_\nu, \quad S^\dagger M_l M_l^\dagger S = M_l M_l^\dagger, \quad S^\dagger M_l^\dagger M_l S = M_l^\dagger M_l, \quad (2.7)$$

where

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad (2.8)$$

is the  $\mu$ - $\tau$  interchange matrix. These equations lead to the corresponding diagonalizing matrices as

$$U_{l_L} = U^{\mu\tau}(\theta_L) Q_L, \quad U_{l_R} = U^{\mu\tau}(\theta_R) Q_R, \quad U_{\nu_L} = U^{\mu\tau}(\theta_\nu) Q_\nu, \quad (2.9)$$

where  $Q_L$ ,  $Q_R$ ,  $Q_\nu$  are diagonal phase matrices and

$$U^{\mu\tau}(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \frac{1}{\sqrt{2}} \sin \theta & \frac{1}{\sqrt{2}} \cos \theta & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \sin \theta & \frac{1}{\sqrt{2}} \cos \theta & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (2.10)$$

Each of the unitary matrices in eq. (2.9) satisfy the conditions depicted in eq. (2.6). Both the kinetic and mass mixing vanish in this case to all orders as long as symmetry in eq. (2.2) remains unbroken.

Eq. (2.6) provides clue to other possible symmetries which can be used to forbid KM. One such well-studied example [45] corresponds to imposing the following symmetry on the leptonic fields

$$L'_e \leftrightarrow L_e'^{CP}, \quad e'_R \leftrightarrow e_R'^{CP}, \quad L'_\mu \leftrightarrow L_\tau'^{CP}, \quad l'_{\mu R} \leftrightarrow l'_{\tau R}{}^{CP}, \quad (2.11)$$

where  $f'^{CP} = \gamma^0 C \bar{f}'^T$ . If this symmetry is respected by the vacuum and Yukawa interactions, then the leptonic mass matrices satisfy

$$S^T M_\nu S = M_\nu^*, \quad S^\dagger M_l M_l^\dagger S = (M_l M_l^\dagger)^*, \quad S^\dagger M_l^\dagger M_l S = (M_l^\dagger M_l)^*. \quad (2.12)$$

The first of the above has been extensively studied in the diagonal basis of the charged leptons [44, 45]. Forbidding kinetic mixing would require that the entire eq. (2.12) be satisfied simultaneously. Above conditions imply [44, 45] that the mixing matrices  $U_{l_{L,R}}$ ,  $U_\nu$  have the form:

$$U_{l_L} = U_{l_L}^{\text{HS}} Q_L, \quad U_{l_R} = U_{l_R}^{\text{HS}} Q_R, \quad U_{\nu L} = U_{\nu L}^{\text{HS}} X_\nu, \quad (2.13)$$

where  $Q_{L,R}$  are diagonal matrices of unphysical phases and  $X_\nu$  is a diagonal matrix with  $X_\nu^2 = 1$ . The matrices  $U_{l_L}^{\text{HS}}$ ,  $U_{l_R}^{\text{HS}}$  and  $U_{\nu L}^{\text{HS}}$  possess the Harrison-Scott form [44]

$$U^{\text{HS}} = \begin{pmatrix} x_1 & x_2 & x_3 \\ z_1 & z_2 & z_3 \\ z_1^* & z_2^* & z_3^* \end{pmatrix}. \quad (2.14)$$

with  $x_{1,2,3}$  real. The above form of  $U_{l_{L,R}}$ ,  $U_{\nu L}$  satisfies eq. (2.6) and the KM cannot arise at the 1-loop level. Eq. (2.12) is more general and can forbid the KM to all orders. This follows from the fact that the neutral current interactions in eq. (2.1) are invariant if the leptonic symmetry, eq. (2.11), is supplemented with the following transformation on the gauge bosons  $B$ ,  $Z'$ :

$$Z'_\mu \rightarrow Z'^\mu, \quad B_\mu \rightarrow -B^\mu. \quad (2.15)$$

Action of this symmetry on  $B_\mu$  corresponds to the standard CP transformation and thus CP invariance of the gauge interactions assures the above mentioned symmetry for the hypercharge current of fermions and scalars. But the  $Z'$  needs to be transformed in the opposite manner compared to  $B$  to make the corresponding term in eq. (2.1) invariant under this generalised CP. This ensures that the KM parameters remain zero to all orders as long as eq. (2.12) holds and the Higgs sector also respects appropriately defined  $\mu$ - $\tau$  reflection symmetry.

### 3 Vanishing kinetic mixing and leptonic mixing

The two examples of symmetries discussed in the previous section which forbid the KM in  $L_\mu - L_\tau$  model have implications on the leptonic mixing. It is known that the  $\mu$ - $\tau$  interchange or reflection symmetry when simultaneously imposed on the charged leptons

and the neutrinos do not lead to phenomenologically viable leptonic mixing. In the case of  $\mu$ - $\tau$  interchange symmetry, the obtained forms of  $U_{lL}$  and  $U_{\nu L}$  given in eq. (2.9) imply that the leptonic mixing matrix  $U_{\text{PMNS}} = U_{lL}^\dagger U_{\nu L}$  has vanishing atmospheric and reactor mixing angles [48]. This does not happen if one uses  $\mu$ - $\tau$  reflection symmetry to forbid KM. But in this case, one gets vanishing leptonic CP violation. This general result can be shown following the arguments given in [49] in a slightly different context. In the case of  $\mu$ - $\tau$  reflection symmetry,  $U_{lL}$  and  $U_{\nu L}$  given in eq. (2.13) diagonalize  $M_l M_l^\dagger$  and  $M_\nu$  of eq. (2.12), respectively. The  $U_{lL}^{\text{HS}}$  and  $U_{\nu L}^{\text{HS}}$  satisfy

$$S U_{lL}^{\text{HS}} = (U_{lL}^{\text{HS}})^*, \quad S U_{\nu L}^{\text{HS}} = (U_{\nu L}^{\text{HS}})^*. \quad (3.1)$$

As a consequence,

$$U_{\text{PMNS}}^* = U_{lL}^\dagger S^2 U_{\nu L} = U_{lL}^T U_{\nu L}^* = U_{\text{PMNS}}, \quad (3.2)$$

and thus leads to a real  $U_{\text{PMNS}}$ . Since  $X_\nu$  in eq. (2.13) is trivial, the Dirac and Majorana phases vanish and there is no CP violation in the lepton sector.  $U_{\text{PMNS}}$  matrix in this case is otherwise general and allows arbitrary values of all the three mixing angles. If the leptonic CP violation is found to be absent, then the  $\mu$ - $\tau$  reflection symmetry can provide an explanation of this and would also forbid KM parameters to all orders. However, one would need to break this symmetry if non-trivial CP violation is to be obtained.

Both the above discussed symmetries forbid KM parameters at the 1-loop level but fail in generating phenomenologically acceptable leptonic mixing. This can be changed by generalizing the definition of  $\mu$ - $\tau$  interchange or reflection symmetries. It is assumed that these symmetries are symmetries of the Yukawa interactions but get broken in such a way that the leptonic mass matrices  $M_l$  and  $M_\nu$  are invariant under different residual symmetries. The idea of using different residual symmetries for the charged leptons and neutrinos is extensively used in constraining leptonic mixing patterns through discrete symmetries (see [50–53] for reviews). Denoting these symmetries by  $S_{l_{L,R}}$  and  $S_{\nu L}$ , we demand

$$S_{lL}^\dagger M_l M_l^\dagger S_{lL} = (M_l M_l^\dagger)^*, \quad S_{lR}^\dagger M_l^\dagger M_l S_{lR} = (M_l^\dagger M_l)^*, \quad S_{\nu L}^T M_\nu S_{\nu L} = M_\nu^*. \quad (3.3)$$

The symmetry operators  $S_{lL}$ ,  $S_{lR}$  and  $S_{\nu L}$  are required to constraint the diagonalizing matrices  $U_{lL}$ ,  $U_{lR}$  and  $U_{\nu L}$  such that each satisfy eq. (2.6) needed to obtain vanishing KM parameters at 1-loop. The most general solution of eq. (2.6) can be written as

$$U = P U^{\text{HS}} Q, \quad (3.4)$$

where  $P$  and  $Q$  are diagonal phase matrices and  $U^{\text{HS}}$  is defined in eq. (2.14). Choosing the above form for  $U_{lL}$ ,  $U_{lR}$  and  $U_{\nu L}$ , we conveniently define

$$U_{lL} = P_L U_L^{\text{HS}} Q_L, \quad U_{lR} = P_R U_R^{\text{HS}} Q_R, \quad U_{\nu L} = P_\nu U_\nu^{\text{HS}} X_\nu, \quad (3.5)$$

with

$$P_L = \text{Diag.}(1, e^{i\phi_{1L}}, e^{i\phi_{2L}}), \quad P_R = \text{Diag.}(1, e^{i\phi_{1R}}, e^{i\phi_{2R}}), \quad P_\nu = \text{Diag.}(1, e^{i\phi_{1\nu}}, e^{i\phi_{2\nu}}), \quad (3.6)$$

and  $Q_L, Q_R$  are diagonal phase matrices.  $X_\nu$  is a trivial diagonal matrix with elements  $\pm 1$  as before. The mass matrices which can be diagonalized by the above unitary matrices have the form:

$$M_\nu = \tilde{U}_{\nu L}^* D_\nu \tilde{U}_{\nu L}^\dagger, \quad M_l M_l^\dagger = \tilde{U}_{lL} |D_l|^2 \tilde{U}_{lL}^\dagger, \quad M_l^\dagger M_l = \tilde{U}_{lR} |D_l|^2 \tilde{U}_{lR}^\dagger. \quad (3.7)$$

The above definitions together with eqs. (3.5) imply

$$S_L \equiv P_L S P_L, \quad S_R \equiv P_R S P_R, \quad S_\nu \equiv P_\nu S P_\nu \quad (3.8)$$

and satisfy eq. (3.3). These symmetries thus represent the generalization of the  $\mu$ - $\tau$  reflection symmetries. One recovers the  $\mu$ - $\tau$  reflection symmetry if the phases satisfy  $\phi_{1a} = -\phi_{2a}$  for  $a = L, R, \nu$ . By construction, the generalized symmetries lead to mixing matrices which assure vanishing KM parameters at 1-loop. Moreover, the newly defined symmetries do not satisfy eq. (3.1) used in proving real  $U_{\text{PMNS}}$  as long as  $P_L \neq P_\nu$  in eq. (3.5). One therefore gets a non-real and general  $U_{\text{PMNS}}$  which allows Dirac CP violation. The Majorana phases still remain zero due to triviality of  $X_\nu$  in eq. (3.5). The action of the above symmetries on the leptonic fields is given by

$$\begin{aligned} \mu'_L &\rightarrow e^{i(\phi_{1L} + \phi_{2L})} \tau'_L{}^{CP}, & \mu'_R &\rightarrow e^{i(\phi_{1R} + \phi_{2R})} \tau'_R{}^{CP}, & \nu'_{\mu L} &\rightarrow e^{i(\phi_{1\nu} + \phi_{2\nu})} \nu'_{\tau L}{}^{CP}, \\ \tau'_L &\rightarrow e^{i(\phi_{1L} + \phi_{2L})} \mu'_L{}^{CP}, & \tau'_R &\rightarrow e^{i(\phi_{1R} + \phi_{2R})} \mu'_R{}^{CP}, & \nu'_{\tau L} &\rightarrow e^{i(\phi_{1\nu} + \phi_{2\nu})} \nu'_{\mu L}{}^{CP}. \end{aligned} \quad (3.9)$$

The neutral current interactions given below in eq. (4.1) are invariant under these transformation if one also transforms the gauge fields as in eq. (2.15). This forbids KM at the tree and 1-loop level. But now the charged current interactions do not remain invariant under these symmetries when  $P_L \neq P_\nu$  in eq. (3.5). This would lead to kinetic mixing at the two loop level in general.

One can analogously define generalisation of the  $\mu$ - $\tau$  symmetry with similar consequences. This is given by

$$\hat{S}_L \equiv P_L S P_L^*, \quad \hat{S}_R \equiv P_R S P_R^*, \quad \hat{S}_\nu \equiv P_\nu S P_\nu^* \quad (3.10)$$

In this case eq. (3.3) is replaced by

$$\hat{S}_L^\dagger M_l M_l^\dagger \hat{S}_L = M_l M_l^\dagger, \quad \hat{S}_R^\dagger M_l^\dagger M_l \hat{S}_R = M_l^\dagger M_l, \quad \hat{S}_\nu^T M_\nu \hat{S}_\nu = M_\nu. \quad (3.11)$$

Again, the neutral current couplings are invariant under this symmetry and do not lead to KM at the tree and 1-loop level but the charged current interactions violate this symmetry in general. The leptonic mixing matrix is quite general in this case and unlike in the case of  $\mu$ - $\tau$  interchange symmetry, one does not get the unwanted result of vanishing  $\theta_{23}$  and  $\theta_{13}$ .

#### 4 Kinetic mixing: general considerations

As discussed in the previous section, the exact  $\mu$ - $\tau$  interchange or reflection symmetry forbidding the KM in gauged  $L_\mu - L_\tau$  is inconsistent with the observed lepton mixing pattern. One therefore needs to break these symmetries either in the charged lepton or in



the neutrino sector. In the absence of such symmetries, the KM gets generated at 1-loop level even if it is assumed to be absent at the tree level. In this section, we first derive a general formula for 1-loop induced kinetic and mass mixing in the SM extended with  $U(1)_X$  gauge symmetry. We then discuss their implications for  $L_\mu - L_\tau$  models.

Let  $f'_{aL}$  and  $f'_{aR}$  with  $a = 1, 2, \dots, n$  be  $n$  copies of left and right-handed fermions with hypercharges  $Y'_{La}$  and  $Y'_{Ra}$ , respectively. The corresponding  $U(1)_X$  charges are  $X'_{La}$  and  $X'_{Ra}$ . These  $n$  copies include three generations of the SM fermions with  $a = i, j = 1, 2, 3$  and  $(n - 3)$  additional fermions with  $a = m = 4, \dots, n$ . The neutral current interactions between these fermions and vector bosons of abelian symmetries are given by

$$-\mathcal{L}_{\text{NC}} = g_Y B_\mu \left( Y'_{La} \bar{f}'_{aL} \gamma^\mu f'_{aL} + Y'_{Ra} \bar{f}'_{aR} \gamma^\mu f'_{aR} \right) + g' Z'_\mu \left( X'_{La} \bar{f}'_{aL} \gamma^\mu f'_{aL} + X'_{Ra} \bar{f}'_{aR} \gamma^\mu f'_{aR} \right), \quad (4.1)$$

where  $g_Y$  and  $g'$  are the gauge couplings corresponding to  $U(1)_Y$  and  $U(1)_X$  gauge groups,<sup>1</sup> respectively. All the  $n$  fermions  $f'_{aL,R}$  of a given charge and helicity mix among themselves. The mass basis, denoted by  $f_{aL}$  and  $f_{aR}$ , is defined by

$$f'_{aL,R} = (\mathcal{U}_{f_{L,R}})_{ab} f_{bL,R}, \quad (4.2)$$

where  $\mathcal{U}_{f_{L,R}}$  are  $n \times n$  matrices. Eq. (4.1) can be rewritten in terms of the mass basis as

$$-\mathcal{L}_{\text{NC}} = g_Y B_\mu \left( Y_{Lab} \bar{f}_{aL} \gamma^\mu f_{bL} + Y_{Rab} \bar{f}_{aR} \gamma^\mu f_{bR} \right) + g' Z'_\mu \left( X_{Lab} \bar{f}_{aL} \gamma^\mu f_{bL} + X_{Rab} \bar{f}_{aR} \gamma^\mu f_{bR} \right), \quad (4.3)$$

where the matrices  $X_{L,R}$  and  $Y_{L,R}$  denote gauge charges in the mass basis. They are obtained as

$$\begin{aligned} X_{L,R} &= \mathcal{U}_{f_{L,R}}^\dagger X'_{L,R} \mathcal{U}_{f_{L,R}}, \\ Y_{L,R} &= \mathcal{U}_{f_{L,R}}^\dagger Y'_{L,R} \mathcal{U}_{f_{L,R}}, \end{aligned} \quad (4.4)$$

where  $X'_{L,R} = \text{Diag.}(X'_{L,R_1}, X'_{L,R_2}, \dots, X'_{L,R_n})$  and  $Y'_{L,R} = \text{Diag.}(Y'_{L,R_1}, Y'_{L,R_2}, \dots, Y'_{L,R_n})$ .

The interactions in eq. (4.3) can contribute to the mixing between the  $B$  and  $Z'$  bosons at loop level through the vacuum polarization effects. Denoting the amplitude of vacuum polarization as  $i\Pi_{BZ'}^{\mu\nu}(q^2)$ , it is parametrized as

$$\Pi_{BZ'}^{\mu\nu}(q^2) = (g^{\mu\nu} q^2 - q^\mu q^\nu) A_{BZ'} + g^{\mu\nu} B_{BZ'}. \quad (4.5)$$

Here, the parameter  $A_{BZ'}$  can be identified with KM while  $B_{BZ'}$  would give rise to mass mixing between the  $B$  and  $Z'$  bosons. Within this framework, 1-loop computation of the vacuum polarization diagrams gives

$$A_{BZ'} = \frac{g_Y g'}{4\pi^2} \left[ -\frac{1}{6} \text{Tr}(Y'_L X'_L + Y'_R X'_R) E + \sum_{a,b} (Y_{Lab} X_{Lba} + Y_{Rab} X_{Rba}) b_2[m_a, m_b, q] \right], \quad (4.6)$$

<sup>1</sup>The hypercharge is normalized such that the electric charge is  $Q = T_3 + Y'$  and  $g_Y = \frac{e}{\cos \theta_W}$ .

and

$$\begin{aligned}
 B_{BZ'} = \frac{g_Y g'}{8\pi^2} & \left[ \left( \frac{1}{2} \text{Tr}(D^2(\{Y_L, X_L\} + \{Y_R, X_R\})) - \text{Tr}(Y_L D X_R D + Y_R D X_L D) \right) E \right. \\
 & - \sum_{a,b} (Y_{Lab} X_{Lba} + Y_{Rab} X_{Rba}) (m_a^2 b_1[m_a, m_b, q] + m_b^2 b_1[m_b, m_a, q]) \\
 & \left. + \sum_{a,b} (Y_{Lab} X_{Rba} + Y_{Rab} X_{Lba}) m_a m_b b_0[m_a, m_b, q] \right]. \tag{4.7}
 \end{aligned}$$

Here,

$$E = \frac{2}{\epsilon} - \gamma + \ln(4\pi) - \ln(\mu^2), \tag{4.8}$$

and  $\epsilon = 4 - d$  in the dimensional regularization scheme. The parameter  $\mu$  is an arbitrary subtraction scale. The terms proportional to  $E$  in both  $A_{BZ'}$  and  $B_{BZ'}$  are divergent in four dimensions.  $m_a$  is the mass of  $a^{\text{th}}$  fermion and  $D = \text{Diag.}(m_1, m_2, \dots, m_n)$ . The loop integration functions  $b_0, b_1, b_2$  are listed as eq. (A.1) in appendix A.

$A_{BZ'}$  and  $B_{BZ'}$  in eqs. (4.6), (4.7) characterize 1-loop contributions to kinetic and mass mixing parameters. More explicitly, at 1-loop:

$$\sin \chi = (\sin \chi)_{\text{tree}} + \sum_f A_{BZ'}, \quad \delta M^2 = (\delta M^2)_{\text{tree}} + \sum_f B_{BZ'}, \tag{4.9}$$

where the sum is over different kind of fermions present in the underlying model. In the presence of non vanishing  $\sin \chi$  or  $\delta M^2$ , the gauge bosons  $B$  and  $Z'$  mix and their mass eigenstates  $\tilde{B}$  and  $\tilde{Z}'$  can be obtained as [38]:

$$\begin{aligned}
 \tilde{B} &= \cos \xi B + \sin(\xi + \chi) Z', \\
 \tilde{Z}' &= -\sin \xi B + \cos(\xi + \chi) Z', \tag{4.10}
 \end{aligned}$$

where

$$\tan 2\xi = \frac{-2 \cos \chi (\delta M^2 - M_B^2 \sin \chi)}{M_{Z'}^2 - M_B^2 \cos 2\chi + 2 \delta M^2 \sin \chi} \tag{4.11}$$

The angle  $\xi$  is a phenomenologically useful parameter which quantifies the overall effect of gauge boson mixing. Non-zero value of  $\xi$  gives rise to deviation in the neutral current couplings associated with the  $Z$  boson from their values predicted in the SM.<sup>2</sup>

As seen from eq. (4.6), the divergent part in  $A_{BZ'}$  vanishes if the fermions  $f_{aL,R}$  have universal hypercharges and  $\text{Tr}(X'_{L,R}) = 0$ . Consequently, finiteness of  $A_{BZ'}$  at 1-loop follows from the charge assignments of fermions and it does not require additional symmetry.<sup>3</sup> This is the case in the standard  $L_\mu - L_\tau$  models used in many works for generating KM at 1-loop through the charged lepton exchanges. One is left with finite and nonzero 1-loop contribution to  $\sin \chi$  in this case for non-vanishing diagonal elements in  $X_L$

<sup>2</sup>For mixing with the standard  $Z$  boson instead of  $B$ , the mixing angle  $\xi$  is obtained by the replacements  $\sin \chi \rightarrow -\sin \theta_W \sin \chi$  and  $M_B \rightarrow M_Z$  in eq. (4.11).

<sup>3</sup>Additional symmetries can play role in finiteness of KM at higher loops [54].

and  $X_R$ . For example, if the charged lepton mass matrix  $M_l$  is diagonal then one obtains the well-known [36] result

$$A_{BZ'} \approx -\frac{g_Y g'}{16\pi^2} \ln \frac{m_\mu^2}{m_\tau^2} \tag{4.12}$$

from eq. (4.6) in the limit  $q^2 \ll m_\mu^2$ . If  $\mu$ - $\tau$  interchange or reflection symmetry is imposed on  $M_l$  then the resulting condition eq. (2.6) leads to vanishing diagonal elements in  $X_{L,R}$  which gives  $A_{BZ'} = 0$ . Finiteness of  $A_{BZ'}$  obtained at 1-loop may not hold at higher loops if there is no  $\mu$ - $\tau$  interchange like symmetry or its breaking is hard.

The divergent part in  $B_{BZ'}$  does not vanish in general. If fermions  $f'_{aL,R}$  carry universal hypercharges  $Y'_{L,R}$ , then their contribution  $B_{BZ'}$  can be written as

$$\frac{g_Y g'}{8\pi^2} (Y'_L - Y'_R) \text{Tr}(D^2(X_L - X_R)) E. \tag{4.13}$$

This piece vanishes only under the specific circumstances: (a) universal masses  $m_a$  since  $\text{Tr}(X_{L,R}) = 0$ , (b) vectorial hypercharge, i.e.  $Y'_L = Y'_R$ , (c) vectorial  $Z'$  current, i.e.  $X_L = X_R$  or (d) generalised  $\mu$ - $\tau$  symmetry as defined by eq. (2.6) for which diagonal elements of  $X_{L,R}$  vanish individually. None of these conditions are automatically satisfied for the charged leptons with a general non-Hermitian mass matrix  $M_l$ . Only if  $M_l$  is Hermitian or possesses one of the symmetries discussed in the previous sections, the divergent piece in  $B_{BZ'}$  vanishes. Otherwise, the 1-loop contribution to  $B_{BZ'}$  is divergent. Thus, in spite of finite and calculable contribution from eq. (4.6), the charged lepton contribution to the  $Z$ - $Z'$  mixing remains incalculable. Similarly, for the neutrino sector, if the fields  $f'_{Li}$ ,  $f'_{Ri}$  represent the standard light Majorana neutrinos with  $f'_R = C\bar{f}'_L{}^T$  then  $Y'_L = -Y'_R$  and  $(X_L)_{ii} = -(X_R)_{ii}$  in eq. (4.13). As a consequence, the light Majorana neutrino contribution to  $B_{BZ'}$  is always divergent unless they are degenerate or the neutrino mass matrix is invariant under one of the symmetries discussed earlier. Since parameter  $B_{BZ'}$  contributes to  $\xi$ , the resultant  $Z$ - $Z'$  mixing remains divergent and incalculable in the minimal set up with general mass matrices for the charged leptons and neutrinos.

There exists one specific scenario for which the 1-loop expression of KM parameter as given in eq. (4.12) holds,  $Z$ - $Z'$  mixing is calculable, and the neutrino mixing is also consistent with the current results. This corresponds to assuming unbroken  $L_\mu - L_\tau$  symmetry for the charged leptons and  $\mu$ - $\tau$  reflection symmetry for the neutrino sector. In this case, neutrinos do not contribute to  $A_{BZ'}$  and  $B_{BZ'}$  at 1-loop as discussed in the previous section, the charged lepton contribution to  $B_{BZ'}$  vanishes and their contribution to  $A_{BZ'}$  is finite and given by eq. (4.12). Unbroken  $\mu$ - $\tau$  reflection symmetry in the neutrino sector predicts maximal atmospheric mixing angle as well as maximal Dirac CP violation.

The divergent part of  $B_{BZ'}$  can be renormalized by introducing suitable counter term as the  $\mu$ - $\tau$  symmetry is already broken in the effective framework. Hence, there is no reason for such counter terms to be not present in the theory. However, in the full ultraviolet completion of the model in which the  $\mu$ - $\tau$  interchange or reflection symmetry is restored, the divergences in the kinetic and mass mixing terms must not arise. In these models, the KM parameters are calculable, and its origin can be linked with the mechanism of

$\mu$ - $\tau$  symmetry breaking. We provide two characteristically different frameworks as the concrete realization of this statement in the next section.

## 5 Models of calculable kinetic mixing

We consider (A) the standard seesaw model and (B) a model with vectorlike charged leptons in which the underlying  $\mu$ - $\tau$  symmetry is broken spontaneously or softly leading to finite  $\sin \chi$  and  $\delta M^2$  at 1-loop. Both of these represent special cases of the general formalism discussed in the last section.

### 5.1 Kinetic mixing in the standard seesaw model

The model is the standard seesaw model augmented with a gauge  $L_\mu - L_\tau$  symmetry and a  $\mu$ - $\tau$  symmetry. Breaking of  $L_\mu - L_\tau$  occurs spontaneously through  $SU(2)_L \times U(1)_Y$  singlet fields. As a consequence, parameter  $\delta M_Z^2$  does not get generated at tree level even after breaking of the  $L_\mu - L_\tau$  and SM gauge symmetries. The charged lepton masses in the model are characterized by the following mass Lagrangian

$$-\mathcal{L}_m^l = \left( \bar{e}'_L \ \bar{\mu}'_L \ \bar{\tau}'_L \right) M_l \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} + \text{h.c.}, \quad (5.1)$$

with

$$M_l = v \begin{pmatrix} \lambda_{ee}^l & \lambda_{e\mu}^l & \lambda_{e\tau}^l \\ \lambda_{\mu e}^l & \lambda_{\mu\mu}^l & \lambda_{\mu\tau}^l \\ \lambda_{\tau e}^l & \lambda_{\tau\mu}^l & \lambda_{\tau\tau}^l \end{pmatrix}. \quad (5.2)$$

Here,  $v$  is the vacuum expectation value (VEV) of the standard model doublet assumed neutral under  $L_\mu - L_\tau$  and  $\mu$ - $\tau$  symmetry. The off-diagonal couplings  $\lambda_{\mu\tau}^l$ ,  $\lambda_{e\mu}^l$ ,  $\lambda_{\tau e}^l$  can be regarded as VEVs of the spurion fields with  $L_\mu - L_\tau$  charge 2,  $-1$  and  $1$ , respectively. It is assumed that these fields break  $L_\mu - L_\tau$  symmetry spontaneously but do not break the  $\mu$ - $\tau$  interchange symmetry which leads to the above form of  $M_l$ . Similarly, the Dirac neutrino mass matrix is also assumed to be invariant under the  $\mu$ - $\tau$  interchange symmetry and has the form

$$m_D = v \begin{pmatrix} \lambda_{11}^D & \lambda_{12}^D & \lambda_{12}^D \\ \lambda_{21}^D & \lambda_{22}^D & \lambda_{23}^D \\ \lambda_{21}^D & \lambda_{23}^D & \lambda_{22}^D \end{pmatrix}. \quad (5.3)$$

Non-zero off-diagonal couplings in  $m_D$  arise because of the spontaneous breaking of  $L_\mu - L_\tau$  symmetry.

Unlike in the case of  $M_l$  and  $m_D$ , the  $\mu$ - $\tau$  symmetry is assumed to be broken by the Majorana masses of RH neutrinos. This can be achieved by introducing an appropriately charged spurions field whose VEV break both the  $\mu$ - $\tau$  and  $L_\mu - L_\tau$  symmetries spontaneously. This allows a completely general form for the RH neutrino mass matrix  $M_R$  and thereby leads to a general lepton mixing matrix. We shall work out the radiatively generated KM parameters for this general matrix. Special cases can be obtained by restricting the structure of  $M_l$  and  $M_R$ . Specific neutrino mass structures and associated

phenomenology has been discussed in a number of papers [6, 55–57] in the context of the  $L_\mu - L_\tau$  symmetry. The neutrino mass Lagrangian is defined as

$$-\mathcal{L}_m^\nu = \frac{1}{2} n_L'^T C \mathcal{M}_\nu n_L' + \text{h.c.}, \quad (5.4)$$

where  $n_L' \equiv (\nu_L', (\nu_R')^c)^T$  is a six dimensional column vector of the left-handed fields. The right-handed components are analogously defined as  $n_R' = (n_L')^c = ((\nu_L')^c, \nu_R')^T$ . The  $6 \times 6$  Majorana neutrino mass matrix is

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix}. \quad (5.5)$$

Six neutrino mass eigenstates are then obtain using the following unitary transformations:

$$n_L' = \mathcal{U} n_L, \quad n_R' = \mathcal{U}^* n_R. \quad (5.6)$$

The chiral components  $n_{L,R}$  of six Majorana mass eigenstates can be identified with the light and heavy neutrino mass eigenstates as:  $n_{iL} = \nu_{iL}$ ,  $n_{(i+3)L} = (\nu_{iR})^c$  and  $n_{iR} = (n_{iL})^c$ . The mixing matrix  $\mathcal{U}$  is required to satisfy

$$\mathcal{U}^T \mathcal{M}_\nu \mathcal{U} = \mathcal{D}_\nu \equiv \text{Diag.}(m_{\nu_i}, M_i) \quad (5.7)$$

where  $m_{\nu_i}, M_i$  are respectively light and heavy neutrino masses.

The neutral current interactions of neutrinos in the  $n_L'$  basis are given by

$$-\mathcal{L}_{\text{NC}} = g_Y B_\mu \tilde{Y}_a \bar{n}'_{aL} \gamma^\mu n'_{aL} + g' Z'_\mu \tilde{X}_a \bar{n}'_{aL} \gamma^\mu n'_{aL}, \quad (5.8)$$

where  $\tilde{Y} = -\frac{1}{2} \text{Diag.}(1, 1, 1, 0, 0, 0)$  and  $\tilde{X} = \text{Diag.}(0, 1, -1, 0, -1, 1)$ . Using the Majorana property  $\bar{n}'_{aL} \gamma^\mu n'_{aL} = -\bar{n}'_{aL} \gamma^\mu n'_{aL} = -\bar{n}'_{aR} \gamma^\mu n'_{aR}$ , the above equation can be cast in the following form:

$$-\mathcal{L}_{\text{NC}} = \frac{g_Y}{2} B_\mu \tilde{Y}_a (\bar{n}'_{aL} \gamma^\mu n'_{aL} - \bar{n}'_{aR} \gamma^\mu n'_{aR}) + \frac{g'}{2} Z'_\mu \tilde{X}_a (\bar{n}'_{aL} \gamma^\mu n'_{aL} - \bar{n}'_{aR} \gamma^\mu n'_{aR}). \quad (5.9)$$

Following the arguments presented between eqs. (4.1) and (4.4) for the general case, we obtain in the mass basis

$$-\mathcal{L}_{\text{NC}} = \frac{g_Y}{2} B_\mu \bar{n} \gamma^\mu \left( \mathcal{U}^\dagger \tilde{Y} \mathcal{U} P_L - \mathcal{U}^T \tilde{Y} \mathcal{U}^* P_R \right) n + \frac{g'}{2} Z'_\mu \bar{n} \gamma^\mu \left( \mathcal{U}^\dagger \tilde{X} \mathcal{U} P_L - \mathcal{U}^T \tilde{X} \mathcal{U} P_R \right) n. \quad (5.10)$$

Eq. (5.9) can be seen as special case of the general expression eq. (4.3) with the identification

$$\begin{aligned} (Y_L)_{ab} &= \frac{1}{2} \left( \mathcal{U}^\dagger \tilde{Y} \mathcal{U} \right)_{ab}, & (X_L)_{ab} &= \frac{1}{2} \left( \mathcal{U}^\dagger \tilde{X} \mathcal{U} \right)_{ab}, \\ (Y_R)_{ab} &= -\frac{1}{2} \left( \mathcal{U}^T \tilde{Y} \mathcal{U}^* \right)_{ab}, & (X_R)_{ab} &= -\frac{1}{2} \left( \mathcal{U}^T \tilde{X} \mathcal{U}^* \right)_{ab}. \end{aligned} \quad (5.11)$$

The above expressions also follow directly from eq. (4.4) by noting that (i) the left and right handed mixing matrices are related as  $\mathcal{U}_L = \mathcal{U}_R^* \equiv \mathcal{U}$  (see eq. (5.6)), and (ii) the  $U(1)$

charges of  $n_R$  and  $n_L$  are opposite to each other. One can use eq. (5.11) to directly obtain KM parameters in the present case. In this we closely follow the treatment of radiative corrections given in [58–60].

It is trivial to see from the comparison with eq. (4.6) that the divergent part in  $A_{BZ'}$  vanishes for the present case as  $\text{Tr}(\tilde{X}\tilde{Y}) = 0$ . To show finiteness of  $B_{BZ'}$ , it is useful to decompose  $\mathcal{U}$  as

$$\mathcal{U} = \begin{pmatrix} V_L \\ V_R^* \end{pmatrix} \quad (5.12)$$

in terms of  $3 \times 6$  matrices  $V_L$  and  $V_R$ . The matrices  $X_{L,R}$  and  $Y_{L,R}$  are then given by

$$X_L = -X_R^* = \frac{1}{2}(V_L^\dagger X_3 V_L - V_R^T X_3 V_R^*), \quad Y_L = -Y_R^* = -\frac{1}{4}V_L^\dagger V_L. \quad (5.13)$$

Here,  $X_3 = \text{Diag.}(0, 1, -1)$ . Eq. (5.7) and unitarity of  $\mathcal{U}$  can be used to derive the relations

$$V_L V_L^\dagger = \mathbf{1}_{3 \times 3}, \quad V_L V_R^T = \mathbf{0}_{3 \times 3}, \quad V_R V_R^\dagger = \mathbf{1}_{3 \times 3}, \quad (5.14)$$

$$V_L \mathcal{D}_\nu V_L^T = \mathbf{0}_{3 \times 3}, \quad V_R \mathcal{D}_\nu V_L^\dagger = m_D, \quad V_R \mathcal{D}_\nu V_R^T = M_R. \quad (5.15)$$

Here,  $\mathbf{1}_{3 \times 3}$  and  $\mathbf{0}_{3 \times 3}$  respectively denote the  $3 \times 3$  identity and null matrix. The expressions for KM parameters follow by substituting eq. (5.11) in the general formula eqs. (4.6), (4.7). Finiteness of  $B_{BZ'}$  then follows from the following identities

$$\frac{1}{2} \text{Tr}(\mathcal{D}_\nu^2(\{Y_L, X_L\} + \{Y_R, X_R\})) = -\frac{1}{4} \text{Tr}(m_D^\dagger m_D X_3), \quad (5.16)$$

$$\text{Tr}(Y_L \mathcal{D}_\nu X_R \mathcal{D}_\nu + Y_R \mathcal{D}_\nu X_L \mathcal{D}_\nu) = -\frac{1}{4} \text{Tr}(m_D m_D^\dagger X_3). \quad (5.17)$$

We have used the definition of  $X_L, Y_L$  and eqs. (5.14), (5.15) in proving above equations. The divergent part in  $B_{BZ'}$  vanishes for  $\mu$ - $\tau$  symmetric  $m_D$  given in eq. (5.3) as both the eqs. (5.16), (5.17) vanish individually. The finite parts can be written as

$$A_{BZ'} = \frac{g_Y g'}{2\pi^2} \sum_{a,b} \text{Re}[Y_{Lab} X_{Lba}] b_2[m_a, m_b, q], \quad (5.18)$$

$$B_{BZ'} = -\frac{g_Y g'}{4\pi^2} \sum_{a,b} \left( \text{Re}[Y_{Lab} X_{Lba}] (m_a^2 b_1[m_a, m_b, q] + m_b^2 b_1[m_b, m_a, q]), \right. \\ \left. + \text{Re}[Y_{Lab} X_{Lba}^*] m_a m_b b_0[m_a, m_b, q] \right). \quad (5.19)$$

The above considerations are valid in general seesaw model without taking the standard limit  $m_D \ll M_R$ . We now consider this limit in order to further simplify the finite contributions to KM parameters.  $\mathcal{U}$  can be written as

$$\mathcal{U} = \begin{pmatrix} 1 - \frac{1}{2}\rho\rho^\dagger & -\rho \\ \rho^\dagger & 1 - \frac{1}{2}\rho^\dagger\rho \end{pmatrix} \begin{pmatrix} K_L & 0 \\ 0 & K_R \end{pmatrix}. \quad (5.20)$$

In the seesaw limit,  $\rho^\dagger \approx -M_R^{-1} m_D$ , while  $K_L$  and  $K_R$  are  $3 \times 3$  matrices which diagonalize the light and heavy neutrino matrices  $m_\nu = -m_D^T M_R^{-1} m_D$  and  $M_R$  respectively. Parameters  $A_{BZ'}$  and  $B_{BZ'}$  can be simplified in a special case of the second and third generations.

Further simplification can be achieved if  $m_D$  is assumed to be invariant under  $L_\mu - L_\tau$  and thus diagonal. In this case, it is proportional to  $2 \times 2$  identity matrix and explicitly

$$m_D \equiv m \mathbf{1}_{2 \times 2}.$$

The light neutrino mass matrix is then given by  $-m^2 M_R^{-1}$  and therefore the matrices  $K_L$  and  $K_R$  are related as  $K_R = K_L^*$ . In this case,  $V_L$  and  $V_R$  defined in eq. (5.12) simplify to

$$V_L = (K_L, -\rho K_R) \approx (K_L, m K_L D_R^{-1}), \quad V_R^* = (\rho^\dagger K_L, K_R) \approx (-m K_L^* D_R^{-1}, K_L^*), \quad (5.21)$$

where  $D_R = \text{Diag.}(M_2, M_3)$ . We parametrise  $2 \times 2$  matrix  $K_L$  as

$$K_L = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \quad (5.22)$$

This together with eq. (5.21) determine the parameters  $(X_L)_{ab}$ ,  $(Y_L)_{ab}$  and lead to

$$\begin{aligned} A_{BZ'} &\approx \frac{g_Y g'}{16\pi^2} \cos 2\theta \left( b_2(m_{\nu_3}, m_{\nu_3}, q) - b_2(m_{\nu_2}, m_{\nu_2}, q) \right. \\ &\quad + \frac{m^2}{M_3^2} (4b_2(M_3, m_{\nu_3}, q) - b_2(M_3, M_3, q) - 3b_2(m_{\nu_3}, m_{\nu_3}, q)) \\ &\quad \left. - \frac{m^2}{M_2^2} (4b_2(M_2, m_{\nu_2}, q) - b_2(M_2, M_2, q) - 3b_2(m_{\nu_2}, m_{\nu_2}, q)) + \mathcal{O}\left(\frac{m^4}{M_{2,3}^4}\right) \right), \\ B_{BZ'} &\approx \frac{g_Y g'}{16\pi^2} \cos 2\theta m^2 \left( b_0(M_3, M_3, q) - 2b_1(M_3, m_{\nu_3}, q) \right. \\ &\quad \left. - b_0(M_2, M_2, q) + 2b_1(M_2, m_{\nu_2}, q) + \mathcal{O}\left(\frac{m^2}{M_{2,3}^2}\right) \right), \end{aligned} \quad (5.23)$$

In the limit  $m_{\nu_{2,3}}^2 \ll |q^2| \ll M_i^2$ , using the approximate solutions of integration functions provided in eqs. (A.2), (A.3), (A.4) in appendix, we obtain

$$\begin{aligned} A_{BZ'} &\approx \frac{g_Y g'}{16\pi^2} \cos 2\theta \left( \frac{\Delta_{\text{atm}}}{-q^2} - \left( \frac{m^2}{M_2^2} - \frac{m^2}{M_3^2} \right) \left( \frac{5}{18} - \frac{1}{2} \ln \frac{-q^2}{\mu^2} \right) \right. \\ &\quad \left. - \frac{m^2}{2M_2^2} \ln \frac{M_2^2}{\mu^2} + \frac{m^2}{2M_3^2} \ln \frac{M_3^2}{\mu^2} \right), \\ B_{BZ'} &\approx -\frac{g_Y g'}{32\pi^2} \cos 2\theta q^2 \left( \frac{m^2}{M_2^2} - \frac{m^2}{M_3^2} \right), \end{aligned} \quad (5.24)$$

at the leading order in  $m^2/M^2$ . The first term in  $A_{BZ'}$  corresponds to the contribution from the effective light neutrino mass matrix and the other two contributions of  $\mathcal{O}(m^2/M^2)$  arise due to light heavy neutrino mixing. All these contributions vanish in the limit of the RH neutrino masses going to infinity.

The parameter  $\theta$  in eq. (5.24) is the neutrino part of the atmospheric mixing angle. The charged lepton contribution to it is maximal because of  $\mu$ - $\tau$  symmetry of  $M_l$  and one

thus gets  $\theta_{23} = \theta - \frac{\pi}{4}$ . One therefore requires small  $\theta$  and hence almost diagonal  $M_R$ . The RH neutrino masses  $M_{2,3}$  in this case are directly linked to the  $L_\mu - L_\tau$  breaking scale. As a consequence, the  $Z'$  mass would be similar to the RH neutrino masses unless  $g'$  is very small. Light  $Z'$  is still a possibility if the RH neutrino mass scale is around TeV, e.g.,  $M_2, M_3 \sim \text{TeV}$  and  $g' \sim 10^{-3}$  would give  $M_{Z'} \sim \text{GeV}$ . The KM is still suppressed by the light neutrino masses. For  $M_{2,3} \sim \text{TeV}$  and  $-q^2 \sim \text{MeV}^2$ , the dominant contribution to  $A_{BZ'}$  comes from the last two terms in eq. (5.24) and is given by

$$A_{BZ'} \sim g_Y g' 3 \cdot 10^{-16} \left( \frac{\text{TeV}}{M_3} \right) .$$

There can be additional contributions to KM from the Higgs sector. Such contributions would vanish in the exact  $\mu$ - $\tau$  symmetric limit. The  $\mu$ - $\tau$  breaking in our case comes from the right-handed neutrino masses, which could be explicit or induced through singlet VEVs. But singlet fields do not directly couple to  $Z$  and cannot induce  $Z - Z'$  mixing. There can be indirect coupling through the quartic interaction  $\lambda \eta^\dagger \eta \phi^\dagger \phi$  of the singlet field  $\eta$  with the SU(2) doublet  $\phi$ . This induced coupling of  $\eta$  to  $Z$  will be suppressed by  $\frac{\langle \phi \rangle}{\langle \eta \rangle}$  and the resulting mixing would also be suppressed.

In the above example, we have assumed  $\mu$ - $\tau$  symmetric  $m_D$  which leads to finite  $B_{BZ'}$  at the 1-loop. Alternatively, the 1-loop divergences in  $B_{BZ'}$  also vanish if  $m_D$  is not  $\mu$ - $\tau$  symmetric but it possesses unbroken  $L_\mu - L_\tau$  symmetry. In this case, the finiteness of  $B_{BZ'}$  follows from cancellation between the contributions (5.16) and (5.17) in eq. (4.7) because of diagonal  $m_D$ . This case is phenomenologically more important since it leads to a non-vanishing contribution even when the RH neutrino masses are taken to infinity. We discuss an explicit example which shows this. Assume again two generations with

$$m_D = \begin{pmatrix} m_2 & 0 \\ 0 & m_3 \end{pmatrix}, \quad M_R = \begin{pmatrix} M_2 & 0 \\ 0 & M_3 \end{pmatrix}. \quad (5.25)$$

$K_{L,R}$  defined in eq. (5.20) are  $2 \times 2$  identity matrices and  $\rho = -m_D M_R^{-1}$  is also diagonal. The  $2 \times 4$  matrices  $V_L, V_R$  can be obtained in this case from eq. (5.21) and one can work out the resulting  $4 \times 4$  matrices  $Y_{L,R}, X_{L,R}$  using eq. (5.13). This leads through eqs. (5.18), (5.19) to the following expressions for  $A_{BZ'}$  and  $B_{BZ'}$ :

$$\begin{aligned} A_{BZ'} &\approx \frac{g_Y g'}{16\pi^2} \left( b_2(m_{\nu_3}, m_{\nu_3}, q) - b_2(m_{\nu_2}, m_{\nu_2}, q) \right. \\ &\quad + \frac{m_3^2}{M_3^2} (4b_2(M_3, m_{\nu_3}, q) - b_2(M_3, M_3, q) - 3b_2(m_{\nu_3}, m_{\nu_3}, q)) \\ &\quad \left. - \frac{m_2^2}{M_2^2} (4b_2(M_2, m_{\nu_2}, q) - b_2(M_2, M_2, q) - 3b_2(m_{\nu_2}, m_{\nu_2}, q)) + \mathcal{O} \left( \frac{m_{2,3}^4}{M_{2,3}^4} \right) \right) \\ B_{BZ'} &\approx \frac{g_Y g'}{16\pi^2} \left( m_3^2 (b_0(M_3, M_3, q) - 2b_1(M_3, m_{\nu_3}, q)) \right. \\ &\quad \left. - m_2^2 (b_0(M_2, M_2, q) - 2b_1(M_2, m_{\nu_2}, q)) + \mathcal{O} \left( \frac{m_{2,3}^2}{M_{2,3}^2} \right) \right), \end{aligned} \quad (5.26)$$



In the limit  $m_{\nu_{2,3}}^2 \ll |q^2| \ll M_{2,3}^2$ , we obtain

$$\begin{aligned}
 A_{BZ'} &\approx \frac{g_Y g'}{16\pi^2} \left( \frac{\Delta_{\text{atm}}}{-q^2} - \left( \frac{m_2^2}{M_2^2} - \frac{m_3^2}{M_3^2} \right) \left( \frac{5}{18} - \frac{1}{2} \ln \frac{-q^2}{\mu^2} \right) - \frac{m_2^2}{2M_2^2} \ln \frac{M_2^2}{\mu^2} + \frac{m_3^2}{2M_3^2} \ln \frac{M_3^2}{\mu^2} \right), \\
 B_{BZ'} &\approx -\frac{g_Y g'}{32\pi^2} \left( 3(m_2^2 - m_3^2) + q^2 \left( \frac{m_2^2}{M_2^2} - \frac{m_3^2}{M_3^2} \right) \right). \tag{5.27}
 \end{aligned}$$

Unlike in the previous case, there is a finite non-decoupling contribution in  $B_{BZ'}$  which does not vanish when the right handed neutrino masses are taken to infinity. This contribution is proportional to the amount of  $\mu$ - $\tau$  breaking,  $m_2^2 - m_3^2$ , in  $m_D$ .  $A_{BZ'}$  is still seen to vanish when the RH neutrino masses go to infinity.

## 5.2 Kinetic mixing in a model with vectorlike charged leptons

In this case, the effective  $3 \times 3$  Majorana<sup>4</sup> neutrino mass matrix  $M_\nu$  is assumed to be invariant under  $\mu$ - $\tau$  interchange symmetry. It is explicitly given as

$$M_\nu = v_\nu \begin{pmatrix} \lambda_{ee}^\nu & \lambda_{e\mu}^\nu & \lambda_{e\mu}^\nu \\ \lambda_{e\mu}^\nu & \lambda_{\mu\mu}^\nu & \lambda_{\mu\tau}^\nu \\ \lambda_{e\mu}^\nu & \lambda_{\mu\tau}^\nu & \lambda_{\mu\mu}^\nu \end{pmatrix}. \tag{5.28}$$

The couplings  $\lambda_{e\mu}^\nu$ ,  $\lambda_{\mu\mu}^\nu$  can be seen as spurions which break  $L_\mu - L_\tau$  symmetry spontaneously but preserve the  $\mu$ - $\tau$  interchange symmetry. Because of the later, the neutrinos by themselves do not induce the KM between the  $B$  and  $Z'$  bosons.

The charged lepton sector is extended by a pair of vectorlike leptons,  $f'_4$  and  $f'_5$ , singlet under  $SU(2)_L$  and with hypercharge  $-1$ . Under the gauged  $L_\mu - L_\tau$  symmetry,  $f'_4$  and  $f'_5$  have charges  $+1$  and  $-1$ , respectively. Further,  $f'_4$  and  $f'_5$  get interchanged under the  $\mu$ - $\tau$  symmetry in addition to the transformations defined in eq. (2.2). After the spontaneous breaking of  $L_\mu - L_\tau$  and electroweak symmetry, the charged lepton mass term in the Lagrangian is given by

$$-\mathcal{L}_m^l = \left( \bar{e}'_L \ \bar{\mu}'_L \ \bar{\tau}'_L \ \bar{f}'_{4L} \ \bar{f}'_{5L} \right) \mathcal{M}_l \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \\ f'_{4R} \\ f'_{5R} \end{pmatrix} + \text{h.c.}, \tag{5.29}$$

where

$$\mathcal{M}_l = \begin{pmatrix} (M_l)_{3 \times 3} & (m_l)_{3 \times 2} \\ (\tilde{m}_l)_{2 \times 3} & (M_f)_{2 \times 2} \end{pmatrix}. \tag{5.30}$$

The matrix  $M_l$  is invariant under  $\mu$ - $\tau$  symmetry and has the same form as in eq. (5.2). The explicit forms of the matrices  $m$  and  $\tilde{m}$  are

$$m_l = \begin{pmatrix} m_{e4} & m_{e4} \\ m_{\mu4} & m_{\mu5} \\ m_{\mu5} & m_{\mu4} \end{pmatrix}, \quad \tilde{m}_l = \begin{pmatrix} m_{4e} & m_{4\mu} & m_{4\tau} \\ m_{4e} & m_{4\tau} & m_{4\mu} \end{pmatrix}, \tag{5.31}$$

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<sup>4</sup>Although we assume neutrinos as Majorana fermions, the same results are obtained if they are Dirac fermions.

Both  $m$  and  $\tilde{m}$  are invariant under  $\mu$ - $\tau$  interchange symmetry. The mass terms  $m_{e4}$ ,  $m_{4e}$ ,  $m_{\mu 5}$  and  $m_{4\tau}$  are spurious which break the  $L_\mu - L_\tau$  symmetry spontaneously. We assume general form for matrix  $M_f$  which breaks the  $\mu$ - $\tau$  interchange symmetry softly unless  $(M_f)_{11} = (M_f)_{22}$  and  $(M_f)_{12} = (M_f)_{21}$ . This soft breaking of  $\mu$ - $\tau$  symmetry in  $M_f$  leads to breaking of the same symmetry in the effective theory obtained after integrating out the vectorlike charged leptons. Therefore, the KM between  $B$  and  $Z'$  gets generated at 1-loop in this setup.

The five mass eigenstates of the charged leptons are obtained using the following unitary transformation.

$$\begin{pmatrix} e'_{L,R} \\ \mu'_{L,R} \\ \tau'_{L,R} \\ f'_{4L,R} \\ f'_{5L,R} \end{pmatrix} = \mathcal{U}_{L,R} \begin{pmatrix} e_{L,R} \\ \mu_{L,R} \\ \tau_{L,R} \\ f_{4L,R} \\ f_{5L,R} \end{pmatrix} \quad (5.32)$$

such that

$$\mathcal{U}_L^\dagger \mathcal{M}_l \mathcal{U}_R = \text{Diag.} (m_e, m_\mu, m_\tau, m_4, m_5) \equiv \mathcal{D}_l. \quad (5.33)$$

For simplification, the  $5 \times 5$  unitary matrices  $\mathcal{U}_{L,R}$  can be represented as

$$\mathcal{U}_{L,R} = \begin{pmatrix} U_{L,R} \\ V_{L,R} \end{pmatrix}, \quad (5.34)$$

where  $U_{L,R}$  and  $V_{L,R}$  are matrices of dimensions  $3 \times 5$  and  $2 \times 5$  respectively. The unitarity of  $\mathcal{U}_{L,R}$  and the relation in eq. (5.33) can be used to obtain the following relations:

$$U_{L,R} U_{L,R}^\dagger = \mathbf{1}_{3 \times 3}, \quad V_{L,R} V_{L,R}^\dagger = \mathbf{1}_{2 \times 2}, \quad U_{L,R} V_{L,R}^\dagger = \mathbf{0}_{3 \times 2}, \quad (5.35)$$

$$U_L \mathcal{D}_l U_R^\dagger = M_l, \quad U_L \mathcal{D}_l V_R^\dagger = m_l, \quad V_L \mathcal{D}_l U_R^\dagger = \tilde{m}_l, \quad V_L \mathcal{D}_l V_R^\dagger = M_f. \quad (5.36)$$

We now discuss the KM between  $B$  and  $Z'$  bosons induced at one loop within this setup. The general formalism developed in section III can be straight forwardly used to compute such mixing. The fermionic currents associated with  $Z'$  and  $B$  bosons in this framework can be read from eqs. (4.1) with  $f_a = \{e, \mu, \tau, f_4, f_5\}$  and

$$X'_L = X'_R = \text{Diag.} (0, 1, -1, 1, -1), \quad Y'_L = \text{Diag.} \left( -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -1, -1 \right), \quad Y'_R = -\mathbf{1}. \quad (5.37)$$

Using eq. (4.4) and definition in eq. (5.34), the gauge couplings in the mass basis are obtained as

$$Y_L = -\frac{1}{2} U_L^\dagger U_L - V_L^\dagger V_L, \quad Y_R = -\mathbf{1}, \quad (5.38)$$

$$X_{L,R} = U_{L,R}^\dagger X_3 U_{L,R} + V_{L,R}^\dagger X_2 V_{L,R}, \quad (5.39)$$

where  $X_2 = \text{Diag.}(1, -1)$ . Eq. (5.37) implies  $\text{Tr}(Y'_L X'_L + Y'_R X'_R) = 0$  making  $A_{BZ'}$  finite in the present framework. Moreover, using eqs. (5.38), (5.39) and the relations obtained in

eqs. (5.35), (5.36) we find

$$\begin{aligned} & \frac{1}{2} \text{Tr}(m^2(\{Y_L, X_L\} + \{Y_R, X_R\})) - \text{Tr}(Y_L m X_R m + Y_R m X_L m) \\ &= \frac{1}{2} \text{Tr} \left( (M_l M_l^\dagger - M_l^\dagger M_l + m_l m_l^\dagger) X_3 - m_l^\dagger m_l X_2 \right), \end{aligned} \quad (5.40)$$

which vanishes identically for the above considered forms of  $M_l$  and  $m_l$ . Therefore, the divergent part in  $B_{BZ'}$  also vanishes making the KM finite and calculable in the underlying framework. The values of  $A_{BZ'}$  and  $B_{BZ'}$  can be explicitly computed using the expressions of finite parts given in eqs. (4.6), (4.7) with  $Y_{L,R}$  and  $X_{L,R}$  obtained in eqs. (5.38), (5.39) for this model.

We explicitly calculate the KM in a specific “seesaw-like” case, i.e.  $M_l < m_l, \tilde{m}_l \ll M_f$ , within this model. The effective mass matrix for the three light charged leptons is obtained as  $M_l^{\text{eff.}} \approx M_l - m_l M_f^{-1} \tilde{m}_l$ . Let  $u_{L,R}$  and  $v_{L,R}$  are matrices which diagonalize  $M_l^{\text{eff.}}$  and  $M_f$ , respectively, such that

$$u_L^\dagger M_l^{\text{eff.}} u_R = \text{Diag.}(m_e, m_\mu, m_\tau), \quad v_L^\dagger M_f v_R = \text{Diag.}(m_4, m_5). \quad (5.41)$$

The  $5 \times 5$  unitary matrices  $\mathcal{U}_{L,R}$  can suitably written as

$$\mathcal{U}_{L,R} \approx \begin{pmatrix} u_{L,R} & -\rho_{L,R} v_{L,R} \\ \rho_{L,R}^\dagger u_{L,R} & v_{L,R} \end{pmatrix}, \quad (5.42)$$

where  $\rho_L \approx -m_l M_f^{-1}$  and  $\rho_R^\dagger \approx -M_f^{-1} \tilde{m}_l$ . Further, we take  $\lambda_{e\mu}^l = \lambda_{\mu e}^l = 0$  and consider the following ansatz for the matrices  $m_l$  and  $\tilde{m}_l$ :

$$m_l = m \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \tilde{m}_l = \tilde{m} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (5.43)$$

The above forms are achieved if  $L_\mu - L_\tau$  symmetry remains unbroken in  $m_l, \tilde{m}_l$ . In this case, the unitary matrices  $u_{L,R}$  and  $v_{L,R}$  can be parametrized as

$$u_{L,R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\theta_{L,R}} & s_{\theta_{L,R}} \\ 0 & -s_{\theta_{L,R}} & c_{\theta_{L,R}} \end{pmatrix}, \quad v_{L,R} = \begin{pmatrix} c_{\phi_{L,R}} & s_{\phi_{L,R}} \\ -s_{\phi_{L,R}} & c_{\phi_{L,R}} \end{pmatrix}, \quad (5.44)$$

where  $c_\theta = \cos \theta$  and  $s_\theta = \sin \theta$ . The general results given in eqs. (4.6), (4.7) are then used to compute the KM using the above simplifications. The leading contributions to kinetic

and mass mixing are obtained as:

$$\begin{aligned}
 A_{BZ'} &\approx -\frac{g_Y g'}{4\pi^2} \left( (c_{2\phi_L} + c_{2\phi_R}) (b_2[m_4, m_4, q] - b_2[m_5, m_5, q]) \right. \\
 &\quad \left. + \frac{1}{2}(c_{2\theta_L} + 2c_{2\theta_R}) (b_2[m_\mu, m_\mu, q] - b_2[m_\tau, m_\tau, q]) + \mathcal{O}\left(\frac{m^2, \tilde{m}^2}{m_{4,5}^2}\right) \right), \\
 B_{BZ'} &\approx \frac{g_Y g'}{16\pi^2} \left( (m^2 - 2\tilde{m}^2) (c_{2\phi_L} - c_{2\phi_R}) (b_0[m_4, m_4, q] - b_0[m_5, m_5, q]) \right. \\
 &\quad \left. + (c_{2\theta_L} - c_{2\theta_R}) (m_\tau^2 b_0[m_\tau, m_\tau, q] - m_\mu^2 b_0[m_\mu, m_\mu, q]) + \mathcal{O}\left(\frac{m^2, \tilde{m}^2}{m_{4,5}^2}\right) \right).
 \end{aligned} \tag{5.45}$$

For  $|q^2| \ll m_5^2, m_4^2$ , the above expressions can further be simplified as

$$\begin{aligned}
 A_{BZ'} &\approx -\frac{g_Y g'}{24\pi^2} \left( (c_{2\phi_L} + c_{2\phi_R}) \ln \frac{m_4^2}{m_5^2} + \frac{1}{2}(c_{2\theta_L} + 2c_{2\theta_R}) \ln \frac{m_\mu^2}{m_\tau^2} \right), \\
 B_{BZ'} &\approx \frac{g_Y g'}{16\pi^2} \left( (m^2 - 2\tilde{m}^2) (c_{2\phi_L} - c_{2\phi_R}) \ln \frac{m_4^2}{m_5^2} \right. \\
 &\quad \left. + (c_{2\theta_L} - c_{2\theta_R}) \left( m_\tau^2 \ln \frac{m_\tau^2}{\mu^2} - m_\mu^2 \ln \frac{m_\mu^2}{\mu^2} \right) \right).
 \end{aligned} \tag{5.46}$$

The first terms in  $A_{BZ'}$  and  $B_{BZ'}$  in eq. (5.46) quantify the 1-loop contribution induced by the vectorlike charged leptons. Since these fermions are charged under both the  $U(1)_Y$  and  $L_\mu - L_\tau$  gauge symmetries, their contribution to KM is nonzero unless  $M_f$  is  $\mu$ - $\tau$  symmetric, i.e.  $\phi_{L,R} = \pi/4$  or  $m_4 = m_5$ . This is in contrast to the standard seesaw case discussed in the previous subsection where the RH neutrinos do not couple to  $B$  and hence they do not induce KM by themselves. The second terms in  $A_{BZ'}$  and  $B_{BZ'}$  correspond to contributions from the SM charged leptons. This along with the other sub-leading contributions in eq. (5.45) vanish in the decoupling limit,  $m_{4,5} \rightarrow \infty$ . Note that  $M_l^{\text{eff.}} = M_l$  is  $\mu$ - $\tau$  symmetric in the same limit which leads to  $\theta_{L,R} = \pi/4$  and vanishing of the charged lepton contributions. In this case, the first terms in  $A_{BZ'}$  and  $B_{BZ'}$  provide non-decoupling contributions to the kinetic and mass mixing respectively.

$M_f$  is a diagonal matrix in the  $L_\mu - L_\tau$  symmetric limit. As a result, the masses of vectorlike leptons need not be linked to the  $L_\mu - L_\tau$  symmetry breaking scale, unlike in the seesaw case discussed in the previous subsection. This allows the  $L_\mu - L_\tau$  breaking scale to be smaller than the vectorlike lepton masses which are required to be large for the phenomenological reasons. The  $Z'$  boson in this case can be light and leave signal in the low energy process. The other advantage of this is that one gets almost diagonal  $M_f$  resulting in  $\phi_{L,R} \approx 0$ . If one also assumes that the elements of  $M_l$  are vanishingly small and the second and the third generation masses arise through the seesaw like contribution

$$M_l^{\text{eff.}} \approx -m_l M_f^{-1} \tilde{m}_l \approx -m \tilde{m} \text{Diag.}(m_4^{-1}, m_5^{-1}) \tag{5.47}$$

then this leads to seesaw contribution which is almost diagonal and results in small  $\theta_{L,R}$ . With a small  $\theta_L$  the atmospheric mixing gets dominant contribution from the  $\mu$ - $\tau$  symmetric neutrino mass matrix and is nearly maximal as required phenomenologically. Further,  $m_4/m_5 \approx m_\tau/m_\mu$  if  $\phi_L$  is vanishingly small. Replacing these in eq. (5.46) results in

$$A_{BZ'} \approx \frac{g_Y g'}{48\pi^2} \ln \frac{m_\mu^2}{m_\tau^2}, \quad B_{BZ'} \approx \mathcal{O} \left( \frac{m^4, \tilde{m}^4}{m_{4,5}^2} \right). \quad (5.48)$$

The  $\mu$ - $\tau$  symmetry in the charged lepton sector is badly broken giving rise to large but finite  $A_{BZ'}$ . The leading order contribution to mass mixing parameter  $B_{BZ'}$  vanish in this case because of  $\phi_L \approx \phi_R$ .

The above setup can straightforwardly be implemented in the quark sector extending  $L_\mu - L_\tau$  symmetry to include the second and third generations of quarks transforming in an analogous way. The  $\mu$ - $\tau$  interchange symmetry can also be generalized as 2-3 interchange symmetry [48]. The up-type quark mass matrix can be assumed invariant under 2-3 interchange symmetry. A similar assumption for the down-type quarks would then imply  $V_{cb} = V_{ub} = 0$  and therefore breaking of 2-3 interchange symmetry would be necessarily required for the realistic quark mixing angles. Such breaking can be incorporated by extending the down-type quark sector by a pair of vectorlike quarks in an analogous way discussed above. One obtains similar expressions for  $A_{BZ'}$  and  $B_{BZ'}$  as in eq. (5.46) with appropriate change in hypercharges and an overall color factor. The difference compared to the leptonic case is that one requires a small deviation from  $\theta_L = \pi/4$  in order to produce realistic quark mixing. This can easily be reproduced through small seesaw-like contribution, and one need not assume vanishing  $M_l$  as it is done in the leptonic case. The mild breaking of 2-3 interchange symmetry gives rise to relatively small kinetic and mass mixing in this case.

## 6 Summary and discussions

The SM extended with gauged  $L_\mu - L_\tau$  symmetry offers phenomenologically rich framework in which the new physics effects can arise directly through the couplings of the second and the third generation leptons with the  $Z'$  boson and indirectly through the KM between the  $Z$  and  $Z'$  bosons. The later makes it possible for the SM quarks and the first generation of leptons to couple to  $Z'$  boson and therefore the KM is of particular interests from the phenomenological considerations. KM in the standard  $L_\mu - L_\tau$  models can be forbidden to all orders if one imposes  $\mu$ - $\tau$  interchange or reflection symmetry under which one of the two gauge bosons also transforms non-trivially. However, the same symmetries do not give phenomenologically viable mixing in the lepton sector. Invariance of leptonic Lagrangian under  $\mu$ - $\tau$  interchange symmetry leads to vanishing atmospheric and reactor mixing angles while the same under  $\mu$ - $\tau$  reflection symmetry implies CP conservation in the lepton sector. We showed that it is possible to create more general versions of these symmetries, which can lead to realistic lepton mixing. However, these symmetries can forbid the KM up to 1-loop level only.

In the absence of  $\mu$ - $\tau$  symmetry, the kinetic and mass mixing in  $L_\mu - L_\tau$  models is given by arbitrary parameters which cannot be determined from the other fundamental

parameters of the theory and can be constrained only from the experimental observations. However, the KM parameters become calculable if  $\mu$ - $\tau$  symmetry is imposed in a full theory, and the mechanism of its breaking is known. The magnitude of KM in this case depends on the details of the new sector responsible for  $\mu$ - $\tau$  breaking. We provided two explicit examples of this in section 5. Both the kinetic mixing parameters are shown to be small and inversely proportional to the right handed neutrino masses in a class of seesaw models in which the  $\mu$ - $\tau$  symmetry breaking is present only in the heavy neutrino sector. The neutrino mass mixing parameter can be large and independent of the right handed neutrino masses if the Dirac mass matrix also break the  $\mu$ - $\tau$  symmetry. On the contrary if  $\mu$ - $\tau$  breaking is introduced through heavy vectorlike charged leptons, the KM parameters are dominantly determined by the new sector, and its magnitude can be large.

Phenomenological consequences of the SM with  $L_\mu - L_\tau$  extensions are widely discussed and used to constrain the parameters  $g'$  and  $M_{Z'}$ . One could divide the tests of this model in two categories, one which exploits KM and use electron or hadron induced interactions. These include a large variety of processes, such as precision electroweak tests, atomic parity violation, beam dump experiments,  $\nu_e - e$  elastic scattering in Borexino, and coherent neutrino-nucleus scattering observed in the COHERENT experiment. Constraints from these experiments mainly for light  $Z'$  are presented in [42]. These constraints do not hold in the type of seesaw model discussed in section 5.1 due to very suppressed  $Z$ - $Z'$  mixing. The other class of tests involve only  $\mu$  and  $\tau$  sector. Anomalous magnetic moment of  $\mu$  and  $\tau$  and the muon neutrino induced trident production through the process  $\nu N \rightarrow \nu N \mu^+ \mu^-$  fall in this category, and have been used to constrain the purely leptonic couplings of  $Z'$ . The latter process is found to be quite constraining and rules out most of the parameter space corresponding to  $M_{Z'} > 400$  MeV which otherwise can be used to explain the discrepancy in  $(g - 2)_\mu$ . It turns out that the neutrino trident process is not a useful probe of models considered here and in [4, 5]. The general  $\mu$ - $\tau$  reflection symmetry requirement in eq. (2.6) imposed to get vanishing KM parameters at 1-loop also implies that the  $Z'$  couplings to leptons are purely off-diagonal in their mass basis. Immediate consequence is that the trident process  $\nu N \rightarrow \nu N \mu^+ \mu^-$  cannot take place at tree level and is unable to constrain the parameters of the model. Instead, the rare tau decays could provide stringent constraint on the model. If  $L_\mu - L_\tau$  symmetry is broken only through Higgs doublet VEV then the rare decay  $\tau \rightarrow \mu Z'$  (for light  $Z'$ ) and the decay  $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$  together rule out the entire space which is responsible for the explanation of  $(g - 2)_\mu$  [4, 5]. Small parameter space is still allowed if SU(2) singlet field is responsible for the  $L_\mu - L_\tau$  breaking as is assumed here. This is analyzed in [5]. The flavour off-diagonal couplings of  $Z'$  to the charged leptons, as obtained in eq. (2.5), depend on the exact structure of the mixing matrices  $U_{lL}$  and  $U_{lR}$ . These matrices have the form given in eq. (2.10) in the limit of the exact  $\mu$ - $\tau$  symmetry. This gives the following couplings of  $Z'$ :

$$g' Z'_\mu (\cos \theta_L \bar{\tau}_L \gamma^\mu \mu_L + \sin \theta_L \bar{\tau}_L \gamma^\mu e_L + \cos \theta_R \bar{\tau}_R \gamma^\mu \mu_R + \sin \theta_R \bar{\tau}_R \gamma^\mu e_R), \quad (6.1)$$

where  $\theta_{L,R}$  are angles entering in definitions of  $U_{lL,lR}$  as given in eq. (2.10). This equation implies non-universality in the decay of  $\tau$  to  $e$  and  $\mu$ . The above equation coincides with the one assumed in [4, 5] for  $\theta_L = \theta_R = 0$ . It is found in [5] that the above coupling can explain

the  $(g - 2)_\mu$  anomaly and be consistent with the observed rare tau decay  $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$  for a very narrow ranges in parameters  $g'$  and  $M_{Z'}$ . We update their analysis considering the latest values of  $(g - 2)_\mu$  from [61] and  $\text{BR}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)$  from [16]. We observe that the positive deviation at  $1.6\sigma$  found in the branching ratio of the decay  $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$  compared to its SM value and the anomaly in  $(g - 2)_\mu$  can be simultaneously reconciled for

$$0.004 \leq g' \leq 0.006 \quad \text{and} \quad 1.12 \text{ GeV} \leq M_{Z'} \leq 1.24 \text{ GeV}$$

which practically coincides with the one already found in [5]. One can obtain  $M_{Z'} \sim g' v_s \sim \mathcal{O}(1)$  GeV for the  $L_\mu - L_\tau$  breaking scale around TeV if the above range in parameters is to be realized. Possible constraint on this model can come at the muon collider [5] through the process  $\mu^+ \mu^- \rightarrow \tau^+ \tau^-$ , search for rare tau decays at Belle II and detection of four charged leptons at colliders as discussed in details in [16].

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## A Loop integrals

The definition of the loop integration functions are as the following [62].

$$\begin{aligned} b_0[m_i, m_j, q] &= \int_0^1 dx \ln(\Delta(m_i, m_j, q)/\mu^2), \\ b_1[m_i, m_j, q] &= \int_0^1 dx x \ln(\Delta(m_i, m_j, q)/\mu^2), \\ b_2[m_i, m_j, q] &= \int_0^1 dx x(1-x) \ln(\Delta(m_i, m_j, q)/\mu^2), \end{aligned} \tag{A.1}$$

where  $\Delta(m_i, m_j, q) = xm_j^2 + (1-x)m_i^2 - x(1-x)q^2$ .  $\mu$  is an arbitrary subtraction scale. The functions  $b_0$  and  $b_2$  are symmetric under the interchange of  $i$  and  $j$ . Also,  $b_1[m_i, m_i, q] = \frac{1}{2}b_0[m_i, m_i, q]$ .

For special cases of interests, the above integrals can be approximated as the following. For  $m^2 \ll q^2$ ,

$$\begin{aligned} b_0[m, m, q] &\approx -2 + \ln\left(-\frac{q^2}{\mu^2}\right) - \frac{m^2}{q^2} \\ b_2[m, m, q] &\approx -\frac{5}{18} + \frac{1}{6} \ln\left(-\frac{q^2}{\mu^2}\right) - \frac{m^2}{q^2}. \end{aligned} \tag{A.2}$$

For  $m^2 \ll q^2 \ll M^2$  ( $\mathcal{O}(m^2/q^2)$  terms not shown),

$$\begin{aligned}
 b_0[m, M, q] &\approx -1 + \ln \frac{M^2}{\mu^2} - \frac{q^2}{2M^2}, \\
 b_1[m, M, q] &\approx -\frac{1}{4} + \frac{1}{2} \ln \frac{M^2}{\mu^2} - \frac{q^2}{6M^2}, \\
 b_1[M, m, q] &\approx -\frac{3}{4} + \frac{1}{2} \ln \frac{M^2}{\mu^2} - \frac{q^2}{3M^2}, \\
 b_2[m, M, q] &\approx -\frac{5}{36} + \frac{1}{6} \ln \frac{M^2}{\mu^2} - \frac{q^2}{12M^2}.
 \end{aligned}
 \tag{A.3}$$

For  $q^2 \ll M^2$ ,

$$\begin{aligned}
 b_0[M, M, q] &\approx \ln \frac{M^2}{\mu^2} - \frac{q^2}{6M^2}, \\
 b_1[M, M, q] &\approx \frac{1}{2} \ln \frac{M^2}{\mu^2} - \frac{q^2}{12M^2}, \\
 b_2[M, M, q] &\approx \frac{1}{6} \ln \frac{M^2}{\mu^2} - \frac{q^2}{30M^2}.
 \end{aligned}
 \tag{A.4}$$

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