Published for SISSA by Springer

Received: *January 10, 2019*
Revised: *March 4, 2019*
Accepted: *March 5, 2019*
Published: *March 8, 2019*

Spontaneous breaking of Weyl quadratic gravity to Einstein action and Higgs potential

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Abstract: We consider the (gauged) Weyl gravity action, quadratic in the scalar curvature ($\tilde{R}$) and in the Weyl tensor ($\tilde{C}_{\mu\nu\rho\sigma}$) of the Weyl conformal geometry. In the *absence* of matter fields, this action has spontaneous breaking in which the Weyl gauge field ω_μ becomes massive (mass $m_\omega \sim$ Planck scale) after "eating" the dilaton in the $\tilde{R}^2$ term, in a Stueckelberg mechanism. As a result, one recovers the Einstein-Hilbert action with a positive cosmological constant and the Proca action for the massive Weyl gauge field ω_μ. Below m_ω this field decouples and Weyl geometry becomes Riemannian. The Einstein-Hilbert action is then just a "low-energy" limit of Weyl quadratic gravity which thus avoids its previous, long-held criticisms. In the presence of matter scalar field ϕ_1 (Higgs-like), with couplings allowed by Weyl gauge symmetry, after its spontaneous breaking one obtains in addition, at low scales, a Higgs potential with spontaneous electroweak symmetry breaking. This is induced by the non-minimal coupling $\xi_1\phi_1^2\tilde{R}$ to Weyl geometry, with Higgs mass $\propto \xi_1/\xi_0$ (ξ_0 is the coefficient of the $\tilde{R}^2$ term). In realistic models ξ_1 must be classically tuned $\xi_1 \ll \xi_0$. We comment on the quantum stability of this value.

Keywords: Classical Theories of Gravity, Effective Field Theories, Higgs Physics, Spontaneous Symmetry Breaking

ArXiv ePrint: 1812.08613

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Article funded by SCOAP^3.

https://doi.org/10.1007/JHEP03(2019)049

Contents

1 Motivation

Introduced a century ago, Weyl's conformal geometry [1–5] is a scalar-vector-tensor theory of gravity that is a generalization of Brans-Dicke-Jordan theory of gravity [6–8]. The latter can be recovered in the limit of vanishing Weyl gauge field. Weyl's conformal geometry was applied to model building beyond SM long ago [9, 10], with recent renewed interest [11–22].

In the *absence* of matter and up to topological terms [22], the Weyl gravity action is the sum of two *quadratic* terms: the square of the Weyl-geometry scalar curvature term, $\tilde{R}^2$, and the square of the Weyl-tensor: $\tilde{C}^2 \equiv \tilde{C}_{\mu\nu\rho\sigma}\tilde{C}^{\mu\nu\rho\sigma}$. These generalise their counterparts R^2 and C^2 of the Riemannian geometry, to include effects due to the Weyl gauge field ω_μ. Indeed, in Weyl geometry $\nabla_\mu g_{\nu\sigma} \propto \omega_\mu g_{\rho\sigma}$, so the Weyl connection coefficients $\tilde{\Gamma}^\rho_{\mu\nu}$ are not determined by the metric alone (as in the Riemannian case), but also depend on ω_μ. Then $\tilde{R}^2$ (or $\tilde{C}^2$) can be expressed in terms of their Riemannian counterparts R^2 (C^2) plus a function of ω_μ, and become equal if $\omega_\mu = 0$ (decoupled); in this limit, Weyl geometry becomes Riemannian. We consider only a torsion-free Weyl geometry.

In section 2 we show that in the *absence* of matter fields, Weyl quadratic gravity has spontaneous breaking via a Stueckelberg mechanism [23] in which the Weyl gauge field ω_μ becomes massive after "eating" the Goldstone (dilaton) field (ϕ_0); ϕ_0 is "extracted" from the $\tilde{R}^2$ term which propagates an extra scalar field. Below the mass m_ω of the Weyl gauge field, $m_\omega \sim$ Planck scale, this field decouples, Weyl geometry becomes Riemannian and we recover the Einstein-Hilbert action and a positive cosmological constant. So Einstein-Hilbert action is a "low-energy" limit of Weyl quadratic gravity which thus avoids previous long-held criticisms against it, see e.g. [2–4]. No additional matter field (scalar, etc.) is required.

The Stueckelberg mechanism of breaking is known in local Weyl-invariant models but these contain, however, *additional* (matter) scalars and are *linear* (rather than quadratic)

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in the scalar curvature $\tilde{R}$ of Weyl geometry [9, 10] (see also recent [15, 16]). In these models the Einstein action follows from the presence of a scalar matter field ϕ with coupling $\phi^2\tilde{R}$, absent in our quadratic Weyl action without matter. To our knowledge, the breaking we study in the "pure" geometric case of $\tilde{R}^2$ and $\tilde{C}^2$-terms only, was not yet discussed. Our goal is to show that this symmetry breaking mechanism is still at work in this case.

In section 3 we also consider the presence of matter fields in addition to Weyl quadratic action. We study the case of the SM Higgs field (ϕ_1) with all dimension-four couplings allowed by Weyl gauge symmetry. We show that the Stueckelberg mechanism is still present, with an additional benefit: the Higgs potential has spontaneous electroweak symmetry breaking. This follows from a Weyl-invariant non-minimal coupling $\xi_1\phi_1^2\tilde{R}$; the higgs mass becomes $\propto \xi_1/\xi_0$ (Planck units), where ξ_0 is the coefficient of the $\tilde{R}^2$ term.

There is also an ultraviolet (UV) motivation to study Weyl gauge symmetry: this can play a role in early cosmology when effective field theory at short distances becomes nearly conformal. Then models with Weyl (gauged) symmetry [9–22], conformal or global scale symmetry [24–45] provide an interesting setup for UV model building. It is possible that Weyl quadratic gravity be a renormalizable theory, similar to (Riemannian) R^2 gravity [46]. Thus Weyl gauge symmetry is interesting for studying the SM in the presence of gravity, with all scales generated spontaneously. This symmetry may even be respected at the quantum level [28–36, 47]. This opens the possibility to solve the hierarchy problem by endowing the SM with spontaneously broken Weyl gauge symmetry [13–15, 22].

2 Spontaneous breaking of Weyl gauge symmetry

2.1 From Weyl conformal geometry to a Riemannian description

Let us review some aspects of Weyl geometry needed when discussing Weyl gauge invariance of the action. A (local) conformal transformation of the metric and of a field ϕ is given by[1]

$$\begin{aligned} g_{\mu\nu} \to g'_{\mu\nu} &= e^{2\,\alpha(x)}\, g_{\mu\nu}, \\ \phi \to \phi' &= e^{-\alpha(x)\,\Delta}\, \phi. \end{aligned} \tag{2.1}$$

Then $g^{\mu\nu\prime} = e^{-2\alpha(x)} g^{\mu\nu}$ and $\sqrt{g'} = e^{4\alpha(x)}\sqrt{g}$ with $g = |\det g_{\mu\nu}|$. $\Delta = 1$ for a scalar field ϕ (3/2 for fermions). To transformation (2.1) one can associate that of a Weyl gauge field ω_μ which, due to (2.1), has geometric origin:

$$\omega_\mu \to \omega'_\mu = \omega_\mu - \frac{2}{q}\,\partial_\mu\alpha(x). \tag{2.2}$$

Eqs. (2.1), (2.2) define our Weyl (gauge) transformations; q is the coupling to scalar field ϕ, with Weyl-covariant derivative $\tilde{D}_\mu\phi = (\partial_\mu - q/2\,\omega_\mu)\phi$. In Weyl geometry $\tilde{\nabla}_\mu g_{\alpha\beta} = -q\,\omega_\mu\, g_{\alpha\beta}$, with $\tilde{\nabla} = dx^\rho\tilde{\nabla}_\rho$ the Weyl covariant derivative computed with the connection $\tilde{\Gamma}^\rho_{\mu\nu}$, where

$$\tilde{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + \frac{q}{2}\Big[\delta^\rho_\mu\,\omega_\nu + \delta^\rho_\nu\,\omega_\mu - g_{\mu\nu}\,\omega^\rho\Big]. \tag{2.3}$$

[1]The conventions used are similar to those in [48], with metric $(+,-,-,-)$.

$\Gamma_{\mu\nu}^{\rho}$ is the Riemannian (Levi-Civita) connection: $\Gamma_{\mu\nu}^{\rho} = (1/2)\, g^{\rho\lambda}\, (\partial_\mu\, g_{\nu\lambda} + \partial_\nu\, g_{\mu\lambda} - \partial_\lambda\, g_{\mu\nu})$. Using eq. (2.3) one verifies that $(\tilde\nabla_\mu + q\,\omega_\mu)\, g_{\alpha\beta} = D_\mu\, g_{\alpha\beta} = 0$, with D_μ the Riemannian covariant derivative (computed with $\Gamma_{\mu\nu}^{\rho}$). Under eqs. (2.1), (2.2), $\tilde\Gamma_{\mu\nu}^{\rho}$ are invariant, which is not true for $\Gamma_{\mu\nu}^{\rho}$. The system is torsion-free (i.e. $\tilde\Gamma_{\mu\nu}^{\rho} = \tilde\Gamma_{\nu\mu}^{\rho}$) and $\tilde\Gamma_{\mu\nu}^{\rho} \to \Gamma_{\mu\nu}^{\rho}$ if $\omega_\mu \to 0$.

Then the Riemann tensor in Weyl geometry, $\tilde R^{\lambda}_{\mu\nu\sigma}$, is generated by the new $\tilde\Gamma_{\mu\nu}^{\rho}$ by a relation similar to that in the Riemannian geometry:

$$\tilde R^{\lambda}_{\mu\nu\sigma} = \partial_\nu \tilde\Gamma^{\lambda}_{\mu\sigma} - \partial_\sigma \tilde\Gamma^{\lambda}_{\mu\nu} + \tilde\Gamma^{\lambda}_{\nu\rho}\, \tilde\Gamma^{\rho}_{\mu\sigma} - \tilde\Gamma^{\lambda}_{\sigma\rho}\, \tilde\Gamma^{\rho}_{\mu\nu}, \tag{2.4}$$

and $\tilde R_{\mu\sigma} = \tilde R^{\lambda}_{\mu\lambda\sigma}$, $\tilde R = g^{\mu\nu} \tilde R_{\mu\nu}$. With (2.3), (2.4), $\tilde R$ is related to its Riemannian version (R):

$$\tilde R = R - 3\, q\, D_\mu \omega^\mu - \frac{3}{2} q^2\, \omega^\mu \omega_\mu. \tag{2.5}$$

Unlike R, under eqs. (2.1), (2.2), $\tilde R$ transforms covariantly $\tilde R' = e^{-2\alpha(x)} \tilde R$, relevant below.

Finally, the Weyl tensor of Weyl geometry $\tilde C_{\mu\nu\rho\sigma}$, like its Riemannian counterpart $C_{\mu\nu\rho\sigma}$, is a function of associated Riemann and Ricci tensors and scalar curvature.[2] Then [22]

$$\tilde C_{\mu\nu\rho\sigma} = C_{\mu\nu\rho\sigma} - \frac{q}{4}\, (g_{\mu\rho}\, F_{\nu\sigma} + g_{\nu\sigma}\, F_{\mu\rho} - g_{\mu\sigma}\, F_{\nu\rho} - g_{\nu\rho}\, F_{\mu\sigma}) + \frac{q}{2}\, F_{\mu\nu}\, g_{\rho\sigma}, \tag{2.6}$$

where $F_{\mu\nu}$ is the field strength of the Weyl gauge field ω_μ; it is given by[3] $F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ and is invariant under transformations (2.1), (2.2), and the same is true for $\sqrt{g}\, \tilde C_{\mu\nu\rho\sigma} \tilde C^{\mu\nu\rho\sigma}$.

2.2 Spontaneous breaking from Weyl gauge symmetry to Einstein gravity

In the absence of matter, the Weyl action has two quadratic terms, with

$$L = L_1 + L_2 \tag{2.7}$$

Here

$$L_1 = \sqrt{g}\, \frac{\xi_0}{4!}\, \tilde R^2, \qquad \xi_0 > 0. \tag{2.8}$$

Under transformations (2.1), (2.2), L_1 is invariant. Another invariant under (2.1), (2.2) is

$$L_2 = \frac{\sqrt{g}}{\eta}\, \tilde C_{\mu\nu\rho\sigma}\, \tilde C^{\mu\nu\rho\sigma} = \frac{\sqrt{g}}{\eta} \left[C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma} + \frac{3\, q^2}{2}\, F_{\mu\nu}\, F^{\mu\nu} \right], \tag{2.9}$$

where η is the coupling in Weyl geometry and in the second step we used eq. (2.6). L_2 is decomposed into the sum of two terms of Riemannian geometry, each of them Weyl invariant. Note the presence of a kinetic term for ω_μ, relevant below. For this term to be canonically normalized in the presence of L_2, we must set $q^2 = -\eta/6$ (so $\eta < 0$).

[2] Explicitly $\tilde C_{\mu\nu\rho\sigma} = \tilde R_{\mu\nu\rho\sigma} - (1/2)(g_{\mu\rho}\, \tilde R_{\nu\sigma} + g_{\nu\sigma} \tilde R_{\mu\rho} - g_{\mu\sigma} \tilde R_{\nu\rho} - g_{\nu\rho} \tilde R_{\mu\sigma}) + (1/6)(g_{\mu\rho}\, g_{\nu\sigma} - g_{\mu\sigma}\, g_{\nu\rho})\, \tilde R$. A similar relation exists for the Weyl tensor $C_{\mu\nu\rho\sigma}$ of the Riemannian case, in terms of $R_{\mu\nu\rho\sigma}$, $R_{\mu\nu}$ and R.

[3] In Weyl geometry $\tilde F_{\mu\nu} = \tilde D_\mu \omega_\nu - \tilde D_\nu \omega_\mu$ and $\tilde D_\mu \omega_\nu = \partial_\mu \omega_\nu - \tilde\Gamma^{\rho}_{\mu\nu} \omega_\rho$. In Riemannian geometry $F_{\mu\nu} = D_\mu \omega_\nu - D_\nu \omega_\mu$ and $D_\mu \omega_\nu = \partial_\mu \omega_\nu - \Gamma^{\rho}_{\mu\nu} \omega_\rho$. Since $\tilde\Gamma^{\rho}_{\mu\nu} = \tilde\Gamma^{\rho}_{\nu\mu}$, $\Gamma^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\nu\mu}$, then $\tilde F_{\mu\nu} = F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\mu \omega_\nu$.

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In the absence of matter fields, $L_{1,2}$ are the only independent terms allowed by Weyl gauge symmetry, up to a topological term that is counterpart to the Riemannian Gauss-Bonnet term and that we do not include here (see e.g. appendix C in [22]).

Returning to L_1, $\tilde{R}^2$ has higher derivatives, so it propagates an additional scalar state, see e.g. [49, 50]. We extract from the $\tilde{R}^2$ term this (dynamical) degree of freedom via a Lagrangian constraint; this brings a term linear in $\tilde{R}$ and an additional scalar ϕ_0. We have

$$L_1 = \sqrt{g}\,\frac{\xi_0}{4!}\Big[-2\,\phi_0^2\,\tilde{R} - \phi_0^4\Big] \tag{2.10}$$

The equation of motion for ϕ_0 gives $\phi_0^2 = -\tilde{R}$. Using this back in (2.10), one recovers L_1 of (2.8), so Lagrangians (2.8) and (2.10) are classically equivalent. Given its definition, ϕ_0 transforms just like any matter scalar under eq. (2.1). Then a (local) shift symmetry exists, as for a Goldstone (dilaton) field: $\ln\phi_0^2 \to \ln\phi_0^2 - 2\alpha(x)$. This results from the Weyl invariance (2.1), (2.2) of the term $\phi_0^2\,\tilde{R}$; note that its Riemannian counterpart ($\phi_0^2\,R$) does not have this symmetry, hence the importance of this step (obviously (2.10) is invariant under (2.1), (2.2)).

Using eq. (2.5) to replace $\tilde{R}$ by its Riemannian version R and an integration by parts, then

$$L_1 = \sqrt{g}\left[-\frac{\xi_0}{12}\,\phi_0^2\,R - \frac{q}{4}\,g^{\mu\nu}\,\omega_\mu\,K_\nu + \frac{q^2}{8}\,g^{\mu\nu}\,K\,\omega_\mu\,\omega_\nu - \frac{\xi_0}{4!}\,\phi_0^4\right] + \text{total derivative.} \tag{2.11}$$

where we introduced

$$K_\nu = \partial_\nu K, \qquad K = \xi_0\,\phi_0^2. \tag{2.12}$$

Let us assume first that L_1 would be the total Lagrangian of our model; its dependence on ω_μ is algebraic and this field can be integrated out. Its equation of motion is

$$\omega_\mu = \frac{1}{q}\,\partial_\mu \ln K \tag{2.13}$$

Inserting this solution back in (2.11) gives

$$L_{\text{eff}} = \sqrt{g}\left\{-\frac{\xi_0}{2}\left[\frac{1}{6}\phi_0^2\,R + g^{\mu\nu}\partial_\mu\phi_0\partial_\nu\phi_0\right] - \frac{\xi_0}{4!}\,\phi_0^4\right\}. \tag{2.14}$$

L_{eff} is however uninteresting: although derived from (2.8) and invariant under (2.1), it has a remaining "fake" conformal symmetry [52, 53]: its associated current is vanishing. This follows the absence of a kinetic term for the Weyl field which allowed its integration.[4,5]

[4]A second issue is that with $\xi > 0$, ϕ_0 becomes ghost-like while if $\xi_0 < 0$ and with $\langle\phi_0\rangle \neq 0$, Newton constant would be negative (with our conventions). The Einstein term in our convention is $(-1/2)\,\sqrt{g}M_p^2\,R$.

[5]A third issue: a conformal transformation (2.1), $\alpha = -\ln(6M_p/\xi_0\phi)$, ($M_p$: Planck scale) on (2.14) removes ϕ from spectrum and we recover the Einstein action from L_{eff}, but then the number of degrees of freedom in Jordan vs Einstein frame does not match, so "something" is missing; see text after eq. (2.25) for our solution.

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This situation is avoided if ω_μ is dynamical, since then the current does not vanish anymore. A Weyl-invariant kinetic term δL_2 for ω_μ

$$\delta L_2 = -\frac{\sqrt{g}}{4}\, g^{\mu\rho}\, g^{\nu\sigma}\, F_{\mu\nu}\, F_{\rho\sigma} \tag{2.15}$$

can be added 'by hand'; but there is no need to do so since L_2 of (2.9) already contains δL_2!

To conclude, hereafter we shall consider that the defining action of our model is

$$L_1 + \delta L_2 \tag{2.16}$$

If one insists to also include the term $C^2 \equiv C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$, then the action of our model is actually $L_1 + L_2$. In both cases, the equation of motion for ω_μ gives

$$\partial^\alpha (F_{\alpha\mu}\sqrt{g}) + \frac{\sqrt{g}}{2}\,\xi_0\,\phi_0\left[\partial_\mu - \frac{q}{2}\omega_\mu\right]\phi_0 = 0. \tag{2.17}$$

By applying ∂^μ with $F_{\alpha\mu}$ antisymmetric in (α,μ), we now find a non-vanishing current

$$\tilde{K}_\mu \equiv \phi_0\,(\partial_\mu - q/2\,\omega_\mu)\phi_0, \qquad \text{with} \qquad \partial^\mu(\sqrt{g}\,\tilde{K}_\mu) = 0, \tag{2.18}$$

so $\tilde{K}_\mu$ is conserved. Further, on the ground state, assuming $\phi_0(x)$=constant, it follows that $\partial^\mu(\sqrt{g}\,\omega_\mu) = 0$, which is a condition similar to that for a Proca (massive) gauge field, leaving three degrees of freedom for ω_μ. In fact, in the case of a Friedmann-Robertson-Walker (FRW) metric, $g_{\mu\nu} = (1, -a^2(t), -a^2(t), -a^2(t))$, with ϕ only t-dependent, the current conservation (in covariant form $D^\mu \tilde{K}_\mu = 0$) leads naturally to ϕ_0=constant [39].

The other equations of motion, of $g^{\mu\nu}$ (after trace) and of ϕ_0, derived from $L_1+\delta L_2$, are

$$\frac{\xi_0}{12}\,\phi_0^2\,R + \frac{q}{4}\,g^{\mu\nu}\,\omega_\mu\,K_\nu - \frac{q^2}{8}\,K\,\omega^\mu\,w_\mu + 2V + \frac{\xi_0}{4}\Box\phi_0^2 = 0 \tag{2.19}$$

and

$$-\frac{\xi_0}{12}\,\phi_0\,R + \frac{q^2}{8}\,\omega_\mu\,\omega^\mu\,\xi_0\,\phi_0 - \frac{1}{2}\,V' + \frac{q}{4\,\sqrt{g}}\,\xi_0\,\phi_0\,\partial_\mu(\sqrt{g}\,\omega^\mu) = 0 \tag{2.20}$$

where we denoted $V \equiv (\xi_0/4!)\,\phi_0^4$. Eq. (2.20) leads to $\langle\phi_0^2\rangle = -\langle\tilde{R}\rangle$ that we already know; thus the ground state has $\tilde{R}$=constant (this is called Weyl gauge [21]). Further, when adding eqs. (2.19), (2.20), with the last one multiplied by ϕ_0, one finds that on the ground state

$$4\,V(\langle\phi_0\rangle) - \phi_0\,V'(\langle\phi_0\rangle) = 0 \tag{2.21}$$

The potential is thus a homogeneous function of fields, as expected (given the symmetry); with our V, it is automatically respected; $\langle\phi_0\rangle$ is thus a parameter, not fixed by theory; in a Weyl-invariant theory only (dimensionless) *ratios* of vev's can be fixed.

To see how ω_μ becomes massive consider a conformal transformation to Einstein frame

$$\hat{g}_{\mu\nu} = \Omega\,g_{\mu\nu}, \qquad \Omega = \frac{\xi_0\,\phi_0^2}{6\,M^2} \tag{2.22}$$

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M is a mass scale present for dimensional reasons;[6] its role is discussed shortly. Then

$$L_1 = \sqrt{\hat g}\left[\frac{-1}{2}\,M^2\,\hat R + \frac{3\,M^2}{4\,\Omega^2}\,\hat g^{\mu\nu}\,(\partial_\mu\Omega)(\partial_\nu\Omega) + \frac{\hat g^{\mu\nu}}{\Omega}\left(\frac{-q}{4}\omega_\mu\,K_\nu + \frac{q^2}{8}K\,\omega_\mu\,\omega_\nu - \frac{\xi_0\,\phi_0^4}{4!\,\Omega}\right)\right]. \tag{2.23}$$

Thus the field ϕ_0 is indeed dynamical, it has a kinetic term. Also note that δL_2 of (2.15) is invariant under (2.22) since the metric part and $F_{\mu\nu}$ are invariant. Further, introduce

$$\omega'_\mu = \omega_\mu - \frac{1}{q}\,\partial_\mu \ln K. \tag{2.24}$$

Since $K = \xi_0\phi_0^2$ then ϕ_0 is absorbed into ω'_μ in (2.24) where also $\partial_\mu \ln K = \partial_\mu \ln\Omega$. Using (2.24), replace ω_μ in L_1 and denote by $F'_{\mu\nu}$ the field strength of ω'_μ. Then the total Lagrangian is

$$L_1 + \delta L_2 = \sqrt{\hat g}\left[-\frac{1}{2}\,M^2\,\hat R - \frac{3\,M^4}{2\,\xi_0}\right] + \sqrt{\hat g}\left[-\frac{1}{4}\,\hat g^{\mu\rho}\,\hat g^{\nu\sigma}\,F'_{\mu\nu}\,F'_{\rho\sigma} + \frac{3\,q^2}{4}\,M^2\,\hat g^{\mu\nu}\omega'_\mu\,w'_\nu\right]. \tag{2.25}$$

It is important to note that there is no kinetic term left for ϕ_0, since it was cancelled by that generated when expressing ω_μ-dependence in eq. (2.23) in terms of ω'_μ. The massless Weyl field has become massive after "eating" the Goldstone mode (ϕ_0), via a Stueckelberg mechanism, *without* a corresponding Higgs mode in the spectrum or a potential. This mechanism essentially re-distributes the degrees of freedom in the action: the initial massless ω_μ and the real scalar ϕ_0 are converted into a single massive Weyl field ω_μ with three degrees of freedom (recall $\partial^\mu(\sqrt{g}\,\omega_\mu) = 0$); so the number of degrees of freedom is indeed conserved when going from the Jordan to the Einstein frame (as it should).

Having checked this conservation, eqs. (2.22), (2.24) may also be seen as a particular Weyl transformation eqs. (2.1), (2.2) of $2\alpha(x) = \ln\Omega$, to the unitary gauge ("gauge fixing") where the Goldstone ϕ_0 is absent (having been "eaten" by now massive ω'_μ) and giving $\hat\phi_0^2 = \phi_0^2/\Omega = 6M^2/\xi_0$ =constant. The unitary gauge being non-renormalizable, one should not use (2.25) for loop calculations, but use instead e.g. (2.11), (2.15).

In eq. (2.25) we obtained the Einstein-Hilbert action, a positive cosmological constant and the Proca action for a massive Weyl field[7] ω'_μ; its mass is related to the Planck scale (M_p)

$$M_p^2 = M^2, \qquad m_\omega^2 = (3/2)\,q^2\,M^2. \tag{2.26}$$

The value of m_ω depends on the gauge coupling q in $L_1 + \delta L_2$, see L_1 of (2.11) and canonically normalised δL_2 of (2.15). Below the scale m_ω the Weyl field decouples and Einstein gravity is obtained as a "low-energy" effective theory limit. The scale M introduced on dimensional grounds in (2.22), remains undetermined by the theory. From (2.11),

[6] One may avoid introducing M if $\hat g_{\mu\nu}$ is dimensionful [26], which may be acceptable since the metric is a dynamical variable of kinetic term found in $\tilde C^2_{\mu\nu\rho\sigma} = C^2_{\mu\nu\rho\sigma} + \ldots$ rather than a simple dimensionless constant.

[7] Our above result is consistent with those in [49–51] where it was shown that 'pure' R^2 gravity in the Riemannian geometry, describes Einstein gravity plus a cosmological constant and a scalar (Goldstone) field. In our case the Goldstone mode is eaten by the Weyl gauge field present in the $\tilde R^2$-term in Weyl geometry.

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$M^2 = \xi_0 \langle\phi_0\rangle^2/6$ which is equally undetermined in a theory invariant under (2.1), (2.2), as discussed.[8]

In the decoupling limit of the massive Weyl gauge field, Weyl geometry "flows" into a Riemannian geometry. This may also be seen dynamically from the conserved current in (2.18) which for a FRW metric is driving ϕ_0 to a constant value [39] (in this case $\propto M$). Below m_ω the Weyl gauge field is absent, therefore the Weyl connection becomes that of the Riemannian geometry, $\tilde\Gamma^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu}$. As a result, long-held criticisms of Weyl quadratic gravity without matter, such as (see e.g. [2–4, 21]): the changing of the norm of a vector under parallel transport on a closed curve, or the changing of the atomic spectral lines, do not apply since such effects are strongly suppressed by a high scale $m_\omega \propto M_p$.

So far our analysis was based on the Lagrangian $L_1 + \delta L_2$. Considering instead the Lagrangian $L = L_1 + L_2$, one has to include the remaining term in the r.h.s. of (2.9), i.e. $C^2 \equiv C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$; this is immediate, since this is invariant under (2.22), (2.24). This term provides the kinetic term for the metric [27] and is needed at the quantum level to renormalize divergences like k^4. However, in this case the coupling q cannot be adjusted at will anymore, being proportional to η (see text after (2.9)). Lowering q too much brings a too light mass $m_g^2 \sim \eta\, M^2$ of the spin-two ghost of the C^2 term, together with its instability.

It is interesting that Lagrangian (2.8), (2.9) dictated by Weyl geometry (no matter) is so rich in structure, encoding Stueckelberg mechanism, dilaton ϕ_0, Einstein action, Proca action for massive ω_μ, a positive cosmological constant and fields kinetic terms and interactions.

3 Adding matter fields

3.1 Spontaneous breaking of Weyl gauge symmetry

Let us now consider the SM scalar sector in addition to Weyl quadratic gravity action, with all dimension-four couplings allowed by the Weyl gauge symmetry. Note that the SM fermions do not have couplings to the Weyl gauge field (in the absence of torsion) [13, 15]. We consider the SM Higgs field and denote by ϕ_1 its neutral component. Then the Lagrangian we study, invariant under (2.1), (2.2), and written in a Weyl geometry language, is

$$\mathcal{L} = \big(L_1 + \delta L_2\big) + \sqrt{g}\,\Big[-\frac{1}{12}\,\xi_1\phi_1^2\,\tilde R + \frac{1}{2}\,g^{\mu\nu}\,\tilde D_\mu\phi_1\tilde D_\nu\phi_1 - V_1\Big] \tag{3.1}$$

with Weyl-covariant derivative $\tilde D_\mu\phi_1 = (\partial_\mu - q/2\,\omega_\mu)\phi_1$ and L_1 and δL_2 of eqs. (2.8), (2.15)

$$L_1 + \delta L_2 = \sqrt{g}\,\frac{\xi_0}{4!}\,\tilde R^2 - \frac{\sqrt{g}}{4}\,g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}. \tag{3.2}$$

The Weyl tensor squared term $C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$ in L_2 of (2.9) is not included in $\mathcal{L}$, but since it is not affecting the transformations below, it can easily be added (replace $\delta L_2 \to L_2$). Also $F_{\mu\nu}$ is the same in both Riemann and Weyl geometry (in the absence of torsion, as here).

[8]In conformal theory only ratios of scales can be predicted in terms of dimensionless couplings. If this symmetry is broken explicitly (by quantum corrections) dimensional transmutation can determine a field vev.

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The only possible form of the Higgs potential V_1 consistent with the symmetry, is

$$V_1 = \frac{\lambda_1}{4!}\,\phi_1^4. \tag{3.3}$$

Using (2.10) and (2.5), one finds, following steps similar to the previous section[9]

$$\mathcal{L} = \delta L_2 + \sqrt{g}\left[-\frac{1}{12}\xi_a\phi_a^2\,R - \frac{q}{4}g^{\mu\nu}\omega_\mu\partial_\nu\mathcal{K} + \frac{q^2}{8}\,g^{\mu\nu}\,\mathcal{K}\,\omega_\mu\,\omega_\nu + \frac{1}{2}\,g^{\mu\nu}\partial_\mu\phi_1\partial_\nu\phi_1 - \mathcal{V}(\phi_0,\phi_1)\right] \tag{3.4}$$

up to a total derivative, with

$$\mathcal{V} = \frac{\xi_0}{4!}\,\phi_0^4 + \frac{\lambda_1}{4!}\,\phi_1^4, \tag{3.5}$$

$$\mathcal{K}_\mu = \partial_\mu\,\mathcal{K}, \qquad \mathcal{K} = \xi_a\phi_a^2 + \phi_1^2, \qquad \text{sum over } a = 0, 1. \tag{3.6}$$

In (3.4) a sum over the repeated index "a" is understood, with $a = 0, 1$. The generalisation of this action to more matter (scalar) fields is immediate.

We perform a conformal transformation to the Einstein frame, to a new metric $\hat{g}_{\mu\nu}$

$$\hat{g}_{\mu\nu} = \Omega\,g_{\mu\nu}, \qquad \Omega = \frac{1}{6M^2}\,(\xi_0\phi_0^2 + \xi_1\phi_1^2) \tag{3.7}$$

One has

$$\mathcal{L} = \sqrt{\hat{g}}\,\Big\{ -\frac{1}{2}\,M^2\,\hat{R} + \frac{3}{4}\,\hat{g}^{\mu\nu}\,M^2\,\partial_\mu\ln\Omega\,\partial_\nu\ln\Omega$$
$$+\frac{\hat{g}^{\mu\nu}}{\Omega}\left[\frac{1}{2}\,\partial_\mu\phi_1\,\partial_\nu\phi_1 - \frac{q}{4}\omega_\mu\,\partial_\nu\,\mathcal{K} + \frac{q^2}{8}\,\mathcal{K}\,\omega_\mu\,\omega^\mu\right] - \frac{1}{4}\,\hat{g}^{\mu\rho}\hat{g}^{\nu\sigma}\,F_{\mu\nu}\,F_{\rho\sigma} - \frac{\mathcal{V}}{\Omega^2}\Big\} \tag{3.8}$$

Further, introduce

$$\omega'_\mu = \omega_\mu - \frac{1}{q}\,\partial_\mu\,\ln\mathcal{K}. \tag{3.9}$$

The kinetic terms in $\mathcal{L}$ for ϕ_0 and ϕ_1 (hereafter $\mathcal{L}_{\text{k.t.}}$) become, after using eqs. (3.7), (3.9)

$$\mathcal{L}_{\text{k.t.}} = \sqrt{\hat{g}}\,\hat{g}^{\mu\nu}\,\frac{1}{8\,\Omega}\left[\frac{6M^2}{\Omega}(\partial_\mu\Omega)\,(\partial_\nu\Omega) - \frac{1}{\mathcal{K}}(\partial_\mu\mathcal{K})\,(\partial_\nu\mathcal{K}) + 4\,(\partial_\mu\phi_1)(\partial_\nu\phi_1)\right] \tag{3.10}$$

$$= \frac{3M^2}{4}\frac{\sqrt{\hat{g}}\,\hat{g}^{\mu\nu}}{1+\mathcal{Z}}(\partial_\mu\ln\mathcal{Z})(\partial_\nu\ln\mathcal{Z}), \quad \text{with} \quad \mathcal{Z} \equiv \xi_0\,\frac{\phi_0^2}{\phi_1^2} + \xi_1. \tag{3.11}$$

We see there is only one kinetic term left in the action, for the new variable $\mathcal{Z}$ which is a combination of initial ϕ_0, ϕ_1. One could introduce polar coordinates fields (ρ, θ), such as

[9] In V_1 there is no classical coupling of ϕ_1 to the dilaton (ϕ_0) hidden in Weyl's quadratic action (3.2) since ϕ_0 is an intrinsic part of our $\tilde{R}^2$ term from which is extracted by "linearisation" of $\tilde{R}^2$, eq. (2.10). Adding to V_1 a term $(\lambda_m/12)\,\phi_0^2\phi_1^2$ is also redundant, since together with (2.10), integrating out ϕ_0 simply restores the original $\tilde{R}^2$ term after a redefinition of initial couplings of ϕ_1: $\lambda_1 \to \lambda_1 + \lambda_m^2/\xi_0$, $\xi_1 \to \xi_1 + \lambda_m$. Also, adding a term $\tilde{\xi}\phi_0^2\tilde{R}$ to (3.2) would introduce an extra (dynamical) degree of freedom beyond the dilaton in $\tilde{R}^2$ term!

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$\phi_0 = (1/\sqrt{\xi_0})\,\rho\sin\theta$ and $\phi_1 = (1/\sqrt{1+\xi_1})\,\rho\cos\theta$. In such basis $\mathcal{Z}$ is an "angular" variable field while $\mathcal{K}$ entering in (3.9) becomes $\mathcal{K} = \rho^2$ and is the "radial" direction in field space.

After transformation (3.9) the terms[10] in (3.8) other than $\mathcal{L}_{\text{k.t.}}$ also depend only on the ratio $\phi_0/\phi_1 \sim \mathcal{Z}$ and not on the radial direction field! To anticipate, this is explained by ω'_μ that must have "eaten" the radial direction in (3.9) (Stueckelberg mechanism), see later.

We bring to canonical form the kinetic term $\mathcal{L}_{\text{k.t.}}$ by replacing

$$\frac{1}{\mathcal{Z}} = \sinh^2 \frac{h}{M\sqrt{6}}, \tag{3.12}$$

where h is our (neutral) Higgs field. This gives that $\mathcal{L}_{\text{k.t.}} = (1/2)\sqrt{\hat{g}}\,\hat{g}^{\mu\nu}(\partial_\mu h)(\partial_\nu h)$.

From $\mathcal{L}$ of (3.8), using (3.11) and notations (3.9), (3.12), we obtain our final Lagrangian

$$\mathcal{L} = \sqrt{\hat{g}}\left[-\frac{M^2}{2}\hat{R} + \frac{\hat{g}^{\mu\nu}}{2}(\partial_\mu h)(\partial_\nu h) - \frac{1}{4}\,\hat{g}^{\mu\rho}\hat{g}^{\nu\sigma}\,F'_{\mu\nu}F'_{\rho\sigma} + \frac{m_\omega^2(h)}{2}\omega'_\mu\,\omega'^\mu - \hat{\mathcal{V}}(h)\right], \tag{3.13}$$

where $F'_{\mu\nu}$ is the field strength of ω'_μ and

$$m_\omega^2(h) = \frac{q^2\mathcal{K}}{4\,\Omega} = \frac{3}{2}\,M^2\,q^2\,\cosh^2\frac{h}{M\sqrt{6}}, \tag{3.14}$$

$$\hat{\mathcal{V}}(h) = \frac{\mathcal{V}}{\Omega^2} = \frac{3\,M^4}{2\;\xi_0}\left[1 - 2\,\xi_1\,\sinh^2\frac{h}{M\sqrt{6}} + \big(\lambda_1\,\xi_0 + \xi_1^2\big)\,\sinh^4\frac{h}{M\sqrt{6}}\right]. \tag{3.15}$$

For small higgs field values $h \ll M$ one has

$$m_\omega^2(h) = \frac{3}{2}\,M^2\,q^2\left[1 + \frac{h^2}{6\,M^2} + \frac{h^4}{108\,M^4} + \mathcal{O}(h^6/M^6)\right] \tag{3.16}$$

$$\hat{\mathcal{V}}(h) = \frac{3\,M^4}{2\,\xi_0} - \frac{\xi_1\,M^2}{2\;\xi_0}\,h^2 + \frac{1}{4!}\left[\lambda_1 + \frac{\xi_1\,(3\xi_1-2)}{3\,\xi_0}\right]\,h^4 + \mathcal{O}(h^6/M^6) \tag{3.17}$$

The first term on the r.h.s. of (3.16) is the mass of the Weyl gauge field ω'_μ, up to additional corrections of order $\mathcal{O}(\langle h\rangle^2/M^2)$ due to the Higgs mechanism itself, if $\langle h\rangle \neq 0$ (see below). Therefore, following eq. (3.9) the "radial" degree of freedom in the field space (dilaton) was "eaten" by the gauge field ω'_μ which has become massive, via the Stueckelberg mechanism. The number of degrees of freedom is conserved: initially we had 2 real scalars (ϕ_0, ϕ_1) and massless ω_μ which were re-arranged into one real scalar h and a massive gauge field ω'_μ.

$\mathcal{L}$ in eq. (3.13) includes the Einstein action with a positive cosmological constant, a massive Weyl gauge field and a potential for the Higgs field h. For a vanishing non-minimal coupling $\xi_1 = 0$, see our starting $\mathcal{L}$ in (3.1), one recovers the initial potential for ϕ_1 plus a cosmological constant similar to that in Weyl quadratic action without matter, eq. (2.25).

Eqs. (3.13) to (3.15) give the Higgs sector for the SM enlarged with Weyl gauge symmetry and can be used for further investigations of this symmetry. These equations bring no restrictions at the classical level for the value of h relative to (otherwise arbitrary) M.

[10]These are the terms in (3.8) proportional to $\mathcal{V}/\Omega^2$ and $\mathcal{K}/\Omega$ and clearly depend only of the ratio ϕ_0/ϕ_1.

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3.2 Electroweak symmetry breaking

From (3.15), (3.17), we see that a non-minimal coupling $\xi_1 \tilde{R}^2 \phi_1^2$, $(\xi_1 > 0)$ induced a negative quadratic term in the potential and spontaneous electroweak symmetry breaking, with

$$m_h^2 = \frac{\xi_1}{\xi_0}\, M^2. \tag{3.18}$$

A hierarchy $m_h^2 \ll M^2$ can be arranged by a classical tuning $\xi_1 \ll \xi_0$ to an ultraweak value of ξ_1. This is a gravitational higgs mechanism (which forbids the presence of TeV-scale squarks!).[11]

Regarding the scale M, in the limit $h \ll M$, can be identified with a constant value of the dilaton $\langle\phi_0\rangle$ (in this limit $\langle\phi_0\rangle \gg \langle\phi_1\rangle$); like M, $\langle\phi_0\rangle$ is a parameter not fixed by the theory and must be tuned to the actual Planck scale value (as mentioned, only ratios of scales can be determined in a Weyl invariant theory). Finally, the vacuum energy is still positive, dominated by the dilaton contribution $\hat{\mathcal{V}}_{\rm min} = 3\, M^4/(2\xi_0)[1 - \xi_1^2/(\xi_0\lambda_1) + \ldots]$.

A study of the quantum corrections to the Higgs mass is beyond the goal of this paper. However, we stress here the role Weyl gauge symmetry may play at the quantum level. Note that classically only the Higgs sector of the SM couples to the Weyl field ω_μ [13, 15]. Using a Weyl-invariant regularization [47] one could answer whether Weyl gauge symmetry can protect ξ_1 and thus the Higgs mass $m_h \propto \xi_1$ against large quantum corrections.[12] Note that each of the terms in our $\mathcal{L}$ of (3.1), (3.2) is separately Weyl gauge invariant, see eqs. (2.1), (2.2). We expect that this symmetry bring some "protection" for the ultraviolet (UV) behaviour of this theory. In particular we expect a better UV behaviour than in Riemannian gravity e.g. Agravity [44, 45] where no similar local symmetry (Weyl, conformal) exists. This motivates a quantum analysis of Weyl quadratic gravity.

In the light of the results in [46] for renormalizability, one would expect a Weyl quadratic theory given by $L_1 + L_2$ of (2.8), (2.9) be renormalizable. One must however pay attention to the analytical continuation from Minkowski to the Euclidean space which is non-trivial in the presence of higher derivative terms [54].

4 Conclusions

We considered the general action of Weyl gravity in the absence of matter, which is the sum of two terms *quadratic* in the curvature scalar ($\tilde{R}$) and in the Weyl tensor ($\tilde{C}_{\mu\nu\rho\sigma}$) of the Weyl conformal geometry, then studied its spontaneous breaking. We also studied the effect of coupling Weyl (quadratic) gravity to a Higgs-like matter sector.

In the absence of matter fields, the Weyl gauge field ω_μ in the action becomes massive, with a mass $m_\omega \sim q\, M_p$ where q is the coupling and M_p the Planck scale. This happens via a Stueckelberg mechanism which is essentially a re-arrangement of the degrees of freedom (without a higgs vev or potential needed): the field ω_μ "eats" a scalar degree of freedom

[11] The quartic term in (3.17) remains positive as long as $3\lambda_1\xi_0 > (2-3\xi_1)\xi_1$ $(\lambda_1 > 0,\, \xi_1 > 0)$ which is easily satisfied for small enough $\xi_1 \ll \xi_0$ required for small Higgs mass. Also note from eq. (3.15) that at large h, the quartic term receives overall positive corrections from the gravitational effects, proportional to ξ_1^2/ξ_0.

[12] Without additional fine tuning beyond the classical one mentioned above.

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(dilaton) "extracted" from the $\tilde{R}^2$ term. The necessary presence of a kinetic term for the Weyl gauge field originates from the term $\tilde{C}_{\mu\nu\rho\sigma}\tilde{C}^{\mu\nu\rho\sigma}$ of Weyl geometry. However, if this is not included in the initial Weyl quadratic action, the gauge kinetic term may be added on its own in Weyl geometry, on symmetry arguments only.

After the Stueckelberg mechanism, one obtains the Einstein-Hilbert action, with a positive cosmological constant and the Proca action for the massive Weyl gauge field (and a Riemannian Weyl-tensor-squared term, if $\tilde{C}_{\mu\nu\rho\sigma}\tilde{C}^{\mu\nu\rho\sigma}$ was included initially). No additional matter scalar field (Higgs, etc.) is needed to this purpose. Below the mass m_ω, the Weyl field ω_μ decouples and Weyl geometry (connection) becomes Riemannian. Therefore, the Einstein-Hilbert action is a "low-energy" effective theory limit of the Weyl quadratic gravity (without matter) which avoids in this way previous, long-held criticisms against it.

This result has consequences for physics at high scale where Weyl gauge symmetry may be present (inflation, black hole physics, conformal supergravity). During the spontaneous breaking of this symmetry the number of degrees of freedom is indeed conserved, also when going to the unitary gauge $\hat{\phi}_0$=constant (unlike in models invariant under "usual" Weyl symmetry, eq. (2.1)). It is remarkable that the simple Weyl quadratic action dictated by Weyl geometry alone is so rich in structure, encoding a Stueckelberg mechanism, the Einstein-Hilbert action, Proca action, positive cosmological constant, the dilaton, the metric and their interactions. Further study of this symmetry should use the Weyl geometry formulation which is easier (than the Riemannian one) since then the scalar curvature transforms covariantly, so each operator respects this symmetry. This places on equal footing, in the Lagrangian, Weyl gauge symmetry and (internal) gauge symmetries.

In the presence of a scalar matter field ϕ_1 (Higgs), the Weyl gauge symmetry allows a non-minimal coupling $\xi_1\phi_1^2\,\tilde{R}$, in addition to the mentioned Weyl quadratic action and to the matter action; the latter is that of the SM Higgs sector with a potential $\lambda_1\phi_1^4/4!$ (as the only one allowed by this symmetry). The Stueckelberg breaking mechanism is still at work and the Weyl gauge field is "eating" the dilaton (the radial direction in the field space of ϕ_0, ϕ_1) which subsequently disappears from the action. At the same time, in the Riemannian limit (ω_μ decoupled), the scalar potential of the remaining Higgs degree of freedom acquires at low energy ($h \ll M$) a negative quadratic term $\propto \xi_1/\xi_0$. This is gravitationally-induced spontaneous electroweak symmetry breaking. It is worth investigating further if the Higgs mass value (ξ_1/ξ_0 in Planck units) is stable at the quantum level in SM with spontaneously broken Weyl gauge symmetry.

Note added. After completing this work we became aware of [55–57] where the connection in a general theory of gravity is shown to acquire mass via a Higgs mechanism, while at low scales is "frozen" to the Levi-Civita connection. This is consistent with our result.

Acknowledgments

We thank Mihai Visinescu (IFIN Bucharest) and Hyun Min Lee (Chung-Ang University, Seoul) for discussions on Weyl geometry and its applications.

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