Continuous spin fields of mixed-symmetry type

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ABSTRACT: We propose a description of continuous spin massless fields of mixed-symmetry type in Minkowski space at the level of equations of motion. It is based on the appropriately modified version of the constrained system originally used to describe massless bosonic fields of mixed-symmetry type. The description is shown to produce generalized versions of triplet, metric-like, and light-cone formulations. In particular, for scalar continuous spin fields we reproduce the Bekaert-Mourad formulation and the Schuster-Toro formulation. Because a continuous spin system inevitably involves infinite number of fields, specification of the allowed class of field configurations becomes a part of its definition. We show that the naive choice leads to an empty system and propose a suitable class resulting in the correct degrees of freedom. We also demonstrate that the gauge symmetries present in the formulation are all Stueckelberg-like so that the continuous spin system is not a genuine gauge theory.

Contents

1 Introduction

Continuous spin massless particles [1] have several interesting properties including the presence of a dimensionful parameter μ and the infinite number of physical degrees of freedom [2–16]. From the conventional higher spin theory perspective [17, 18] the most striking and intriguing feature is that the continuous spin dynamics can be defined on the space of fields which is the sum of Fronsdal-like rank-s fields with $s = 0, ..., \infty$. The corresponding gauge invariant action functional on Minkowski space $\mathbb{R}^{d-1,1}$ and AdS_d space is the infinite sum of Fronsdal rank-s actions with off-diagonal terms proportional to the continuous spin parameter μ [4, 7]. The gauge transformations are the standard Fronsdal

transformations deformed by Stueckelberg-like terms also proportional to μ. Moreover, the continuous spin fields can consistently interact with massive higher spin fields [13, 14].

From the group-theoretical perspective continuous spin particles correspond to infinite-dimensional massless UIRs of the Poincare algebra $iso(d-1,1)$, induced from infinite-dimensional UIRs of $iso(d-2)$ subalgebra [1, 19, 20]. The associated quantum numbers are the standard mass $m = 0$, the continuous spin parameter $\mu \neq 0$, and (half-)integer spin numbers $(s_1, ..., s_p)$, where $p = [\frac{d-3}{2}]$. The description of [4, 7] was derived in the case of scalar representation. In the mixed-symmetry case the continuous spin dynamics on $\mathbb{R}^{d-1,1}$ was described at the level of equations of motion [2] as the particular contraction of the Fronsdal massless equations in $\mathbb{R}^{d,1}$ space.[1]

In this paper we study equations of motion for mixed-symmetry continuous spin fields in Minkowski space. To this end we use the generating formulation elaborated in [21–24] based on a first-quantized constrained system whose representation space is interpreted as the space of field configurations. We show that the continuous spin system corresponds to an appropriate deformation of the constraints, which is parameterized by the continuous spin parameter μ. This allows us to formulate the triplet-like formulation of the continuous spin dynamics that generalizes the standard triplet formulation [22, 25–27]. Taking the triplet form as a starting point we derive the metric-like description which generalizes Fronsdal or Labastida formulations [28, 29].[2] Under the usual assumptions we also arrive at a light-cone formulation by eliminating quartets.

Because a continuous spin system inevitably involves infinite number of fields, specification of the allowed class of field configurations becomes a part of its definition. Leaving aside space-time behavior of the fields we concentrate on the target space which is the space of "functions" in auxiliary oscillator variables. The simplest choice which is polynomials in oscillators is not compatible with the deformed constraints in the sense that in contrast to the usual helicity spin case the constraints do not admit polynomial solutions. The way out would be to allow for formal series but as we demonstrate in this work with this choice gauge symmetry kills all the degrees of freedom. Nevertheless, it turns out that it is possible to identify a certain subspace of the formal series in which, on one hand, deformed constraints admit nontrivial solutions while, on the other hand, gauge symmetry does not kill everything. We then demonstrate by performing the light-cone analysis that the system indeed propagates correct degrees of freedom.

The paper is organized as follows. In section 2 we formulate the continuous spin dynamics in terms of the BRST first quantized system. To identify the continuous spin parameter we explicitly compute both the quadratic and quartic Casimir operators of the Poincare algebra. In section 3 we develop the triplet formulation of the continuous spin dynamics.[3] Here we briefly review the homological reduction technique that we use to derive other forms of the continuous spin dynamics. In this section we also find the metric-like formulation which in the scalar continuous spin case reproduces the Bekaert-Mourad

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[1]For the frame-like Lagrangian formulation of continuous spin $(s_1, 0, ..., 0)$ fields in AdS_d see [15], the light-cone dynamics of continuous spin $(s_1, 0, ..., 0)$ fields in AdS_5 space was considered in [16].

[2]Free massless higher spin fields were discussed within various formulations, see e.g. [30–41].

[3]BRST formulation of the continuous spin dynamics was previously discussed in [3, 42].

equations [2] and the Schuster-Toro equations [4] for an infinite collection of Fronsdal-like tensor fields of all ranks. In section 4 we show how the triplet equations reduce to the light-cone equations. In particular, we explicitly describe the infinite-dimensional field space in terms of $o(d-2)$ tensors and calculate the Casimir operators of the Wigner algebra $iso(d-2)$. In section 5 we study the Weyl and gauge modules and show that both of them are trivial unless one assumes specific functional class in the sector of oscillator variables. Within this class the gauge module remains trivial while the Weyl module becomes non-trivial thereby indicating the system propagates physical degrees of freedom. Appendices A–C contain auxiliary and technical statements.

2 Equations of motion for continuous spin fields

In this section we formulate the continuous spin dynamics as viewed from the constraint algebra perspective. The underlying constrained system is the modified version of that proposed in the helicity spin case [22], where some of the constraints are deformed by constant terms associated to the continuous spin parameter.

2.1 Auxiliary variables and constraints

A continuous spin massless system can be represented in terms of generating function $\phi(x, a)$, where x^b, $b = 0, \ldots, d-1$ are Cartesian coordinates in Minkowski space $\mathbb{R}^{d-1,1}$ with the metric $\eta_{ab} = (+ - \cdots -)$, and a_i^b are auxiliary commuting variables, $i = 1, \ldots, n$.

The Poincare algebra $iso(d-1, 1)$ basis elements are realized as

$$P_a = \frac{\partial}{\partial x^a} \ , \qquad M_{ab} = x_a \frac{\partial}{\partial x^b} - x_b \frac{\partial}{\partial x^a} + a_{ai} \frac{\partial}{\partial a_i^b} - a_{bi} \frac{\partial}{\partial a_i^a} \ . \tag{2.1}$$

Let us introduce notation

$$\Box = \frac{\partial^2}{\partial x^b \partial x_b} \ , \qquad D_i^\dagger = a_i^b \frac{\partial}{\partial x^b} \ , \qquad D^i = \frac{\partial^2}{\partial a_i^b \partial x_b} \ ,$$

$$T^{ij} = \frac{\partial^2}{\partial a_{ib} \partial a_j^b} \ , \qquad T_{ij}^\dagger = a_i^b a_{bj} \ , \qquad N_i{}^j = a_i^b \frac{\partial}{\partial a_j^b} \ , \qquad N_i = a_i^b \frac{\partial}{\partial a_i^b} \ . \tag{2.2}$$

The above operators form a subalgebra in $sp(2n + 2)$ algebra which is dual to the Lorentz algebra formed by generators M_{ab} in the sense of Howe [43]. The space of formal series in $a_I^b = (x^b, a_i^b)$ is an $iso(d-1, 1) \oplus sp(2n + 2)$ bimodule. This algebraic framework was employed in [22] to describe usual mixed-symmetry gauge fields.

The continuous spin system is described in terms of the (suitably modified) constraints from (2.2). Moreover, all the constraints but $D_i^\dagger$ are imposed directly while constraints $D_i^\dagger$ are imposed in a dual way as generators of gauge transformations.

Differential constraints. These are the same as in the helicity spin case,

$$\Box \phi = 0 \ , \qquad D^i \phi = 0 \ , \qquad i = 1, \ldots, n \ . \tag{2.3}$$

Algebraic constraints. We impose the modified trace and Young symmetry constraints

$$\left(T^{ij} + \nu^{ij}\right)\phi = 0\,, \qquad\qquad \nu^{ij} = \nu\,\delta^{1i}\delta^{1j}\,, \qquad\qquad i,j = 1,\ldots,n\,, \qquad (2.4)$$

$$N_i{}^j\phi = 0 \quad i < j\,, \qquad\qquad N_i\phi = s_i\phi\,, \qquad\qquad i,j = 2,\ldots,n\,, \qquad (2.5)$$

where the spin weights $s_i \geqslant 0$ are non-negative integers and $\nu \in \mathbb{R}$.

Gauge equivalence. The gauge transformations are given by

$$\delta\phi = \left(D_i^\dagger + \mu_i\right)\chi^i\,, \qquad \mu_i = \mu\,\delta_{1i}\,, \qquad i = 1,\ldots,n\,, \qquad (2.6)$$

where χ^i are the gauge parameters satisfying the off-shell constraints that follow from (2.3)–(2.5) and $\mu \in \mathbb{R}$.

We note that the constraints are consistent provided that the parameter matrices ν^{ij} and μ^i are fixed as in (2.4) and (2.6). In section 2.3 we show that $\nu\mu^2$ is the value of the quartic Casimir operator of the Poincare algebra $iso(d-1,1)$ so that fixing e.g. $\nu = 1$ we find out that μ is the continuous spin parameter. Note also that a similar constraint system was discussed in [2].

In contrast to the constrained system describing the helicity spin fields the employed constraints are not the highest weight conditions of $sp(2n+2)$ algebra. On the contrary, the deformed constraints are typical for the theory of coherent states, where the states are defined as eigenstates of the annihilation operator (e.g., T^{ij} in our case). It follows that such elements do not diagonalize the particle number operator anymore (missing N_1 in our case) and are represented as infinite power series in auxiliary variables.

As the formulation involves operators that mix tensors of different rank the system non-trivially involves infinite number of fields and, hence, it is important to specify which functions in a_i^b are allowed because this defines the field content of the theory. The choice to be motivated later (see section 5) is as follows: we take formal series in a_i^b satisfying the additional admissibility condition. A series f is admissible if its trace decomposition

$$f = f_0 + f_1^{ij}T_{ij}^\dagger + f_2^{ij,kl}T_{ij}^\dagger T_{kl}^\dagger + \ldots\,, \qquad T^{ij}f_p^{\cdots} = 0\,, \qquad (2.7)$$

is such that all coefficients are polynomials of finite order (i.e. for a given f there exists such $N \in \mathbb{N}$ that all f_r are of order not exceeding N). It is clear that both Poincare and $sp(2n+2)$ algebras preserve the space of admissible elements. This space serves as the target space of the system.

2.2 BRST operator

In what follows we actively employ BRST first quantized formalism, see e.g. [21, 22] for more details and original references. To this end, we introduce the anticommuting ghost variable b^i with the ghost number $\mathrm{gh}\,(b_i) = -1$, and then split both the auxiliary commuting variables and ghost variables as $a_i^b = (a^b, a_\alpha^b)$ and $b^i = (b, b^\alpha)$, where $\alpha = 1,\ldots,n-1$. Fields of the system as well as (higher-order) gauge parameters are encoded in the generating

function $\Psi(x, a|b)$. It can be expanded in b_i so that homogenous components have definite ghost degrees, i.e.

$$\Psi = \sum_{n \geqslant 0} \Psi^{(-n)} , \qquad \mathrm{gh}\left(\Psi^{(-n)}\right) = -n . \tag{2.8}$$

According to the usual prescription fields are encoded in degree 0 component, gauge parameters in degree -1 component, etc.

The system (2.3)–(2.6) can now be written in the BRST form. All constraints remain the same except for the Young symmetry and spin conditions (2.5) that receive ghost extensions. Namely,

$$\mathcal{N}_\alpha{}^\beta \Psi = 0 \quad \alpha < \beta , \qquad \mathcal{N}_\alpha \Psi = s_\alpha \Psi , \qquad \alpha, \beta = 1, ..., n-1 , \tag{2.9}$$

where

$$\mathcal{N}_\alpha{}^\beta = N_\alpha{}^\beta + b_\alpha \frac{\partial}{\partial b_\beta} \qquad \text{and} \qquad \mathcal{N}_\alpha = N_\alpha + b_\alpha \frac{\partial}{\partial b_\alpha} , \tag{2.10}$$

and no sum over α in the last formula is implied. In this way the above constraints simultaneously impose the conditions on fields as well as on gauge parameters.

The BRST operator is given by

$$\mathcal{Q} = \left(D_i^\dagger + \mu_i\right) \frac{\partial}{\partial b_i} \equiv \left(D^\dagger + \mu\right) \frac{\partial}{\partial b} + D_\alpha^\dagger \frac{\partial}{\partial b_\alpha} , \qquad Q^2 = 0 . \tag{2.11}$$

The BRST invariance of the constraints fixes parameters ν^{ij} and μ^i to be proportional to arbitrary ν and μ.[4] To reproduce the gauge transformation (2.6) we identify fields $\phi(x, a)$ as ghost number zero components $\phi(x, a) = \Psi^{(0)}(x, a)$ and gauge parameters $\chi^i(x, a)$ as ghost minus one components $b_i \chi^i(x, a) = \Psi^{(-1)}(x, a|b)$, cf. (2.8). Then, the gauge transformation $\delta \Psi^{(0)} = \mathcal{Q} \Psi^{(-1)}$ yields (2.6), where parameters $\chi^i(x, a)$ satisfy the same constraints (2.3), (2.4), while (2.5) are appropriately modified.

2.3 Evaluating the Casimir operators

Our formulation involves parameters μ, ν and $(n-1)$ spin weights $s_1, ..., s_{n-1}$. In d dimensions $n = [\frac{d-3}{2}]$ that allows describing all possible finite-dimensional modules of the short little algebra $o(d-3) \subset iso(d-1,1)$ [19].

To characterize $iso(d-1,1)$ representations underlying the system (2.3)–(2.6) we analyze the Casimir operators of the Poincare algebra (2.1) briefly reviewed in appendix A. The quadratic Casimir operator $C_2 = P_a P^a \approx 0$ vanishes on-shell because of the constraint (2.3). Then, the quartic Casimir operator $C_4 = (M_{ab} P^b)^2$ equals

$$C_4 \, \phi(x, a) = -D_i^\dagger D_j^\dagger T^{ij} \, \phi(x, a) \approx \mu^2 \nu \, \phi(x, a) , \tag{2.12}$$

where we used the differential constraints (2.3), trace constrains (2.4) and the equivalence relation $\phi \sim \phi + \mathcal{Q}\chi$ with the gauge parameter expressed in terms of the field ϕ.[5] From

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[4]To this end, we compute $[Q, \mathcal{N}_i{}^j] = \mu_i \frac{\partial}{\partial b^j}$ and $[T^{mn}, \mathcal{N}_i{}^j] = \delta_i^n T^{mj} + \delta_i^m T^{nj}$ and imposing the constraints we find out that the parameters must be as in (2.4) and (2.6).

[5]The analogous consideration of the Casimir operators on the equivalence classes in the case of $o(d-1,2)$ algebra and AdS_d gauge fields can be found in [24].

our analysis in appendix A it follows that any higher order Casimir operator is also proportional to $\mu^2\nu$.

Thus, we see that the model propagates continuous spin particles, in which case fixing $\nu = 1$ we identify μ as the continuous spin parameter. On the other hand, we stress that such a split between deformation parameters μ and ν is artificial and only their combination $\mu^2\nu$ has invariant meaning.

3 Triplet formulation

Now we extend the triplet formulation of helicity spin fields of symmetric and mixed-symmetry type [22, 25–27] to the continuous spin case. To this end, we introduce additional anti-commuting ghost variables c_0, c_i, $i = 1, \ldots, n$, with positive ghost numbers $\mathrm{gh}\,(c_0) = 1$, $\mathrm{gh}\,(c_i) = 1$. These variables are associated with the differential constraints (2.3).

Then, the BRST operator (2.11) can be extended as follows

$$\Omega = c_0\Box + c_i D^i + \left(D_i^\dagger + \mu_i\right)\frac{\partial}{\partial b_i} - c_i\frac{\partial}{\partial b_i}\frac{\partial}{\partial c_0}\,, \tag{3.1}$$

where $\mu_i = \mu\delta_{i1}$. It is defined on the subspace of $\Psi = \Psi(x, a|c, b)$ singled out by the BRST extended trace constraints

$$(\mathcal{T} + \nu)\Psi = 0\,, \qquad \mathcal{T}^\alpha\Psi = 0\,, \qquad \mathcal{T}^{\alpha\beta}\Psi = 0\,, \tag{3.2}$$

as well as the Young symmetry and the spin weight constraints

$$\mathcal{N}_\alpha{}^\beta\Psi = 0 \quad \alpha < \beta\,, \qquad \mathcal{N}_\alpha\Psi = s_\alpha\Psi\,. \tag{3.3}$$

The extended constraints read explicitly as

$$\mathcal{T}^{ij} = T^{ij} + \frac{\partial}{\partial c_i}\frac{\partial}{\partial b_j} + \frac{\partial}{\partial c_j}\frac{\partial}{\partial b_i}\,, \qquad \mathcal{N}_\alpha{}^\beta = N_\alpha{}^\beta + b_\alpha\frac{\partial}{\partial b_\beta} + c_\alpha\frac{\partial}{\partial c_\beta}\,,$$
$$\mathcal{N}_\alpha = N_\alpha + b_\alpha\frac{\partial}{\partial b_\alpha} + c_\alpha\frac{\partial}{\partial c_\alpha}\,, \qquad \alpha, \beta = 1, ..., n-1\,, \tag{3.4}$$

where in the last expression no summation over α is implied. Note that the BRST operator (3.1) is nilpotent on the entire space of unconstrained fields and not only on the subspace singled out by (3.2) and (3.3). On the entire space BRST operator (3.1) describes a reducible system whose analog in the helicity case is well known in the literature [25–27] and is relevant in the context of tensionless strings.

Expanding Ψ into homogeneous components of definite ghost degree we concentrate on the vanishing degree component $\Psi^{(0)}$. Representing then $\Psi^{(0)}$ as $\Psi^{(0)} = \Phi + c_0 C$ we introduce component fields entering $\Phi = \Phi(x, a|b, c)$ and $C = C(x, a|b, c)$ according to

$$\Phi = \sum_{k=0}^{n-1} c_{i_1}...c_{i_k}b_{j_1}...b_{j_k}\Phi^{i_1...i_k|j_1...j_k}\,, \qquad C = \sum_{k=0}^{n-2} c_{i_1}...c_{i_k}b_{j_1}...b_{j_{k+1}}C^{i_1...i_k|j_1...j_{k+1}}\,. \tag{3.5}$$

These component fields can be identified as generalized triplet fields [22, 25, 27]. The corresponding gauge transformation reads

$$\delta\Psi^{(0)} = \Omega\Psi^{(-1)} \, , \tag{3.6}$$

where the ghost number -1 parameters $\Psi^{(-1)} = \Lambda + c_0\Upsilon$ are given by

$$\Lambda = \sum_{k=0}^{n-2} c_{i_1}...c_{i_k} b_{j_1}...b_{j_{k+1}} \Lambda^{i_1...i_k|j_1...j_{k+1}} \, , \qquad \Upsilon = \sum_{k=0}^{n-3} c_{i_1}...c_{i_k} b_{j_1}...b_{j_{k+2}} \Upsilon^{i_1...i_k|j_1...j_{k+2}} \, . \tag{3.7}$$

Analogously, the ghost number $-k$ component $\Psi^{(k)}$ encodes $(k-1)$-th level reducibility parameters.

Finally, the triplet equations of motion for continuous spin fields have the form

$$\Omega\Psi^{(0)} = 0 \, . \tag{3.8}$$

By construction, (3.8) is invariant with respect to the gauge transformation (3.6), where both fields and parameters are constrained by (3.2)–(3.3). We note that the BRST operator (3.1) for the continuous spin system differs from the BRST operator for the helicity spin system [22] by adding the term proportional to μ, i.e. $\Omega \to \Omega + \mu\frac{\partial}{\partial b}$.

3.1 Homological reduction

In this section we shortly review the homological reduction that can be applied to any gauge system defined by a BRST operator [21].[6] In the formal language the linear gauge system can be defined as a pair $(\mathcal{H}, \Omega)$, where $\mathcal{H}$ is the representation space of the BRST first-quantized system and Ω is the nilpotent BRST operator. In addition, it is assumed that $\mathcal{H}$ is graded by ghost degree in such a way that Ω carries degree 1. Ghost degree zero elements $\Psi^{(0)}$ of $\mathcal{H}$ are identified with field configurations while those at negative ghost degree with (higher order) gauge parameters. As before, the equations of motion are $\Omega\Psi^{(0)} = 0$ while gauge transformation are $\delta\Psi^{(0)} = \Omega\Psi^{(-1)}$. Similarly one defines higher order gauge transformations (also known as reducibility relations).

Suppose $\mathcal{H}$ can be split into three subspaces: $\mathcal{H} = \mathcal{E} \oplus \mathcal{F} \oplus \mathcal{G}$. Let $\overset{\mathcal{GF}}{\Omega} : \mathcal{F} \to \mathcal{G}$ denotes Ω restricted to $\mathcal{F}$ and projected to $\mathcal{G}$, i.e. $\overset{\mathcal{GF}}{\Omega} f = (\Omega f)|_{\mathcal{G}}$, where $f \in \mathcal{F}$. If $\overset{\mathcal{GF}}{\Omega} : \mathcal{F} \to \mathcal{G}$ is invertible then all the fields associated with $\mathcal{F}$ and $\mathcal{G}$ are generalized auxiliary fields. The notion of generalized auxiliary fields was introduced in [45] and extended to the case of not necessarily Lagrangian systems in [21]. In particular, among generalized auxiliary fields one finds usual auxiliary fields and Stueckelberg fields as well as the associated ghosts and antifields.

Generalized auxiliary fields can be eliminated, resulting in a new gauge theory or, rather, another formulation of the same theory. Theories related through elimination/addition of generalized auxiliary fields are considered equivalent. Note that typically

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[6]In the context of the unfolded formulation of higher spin fields the homological technique to identify auxiliary fields and Stueckelberg variables was proposed in [30, 44]

one is interested in local gauge field theories in which case one requires that generalized auxiliary fields can be eliminated algebraically. In the present case this corresponds to requiring $\overset{\mathcal{GF}}{\Omega}$ to be algebraic. Eliminating generalized auxiliary fields associated to $\mathcal{F}, \mathcal{G}$ gives a reduced theory $(\mathcal{E}, \widetilde{\Omega})$, where $\mathcal{E}$ is the representation space of the reduced BRST operator $\widetilde{\Omega}$ given by

$$\widetilde{\Omega} = \left(\overset{\mathcal{EE}}{\Omega} - \overset{\mathcal{EF}}{\Omega} (\overset{\mathcal{GF}}{\Omega})^{-1} \overset{\mathcal{GE}}{\Omega} \right) . \tag{3.9}$$

It is often useful to identify $\mathcal{E}$ as a cohomology of a certain piece of Ω. More specificaly, suppose that $\mathcal{H}$ admits an additional grading

$$\mathcal{H} = \bigoplus_{-N}^{\infty} \mathcal{H}_i \, , \tag{3.10}$$

with N finite integer and such that Ω decomposes into homogeneous componets as follows

$$\Omega = \Omega_{-1} + \Omega_0 + \Omega_1 + \dots \, . \tag{3.11}$$

It follows that the lowest grade part of the BRST operator Ω_{-1} defines the decomposition introduced above

$$\mathcal{E} \oplus \mathcal{G} = \operatorname{Ker} \Omega_{-1} \, , \qquad \mathcal{G} = \operatorname{Im} \Omega_{-1} \, , \qquad \mathcal{E} = \frac{\operatorname{Ker} \Omega_{-1}}{\operatorname{Im} \Omega_{-1}} \equiv H(\Omega_{-1}) \, . \tag{3.12}$$

Note that by construction such $\overset{\mathcal{GF}}{\Omega}$ is invertible.

In what follows the homological reduction technique is applied to find equivalent forms of the triplet formulation (3.1): the metric-like formulation that generalizes Fronsdal and Labastida formulations to the continuous spin case, and the light-cone formulation.

3.2 Metric-like formulation

Let the additional grading (3.10) be a homogeneity degree in c_0. Then, BRST operator (3.1) can be decomposed as $\Omega = \Omega_{-1} + \Omega_0 + \Omega_1$ with

$$\Omega_{-1} = -c_i \frac{\partial}{\partial b_i} \frac{\partial}{\partial c_0} \, , \qquad \Omega_0 = c_i D^i + \left(D_i^\dagger + \mu_i \right) \frac{\partial}{\partial b_i} \, , \qquad \Omega_1 = c_0 \square \, . \tag{3.13}$$

The theory can be consistently reduced to the subspace $\mathcal{E} = H(\Omega_{-1})$ using the homological technique reviewed in section 3.1. Note that the μ-deformation term enters only Ω_0 and, therefore, the cohomology $H(\Omega_{-1})$ remains the same as in the helicity spin case [22].

The cohomology $H(\Omega_{-1})$ in ghost degree 0 and -1 can be explicitly described in terms of the lowest expansion components in ghosts c_i and b^i. More precisely, denoting the lowest component in (3.5) as φ we find $\Phi = \varphi + \dots$, where the ellipses denote the ghost contributions expressed in terms of traces of φ. Analogously, we denote by χ^i the lowest component of the gauge parameter Λ^i in (3.7). From (3.2)–(3.4) it then follows that φ and χ^i satisfy the modified trace conditions

$$\mathbb{T}^{(ij}\mathbb{T}^{kl)}\varphi = 0 \, , \qquad \mathbb{T}^{(ij}\chi^{k)} = 0 \, , \tag{3.14}$$

where we introduced the notation $\mathbb{T}^{ij} \equiv T^{ij} + \nu \, \delta^{i1}\delta^{j1}$. Young symmetry and spin weight conditions take then the form

$$N_\alpha{}^\beta \varphi = 0 \quad \text{at} \quad \alpha < \beta \qquad \text{and} \qquad N_\alpha \varphi = s_\alpha \varphi \,, \tag{3.15}$$

and

$$N_\alpha{}^\beta \chi^\gamma + \delta_\alpha^\gamma \chi^\beta = 0 \quad \text{at} \quad \alpha < \beta \qquad \text{and} \qquad N_\alpha \chi = s_\alpha \chi \,, \quad N_\alpha \chi^\alpha = (s_\alpha - 1)\chi^\alpha \,, \tag{3.16}$$

where $N_\alpha{}^\beta$ and N_α are given by (2.2) and no sum over α in the last equation of (3.16) is implied. Let us note that the Young and weight conditions are imposed in the sector of a_α^b variables only. However, there are cross-trace conditions in (3.14) that mix up expansion coefficients in a^b and a_α^b.

Now, using the above cohomological results we can straightforwardly find the reduced equations of motion. Introducing operator Z via $\Omega_{-1} \equiv -\frac{\partial}{\partial c_0} Z$ the original triplet equations (3.8) can be cast into the form

$$\Box \Phi - \Omega_0 C = 0 \,, \qquad \Omega_0 \Phi - ZC = 0 \,, \tag{3.17}$$

where fields Φ and C are defined by the expansion (3.5). It follows from the stucture of $H(\Omega_{-1})$ that C is an auxiliary field and, therefore, using the second equation in (3.17) it can be expressed in terms of $\Omega_0 \Phi$. In other words, C is given by derivatives of Φ, while Φ itself is reduced to the lowest component φ. It follows (3.17) take the form

$$\Box \varphi - \left(D_i^\dagger + \mu_i \right) C^i = 0 \,, \qquad D^i \varphi - \left(D_j^\dagger + \mu_j \right) \Phi^{i|j} - C^i = 0 \,, \tag{3.18}$$

where the component $\Phi^{i|j}$ can be expressed via φ by virtue of the trace conditions (3.2) as $\Phi^{i|j} = \frac{1}{2}\mathbb{T}^{ij}\varphi$. Eliminating the auxiliary field C^i we finally arrive at the reduced equations of motion

$$\left[\Box - \left(D_i^\dagger + \mu_i \right) D^i + \frac{1}{2} \left(D_i^\dagger + \mu_i \right) \left(D_j^\dagger + \mu_j \right) \left(T^{ij} + \nu^{ij} \right) \right] \varphi = 0 \,, \tag{3.19}$$

which are invariant with respect to the gauge transformations

$$\delta \varphi = \left(D_i^\dagger + \mu_i \right) \chi^i \,. \tag{3.20}$$

Here, fields and gauge parameters are subject to the algebraic conditions (3.14)–(3.16). Note that setting $\mu, \nu = 0$ we reproduce the Labastida formulation [29].

3.3 Scalar continuous spin case

There is an interesting formulation of the continuous spin dynamics due to Schuster and Toro, where the field content is that of infinite sum of Fronsdal systems while the equations and gauge symmetries involve off-diagonal terms proportional to the parameter μ [4]. In this section we show that the Schuster-Toro equations can be obtained from the metric-like equations of the previous section by choosing $n = 1$. In this case the spin weights vanish $s_i = 0$, $i = 1, 2, \ldots$ so that we deal with the scalar continuous spin system.

The reduced equations of motion (3.19) take the form

$$\Box\varphi - \left(D^\dagger + \mu\right)D\varphi + \frac{1}{2}\left(D^\dagger + \mu\right)^2 (T + \nu)\varphi = 0 \,. \tag{3.21}$$

These equations were proposed by Bekaert and Mourad in [2]. By construction, these are invariant under transformations

$$\delta\varphi = D^\dagger\epsilon + \mu\epsilon \,, \tag{3.22}$$

supplemented with the deformed trace conditions

$$(T + \nu)^2\varphi = 0 \,, \qquad (T + \nu)\epsilon = 0 \,. \tag{3.23}$$

Note that there are no spin weight conditions in this case. However, the dynamics cannot be restricted to the spin-s subspace since the deformed trace constraints are incompatible with the spin-s weight condition $N\phi = s\phi$. Nonetheless, sending both ν and μ to zero we reproduce a direct sum of the Fronsdal equations for all integer spins.

The deformed trace conditions can be explicitly solved in terms of tensors subjected to the standard trace conditions. Namely, in appendix B we demonstrate that fields and parameters can be equivalently represented as

$$\varphi = \sum_{n,m=0}^\infty \beta_{m,n}\left(T^\dagger\right)^m \varphi_{(n)} \,, \qquad \epsilon = \sum_{n,m=0}^\infty \beta_{m,n+1}\left(T^\dagger\right)^m \epsilon_{(n)} \,, \tag{3.24}$$

where $T^\dagger = a^b a_b$ is the trace creation operator, the rank-n tensors on the right-hand sides satisfy the Fronsdal conditions

$$T^2\varphi_{(n)} = 0 \,, \qquad T\epsilon_{(n)} = 0 \,, \tag{3.25}$$

while the coefficients $\beta_{m,n}$ are explicitly given by (B.5). We conclude that original φ and ϵ are replaced now by infinite collections of Fronsdal (single and double traceless) tensors of ranks running from zero to infinity.

In appendix C we explicitly show that in the Fronsdal basis (3.25) the metric-like equations (3.21) take the Schuster-Toro form [4, 7]

$$-\Box\varphi_{(n)} + D^\dagger G_{(n-1)} + \mu\left[G_{(n)} + d_n T^\dagger G_{(n-2)}\right] = 0 \,, \qquad n = 0, 1, 2, \dots \,. \tag{3.26}$$

Here,

$$G_{(n)} = A_{(n)} + \mu\, c_n B_{(n)} \,, \tag{3.27}$$

with the derivative and algebraic terms combined into

$$A_{(n)} = D\varphi_{(n+1)} - \frac{1}{2}D^\dagger T\varphi_{(n+1)} \,, \qquad B_{(n)} = \varphi_{(n)} + a_n T^\dagger T\varphi_{(n)} + b_n T\varphi_{(n+2)} \,, \tag{3.28}$$

where the coefficients are given by

$$\begin{aligned} a_n &= -\frac{1}{2d + 2n - 8} \,, & b_n &= \frac{d + 2n - 2}{2\nu} \,, \\ c_n &= -\frac{1}{2b_n} \,, & d_n &= -\frac{\nu}{(d + 2n - 4)(d + 2n - 6)} \,. \end{aligned} \tag{3.29}$$

We note that $A_{(n)}$ and $B_{(n)}$ as well as $G_{(n)}$ are traceless. These combinations of fields and their derivatives are convenient to build the double-traceless operator (3.26).

The gauge transformation (3.22) reads

$$\delta\varphi_{(n)} = D^\dagger \epsilon_{(n-1)} + \mu \left[\epsilon_{(n)} + d_n T^\dagger \epsilon_{(n-2)} \right] . \tag{3.30}$$

This is the Stueckelberg-like transformation law with three different rank traceless gauge parameters, which is typical for massive higher spin theories [32].

4 Light-cone formulation for continuous spin fields

To formulate the light-cone dynamics we start from the triplet formulation and eliminate unphysical degrees of freedom by means of the homological reduction. This is achieved by using the suitable grading [46] (see also [22, 47]) such that the so-called quartets form the subspace $\mathcal{F} \oplus \mathcal{G}$ while the cohomology of the lowest degree piece of the BRST operator form the complementary subspace $\mathcal{E}$ describing configurations of $o(d-2)$ tensor fields on the light-cone.

The quartet grading is defined by[7]

$$\deg a_i^\pm = \pm 2 \,, \qquad \deg a_i^m = 0 \,, \qquad \deg c_0 = 0 \,, \qquad \deg c_i = 1 \,, \qquad \deg b^i = -1 \,. \tag{4.1}$$

Note that in the assumed functional space this grading is bounded from both above and below because $\deg T^\dagger = 0$ so that only the coefficients of the series (2.7) may contribute to the degree of an element. By the assumption for a given element these coefficients are polynomials of finite order in a-oscillators and hence the degree is finite. Thus, the conditions for the applicability of the homological reduction technique are satisfied.

The triplet BRST operator (3.1) decomposes into the homogeneous degree components as $\Omega = \Omega_{-1} + \Omega_0 + \Omega_1 + \Omega_2 + \Omega_3$, where

$$\Omega_{-1} = p^+ \left(c_i \frac{\partial}{\partial a_i^+} + a_i^- \frac{\partial}{\partial b_i} \right) \,, \qquad \Omega_0 = c_0 \left(2p^+ p^- + p_m p^m \right) \,,$$

$$\Omega_1 = c_i p^m \frac{\partial}{\partial a_i^m} + p^+ a_i^- \frac{\partial}{\partial b_i} + \mu \frac{\partial}{\partial b} \,, \quad \Omega_2 = -c_i \frac{\partial}{\partial b_i} \frac{\partial}{\partial c_0} \,, \quad \Omega_3 = p^- \left(c_i \frac{\partial}{\partial a_i^-} + a_i^+ \frac{\partial}{\partial b_i} \right) . \tag{4.2}$$

We note that Ω_{-1} is the de Rham differential in variables c_i, b_i and a_i^-, a_i^+. Thus, $H(\Omega_{-1})$ identified with dynamical fields consists of elements depending on transverse oscillators only $\phi = \phi(x | a_i^m)$ [22, 47]. Using the homological technique of section 3.1 one can explicitly show that the reduced BRST charge reads

$$\widetilde{\Omega} = c_0 \left(2p^+ p_- + p^m p_m \right) \,, \tag{4.3}$$

so that the equations of motion reduce to the mass-shell condition $p^2 = 0$.

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[7]The light-cone coordinates are $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^{d-1})$ and x^m, where $m = 1, ..., d-2$. $o(d-2)$ indices are denoted by letters from the middle of the alphabet $m, n, k, l, p, s, ...$ The scalar product reads $A_c B^c = A^+ B^- + A^- B^+ - A^m B^m$.

The light-cone off-shell constraints following from (3.2)–(3.4) read

$$\left(\widetilde{T} + \nu\right)\phi = 0 \ , \qquad \widetilde{T}^{\alpha}\phi = 0 \ , \qquad \widetilde{T}^{\alpha\beta}\phi = 0 \ , \tag{4.4}$$

the Young symmetry and spin weight conditions

$$\widetilde{N}_{\alpha}{}^{\beta}\phi = 0 \quad \alpha < \beta \ , \qquad \widetilde{N}_{\alpha}\phi = s_{\alpha}\phi \ , \qquad \alpha, \beta = 1, \ldots, n-1 \ , \tag{4.5}$$

where

$$\widetilde{T}^{ij} = \frac{\partial^2}{\partial a_i^m \partial a_{jm}} \ , \qquad \widetilde{N}_{\alpha}{}^{\beta} = a_{\alpha}^m \frac{\partial}{\partial a^{\beta m}} \ , \qquad \widetilde{N}_{\alpha} = a_{\alpha}^m \frac{\partial}{\partial a_{\alpha}^m} \ , \tag{4.6}$$

no sum over α in $\widetilde{N}_{\alpha}$. The light-cone BRST operator (4.3) obviously acts in the subspace.

Poincare algebra. The Poincare generators (2.1) in the light-cone basis split into two groups: kinematical $G_{kin} = (P^+, P^m, M^{+m}, M^{+-}, M^{mk})$ and dynamical $G_{dyn} = (P^-, M^{-k})$. After quartet reduction both types of generators act in the subspace, $\widetilde{G}_{kin}$ and $\widetilde{G}_{dyn}$. We find out that the reduced kinematical generators $\widetilde{G}_{kin}$ take the standard form, while the reduced dynamical generators $\widetilde{G}_{dyn}$ are given by

$$\widetilde{P}^- = -\frac{p^k p_k}{2p^+} \ , \qquad \widetilde{M}^{-m} = -\frac{\partial}{\partial p^+} p^m - \frac{\partial}{\partial p_m} \frac{p^k p_k}{2p^+} + \frac{1}{p^+}\left(S^{mk}p_k + H^m\right) \ , \tag{4.7}$$

where S^{mn} and H^m read

$$S^{mn} = a_{\alpha}^m \frac{\partial}{\partial a_n^{\alpha}} + a^m \frac{\partial}{\partial a_n} - (m \leftrightarrow n) \ , \qquad H_n = \mu \frac{\partial}{\partial a^n} \ . \tag{4.8}$$

The elements S^{kl} and H^n satisfy the $iso(d-2)$ commutation relations

$$[S^{kl}, S^{ps}] = \delta^{kp}S^{ls} + 3 \text{ terms} \ , \qquad [S^{kl}, H^n] = \delta^{kn}H^l - \delta^{ln}H^k \ , \qquad [H^k, H^l] = 0 \ . \tag{4.9}$$

Note that this oscillator realization of $iso(d-2)$ algebra is analogous to the realization of $iso(d-1,1)$ employed in [22] in describing mixed-symmetry helicity fields on Minkowski space.

Casimir operators. To characterize the light-cone representation with given weights we compute the Casimir operators of the $iso(d-2)$ algebra (4.9), cf. discussion in section 2.3. Using (4.8) we immediately see that the second order $iso(d-2)$ Casimir operator is given by

$$c_2 \equiv H^2 \approx \mu^2 \nu \ , \tag{4.10}$$

where we denoted $H^2 \equiv H_m H^m$ and used the modified trace constraint (4.4), cf. (2.12).

Using both the (modified) trace conditions (4.4) and Young and spin weight conditions (4.5) we explicitly compute the quartic Casimir operator

$$c_4 \equiv H^2 S^2 - 2(HS)^2 \approx \mu^2 \nu \sum_{\alpha=1}^{n-1} s_{\alpha}(s_{\alpha} + d - 2\alpha - 3) \ , \tag{4.11}$$

where $H^2 = H^m H_m$, $S^2 = S_{mn}S^{mn}$, $(HS)^m = H_n S^{nm}$. Higher order Casimir operators can be found analogously (see the general discussion in appendix A).

Note that evaluating Casimir operators (4.10) and (4.11) we do not need to solve the constraints (4.4)–(4.5) explicitly. The Poincare algebra generators projected onto the subspace singled out by the off-shell constraints are quite complicated. In particular, $iso(d-2)$ subalgebra (4.8) is now differently realized: S^{mn} still act as rotations, while translations H^m are non-trivially projected onto the subspace.[8]

4.1 Spin-s case

Let us analyze the continuous spin representation labeled by $(s, 0, ..., 0)$ in more detail. In this case there are two oscillators (a, a_1^m) and the trace constraints read

$$\left(\widetilde{T} + \nu\right)\phi = 0 , \qquad \widetilde{T}^1 \phi = 0 , \qquad \widetilde{T}^{11}\phi = 0 , \tag{4.12}$$

where

$$\phi = \sum_{p=0}^{\infty} \phi_{m_1...m_p | n_1...n_s} \, a^{m_1} \cdots a^{m_p} a_1^{n_1} \cdots a_1^{n_s} , \tag{4.13}$$

and the spin weight condition $\widetilde{N}_1 \phi = s\phi$ has been taken into account.

Let $Y(k, l)$ denote a traceless $o(d-2)$ tensor associated to the Young diagram with k indices in the first row and l indices in the second row. Then, the solution to (4.12) is given by

$$\phi : \qquad \bigoplus_{l=0}^{s} \bigoplus_{k=s}^{\infty} Y(k, l) . \tag{4.14}$$

It is obtained by tensoring two independent traceless rank-p and rank-s tensors in (4.13) and subtracting cross-traces.

When $s = 0$ the space (4.14) is an infinite chain of totally symmetric $o(d-2)$ traceless tensors [4, 7, 13]. In this case, (4.14) directly follows from the $o(d-2)$ version of proposition B.1. For $s \neq 0$ the space (4.14) is a light-cone version of the covariant formulation discussed in [9].

In particular, let us consider $d = 5$. Using the fact that $o(3)$ traceless tensors satisfy the Hodge duality relations $Y(k, 1) \sim Y(k, 0)$ and $Y(k, m) = 0$ at $m > 1$ we find out that in this case (4.14) is the representation space described in [19] (see also [16]), i.e. two infinite chains of traceless $o(3)$ tensors $Y(k, 0)$ with $k = s, s+1, ..., \infty$. The quartic Casimir operator (4.11) in five dimensions can be represented as $c_4 = W^2$, where $W = \epsilon_{klm}H^k S^{lm}$. Thus, we reproduce the result of [19] that $W = \pm\,\mu\,s$.

4.2 Mixed-symmetry case

Here we describe the space of light-cone continuous spin fields for arbitrary spins $(s_1, ..., s_n)$. Let $Y(l_1, ..., l_p)$ denote a traceless $o(d-2)$ tensor with indices described by Young diagram

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[8]In the spin-0 case the explicit realization can be found in [13] and in the spin-s case in $d = 5$ in [16].

with p rows of lengths l_j, $j = 1, ..., p$. Then, the solution to the algebraic constraints (4.4) and (4.5) is given by

$$\phi \; : \qquad \bigoplus_{k=s_1}^{\infty} \bigoplus_{l_\alpha \leqslant s_\alpha} Y(k, l_1, ..., l_{n-1}) \,. \tag{4.15}$$

In other words, the solution space is given by a finite collection of infinite chains of Young diagrams with the length of the first row running from s_1 to infinity. The chains differ from each other by the form of the lower part of diagrams which is defined by all admissible (consistent with the Young symmetry conditions) lengths $l_\alpha = 0, 1, ..., s_\alpha$.

5 Weyl and gauge modules

An important invariant information about a given linear gauge system is encoded in the space of gauge inequivalent formal solutions to the equations of motion, known as Weyl module, and the space of (higher-order) global reducibility parameters, known as a gauge module. These spaces are typically seen as modules over the space-time global symmetry algebra. In particular, if the gauge module vanishes the system is non-gauge, i.e. all the gauge symmetries are Stueckelberg-like. Note also, that if the gauge module vanishes and the space-time global symmetries (e.g. Poincare or AdS) act transitively, the system is entirely determined by the Weyl module structure. This property is manifest in the unfolded approach.[9]

We are now interested in the gauge and Weyl modules of the continuous spin system. To analyze formal solutions in this section we replace space-time coordinates x^a by formal coordinates y^a. In particular, in this section it is implicitly assumed that in all the expressions for fields, parameters, operators, etc. x^a is replaced with y^a. The gauge and Weyl modules can be defined as the cohomology $H^k(Q)$ of the BRST operator

$$Q = \left(D_i^\dagger + \mu_i \right) \frac{\partial}{\partial b_i} \,, \qquad \text{where} \qquad D_i^\dagger = a_a^i \frac{\partial}{\partial y^a} \,, \qquad i = 1, ..., n \,, \tag{5.1}$$

acting in the formal series singled out by the modified trace and Young symmetry and spin weight constraints (2.3)–(2.5). The Weyl module is the zero ghost number cohomology $H^0(Q)$, the gauge module is a collection of modules identified with negative ghost degree cohomology $H^k(Q)$ at $k < 0$ [22–24].

5.1 Scalar continuous spin fields

To begin with we consider the scalar continuous system ($n = 1$) taking as a functional class formal series in formal variable y^a and auxiliary variable a^b. To compute the cohomology we first reduce the problem to the subspace

$$(T + \nu)\phi = \Box\phi = D\phi = 0 \,. \tag{5.2}$$

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[9]For a review of the unfolded approach, see e.g. [18]. Note that the unfolded description for continuous spin fields was implicitly discussed in [48]. Within the present framework, more details on the gauge and Weyl modules can be found in [49, 50] and references therein.

The Weyl module is then the quotient of this space modulo the image of $D^\dagger + \mu$ (i.e. its cokernel), while the gauge module is the kernel of $D^\dagger + \mu$.

Let us first characterize the subspace (5.2) differently. We have

Proposition 5.1 *Let $\phi_0(y, a)$ be totally traceless, i.e.*

$$T\phi_0 = \Box\phi_0 = D\phi_0 = 0 \,. \tag{5.3}$$

Then, there exists a unique ϕ such that $\Pi\phi = \phi_0$, where ϕ satisfies (5.2), and Π denotes the projector to the totally traceless component. This gives an isomorphism between the space (5.2) and the space of totally traceless elements (5.3).

The proof makes use of the cohomology statements from [21]. The present proposition is analogous to the proposition B.1.

Using the above isomorphism the action of $D^\dagger + \mu$ can be written in terms of totally traceless ϕ_0 so that the problem is reformulated as that of computing kernel and cokernel of $D^\dagger + \mu$ in the space (5.3).

Proposition 5.2 *In the space of totally traceless formal series in y, a we have*

$$\ker\left(D^\dagger + \mu\right) = 0 \,, \qquad \mathrm{coker}\left(D^\dagger + \mu\right) = 0 \,. \tag{5.4}$$

The first part is trivial. The second is equivalent to the fact that any ϕ_0 can be represented as $(D^\dagger + \mu)\chi$. In the space of formal series $\chi = (D^\dagger + \mu)^{-1}\phi_0$, where the inverse gauge generator is the Neumann series

$$\frac{1}{D^\dagger + \mu} \;=\; \mu^{-1} - \mu^{-2}\, D^\dagger + \mu^{-3}\, D^\dagger D^\dagger - \mu^{-4}\, D^\dagger D^\dagger D^\dagger + \dots \,. \tag{5.5}$$

It follows from the above statement that formal power series in y^a and a^b is not a satisfactory functional class as the system is empty with such a choice. Although the gauge module $\ker(D^\dagger + \mu)$ remains trivial even if one restricts to a subspace of formal series, the Weyl module can be made nontrivial if in proposition 5.2 one restricts to polynomials in a with coefficients in formal series in y (i.e. ϕ_0 belongs to the natural class employed when studying the Fronsdal system).

This can be achieved if before reducing to totally traceless elements one restricts to the following class: formal series in y^a and a^b such that $\Pi\phi$ is a polynomial in a^b with coefficients in formal series in y^a. Indeed, with this choice proposition 5.1 remains correct as the space of solutions of (5.2) in this functional space is isomorphic to totally traceless elements which are polynomials in a^b with coefficients in formal series in y^a.

Getting back to the Weyl module it is straightforward to see that it becomes nontrivial. Indeed, the formal inverse (5.5) of $D^\dagger + \mu$ does not preserve polynomials in a^b as it contains an infinite series in a^b. On the other hand, any polynomial in y^a is in the image of $D^\dagger + \mu$ because the series terminates in this case.

Although it is not clear how to characterize the Weyl module explicitly let us give an example of a nontrivial element. Let k^a be a light-like constant vector, i.e. $k^a k_a = 0$. Consider the "formal plane wave"

$$\phi_0 = \exp\left(ik_a y^a\right) : \qquad T\phi_0 = \Box\phi_0 = D\phi_0 = 0 \,. \tag{5.6}$$

It is a polynomial in a (zero degree), while $(D^\dagger + \mu)^{-1}\phi_0$ is a formal series in a. We conclude that the element (5.6) does not belong to the functional class and, therefore, defines a nontrivial cohomology.

Finally, to arrive at the functional class introduced from the very beginning in section 2.1 one observes that one can equivalently require only the traceless (in a-oscillators) component of ϕ to be polynomial in a^b. Also, for the subspace to be closed with respect to all the $sp(2n+2)$ generators one should require all traces to be polynomial in a^b. As we shown in section 4 this is also necessary for the consistent light-cone reduction of the system. The above considerations motivate the functional class choice made in section 2.1.

5.2 Mixed-symmetry continuous spin fields

Let us now turn to the general case $n \geqslant 1$. To compute the cohomology $H(Q)$ we have the following generalization of the proposition 5.1.

Proposition 5.3 *Let $\phi_0(y, a)$ be totally traceless, i.e. $T^{ij}\phi_0 = \square\phi_0 = D^i\phi_0 = 0$. Then, there exists a unique ϕ such that*

$$\Pi\phi = \phi_0, \qquad (T+\nu)\phi = T^\alpha\phi = T^{\alpha\beta}\phi = \square\phi = D^i\phi = 0, \tag{5.7}$$

where Π denotes the projector to the totally traceless component. This gives an isomorphism between the space $(T+\nu)\phi = T^\alpha\phi = T^{\alpha\beta}\phi = \square\phi = D^i\phi = 0$ and the space of totally traceless elements.

The proof is analogous to that of proposition 5.1 and involves a cohomology statement from [22].

Using the isomorphism (5.7) one can reformulate the problem in terms of totally traceless elements. Let us explicitly consider the first nontrivial case $n = 2$, where only two oscillators a, a_1 are present. The traceless general element $\Phi = \phi + \psi b + \psi^1 b_1 + \chi b b_1$ satisfies BRST extended spin condition (2.9) and belongs to the functional class defined in section 2.1.

Gauge module. In degree -2 one gets the cocylce condition

$$\left(D^\dagger + \mu\right)\chi = 0, \qquad D_1^\dagger\chi = 0, \tag{5.8}$$

so that the cohomology is empty, cf. (5.5).

Now, in degree -1 the cocycle and coboundary conditions read as

$$\left(D^\dagger + \mu\right)\psi + D_1^\dagger\psi^1 = 0, \qquad \psi \sim \psi + D_1^\dagger\lambda, \quad \psi^1 \sim \psi^1 - \left(D^\dagger + \mu\right)\lambda, \tag{5.9}$$

where degree -2 parameter is $\lambda b b_1$. Suppose we are given with a nontrivial solution $\psi b + \psi^1 b_1$. It follows that

$$\psi = -\left(D^\dagger + \mu\right)^{-1} D_1^\dagger\psi^1, \tag{5.10}$$

and, hence, there exists ℓ such that $(D^\dagger)^\ell(D_1^\dagger\psi^1) = 0$ (otherwise ψ is an infinite series in a). This in turn implies that ψ_1 is polynomial in y. Being polynomial it can always be represented as $(D^\dagger+\mu)\kappa$ for some κ and, hence, one can assume $\psi^1 = 0$. It then follows that $(D^\dagger + \mu)\psi = 0$ and, hence, $\psi = 0$ as $\ker(D^\dagger + \mu) = 0$. We conclude that the cohomology is empty.

Weyl module. In degree 0 the cocycle condition is trivial, while the coboundary condition gives

$$\phi \sim \phi + \left(D^\dagger + \mu\right)\xi + D_1^\dagger \xi^1 \,, \tag{5.11}$$

where degree -1 parameter is $\xi b + \xi^1 b_1$. Had we taken as a functional class all formal series in a-oscillators, proposition 5.2 would have implied that the cohomology is empty. However, using the functional class introduced in section 2.1 the Weyl module is non-vanishing similarly to the scalar case. Indeed, the following analog of (5.6) gives an example of a nontrivial element of the Weyl module:

$$\phi_0 = \Pi\left(g^{s_1}(a_1)\exp(ik_a y^a)\right) \,, \tag{5.12}$$

where $g^{s_1}(a_1)$ is a degree s_1 polynomial in a_1 and Π denotes a projector to totally traceless component. Similar reasoning show that ϕ_0 is nontrivial in cohomology for $g^{s_1}(a_1)$ of general position.

The extension of the above statements to the general mixed-symmetry case is straightforward. We conclude that with the properly chosen functional class for spin oscillators, generic massless continuous spin fields are not genuine gauge fields. In particular, the gauge fields present in various formulations of continuous spin systems should be Stueckelberg fields. At the same time the Weyl module is nontrivial in agreement with the light-cone formulation of section 4.

6 Conclusion

In this paper we developed the BRST-based approach to continuous spin fields of arbitrary mixed-symmetry type in Minkowski space. Using the Howe duality between $o(d-1,1)$ Lorentz algebra and $sp(2n+2)$ symplectic algebra we formulated a set of Poincare invariant constraints underlying the continuous spin dynamics. The constraint set consists of both the algebraic and differential conditions and can be viewed as the μ-deformation of the helicity spin case, where μ is the continuous spin parameter.

Implementing differential constraints via the BRST operator and imposing algebraic constraints directly we arrive at the triplet formulation for continuous spin. The resulting equations of motion (3.8), (3.1) have a simple form even in the general mixed-symmetry case. Using the homological reductions of the triplet BRST operator we found the metric-like formulation (3.19) that generalizes the Schuster-Toro description of the scalar continuous spin fields. On the other hand, the resulting metric-like formulation is the μ-deformation of the Labastida equations.

Applying the so-called quartet mechanism we can get rid of the unphysical components of the oscillators to obtain the light-cone form of the continuous spin dynamics. In particular, we explicitly built the $iso(d-2)$ Wigner little algebra and computed its second and fourth Casimir operators.

Our formulation can be naturally extended in several ways. First of all, we completely left aside the Lagrangian formulation for the equations we considered in this paper. Formally, the triplet BRST operator (3.1) is not Hermitian with respect to the standard inner

product contrary to the helicity spin case. On the other hand, the Schuster-Toro equations are known to be variational [4, 7] and, hopefully, there is a modified inner product that would determine the action functional. Also, the continuous spin fields are known to consistently propagate on the (A)dS background [7–9, 11, 15, 48]. Thus, the AdS formulation for arbitrary mixed-symmetry continuous spin fields along the lines of [23, 24] seems to exist though a group-theoretical description of such fields is yet to be found. Furthermore, it would be interesting to extend the recent results on interacting scalar continuous fields [13, 14] and elaborate on cubic vertices for mixed-symmetry fields using either the metric-like formulation [51] or the frame-like formulation [52–54]. Finally, the approach and results of this work should extend smoothly to the case of fermionic continuous spin fields.

Acknowledgments

A   Casimir operators of the Poincare algebra

Let us consider the Casimir operators C_{2p}, where $p = 1, 2, \dots$. We introduce the generalized Pauli-Lubanski tensors

$$W_{m_1 \dots m_k} = \epsilon_{m_1 \dots m_k a_{k+1} \dots a_d} P^{a_{k+1}} M^{a_{k+2} a_{k+3}} \dots M^{a_{d-1} a_d} , \qquad (A.1)$$

where $\epsilon_{m_1 \dots m_d}$ is the $o(d-1, 1)$ Levi-Civita tensor, $k = 1, 3, \dots, d-3$ for even d and $k = 0, 2, \dots, d-3$ for odd d. The Pauli-Lubanski tensors covariantly transform under Lorentz subalgebra $o(d-1, 1)$ and satisfy $[P_a, W_{m_1 \dots m_k}] = 0$ that allows us to represent the Casimir operators as follows

$$C_{2p} = W_{m_1 \dots m_{p-1}} W^{m_1 \dots m_{p-1}} . \qquad (A.2)$$

Basis elements of the Wigner little algebras can be read off from the tensor $W_{m_1 \dots m_{d-3}}$, which is the direct counterpart of the original $d = 4$ Pauli-Lubanski vector. In the massive case we choose the standard momentum representative of the $P^2 = m^2$ condition as $P^m = (m, 0, \dots, 0)$. Then, $W_{m_1 \dots m_{d-3}}$ can be split into components $W_{i_1 \dots i_{d-3}} = m \epsilon_{i_1 \dots i_{d-1}} M^{i_{d-2} i_{d-1}}$ which are simply the dualized $o(d-1)$ basis elements. The massless case is more intricate. Here, we choose the standard momentum representative $P^m = (\varkappa, 0, \dots, 0, \varkappa)$. Then, $W_{m_1 \dots m_{d-3}}$ similarly splits into $o(d-2)$ rotations and $d-2$ translations that altogether form $iso(d-2)$ algebra.

For arbitrary representations the Casimir operators can be rather complicated, but in the massless case $P^2 = 0$ they are drastically simplified. First of all, we observe that all the Casimir operators are bilinear in momenta, $C_{2p} \sim F^{mn}(M) P_m P_n$, where F is a polynomial

of Lorentz generators. Moreover, one can show that in the massless case momenta P_m enter only via combination $M_{ab}P^b$. Denoting $\pi_a = M_{ab}P^b$ we find the general expression

$$C_{2p} \approx \left[a_{p,0} + a_{p,2}\, M^2 + ... + a_{p,2p-4}\, M^{2p-4}\right] \pi_a \pi^a \,, \tag{A.3}$$

where $\alpha_{p,i}$ are numerical coefficients, M^{2k} are order $2k$ polynomials in M_{ab}, the weak equality means that evaluating (A.2) we use the $P^2 = 0$ condition. For example, the quartic Casimir operator[10] is given by $C_4 \sim \pi_a \pi^a$. It is clear then that $C_2 = 0$ defines masslessness, C_4 yields the continuous spin value μ^2, while a number of independent spin weights equals to that of the Casimir operators, $C_6, C_8, ...$. In other words, a continuous spin representation is characterized by the parameter μ and (half-)integers $s_1, ..., s_r$, where $r \leqslant [\frac{d-2}{2}] - 1$. We see that contrary to the helicity spin case there are one less spin weights.

B   Trace decompositions

In what follows we solve the deformed trace conditions in the case of totally symmetric tensors. Let $F = F(a)$ be a generating series

$$F = \sum_{n=0}^{\infty} F_{m_1...m_n}\, a^{m_1} \cdots a^{m_n} \equiv \sum_{n=0}^{\infty} F_{(n)} \,, \tag{B.1}$$

where $NF_{(n)} = nF_{(n)}$. Let us consider single or double trace conditions

$$(T + \nu)F = 0 \,, \tag{B.2}$$

$$(T + \nu)^2 F = 0 \,. \tag{B.3}$$

Proposition B.1 *Solutions to* (B.2) *and* (B.3) *are respectively given by*

$$F = \sum_{n,m=0}^{\infty} \alpha_{m,n} \left(T^\dagger\right)^m f_{(n)} \,, \qquad T f_{(n)} = 0 \,, \qquad \alpha_{m,n} = \alpha_n \frac{4^{-m+1}\nu^{m-1}}{m!\left(\frac{d}{2} + n - 1\right)_{m-1}} \,, \tag{B.4}$$

and

$$F = \sum_{n,m=0}^{\infty} \beta_{m,n} \left(T^\dagger\right)^m f_{(n)} \,, \qquad T^2 f_{(n)} = 0 \,, \qquad \beta_{m,n} = \beta_n \frac{4^{-m+1}\nu^{m-1}}{m!\left(\frac{d}{2} + n\right)_{m-1}} \,, \tag{B.5}$$

where $\alpha_n = \alpha_n(n,d)$ *and* $\beta_n = \beta_n(n,d)$ *are arbitrary prefactors depending on rank and dimensionality, and* $(a)_m$ *is the Pochhammer symbol. Up to prefactors the coefficients are related as* $\alpha_{m,n} = \beta_{m,n+1}$.

To prove the proposition we reformulate the deformed trace conditions as recursive equations on the coefficients $\alpha_{m,n}$ and $\beta_{m,n}$. The general solution is parameterized by n-dependent constants. Let us note that the solution exists when e.g. $\alpha_n = 0$ at $\forall n \neq n_0$.

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[10]An explicit expression reads $C_4 = P^2 M^2 - 2\pi_a \pi^a \approx -2\pi_a \pi^a$, see e.g. ref. [20].

C Reproducing the Schuster-Toro equations

Let us consider first the deformed gauge transformation (3.22). In the Fronsdal basis (B.4), (B.5) n-th rank Fronsdal field is transformed as

$$\delta\varphi_{(n)} = D^\dagger \epsilon_{(n-1)} + \rho_n\, \epsilon_{(n)} + \gamma_n\, T^\dagger \epsilon_{(n-2)} \,, \tag{C.1}$$

where the coefficients are proportional to the continuous spin parameter μ. To find them we have to solve the system of recurrent equations coming from representing both sides of (3.22) in the Fronsdal basis. Fixing the number $\#a_i^\mu = p$ we find that

$$\beta_{0,p}\delta\varphi_{(p)} = \sum_{k=0}^{\frac{p-1}{2}} \alpha_{k,p-2k-1} \left(T^\dagger\right)^k \epsilon_{(p-2k-1)} + \mu \sum_{k=0}^{\frac{p}{2}} \alpha_{k,p-2k} \left(T^\dagger\right)^k D^\dagger \epsilon_{(p-2k)} - \\ - \sum_{k=0}^{\frac{p-1}{2}} \alpha_{k,p-2k} \left(T^\dagger\right)^k D^\dagger \delta\varphi_{(p-2k)} \,. \tag{C.2}$$

Substituting (C.1) into (C.2) and using the property $\alpha_{n,s} = \beta_{n,s+1}$ (see proposition B.1) we find that in the p-th order

$$\rho_p = \frac{\alpha_{0,p}}{\beta_{0,p}} \,, \qquad \gamma_p = \frac{\alpha_{1,p-2} - \beta_{1,p-2}\rho_{p-2}}{\beta_{0,p}} \,, \tag{C.3}$$

while in the lower orders we find the system

$$\alpha_{k,p-2k} - \beta_{k,p-2k}\rho_{p-2k} - \beta_{k-1,p-2k+2}\gamma_{p-2k+2} = 0 \,, \qquad k = 1, 2, ..., \frac{p}{2} \,. \tag{C.4}$$

Fixing normalization constants in (B.5) as $\beta_n = \frac{d}{d+2n-2}$ we find that

$$\rho_n = \mu \,, \qquad \gamma_n = -\frac{\mu\nu}{(d+2n-4)(d+2n-6)} \,, \tag{C.5}$$

cf. (3.30), and that the equation system (C.4) is identically satisfied.

The equations of motion (3.21) can be analyzed along the same lines. Substituting the Fronsdal decomposition (3.24) into (3.21) we find that the equations take the form

$$\sum_{s,n=0}^{\infty} \beta_{n,s} \left(T^\dagger\right)^n F_{(s)} + \sum_{s,n=0}^{\infty} \zeta_{n,s} \left(T^\dagger\right)^n D\varphi_{(s)} + \sum_{s,n=0}^{\infty} \gamma_{n,s} \left(T^\dagger\right)^n D^\dagger \varphi_{(s)} + \\ + \sum_{s,n=0}^{\infty} \rho_{n,s} \left(T^\dagger\right)^n D^\dagger T \varphi_{(s)} + \sum_{s,n=0}^{\infty} \tau_{n,s} \left(T^\dagger\right)^n \varphi_{(s)} + \sum_{s,n=0}^{\infty} \varkappa_{n,s} \left(T^\dagger\right)^n T \varphi_{(s)} = 0 \,, \tag{C.6}$$

where

$$F_{(n)} = \left[\Box - D^\dagger D + \frac{1}{2} D^\dagger D^\dagger T\right] \varphi_{(n)} \,, \tag{C.7}$$

is the standard Fronsdal kinetic operator, the coefficients are given by

$$\zeta_{n,s} = -\mu\beta_{n,s} \,, \qquad \gamma_{n,s} = 2\mu(n+1)\beta_{n+1,s} \,, \qquad \rho_{n,s} = \mu\beta_{n,s} \,, \\ \tau_{n,s} = 2\mu^2(n+1)\beta_{n+1,s} \,, \qquad \varkappa_{n,s} = \frac{1}{2}\mu^2\beta_{n,s} \,. \tag{C.8}$$

We want to represent (C.6) as

$$\sum_{n,m=0}^{\infty} \beta_{m,n} \left(T^\dagger\right)^m \left[F_{(n)} + E_{(n)}\right] = 0 \, , \qquad (C.9)$$

so that the equations for the n-th rank contribution take the form $F_{(n)} + E_{(n)} = 0$, where the μ-correction is double traceless, $T^2 E_{(n)} = 0$. The form of (C.6) suggests the following general expression

$$E_{(p)} = a_p D\varphi_{(p+1)} + b_p D^\dagger T \varphi_{(p+1)} + c_p D^\dagger \varphi_{(p-1)} + d_p D^\dagger T^\dagger T \varphi_{(p-1)} + e_p T^\dagger D\varphi_{(p-1)} +$$
$$+ \, \bar{a}_p \varphi_{(p)} + \bar{b}_p T^\dagger \varphi_{(p-2)} + \bar{c}_p T \varphi_{(p+2)} + \bar{d}_p T^\dagger T \varphi_{(p)} + \bar{e}_p T^\dagger T^\dagger \varphi_{(p-4)} \, ,$$
$$(C.10)$$

where coefficients are not independent and related by the double trace condition imposed both on $\varphi_{(k)}$ and $E_{(l)}$. Now, let us introduce particular traceless combinations, cf. (3.28),

$$A_{(n)} = D\varphi_{(n+1)} - \frac{1}{2} D^\dagger T \varphi_{(n+1)} \, , \qquad B_{(n)} = \varphi_{(n)} + y_n T^\dagger T \varphi_{(n)} + z_n T \varphi_{(n+2)} \, , \quad (C.11)$$

where the coefficients are fixed by the zero trace condition as $y_n = -(2d + 2n - 8)^{-1}$ and $\forall z_n$. Then, we can represent (C.10) as

$$E_{(p)} = \widetilde{\varkappa}_p A_{(p)} + \widetilde{\tau}_p T^\dagger A_{(p-2)} + \widetilde{\rho}_p D^\dagger B_{(p-1)} + \bar{\rho}_p B_{(p)} + \bar{\varkappa}_p T^\dagger B_{(p-2)} \, , \qquad (C.12)$$

while the Fronsdal operator takes the form $F_{(n)} = \Box \varphi_{(p)} - D^\dagger A_{(p-1)}$. The coefficients $\widetilde{\varkappa}_p, \widetilde{\tau}_p, \widetilde{\rho}_p, \bar{\rho}_p, \bar{\varkappa}_p$ and z_n are expressed via a_p, b_p, c_p, d_p, e_p and $\bar{a}_p, \bar{b}_p, \bar{c}_p, \bar{d}_p, \bar{e}_p$ and y_n. Substituting (C.12) into (C.9) and performing the analysis similar to what we did for the gauge transformations we find that coefficients in (C.12) are fixed as follows

$$\widetilde{\varkappa}_p = -\mu \, , \qquad \widetilde{\tau}_p = \frac{\mu\nu}{(d+2p-4)(d+2p-6)} \, , \qquad \widetilde{\rho}_p = \frac{\mu\nu}{d+2p-4} \, ,$$
$$\bar{\rho}_p = \frac{\mu^2\nu}{d+2p-2} \, , \qquad \bar{\varkappa}_p = \frac{\mu^2\nu^2}{(d+2p-4)(d+2p-6)^2} \, , \qquad z_p = \frac{d+2p-2}{2\nu} \, .$$
$$(C.13)$$

Reorganizing the equation (C.12) and the Fronsdal operator in terms of $G_{(p)}$ (3.27) we finally arrive at the Schuster-Toro equations (3.26).

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