

Static BPS black holes in AdS_4 with general dyonic charges

Nick Halmagyi

*Sorbonne Universités, UPMC Paris 06, UMR 7589, LPTHE,
75005, Paris, France
CNRS, UMR 7589, LPTHE,
75005, Paris, France*

E-mail: halmagyi@lpthe.jussieu.fr

ABSTRACT: We complete the study of static BPS, asymptotically AdS_4 black holes within $\mathcal{N} = 2$ FI-gauged supergravity and where the scalar manifold is a symmetric very special Kähler manifold. We find the analytic form for the general solution to the BPS equations, the horizon appears as a double root of a particular quartic polynomial whereas in previous work this quartic polynomial further factored into a pair of double roots. A new and distinguishing feature of our solutions is that the phase of the supersymmetry parameter varies throughout the black hole. The general solution has $2n_v$ independent parameters; there are two algebraic constraints on $2n_v + 2$ charges, matching our previous analysis on BPS solutions of the form $\text{AdS}_2 \times \Sigma_g$. As a consequence we have proved that every BPS geometry of this form can arise as the horizon geometry of a BPS AdS_4 black hole. When specialized to the STU-model our solutions uplift to M-theory and describe a stack of M2-branes wrapped on a Riemann surface in a Calabi-Yau fivefold with internal angular momentum.

KEYWORDS: Gauge-gravity correspondence, Black Holes in String Theory, AdS-CFT Correspondence, Supergravity Models

ARXIV EPRINT: [1408.2831](https://arxiv.org/abs/1408.2831)

Contents

1	Introduction	1
2	The equations and the Ansatz	2
3	The solution	5
3.1	The general solution	5
3.2	The limit $I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q}) = I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q}) = 0$	8
4	Conclusions	8
A	Special geometry background	10
A.1	Generalities	10
A.2	The quartic invariant	11
A.3	Identities using the quartic invariant	12

1 Introduction

The study of BPS black holes in flat space has provided important insights into the nature of quantum gravity [1] and it is natural to search for a similar understanding of BPS black holes in asymptotically AdS space. Indeed there is hope that one might ultimately be able to use the holographic duality of the background AdS space [2–4] to understand the dynamics of black holes and these holographic dualities are certainly under the best control for supersymmetric theories.

Whereas the complete solution for asymptotically flat BPS black holes in four dimensional $\mathcal{N} = 2$ ungauged supergravity was found some time ago [5, 6] it is only somewhat recently that there has been progress in charting out the solution space of asymptotically AdS₄ static BPS black holes in gauged $\mathcal{N} = 2$ supergravity. In this work we complete the study of the solution space of static BPS black holes in $\mathcal{N} = 2$ FI-gauged supergravity.¹ To be more precise we require that the vector-multiplet scalar manifold \mathcal{M}_v is homogenous and very special Kähler or in other words that it is a coset and that there is a symplectic duality frame where the prepotential is cubic.

A notable development in this field was the solution of Cacciatori-Klemm [7] who studied a restricted set of background charges in the STU-model and found a class of solutions depending on four charges with one constraint, leaving a three-dimensional solution space.² This constraint is a BPS refinement of the Dirac quantization condition due to the charged

¹FI-gauged supergravity refers to the Fayet-Iliopoulos gauging, namely where the $U(1)_R \subset SU(2)_R$ is gauged.

²This solution was further studied in [8, 9].

gravitini and is of course absent in ungauged supergravity and the study of asymptotically flat black holes.

We have recently considered [10] these general $\mathcal{N} = 2$ FI-gauged supergravity theories and exactly solved the BPS conditions for geometries of the form $\text{AdS}_2 \times \Sigma_g$ where $\Sigma_g \in \{S^2, \mathbb{R}^2, \mathbb{H}^2\}$ is (a covering space of) a Riemann surface of genus g . We found that the entropy of the solution is related to the famous quartic invariant of special geometry and indeed this invariant will prove to be indispensable in our current analysis as well. An interesting result from the analysis of horizon geometries was that the solution space is³ $2n_v$ -dimensional; there are $n_v + 1$ magnetic charges p^Λ , along with $n_v + 1$ electric charges q_Λ and there are two constraints from the BPS conditions. One constraint is the Dirac quantization condition mentioned earlier but in addition there is another constraint given by (3.25) below. Consequently it was conjectured that the solution space of BPS black holes is also $2n_v$ -dimensional. In this work we prove this conjecture by constructing these solutions explicitly.

In previous work we have generalized the solution of [7] in two ways; for theories with a symmetric very special Kähler scalar manifold we presented axion-free solutions which depend on n_v independent charges and give the CK solution when restricted to the STU model. In addition, in [11] we showed how to use unbroken duality symmetries to generate two new charges in the STU-model. For the STU-model in particular, this gives a five-dimensional solution space whereas the analysis of [10] predicts that there should be a six-dimensional solution space. Recently it was shown [12] that this family of solutions with non-trivial axions can be generalized from the STU-model to theories where \mathcal{M}_v is a symmetric very special Kähler manifold but still, the dimension of the solution space was $2n_v - 1$, one less than conjectured in [10]. The final dimension of the solution space indeed exists as we constructively prove in the current work.

This paper is organized as follows. In section 2 we review the BPS equations and outline our ansatz. In section 3 we present our solution to the BPS equations. In appendix A we summarize various details of special geometry which are needed for the current work.

2 The equations and the Ansatz

The standard ansatz for static AdS_4 black holes is

$$ds_4^2 = -e^{2U} dt^2 + e^{-2U} dr^2 + e^{2(V-U)} d\Sigma_g^2 \tag{2.1}$$

where $d\Sigma_g^2$ is the uniform metric on (S^2, T^2, \mathbb{H}^2) with corresponding curvature $\kappa = (1, 0, -1)$. The scalar fields are radially dependent $z^i = z^i(r)$ and the gauge fields just contribute through the conserved charges

$$p^\Lambda = \frac{1}{4\pi} \int_{\Sigma_g} F^\Lambda, \quad q_\Lambda = \frac{1}{4\pi} \int_{\Sigma_g} G_\Lambda \tag{2.2}$$

where the dual field strength is given by

$$G_\Lambda = \mathcal{R}_{\Lambda\Sigma} F^\Sigma - \mathcal{I}_{\Lambda\Sigma} *_4 F^\Sigma. \tag{2.3}$$

³ n_v is the number of vector multiplets.

The charges and gauging parameters⁴ are naturally assembled into symplectic vectors

$$\mathcal{Q} = \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix}, \quad \mathcal{G} = \begin{pmatrix} g^\Lambda \\ g_\Lambda \end{pmatrix}. \quad (2.4)$$

A static BPS AdS₄ black hole amounts to a domain wall interpolating between AdS₄ in the UV to AdS₂ × Σ_g in the IR, the effective cosmological constant varies along this flow due to the non-trivial profiles for scalar fields which contribute to the cosmological constant through the scalar potential.

For a particular \mathcal{G} and \mathcal{Q} in the STU-model,⁵ in [7] the equations for $\frac{1}{4}$ -BPS black holes were found to simplify considerably and the exact solution was found. These solutions can be lifted in M-theory and interpreted as wrapped M2-branes along the lines of the work of Maldacena-Nunez [13]. Indeed restricted classes of such wrapped M2-branes had already been considered numerically in [14] but the solution of [7] was the first exact supergravity solution⁶ in any dimension of a stack of wrapped branes which has an AdS factor in the IR.

In four dimensions one has available the tools of special geometry and in [8] a duality covariant form of the BPS equations was derived:⁷

$$2e^{2V} \partial_r \left[\text{Im}(e^{-i\psi} e^{-U} \mathcal{V}) \right] = 8e^{2(V-U)} \text{Re}(e^{-i\psi} \mathcal{L}) \text{Re}(e^{-i\psi} \mathcal{V}) - \mathcal{Q} - e^{2(V-U)} \Omega \mathcal{M} \mathcal{G} \quad (2.6)$$

$$\partial_r(e^V) = 2e^{V-U} \text{Im}(e^{-i\psi} \mathcal{L}) \quad (2.7)$$

$$\psi' + A_r = -e^{U-2V} \text{Re}(e^{-i\psi} \mathcal{Z}) - e^{-U} \text{Im}(e^{-i\psi} \mathcal{L}) \quad (2.8)$$

where ψ is the phase of the supersymmetry parameter and we have defined the standard symplectic scalars

$$\mathcal{L} = \langle \mathcal{G}, \mathcal{V} \rangle, \quad \mathcal{Z} = \langle \mathcal{Q}, \mathcal{V} \rangle. \quad (2.9)$$

In addition one must impose the Dirac quantization condition specialized for these BPS equations:

$$\langle \mathcal{G}, \mathcal{Q} \rangle = -\kappa. \quad (2.10)$$

The equation (2.6) is a real symplectic vector with $2n_v + 2$ components which can be extracted by contracting (2.6) with $\{\mathcal{V}, \bar{\mathcal{V}}, D_i \mathcal{V}, D_{\bar{i}} \bar{\mathcal{V}}\}$. In this way, one obtains differential equations for the phase ψ , the metric mode e^U and the n_v complex scalars z^i . Comparing the resulting equation for ψ with (2.8) one obtains the algebraic constraint

$$\text{Re}(e^{-i\psi} \mathcal{L}) = e^{2(V-U)} \text{Im}(e^{-i\psi} \mathcal{Z}) \quad (2.11)$$

⁴The gravitini are charged with respect to the U(1) gauge fields A_μ^Λ by \mathcal{G} , it is always possible to find a symplectic frame where these couplings are purely electric and so the gravitini are just minimally coupled to the A_μ^Λ .

⁵In the symplectic frame where the gauging parameters are purely electric, the charges studied in [7] were purely magnetic.

⁶We are excluding the elementary solutions with constant scalar fields which for four dimensional black holes were considered in [15].

⁷We use the standard notation for the sections

$$\mathcal{V} = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix} = e^{K/2} \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix}. \quad (2.5)$$

which can then be enforced at the expense of (2.8). To be clear, an equivalent complete set of BPS conditions is (2.6), (2.7), (2.10), (2.11) and this is the set we will solve in the next section. It may be interesting to note that the constraint (2.11) is a generalization of the constraint $0 = \text{Im}(e^{-i\psi}\mathcal{Z})$ found for single-center half-BPS black holes in ungauged supergravity [16].

Using these equations, the analytic solution of [7] was generalized to theories where \mathcal{M}_v is a symmetric very special Kähler manifold in [17]. In these examples, the axions vanish identically and the complex symplectic sections become either real or imaginary leading to a sharp simplification of the equations (2.6)–(2.7) and in these example both sides of (2.11) vanish identically. The solutions of [17] are parameterized by n_v charges out of the $2n_v + 2$ components of \mathcal{Q} and in [11] it was observed that one can use unbroken duality symmetries to generate axions.

The BPS equation (2.6) was recently refined in [12] using the quartic invariant I_4 and its derivative,⁸ which further simplifies the analysis of the models with general gauging parameters \mathcal{G} and charges \mathcal{Q} . Specifically, using the identity (A.19) one immediately finds that (2.6), (2.7) and (2.11) become

$$0 = 2e^V \partial_r (\text{Im}\tilde{\mathcal{V}}) - I'_4(\text{Im}\tilde{\mathcal{V}}, \text{Im}\tilde{\mathcal{V}}, \mathcal{G}) + \mathcal{Q} \tag{2.12}$$

$$(e^V)' = 2\langle \mathcal{G}, \text{Im}\tilde{\mathcal{V}} \rangle \tag{2.13}$$

$$\langle \mathcal{G}, \text{Re}\tilde{\mathcal{V}} \rangle = e^{2(V-U)} \langle \mathcal{Q}, \text{Im}\tilde{\mathcal{V}} \rangle \tag{2.14}$$

where

$$\tilde{\mathcal{V}} = e^{V-U} e^{-i\psi} \mathcal{V}. \tag{2.15}$$

The key step in arriving at (2.12) is to remove the pesky $\Omega\mathcal{M}\mathcal{G}$ and $\text{Re}\mathcal{V}$ terms in (2.6) leaving an equation purely in terms of $\text{Im}\tilde{\mathcal{V}}$ and e^V . The main result of the current work is to solve (2.12)–(2.14) in complete generality subject to the boundary conditions of AdS_4 in the UV and $\text{AdS}_2 \times \Sigma_g$ in the IR.

The key insight of Cacciatori and Klemm in [7] was the ansatz for e^V :

$$e^V|_{\text{CK}} = \frac{r^2}{R} - v_0, \tag{2.16}$$

where R is the radius of AdS_4 and $v_0 > 0$. In (2.16) e^V is completely fixed by the UV and IR boundary conditions whereas in principle e^V could be a power series in $\frac{1}{r}$ and indeed that is what we will find.⁹ In this work we generalize (2.16) to the following ansatz¹⁰

$$e^V = r\sqrt{v_4 r^2 + v_3 r + v_2}. \tag{2.17}$$

Clearly e^{2V} has a double root at $r = 0$, a familiar feature of charged extremal black holes namely that the two horizons coincide, the remaining two roots however need not coincide.

⁸To be precise this rewriting is only valid when \mathcal{M}_v is a symmetric very special Kähler manifold, we have included substantial background about the quartic invariant I_4 and its derivative I'_4 in appendix A.2.

⁹This contradicts the analysis in [12] where such solutions were excluded.

¹⁰We have shifted the radial co-ordinate to move the horizon to $r = 0$.

The ansatz for $\text{Im}\tilde{\mathcal{V}}$ requires further inspiration, we find that the following is ultimately justified by our success:

$$\text{Im}\tilde{\mathcal{V}} = e^{-V} \left[A_1 r + A_2 r^2 + A_3 r^3 \right], \quad (2.18)$$

where A_i are the symplectic vectors which will be explicitly solved for.

3 The solution

3.1 The general solution

When evaluated on the ansatz (2.17) and (2.18) and expanded order by order, the BPS equation (2.12) yields

$$I'_4(A_3, A_3, \mathcal{G}) = 2\langle \mathcal{G}, A_3 \rangle A_3 \quad (3.1)$$

$$I'_4(A_2, A_3, \mathcal{G}) = 2\langle \mathcal{G}, A_2 \rangle A_3 \quad (3.2)$$

$$2I'_4(A_1, A_3, \mathcal{G}) + I'_4(A_2, A_2, \mathcal{G}) = \langle \mathcal{G}, A_3 \rangle (\mathcal{Q} - 2A_1) + 8\langle \mathcal{G}, A_1 \rangle A_3 + \frac{4}{3}\langle \mathcal{G}, A_2 \rangle A_2 \quad (3.3)$$

$$3I'_4(A_1, A_2, \mathcal{G}) = 6\langle \mathcal{G}, A_1 \rangle A_2 + 2\langle \mathcal{G}, A_2 \rangle (\mathcal{Q} - A_1) \quad (3.4)$$

$$I'_4(A_1, A_1, \mathcal{G}) = 2\langle \mathcal{G}, A_1 \rangle \mathcal{Q} \quad (3.5)$$

where we have used the solution to (2.13)

$$v_{i+1} = \frac{4}{i+1} \langle \mathcal{G}, A_i \rangle. \quad (3.6)$$

The constraint (2.14) expanded order by order in r gives

$$\langle A_3, A_2 \rangle = 0 \quad (3.7)$$

$$2\langle A_1, A_3 \rangle = \langle \mathcal{Q}, A_3 \rangle \quad (3.8)$$

$$\langle A_1, A_2 \rangle = \langle \mathcal{Q}, A_2 \rangle \quad (3.9)$$

$$0 = \langle \mathcal{Q}, A_1 \rangle. \quad (3.10)$$

It should be noted that this whole system of equations is vastly overdetermined and for a solution to exist the system must be highly redundant. One starts solving this system by first re-analyzing the UV (3.1) and IR (3.5), (3.10) but the intermediate equations involve A_2 in nontrivial ways.

The UV boundary conditions for AdS_4 provide A_3 as the solution to (3.1) and one finds

$$A_3 = \frac{6I'_4(\mathcal{G})}{\sqrt{I_4(\mathcal{G})}}, \quad v_4 = \frac{1}{R_{\text{AdS}_4}^2} = \sqrt{I_4(\mathcal{G})} \quad (3.11)$$

where we have appealed to the analysis of [17] to fix the overall normalization.

The BPS equations are non-linear in terms of the A_i and are somewhat challenging to solve directly. Making use of the various relations contained in appendix A.2 we have found the following explicit solution:

$$A_1 = a_1 I'_4(\mathcal{G}) + a_2 I'_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}) + a_3 I'_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}) + a_4 I'_4(\mathcal{Q}), \quad (3.12)$$

where the constants a_j are given by

$$a_1 = \frac{I_4(\mathcal{Q})I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})^2 - I_4(\mathcal{G})I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})^2}{I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})^2(\kappa I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q}) + 2\langle I'_4(\mathcal{G}), I'_4(\mathcal{Q}) \rangle)} \quad (3.13)$$

$$a_2 = -\frac{1}{12} \frac{I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})}{\kappa I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q}) + 2\langle I'_4(\mathcal{G}), I'_4(\mathcal{Q}) \rangle} \quad (3.14)$$

$$a_3 = \frac{I_4(\mathcal{G})I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})^2 - I_4(\mathcal{Q})I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})^2}{2I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})(\kappa I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q}) + 2\langle I'_4(\mathcal{G}), I'_4(\mathcal{Q}) \rangle)} \quad (3.15)$$

$$a_4 = \frac{1}{6} \frac{I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})}{\kappa I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q}) + 2\langle I'_4(\mathcal{G}), I'_4(\mathcal{Q}) \rangle} \quad (3.16)$$

Our solution for A_2 is somewhat more complicated, the ansatz is the same as for A_1

$$A_2 = b_1 I'_4(\mathcal{G}) + b_2 I'_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}) + b_3 I'_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}) + b_4 I'_4(\mathcal{Q}) \quad (3.17)$$

and we find that the constants b_j are given by

$$b_1 = \frac{2b_3}{3} \left[\frac{I_4(\mathcal{Q})I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})}{I_4(\mathcal{G})I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})} - \frac{2I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})}{I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})} + \frac{\kappa I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})^2}{18 I_4(\mathcal{G})\Pi_3} \right] \quad (3.18)$$

$$b_2 = -\frac{b_3}{6} \frac{I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})}{I_4(\mathcal{G})} \frac{\Pi_1}{\Pi_3} \quad (3.19)$$

$$b_3^2 = -\frac{9}{16} I_4(\mathcal{G})^{3/2} I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q}) I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q}) \Pi_3^2 \cdot [2\kappa I_4(\mathcal{Q})I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})^2 + I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})\Pi_2]^{-1} \\ [-\kappa I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})^2 I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})\Pi_1 + 18I_4(\mathcal{G})\Pi_3^2]^{-1} \quad (3.20)$$

$$b_4 = -\frac{2b_3\kappa}{3} \frac{I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})}{\Pi_3} \quad (3.21)$$

where we have defined

$$\begin{aligned} \Pi_1 &= 2I_4(\mathcal{Q})I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})\kappa + I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})\langle I'_4(\mathcal{G}), I'_4(\mathcal{Q}) \rangle \\ \Pi_2 &= 2I_4(\mathcal{Q})I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})\kappa + I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})\langle I'_4(\mathcal{G}), I'_4(\mathcal{Q}) \rangle \\ \Pi_3 &= 4I_4(\mathcal{Q})I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})\kappa + I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})\langle I'_4(\mathcal{G}), I'_4(\mathcal{Q}) \rangle. \end{aligned} \quad (3.22)$$

Note that the BPS equations have a symmetry $A_2 \rightarrow -A_2$ and as such the overall sign of b_3 in our expressions is undetermined by the BPS conditions and must be fixed by demanding regularity of the solution. A priori one might have expected that the ansatz for A_i involved terms proportional to the symplectic vectors \mathcal{G} and \mathcal{Q} , we have considered such terms and found their coefficients to vanish.

With these expressions for A_i we get the following result for the expansion of e^V :

$$v_2 = -\frac{I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})^3 I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})^3 + 36 [I_4(\mathcal{Q})I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})^2 - I_4(\mathcal{G})I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})^2]^2}{9I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})^2 I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})^2 (\kappa I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q}) + 2\langle I'_4(\mathcal{G}), I'_4(\mathcal{Q}) \rangle)} \quad (3.23)$$

$$v_3 = b_3 \left[3I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q}) - \frac{I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})^2}{6I_4(\mathcal{G})} - \frac{24I_4(\mathcal{G})I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})}{I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})} \right. \\ \left. + \frac{\kappa I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})^2 I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q}) I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})}{12I_4(\mathcal{G})\Pi_3} \right]. \quad (3.24)$$

In addition, the constraint (2.14) in the IR (3.10) gives the constraint found in [10]:

$$0 = 4I_4(\mathcal{G})I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})^2 + 4I_4(\mathcal{Q})I_4(\mathcal{Q}, \mathcal{G}, \mathcal{G}, \mathcal{G})^2 - I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q})I_4(\mathcal{Q}, \mathcal{G}, \mathcal{G}, \mathcal{G}). \quad (3.25)$$

Since this constraint is independent of the radius it must be imposed on the whole black hole. We find that with this constraint enforced, the above solution solves each of (3.7)–(3.10). The expressions above for $\{a_i, b_j, v_k\}$ have been evaluated subject to (3.25) although in principle one could have used (3.25) to eliminate $I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q})$. We have used what we find to be a more concise presentation.

We emphasize that solving for $\text{Im}\tilde{\mathcal{V}}$ constitutes a complete solution of the problem, from which the metric functions (e^U, e^V) , the scalar fields z^i and the phase of the supersymmetry parameter ψ can all be obtained as follows. Using the identity (which follows from (A.19) evaluated with $A \rightarrow \text{Im}\tilde{\mathcal{V}}$):

$$I'_4(\text{Im}\tilde{\mathcal{V}}, \text{Im}\tilde{\mathcal{V}}, \text{Im}\tilde{\mathcal{V}}) = 4\text{Im}[\langle \text{Im}\tilde{\mathcal{V}}, \tilde{\mathcal{V}} \rangle] \text{Im}\tilde{\mathcal{V}} + 8\text{Re}[\langle \text{Im}\tilde{\mathcal{V}}, \tilde{\mathcal{V}} \rangle] \text{Re}\tilde{\mathcal{V}} - e^{2(V-U)}\Omega\mathcal{M}\text{Im}\tilde{\mathcal{V}} \quad (3.26)$$

we find

$$\text{Re}\tilde{\mathcal{V}} = -\frac{I'_4(\text{Im}\tilde{\mathcal{V}})}{2\sqrt{I_4(\text{Im}\tilde{\mathcal{V}})}}. \quad (3.27)$$

Having the expression for the complex sections $\tilde{\mathcal{V}}$ it is straightforward to read off the complex scalar fields:

$$z^i = \frac{\tilde{\mathcal{L}}^i}{\tilde{\mathcal{L}}^0}. \quad (3.28)$$

Contracting (3.27) with $\text{Im}\tilde{\mathcal{V}}$ we get the remaining metric function from

$$e^{2(V-U)} = 4\sqrt{I_4(\text{Im}\tilde{\mathcal{V}})}. \quad (3.29)$$

The phase of the supersymmetry parameter ψ can be extracted from

$$e^{2i\psi} = \frac{\tilde{\mathcal{L}}^0}{\tilde{\mathcal{L}}^0} \quad (3.30)$$

and is generically non-constant as opposed to all previous solutions in [7, 11, 12, 17] where the supersymmetry phase is constant. Note that the numerical solution presented in [18] for gauged supergravity with hypermultiplets also has a non-constant phase for the spinor.

While the explicit solution (3.13)–(3.16) and (3.18)–(3.21) is satisfying in the sense that it is exhaustive within the class of theories considered, its explicit form is not exactly edifying. The crucial step in finding these solutions was starting from the metric ansatz in (2.17), which for various reasons this was incorrectly excluded in [12]. It seems plausible that using the tools of special geometry one could find a more concise representation of this solution, which may possibly be required for this solution to have applications to generalizations discussed in the introduction.

3.2 The limit $I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q}) = I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q}) = 0$

To observe the limit $I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q}) = I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q}) = 0$, it is useful to note that the constraint (3.25) implies

$$\frac{I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})}{I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})} = \frac{I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q}) \pm \sqrt{I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q})^2 - 64I_4(\mathcal{G})I_4(\mathcal{Q})}}{8I_4(\mathcal{G})} \quad (3.31)$$

which is finite in this limit. We find the following explicit expressions

$$a_1 = \frac{a_3(\sqrt{I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q})^2 - 64I_4(\mathcal{Q})I_4(\mathcal{G})} - I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q}))}{24I_4(\mathcal{G})} \quad (3.32)$$

$$a_2 = 0 \quad (3.33)$$

$$a_3 = -\frac{\sqrt{I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q})^2 - 64I_4(\mathcal{Q})I_4(\mathcal{G})}}{8(\kappa I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q}) + 2\langle I_4(\mathcal{G}), I_4(\mathcal{Q}) \rangle)} \quad (3.34)$$

$$a_4 = 0 \quad (3.35)$$

as well as

$$b_1 = \frac{a_3(3\sqrt{I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q})^2 - 64I_4(\mathcal{Q})I_4(\mathcal{G})} - I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q}))}{24I_4(\mathcal{G})} \quad (3.36)$$

$$b_2 = 0 \quad (3.37)$$

$$b_3 = \frac{I_4(\mathcal{G})^{1/4}}{4\sqrt{-\kappa I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q}) - 2\langle I_4(\mathcal{G}), I_4(\mathcal{Q}) \rangle}} \quad (3.38)$$

$$b_4 = 0 \quad (3.39)$$

and

$$v_2 = \frac{64I_4(\mathcal{G})I_4(\mathcal{Q}) - I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q})^2}{4(\kappa I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q}) + 2\langle I_4(\mathcal{G}), I_4(\mathcal{Q}) \rangle)} \quad (3.40)$$

$$v_3 = \frac{3I_4(\mathcal{G})^{1/4}\sqrt{64I_4(\mathcal{G})I_4(\mathcal{Q}) - I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q})^2}}{4\sqrt{\kappa I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q}) + 2\langle I_4(\mathcal{G}), I_4(\mathcal{Q}) \rangle}}. \quad (3.41)$$

This limit corresponds to those solutions which can be obtained by a duality transformation from the solutions in [17], a distinguishing feature is that the phase of the supersymmetry parameter (which is duality invariant) is constant.

4 Conclusions

We have solved the general static BPS black hole in $\mathcal{N} = 2$ FI-gauged supergravity and found that the full solution depends on $2n_v$ out of $2n_v + 2$ charges. The two constraints are given by the BPS Dirac quantization condition (2.10) as well as (3.25), where the latter has its origin in (2.11). When restricted to the STU-model, all these solutions lift to M-theory via an embedding [19] of the STU-model into the $\mathcal{N} = 8$ de-Wit Nicolai theory [20]. In principle this uplift is known [21, 22] but in practice it is quite complicated. Simple formulae for this uplift are available in [19] for the STU-model when the axions vanish but our solutions have non-trivial axions and there is thus no simple uplift formula. In the symplectic frame discussed in [17] the magnetic charges uplift to Chern numbers of fibration of S^7 over Σ_g (the structure group of the S^7 bundle is reduced from $SO(8)$ to $U(1)^4$) whereas in this frame the electric charges lift to angular momentum around circles in S^7 .

While our results are limited to BPS black holes, a solution for (not necessarily BPS) asymptotically AdS₄ black holes in the STU-model was presented in [23] depending on all eight charges. This was performed with a view towards embedding these black holes in the $\mathcal{N} = 8$ theory¹¹ using the embedding of [19]. In principle the solutions of the current work, when restricted to the STU-model should be contained in [23] but establishing this map does not appear to be straightforward. Presumably making this connection clear would aid our understanding of these solutions in general.

We hope that the solution obtained in this work will shine light on the construction of more general BPS black holes in gauged supergravity. One might find a useful comparison to the work of Demianski-Plebanski [32] where they found a very general black hole solution to the Einstein-Maxwell theory depending on mass, nut charge, electric and magnetic charge as well as rotation and acceleration, with and without a cosmological constant. This remarkable solution unified a slew of existing works in one framework (see [33] for the most recent analysis of the supersymmetry of the PD solution). It seems to be a reasonable hope that a similar solution but with non-constant scalars can be obtained in FI-gauged supergravity, at least in the BPS sector, and we hope that the methods employed in this work, which maintain the full symmetries of the theory will be useful in this regard.¹² Having said that, one should keep in mind an interesting feature which emerged in the work [23]; while the full black hole solution space is continuously connected, allowing for a tuning to zero of the cosmological constant, the BPS sector is not. To pass continuously from a BPS asymptotically AdS₄ black hole to a BPS asymptotically flat black hole, one must pass through the non-BPS sector.

We also have in mind black holes in theories with hypermultiplets [18]. Hypermultiplets do not add any charges, although they may give mass to vector fields, in a sense they are just an additional scalar field sector, and as such one might hope that ultimately there are analytic solutions analogous to those presented here with non-trivial profiles for the hyper-scalars. This would have interesting applications since the M-theory lift of the many known $\mathcal{N} = 2$ gauged supergravities with hypermultiplets often involve wrapped M5 branes in addition to wrapped M2-branes [38, 39]. There are also known embeddings into M-theory of such theories which just contain the dynamics of M2-branes [40].

Acknowledgments

I would like to thank Stefanos Katmadas for bringing his work to my attention and for discussions as well as Alessandra Gnechi, Thomas Vanel and Harold Erbin for collaborations on closely related material. I offer my warm appreciation to the tranquility of the Parisian summer during which time these results were obtained. This work was conducted within the framework of the ILP LABEX (ANR-10-LABX-63) supported by French state funds managed by the ANR within the Investissements d’Avenir programme under reference ANR-11-IDEX-0004-02

¹¹See also [24–31] for other works on non-BPS black holes in $\mathcal{N} = 2$ gauged supergravity.

¹²See [23, 34–37] for results on rotating black holes in gauged supergravity.

A Special geometry background

A.1 Generalities

We will use the conventions where the prepotential is given by

$$\mathcal{F} = -\frac{d_{ijk}X^iX^jX^k}{X^0}, \quad (\text{A.1})$$

the metric is

$$g_{ij} = -\frac{3}{2}\frac{d_{y,ij}}{d_y} + \frac{9}{4}\frac{d_{y,i}d_{y,j}}{d_y^2}, \quad (\text{A.2})$$

where

$$X^\Lambda = \begin{pmatrix} 1 \\ z^i \end{pmatrix}, \quad z^i = x^i + iy^i \quad (\text{A.3})$$

and the covariant tensor is given by

$$\widehat{d}^{ijk} = \frac{g^{il}g^{jm}g^{kn}d_{ijk}}{d_y^2}. \quad (\text{A.4})$$

When \mathcal{M}_v is a symmetric very special Kähler manifolds the tensor \widehat{d}^{ijk} has constant entries and satisfies

$$\widehat{d}^{ijk}d_{jl(m}d_{mp)k} = \frac{16}{27}\left[\delta_l^i d_{mnp} + 3\delta_{(m}^i d_{mp)l}\right]. \quad (\text{A.5})$$

The sections are given by

$$\mathcal{V} = \begin{pmatrix} L^\Lambda \\ M_\Lambda \end{pmatrix} = e^{K/2} \begin{pmatrix} X^\Lambda \\ F_\Lambda \end{pmatrix} \quad (\text{A.6})$$

and satisfy¹³

$$\langle \mathcal{V}, \bar{\mathcal{V}} \rangle = -i, \quad \langle D_i \mathcal{V}, D_{\bar{j}} \bar{\mathcal{V}} \rangle = ig_{i\bar{j}} \quad (\text{A.7})$$

and any symplectic vector can be expanded in these sections. For example the charges are expanded as

$$\mathcal{Q} = i\bar{\mathcal{Z}}\mathcal{V} - i\mathcal{Z}\bar{\mathcal{V}} + i\bar{\mathcal{Z}}^{\bar{i}}D_{\bar{i}}\bar{\mathcal{V}} - i\bar{\mathcal{Z}}^i D_i \mathcal{V} \quad (\text{A.8})$$

where

$$\mathcal{Z} = \langle \mathcal{Q}, \mathcal{V} \rangle, \quad \mathcal{Z}_i = \langle \mathcal{Q}, D_i \mathcal{V} \rangle. \quad (\text{A.9})$$

We also have a complex structure on the symplectic bundle over \mathcal{M}_v :

$$\Omega \mathcal{M} \mathcal{V} = -i\mathcal{V}, \quad \Omega \mathcal{M} (D_i \mathcal{V}) = iD_i \mathcal{V} \quad (\text{A.10})$$

where

$$\Omega = \begin{pmatrix} 0 & -\mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \quad \mathcal{M} = \begin{pmatrix} 1 & -\mathcal{R} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathcal{I} & 0 \\ 0 & \mathcal{I}^{-1} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\mathcal{R} & 1 \end{pmatrix} \quad (\text{A.11})$$

and $\mathcal{N} = \mathcal{R} + i\mathcal{I}$ is the standard matrix which gives the kinetic and topological terms in the action for the gauge fields.

¹³The symplectic inner product is $\langle A, B \rangle = B^\Lambda A_\Lambda - B_\Lambda A^\Lambda$.

A.2 The quartic invariant

Homogeneous very special Kähler manifolds were classified by de Wit and Van Proeyen [41] (see also [42–44]) and there are several infinite families as well as numerous sporadic examples. For each of these manifolds one can define the quartic invariant:

$$\begin{aligned}
 I_4(\mathcal{Q}) &= \frac{1}{4!} t^{MNPQ} \mathcal{Q}_M \mathcal{Q}_N \mathcal{Q}_P \mathcal{Q}_Q \\
 &= - (p^0 q_0 + p^i q_i)^2 - 4q_0 d_{ijk} p^i p^j p^k + \frac{1}{16} p^0 \widehat{d}^{ijk} q_i q_j q_k + \frac{9}{16} d_{ijk} \widehat{d}^{ilm} p^i p^k q_l q_m. \quad (\text{A.12})
 \end{aligned}$$

Recall that the indices take values $\Lambda = 0, \dots, n_v$ and $i = 1, \dots, n_v$. Then the indices $\{M, N, P, Q\}$ take both Λ indices up and own, for example

$$\mathcal{Q}_M = \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix}. \quad (\text{A.13})$$

Homogeneous spaces are cosets G/H and $I_4(\mathcal{Q})$ is invariant under the global symmetries G of the coset. In the work of de Wit and Van Proeyen one can find the explicit embedding of G into the symplectic group $\text{Sp}(2n_v + 2, \mathbb{R})$ which then acts in a straightforward manner on $I_4(\mathcal{Q})$. The first steps incorporating this quartic invariant into the lexicon of BPS black holes were taken in [45–47] it is quite remarkable how integral it has become. Some more recent references which utilize it are [48–50].

Using the four index tensor t^{MNPQ} one can also define I_4 evaluated on four distinct symplectic vectors as well as its derivative I'_4 which is itself a symplectic vector. We essentially use the same normalizations as in [12]:

$$I_4(A, B, C, D) = t^{MNPQ} A_M B_N C_P D_Q \quad (\text{A.14})$$

$$I'_4(A, B, C)_M = \Omega_{MN} t^{NPQR} A_N B_Q C_R \quad (\text{A.15})$$

$$I_4(A) = \frac{1}{4!} t^{MNPQ} A_M A_N A_P A_Q \quad (\text{A.16})$$

$$I'_4(A)_M = \frac{1}{3!} \Omega_{MN} t^{NPQR} A_P A_Q A_R. \quad (\text{A.17})$$

We then have

$$24I_4(A) = I_4(A, A, A, A), \quad 6I'_4(A) = I'_4(A, A, A). \quad (\text{A.18})$$

A useful identity which is valid when \mathcal{M}_v is a symmetric space and which plays the key role in deriving the form of the BPS equations given in (2.12) and (2.13) is

$$I'_4(A, \text{Im}\mathcal{V}, \text{Im}\mathcal{V}) = 4\text{Im}[\langle A, \mathcal{V} \rangle] \text{Im}\mathcal{V} + 8\text{Re}[\langle A, \mathcal{V} \rangle] \text{Re}\mathcal{V} - \Omega \mathcal{M} A. \quad (\text{A.19})$$

Using this identity and replacing $A \rightarrow \text{Im}\mathcal{V}$ we derive the useful expressions

$$\text{Re}\mathcal{V} = - \frac{I'_4(\text{Im}\mathcal{V})}{2\sqrt{I_4(\text{Im}\mathcal{V})}}, \quad (\text{A.20})$$

and

$$I_4(\text{Im}\mathcal{V}) = \frac{1}{16}. \quad (\text{A.21})$$

A.3 Identities using the quartic invariant

The components of t^{MNPQ} are

$$t_{00}^{00} = -4, \quad t_{0j}^{0i} = -2\delta_j^i, \quad t_{kl}^{ij} = -4\delta_k^{(i}\delta_l^{j)} + \frac{9}{4}d_{klm}\tilde{d}^{ijm}, \quad (\text{A.22})$$

$$t_0^{ijk} = -\frac{3}{8}\tilde{d}^{ijk}, \quad t_{ijk}^0 = 24d_{ijk} \quad (\text{A.23})$$

from which with some effort one can derive the following identities. First we have the scalar identities

$$\langle I'_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}), I'_4(\mathcal{G}) \rangle = 8I_4(\mathcal{G})\langle \mathcal{G}, \mathcal{Q} \rangle \quad (\text{A.24})$$

$$\langle I'_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}), I'_4(\mathcal{G}) \rangle = \frac{2}{3}I_4(\mathcal{Q}, \mathcal{G}, \mathcal{G}, \mathcal{G})\langle \mathcal{G}, \mathcal{Q} \rangle \quad (\text{A.25})$$

$$\langle I'_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}), I'_4(\mathcal{Q}, \mathcal{G}, \mathcal{G}) \rangle = 4I_4(\mathcal{Q}, \mathcal{Q}, \mathcal{G}, \mathcal{G})\langle \mathcal{G}, \mathcal{Q} \rangle - 12\langle I'_4(\mathcal{Q}), I'_4(\mathcal{G}) \rangle. \quad (\text{A.26})$$

Then to evaluate the l.h.s. of (3.1)–(3.5) we need to expand various symplectic vectors in terms of the set

$$\{\mathcal{G}, \mathcal{Q}, I'_4(\mathcal{G}), I'_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}), I'_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}), I'_4(\mathcal{Q})\}. \quad (\text{A.27})$$

After much algebra we found the following expressions:

$$\begin{aligned} I'_4(I'_4(\mathcal{G}), I'_4(\mathcal{G}), \mathcal{G}) &= 8I_4(\mathcal{G})I'_4(\mathcal{G}) \\ I'_4(I'_4(\mathcal{G}), I'_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}), \mathcal{G}) &= 2I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})I'_4(\mathcal{G}) + 16I_4(\mathcal{G})\langle \mathcal{G}, \mathcal{Q} \rangle \mathcal{G} \\ I'_4(I'_4(\mathcal{G}), I'_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}), \mathcal{G}) &= \frac{4}{3}I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})\langle \mathcal{G}, \mathcal{Q} \rangle \mathcal{G} + 2I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q})I'_4(\mathcal{G}) \\ I'_4(I'_4(\mathcal{G}), I'_4(\mathcal{Q}), \mathcal{G}) &= 2\langle I'_4(\mathcal{G}), I'_4(\mathcal{Q}) \rangle \mathcal{G} + \frac{1}{3}I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})I'_4(\mathcal{G}) \\ I'_4(I'_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}), I'_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}), \mathcal{G}) &= 8I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q})I'_4(\mathcal{G}) + \frac{4}{3}I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})I'_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}) \\ &\quad - 16I_4(\mathcal{G})I'_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}) + 64I_4(\mathcal{G})\langle \mathcal{G}, \mathcal{Q} \rangle \mathcal{Q} \\ &\quad + \frac{16}{3}I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})\langle \mathcal{G}, \mathcal{Q} \rangle \mathcal{G} \\ I'_4(I'_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}), I'_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}), \mathcal{G}) &= \frac{16}{3}I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})I'_4(\mathcal{G}) + 2I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q})I'_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}) \\ &\quad - \frac{2}{3}I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})I'_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}) - 32I_4(\mathcal{G})I'_4(\mathcal{Q}) \\ &\quad + 8\left[I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q})\langle \mathcal{G}, \mathcal{Q} \rangle - \langle I'_4(\mathcal{G}), I'_4(\mathcal{Q}) \rangle\right] \mathcal{G} \\ &\quad + \frac{16}{3}I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})\langle \mathcal{G}, \mathcal{Q} \rangle \mathcal{Q} \\ I'_4(I'_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}), I'_4(\mathcal{Q}), \mathcal{G}) &= -\frac{2}{3}I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})I'_4(\mathcal{Q}) + 8\langle I'_4(\mathcal{G}), I'_4(\mathcal{Q}) \rangle \mathcal{Q} \\ &\quad + \frac{4}{3}I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})\langle \mathcal{G}, \mathcal{Q} \rangle \mathcal{G} + 16I_4(\mathcal{Q})I'_4(\mathcal{G}) \\ &\quad + \frac{1}{3}I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})I'_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}) \end{aligned}$$

$$\begin{aligned}
 I_4'(I_4'(\mathcal{G}, \mathcal{Q}, \mathcal{Q}), I_4'(\mathcal{G}, \mathcal{Q}, \mathcal{Q}), \mathcal{G}) &= 32I_4(\mathcal{Q})I_4'(\mathcal{G}) + \frac{8}{3}I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})I_4'(\mathcal{G}, \mathcal{G}, \mathcal{Q}) \\
 &\quad - \frac{16}{3}I_4(\mathcal{G}, \mathcal{G}, \mathcal{G}, \mathcal{Q})I_4'(\mathcal{Q}) + 16I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q})\langle \mathcal{G}, \mathcal{Q} \rangle \mathcal{Q} \\
 &\quad + \frac{16}{3}I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})\langle \mathcal{G}, \mathcal{Q} \rangle \mathcal{G} - 32\langle I_4'(\mathcal{G}), I_4'(\mathcal{Q}) \rangle \mathcal{Q} \\
 I_4'(I_4'(\mathcal{G}, \mathcal{Q}, \mathcal{Q}), I_4'(\mathcal{Q}), \mathcal{G}) &= 8I_4(\mathcal{Q})I_4'(\mathcal{G}, \mathcal{G}, \mathcal{Q}) + \frac{1}{3}I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})I_4'(\mathcal{G}, \mathcal{Q}, \mathcal{Q}) \\
 &\quad - 2I_4(\mathcal{G}, \mathcal{G}, \mathcal{Q}, \mathcal{Q})I_4'(\mathcal{Q}) + \frac{8}{3}I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})\langle \mathcal{G}, \mathcal{Q} \rangle \mathcal{Q} \\
 &\quad + 16I_4(\mathcal{Q})\langle \mathcal{G}, \mathcal{Q} \rangle \mathcal{G} \\
 I_4'(I_4'(\mathcal{Q}), I_4'(\mathcal{Q}), \mathcal{G}) &= 4I_4(\mathcal{Q})I_4'(\mathcal{G}, \mathcal{Q}, \mathcal{Q}) - \frac{2}{3}I_4(\mathcal{G}, \mathcal{Q}, \mathcal{Q}, \mathcal{Q})I_4'(\mathcal{Q}) \\
 &\quad + 16I_4(\mathcal{Q})\langle \mathcal{G}, \mathcal{Q} \rangle \mathcal{Q}.
 \end{aligned}$$

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] A. Strominger and C. Vafa, *Microscopic origin of the Bekenstein-Hawking entropy*, *Phys. Lett. B* **379** (1996) 99 [[hep-th/9601029](#)] [[INSPIRE](#)].
- [2] J.M. Maldacena, *The large- N limit of superconformal field theories and supergravity*, *Int. J. Theor. Phys.* **38** (1999) 1113 [[hep-th/9711200](#)] [[INSPIRE](#)].
- [3] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from noncritical string theory*, *Phys. Lett. B* **428** (1998) 105 [[hep-th/9802109](#)] [[INSPIRE](#)].
- [4] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [[hep-th/9802150](#)] [[INSPIRE](#)].
- [5] K. Behrndt, D. Lüster and W.A. Sabra, *Stationary solutions of $N = 2$ supergravity*, *Nucl. Phys. B* **510** (1998) 264 [[hep-th/9705169](#)] [[INSPIRE](#)].
- [6] B. Bates and F. Denef, *Exact solutions for supersymmetric stationary black hole composites*, *JHEP* **11** (2011) 127 [[hep-th/0304094](#)] [[INSPIRE](#)].
- [7] S.L. Cacciatori and D. Klemm, *Supersymmetric AdS_4 black holes and attractors*, *JHEP* **01** (2010) 085 [[arXiv:0911.4926](#)] [[INSPIRE](#)].
- [8] G. Dall'Agata and A. Gnecci, *Flow equations and attractors for black holes in $N = 2$ U(1) gauged supergravity*, *JHEP* **03** (2011) 037 [[arXiv:1012.3756](#)] [[INSPIRE](#)].
- [9] K. Hristov and S. Vandoren, *Static supersymmetric black holes in AdS_4 with spherical symmetry*, *JHEP* **04** (2011) 047 [[arXiv:1012.4314](#)] [[INSPIRE](#)].
- [10] N. Halmagyi, *BPS Black Hole Horizons in $N = 2$ Gauged Supergravity*, *JHEP* **02** (2014) 051 [[arXiv:1308.1439](#)] [[INSPIRE](#)].
- [11] N. Halmagyi and T. Vanel, *AdS Black Holes from Duality in Gauged Supergravity*, *JHEP* **04** (2014) 130 [[arXiv:1312.5430](#)] [[INSPIRE](#)].

- [12] S. Katmadas, *Static BPS black holes in U(1) gauged supergravity*, *JHEP* **09** (2014) 027 [[arXiv:1405.4901](#)] [[INSPIRE](#)].
- [13] J.M. Maldacena and C. Núñez, *Supergravity description of field theories on curved manifolds and a no go theorem*, *Int. J. Mod. Phys. A* **16** (2001) 822 [[hep-th/0007018](#)] [[INSPIRE](#)].
- [14] J.P. Gauntlett, N. Kim, S. Pakis and D. Waldram, *Membranes wrapped on holomorphic curves*, *Phys. Rev. D* **65** (2002) 026003 [[hep-th/0105250](#)] [[INSPIRE](#)].
- [15] M.M. Caldarelli and D. Klemm, *Supersymmetry of Anti-de Sitter black holes*, *Nucl. Phys. B* **545** (1999) 434 [[hep-th/9808097](#)] [[INSPIRE](#)].
- [16] F. Denef, *Supergravity flows and D-brane stability*, *JHEP* **08** (2000) 050 [[hep-th/0005049](#)] [[INSPIRE](#)].
- [17] A. Gneccchi and N. Halmagyi, *Supersymmetric black holes in AdS₄ from very special geometry*, *JHEP* **04** (2014) 173 [[arXiv:1312.2766](#)] [[INSPIRE](#)].
- [18] N. Halmagyi, M. Petrini and A. Zaffaroni, *BPS black holes in AdS₄ from M-theory*, *JHEP* **08** (2013) 124 [[arXiv:1305.0730](#)] [[INSPIRE](#)].
- [19] M. Cvetič et al., *Embedding AdS black holes in ten-dimensions and eleven-dimensions*, *Nucl. Phys. B* **558** (1999) 96 [[hep-th/9903214](#)] [[INSPIRE](#)].
- [20] B. de Wit and H. Nicolai, *N = 8 Supergravity*, *Nucl. Phys. B* **208** (1982) 323 [[INSPIRE](#)].
- [21] B. de Wit and H. Nicolai, *The Consistency of the S⁷ Truncation in D = 11 Supergravity*, *Nucl. Phys. B* **281** (1987) 211 [[INSPIRE](#)].
- [22] H. Nicolai and K. Pilch, *Consistent Truncation of D = 11 Supergravity on AdS₄ × S⁷*, *JHEP* **03** (2012) 099 [[arXiv:1112.6131](#)] [[INSPIRE](#)].
- [23] D.D.K. Chow and G. Compère, *Dyonic AdS black holes in maximal gauged supergravity*, *Phys. Rev. D* **89** (2014) 065003 [[arXiv:1311.1204](#)] [[INSPIRE](#)].
- [24] D. Klemm and O. Vaughan, *Nonextremal black holes in gauged supergravity and the real formulation of special geometry*, *JHEP* **01** (2013) 053 [[arXiv:1207.2679](#)] [[INSPIRE](#)].
- [25] D. Klemm and O. Vaughan, *Nonextremal black holes in gauged supergravity and the real formulation of special geometry II*, *Class. Quant. Grav.* **30** (2013) 065003 [[arXiv:1211.1618](#)] [[INSPIRE](#)].
- [26] A. Gneccchi and C. Toldo, *On the non-BPS first order flow in N = 2 U(1)-gauged Supergravity*, *JHEP* **03** (2013) 088 [[arXiv:1211.1966](#)] [[INSPIRE](#)].
- [27] K. Hristov, C. Toldo and S. Vandoren, *Phase transitions of magnetic AdS₄ black holes with scalar hair*, *Phys. Rev. D* **88** (2013) 026019 [[arXiv:1304.5187](#)] [[INSPIRE](#)].
- [28] A. Gneccchi and C. Toldo, *First order flow for non-extremal AdS black holes and mass from holographic renormalization*, *JHEP* **10** (2014) 075 [[arXiv:1406.0666](#)] [[INSPIRE](#)].
- [29] S. Barisch, G. Lopes Cardoso, M. Haack, S. Nampuri and N.A. Obers, *Nernst branes in gauged supergravity*, *JHEP* **11** (2011) 090 [[arXiv:1108.0296](#)] [[INSPIRE](#)].
- [30] K. Goldstein, S. Nampuri and Á. Véliz-Ororio, *Heating up branes in gauged supergravity*, *JHEP* **08** (2014) 151 [[arXiv:1406.2937](#)] [[INSPIRE](#)].
- [31] H. Lü, Y. Pang and C.N. Pope, *AdS Dyonic Black Hole and its Thermodynamics*, *JHEP* **11** (2013) 033 [[arXiv:1307.6243](#)] [[INSPIRE](#)].

- [32] J.F. Plebanski and M. Demianski, *Rotating, charged and uniformly accelerating mass in general relativity*, *Annals Phys.* **98** (1976) 98 [INSPIRE].
- [33] D. Klemm and M. Nozawa, *Supersymmetry of the C-metric and the general Plebanski-Demianski solution*, *JHEP* **05** (2013) 123 [arXiv:1303.3119] [INSPIRE].
- [34] V.A. Kostelecky and M.J. Perry, *Solitonic black holes in gauged $N = 2$ supergravity*, *Phys. Lett. B* **371** (1996) 191 [hep-th/9512222] [INSPIRE].
- [35] Z.-W. Chong, M. Cvetič, H. Lü and C.N. Pope, *Charged rotating black holes in four-dimensional gauged and ungauged supergravities*, *Nucl. Phys. B* **717** (2005) 246 [hep-th/0411045] [INSPIRE].
- [36] D. Klemm, *Rotating BPS black holes in matter-coupled AdS_4 supergravity*, *JHEP* **07** (2011) 019 [arXiv:1103.4699] [INSPIRE].
- [37] A. Gnecci, K. Hristov, D. Klemm, C. Toldo and O. Vaughan, *Rotating black holes in 4d gauged supergravity*, *JHEP* **01** (2014) 127 [arXiv:1311.1795] [INSPIRE].
- [38] A. Donos and J.P. Gauntlett, *Supersymmetric quantum criticality supported by baryonic charges*, *JHEP* **10** (2012) 120 [arXiv:1208.1494] [INSPIRE].
- [39] D. Cassani, P. Koerber and O. Varela, *All homogeneous $N = 2$ M-theory truncations with supersymmetric AdS_4 vacua*, *JHEP* **11** (2012) 173 [arXiv:1208.1262] [INSPIRE].
- [40] N. Bobev, N. Halmagyi, K. Pilch and N.P. Warner, *Supergravity Instabilities of Non-Supersymmetric Quantum Critical Points*, *Class. Quant. Grav.* **27** (2010) 235013 [arXiv:1006.2546] [INSPIRE].
- [41] B. de Wit and A. Van Proeyen, *Special geometry, cubic polynomials and homogeneous quaternionic spaces*, *Commun. Math. Phys.* **149** (1992) 307 [hep-th/9112027] [INSPIRE].
- [42] B. de Wit, F. Vanderseypen and A. Van Proeyen, *Symmetry structure of special geometries*, *Nucl. Phys. B* **400** (1993) 463 [hep-th/9210068] [INSPIRE].
- [43] B. de Wit and A. Van Proeyen, *Hidden symmetries, special geometry and quaternionic manifolds*, *Int. J. Mod. Phys. D* **3** (1994) 31 [hep-th/9310067] [INSPIRE].
- [44] B. de Wit and A. Van Proeyen, *Isometries of special manifolds*, [hep-th/9505097] [INSPIRE].
- [45] M. Cvetič and A.A. Tseytlin, *Solitonic strings and BPS saturated dyonic black holes*, *Phys. Rev. D* **53** (1996) 5619 [Erratum *ibid.* **D 55** (1997) 3907] [hep-th/9512031] [INSPIRE].
- [46] R. Kallosh and B. Kol, *E_7 symmetric area of the black hole horizon*, *Phys. Rev. D* **53** (1996) 5344 [hep-th/9602014] [INSPIRE].
- [47] M. Cvetič and C.M. Hull, *Black holes and U duality*, *Nucl. Phys. B* **480** (1996) 296 [hep-th/9606193] [INSPIRE].
- [48] B.L. Cerchiai, S. Ferrara, A. Marrani and B. Zumino, *Duality, Entropy and ADM Mass in Supergravity*, *Phys. Rev. D* **79** (2009) 125010 [arXiv:0902.3973] [INSPIRE].
- [49] S. Ferrara, A. Marrani and A. Yeranyan, *On Invariant Structures of Black Hole Charges*, *JHEP* **02** (2012) 071 [arXiv:1110.4004] [INSPIRE].
- [50] G. Bossard and S. Katmadas, *Duality covariant non-BPS first order systems*, *JHEP* **09** (2012) 100 [arXiv:1205.5461] [INSPIRE].