Published for SISSA by Springer

Received: *November 11, 2013*
Revised: *January 15, 2014*
Accepted: *January 24, 2014*
Published: *March 5, 2014*

A dynamical theory for linearized massive superspin 3/2[1]

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Abstract: We present a new theory of free massive superspin $Y = 3/2$ irreducible representation of the $4D$, $\mathcal{N} = 1$ Super-Poincaré group, which has linearized non-minimal supergravity (superhelicity $Y = 3/2$) as it's massless limit. The new results will illuminate the underlying structure of auxiliary superfields required for the description of higher massive superspin systems.

Keywords: Space-Time Symmetries, Supersymmetric Effective Theories

ArXiv ePrint: 1310.7387

[1]Supported in part by National Science Foundation Grant PHY-09-68854.

doi:10.1007/JHEP03(2014)030

Contents

1 Introduction

After four decades of exploring the topic of supersymmetry (SUSY), the problem of writing a manifestly susy-invariant action that describes a free, off-shell massive arbitrary superspin irreducible representation of the Super-Poincaré group still possesses puzzles. Although the non-supersymmetric case of massive higher spin theory has been developed [1, 2] and is well understood, the off-shell supersymmetric case has yet to be understood with a comparable level of clarity. There has been progress for on-shell supersymmetry [3], but these results don't capture the rich off-shell structure of supersymmetric theories. There is a need for a manifestly susy invariant theory of massive integer and half-integer superspins which includes all the auxiliary superfields a theory of this nature is expected to possess.

Progress in this direction was made with the works presented in [4–6] where free massive irreducible representations of superspin 1 and 3/2 were constructed. These results provided a proof of concept that constructions like these are possible, but they don't shed light to the heart of the problem which is to determine the set of auxiliary superfields required to describe an arbitrary superspin system with a proper massless limit. Specifically in [4] the focus was on massive extension of linearized old-minimal supergravity and new-minimal supergravity. These theories do not generalize to the arbitrary spin case, therefore the results obtained do not provide clues about the underlying structure of the auxiliary superfields for the general case.

This is not the case with the work presented in [6] where a free massive extension of linearized non-minimal supergravity is derived. Linearized non-minimal supergravity supermultiplet is a member of a tower of irreducible representations that can be extended to the arbitrary super-helicity and that makes it a good starting point. However their construction uses a lagrange multiplier technique in order to impose constraints that were not derived in a dynamical way.

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We will show that there is an alternative formulation of the theory where all the constraints required, for the description of a free massive irreducible representation of $Y = 3/2$, are dynamically generated from the equations of motion of a set of superfields $\{H_{\alpha\dot{\alpha}},\ \chi_\alpha,\ u_\alpha\}$. Superfields $H_{\alpha\dot{\alpha}},\ \chi_\alpha$ in the massless limit form the free linearized non-minimal theory (superhelicity $Y = 3/2$) and u_α is an auxiliary superfield that decouples when $m \to 0$.

Finally, the theory presented here is a free theory without interactions. The fully interacting, non-linear problem is still an open and very difficult one and it is one of the motivations for this kind of investigations. In a realistic approach we can not talk about interactions if we haven't established the free theories first. The results presented here extend our understanding for the free massive theory of the non-minimal superspin 3/2 supermultiplet, which is the first non-trivial. Furthermore we provide clues for some of the degrees of freedom that must be present in the non-linear, interacting theory. These are the superfields that have auxiliary status in the free linearized theory.

Our presentation is organized as follows: in section 2, we quickly review the representation theory of the $4D$, $\mathcal{N} = 1$ Super-Poincaré group for a free massive arbitrary superspin system. In section 3, we present the constraints imposed in the theory in order to have a proper massless limit. In the following section 4, we start with a warm up exercise by quickly reproducing the massive theory for superspin $Y = 1/2$. In the last section 5 we present the new massive theory for $Y = 3/2$.

2 Arbitrary superspin representation theory

The irreducible representations of the Super-Poincaré group are labeled by it's two Casimir operators. The first one is the mass and the other one is a supersymmetric extension of the Poincaré Spin operator. For the massive case the Super-spin casimir operator takes the form

$$C_2 = \frac{W^2}{m^2} + \left(\frac{3}{4} + \lambda\right) P_{(o)} \quad , \tag{2.1}$$

where W^2 is the ordinary spin operator (the square of the Pauli-Lubanski vector), $P_{(o)}$ is the projection operator $P_{(o)} = -\frac{1}{m^2}\mathrm{D}^\gamma\bar{\mathrm{D}}^2\mathrm{D}_\gamma$ and the parameter λ satisfies the equation

$$\lambda^2 + \lambda = \frac{W^2}{m^2} \quad . \tag{2.2}$$

In order to diagonalize C_2 we want to diagonalize both $W^2, P_{(o)}$. The superfield $\Phi_{\alpha(n)\dot{\alpha}(m)}$ that does this

$$\begin{aligned} &W^2\Phi_{\alpha(n)\dot{\alpha}(m)} = j(j+1)m^2\Phi_{\alpha(n)\dot{\alpha}(m)},\ j = \frac{n+m}{2} \quad , \\ &P_{(o)}\Phi_{\alpha(n)\dot{\alpha}(m)} = \Phi_{\alpha(n)\dot{\alpha}(m)} \quad , \end{aligned} \tag{2.3}$$

and describes the highest possible representation (highest superspin)

$$\begin{aligned} &\lambda = \frac{n+m}{2} \quad , \\ &C_2\Phi_{\alpha(n)\dot{\alpha}(m)} = Y(Y+1)\Phi_{\alpha(n)\dot{\alpha}(m)},\ Y = \frac{n+m+1}{2} \quad , \end{aligned} \tag{2.4}$$

has to satisfy the following:

$$
\begin{aligned}
\mathrm{D}^2\Phi_{\alpha(n)\dot{\alpha}(m)} &= 0 \ , \\
\bar{\mathrm{D}}^2\Phi_{\alpha(n)\dot{\alpha}(m)} &= 0 \ , \\
\mathrm{D}^{\gamma}\Phi_{\gamma\alpha(n-1)\dot{\alpha}(m)} &= 0 \ , \\
\partial^{\gamma\dot{\gamma}}\Phi_{\gamma\alpha(n-1)\dot{\gamma}\dot{\alpha}(m-1)} &= 0 \ , \\
\Box\Phi_{\alpha(n)\dot{\alpha}(m)} &= m^2\Phi_{\alpha(n)\dot{\alpha}(m)} \ ,
\end{aligned}
\tag{2.5}
$$

where all dotted and undotted indices are fully symmetrized and the spin content of this supermultiplet is $j = Y + 1/2,\ Y,\ Y,\ Y - 1/2$.

A superfield that describes a superspin Y system has index structure such that $n+m = 2Y - 1$ where n, m are integers. This Diophantine equation has a finite number of different solutions for (n, m) pairs but the corresponding superfields are all equivalent because we can use the $\partial_{\beta\dot{\beta}}$ operator to convert one kind of index to another. So we can pick one of them to represent the entire class.

One last comment has to be made about the reality of the representation. The reality condition imposed on the superfield differs with the character of the superfield. The bosonic superfields, have even total number of indices therefore describe half-integer superspin systems, $Y = s + 1/2$. In this case we can pick to have $n = m = s$ ($H_{\alpha(s)\dot{\alpha}(s)}$) and the reality condition is $H_{\alpha(s)\dot{\alpha}(s)} = \bar{H}_{\alpha(s)\dot{\alpha}(s)}$. On the other hand the fermionic superfields have odd total number of indices and describe integer superpsin systems, $Y = s + 1$. For that case we can pick $n = s + 1,\ m = s$ ($\Psi_{\alpha(s+1)\dot{\alpha}(s)}$) and the reality condition is the Dirac equation $i\partial_{\alpha_{s+1}}{}^{\dot{\alpha}_{s+1}}\bar{\Psi}_{\alpha(s)\dot{\alpha}(s+1)} + m\Psi_{\alpha(s+1)\dot{\alpha}(s)}$.

3 The massless limit

Representation theory tells us the type of superfield and constraints we need in order to describe a specific irreducible representation. We would like to have a dynamical way to derive these constraints, through an action. Very quickly we realize that, for that to happen we need a set of auxiliary superfields to help us generate these constraints, as in the non-supersymmetric free massive arbitrary spin story. To find the minimum number and type of these auxiliary superfields needed, is the heart of the problem. That sounds like an intuitive trial-and-error process, but there is a hidden clue and that is the massless limit of the theory. We demand the massless limit of our massive theory to give the corresponding massless irreducible representation.

The list of available massless highest superhelicity irreducible representations was presented in [7–10], and [11]. There is one infinite tower for theories of integer superhelicity and two different infinite towers for theories of half integer super-helicities (figure 1). However there are a few theories that don't fall in one of the three infinite towers, like the old minimal, new minimal and new-new minimal supermultiplets. These are special cases that can not be generalized to the arbitrary superhelicity. If our goal is towards the construction

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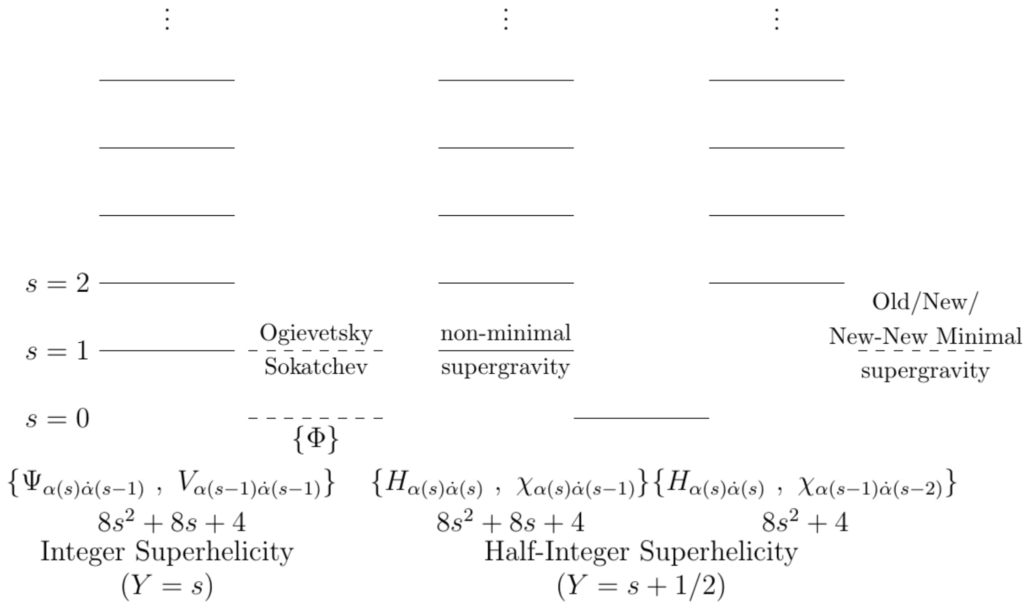

Figure 1. Towers of Massive Higher Spin Supermultiplets

of an arbitrary massive superspin supermultiplet, then it is obvious that we should start with massless theories that are members of an infinite tower and not a special case.

The conclusion is that the construction of massive theories must start with the corresponding massless action, the addition of some extra auxiliary superfields (if necessary) and appropriate (self)interaction terms proportional to m and m^2, so the massless theory decouples in the massless limit.

4 Warming up with $Y = 1/2$

So if we want to construct the theory of superspin 1/2 we start with the theory of superhelicity 1/2, add terms proportional to m and m^2 and check if we can generate the desired constraints. If not then we add extra auxiliary fields until we do. The starting action is:

$$S = \int d^8z \Big\{ \; a_1 \, H \mathrm{D}^{\gamma} \bar{\mathrm{D}}^2 \mathrm{D}_{\gamma} H \; + \; a_2 m \; H \left(\mathrm{D}^2 H + \bar{\mathrm{D}}^2 H \right) \; + \; a_3 m^2 \; H^2 \Big\} \quad . \tag{4.1}$$

To describe $Y = \frac{1}{2}$, H must satisfy $\mathrm{D}^2 H = 0$ and $\Box H = m^2 H$. The equation of motion is

$$\mathcal{E}^{(H)} = \frac{\delta S}{\delta H} = 2a_1 \mathrm{D}^{\gamma} \bar{\mathrm{D}}^2 \mathrm{D}_{\gamma} H + 2a_2 m \left(\mathrm{D}^2 H + \bar{\mathrm{D}}^2 H \right) + 2a_3 m^2 H \quad , \tag{4.2}$$

which gives

$$\mathrm{D}^2 \mathcal{E}^{(H)} = 2a_2 m \mathrm{D}^2 \bar{\mathrm{D}}^2 H + 2a_3 m^2 \mathrm{D}^2 H \quad , \tag{4.3}$$

so by choosing $a_2 = 0, a_3 \neq 0$ we find $\mathrm{D}^2 H = 0 \rightsquigarrow \bar{\mathrm{D}}^2 H = 0$ (reality) and if this is substituted back into $\mathcal{E}^{(H)}$ we get $\Box H = \frac{a_3}{a_1} m^2 H$ which fixes $a_3 = a_1$ for compatibility with the Klein-Gordon equation.

There is also another way to obtain these results and that is à la Stückelberg. The observation is that at least on-shell the massive superspin $\frac{1}{2}$ can be seen as the result of the combination of the massless superhelicity $\frac{1}{2}$ plus the massless superhelicity 0. So we start with the actions for superhelicity $\frac{1}{2}$ and 0 and we introduce (self)interaction terms proportional to m and m^2

$$S = \int d^8z \Big\{ a_1 \; H\bar{\mathrm{D}}^{\dot{\gamma}}\mathrm{D}^2\bar{\mathrm{D}}_{\dot{\gamma}}H + a_2 m \, H \left(\mathrm{D}^2 H + \bar{\mathrm{D}}^2 H\right) + a_3 m^2 \; H^2 \\ + \gamma m \; H\left(\Phi + \bar{\Phi}\right) \; + \; b_1 \; \Phi\bar{\Phi} \Big\} \; + \; \int d^6z \; b_2 m \; \Phi\Phi \quad . \tag{4.4}$$

The equations of motion are:

$$\mathcal{E}^{(H)} = 2a_1 \mathrm{D}^{\gamma}\bar{\mathrm{D}}^2\mathrm{D}_{\gamma}H + 2a_2 m\left(\mathrm{D}^2 H + \bar{\mathrm{D}}^2 H\right) + \gamma m\left(\Phi + \bar{\Phi}\right) + 2a_3 m^2 H \quad , \tag{4.5}$$

$$\mathcal{E}^{(\Phi)} = -b_1 \bar{\mathrm{D}}^2\bar{\Phi} - \gamma m \bar{\mathrm{D}}^2 H + 2b_2 m \Phi \quad . \tag{4.6}$$

If we manage to show that on-shell $\Phi = 0$ then $\mathcal{E}^{(\Phi)} = 0 \rightsquigarrow \mathrm{D}^2 H = 0 \rightsquigarrow \Box H = m^2 H$ ($a_3 = a_1$). With that goal in mind we attempt to eliminate H from the equation of Φ and choose coefficients in such a way to find $\Phi = 0$. We begin by defining $I = \bar{\mathrm{D}}^2\mathcal{E}^{(H)} + m\mathcal{E}^{(\Phi)}$ and then notice

$$I = \bar{\mathrm{D}}^2\mathcal{E}^{(H)} + m\mathcal{E}^{(\Phi)} \; = \; (\gamma - b_1)\, m \; \bar{\mathrm{D}}^2\bar{\Phi} \; + \; (2a_3 - \gamma)\, m^2 \; \bar{\mathrm{D}}^2 H \\ + 2a_2 \; \bar{\mathrm{D}}^2\mathrm{D}^2 H \; + \; 2b_2 m^2 \; \Phi \quad . \tag{4.7}$$

If we choose $\gamma = b_1 = 2a_3 = 2a_1, \; a_2 = 0$ we obtain $I = 2b_2 m^2 \; \Phi$. Now we can follow two possible routes

1. $b_2 \neq 0$: b_2 can be anything besides zero and in that case on-shell $I = 0 \rightsquigarrow \Phi = 0$ we find all the desired constraints for H and the action is

$$S = \int d^8z \Big\{ c \; H\bar{\mathrm{D}}^{\dot{\gamma}}\mathrm{D}^2\bar{\mathrm{D}}_{\dot{\gamma}}H \; + cm^2 \, H^2 + 2cm \; H\left(\Phi + \bar{\Phi}\right) \\ + 2c \; \Phi\bar{\Phi} \Big\} + \int d^6z \; b_2 m \; \Phi\Phi \quad . \tag{4.8}$$

2. $b_2 = 0$: if we set b_2 to zero, then I identically vanish. That means the $\bar{\mathrm{D}}^2\mathcal{E}^{(H)} + m\mathcal{E}^{(\Phi)} = 0$ can be treated as a Bianchi identity and the corresponding action is invariant under a symmetry. The symmetry of the action that generates the above Bianchi identity is

$$\delta_G H \sim \bar{\mathrm{D}}^2 L + \mathrm{D}^2\bar{L} \; , \tag{4.9}$$

$$\delta_G \Phi \sim m\bar{\mathrm{D}}^2 L \quad . \tag{4.10}$$

Due to this symmetry, the chiral superfield Φ can be gauged away completely and therefore it's equation of motion (or the Bianchi identity) will give the desired constraint of $\mathrm{D}^2 H = 0$. The action for this case is

$$S = \int d^8z \Big\{ c \; H\bar{\mathrm{D}}^{\dot{\gamma}}\mathrm{D}^2\bar{\mathrm{D}}_{\dot{\gamma}}H \; + cm^2 \, H^2 \; + 2cm \; H\left(\Phi + \bar{\Phi}\right) \; + \; 2c \; \Phi\bar{\Phi} \Big\} \quad , \tag{4.11}$$

and the gauge fixed action is identical with the action obtained from the first derivation. We would like to know if similar 'Stückelberg' constructions can occur for the higher superspin theories, like it is the case for the higher spin theories

5 New massive $Y = 3/2$ theory

Now we will follow a similar strategy to build a theory of superspin $\frac{3}{2}$. The starting point is the theory of superhelicity $\frac{3}{2}$, in specific we choose the theory of non-minimal supergravity ($s = 1$ in [9]). Linear Non-minimal supergravity is formulated in terms of a real bosonic superfield $H_{\alpha\dot{\alpha}}$ and a fermionic superfield χ_α. We will add mass corrections to that action and check if 1) we can make χ_α vanish on-shell (auxiliary status) and 2) we can generate the constraints on $H_{\alpha\dot{\alpha}}$ demanded by representation theory $\mathrm{D}^\alpha H_{\alpha\dot{\alpha}} = 0$, $\Box H_{\alpha\dot{\alpha}} = m^2 H_{\alpha\dot{\alpha}}$. The starting action is given by

$$S = \int d^8z \Big\{ H^{\alpha\dot{\alpha}}\mathrm{D}^\gamma\bar{\mathrm{D}}^2\mathrm{D}_\gamma H_{\alpha\dot{\alpha}} \qquad + a_1 m H^{\alpha\dot{\alpha}}(\bar{\mathrm{D}}_{\dot{\alpha}}\chi_\alpha - \mathrm{D}_\alpha\bar{\chi}_{\dot{\alpha}}) \\ - 2\, H^{\alpha\dot{\alpha}}\bar{\mathrm{D}}_{\dot{\alpha}}\mathrm{D}^2\chi_\alpha + c.c. \quad + a_2 m H^{\alpha\dot{\alpha}}(\mathrm{D}^2 H_{\alpha\dot{\alpha}} + \bar{\mathrm{D}}^2 H_{\alpha\dot{\alpha}}) \\ - 2\, \chi^\alpha\mathrm{D}^2\chi_\alpha + c.c. \qquad + a_3 m\chi^\alpha\chi_\alpha + c.c. \\ + 2\, \chi^\alpha\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot{\alpha}}\bar{\chi}_{\dot{\alpha}} \qquad + a_4 m^2 H^{\alpha\dot{\alpha}}H_{\alpha\dot{\alpha}} \Big\} \quad , \tag{5.1}$$

and the equations of motion are:

$$\mathcal{E}^{(H)}_{\alpha\dot{\alpha}} = 2\mathrm{D}^\gamma\bar{\mathrm{D}}^2\mathrm{D}_\gamma H_{\alpha\dot{\alpha}} + 2(\mathrm{D}_\alpha\bar{\mathrm{D}}^2\bar{\chi}_{\dot{\alpha}} - \bar{\mathrm{D}}_{\dot{\alpha}}\mathrm{D}^2\chi_\alpha) + a_1 m(\bar{\mathrm{D}}_{\dot{\alpha}}\chi_\alpha - \mathrm{D}_\alpha\bar{\chi}_{\dot{\alpha}}) \\ + 2a_2 m(\mathrm{D}^2 H_{\alpha\dot{\alpha}} + \bar{\mathrm{D}}^2 H_{\alpha\dot{\alpha}}) + 2a_4 m^2 H_{\alpha\dot{\alpha}} \ , \tag{5.2}$$

$$\mathcal{E}^{(\chi)}_\alpha = -4\mathrm{D}^2\chi_\alpha + 2\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot{\alpha}}\bar{\chi}_{\dot{\alpha}} - 2\mathrm{D}^2\bar{\mathrm{D}}^{\dot{\alpha}}H_{\alpha\dot{\alpha}} + a_1 m\bar{\mathrm{D}}^{\dot{\alpha}}H_{\alpha\dot{\alpha}} + 2a_3 m\chi_\alpha \quad . \tag{5.3}$$

Now we may use these equations and attempt to remove any $H_{\alpha\alpha}$-dependence to derive one equation that depends solely on χ_α. That will tell us if we can pick coefficients in a way that χ_α vanishes on-shell. Consider the following linear combination of equations of motion where each such equation of motion is obtained by the variation of the action with regard to the respective superfields indicated by the subscripts in the first equation below:

$$I_\alpha = A\mathrm{D}^2\bar{\mathrm{D}}^{\dot{\alpha}}\mathcal{E}^{(H)}_{\alpha\dot{\alpha}} + B\mathrm{D}^2\bar{\mathrm{D}}^2\mathcal{E}^{(\chi)}_\alpha + m^2\mathcal{E}^{(\chi)}_\alpha \\ = -2\,(A+B)\,\Box\mathrm{D}^2\bar{\mathrm{D}}^{\dot{\alpha}}H_{\alpha\dot{\alpha}} \quad + 2\,(A+B)\,\mathrm{D}^2\bar{\mathrm{D}}^2\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot{\alpha}}\bar{\chi}_{\dot{\alpha}} \quad - Aa_1 m\mathrm{D}^2\bar{\mathrm{D}}^{\dot{\alpha}}\mathrm{D}_\alpha\bar{\chi}_{\dot{\alpha}} \\ + 2\,(Aa_4 - 1)\,m^2\mathrm{D}^2\bar{\mathrm{D}}^{\dot{\alpha}}H_{\alpha\dot{\alpha}} - 4\,(A+B)\,\Box\mathrm{D}^2\chi_\alpha \qquad - 4m^2\mathrm{D}^2\chi_\alpha \\ + (a_1)\,m^3\bar{\mathrm{D}}^{\dot{\alpha}}H_{\alpha\dot{\alpha}} \qquad + 2\,(Aa_1 + Ba_3)\,m\mathrm{D}^2\bar{\mathrm{D}}^2\chi_\alpha \quad + 2m^2\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot{\alpha}}\bar{\chi}_{\dot{\alpha}} \\ + 2a_3 m^3\chi_\alpha \quad . \tag{5.4}$$

The following choice of coefficients will remove any $H_{\alpha\dot{\alpha}}$ dependence from the equation above:

$$A + B = 0 \ , \quad Aa_4 - 1 = 0 \ , \quad a_1 = 0 \quad , \tag{5.5}$$

and imposing these leads to the form of I_α to be given by

$$I_\alpha \;= -4m^2\mathrm{D}^2\chi_\alpha + 2Ba_3m\mathrm{D}^2\bar{\mathrm{D}}^2\chi_\alpha \;+ 2m^2\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot\alpha}\bar\chi_{\dot\alpha} + 2a_3m^3\chi_\alpha \quad . \tag{5.6}$$

From this we see there is no choice of coefficients that will make χ_α vanish on-shell. Therefore we must introduce an auxiliary superfield. Its purpose will be to impose a constraint on χ_α when it vanishes. That constraint will be used to simplify the above expression for I_α and set χ_α to zero. But a more careful examination of I_α will convince us that there is no unique constraint on χ_α that will make all terms (except the last one) vanish. The inescapable conclusion is that we have to treat $\chi_\alpha = 0$ as the desired constraint. This suggests that we must introduce a spinorial superfield u_α that couples with χ_α through only a mass term $\sim mu^\alpha\chi_\alpha$. Hence when $u_\alpha = 0$ then immediately we see $\chi_\alpha = 0$.

We must update the action with the introduction of a few new terms: the interaction term $mu^\alpha\chi_\alpha$ and the kinetic energy terms for u_α (the most general quadratic action). The new action is

$$\begin{aligned}
S = \int d^8z \Big\{ & H^{\alpha\dot\alpha}\mathrm{D}^\gamma\bar{\mathrm{D}}^2\mathrm{D}_\gamma H_{\alpha\dot\alpha} & & & & + \gamma m u^\alpha\chi_\alpha + c.c. \\
& - 2\; H^{\alpha\dot\alpha}\bar{\mathrm{D}}_{\dot\alpha}\mathrm{D}^2\chi_\alpha + c.c. & & + a_2 m H^{\alpha\dot\alpha}\mathrm{D}^2 H_{\alpha\dot\alpha} + c.c. & & + b_1 u^\alpha\mathrm{D}^2 u_\alpha + c.c. \\
& - 2\; \chi^\alpha\mathrm{D}^2\chi_\alpha + c.c. & & + a_3 m\chi^\alpha\chi_\alpha + c.c. & & + b_2 u^\alpha\bar{\mathrm{D}}^2 u_\alpha + c.c. \\
& + 2\; \chi^\alpha\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot\alpha}\bar\chi_{\dot\alpha} & & + a_4 m^2 H^{\alpha\dot\alpha}H_{\alpha\dot\alpha} & & + b_3 u^\alpha\bar{\mathrm{D}}^{\dot\alpha}\mathrm{D}_\alpha\bar u_{\dot\alpha} \\
& & & & & + b_4 u^\alpha\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot\alpha}\bar u_{\dot\alpha} \\
& & & & & + b_5 m u^\alpha u_\alpha \Big\} \quad ,
\end{aligned} \tag{5.7}$$

and the updated equations of motion are

$$\begin{aligned}
\mathcal{E}^{(H)}_{\alpha\dot\alpha} &= 2\mathrm{D}^\gamma\bar{\mathrm{D}}^2\mathrm{D}_\gamma H_{\alpha\dot\alpha} + 2(\mathrm{D}_\alpha\bar{\mathrm{D}}^2\bar\chi_{\dot\alpha} - \bar{\mathrm{D}}_{\dot\alpha}\mathrm{D}^2\chi_\alpha) + 2a_2m(\mathrm{D}^2H_{\alpha\dot\alpha} + \bar{\mathrm{D}}^2H_{\alpha\dot\alpha}) \\
&\quad + 2a_4m^2H_{\alpha\dot\alpha} \ ,
\end{aligned} \tag{5.8}$$

$$\mathcal{E}^{(\chi)}_\alpha = -4\mathrm{D}^2\chi_\alpha + 2\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot\alpha}\bar\chi_{\dot\alpha} - 2\mathrm{D}^2\bar{\mathrm{D}}^{\dot\alpha}H_{\alpha\dot\alpha} + 2a_3m\chi_\alpha + \gamma m u_\alpha \quad , \tag{5.9}$$

$$\mathcal{E}^{(u)}_\alpha = 2b_1\mathrm{D}^2u_\alpha + 2b_2\bar{\mathrm{D}}^2u_\alpha + b_3\bar{\mathrm{D}}^{\dot\alpha}\mathrm{D}_\alpha\bar u_{\dot\alpha} + b_4\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot\alpha}\bar u_{\dot\alpha} + 2b_5mu_\alpha + \gamma m\chi_\alpha \quad . \tag{5.10}$$

Now we repeat the process of eliminating $H_{\alpha\dot\alpha}$, but since u_α doesn't couple to $H_{\alpha\dot\alpha}$ nothing will be changed regarding the $H_{\alpha\dot\alpha}$-dependent terms. The same choice of coefficients as in (24) must be made to remove $H_{\alpha\dot\alpha}$. So the updated expression for I_α is

$$\begin{aligned}
I_\alpha =& 2Ba_3m\mathrm{D}^2\bar{\mathrm{D}}^2\chi_\alpha & & - 4m^2\mathrm{D}^2\chi_\alpha \\
& + B\gamma m\mathrm{D}^2\bar{\mathrm{D}}^2u_\alpha & & + 2m^2\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot\alpha}\bar\chi_{\dot\alpha} \\
& + \gamma m^3u_\alpha & & + 2a_3m^3\chi_\alpha \quad .
\end{aligned} \tag{5.11}$$

Now we want to use the equation of motion of u_α to remove any dependences on χ_α in order to derive an equation of u_α. For that we calculate the updated version of I_α which we denote by J_α whose explicit form is given by

$$
\begin{aligned}
J_\alpha \;=\;& I_\alpha + mK\mathrm{D}^2\mathcal{E}^{(u)}_\alpha + m\Lambda\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot\alpha}\bar{\mathcal{E}}^{(u)}_{\dot\alpha} \\
=\;& [2Ba_3]\mathrm{D}^2\bar{\mathrm{D}}^2\chi_\alpha && + [B\gamma + 2Kb_2 + \Lambda b_3]m\mathrm{D}^2\bar{\mathrm{D}}^{\dot\alpha}u_\alpha \\
& - [4 - K\gamma]m^2\mathrm{D}^2\chi_\alpha && + [Kb_3 + 2\Lambda b_2]m\mathrm{D}^2\bar{\mathrm{D}}^{\dot\alpha}\mathrm{D}_\alpha\bar{u}_{\dot\alpha} \\
& + [2 + \Lambda\gamma]m^2\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot\alpha}\bar{\chi}_{\dot\alpha} && + [\Lambda(2b_4 - b_3)]\mathrm{D}_\alpha\bar{\mathrm{D}}^2\mathrm{D}^\beta u_\beta \\
& + [2a_3]m^3\chi_\alpha && + \gamma m^3 u_\alpha \\
& + [Kb_5]m^2\mathrm{D}^2u_\alpha && + [\Lambda b_5]m^2\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot\alpha}\bar{u}_{\dot\alpha} \quad .
\end{aligned}
\tag{5.12}
$$

If we choose

$$
a_3 = 0 \;, \quad -4 + K\gamma = 0 \;, \quad 2 + \Lambda\gamma = 0 \quad , \tag{5.13}
$$

we derive an equation of motion for u_α in the form

$$
\begin{aligned}
J_\alpha =& [B\gamma + 2Kb_2 + \Lambda b_3]m\mathrm{D}^2\bar{\mathrm{D}}^{\dot\alpha}u_\alpha \; + [Kb_5]m^2\mathrm{D}^2u_\alpha \\
& + [Kb_3 + 2\Lambda b_2]m\mathrm{D}^2\bar{\mathrm{D}}^{\dot\alpha}\mathrm{D}_\alpha\bar{u}_{\dot\alpha} \; + [\Lambda b_5]m^2\mathrm{D}_\alpha\bar{\mathrm{D}}^{\dot\alpha}\bar{u}_{\dot\alpha} \\
& + [\Lambda(2b_4 - b_3)]\mathrm{D}_\alpha\bar{\mathrm{D}}^2\mathrm{D}^\beta u_\beta \\
& + \gamma m^3 u_\alpha
\end{aligned}
\tag{5.14}
$$

Now we are in position to choose coeffecients so as to make u_α vanish on-shell by selecting

$$
B\gamma + 2Kb_2 + \Lambda b_3 = 0\,,\;\; Kb_3 + 2\Lambda b_2 = 0\,,\;\; 2b_4 - b_3 = 0\,,\;\; b_5 = 0\,,\;\; \gamma \neq 0 \tag{5.15}
$$

Since $u_\alpha = 0$ on-shell, now we can reverse the arguments. Its equation of motion will give $\chi_\alpha = 0$ and that will put constraints on $H_{\alpha\dot\alpha}$: $\mathrm{D}^2\bar{\mathrm{D}}^{\dot\alpha}H_{\alpha\dot\alpha} = 0$

$$
\begin{aligned}
\mathcal{E}^{(H)}_{\alpha\dot\alpha} &= 2\mathrm{D}^\gamma\bar{\mathrm{D}}^2\mathrm{D}_\gamma H_{\alpha\dot\alpha} + 2a_2m(\mathrm{D}^2H_{\alpha\dot\alpha} + \bar{\mathrm{D}}^2H_{\alpha\dot\alpha}) + 2a_4m^2H_{\alpha\dot\alpha} \quad , \\
\mathcal{E}^{(\chi)}_\alpha &= -2\mathrm{D}^2\bar{\mathrm{D}}^{\dot\alpha}H_{\alpha\dot\alpha} \quad .
\end{aligned}
\tag{5.16}
$$

Finally because of $\mathrm{D}^2\bar{\mathrm{D}}^{\dot\alpha}H_{\alpha\dot\alpha} = 0$ we see that

$$
\mathrm{D}^\alpha\mathcal{E}^{(H)}_{\alpha\dot\alpha} = 2a_2m\mathrm{D}^\alpha\bar{\mathrm{D}}^2H_{\alpha\dot\alpha} + 2a_4m^2\mathrm{D}^\alpha H_{\alpha\dot\alpha} \quad . \tag{5.17}
$$

For $a_2 = 0,\; a_4 \neq 0$ this gives $\mathrm{D}^\alpha H_{\alpha\dot\alpha} = 0$. Thus the equation of motion for $H_{\alpha\dot\alpha}$ becomes the Klein-Gordon equation with $a_4 = 1$

$$
\Box H_{\alpha\dot\alpha} = m^2H_{\alpha\dot\alpha} \tag{5.18}
$$

To complete the analysis we look for the consistency and non-trivial solution of the systems of equations (20), (28), (30), $a_2 = 0$, and $a_4 = 1$. A solution exists and it is

$$
\begin{aligned}
&a_1 = 0 \ , \ b_1 = \text{free, can be set to zero} \ , \ \gamma = 1 \ , \ \Lambda = -2 \ , \\
&a_2 = 0 \ , \ b_2 = \frac{1}{6} \ , \qquad\qquad\qquad , \ A = 1 \ , \\
&a_3 = 0 \ , \ b_3 = \frac{1}{6} \ , \qquad\qquad\qquad , \ B = -1 \ , \\
&a_4 = 1 \ , \ b_4 = \frac{1}{12} \ , \qquad\qquad\qquad , \ K = 4 \ , \\
&\qquad\quad\ b_5 = 0 \ .
\end{aligned} \tag{5.19}
$$

The final action takes the form

$$
\begin{aligned}
S = \int d^8z \Big\{ & H^{\alpha\dot{\alpha}} \mathrm{D}^{\gamma} \bar{\mathrm{D}}^2 \mathrm{D}_{\gamma} H_{\alpha\dot{\alpha}} && + m u^{\alpha} \chi_{\alpha} + c.c. \\
& - 2\ H^{\alpha\dot{\alpha}} \bar{\mathrm{D}}_{\dot{\alpha}} \mathrm{D}^2 \chi_{\alpha} + c.c. && + \frac{1}{6} u^{\alpha} \bar{\mathrm{D}}^2 u_{\alpha} + c.c. \\
& - 2\ \chi^{\alpha} \mathrm{D}^2 \chi_{\alpha} + c.c. && + \frac{1}{6} u^{\alpha} \bar{\mathrm{D}}^{\dot{\alpha}} \mathrm{D}_{\alpha} \bar{u}_{\dot{\alpha}} \\
& + 2\ \chi^{\alpha} \mathrm{D}_{\alpha} \bar{\mathrm{D}}^{\dot{\alpha}} \bar{\chi}_{\dot{\alpha}} && + \frac{1}{12} u^{\alpha} \mathrm{D}_{\alpha} \bar{\mathrm{D}}^{\dot{\alpha}} \bar{u}_{\dot{\alpha}} \\
& + m^2 H^{\alpha\dot{\alpha}} H_{\alpha\dot{\alpha}} \Big\} \quad .
\end{aligned} \tag{5.20}
$$

This is the superspace action that describes a superspin $Y = \frac{3}{2}$ system with the minimum number of auxiliary superfields and has a massless limit that gives the free linearized non-minimal supergravity. This action is a representative of a family of actions that are all equivalent and connected through superfields redefinitions of the form

$$\chi_a \to \chi_\alpha + z_1 u_\alpha + w_1 \bar{\mathrm{D}}^{\dot{\alpha}} H_{\alpha\dot{\alpha}} \tag{5.21}$$

$$u_\alpha \to u_\alpha + z_2 \chi_\alpha + w_2 \bar{\mathrm{D}}^{\dot{\alpha}} H_{\alpha\dot{\alpha}}, \ \text{where } z_i, w_i \text{ are complex} \tag{5.22}$$

6 Summary and conclusions

We started with the $\frac{3}{2}$ superhelicity theory of free linearized non-minimal supergravity, formulated in terms of a real vector superfield $H_{\alpha\dot{\alpha}}$ and a fermionic compensator χ_α. We then added mass terms to it in an attempt to discover a theory for massive superspin $\frac{3}{2}$ system, only to find that it is not possible and we need the help of an extra fermionic auxiliary superfield u_α which must couple only to χ_α through a mass term. Finally using the equations of motion we manage to show that on-shell $u_\alpha = 0 \rightsquigarrow \chi_\alpha = 0 \rightsquigarrow \mathrm{D}^{\alpha} H_{\alpha\dot{\alpha}} = 0 \rightsquigarrow \Box H_{\alpha\dot{\alpha}} = m^2 H_{\alpha\dot{\alpha}}$

We have managed to derive yet another formulation of free massive supergravity supermultiplet and most importantly probe the set of auxiliary superfields required for the construction of higher superspin theories. The fermionic superfield u_α is the first non-trivial auxiliary superfield needed beyond the massless theory. As we go to even higher superspin values we should discover more and more of these objects. The hope is that after the study of some non-trivial low superspin examples, such as the one demonstrated here, we will have a deeper understanding on the number, type and role of these auxiliary objects. When that happens we might be in a position to construct the arbitrary massive superspin irreducible representation in an inductive manner.

Acknowledgments

This research has been supported in part by NSF Grant PHY-09-68854, the J. S. Toll Professorship endowment and the UMCP Center for String & Particle Theory.

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