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Higher spin superfield interactions with complex linear supermultiplet: conserved supercurrents and cubic vertices

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Abstract: We continue the program of constructing cubic interactions between matter and higher spin supermultiplets. In this work we consider a complex linear superfield and we find that it can have cubic interactions only with supermultiplets with propagating spins $j = s + 1$, $j = s + 1/2$ for any non-negative integer s (half-integer superspin supermultiplets). We construct the higher spin supercurrent and supertrace, these compose the canonical supercurrent multiplet which generates the cubic interactions. We also prove that for every s there exist an alternative minimal supercurrent multiplet, with vanishing supertrace. Furthermore, we perform a duality transformation in order to make contact with the corresponding chiral theory. An interesting result is that the dual chiral theory has the same coupling constant with the complex linear theory only for odd values of s, whereas for even values of s the coupling constants for the two theories have opposite signs. Additionally we explore the component structure of the supercurrent multiplet and derive the higher spin currents. We find two bosonic currents for spins $j = s$ and $j = s + 1$ and one fermionic current for spin $j = s + 1/2$.

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Contents

1 Introduction

In a recent paper [1] the cubic interactions between the chiral supermultiplet and higher spin supermultiplets were constructed via Noether's procedure. The conclusion was that both massive and massless chiral superfields couple only to half-integer superspin supermultiplets $(s+1, s+1/2)$ with an interesting twist that the integer s must be odd for the case of massive chirals. Furthermore, excplicit expressions were given for the supercurrent multiplet which includes the higher spin supercurrent $\mathcal{J}_{\alpha(s)\dot{\alpha}(s)}$,[1] and the higher spin supertrace $\mathcal{T}_{\alpha(s-1)\dot{\alpha}(s-1)}$.

This paper is the continuation of that program for a different matter supermultiplet. In this work, the role of matter will be played by the complex linear supermultiplet and the goal is to (i) find the spin selection rules for consistent cubic interactions between the complex linear and the $4D$, $\mathcal{N}=1$ super-Poincaré higher spin supermultiplets and (ii) give explicit expressions for the higher spin supercurrent supermultiplets.

The existence of *variant* descriptions of supersymmetric theories has been well documented [2] and one of the most well known examples are the minimal and non-minimal formulations of $4D$, $\mathcal{N}=1$ supergravity. The complex linear supermultiplet is another well known, non-minimal, description of the scalar multiplet. In a previous paper [3] we derived the supercurrent multiplets for various free and interacting (including higher derivatives) theories of a complex linear superfield (Σ) by investigating the linearized coupling to supergravity generated by linearized superdiffeomorphisms.

[1]The notation $\alpha(s)$ represents a string of s undotted, symmetric indices α, $\alpha_1\alpha_2\ldots\alpha_s$.

In this paper, we generalize the linearized superdiffeomorphism transformation of Σ in order to include higher rank parameters in a manner consistent with the linearity constraint of Σ ($\bar{\mathrm{D}}^2\Sigma = 0$). Using this *higher spin* transformation of Σ, we perform a perturbative Noether's procedure in order to construct an invariant theory, up to first order in coupling constant. This will reveal the cubic interactions of Σ with the $4D$, $\mathcal{N} = 1$ super-Poincaré higher spin supermultiplets. From that action we can read off the corresponding higher spin supercurrent multiplet which generates the cubic vertices.

The results we find are that a complex linear superfield can have cubic interactions only with half-integer superspin supermultiplets $(s+1, s+1/2)$ exactly like a chiral superfield. Moreover, we provide explicit expressions for the both the *canonical* and *minimal* supercurrent multiplets. Additionally, we investigate the duality transformation between the complex linear and the chiral in the presence of these higher spin cubic interactions. Interestingly enough we find that the *charge* (the coupling constant that controls these cubic interactions) of the dual theory differs from the charge of complex linear by a sign that depends on the value of s. Specifically, for even values of s they are opposite and for odd values of s are the same. We also calculate the set of component higher spin currents. There are two bosonic conserved currents, one for spin $j = s$ and one for spin $j = s+1$ and there is a fermionic current for spin $j = s+1/2$. These currents agree with the expressions derived from the chiral theory presented in [1].

The paper is organized in the following manner. In section 2 we consider a family of first order transformations for the complex linear superfield and their compatibility with the linearity condition. In section 3 we use them for Noether's procedure in order to construct an invariant action. This will force us to introduce the cubic interactions between Σ and higher spin. These interactions are generated by the *canonical* higher spin supercurrent multiplet which includes the higher spin supercurrent and supertrace. In section 4 we prove that always exist an alternative *minimal* supercurrent multiplet in which the higher spin supertrace vanishes. This is possible due to (*i*) the freedom of choosing appropriate improvement terms, (*ii*) the freedom to absorb trivial terms by redefining Σ and (*iii*) the freedom in defining the supercurrent and supertrace up to a particular equivalence relation. In section 5 we discuss the superspace conservation equations for the supercurrent multiplets and in section 6 we compare the results found for the complex linear with the results for chiral, by performing the duality between them. In section 7 we extract the component higher spin currents and in section 8 we summarize our results.

For additional developments on the topic of supersymmetric higher spin supercurrents the reader can refer to [4, 5] and for non-supersymmetric higher spin to [6].

2 First order transformation for complex linear superfield

A cubic interaction between two types of fields can be written in the form jh, where j is a current constructed from matter fields ϕ and h is a set of gauge fields. Because the gauge field h is defined up to a gauge redundancy, the current j must be conserved. Noether's method is a systematic and perturbative method for constructing invariant theories that respect these gauge redundancies and therefore generate the appropriate interactions between matter and gauge fields [7]. In this approach one expands the action $S[\phi,h]$ and the transformation of fields in a power series of a coupling constant g

$$S[\phi,h] = S_0[\phi] + gS_1[\phi,h] + g^2S_2[\phi,h] + \dots\,, \tag{2.1}$$
$$\delta\phi = \delta_0[\xi] + g\delta_1[\phi,\xi] + g^2\delta_2[\phi,\xi] + \dots\,, \tag{2.2}$$
$$\delta h = \delta_0[\zeta] + g\delta_1[h,\zeta] + g^2\delta_2[h,\zeta] + \dots \tag{2.3}$$

where $S_i[\phi,h]$ includes the interaction terms of order $i+2$ in the number of fields and δ_i is the part of transformation with terms of order i in the number of fields. The requirement for invariance up to order g^1 (cubic interactions) is

$$g\frac{\delta S_0}{\delta\phi}\delta_1\phi + g\frac{\delta S_1}{\delta\phi}\delta_0\phi + g\frac{\delta S_1}{\delta h}\delta_0 h = 0\,. \tag{2.4}$$

In this work, the role of matter will be played by the complex linear supermultiplet, described by a complex linear superfield Σ ($\bar{\mathrm{D}}^2\Sigma = 0$). For the role of gauge fields we consider the massless, higher spin, irreducible representations of the $4D$, $\mathcal{N}=1$, super-Poincaré algebra. These were first introduced in [8–10] and later given in a superspace formulation [11–13] and further developed in [14–17]. Higher spin supermultiplets are parametrized by a quantum number called *superspin* Y, which is a supersymmetric extension of spin and it takes integer s and half-integer $s+1/2$ values. Massless higher spin supermultiplets contain two irreducible representations of the Poincaré algebra with spins j and $j-1/2$, therefore we denote the entire supermultiplet by $(j, j-1/2)$. We briefly remind the reader of the various superspace descriptions of free super-Poincaré higher spin supermultiplets:

1. The integer superspin $Y = s$ supermultiplets $(s+1/2, s)$ [11][2] are described by a pair of superfields $\Psi_{\dot\alpha(s)\dot\alpha(s-1)}$ and $V_{\alpha(s-1)\dot\alpha(s-1)}$ with the following zero order gauge transformations

$$\delta_0\Psi_{\alpha(s)\dot\alpha(s-1)} = -\mathrm{D}^2L_{\alpha(s)\dot\alpha(s-1)} + \frac{1}{(s-1)!}\bar{\mathrm{D}}_{(\dot\alpha_{s-1}}\Lambda_{\alpha(s)\dot\alpha(s-2))}\,, \tag{2.5a}$$
$$\delta_0V_{\alpha(s-1)\dot\alpha(s-1)} = \mathrm{D}^{\alpha_s}L_{\alpha(s)\dot\alpha(s-1)} + \bar{\mathrm{D}}^{\dot\alpha_s}\bar{L}_{\alpha(s-1)\dot\alpha(s)}\,. \tag{2.5b}$$

2. The half-integer superspin $Y = s+1/2$ supermultiplets $(s+1, s+1/2)$ [12][2] have two formulations, the transverse and the longitudinal. The transverse description uses the pair of superfields $H_{\alpha(s)\dot\alpha(s)}$, $\chi_{\alpha(s)\dot\alpha(s-1)}$ with the following zero order gauge transformations

$$\delta_0H_{\alpha(s)\dot\alpha(s)} = \frac{1}{s!}\mathrm{D}_{(\alpha_s}\bar{L}_{\alpha(s-1))\dot\alpha(s)} - \frac{1}{s!}\bar{\mathrm{D}}_{(\dot\alpha_s}L_{\alpha(s)\dot\alpha(s-1))}\,, \tag{2.6a}$$
$$\delta_0\chi_{\alpha(s)\dot\alpha(s-1)} = \bar{\mathrm{D}}^2L_{\alpha(s)\dot\alpha(s-1)} + \mathrm{D}^{\alpha_{s+1}}\Lambda_{\alpha(s+1)\dot\alpha(s-1)} \tag{2.6b}$$

[2]We are following the reformulation given in [15].

whereas the longitudinal description uses the superfields $H_{\alpha(s)\dot\alpha(s)}$, $\chi_{\alpha(s-1)\dot\alpha(s-2)}$ with

$$\delta_0 H_{\alpha(s)\dot\alpha(s)} = \frac{1}{s!}\mathrm{D}_{(\alpha_s}\bar{L}_{\alpha(s-1))\dot\alpha(s)} - \frac{1}{s!}\bar{\mathrm{D}}_{(\dot\alpha_s}L_{\alpha(s)\dot\alpha(s-1))}\,, \tag{2.7a}$$

$$\delta_0\chi_{\alpha(s-1)\dot\alpha(s-2)} = \bar{\mathrm{D}}^{\dot\alpha_{s-1}}\mathrm{D}^{\alpha_s}L_{\alpha(s)\dot\alpha(s-1)} + \frac{s-1}{s}\mathrm{D}^{\alpha_s}\bar{\mathrm{D}}^{\dot\alpha_{s-1}}L_{\alpha(s)\dot\alpha(s-1)} + \frac{1}{(s-2)!}\bar{\mathrm{D}}_{(\dot\alpha_{s-2}}J_{\alpha(s-1)\dot\alpha(s-3))}\,. \tag{2.7b}$$

The starting point of our analysis is the free action for a complex linear superfield

$$S_0 = -\int d^8z\ \bar\Sigma\ \Sigma \tag{2.8}$$

and the zeroth order transformation of Σ, $\delta_0\Sigma = 0$. Hence according to (2.4), the cubic interactions of the complex linear superfield with higher spin supermultiplets, described by the $S_1[\Sigma,\mathcal{A}]$,[3] must satisfy:

$$\frac{\delta S_0}{\delta\Sigma}\delta_1\Sigma + \frac{\delta S_1}{\delta\mathcal{A}}\delta_0\mathcal{A} = 0\,. \tag{2.9}$$

Therefore it is important to find the first order, *higher spin* transformation of Σ ($\delta_1\Sigma$). Motivated from the structure of the transformation of Σ under superdiffeomorphisms presented in [3] we write the following ansatz[4]

$$\begin{aligned}\delta_1\Sigma = g\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}\Big\{ & A_l^{\alpha(k+1)\dot\alpha(k)}\ \Box^l\ \mathrm{D}_{\alpha_{k+1}}\bar{\mathrm{D}}_{\dot\alpha_k}\mathrm{D}_{\alpha_k}\ldots\bar{\mathrm{D}}_{\dot\alpha_1}\mathrm{D}_{\alpha_1}\Sigma\\ &+ B_l^{\alpha(k)\dot\alpha(k+1)}\ \Box^l\ \bar{\mathrm{D}}_{\dot\alpha_{k+1}}\mathrm{D}_{\alpha_k}\bar{\mathrm{D}}_{\dot\alpha_k}\ldots\mathrm{D}_{\alpha_1}\bar{\mathrm{D}}_{\dot\alpha_1}\Sigma\\ &+ \Gamma_l^{\alpha(k)\dot\alpha(k)}\ \Box^l\ \bar{\mathrm{D}}_{\dot\alpha_k}\mathrm{D}_{\alpha_k}\ldots\bar{\mathrm{D}}_{\dot\alpha_1}\mathrm{D}_{\alpha_1}\Sigma\\ &+ \Delta_l^{\alpha(k)\dot\alpha(k)}\ \Box^l\ \mathrm{D}_{\alpha_k}\bar{\mathrm{D}}_{\dot\alpha_k}\ldots\mathrm{D}_{\alpha_1}\bar{\mathrm{D}}_{\dot\alpha_1}\Sigma\Big\}\end{aligned} \tag{2.10}$$

which depends on four infinite families of parameters $\{A^l_{\alpha(k+1)\dot\alpha(k)}, B^l_{\alpha(k)\dot\alpha(k+1)}, \Gamma^l_{\alpha(k)\dot\alpha(k)}, \Delta^l_{\alpha(k)\dot\alpha(k)}\}$ with independently symmetrized dotted and undotted indices. To have this transformation consistent with the linearity of Σ ($\bar{\mathrm{D}}^2\Sigma = 0$), we must have

$$A^l_{\alpha(k+1)\dot\alpha(k)} = -\bar{\mathrm{D}}^{\dot\alpha_{k+1}}\Gamma^l_{\alpha(k+1)\dot\alpha(k+1)} + \frac{1}{k+2}\bar{\mathrm{D}}^{\dot\alpha_{k+1}}\Delta^l_{\alpha(k+1)\dot\alpha(k+1)}\,, \quad k = 0,1\ldots, \tag{2.11a}$$

$$\frac{1}{(k+1)!}\bar{\mathrm{D}}_{(\dot\alpha_{k+1}}\Delta^l_{\alpha(k)\dot\alpha(k))} = -\bar{\mathrm{D}}^2 B^l_{\alpha(k)\alpha(k+1)}\,, \quad k = 1,2,\ldots, \tag{2.11b}$$

$$\bar{\mathrm{D}}_{\dot\alpha}(\Gamma^l + \Delta^l) = -\bar{\mathrm{D}}^2 B^l_{\dot\alpha}\,. \tag{2.11c}$$

---

[3] Where $\mathcal{A}$ is the set of superfields that participate in the description of higher spin supermultiplets.

[4] We use the conventions of *Superspace* [18], $\{\mathrm{D}_\alpha, \bar{\mathrm{D}}_{\dot\alpha}\} = i\partial_{\alpha\dot\alpha}$, $\mathrm{D}^\alpha\mathrm{D}_\alpha = 2\mathrm{D}^2$ and $\bar{\mathrm{D}}^{\dot\alpha}\bar{\mathrm{D}}_{\dot\alpha} = 2\bar{\mathrm{D}}^2$.

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It is worth mentioning that (2.10) is not the most general transformation linear in Σ one can write. For example we can have D^2, $\bar{\mathrm{D}}^2\mathrm{D}^2$ or $\bar{\mathrm{D}}^2\mathrm{D}_{\alpha_{k+1}}$ appropriately placed in the various terms. These extra terms will modify the above constraints, nevertheless we have verified that they do not introduce extra structure regarding the coupling to higher spin supermultiplets, so we will not consider them. Solving (2.11a) we conclude that the parameters $B^l_{\alpha(k)\dot{\alpha}(k+1)}$, $\Gamma^l_{\alpha(k)\dot{\alpha}(k)}$ are unconstrained, whereas $A^l_{\alpha(k+1)\dot{\alpha}(k)}$, $\Delta^l_{\alpha(k)\dot{\alpha}(k)}$ are given by

$$A^l_{\alpha(k+1)\dot{\alpha}(k)} = -\bar{\mathrm{D}}^{\dot{\alpha}_{k+1}}\Gamma^l_{\alpha(k+1)\dot{\alpha}(k+1)} + \frac{1}{k+1}\bar{\mathrm{D}}^2\ell^l_{\alpha(k+1)\dot{\alpha}(k)}\,, \qquad k=0,1\dots, \tag{2.12a}$$

$$\Delta^l_{\alpha(k)\dot{\alpha}(k)} = \bar{\mathrm{D}}^{\dot{\alpha}_{k+1}}B^l_{\alpha(k)\dot{\alpha}(k+1)} + \frac{1}{k!}\bar{\mathrm{D}}_{(\dot{\alpha}_k}\ell^l_{\alpha(k)\dot{\alpha}(k-1))}\,, \qquad k=1,2,\dots, \tag{2.12b}$$

$$\Gamma^l + \Delta^l = \bar{\mathrm{D}}^{\dot{\alpha}}B^l_{\dot{\alpha}} + \bar{\mathrm{D}}^2\ell^l\,, \tag{2.12c}$$

where $\ell^l_{\alpha(k+1)\dot{\alpha}(k)}$ and ℓ^l are arbitrary, unconstrained superfields. A useful observation is that the constraints (2.11a) do not mix the different l-levels and all the determined parameters (2.12a) are functions of other parameters of the same level. Therefore, the l label does not provide any further structure and we can simplify (2.10) by confining ourselves in the $l=0$ level.[5] Therefore, the transformation we consider is

$$\begin{aligned}
\delta_1\Sigma = g\sum_{k=0}^{\infty}\Big\{&\bar{\mathrm{D}}^2\ell^{\alpha(k+1)\dot{\alpha}(k)}\ \frac{i^k}{k+1}\ \mathrm{D}_{\alpha_{k+1}}\partial_{\alpha_k\dot{\alpha}_k}\dots\partial_{\alpha_1\dot{\alpha}_1}\Sigma\\
&+\frac{1}{(k+1)!}\ \bar{\mathrm{D}}^{(\dot{\alpha}_{k+1}}\ell^{\alpha(k+1)\dot{\alpha}(k))}\ i^k\ \mathrm{D}_{\alpha_{k+1}}\bar{\mathrm{D}}_{\dot{\alpha}_{k+1}}\partial_{\alpha_k\dot{\alpha}_k}\dots\partial_{\alpha_1\dot{\alpha}_1}\Sigma\\
&+B^{\alpha(k)\dot{\alpha}(k+1)}\ i^k\ \bar{\mathrm{D}}_{\dot{\alpha}_{k+1}}\partial_{\alpha_k\dot{\alpha}_k}\dots\partial_{\alpha_1\dot{\alpha}_1}\Sigma\\
&-\bar{\mathrm{D}}_{\dot{\alpha}_{k+2}}B^{\alpha(k+1)\dot{\alpha}(k+2)}\ i^k\ \mathrm{D}_{\alpha_{k+1}}\bar{\mathrm{D}}_{\dot{\alpha}_{k+1}}\partial_{\alpha_k\dot{\alpha}_k}\dots\partial_{\alpha_1\dot{\alpha}_1}\Sigma\\
&+\Gamma^{\alpha(k+1)\dot{\alpha}(k+1)}\ i^k\ \bar{\mathrm{D}}_{\dot{\alpha}_{k+1}}\mathrm{D}_{\alpha_{k+1}}\partial_{\alpha_k\dot{\alpha}_k}\dots\partial_{\alpha_1\dot{\alpha}_1}\Sigma\\
&+\bar{\mathrm{D}}_{\dot{\alpha}_{k+1}}\Gamma^{\alpha(k+1)\dot{\alpha}(k+1)}\ i^k\ \mathrm{D}_{\alpha_{k+1}}\partial_{\alpha_k\dot{\alpha}_k}\dots\partial_{\alpha_1\dot{\alpha}_1}\Sigma\Big\}\\
+g\ \left(\bar{\mathrm{D}}^2\ell - \bar{\mathrm{D}}_{\dot{\alpha}}B^{\dot{\alpha}}\right)\ \Sigma\,.&
\end{aligned} \tag{2.13}$$

Additionally, if we want to make contact with the superdiffeomorphism transformation, we choose

$$\Gamma_{\alpha(k)\alpha(k)} = \Delta_{\alpha(k)\dot{\alpha}(k)}\,, \qquad k=1,2,\dots\,. \tag{2.14}$$

so that the $\Gamma_{\alpha(k)\dot{\alpha}(k)}$ with $\Delta_{\alpha(k)\dot{\alpha}(k)}$ terms for $k\geq 1$ combine to give spacetime derivatives

[5] From this point forward we drop the l label.

resulting in

$$
\begin{aligned}
\delta_1\Sigma = -g\sum_{k=0}^{\infty}\Big\{ & \bar{\mathrm{D}}^2\ell^{\alpha(k+1)\dot\alpha(k)}\; i^k\; \mathrm{D}_{\alpha_{k+1}}\partial_{\alpha_k\dot\alpha_k}\dots\partial_{\alpha_1\dot\alpha_1}\Sigma \\
& -\frac{1}{(k+1)!}\bar{\mathrm{D}}^{(\dot\alpha_{k+1}}\ell^{\alpha(k+1)\dot\alpha(k))}\; i^{k+1}\;\partial_{\alpha_{k+1}\dot\alpha_{k+1}}\dots\partial_{\alpha_1\dot\alpha_1}\Sigma \\
& - B^{\alpha(k)\dot\alpha(k+1)}\; i^k\;\bar{\mathrm{D}}_{\dot\alpha_{k+1}}\partial_{\alpha_k\dot\alpha_k}\dots\partial_{\alpha_1\dot\alpha_1}\Sigma \\
& + \bar{\mathrm{D}}_{\dot\alpha_{k+2}}B^{\alpha(k+1)\dot\alpha(k+2)}\; i^{k+1}\;\partial_{\alpha_{k+1}\dot\alpha_{k+1}}\dots\partial_{\alpha_1\dot\alpha_1}\Sigma\Big\} \\
+ g(\bar{\mathrm{D}}^2\ell - \bar{\mathrm{D}}_{\dot\alpha}B^{\dot\alpha})\;\Sigma\,. &
\end{aligned}
\tag{2.15}
$$

3 Higher spin supercurrent multiplet

Having found the appropriate first order transformation for the complex linear superfield, we use it to perform Noether's procedure for the cubic order terms, as described in section 2 and construct the higher spin supercurrent multiplet. Starting from the free theory (2.8) we calculate its variation under (2.15):

$$
\begin{aligned}
\delta_1 S_0 = g\int d^8z\sum_{k=0}^{\infty}\Big\{ & \bar{\mathrm{D}}^2\ell^{\alpha(k+1)\dot\alpha(k)}\; i^k\;\mathrm{D}_{\alpha_{k+1}}\partial_{\alpha_k\dot\alpha_k}\dots\partial_{\alpha_1\dot\alpha_1}\Sigma\;\bar\Sigma + \text{c.c.} \\
& + \bar{B}^{\alpha(k+1)\dot\alpha(k)}\;(-i)^k\;\mathrm{D}_{\alpha_{k+1}}\Sigma\;\partial_{\alpha_k\dot\alpha_k}\dots\partial_{\alpha_1\dot\alpha_1}\bar\Sigma + \text{c.c.} \\
& - \frac{1}{(k+1)!}\bar{\mathrm{D}}^{(\dot\alpha_{k+1}}\ell^{\alpha(k+1)\dot\alpha(k))}\; i^{k+1}\;\partial_{\alpha_{k+1}\dot\alpha_{k+1}}\dots\partial_{\alpha_1\dot\alpha_1}\Sigma\;\bar\Sigma + \text{c.c.}\Big\} \\
- g\int d^8z\;(\bar{\mathrm{D}}^2\ell + \mathrm{D}^2\bar\ell)\;\Sigma\;\bar\Sigma\,. &
\end{aligned}
\tag{3.1}
$$

At this point it is tempting to choose the $B_{\alpha(k)\dot\alpha(k+1)}$ parameter to be a function of the $\ell_{\alpha(k+1)\dot\alpha(k)}$ parameter. In principle we can write all possible terms allowed by engineering dimensions and index structure:

$$
\bar{B}_{\alpha(k+1)\dot\alpha(k)} = d_1\bar{\mathrm{D}}^2\ell_{\alpha(k+1)\dot\alpha(k)} + \frac{d_2}{(k+1)!}\mathrm{D}_{(\alpha_{k+1}}\bar{\mathrm{D}}^{\dot\alpha_{k+1}}\bar\ell_{\alpha(k))\dot\alpha(k+1)} + \frac{d_3}{(k+1)!}\bar{\mathrm{D}}^{\dot\alpha_{k+1}}\mathrm{D}_{(\alpha_{k+1}}\bar\ell_{\alpha(k))\dot\alpha(k+1)}\,. \tag{3.2}
$$

A similar step was done in [3] and led to the coupling of the complex linear theory to the various formulations of supergravity depending on the values of d_1, d_2, d_3. In this analysis supergravity would correspond to $k = 0$. Notice that in (3.1) the coefficients of $\bar{B}_{\alpha(k+1)\dot\alpha(k)}$ and $\bar{\mathrm{D}}^2\ell_{\alpha(k+1)\dot\alpha(k)}$ do not match for $k \geq 1$, hence we can not repeat the same arguments in [3] for higher spin coupling. There is only one viable option

$$
\bar{B}_{\alpha(k+1)\dot\alpha(k)} = -\bar{\mathrm{D}}^2\ell_{\alpha(k+1)\dot\alpha(k)}\,. \tag{3.3}
$$

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By doing this, the variation of the action can be written in the following way

$$
\begin{aligned}
\delta_1 S_0 = g \int d^8z \sum_{k=0}^{\infty} \Big\{ & \bar{\mathrm{D}}^2 \ell^{\alpha(k+1)\dot\alpha(k)}\ i^k \left[\partial^{(k)}\mathrm{D}\Sigma\ \bar\Sigma + (-1)^{k-1}\ \mathrm{D}\Sigma\ \partial^{(k)}\bar\Sigma\right] + \text{c.c.} \\
& - \frac{1}{(k+1)!}\bar{\mathrm{D}}^{(\dot\alpha_{k+1}}\ell^{\alpha(k+1)\dot\alpha(k))}\ i^{k+1}\ \partial^{(k+1)}\Sigma\ \bar\Sigma + \text{c.c.}\Big\} \\
- g \int d^8z\ (\bar{\mathrm{D}}^2\ell + \mathrm{D}^2\bar\ell)\ \Sigma\ \bar\Sigma &
\end{aligned}
\tag{3.4}
$$

where for simplicity we suppress the uncontracted indices, their symmetrization and the symmetrization factors when they are appropriate. The symbol $\partial^{(k)}$ denotes a string of k spacetime derivatives. Moreover, we can use the following identity

$$
\partial^{(k)}A\ B + (-1)^{k-1}\ A\ \partial^{(k)}B = \partial\left\{\sum_{n=0}^{k-1}(-1)^n\ \partial^{(k-1-n)}A\ \partial^{(n)}B\right\}
\tag{3.5}
$$

which holds for arbitrary (super)functions A and B and simplify (3.4) to:

$$
\begin{aligned}
\delta_1 S_0 = -g \int d^8z \sum_{k=0}^{\infty} \Big\{ & \bar{\mathrm{D}}^2 \ell^{\alpha(k+1)\dot\alpha(k)}\ \mathcal{T}_{\alpha(k+1)\dot\alpha(k)} + \text{c.c.} \\
& - \frac{1}{(k+1)!}\bar{\mathrm{D}}^{(\dot\alpha_{k+1}}\ell^{\alpha(k+1)\dot\alpha(k))}\ \mathcal{J}_{\alpha(k+1)\dot\alpha(k+1)} + \text{c.c.}\Big\} \\
+ g \int d^8z\ (\bar{\mathrm{D}}^2\ell + \mathrm{D}^2\bar\ell)\ \mathcal{J}\,. &
\end{aligned}
\tag{3.6}
$$

where

$$
\mathcal{J}_{\alpha(k+1)\dot\alpha(k+1)} = -i^{(k+1)}\ \partial^{(k+1)}\Sigma\ \bar\Sigma + \frac{1}{(k+1)!}\mathrm{D}_{(\alpha_{k+1}}\bar{\mathrm{D}}^2\bar U_{\alpha(k))\dot\alpha(k+1)} + \frac{k+1}{(k+2)!}\bar{\mathrm{D}}_{(\dot\alpha_{k+1}}W_{\alpha(k+1)\dot\alpha(k))}\,,
\tag{3.7a}
$$

$$
\begin{aligned}
\mathcal{T}_{\alpha(k+1)\dot\alpha(k)} = -i^{(k-1)}\ \mathrm{D}\bar{\mathrm{D}}\left\{\sum_{n=0}^{k-1}(-1)^n\ \partial^{(k-1-n)}\mathrm{D}\Sigma\ \partial^{(n)}\bar\Sigma\right\} + \frac{1}{(k+1)!}\mathrm{D}_{(\alpha_{k+1}}\bar{\mathrm{D}}^{\dot\alpha_{k+1}}\bar U_{\alpha(k))\dot\alpha(k+1)} & \\
+ W_{\alpha(k+1)\dot\alpha(k)}\,, &
\end{aligned}
\tag{3.7b}
$$

$$
\mathcal{J} = -\Sigma\bar\Sigma\,.
\tag{3.7c}
$$

The superfields $W_{\alpha(k+1)\dot\alpha(k)}$ and $\bar U_{\alpha(k)\dot\alpha(k+1)}$ are improvement terms that we can add, as discussed in [1]. Also, it is important to keep in mind that the $\mathcal{J}_{\alpha(k+1)\dot\alpha(k+1)}$, $\mathcal{T}_{\alpha(k+1)\dot\alpha(k)}$ and $\mathcal{J}$ are not defined uniquely but they satisfy an equivalence relation. For example

$$
\begin{aligned}
\mathcal{T}_{\alpha(k+1)\dot\alpha(k)} \sim \mathcal{T}_{\alpha(k+1)\dot\alpha(k)} &+ \bar{\mathrm{D}}_{(\dot\alpha_k}P^{(1)}_{\alpha(k+1)\dot\alpha(k-1))} + \bar{\mathrm{D}}^{\dot\alpha_{k+1}}P^{(2)}_{\alpha(k+1)\dot\alpha(k+1)} \\
&+ \mathrm{D}_{(\alpha_{k+1}}\bar{\mathrm{D}}^2 P^{(3)}_{\alpha(k))\dot\alpha(k)} + \mathrm{D}^{\alpha_{k+2}}\bar{\mathrm{D}}^2 P^{(4)}_{\alpha(k+2)\dot\alpha(k)}\,.
\end{aligned}
\tag{3.8}
$$

for arbitrary superfields $P^{(1)}$, $P^{(2)}$, $P^{(3)}$, $P^{(4)}$.

Using (3.5) we can express the $-i^{k+1}\partial^{k+1}\Sigma\,\bar{\Sigma}$ in the following manner

$$-i^{(k+1)}\ \partial^{k+1}\Sigma\,\bar{\Sigma} = X_{\alpha(k+1)\dot{\alpha}(k+1)} + \frac{1}{(k+1)!^2}\bar{\mathrm{D}}_{(\dot{\alpha}_{k+1}}\mathrm{D}_{(\alpha_{k+1}}Z_{\alpha(k))\dot{\alpha}(k))} \tag{3.9}$$

where $X_{\alpha(k+1)\dot{\alpha}(k+1)}$ and $Z_{\alpha(k)\dot{\alpha}(k)}$ are real superfields given by the following expressions:

$$\begin{aligned} X_{\alpha(k+1)\dot{\alpha}(k+1)} = & -\frac{i^{k+1}}{2}\left[\partial^{(k+1)}\Sigma\,\bar{\Sigma} + (-1)^{k+1}\ \Sigma\ \partial^{(k+1)}\bar{\Sigma}\right] \\ & -\frac{i^k}{2}\sum_{n=0}^{k}(-1)^n\left\{\partial^{(k-n)}[\mathrm{D},\bar{\mathrm{D}}]\Sigma\ \partial^{(n)}\bar{\Sigma} + \partial^{(k-n)}\Sigma\ \partial^{(n)}[\mathrm{D},\bar{\mathrm{D}}]\bar{\Sigma}\right\} \\ & -i^k\sum_{n=0}^{k}(-1)^n\left\{\partial^{(k-n)}\mathrm{D}\Sigma\ \partial^{(n)}\bar{\mathrm{D}}\bar{\Sigma} - \partial^{(k-n)}\bar{\mathrm{D}}\Sigma\ \partial^{(n)}\mathrm{D}\bar{\Sigma}\right\}, \end{aligned} \tag{3.10a}$$

$$Z_{\alpha(k)\dot{\alpha}(k)} = -i^k\sum_{n=0}^{k}(-1)^n\ \partial^{(k-n)}\Sigma\ \partial^{(n)}\bar{\Sigma}\,. \tag{3.10b}$$

Because of (3.9), it is straight forward to prove that there is always a choice for the improvement term $W_{\alpha(k+1)\dot{\alpha}(k)}$ that makes $\mathcal{J}_{\alpha(k+1)\dot{\alpha}(k)}$ real. Specifically, by selecting

$$W_{\alpha(k+1)\dot{\alpha}(k)} = -\frac{k+2}{k+1}\mathrm{D}^2U_{\alpha(k+1)\dot{\alpha}(k)} - \frac{k+2}{k+1}\frac{1}{(k+1)!}\mathrm{D}_{(\alpha_{k+1}}Z_{\alpha(k))\dot{\alpha}(k)} \tag{3.11}$$

we get

$$\mathcal{J}_{\alpha(k+1)\dot{\alpha}(k+1)} = X_{\alpha(k+1)\dot{\alpha}(k)} + \frac{1}{(k+1)!}\mathrm{D}_{(\alpha_{k+1}}\bar{\mathrm{D}}^2\bar{U}_{\alpha(k))\dot{\alpha}(k+1)} - \frac{1}{(k+1)!}\bar{\mathrm{D}}_{(\dot{\alpha}_{k+1}}\mathrm{D}^2U_{\alpha(k+1)\dot{\alpha}(k))}\,, \tag{3.12a}$$

$$\mathcal{T}_{\alpha(k+1)\dot{\alpha}(k)} = \frac{1}{(k+1)!}\mathrm{D}_{(\alpha_{k+1}}\mathcal{T}_{\alpha(k))\dot{\alpha}(k)}\,, \tag{3.12b}$$

$$\begin{aligned} \mathcal{T}_{\alpha(k)\dot{\alpha}(k)} = & -i^{(k-1)}\ \bar{\mathrm{D}}\left\{\sum_{n=0}^{k-1}(-1)^n\ \partial^{(k-1-n)}\mathrm{D}\Sigma\ \partial^{(n)}\bar{\Sigma}\right\} - \frac{k+2}{k+1}Z_{\alpha(k)\dot{\alpha}(k)} \\ & + \frac{k+2}{k+1}\ \mathrm{D}^{\alpha_{k+1}}U_{\alpha(k+1)\dot{\alpha}(k)} + \bar{\mathrm{D}}^{\dot{\alpha}_{k+1}}\bar{U}_{\alpha(k)\dot{\alpha}(k+1)}\,. \end{aligned} \tag{3.12c}$$

The reality of $\mathcal{J}_{\alpha(k+1)\dot{\alpha}(k+1)}$ and the fact that $\mathcal{T}_{\alpha(k+1)\dot{\alpha}(k)}$ becomes a total spinorial derivative (3.12b), allows us to modify (3.6) in the following way:

$$\begin{aligned} \delta_1 S_0 = & -g\int d^8z\sum_{k=0}^{\infty}\left\{\left[\bar{\mathrm{D}}^2\ell^{\alpha(k+1)\dot{\alpha}(k)} - \mathrm{D}_{\alpha_{k+2}}\lambda^{\alpha(k+2)\dot{\alpha}(k)}\right]\ \frac{1}{(k+1)!}\mathrm{D}_{(\alpha_{k+1}}\mathcal{T}_{\alpha(k))\dot{\alpha}(k)} + \text{c.c.}\right. \\ & \left. + \left[\frac{1}{(k+1)!}\mathrm{D}^{(\alpha_{k+1}}\bar{\ell}^{\alpha(k))\dot{\alpha}(k+1)} - \frac{1}{(k+1)!}\bar{\mathrm{D}}^{(\dot{\alpha}_{k+1}}\ell^{\alpha(k+1)\dot{\alpha}(k))}\right]\ \mathcal{J}_{\alpha(k+1)\dot{\alpha}(k+1)}\right\} \\ & + g\int d^8z\ (\bar{\mathrm{D}}^2\ell + \mathrm{D}^2\bar{\ell})\ \mathcal{J}\,. \end{aligned} \tag{3.13}$$

The terms inside the square brackets are exactly the zeroth order transformations that appear in (2.6a). Hence, in order to get an invariant theory, we must add the following cubic interaction terms between $(s+1, s+1/2)$ higher spin supermultiplets (2.6a) and the complex linear superfield

$$
\begin{aligned}
S_{\text{HS-}\Sigma\text{ cubic interactions}} = g\int d^8z \sum_{k=0}^{\infty}\Big\{ & H^{\alpha(k+1)\dot\alpha(k+1)}\mathcal{J}_{\alpha(k+1)\dot\alpha(k+1)} \\
& +\chi^{\alpha(k+1)\dot\alpha(k)}\mathrm{D}_{\alpha_{k+1}}\mathcal{T}_{\alpha(k)\dot\alpha(k)}+\bar\chi^{\alpha(k)\dot\alpha(k+1)}\bar{\mathrm{D}}_{\dot\alpha_{k+1}}\bar{\mathcal{T}}_{\alpha(k)\dot\alpha(k)}\Big\} \\
& - g\int d^8z\, V\mathcal{J}
\end{aligned} \tag{3.14}
$$

where V $(\delta_0 V = \bar{\mathrm{D}}^2\ell + \mathrm{D}^2\bar\ell)$ is the real prepotential used for the description of the vector supermultiplet $(1, 1/2)$.

Superfields $\mathcal{J}_{\alpha(k+1)\dot\alpha(k+1)}$ (3.12a) and $\mathcal{T}_{\alpha(k)\dot\alpha(k)}$ (3.12c) are the Σ-generated higher spin supercurrent and higher spin supertrace respectively and together they form the higher spin supercurrent multiplet $\{\mathcal{J}_{\alpha(k+1)\dot\alpha(k+1)}, \mathcal{T}_{\alpha(k)\dot\alpha(k)}\}$. The results we get are similar to the case of the chiral superfield described in [1] and [19]. Specifically, we find that a single, free, massless, complex linear supermultiplet couples only to half integer superspin supermultiplets with a preference to the transverse formulation (2.6a) which is the only formulation of half-integer superspins that can be elevated to $\mathcal{N}=2$ theories. Furthermore, like in the chiral case the higher spin supercurrent and supertrace include higher derivative terms, as expected from [20].

4 Minimal higher spin supercurrent multiplet

In the previous section we constructed what is known as the *canonical* supercurrent multiplet [21]. In this section we will prove that for every value of the non-negative integer k there is a unique, alternative higher spin supercurrent multiplet, called the *minimal* supercurrent multiplet defined by $\mathcal{T}^{\text{min}}_{\alpha(k)\dot\alpha(k)}=0$. This will be possible due to (i) the freedom in choosing the unconstrained improvement term $U_{\alpha(k+1)\dot\alpha(k)}$, (ii) the freedom in the definition of $\mathcal{T}_{\alpha(k+1)\dot\alpha(k)}$ (3.8) and (iii) the freedom to absorb trivial terms by redefining Σ.

For an arbitrary superfield $\mathcal{Y}_{\dot\alpha}$, the combination $\bar{\mathrm{D}}^{\dot\alpha}\mathcal{Y}_{\dot\alpha}$ is a complex linear superfield. Therefore, one can consider the redefinition of Σ

$$
\Sigma = \hat\Sigma + g\,\bar{\mathrm{D}}^{\dot\alpha}\mathcal{Y}_{\dot\alpha} \tag{4.1}
$$

which will modify the free theory in the following manner

$$
S_0 = -\int d^8z\,\hat\Sigma\,\bar{\hat\Sigma} + g\int d^8z\,\bar{\mathcal{Y}}^{\alpha}\,\mathrm{D}_\alpha\hat\Sigma + g\int d^8z\,\mathcal{Y}^{\dot\alpha}\,\bar{\mathrm{D}}_{\dot\alpha}\bar{\hat\Sigma} + g^2\int d^8z\,\mathcal{Y}^{\dot\alpha}\,\bar{\mathrm{D}}_{\dot\alpha}\mathrm{D}^{\alpha}\bar{\mathcal{Y}}_{\alpha} \tag{4.2}
$$

where g is the perturbative parameter. Because we are working up to order g^1 (cubic interactions) we can ignore the last term in (4.2) and therefore conclude that terms in the action that depend on $\mathrm{D}\Sigma$ or $\bar{\mathrm{D}}\bar\Sigma$ (these are the on-shell equation of motion for the free theory) are *trivial* terms and can be absorbed by an appropriate redefinition of Σ like (4.1). This realization simplifies the calculation of the supercurrent multiplet. For example, if we consider the transformation of Σ (2.13) before fixing $\Gamma_{\alpha(k)\dot\alpha(k)}$ and $B_{\alpha(k)\dot\alpha(k+1)}$ then we get:

$$
\begin{aligned}
\delta_1 S_0 = -g\int d^8z \sum_{k=0}^{\infty}\Big\{ &\bar{\mathrm{D}}^2\ell^{\alpha(k+1)\dot\alpha(k)}\ \frac{i^k}{k+1}\ \partial^{(k)}\mathrm{D}\Sigma\ \bar\Sigma \\
&+\frac{1}{(k+1)!}\bar{\mathrm{D}}^{(\dot\alpha_{k+1}}\ell^{\alpha(k+1)\dot\alpha(k))}\ i^{(k+1)}\ \partial^{(k+1)}\Sigma\ \bar\Sigma \\
&-\frac{1}{(k+1)!}\bar{\mathrm{D}}^{(\dot\alpha_{k+1}}\ell^{\alpha(k+1)\dot\alpha(k))}\ i^{k}\ \partial^{(k)}\bar{\mathrm{D}}\mathrm{D}\Sigma\ \bar\Sigma \\
&-B^{\alpha(k)\dot\alpha(k+1)}\ i^k\ \partial^{(k)}\Sigma\ \bar{\mathrm{D}}\bar\Sigma \\
&+B^{\alpha(k+1)\dot\alpha(k+2)}\ i^k\ \partial^{(k)}\bar{\mathrm{D}}\mathrm{D}\Sigma\ \bar{\mathrm{D}}\bar\Sigma \\
&+\Gamma^{\alpha(k+1)\dot\alpha(k+1)}\ i^k\ \partial^{(k)}\mathrm{D}\Sigma\ \bar{\mathrm{D}}\bar\Sigma\Big\} + \text{c.c.} \\
-\, g\int d^8z\ (\bar{\mathrm{D}}^2\ell + \mathrm{D}^2\bar\ell)\ \Sigma\ \bar\Sigma\,. &
\end{aligned}
\tag{4.3}
$$

Observe that the contribution of the $B_{\alpha(k)\dot\alpha(k+1)}$ and $\Gamma_{\alpha(k)\dot\alpha(k)}$ terms is trivial in the sense explained previously, hence we can remove them by an appropriate redefinition of Σ. This is very satisfying because, unlike the chiral case, the first order transformation of Σ (2.13) is far from unique. It includes two infinite families of unconstrained superfields. Despite that, as we see at the level of the action, these unconstrained parameters do not give non-trivial contributions. Therefore, the variation of the action is completely fixed without any freedom left:

$$
\begin{aligned}
\delta_1 S_0 = -g\int d^8z \sum_{k=0}^{\infty}\left[-\frac{1}{(k+1)!}\bar{\mathrm{D}}^{(\dot\alpha_{k+1}}\ell^{\alpha(k+1)\dot\alpha(k))}\right]\ \mathcal{J}^{\min}_{\alpha(k+1)\dot\alpha(k+1)} + \text{c.c.} \\
+\, g\int d^8z\ (\bar{\mathrm{D}}^2\ell + \mathrm{D}^2\bar\ell)\ \mathcal{J}
\end{aligned}
\tag{4.4}
$$

where

$$
\begin{aligned}
\mathcal{J}^{\min}_{\alpha(k+1)\dot\alpha(k+1)} = &-i^{(k+1)}\ \partial^{(k+1)}\Sigma\ \bar\Sigma \\
&+\frac{1}{(k+1)!}\mathrm{D}_{(\alpha_{k+1}}\bar{\mathrm{D}}^2\bar U_{\alpha(k))\dot\alpha(k+1)} - \frac{k+1}{(k+2)!(k+1)!)}\bar{\mathrm{D}}_{(\dot\alpha_{k+1}}\mathrm{D}_{(\alpha_{k+1}}\bar{\mathrm{D}}^{\dot\beta}\bar U_{\alpha(k))\dot\beta\dot\alpha(k))}\,,
\end{aligned}
\tag{4.5a}
$$

$$
\mathcal{J} = -\Sigma\ \bar\Sigma\,. \tag{4.5b}
$$

The superfield $U_{\alpha(k+1)\dot\alpha(k)}$ is an improvement term similar to the improvement terms appearing in (3.7a). We can deduce its presence by constraining the $W_{\alpha(k+1)\dot\alpha(k)}$ superfield in (3.7b) to cancel the $U_{\alpha(k+1)\dot\alpha(k)}$ contribution, giving $W_{\alpha(k+1)\dot\alpha(k)} = -\frac{1}{(k+1)!}\mathrm{D}_{(\alpha_{k+1}}\bar{\mathrm{D}}^{\dot\alpha_{k+1}}\cdot\bar U_{\alpha(k))\dot\alpha(k+1)}$. Notice, there is no $\mathcal{T}_{\alpha(k+1)\dot\alpha(k)}$ contribution like in (3.6) which means we are already in the *minimal* setup of the supercurrent multiplet. Also, we must keep in mind that the above definition of $\mathcal{J}^{\min}_{\alpha(k+1)\dot\alpha(k+1)}$ satisfies an equivalence relation

$$\mathcal{J}^{\min}_{\alpha(k+1)\dot\alpha(k+1)} \sim \mathcal{J}^{\min}_{\alpha(k+1)\dot\alpha(k+1)} + \bar{\mathrm{D}}^{\dot\alpha_{k+2}}\Theta^{(1)}_{\alpha(k+1)\dot\alpha(k+2)} + \bar{\mathrm{D}}^2\Theta^{(2)}_{\alpha(k+1)\dot\alpha(k+1)}\,. \tag{4.6}$$

for arbitrary $\Theta^{(1)}$ and $\Theta^{(2)}$ superfields. Furthermore, using (3.9) we can rewrite (4.5a)

$$\begin{aligned}\mathcal{J}^{\min}_{\alpha(k+1)\dot\alpha(k+1)} = X_{\alpha(k+1)\dot\alpha(k+1)} &+ \frac{1}{(k+1)!}\mathrm{D}_{(\alpha_{k+1}}\bar{\mathrm{D}}^2\bar{U}_{\alpha(k))\dot\alpha(k+1)} - \frac{1}{(k+1)!}\bar{\mathrm{D}}_{(\dot\alpha_{k+1}}\mathrm{D}^2U_{\alpha(k)\dot\alpha(k+1))} \\ &+ \frac{1}{(k+1)!^2}\bar{\mathrm{D}}_{(\dot\alpha_{k+1}}\mathrm{D}_{(\alpha_{k+1}}\left[Z_{\alpha(k)\dot\alpha(k)} - \frac{k+1}{k+2}\bar{\mathrm{D}}^{\dot\beta}\bar{U}_{\alpha(k))\dot\beta\dot\alpha(k))} - \mathrm{D}^{\beta}U_{\beta\alpha(k))\dot\alpha(k))}\right].\end{aligned} \tag{4.7}$$

Thus, we can make $\mathcal{J}^{\min}_{\alpha(k+1)\dot\alpha(k+1)}$ real if and only if we can find a superfield $U_{\alpha(k+1)\dot\alpha(k)}$ that satisfies the following constraint:

$$\begin{aligned}Z_{\alpha(k)\dot\alpha(k)} &= \mathrm{D}^{\alpha_{k+1}}U_{\alpha(k+1)\dot\alpha(k)} + \frac{k+1}{k+2}\bar{\mathrm{D}}^{\dot\alpha_{k+1}}\bar{U}_{\alpha(k)\dot\alpha(k+1)} \\ &\quad + \frac{1}{k!}\mathrm{D}_{(\alpha_k}\zeta^{(1)}_{\alpha(k-1))\dot\alpha(k)} + \mathrm{D}^2\zeta^{(2)}_{\alpha(k)\dot\alpha(k)} + \bar{\mathrm{D}}^2\zeta^{(3)}_{\alpha(k)\dot\alpha(k)} + \zeta^{(4)}_{\alpha(k)\dot\alpha(k)}(\mathrm{D}\Sigma,\bar{\mathrm{D}}\bar\Sigma)\end{aligned} \tag{4.8}$$

for some superfields $\zeta^{(1)}_{\alpha(k-1)\dot\alpha(k)}$, $\zeta^{(2)}_{\alpha(k)\dot\alpha(k)}$, $\zeta^{(3)}_{\alpha(k)\dot\alpha(k)}$ and a term $\zeta^{(4)}_{\alpha(k)\dot\alpha(k)}$ that depends only on $\mathrm{D}\Sigma$ and $\bar{\mathrm{D}}\bar\Sigma$. This condition corresponds to setting the supertrace (3.12c) to zero, which is exactly the demand for a minimal supercurrent multiplet.

Consider the following ansatz for a solution of (4.8)

$$U_{\alpha(k+1)\dot\alpha(k)} = \sum_{p=0}^{k} f_p\;\partial^{(p)}\Sigma\;\partial^{k-p}\lambda \tag{4.9}$$

with $\bar\lambda_{\dot\alpha}$ being the unconstrained prepotential of the complex linear superfield Σ ($\Sigma = \bar{\mathrm{D}}^{\dot\alpha}\bar\lambda_{\dot\alpha}$, $\bar\Sigma = \mathrm{D}^{\alpha}\lambda_{\alpha}$). It is straight forward to show that:[6]

$$\begin{aligned}\mathrm{D}^{\alpha_{k+1}}U_{\alpha(k+1)\dot\alpha(k)} &= \sum_{p=0}^{k} f_p\;\frac{k-p+1}{k+1}\;\partial^p\Sigma\;\partial^{(k-p)}\bar\Sigma \\ &\quad - i\sum_{p=0}^{k-1} f_p\;\frac{p+1}{k+1}\;\partial^p\bar{\mathrm{D}}\Sigma\;\partial^{(k-p)}\mathrm{D}\bar\Sigma\;+\mathrm{D}^2\left[\dots\right] + \mathcal{O}(\mathrm{D}\Sigma,\bar{\mathrm{D}}\bar\Sigma)\end{aligned} \tag{4.10}$$

and

$$\begin{aligned}\mathrm{D}^{\alpha_{k+1}}U_{\alpha(k+1)\dot\alpha(k)} + \frac{k+1}{k+2}\;\bar{\mathrm{D}}^{\dot\alpha_{k+1}}\bar{U}_{\alpha(k)\dot\alpha(k+1)} &= \left\{f_0 + f^*_k\frac{1}{k+2}\right\}\;\Sigma\;\partial^{(k)}\bar\Sigma \\ &\quad + \sum_{p=1}^{k}\left\{f_p + f^*_{k-p}\frac{p+1}{k+2} - f^*_{k-p+1}\frac{k-p+1}{k+2}\right\}\partial^{(p)}\Sigma\;\partial^{(k-p)}\bar\Sigma \\ &\quad + \frac{1}{k!}\mathrm{D}_{(\alpha_k}\left[\dots\right] + \mathrm{D}^2\left[\dots\right] + \bar{\mathrm{D}}^2\left[\dots\right] + \mathcal{O}(\mathrm{D}\Sigma,\bar{\mathrm{D}}\bar\Sigma)\,.\end{aligned} \tag{4.11}$$

[6]We remind the reader of our convention to suppress uncontracted indices, their symmetrization and appropriate factors.

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From the above and equation (3.10b), we get that condition (4.8) takes the form

$$(k+2)f_0 + f_k^* = (-1)^{k+1}\ i^k\ (k+2)\,, \tag{4.12a}$$

$$(k+2)f_p + (p+1)f_{k-p}^* - (k-p+1)f_{k-p+1}^* = (-1)^{k-p+1}\ i^k\ (k+2)\,, \qquad p=1,\dots,k\,. \tag{4.12b}$$

This is a system of $k+1$ linear equations for the $k+1$ complex variables f_p, $p=0,1,\dots,k$. If we introduce a new set of variables $\hat{f}_p$ defined as

$$f_p = (-1)^{k+p+1}\ i^k + (-1)^k\ \hat{f}_{k-p} \tag{4.13}$$

then the (4.12a) system of equations takes the form

$$(k+2)\hat{f}_k + \hat{f}_0^* = i^k\,, \tag{4.14a}$$

$$(k+2)\hat{f}_p + (k-p+1)\hat{f}_{k-p}^* - (p+1)\hat{f}_{k-p-1}^* = (-1)^{k+p}\ i^k\ (k+2)\,, \qquad p=0,\dots,k-1\,. \tag{4.14b}$$

This is the same system of equations which appeared in [1] in the process of removing the supertrace. There it was proved that for every k, this system of equations has a solution, hence we conclude that the condition (4.8) can always be satisfied. Using the solution coming from [1], we can write

$$f_p = (-1)^{k+p+1}\ i^k \left[1 - \frac{\sum\limits_{j=0}^{p}\binom{k+j+1}{p}\binom{k+1-j}{p-j}}{\binom{2k+3}{k+2}}\right]\,, \qquad p=0,1,\dots,k\,. \tag{4.15}$$

Therefore, due to (4.4) with a real $\mathcal{J}^{\rm min}_{\alpha(k+1)\dot\alpha(k+1)}$ we must add the following interaction terms

$$S_{\text{HS-}\Sigma\text{ minimal cubic interactions}} = g\int d^8z\,\sum_{k=0}^{\infty} H^{\alpha(k+1)\dot\alpha(k+1)}\mathcal{J}^{\rm min}_{\alpha(k+1)\dot\alpha(k+1)} - g\int d^8z\ V\mathcal{J}\,. \tag{4.16}$$

The conclusion is that, similar to the chiral case, for any value of k, we can go from the *canonical* $\{\mathcal{J}_{\alpha(k+1)\dot\alpha(k+1)}, \mathcal{T}_{\alpha(k)\dot\alpha(k)}\}$ (3.12a), (3.12c) to the *minimal* higher spin supercurrent multiplet $\{\mathcal{J}^{\rm min}_{\alpha(k+1)\dot\alpha(k+1)}, 0\}$. To get the explicit expression for $\mathcal{J}^{\rm min}_{\alpha(k+1)\dot\alpha(k+1)}$ we combine (3.10a), (3.12a), (4.9)

$$\begin{aligned}\mathcal{J}^{\rm min}_{\alpha(k+1)\dot\alpha(k+1)} = & -i\left\{f_0^* + i^k\right\}\ \partial^{(k+1)}\Sigma\ \bar\Sigma + i\left\{f_0 + (-i)^k\right\}\ \Sigma\ \partial^{(k+1)}\bar\Sigma \\ & + i\sum_{p=1}^{k}\left\{f_p - f_{k+1-p}^* + (-1)^{k+p}\ i^k\right\}\ \partial^{(p)}\Sigma\ \partial^{(k+1-p)}\bar\Sigma \\ & + \sum_{p=0}^{k}\left\{f_p + f_{k-p}^* + (-1)^{k+p}\ i^k\right\}\ \partial^{(p)}\bar{\rm D}\Sigma\ \partial^{(k-p)}{\rm D}\bar\Sigma\,.\end{aligned} \tag{4.17}$$

The first three ($k=0$, $k=1$, $k=2$) minimal supercurrents are:

$$\mathcal{J}^{\rm min}_{\alpha\dot\alpha} = -\frac{i}{3}\,\partial_{\alpha\dot\alpha}\Sigma\;\bar\Sigma + \frac{i}{3}\,\Sigma\;\partial_{\alpha\dot\alpha}\bar\Sigma - \frac{1}{3}\,\bar{\rm D}\Sigma\;{\rm D}\bar\Sigma\,, \tag{4.18}$$

$$\mathcal{J}^{\rm min}_{\alpha\beta\dot\alpha\dot\beta} = \frac{1}{10}\,\partial^{(2)}\Sigma\;\bar\Sigma + \frac{1}{10}\,\Sigma\;\partial^{(2)}\bar\Sigma - \frac{2}{5}\,\partial\Sigma\;\partial\bar\Sigma + \frac{i}{5}\,\bar{\rm D}\Sigma\;\partial{\rm D}\bar\Sigma - \frac{i}{5}\,\partial\bar{\rm D}\Sigma\;{\rm D}\bar\Sigma\,, \tag{4.19}$$

$$\begin{aligned}\mathcal{J}^{\rm min}_{\alpha\beta\gamma\dot\alpha\dot\beta\dot\gamma} = {}&\frac{i}{35}\,\partial^{(3)}\Sigma\;\bar\Sigma - \frac{i}{35}\,\Sigma\;\partial^{(3)}\bar\Sigma + \frac{9i}{35}\,\partial\Sigma\;\partial^{(2)}\bar\Sigma - \frac{9i}{35}\,\partial^{(2)}\Sigma\;\partial\bar\Sigma \\ &+ \frac{3}{35}\,\bar{\rm D}\Sigma\;\partial^{(2)}{\rm D}\bar\Sigma - \frac{9}{35}\,\partial\bar{\rm D}\Sigma\;\partial{\rm D}\bar\Sigma + \frac{3}{35}\,\partial^{(2)}\bar{\rm D}\Sigma\;{\rm D}\bar\Sigma\,.\end{aligned} \tag{4.20}$$

The above expressions (4.17), (4.18), (4.19), (4.20) for the minimal higher spin supercurrent of a complex linear superfield have striking similarities with the minimal higher spin supercurrent for a chiral superfield. The detailed connection will be established in section 6.

5 Conservation equation

The off-shell invariance of the action $S = S_0 + S_{\text{HS-}\Sigma\text{ cubic interactions}}$ as constructed in section 3, up to order g can be expressed in terms of a set of Bianchi identities. Using them together with the on-shell equations of motion of Σ we can show that the canonical higher spin supercurrent multiplet satisfies the following conservation equation:

$$\bar{\rm D}^{\dot\alpha_{k+1}}\mathcal{J}_{\alpha(k+1)\dot\alpha(k+1)} = \frac{1}{(k+1)!}\bar{\rm D}^2{\rm D}_{(\alpha_{k+1}}\mathcal{T}_{\alpha(k))\dot\alpha(k)}\,. \tag{5.1}$$

One can show that expressions (3.12a) and (3.12c) automatically satisfy (5.1), given the free theory equation of motion ${\rm D}_\alpha\Sigma = 0$. Similarly, the minimal higher spin supercurrent multiplet (4.17) satisfies the conservation equation

$$\bar{\rm D}^{\dot\alpha_{k+1}}\mathcal{J}^{\rm min}_{\alpha(k+1)\dot\alpha(k+1)} = 0\,. \tag{5.2}$$

We can use this property to find a simpler expression for $\mathcal{J}^{\rm min}_{\alpha(k+1)\dot\alpha(k+1)}$. Using as an ansatz the structure that appears in (4.17) we can write

$$\mathcal{J}^{\rm min}_{\alpha(s)\dot\alpha(s)} = \sum_{p=0}^{s} a_p\;\partial^{(p)}\Sigma\;\partial^{(s-p)}\bar\Sigma + \sum_{p=0}^{s-1} b_p\;\partial^{(p)}\bar{\rm D}\Sigma\;\partial^{(s-p-1)}{\rm D}\bar\Sigma \tag{5.3}$$

and now we impose on this quantity two necessary conditions, reality and on-shell conservation equation (5.2). The reality of $\mathcal{J}^{\rm min}_{\alpha(s)\dot\alpha(s)}$ gives the constraints

$$a_p = a^*_{s-p}\,, \qquad p = 0,\dots,s\,, \tag{5.4a}$$

$$b_p = b^*_{s-p-1}\,, \qquad p = 0,\dots,s-1\,. \tag{5.4b}$$

The conservation of $\mathcal{J}^{\rm min}_{\alpha(s)\dot\alpha(s)}$ (using ${\rm D}_\alpha\Sigma = 0$) gives:

$$b_p\,\frac{p+1}{s} - i\;a_p\,\frac{s-p}{s} = 0\,, \qquad p = 0,\dots,s-1\,. \tag{5.5}$$

These two constraints (5.4a), (5.5) are enough to fix coefficients a_p, b_p up to a real proportionality constant c

$$a_p = c\, i^s\, (-1)^p \binom{s}{p}^2 , \qquad\qquad p = 0, \dots, s\,, \tag{5.6a}$$

$$b_p = c\, i^{s+1}\, (-1)^p \binom{s}{p}^2 \frac{s-p}{p+1} , \qquad p = 0, \dots, s-1\,. \tag{5.6b}$$

The overall constant of proportionality c, can be fixed by comparing with (4.17). The result is

$$\mathcal{J}^{\rm min}_{\alpha(s)\dot\alpha(s)} = -\frac{(-i)^s}{\binom{2s+1}{s+1}} \sum_{p=0}^{s} (-1)^p \binom{s}{p}^2 \left\{ \partial^{(p)}\Sigma\, \partial^{(s-p)}\bar\Sigma + i \binom{s-p}{p+1} \partial^{(p)}\bar{\rm D}\Sigma\, \partial^{(s-p-1)}{\rm D}\bar\Sigma \right\} . \tag{5.7}$$

6 Higher spin supercurrents via complex linear — chiral duality

We have already noticed that (i) the complex linear supermultiplet couples only to half-integer superspin supermultiplets $(s+1, s+1/2)$, like the chiral superfield does and (ii) the corresponding minimal higher spin supercurrents for the complex linear superfield and the chiral superfield have many similarities. In order to find what is their precise connection, we perfom the well known duality procedure that maps one to the other.

As a starting point, we consider the auxiliary action

$$S = -\int d^8z\; \bar\sigma\; \sigma + \int d^8z\; \Phi\; \sigma + \int d^8z\; \bar\Phi\; \bar\sigma + g \int d^8z \sum_{s=0}^{\infty} H^{\alpha(s)\dot\alpha(s)} \mathcal{J}^{\rm min}_{\alpha(s)\dot\alpha(s)} \tag{6.1}$$

where σ is an unconstrained, complex, scalar superfield, Φ is a chiral superfield and $\mathcal{J}^{\rm min}_{\alpha(s)\dot\alpha(s)}$ is given by (5.7) with Σ and $\bar\Sigma$ replaced by σ and $\bar\sigma$ respectively. It is straightforward to see that once we integrate out Φ, σ is promoted to a complex linear superfield Σ and we recover $S_0 + S_{\text{HS-}\Sigma\text{ minimal cubic interactions}}$ with the correct higher spin supercurrents. Now, if we instead integrate out σ, we get

$$S = \int d^8z\; \bar\Phi\; \Phi + g \int d^8z \sum_{s=0}^{\infty} H^{\alpha(s)\dot\alpha(s)} \mathcal{J}^{\rm min}_{\alpha(s)\dot\alpha(s)}\Big|_{\sigma=\bar\Phi} . \tag{6.2}$$

However, the quantity $\mathcal{J}^{\rm min}_{\alpha(s)\dot\alpha(s)}\Big|_{\sigma=\bar\Phi}$ can be written as

$$\begin{aligned}\mathcal{J}^{\rm min}_{\alpha(s)\dot\alpha(s)}\Big|_{\sigma=\bar\Phi} &= (-1)^{s+1} \frac{(-i)^s}{\binom{2s+1}{s+1}} \sum_{p=0}^{s} (-1)^p \binom{s}{p}^2 \left\{ \partial^{(p)}\Phi\, \partial^{(s-p)}\bar\Phi + i \binom{s-p}{p+1} \partial^{(p)}{\rm D}\Phi\, \partial^{(s-p-1)}\bar{\rm D}\bar\Phi \right\} \\ &= (-1)^{s+1} \mathcal{J}^{\rm min\ (\Phi)}_{\alpha(s)\dot\alpha(s)} \end{aligned} \tag{6.3}$$

where $\mathcal{J}^{\rm min\ (\Phi)}_{\alpha(s)\dot\alpha(s)}$ is the minimal higher spin supercurrent constructed out of a chiral superfield [1, 19].

The result of the duality transformation is a chiral theory with cubic interactions to higher spin supermultiplets, given by

$$S = \int d^8z\ \bar{\Phi}\ \Phi + g \int d^8z \sum_{s=0}^{\infty}\ (-1)^{s+1}\ H^{\alpha(s)\dot{\alpha}(s)} \mathcal{J}^{\min\ (\Phi)}_{\alpha(s)\dot{\alpha}(s)}\,. \tag{6.4}$$

Notice that there is a difference in the coupling constant or *charge* that controls the cubic interactions with higher spin supermultiplets. We started by fixing all the higher spin charges of the complex linear theory to be the same [g] and we found that the corresponding charge of the dual chiral theory is spin dependent [$(-1)^{s+1}g$] and alternates in sign between even and odd values of s. For odd values of s both theories have the same sign charge and for even values of s they have opposite sign charge.

It is known that for the case of coupling to the vector multiplet ($s = 0$), the chiral and complex linear superfield have opposite sign charge. This can be easily understood be observing the neutrality of the $\Phi\ \sigma$ term in (6.1). Our analysis indicates that this behavior extents to arbitrary high spin supermultiplets $(s+1, s+1/2)$ with even s. For the supergravity case ($s = 1$) and all higher spin supermultiplets $(s+1, s+1/2)$ with s odd, both charges have the same sign. We remind the reader that the highest propagating spin of the $(s+1, s+1/2)$ supermultiplet is $j = s+1$, hence it would be interesting to determine whether this result has a connection with the fact that odd j spins can have repulsive interactions and even j spins have only attractive interactions [22].

7 Higher spin component currents for the complex linear superfield

To conclude our analysis, we would like to extract the higher spin component currents contained inside the supercurrent multiplet. The easiest way to identify them, is through their conservation equation. For this reason, we will project the superspace conservation equation (5.2) to components.

First of all notice that, due to (5.2) and the reality of $\mathcal{J}^{\min}_{\alpha(s)\dot{\alpha}(s)}$, the entire superfield satisfies a spacetime conservation equation

$$\partial^{\alpha_s\dot{\alpha}(s)} \mathcal{J}^{\min}_{\alpha(s)\dot{\alpha}(s)} = 0 \tag{7.1}$$

hence all of its components will be conserved. However, because of (5.2) not all of these components are independent. The independent ones are:

$$\mathcal{J}^{\min\ (0,0)}_{\alpha(s)\dot{\alpha}(s)} \sim \mathcal{J}^{\min}_{\alpha(s)\dot{\alpha}(s)}\Big|_{\substack{\theta=0\\ \bar{\theta}=0}}\,, \tag{7.2}$$

$$\mathcal{J}^{\min\ (1,0)}_{\alpha(s+1)\dot{\alpha}(s)} \sim \frac{1}{(s+1)!} \mathrm{D}_{(\alpha_{s+1}} \mathcal{J}^{\min}_{\alpha(s))\dot{\alpha}(s)}\Big|_{\substack{\theta=0\\ \bar{\theta}=0}}\,, \tag{7.3}$$

$$\mathcal{J}^{\min\ (1,1)}_{\alpha(s+1)\dot{\alpha}(s+1)} \sim \frac{1}{(s+1)!(s+1)!} \left[\mathrm{D}_{(\alpha_{s+1}}, \bar{\mathrm{D}}_{(\dot{\alpha}_{s+1}}\right] \mathcal{J}^{\min}_{\alpha(s))\dot{\alpha}(s))}\Big|_{\substack{\theta=0\\ \bar{\theta}=0}}\,. \tag{7.4}$$

and they are all conserved on-shell.

The first one (7.2) is a bosonic, integer spin ($j = s$), R-symmetry current. Using (5.7) we find that it is proportional to

$$\mathcal{J}^{\min\ (0,0)}_{\alpha(s)\dot{\alpha}(s)} \sim -(-i)^s \sum_{p=0}^{s} (-1)^p \binom{s}{p}^2 \left\{ \partial^{(p)} A\, \partial^{(s-p)} \bar{A} + i \left(\frac{s-p}{p+1}\right) \partial^{(p)} \bar{\psi}\, \partial^{(s-p-1)} \psi \right\} \tag{7.5}$$

where $\Sigma| = A$ and $\bar{\mathrm{D}}_{\dot{\alpha}}\Sigma| = \bar{\psi}_{\dot{\alpha}}$. The second one (7.3) is a fermionic, half-integer ($j = s+1/2$) current

$$\mathcal{J}^{\min\ (1,0)}_{\alpha(s+1)\dot{\alpha}(s)} \sim -(-i)^s \sum_{p=0}^{s} (-1)^p \binom{s}{p}^2 \frac{s+1}{s-p+1}\, \partial^{(p)} A\, \partial^{(s-p)} \psi\,. \tag{7.6}$$

The last one (7.4) is a bosonic, integer spin ($j = s+1$) current

$$\begin{aligned} \mathcal{J}^{\min\ (1,1)}_{\alpha(s+1)\dot{\alpha}(s+1)} \sim (-i)^s \sum_{p=0}^{s} (-1)^p \binom{s}{p}^2 \Bigg\{ & i\, \partial^{(p)} A\, \partial^{(s+1-p)} \bar{A} - i \left[\frac{2s-p+1}{p+1}\right] \partial^{(p+1)} A\, \partial^{(s-p)} \bar{A} \\ & + \left[\frac{s+p+2}{p+1}\right] \partial^{(p)} \bar{\psi}\, \partial^{(s-p)} \psi - \left[\frac{s-p}{p+1}\right] \partial^{(p+1)} \bar{\psi}\, \partial^{(s-p-1)} \psi \Bigg\}\,. \end{aligned} \tag{7.7}$$

All these currents are proportional to the ones constructed from the chiral theory in [1].

8 Summary and discussion

Let us briefly summarize our results. This work is the continuation of [1] and it aims to investigate the possibility of cubic interactions between a single, massless, complex linear supermultiplet and $4D$, $\mathcal{N} = 1$ higher spin supermultiplets. Using Noether's method we derive explicit expressions for the higher spin supercurrent multiplet that gives rise to such interactions. In section 2 we give a first order transformation for the complex linear superfield. Its compatibility with the linearity condition will give a set of constraints to the transformation parameters, the solution of which gives structures similar to the zeroth order gauge transformations of some higher spin supermultiplets. The outcomes of Noether's procedure are:

1. Cubic interactions exist only for the half-integer superspin supermultiplets ($s + 1$, $s + 1/2$).

2. The *canonical* higher spin supercurrent multiplet $\{\mathcal{J}_{\alpha(k+1)\dot{\alpha}(k+1)}, \mathcal{T}_{\alpha(k)\dot{\alpha}(k)}\}$ includes the supercurrent (3.12a) and supertrace (3.12c). Both of them generate the interaction terms (3.14) and satisfy the superspace conservation equation (5.1).

3. For every value of k, there exist an improvement term that will take us from the *canonical* supercurrent multiplet to the *minimal* supercurrent multiplet defined by $\mathcal{T}^{\min}_{\alpha(k)\dot{\alpha}(k)} = 0$ which includes the minimal higher spin supercurrent $\mathcal{J}^{\min}_{\alpha(k+1)\dot{\alpha}(k+1)}$ given by (4.17), (5.7). It satisfies the superspace conservation equation (5.2) and the cubic interactions it generates are (4.16).

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These results have similarities with the results presented in [1], where the cubic interactions between higher spin and chiral supermultiplets are constructed. For that reason we check the well known duality between a chiral and complex linear theory in the presence of these higher spin cubic interactions. The result is:

4. The duality holds with an interesting twist. If the *charge* that controls the cubic interaction of a complex linear superfield to the higher spin supermultiplet $(s+1, s+1/2)$ is g, then the corresponding *charge* for the chiral theory is $(-1)^{s+1}g$. Therefore, for odd values of s (such as supergravity, $s=1$) both the chiral and complex linear superfields have the same sign higher spin charge. However, for even values of s (such as the vector multiplet, $s=0$) they have opposite sign higher spin charge.

Finally, we focus at the component level of the supercurrent multiplet and identify the various spacetime conserved higher spin currents. There are three higher spin currents:

5. There is a bosonic, spin $j=s$ current $\mathcal{J}^{\text{min }(0,0)}_{\alpha(s)\dot{\alpha}(s)}$ (7.2), (7.5) which corresponds to the θ independent component of $\mathcal{J}^{\text{min}}_{\alpha(k+1)\dot{\alpha}(k+1)}$. This current corresponds to an R-symmetry.

6. There is a second bosonic, spin $j=s+1$ current $\mathcal{J}^{\text{min }(1,1)}_{\alpha(s+1)\dot{\alpha}(s+1)}$ (7.4), (7.7) corresponding to the $\theta\bar{\theta}$ component.

7. There is a fermionic, spin $j=s+1/2$ current $\mathcal{J}^{\text{min }(1,0)}_{\alpha(s+1)\dot{\alpha}(s)}$ (7.3), (7.6), corresponding to the θ component.

Notice that the bosonic currents have two independent contributions, one coming from the bosonic sector (complex scalar) and another coming from the fermionic sector (spinor). These two contributions have been discovered independently by studying non-supersymmetric theories [23–25]. However, the fermionic current has a single contribution that depends on both the boson and the fermion, hence the discovery of such a higher spin current appears naturally in supersymmetric theories. The first time this type of current appeared was in [1], where the complex scalar and spinor are defined as components of a chiral superfield. In this work we find the analogue expressions for the case of using a complex linear superfield to define them. The results we get are proportional to the ones found in [1] and consistent with the duality transformation discussed in section 6.

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