New physics in $s \rightarrow d$ semileptonic transitions: rare hyperon vs. kaon decays

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ABSTRACT: We investigate the potential of rare hyperon decays to probe the short distance structure in the $s \rightarrow d\nu\bar{\nu}$ and $s \rightarrow d\ell^+\ell^-$ transitions. Hyperon decays into neutrinos ($B_1 \rightarrow B_2\nu\bar{\nu}$) can be reliably predicted by using form factors determined in baryon chiral perturbation theory. Their decay rates are sensitive to different short-distance operators, as compared to their kaon counterparts, and the corresponding branching fractions are in the range of $10^{-14} \sim 10^{-13}$ in the standard model. In the context of the low-energy effective theory, we find that the anticipated BESIII measurements of the $B_1 \rightarrow B_2\nu\bar{\nu}$ decays would lead to constraints on new physics in the purely axial vector $\bar{d}\gamma_\mu\gamma_5 s$ current that are stronger than the present limits from their kaon siblings $K \rightarrow \pi\pi\nu\bar{\nu}$. On the other hand, although hyperon decays into charged leptons are dominated by long-distance hadronic contributions, angular observable such as the leptonic forward-backward asymmetry is sensitive to the interference between long- and short-distance contributions. We discuss the sensitivity to new physics of a potential measurement of this observable in comparison with observables in the kaon decays $K_L \rightarrow \mu^+\mu^-$ and $K^+ \rightarrow \pi^+\mu^+\mu^-$. We conclude that the current kaon bounds are a few orders of magnitude better than those that could be obtained from $\Sigma^+ \rightarrow p\mu^+\mu^-$ except for two scenarios with new physics in the $(\bar{d}\gamma^\mu s)(\bar{\ell}\gamma_\mu\gamma_5\ell)$ and $(\bar{d}\gamma^\mu\gamma_5 s)(\bar{\ell}\gamma_\mu\ell)$ currents. Finally, we point out that the loop effects from renormalization group evolution are important in this context, when relating the low-energy effective field theory to new physics models in the UV.

KEYWORDS: Beyond Standard Model, Effective Field Theories, Kaon Physics, Nonperturbative Effects

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1 Introduction

The flavor-changing neutral current (FCNC) transitions $s \rightarrow d\nu\bar{\nu}$ and $s \rightarrow d\ell^+\ell^-$ are loop-induced and suppressed by the GIM mechanism [1] in the standard model (SM). Therefore they are ideal processes to test the SM and search for new physics (NP). Considerable effort has been invested in the kaon decays induced by these transitions (see refs. [2, 3] for comprehensive reviews). Recently, the neutrino modes $K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$ were measured by the NA62 Collaboration [4, 5] and KOTO Collaboration [6, 7]. The best upper limits of the corresponding branching fractions at 90% confidence level (CL) are, respectively:

$$\text{BR}(K^+ \rightarrow \pi^+\nu\bar{\nu}) < 1.78 \times 10^{-10}, \quad (1.1)$$

$$\text{BR}(K_L \rightarrow \pi^0\nu\bar{\nu}) < 3.0 \times 10^{-9}. \quad (1.2)$$

Both measurements show a slight excess with respect to the SM expectations $\sim 10^{-11}$ (see section 3.2 and the most recent predictions [8, 9]), which leaves little room for NP from

vectorial couplings in the $s \rightarrow d$ currents. On the other hand, the experimental upper bounds of $K^+ \rightarrow \pi^+ \pi^0 \nu \bar{\nu}$ [10] and $K_L \rightarrow \pi^0 \pi^0 \nu \bar{\nu}$ [11] at 90% CL,

$$\text{BR}(K^+ \rightarrow \pi^+ \pi^0 \nu \bar{\nu}) < 4.3 \times 10^{-5}, \quad (1.3)$$

$$\text{BR}(K_L \rightarrow \pi^0 \pi^0 \nu \bar{\nu}) < 8.1 \times 10^{-7}, \quad (1.4)$$

are still very far from the SM predictions $\sim 10^{-15} - 10^{-13}$ (see section 3.2 and refs. [12–14]). Thus, these decays provide only loose constraints on NP from axial-vectorial couplings which are not directly accessible in $K \rightarrow \pi \nu \bar{\nu}$. In addition, the charged-lepton modes $K_L \rightarrow \mu^+ \mu^-$ and $K^+ \rightarrow \pi^+ \mu^+ \mu^-$, whose decay rates are dominated by long-distance contributions, cannot probe all the possible NP interactions in the $s \rightarrow d \ell^+ \ell^-$ decays. Therefore, it would be important to find other probes and measurements that allow us to thoroughly search for NP in these strangeness-changing transitions.

Hyperons, which carry net baryon number one and half-integer spin, can lead to substantially different decay modes, observables, and sensitivity to the underlying structure of the $s \rightarrow d \nu \bar{\nu}$ and $s \rightarrow d \ell^+ \ell^-$ transitions. On the experimental side, the BESIII Collaboration has started a campaign of measurements of the $B_1 \rightarrow B_2 \nu \bar{\nu}$ decays by collecting hyperon-antihyperon resonant pair production at the J/ψ peak [15–17]. Thus, this experiment can be run as a “hyperon factory” in the characteristically clean environment of an e^+e^- collider, enabling the measurement of several semileptonic rare decays. In addition, the LHCb Collaboration has also started an ambitious program with the measurement of the branching fraction of the $\Sigma^+ \rightarrow p \mu^+ \mu^-$ decay [18] (after first evidence reported by the HyperCP Collaboration [19]), which is expected to be followed by measurements of angular asymmetries after Run 2 of the LHC [20, 21].

Until quite recently, little theoretical attention has been paid to rare hyperon decays. In a series of works [22–27], the authors have studied the rare hyperon decays $B_1 \rightarrow B_2 \nu \bar{\nu}$ and $\Sigma^+ \rightarrow p \mu^+ \mu^-$. However, only in the decays with missing energy these have been compared to the corresponding kaon modes [22, 25, 26].

In this work we systematically investigate the potential of rare hyperon decays to probe the $s \rightarrow d$ short-distance structure in both neutrino and charged-lepton modes, putting special emphasis in the complementarity with the corresponding decays of their kaon siblings. In comparison with previous works, we also improve the nonperturbative inputs required for the form factors entering these decays by using model-independent methods such as isospin symmetry in $K \rightarrow \pi \pi \nu \bar{\nu}$ or baryon chiral perturbation theory (BChPT) in the hyperon decays. Finally, we discuss the importance of renormalization group evolution (RGE) effects in connecting new physics models to the low energy effective field theory at the electroweak scale.

In section 2, we first provide predictions for the branching ratios of the neutrino modes in the SM and study their sensitivity to effective operators of the weak Hamiltonian in the presence of NP. In section 3 we discuss the bounds obtained from the $B_1 \rightarrow B_2 \nu \bar{\nu}$ decays, which are compared with those stemming from the kaon decays $K \rightarrow \pi \nu \bar{\nu}$ and $K \rightarrow \pi \pi \nu \bar{\nu}$. Decays into charged leptons are dominated by long-distance contributions but are also phenomenologically richer, with observables in the angular distributions potentially sensitive to

short-distance structure. In section 4 we show that, indeed, the leptonic forward-backward asymmetry A_{FB} is such an observable [24, 28] and it is sensitive to the interference between the long- and short-distance contributions. Finally, we present predictions and compare the sensitivity to NP of the $\Sigma^+ \rightarrow p\mu^+\mu^-$ decay to that of the rare kaon decays. In section 5, we discuss the importance of renormalization group evolution effects. A short summary and outlook is given in section 6.

2 Low-energy effective Hamiltonian

Semileptonic rare $s \rightarrow d$ decays in the SM are described by the following effective Hamiltonian [2, 3]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \lambda_t \left(\sum_{i=1}^{10} C_i O_i + \sum_{\ell=e,\mu,\tau} C_{\nu_\ell}^L O_{\nu_\ell}^L \right), \quad (2.1)$$

where G_F is the Fermi constant, C_i and C_{ν_ℓ} are the Wilson coefficients (WCs), and $\lambda_q = V_{qs}V_{qd}^*$ is defined by the product of the two Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The subscript q represents u, c , or t quark. The operators $O_{1,2}$ are the “current-current” Fermi interactions from tree-level W exchanges, O_{3-6} are the QCD penguin-loop contributions, and O_8 is the “chromomagnetic” operator. The remaining operators will be explained below and one can find more details in refs. [2, 29].

Decays into neutrinos are described by the following operator

$$O_{\nu_\ell}^L = \alpha \left(\bar{d} \gamma_\mu (1 - \gamma_5) s \right) (\bar{\nu}_\ell \gamma^\mu (1 - \gamma_5) \nu_\ell), \quad (2.2)$$

and Wilson coefficient

$$C_{\nu_\ell}^L = \frac{1}{2\pi \sin^2 \theta_w} \left(\frac{\lambda_c}{\lambda_t} X_c^\ell + X_t \right). \quad (2.3)$$

Here α and $\sin^2 \theta_w$ denote the electromagnetic coupling and the weak mixing angle, respectively. The contributions of the charm- and top-quark are characterized by X_c^ℓ and X_t , while that of the up-quark is much suppressed by the GIM mechanism. The first term in eq. (2.3) has some mild sensitivity to the charged-lepton flavor in the box-diagram and therefore it has the corresponding label in the Wilson coefficient. The ℓ stands for e, μ , and τ lepton respectively. In the numerical analysis, we take $X_t = 1.48$ considering the next-to-leading order (NLO) QCD corrections [2, 30] and two-loop electroweak corrections [31]. For X_c^ℓ , we take $X_c^e \simeq X_c^\mu = 1.04 \times 10^{-3}$ and $X_c^\tau = 0.70 \times 10^{-3}$ obtained by the renormalization group (RG) calculation with the next-to-leading logarithmic approximation [2, 30].¹ The $\alpha/\sin^2 \theta_w$ is a renormalization scheme dependent quantity. The scheme dependence can be removed by considering higher order electroweak effects in $K \rightarrow \pi\nu\bar{\nu}$. Here we adopt the modified minimal subtraction scheme ($\overline{\text{MS}}$) definitions of α and $\sin^2 \theta_w$, i.e., $\alpha = \alpha_{\overline{\text{MS}}}(M_Z) = 1/127.952$ and $\sin^2 \theta_w = \sin^2 \hat{\theta}_w^{\overline{\text{MS}}}(M_Z) = 0.23121$ from the Particle

¹We note that the combination of $\frac{1}{\lambda^4}(\frac{2}{3}X_c^e + \frac{1}{3}X_c^\tau) = 0.36$ is consistent with that determined in refs. [32–34], which considered NNLO QCD and two-loop electroweak corrections.

Data Group (PDG) review [35]. These higher order electroweak corrections in $K \rightarrow \pi \nu \bar{\nu}$ with the $\overline{\text{MS}}$ scheme are found below 2% [31, 36]. We take the Wolfenstein parameters for the CKM matrix and the Fermi constant G_F from the most recent pdgLive [35].

The four-quark operators contributing to the $s \rightarrow d\ell^+\ell^-$ transitions, $O_{7,9,10}$, are defined as follows

$$\begin{aligned} O_7 &= i \frac{e}{4\pi} m_s \bar{d} \sigma_{\mu\nu} (1 + \gamma_5) s F^{\mu\nu}, \\ O_9 &= \alpha \left(\bar{d} \gamma_\mu (1 - \gamma_5) s \right) \left(\bar{\ell} \gamma^\mu \ell \right), \quad O_{10} = \alpha \left(\bar{d} \gamma_\mu (1 - \gamma_5) s \right) \left(\bar{\ell} \gamma^\mu \gamma_5 \ell \right), \end{aligned} \quad (2.4)$$

where $\alpha = e^2/4\pi$ and $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2$ [37]. These Lagrangians stem from electromagnetic penguin-, electroweak penguin-, and box-loop diagrams. The Wilson coefficients can be expressed as $C_7 = \frac{C_\gamma^c}{4\pi} \frac{\lambda_c}{\lambda_t}$, $C_9 = z_{7V} \frac{\lambda_u}{\lambda_t} - y_{7V}$, and $C_{10} = -y_{7A}$. The Wilson coefficient C_7 is dominated by the charm penguin-loop contribution. The terms $y_{7V,7A}$ are produced by integrating out the top quark at $\mu = m_W$ while $z_{7V}(\mu)$ are the corresponding ones by integrating out the charm quark at $\mu = m_c$. Both coefficients z_{7V} and y_{7V} depend on the renormalization scale through the mixing of four-quark operators in QCD, which must be canceled at the observable level by long-distance contributions. The axial vector coupling to the leptons, y_{7A} , is not renormalized in QCD. The long-distance interactions dominating the $s \rightarrow d\ell^+\ell^-$ modes couple vectorially to the leptons and are difficult to calculate from first principles. Specifically, the long-distance amplitudes for the $B_1 \rightarrow B_2 \ell^+\ell^-$ decays can be parameterized as [23]

$$\mathcal{M}_{\text{LD}} = -\frac{e^2 G_F}{q^2} \bar{B}_2 \sigma_{\mu\nu} q^\nu (a + b\gamma_5) B_1 \bar{\ell}^- \gamma^\mu \ell^+ - e^2 G_F \bar{B}_2 \gamma_\mu (c + d\gamma_5) B_1 \bar{\ell}^- \gamma^\mu \ell^+, \quad (2.5)$$

where a , b , c and d are complex form factors, and B_1 and B_2 denote the initial and final octet baryons, respectively. For the WCs in the $s \rightarrow d\ell^+\ell^-$ transitions, we use the NLO QCD values $z_{7V}(1 \text{ GeV}) = -0.046$, $y_{7V}(1 \text{ GeV}) = 0.735$, and $y_{7A} = y_{7A}(m_W) = -0.700$ obtained by choosing the “naive dimensional regularization” scheme with γ_5 taken to be anticommuting [30].² It has been pointed out in ref. [30] that the operator O_7 does not influence the WCs of other operators since its contribution is suppressed by an additional factor m_s . Here we take the value $C_\gamma^c = 0.13$ derived from the short-distance contribution of the weak radiative transition $s \rightarrow d\gamma$ [23, 38].

Beyond the SM and for scenarios without right-handed neutrinos, the only new operator that can contribute to $s \rightarrow d\nu\bar{\nu}$ is $O_{\nu\ell}^R$, which is the same as eq. (2.2) but with a chiral flip in the quark current. In the decays into charged leptons, we can also have semileptonic operators with a chiral flip in the quark current ($O'_{9,10}$) or (pseudo) scalar operators, but not tensors if NP is significantly heavier than the electroweak scale [39].³ However, tensor

²Another possible choice is the 't-Hooft-Veltman scheme with non-anticommuting γ_5 .

³In our convention, these operators are written as follows:

$$\begin{aligned} O_S &= \alpha \left(\bar{d}(1 + \gamma_5)s \right) \left(\bar{\ell}\ell \right), & O'_S &= \alpha \left(\bar{d}(1 - \gamma_5)s \right) \left(\bar{\ell}\ell \right), \\ O_P &= \alpha \left(\bar{d}(1 + \gamma_5)s \right) \left(\bar{\ell}\gamma_5\ell \right), & O'_P &= \alpha \left(\bar{d}(1 - \gamma_5)s \right) \left(\bar{\ell}\gamma_5\ell \right). \end{aligned}$$

	Λn	$\Sigma^+ p$	$\Xi^- \Sigma^-$	$\Xi^0 \Sigma^0$	$\Xi^0 \Lambda$
$f_1(0)$	-1.22(6)	-1.00(5)	1.00(5)	-0.71(4)	1.22(6)
$g_1(0)$	-0.88(2)	0.33(2)	1.22(4)	-0.86(3)	0.21(4)

Table 1. Values of vector and axial vector couplings, $f_1(0)$ and $g_1(0)$, for the $B_1 \rightarrow B_2 \nu \bar{\nu}$ decays studied in this work. The uncertainty in $g_1(0)$ stems from experimental data [45]. The one in $f_1(0)$ corresponds to $\mathcal{O}(\delta^2)$ corrections estimated as a 5% relative uncertainty by comparing the SU(3)-symmetric prediction with explicit calculations in LQCD [47].

operators are nonvanishing in the nonlinear effective field theory of ref. [40]. In this work, we assume that all the WCs are real as we refrain from discussing CP violation.

3 $s \rightarrow d \nu \bar{\nu}$ processes

3.1 $B_1 \rightarrow B_2 \nu \bar{\nu}$ decays

Let us first investigate the rare hyperon decays into neutrinos. As pointed out in refs. [27, 41], the vector transition matrix elements for the decuplet-to-octet transitions are not well determined because of lack of experimental data for the electric-quadrupole and magnetic-dipole transition form factors. Therefore, in the following we only study five rare octet-hyperon decays: $\Lambda \rightarrow n \nu \bar{\nu}$, $\Sigma^+ \rightarrow p \nu \bar{\nu}$, $\Xi^- \rightarrow \Sigma^- \nu \bar{\nu}$, $\Xi^0 \rightarrow \Sigma^0 \nu \bar{\nu}$, and $\Xi^0 \rightarrow \Lambda \nu \bar{\nu}$. Following ref. [42], one can obtain the total decay width in the presence of NP, which, expanded up to NLO in terms of the SU(3) breaking parameter δ , reads

$$\Gamma = \sum_{\ell=e,\mu,\tau} \frac{\alpha^2 G_F^2 |\lambda_t|^2 f_1(0)^2 \Delta^5}{60\pi^3} \cdot \left[\left(1 - \frac{3}{2}\delta\right) |C_{\nu_\ell}^L + C_{\nu_\ell}^R|^2 + 3 \left(1 - \frac{3}{2}\delta\right) \frac{g_1(0)^2}{f_1(0)^2} |C_{\nu_\ell}^L - C_{\nu_\ell}^R|^2 + \mathcal{O}(\delta^2) \right], \quad (3.1)$$

where $\Delta = M_1 - M_2$ and $\delta = \Delta/M_1$. NP can enter with both chiralities $C_{\nu_\ell}^L = C_{\nu_\ell}^{L,\text{SM}} + \delta C_{\nu_\ell}^L$ and $C_{\nu_\ell}^R$. The parameterization of relevant hadronic matrix elements can be found in appendix A.

The $B_1 \rightarrow B_2$ form factors can be related to those entering the semileptonic hyperon decays $B_1 \rightarrow B_2 \ell^- \bar{\nu}$ [43] using isospin symmetry. As shown in eq. (3.1), only the vector and axial vector couplings, $f_1(0)$ and $g_1(0)$, are necessary for our predictions.⁴ For consistency, the axial vector couplings $g_1(0)$ are obtained by including the $\mathcal{O}(\delta)$ corrections calculated in chiral perturbation theory [45]. The vector couplings $f_1(0)$ are the SU(3) symmetric values which are protected from $\mathcal{O}(\delta)$ corrections by the Ademollo-Gatto theorem [46]. For the sake of convenience, we collect these results in table 1. In addition, at $\mathcal{O}(\delta)$ one has in eq. (3.1) the contribution from the weak-magnetic form factor $g_2(0)$, which is neglected because $g_2(0) \sim \mathcal{O}(\delta)$.

⁴The q^2 -dependent scales can be expanded in powers of $q^2/M_X^2 \sim \delta^2$, where $M_X \sim 1 \text{ GeV}$ is a hadronic scale related to the mass of the resonance coupling to a given current [44].

Decay modes	Λn	$\Sigma^+ p$	$\Xi^- \Sigma^-$	$\Xi^0 \Sigma^0$	$\Xi^0 \Lambda$
$10^{13} \times \text{BR}(B_1 \rightarrow B_2 \nu \bar{\nu})^{\text{SM}}$	6.26(16)	3.49(16)	1.10(1)	0.89(1)	5.52(13)
$10^6 \times \text{BR}(B_1 \rightarrow B_2 \nu \bar{\nu})^{\text{BESIII}}$	< 0.3	< 0.4	—	< 0.9	< 0.8
$10^3 \times \delta C_{\nu_\ell}^L + C_{\nu_\ell}^R $	< 1.6	< 1.7	—	< 10	< 1.8
$10^3 \times \delta C_{\nu_\ell}^L - C_{\nu_\ell}^R $	< 1.3	< 3.4	—	< 5.2	< 8.6

Table 2. Predicted branching fractions of the $B_1 \rightarrow B_2 \nu \bar{\nu}$ decays valid up to NLO corrections in the SU(3)-flavor-breaking expansion. The combinations of the two Wilson coefficients are constrained by the anticipated BESIII data [15] at 90% CL. The theoretical uncertainties of branching fractions come from the missing $\mathcal{O}(\delta^2)$ terms in eq. (3.1).

In table 2, we show the results for the semileptonic rare hyperon decays into neutrinos. We first show the predicted branching fractions of $B_1 \rightarrow B_2 \nu \bar{\nu}$ valid up to NLO corrections in the SU(3)-flavor-breaking expansion. The theoretical uncertainties of these branching fractions originate from the naive-dimensional estimate of the missing $\mathcal{O}(\delta^2)$ terms in eq. (3.1). In addition, we find that our branching fractions of the $B_1 \rightarrow B_2 \nu \bar{\nu}$ decays in the SM are 10% $\sim$ 60% smaller than those given in refs. [22, 25–27]. This could be attributed to the improved nonperturbative inputs for the form factors.

Next, we present the ranges of NP contributions at 90% CL allowed by the constraints of the BESIII projections for the near future [15]. One sees that the BESIII projections do not include the $\Xi^- \rightarrow \Sigma^- \nu \bar{\nu}$ decay because it cannot be detected at BESIII due to the ineffectiveness of the “tag technique” [15] for this mode. From the constraints of the anticipated BESIII data, we find that among the five $B_1 \rightarrow B_2 \nu \bar{\nu}$ decays, the $\Lambda \rightarrow n \nu \bar{\nu}$ decay would be the one most sensitive to new physics.

3.2 $K \rightarrow \pi \nu \bar{\nu}$ and $K \rightarrow \pi \pi \nu \bar{\nu}$ decays

The $K \rightarrow \pi \nu \bar{\nu}$ and $K \rightarrow \pi \pi \nu \bar{\nu}$ decays also probe the $s \rightarrow d \nu \bar{\nu}$ processes at the quark level. Theoretically, the form factors of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ can be related via isospin symmetry to the semileptonic charged-current $K^+ \rightarrow \pi^0 e^+ \nu_e$ decay, which are well known experimentally. Using the effective Hamiltonian of eq. (2.1) and applying the isospin symmetry relations (see, e.g., ref. [29]), one can easily obtain the branching fractions of the two $K \rightarrow \pi \nu \bar{\nu}$ processes in the presence of NP as

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \frac{2\alpha^2 |\lambda_t|^2 \text{BR}(K^+ \rightarrow \pi^0 e^+ \nu_e)}{|V_{us}|^2} \sum_{\ell=e,\mu,\tau} |C_{\nu_\ell}^L + C_{\nu_\ell}^R|^2, \quad (3.2)$$

$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \frac{2\alpha^2 \tau_{K_L} \text{BR}(K^+ \rightarrow \pi^0 e^+ \nu_e)}{\tau_{K^+} |V_{us}|^2} \sum_{\ell=e,\mu,\tau} \left(\text{Im}[(C_{\nu_\ell}^L + C_{\nu_\ell}^R) \lambda_t] \right)^2, \quad (3.3)$$

where τ_{K_L} and τ_{K^+} are the lifetimes of the K_L and K^+ mesons and $\lambda_t = V_{ts} V_{td}^*$. For the branching fraction of $K^+ \rightarrow \pi^0 e^+ \nu_e$, we take the value $(5.07 \pm 0.04) \times 10^{-2}$ from the PDG average [35]. Throughout the present work, we do not consider isospin breaking effects.

In this work, we study the $K^+ \rightarrow \pi^+ \pi^0 \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \pi^0 \nu \bar{\nu}$ decays in helicity basis (details can be found in appendix B). The form factors for the $K_L \rightarrow \pi^0 \pi^0$ and

$K^+ \rightarrow \pi^+\pi^0$ transitions only contain $I = 0$ and $I = 1$ components (of the $\pi\pi$ pair) for S - and P -waves, respectively. The contributions of D - and higher partial waves between the two pions in these form factors have been neglected. These form factors can be connected to those measured in the charged current $K^+ \rightarrow \pi^+\pi^-e^+\nu$ decay by isospin symmetry as follows [12, 13, 48]:

$$\begin{aligned}\langle \pi^+\pi^0 | (\bar{s}d)_{V-A} | K^+ \rangle &= -\sqrt{2} \langle (\pi^+\pi^-)_{I=1} | (\bar{s}u)_{V-A} | K^+ \rangle, \\ \langle \pi^0\pi^0 | (\bar{s}d)_{V-A} | K^0 \rangle &= \langle (\pi^+\pi^-)_{I=0} | (\bar{s}u)_{V-A} | K^+ \rangle.\end{aligned}\quad (3.4)$$

All the form factors for the $K^+ \rightarrow \pi^+\pi^-$ decay can be described as a series expansion of the dimensionless invariants $q^2 = s_\pi/4m_\pi^2 - 1$ and $s_\ell/4m_\pi^2$, and the coefficients of relevant expansions, as well as the difference of the rescattering phases $\delta = \delta_s - \delta_p$ have been determined experimentally from the angular analysis of the $K^+ \rightarrow \pi^+\pi^-e^+\nu$ decay [49, 50].⁵

Therefore, the branching fractions of the four-body $K \rightarrow \pi\pi\nu\bar{\nu}$ decays as functions of the two Wilson coefficients can be expressed as

$$\text{BR}(K^+ \rightarrow \pi^+\pi^0\nu\bar{\nu}) = \sum_{\ell=e,\mu,\tau} \left(3.13 \times 10^{-3} \left| C_{\nu\ell}^L + C_{\nu\ell}^R \right|^2 + 1.41 \left| C_{\nu\ell}^L - C_{\nu\ell}^R \right|^2 \right) \times 10^{-15}, \quad (3.5)$$

$$\text{BR}(K_L \rightarrow \pi^0\pi^0\nu\bar{\nu}) = \sum_{\ell=e,\mu,\tau} \left(\text{Re} \left[(-6.35 - 2.72i) \left(C_{\nu\ell}^L - C_{\nu\ell}^R \right) \right] \right)^2 \times 10^{-15}, \quad (3.6)$$

where the numerical coefficients have been obtained by integrating over phase space. In eqs. (3.5) and (3.6), we do not include hadronic uncertainties stemming from form factors. Compared with the exact numerical calculation using the formulae given in appendix B, we find that the hadronic uncertainties induce effects of less than 1% on the branching fractions of the $K \rightarrow \pi\pi\nu\bar{\nu}$ decays.

In table 3, we show the branching fractions of the $K \rightarrow \pi\nu\bar{\nu}$ and $K \rightarrow \pi\pi\nu\bar{\nu}$ decays in the SM and the ranges of NP allowed by the experiment measurements at 90% CL. For the two $K \rightarrow \pi\nu\bar{\nu}$ decays, the SM predictions are consistent with those of ref. [8] and are close to the current experiment measurements. From eqs. (3.2) and (3.3), one can easily see that the theoretical uncertainties for the branching fractions of the $K \rightarrow \pi\nu\bar{\nu}$ decays originate from the current experiment measurement of $\text{BR}(K^+ \rightarrow \pi^0e^+\nu_e) = (5.07 \pm 0.04) \times 10^{-2}$ in the absence of isospin breaking effects. Both decay modes, especially $K^+ \rightarrow \pi^+\nu\bar{\nu}$, constrain strongly $\delta C_{\nu\ell}^L + C_{\nu\ell}^R$ but they are not sensitive to the combination of $\delta C_{\nu\ell}^L - C_{\nu\ell}^R$. On the other hand, following ref. [52] one can easily derive by means of isospin symmetry a Grossman-Nir (GN) bound,

$$\text{BR}(K_L \rightarrow \pi^0\nu\bar{\nu})_{\text{exp}} \leq 4.3 \text{BR}(K^+ \rightarrow \pi^+\nu\bar{\nu})_{\text{exp}}. \quad (3.7)$$

Using eq. (1.1), the standard GN bound gives $\text{BR}(K_L \rightarrow \pi^0\nu\bar{\nu})_{\text{exp}} \leq 7.7 \times 10^{-10}$. This bound is stronger than the current direct bounds of 3.0×10^{-9} and 4.9×10^{-9} at 90% CL from the KOTO experiment [6, 7].

⁵See also ref. [51] for the analysis of $K^+ \rightarrow \pi^0\pi^0$, but this process has less statistics and we will not consider it in our present study.

Decay modes	$K^+\pi^+$	$K_L\pi^0$	$K^+\pi^+\pi^0$	$K_L\pi^0\pi^0$
BR^{SM}	$8.55(4)\times 10^{-11}$	$2.89(1)\times 10^{-11}$	$8.35(22)\times 10^{-15}$	$2.59(3)\times 10^{-13}$
BR^{Expt}	$< 1.78\times 10^{-10}$	$< 3.0\times 10^{-9}$	$< 4.3\times 10^{-5}$	$< 8.1\times 10^{-7}$
$\delta C_{\nu_\ell}^L + C_{\nu_\ell}^R$	$(-3.4, 0.6)$	$(-11.5, 9.4)$	$(-2.2, 2.2)\times 10^6$	—
$\delta C_{\nu_\ell}^L - C_{\nu_\ell}^R$	—	—	$(-1.1, 1.1)\times 10^5$	$(-2.7, 2.7)\times 10^3$

Table 3. Predictions in the SM for the branching fractions of $K \rightarrow \pi\nu\bar{\nu}$ and $K \rightarrow \pi\pi\nu\bar{\nu}$. The constraints on the combinations of two Wilson coefficients are derived from the experimental measurements at 90% CL [4, 6, 10, 11].

Figure 1. 90% CL constraints on $\delta C_{\nu_\ell}^L + C_{\nu_\ell}^R$ and $\delta C_{\nu_\ell}^L - C_{\nu_\ell}^R$ from the BESIII projections in the $B_1 \rightarrow B_2\nu\bar{\nu}$ decays (region in red) and the current experiment data in the kaon decay modes (regions in green and blue).

For the $K^+ \rightarrow \pi^+\pi^0\nu\bar{\nu}$ decay, our prediction for the branching fraction in the SM is 20% smaller than that of ref. [12] while it is 40% larger than that of ref. [13] for the $K_L \rightarrow \pi^0\pi^0\nu\bar{\nu}$ decay, due to different inputs for the form factors. The theoretical uncertainties of the $K \rightarrow \pi\pi\nu\bar{\nu}$ branching fractions in table 3 originate from form factors introduced in appendix B. It is clear that the $K \rightarrow \pi\pi\nu\bar{\nu}$ decays determine poorly $\delta C_{\nu_\ell}^L + C_{\nu_\ell}^R$ but they can better constrain $\delta C_{\nu_\ell}^L - C_{\nu_\ell}^R$. In addition, the $K_L \rightarrow \pi^0\pi^0\nu\bar{\nu}$ decay provides a constraint on NP more stringent than the $K^+ \rightarrow \pi^+\pi^0\nu\bar{\nu}$ decay.

In figure 1, we compare the $B_1 \rightarrow B_2\nu\bar{\nu}$ decays with their kaon siblings and conclude that $\delta C_{\nu_\ell}^L + C_{\nu_\ell}^R$ can be well determined by the kaon modes. However future measurements of the $B_1 \rightarrow B_2\nu\bar{\nu}$ at BESIII could lead to better constraints on $\delta C_{\nu_\ell}^L - C_{\nu_\ell}^R$ compared to their kaon siblings. Therefore, it would be interesting to search for these semileptonic rare hyperon decays at BESIII.

4 $s \rightarrow d\ell^+\ell^-$ processes

4.1 $B_1 \rightarrow B_2\ell^+\ell^-$ decays

In this subsection, we investigate the rare hyperon decays into charged leptons. Using eqs. (2.1), (2.4) and (2.5) we calculate the relevant observables in helicity basis, because

then the interference between the long-distance and short-distance contributions in the leptonic forward-backward asymmetry becomes rather transparent (see appendix C for more details). Furthermore, following ref. [42], we expand the short-distance contributions in terms of the SU(3)-flavor-breaking expansion up to $\mathcal{O}(\delta)$ and keep the long-distance amplitudes unchanged, and then we obtain

$$\frac{d\Gamma}{d\cos\theta_\ell} = \mathcal{N}_f \left[I_1 + I_2 \cos\theta_\ell + I_3 \cos^2\theta_\ell \right], \quad (4.1)$$

with

$$\begin{aligned} I_1 &= \left(\frac{137.06}{\Delta^2 f_1(0)^2} \right) \left(1 - \frac{3}{2}\delta \right) \left| \frac{a}{\lambda_t} \right|^2 + \left(\frac{58.50}{f_1(0)^2} \right) \left(1 - \frac{3}{2}\delta \right) \left| \frac{c}{\lambda_t} \right|^2 \\ &\quad + \left(\frac{1221.67}{\Delta^2 f_1(0)^2} \right) \left(1 - \frac{3}{2}\delta \right) \left| \frac{b}{\lambda_t} \right|^2 + \left(\frac{974.60}{f_1(0)^2} \right) \left(1 - \frac{3}{2}\delta \right) \left| \frac{d}{\lambda_t} \right|^2 \\ &\quad + \left(\frac{168.52}{\Delta f_1(0)^2} \delta \right) \text{Re} \left[\frac{ac^*}{\lambda_t \lambda_t^*} \right] - \left(\frac{2199.79}{\Delta f_1(0)^2} \right) \left(1 - \frac{3}{2}\delta \right) \text{Re} \left[\frac{bd^*}{\lambda_t \lambda_t^*} \right], \\ I_2 &= (C_S + C'_S) \text{Re} \left[\left(\frac{2.32}{\Delta f_1(0)} \delta \right) \frac{f_S(0)}{f_1(0)} \left(\frac{a}{\lambda_t} \right)^* + \left(\frac{4.64}{f_1(0)} \right) \left(1 - \frac{3}{2}\delta \right) \frac{f_S(0)}{f_1(0)} \left(\frac{c}{\lambda_t} \right)^* \right] \\ &\quad + (C_S - C'_S) \text{Re} \left[\left(\frac{2.32}{\Delta f_1(0)} \delta \right) \frac{g_P(0)}{f_1(0)} \left(\frac{b}{\lambda_t} \right)^* - \left(\frac{2.32}{f_1(0)} \delta \right) \frac{g_P(0)}{f_1(0)} \left(\frac{d}{\lambda_t} \right)^* \right] \\ &\quad + (C_{10} + C'_{10}) \text{Re} \left[\left(\frac{5.51}{\Delta f_1(0)} \delta \right) \left(\frac{f_1(0) + 2f_2(0)}{f_1(0)} \right) \left(\frac{b}{\lambda_t} \right)^* \right. \\ &\quad \left. - \left(\frac{4.68}{f_1(0)} \delta \right) \left(\frac{f_1(0) + 2f_2(0)}{f_1(0)} \right) \left(\frac{d}{\lambda_t} \right)^* \right] \\ &\quad + (C_{10} - C'_{10}) \text{Re} \left[\left(\frac{11.02}{\Delta f_1(0)} \right) \frac{g_1(0)}{f_1(0)} \left(1 - \frac{3}{2}\delta \right) \left(\frac{a}{\lambda_t} \right)^* + \left(\frac{4.68}{f_1(0)} \delta \right) \frac{g_1(0)}{f_1(0)} \left(\frac{c}{\lambda_t} \right)^* \right], \\ I_3 &= \left(\frac{8.30}{\Delta^2 f_1(0)^2} \right) \left(1 - \frac{3}{2}\delta \right) \left| \frac{a}{\lambda_t} \right|^2 - \left(\frac{6.99}{f_1(0)^2} \right) \left(1 - \frac{3}{2}\delta \right) \left| \frac{c}{\lambda_t} \right|^2 \\ &\quad + \left(\frac{9.00}{\Delta^2 f_1(0)^2} \right) \left(1 - \frac{3}{2}\delta \right) \left| \frac{b}{\lambda_t} \right|^2 - \left(\frac{7.00}{f_1(0)^2} \right) \left(1 - \frac{3}{2}\delta \right) \left| \frac{d}{\lambda_t} \right|^2, \end{aligned} \quad (4.2)$$

where $\Delta = M_1 - M_2$, $\delta = \Delta/M_1$ and $\mathcal{N}_f = G_F^2 \alpha^2 |\lambda_t|^2 \Delta^5 f_1(0)^2 / (2048\pi^3)$. The NP contributions are characterized by $\delta C_i = C_i - C_i^{\text{SM}}$ and C'_i . We have neglected short-distance contributions when long-distance contributions are present. The θ_ℓ is the angle between the three-momenta of the final state particle B_2 and the direction of flight of the ℓ^- measured in the dilepton rest frame. The local form factors f_1 , f_2 , g_1 , f_S , and g_P appear in the parameterized hadronic matrix elements explicitly given in appendix A. The form factors a , b , c , and d characterizing long-distance contributions stem from eq. (2.5). The decay width and leptonic forward-backward asymmetry for the $B_1 \rightarrow B_2 \ell^+ \ell^-$ decays can be expressed in terms of I_1 , I_2 , and I_3 , which are

$$\Gamma_{\Sigma^+} = 2\mathcal{N}_f \left(I_1 + \frac{1}{3}I_3 \right), \quad (4.3)$$

$$A_{FB, \Sigma^+} = \frac{\mathcal{N}_f \cdot I_2}{\Gamma_{\Sigma^+}}. \quad (4.4)$$

	$a \times 10^{-3}$	$b \times 10^{-3}$	$\text{BR} \times 10^8$	$A_{FB} \times 10^5$
Case 1	$13.3 + 2.85i$	$-6.0 - 1.84i$	1.6	-1.4
Case 2	$-13.3 + 2.85i$	$6.0 - 1.84i$	3.3	0.2
Case 3	$6.0 + 2.85i$	$-13.3 - 1.84i$	5.2	0.6
Case 4	$-6.0 + 2.85i$	$13.3 - 1.84i$	8.9	-0.5

Table 4. Predicted branching fraction and leptonic forward-backward asymmetry A_{FB} in the SM for the $\Sigma^+ \rightarrow p\mu^+\mu^-$ decay, where a and b are in units of GeV.

In particular, for the $\Sigma^+ \rightarrow p\mu^+\mu^-$ decay, the form factors $f_S(0)$ and $g_P(0)$ from short-distance contributions can be obtained using the conservation of vector current (CVC) and the partial conservation of axial current (PCAC) in QCD:

$$f_S(0) = \frac{M_{\Sigma^+} - M_p}{m_s - m_d} f_1(0), \quad (4.5)$$

$$g_P(0) = \frac{M_{\Sigma^+} + M_p}{m_s + m_d} g_1(0), \quad (4.6)$$

where $f_1(0)$ and $g_1(0)$ are listed in table 1. The $f_2(0)/f_1(0)$ has been measured at Fermilab [53]. Nevertheless, we take the SU(3) symmetric prediction $f_2(0) = -(2\mu_n + \mu_p - 1)$ in the following analysis, where μ_p and μ_n are the proton and neutron anomalous magnetic moments which are calculated precisely up to NLO in covariant baryon chiral perturbation theory (BChPT) with the extended-on-mass-shell renormalization (EOMS) scheme [54, 55]. All the form factors above are evaluated at $\mu = 1$ GeV.

In addition, the four complex form factors a , b , c , and d which stem from long-distance contributions have been estimated in the heavy baryon chiral perturbation theory [56], relativistic baryon chiral perturbation theory [57], and vector-meson-dominance model [23]. In our analysis we take the latest relativistic results used in refs. [23, 24], where $c(0) = (-0.65 + 2.75i) \times 10^{-2}$ and $d(0) = (-83 + 3.8i) \times 10^{-4}$. The real parts of the form factors a and b cannot be uniquely determined at present from experimental data. For $\text{Re}[a(q^2)]$ and $\text{Re}[b(q^2)]$, their values at $q^2 = 0$ can be extracted from the experimental branching fractions and asymmetry parameters using their imaginary parts as inputs. Therefore, it yields four sets of solutions for a and b , which are listed in table 4.⁶

In table 4, we predict the branching fraction and the leptonic forward-backward asymmetry A_{FB} in the SM for the $\Sigma^+ \rightarrow p\mu^+\mu^-$ decay, which are

$$1.6 \times 10^{-8} \leq \text{BR}(\Sigma^+ \rightarrow p\mu^+\mu^-) \leq 8.9 \times 10^{-8}, \quad (4.7)$$

$$-1.4 \times 10^{-5} \leq A_{FB}(\Sigma^+ \rightarrow p\mu^+\mu^-) \leq 0.6 \times 10^{-5}, \quad (4.8)$$

where the theoretical uncertainties originate mainly from long-distance form factors a , b , c , and d . Compared with the results of refs. [23, 24], one finds that the predictions for the branching fraction are consistent, however there is a sign difference for the leptonic

⁶In ref. [23], it is shown that the form factors a , b , c and d have fairly mild q^2 -dependence. Therefore, it is reasonable to use their values at $q^2 = 0$ in our analysis.

forward-backward asymmetry A_{FB} in cases 1 and 2 due to the different definitions for the angle θ_ℓ . In addition, we note that our predicted A_{FB} are smaller than those of refs. [23, 24] in cases 3 and 4. The reason can be traced back to the fact that $f_2(0)$, multiplied by the long-distance contribution, is counted as of order δ (i.e. NLO) in the present work, as can be seen in eq. (4.2). However, the contribution from $f_2(0)$ is not included in refs. [23, 24].⁷ We also find that only the branching fraction predicted in case 1 and case 2 are in agreement with the current LHCb's BR measurement [18]. Therefore, in the following, we focus on the contributions of NP to A_{FB} with the inputs given in cases 1 and 2.

In ref. [24], the authors have studied the sensitivity of the leptonic forward-backward asymmetry A_{FB} to the two new interactions $(\bar{d}\gamma^\mu s)(\bar{\ell}\gamma_\mu\gamma_5\ell)$ and $(\bar{d}\gamma^\mu\gamma_5 s)(\bar{\ell}\gamma_\mu\gamma_5\ell)$ but this was not compared with the corresponding bounds from kaon decays. From eq. (4.2), one can see that the observable $A_{FB}(\Sigma^+ \rightarrow p\mu^+\mu^-)$ is sensitive to four operators $(\bar{d}\gamma^\mu s)(\bar{\ell}\gamma_\mu\gamma_5\ell)$, $(\bar{d}\gamma^\mu\gamma_5 s)(\bar{\ell}\gamma_\mu\gamma_5\ell)$, $(\bar{d}s)(\bar{\ell}\ell)$ and $(\bar{d}\gamma_5 s)(\bar{\ell}\ell)$ because of the interference of long-distance and short-distance contributions. In this work, we will investigate the sensitivity of $A_{FB}(\Sigma^+ \rightarrow p\mu^+\mu^-)$ to the four types of new interactions and compare it with the corresponding kaon decays. For such a purpose, we study the $K_L \rightarrow \mu^+\mu^-$ and $K^+ \rightarrow \pi^+\mu^+\mu^-$ decays.

4.2 $K_L \rightarrow \mu^+\mu^-$ and $K^+ \rightarrow \pi^+\mu^+\mu^-$ decays

Following ref. [58], one can easily obtain the branching fraction of the $K_L \rightarrow \mu^+\mu^-$ decay as follows:

$$\text{BR}(K_L \rightarrow \mu^+\mu^-) = \left[6.7 + \left(\frac{0.08\alpha\pi^2}{\sqrt{2}G_F m_s m_\mu} \text{Im} [\lambda_t^*(C_S - C'_S)^*] \right)^2 + (1.1(C'_{10} - C_{10})^* + \frac{0.10\alpha\pi^2}{\sqrt{2}G_F m_s m_\mu} \text{Re} [\lambda_t^*(C_P - C'_P)^*] - 0.2 \pm 0.4_{-0.5}^{+0.5})^2 \right] \times 10^{-9}. \quad (4.9)$$

The differential distribution of the $K^+ \rightarrow \pi^+\mu^+\mu^-$ decay is conveniently expressed as,

$$\frac{d\Gamma}{dq^2 d\cos\theta_\mu} = \Gamma_0 \beta_\mu \sqrt{\lambda(q^2)} \left[A(q^2) + B(q^2) \cos\theta_\mu + C(q^2) \cos^2\theta_\mu \right], \quad (4.10)$$

with

$$\begin{aligned} A(q^2) &= q^2 \left(\beta_\mu^2 |F_S(q^2)|^2 + |F_P(q^2)|^2 \right) + \frac{\lambda(q^2)}{4} \left(|F_A(q^2)|^2 + |F_V(q^2)|^2 \right) \\ &\quad + 4m_\mu^2 m_{K^+}^2 |F_A(q^2)|^2 + 2m_\mu \left(m_{K^+}^2 - m_{\pi^+}^2 + q^2 \right) \text{Re} [F_P(q^2) F_A(q^2)^*], \\ B(q^2) &= 2m_\mu \beta_\mu \sqrt{\lambda(q^2)} \text{Re} [F_S(q^2) F_V(q^2)^*], \\ C(q^2) &= -\frac{\beta_\mu^2 \lambda(q^2)}{4} \left(|F_A(q^2)|^2 + |F_V(q^2)|^2 \right), \end{aligned} \quad (4.11)$$

⁷If we set $f_2(0) = 0$ in eq. (4.2), we recover the results of refs. [23, 24].

and

$$\begin{aligned}
 F_V(q^2) &= 2(C_9 + C_9')^* f_+(q^2) + \frac{4m_s}{m_{K^+} + m_{\pi^+}} f_T(q^2) C_7^* + F_{V\gamma}(q^2), \\
 F_A(q^2) &= 2(C_{10} + C_{10}')^* f_+(q^2), \\
 F_S(q^2) &= \frac{m_{K^+}^2 - m_{\pi^+}^2}{m_s - m_d} f_0(q^2) (C_S + C_S')^*, \\
 F_P(q^2) &= \frac{m_{K^+}^2 - m_{\pi^+}^2}{m_s - m_d} f_0(q^2) (C_P + C_P')^* - 2m_\mu (C_{10} + C_{10}')^* \\
 &\quad \cdot \left[f_+(q^2) - \frac{m_{K^+}^2 - m_{\pi^+}^2}{q^2} (f_0(q^2) - f_+(q^2)) \right], \tag{4.12}
 \end{aligned}$$

where $\Gamma_0 = G_F^2 \alpha^2 |\lambda_t|^2 / (512\pi^3 m_{K^+}^3)$, $\beta_\mu = \sqrt{1 - 4m_\mu^2/q^2}$, and $\lambda(q^2) = q^4 + m_{K^+}^4 + m_{\pi^+}^4 - 2(m_{K^+}^2 m_{\pi^+}^2 + m_{K^+}^2 q^2 + m_{\pi^+}^2 q^2)$. The function $F_{V\gamma}(q^2)$ is the matrix element corresponding to the long-distance contribution, which is dominated by the one-photon exchange transition $K^+ \rightarrow \pi^+ \gamma^* \rightarrow \pi^+ \mu^+ \mu^-$. As shown in ref. [59], one has

$$F_{V\gamma}(q^2) = - \left[\left(a_+ + b_+ \frac{q^2}{m_{K^+}^2} \right) + \frac{1}{m_{K^+}^2 G_F} W_+^{\pi\pi}(q^2) \right] \frac{\sqrt{2}}{2\pi\lambda_t^*}, \tag{4.13}$$

and the real parameters a_+ and b_+ have been determined by the NA48/2 experiment [60]. The non-analytic term $W_+^{\pi\pi}(q^2)$ stands for the pion-loop contribution, and its expression can be found in ref. [59]. The decay width and forward-backward asymmetry are,

$$\Gamma_{K^+} = 2\Gamma_0 \int_{q_{\min}^2}^{q_{\max}^2} \beta_\mu \sqrt{\lambda(q^2)} \left(A(q^2) + \frac{1}{3} C(q^2) \right) dq^2, \tag{4.14}$$

$$A_{FB, K^+} = \frac{\Gamma_0 \int_{q_{\min}^2}^{q_{\max}^2} \beta_\mu \sqrt{\lambda(q^2)} B(q^2) dq^2}{\Gamma_{K^+}}, \tag{4.15}$$

where $q_{\min}^2 = 4m_\mu^2$ and $q_{\max}^2 = (m_{K^+} - m_{\pi^+})^2$.

In the above expressions, the form factors $f_+(q^2)$, $f_0(q^2)$, and $f_T(q^2)$ stem from the standard decompositions of the hadronic matrix elements [61], and they can be related to those of the charge-current $K^+ \rightarrow \pi^0$ and neutral-current $K^0 \rightarrow \pi^0$ transitions using isospin symmetry [29]. We use the quadratic and linear parametrizations for the vector and scalar form-factors, respectively:

$$\begin{aligned}
 f_+(q^2) &= \sqrt{2} f_+^{K^+\pi^0}(0) \left(1 + \lambda'_+ \frac{q^2}{m_{\pi^+}^2} + \lambda''_+ \frac{q^4}{2m_{\pi^+}^4} \right), \\
 f_0(q^2) &= \sqrt{2} f_+^{K^+\pi^0}(0) \left(1 + \lambda'_0 \frac{q^2}{m_{\pi^+}^2} \right), \tag{4.16}
 \end{aligned}$$

where $f_+^{K^+\pi^0}(0) = 0.9706(27)$ is obtained from the $N_f = 2 + 1 + 1$ lattice QCD simulations [62], and λ'_0 , λ'_+ , and λ''_+ can be found in the PDG review [35]. For the tensor form factor, we employ the result from lattice QCD [63]:

$$f_T(q^2) = \frac{2f_T^{K^0\pi^0}(0)}{1 - s_T^{K^0\pi^0} q^2}, \tag{4.17}$$

where $f_T^{K^0\pi^0}(0) = 0.417(15)$ and $s_T^{K^0\pi^0} = 1.10(14) \text{ GeV}^{-2}$.

	$\text{BR}(K_L \rightarrow \mu^+ \mu^-)$	$A_{FB}(K^+ \rightarrow \pi^+ \mu^+ \mu^-)$	$A_{FB}(\Sigma^+ \rightarrow p \mu^+ \mu^-)$	
			Case 1	Case 2
SM predictions	$7.64(73) \times 10^{-9}$	0	-1.4×10^{-5}	0.2×10^{-5}
Data	$6.84(11) \times 10^{-9}$	$(-2.3, 2.3) \times 10^{-2}$	$(-2.3, 2.3) \times 10^{-2}$	
$C_S + C'_S$	—	$(-1.7, 1.7)$	$(-6.8, 6.8) \times 10^2$	$(-9.3, 9.3) \times 10^3$
$C_S - C'_S$	$(-0.12, 0.12)$	—	$(-1.3, 1.3) \times 10^3$	$(-1.8, 1.8) \times 10^3$
$\delta C_{10} + C'_{10}$	—	—	$(-1.2, 1.2) \times 10^3$	$(-1.8, 1.8) \times 10^3$
$\delta C_{10} - C'_{10}$	$(-2.35, 0.59)$	—	$(-5.8, 5.8) \times 10^2$	$(-1.5, 1.5) \times 10^3$

Table 5. Observables predicted in the SM and constraints on NP at 90% CL derived from the experimental measurements [35, 60]. Here, we are assuming a hypothetical measurement of $A_{FB}(\Sigma^+ \rightarrow p \mu^+ \mu^-)$ that is identical to $A_{FB}(K^+ \rightarrow \pi^+ \mu^+ \mu^-)$.

Using eqs. (4.9) and (4.14), we first obtain the branching fractions of $K_L \rightarrow \mu^+ \mu^-$ and $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ in the SM,

$$\text{BR}(K_L \rightarrow \mu^+ \mu^-) = 7.64(73) \times 10^{-9}, \quad (4.18)$$

$$\text{BR}(K^+ \rightarrow \pi^+ \mu^+ \mu^-) = 9.5(7) \times 10^{-8}, \quad (4.19)$$

which are consistent with the current PDG averages at 1σ confidence level [35],

$$\text{BR}(K_L \rightarrow \mu^+ \mu^-)_{\text{exp}} = 6.84(11) \times 10^{-9}, \quad (4.20)$$

$$\text{BR}(K^+ \rightarrow \pi^+ \mu^+ \mu^-)_{\text{exp}} = 9.4(6) \times 10^{-8}. \quad (4.21)$$

We need to emphasize that a precise SM prediction for the branching fraction of the $K_L \rightarrow \mu^+ \mu^-$ decay is complicated due to the presence of poorly known long-distance contributions. However, the $K_L \rightarrow \mu^+ \mu^-$ decay can still be used to test the SM and probe NP in a clean way as demonstrated in refs. [64, 65]. In addition, from the definitions given in eqs. (4.14) and (4.15), one clearly sees that the branching fraction of the $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay is insensitive to NP due to the pollution of long-distance hadronic effects. On the other hand, the leptonic forward-backward asymmetry A_{FB} of the $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay provides a clean null test of the SM, becoming very sensitive to the new interaction $(\bar{d}s)(\bar{\ell}\ell)$ because of the interference of long-distance and short-distance contributions. Therefore, we study the sensitivity of the branching fraction of the $K_L \rightarrow \mu^+ \mu^-$ decay and the leptonic forward-backward asymmetry A_{FB} of the $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay to NP.

From table 5 one finds that current experimental measurements in the kaon modes can constrain well different scenarios of NP. We note that the leptonic forward-backward asymmetry A_{FB} of $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ is of the order of $\sim 10^{-2}$ if the combination of scalar WCs is $\sim \mathcal{O}(1)$. This conclusion is in agreement with those of refs. [66, 67]. However, we note that the kaon modes are not directly sensitive to all the NP structures, i.e., $\delta C_{10} + C'_{10}$. Therefore, in order to be sensitive to this combination of WCs one needs to turn to the $\Sigma^+ \rightarrow p \mu^+ \mu^-$ decay. In table 5 we also study the sensitivity of the leptonic forward-backward asymmetry A_{FB} of the $\Sigma^+ \rightarrow p \mu^+ \mu^-$ decay to different NP structures and assuming a future experimental measurement equivalent to the kaonic one. We find

that the sensitivity of this observable to NP can be $\sim 10^{-3}$ times weaker than the bounds derived from the kaon modes.

In order to understand better the sensitivity of the hyperon decay to NP, we obtain numerical formulas for the different observables. The decay widths of $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ and $\Sigma^+ \rightarrow p \mu^+ \mu^-$ as functions of C_9 and C'_9 are:

$$\Gamma_{K^+} = \left(1.36 \cdot 10^{-6} |C_9 + C'_9|^2 + 5.07 \cdot 10^{-2} \right) \times 10^{-22} \text{ GeV}, \quad (4.22)$$

$$\Gamma_{\Sigma^+} = \left(7.82 \cdot 10^{-8} |C_9 + C'_9|^2 + 3.77 \cdot 10^{-8} |C_9 - C'_9|^2 + 1.33 \right) \times 10^{-22} \text{ GeV}, \quad (4.23)$$

where we have neglected the interference terms between long- and short-distance contributions. The long-distance contribution to the decay rate for the hyperon is roughly a factor $\sim \mathcal{O}(100)$ larger than for the kaon.⁸ This must indeed be the case given that the K^+ has a lifetime longer than the Σ^+ by the same amount while both decays have similar BRs. On the other hand, and despite the long-distance effects it is still possible to roughly estimate the bounds on $\delta C_9 + C'_9$ and $\delta C_9 - C'_9$ from the branching fraction of $\Sigma^+ \rightarrow p \mu^+ \mu^-$. If we assume that the short-distance contribution saturates the experimental branching fraction $\text{BR}(\Sigma^+ \rightarrow p \mu^+ \mu^-)_{\text{exp}} = \left(2.2^{+1.8}_{-1.3} \right) \times 10^{-8}$ one obtains upper limits in the ballpark of 10^2 for $\delta C_9 + C'_9$ and $\delta C_9 - C'_9$. Applying the same argument for the $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ decay one obtains an upper bound of order 10 on $\delta C_9 + C'_9$ from the experimental measurement $\text{BR}(K^+ \rightarrow \pi^+ \mu^+ \mu^-)_{\text{exp}} = 9.4(6) \times 10^{-8}$.

Similarly, one can obtain numerical formulas for A_{FB} of $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ and $\Sigma^+ \rightarrow p \mu^+ \mu^-$ as functions of C_S , C'_S , C_{10} and C'_{10} , which read

$$A_{FB,K^+} = -1.38 \times 10^{-2} (C_S + C'_S), \quad (4.24)$$

$$A_{FB,\Sigma^+} = \left[3.39 (C_S + C'_S) + 1.76 (C_S - C'_S) + 1.94 (C_{10} + C'_{10}) - 3.96 (C_{10} - C'_{10}) \right] \times 10^{-5}, \quad (4.25)$$

where we have chosen case 1 for the numerics of $\Sigma^+ \rightarrow p \mu^+ \mu^-$. As already anticipated by the results shown in table 5, A_{FB} for hyperons suffers from a loss of sensitivity of 2–3 orders of magnitude compared to the one of kaons. This can be partly explained by the different importance of the long-distance contributions to the amplitude, which appear linearly in the numerators while quadratically in the denominators of these *normalized* observables.

5 Renormalization group evolution effects

As discussed in section 3 and section 4, hyperon decays can constrain some combinations of WCs which are not constrained by the kaon modes. It is natural to ask what NP models could be tested by hyperon decays. For such a purpose, the ideal scenario is that NP only contributes to specific combinations of WCs at tree level. However, it is well known that in general loop effects will change this situation. In order to investigate the importance of

⁸The phase-space factor in the rate for $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ is ~ 10 times larger than that for $\Sigma^+ \rightarrow p \mu^+ \mu^-$. Hence, the long-distance contribution to the hyperon decay is larger than that to the kaon decay by a factor ~ 10 –100 at the *amplitude* level.

this we calculate the RGE effects in the standard model effective field theory (SMEFT), which allows us to treat a wide variety of phenomena spanning different energy regimes, from Higgs physics to kaon decays, in a systematic and model-independent way. In the following analysis, we assume that the electroweak symmetry breaking is linearly realized, keeping the $SU(2)_L \times U(1)_Y$ relations [39]. The SMEFT Lagrangian describing the NP contributions to down-quark FCNC semi-leptonic decays is [68]

$$\mathcal{L}_{\text{NP}} = \frac{1}{\Lambda} \sum_i C_i Q_i, \quad (5.1)$$

with

$$\begin{aligned} Q_{\ell q}^{(1),ij\alpha\beta} &= (\bar{q}_L^j \gamma^\mu q_L^i) (\bar{\ell}_L^\beta \gamma_\mu \ell_L^\alpha), & Q_{\ell q}^{(3),ij\alpha\beta} &= (\bar{q}_L^j \vec{\tau} \gamma^\mu q_L^i) (\bar{\ell}_L^\beta \vec{\tau} \gamma_\mu \ell_L^\alpha), \\ Q_{\ell d}^{ij\alpha\beta} &= (\bar{d}_R^j \gamma^\mu d_R^i) (\bar{\ell}_L^\beta \gamma_\mu \ell_L^\alpha), & Q_{qe}^{ij\alpha\beta} &= (\bar{q}_L^j \gamma^\mu q_L^i) (\bar{e}_R^\beta \gamma_\mu e_R^\alpha), \\ Q_{ed}^{ij\alpha\beta} &= (\bar{d}_R^j \gamma^\mu d_R^i) (\bar{e}_R^\beta \gamma_\mu e_R^\alpha), & Q_{\ell edq}^{ij\alpha\beta} &= (\bar{\ell}_L e_R) (\bar{d}_R q_L), \\ Q_{\ell edq}^{ij\alpha\beta} &= (\bar{e}_R \ell_L) (\bar{q}_L d_R), \end{aligned} \quad (5.2)$$

where Greek letters α, β and Latin letters i, j stand for lepton flavor indices and quark flavor indices, respectively, q_L and ℓ_L are the quark and lepton doublets, e_R and d_R are the right-handed charged leptons and down-type quarks singlets. The scale Λ is the new physics scale. Following the convention of ref. [29], the SM fields for quarks and leptons are denoted as $q_L \sim (\mathbf{3}, \mathbf{2}, \frac{1}{6})$, $d_R \sim (\mathbf{3}, \mathbf{1}, -\frac{1}{3})$, $\ell_L \sim (\mathbf{1}, \mathbf{2}, -\frac{1}{2})$, and $e_R \sim (\mathbf{1}, \mathbf{1}, -1)$, respectively.

For the $s \rightarrow d\mu^+\mu^-$ and $s \rightarrow d\nu\bar{\nu}$ transitions, we work in the basis where the down-type quark mass matrix is diagonal. At the electroweak scale $v = (\sqrt{2}G_F)^{-1/2} \approx 246$ GeV, the NP contributions to the Wilson coefficients in the low-energy effective Lagrangian can be expressed in terms of those in the SMEFT Lagrangian as follows:

$$\begin{aligned} [\delta C_9]_{sd\mu\mu} &= \frac{2\pi}{e^2\lambda_t} \frac{v^2}{\Lambda^2} [C_{\ell q}^{(1)} + C_{\ell q}^{(3)} + C_{qe}]_{sd\mu\mu}, & [C'_9]_{sd\mu\mu} &= \frac{2\pi}{e^2\lambda_t} \frac{v^2}{\Lambda^2} [C_{ed} + C_{\ell d}]_{sd\mu\mu}, \\ [\delta C_{10}]_{sd\mu\mu} &= \frac{2\pi}{e^2\lambda_t} \frac{v^2}{\Lambda^2} [-C_{\ell q}^{(1)} - C_{\ell q}^{(3)} + C_{qe}]_{sd\mu\mu}, & [C'_{10}]_{sd\mu\mu} &= \frac{2\pi}{e^2\lambda_t} \frac{v^2}{\Lambda^2} [C_{ed} - C_{\ell d}]_{sd\mu\mu}, \\ [C_S]_{sd\mu\mu} &= -[C_P]_{sd\mu\mu} = \frac{2\pi}{e^2\lambda_t} \frac{v^2}{\Lambda^2} [C_{\ell edq}]_{sd\mu\mu}, & [C'_S]_{sd\mu\mu} &= [C'_P]_{sd\mu\mu} = \frac{2\pi}{e^2\lambda_t} \frac{v^2}{\Lambda^2} [C'_{\ell edq}]_{sd\mu\mu}, \\ [\delta C_{\nu\ell}^L]_{sd\nu\bar{\nu}} &= \frac{2\pi}{e^2\lambda_t} \frac{v^2}{\Lambda^2} [C_{\ell q}^{(1)} - C_{\ell q}^{(3)}]_{sd\nu\bar{\nu}}, & [C_{\nu\ell}^R]_{sd\nu\bar{\nu}} &= \frac{2\pi}{e^2\lambda_t} \frac{v^2}{\Lambda^2} [C_{\ell d}]_{sd\nu\bar{\nu}}. \end{aligned} \quad (5.3)$$

Let us first take the $s \rightarrow d\nu\bar{\nu}$ process as an example. We consider a Z' model in which a single $Z' \sim (\mathbf{1}, \mathbf{1}, 0)$ gauge boson couples to left-handed leptons. The Lagrangian for this model is

$$\mathcal{L}_{Z'} = (g_L^{ij} \bar{q}_L^j \gamma^\mu q_L^i + g_R^{ij} \bar{d}_R^j \gamma^\mu d_R^i + g_L^{\alpha\beta} \bar{\ell}_L^\beta \gamma^\mu \ell_L^\alpha) Z'_\mu, \quad (5.4)$$

which does not contribute to the WCs $C_{\ell q}^{(3)}$, C_{qe} , C_{ed} and $C_{\ell edq}$ of the SMEFT. Furthermore, by fine-tuning the couplings $g_L^{sd} g_L^{\nu\bar{\nu}} = -g_R^{sd} g_L^{\nu\bar{\nu}}$, it yields a purely axial coupling to quarks at

tree level, i.e., $[\delta C_{\nu_\ell}^L(\Lambda) + C_{\nu_\ell}^R(\Lambda)]_{sd\nu\bar{\nu}} = 0$ but $[\delta C_{\nu_\ell}^L(\Lambda) - C_{\nu_\ell}^R(\Lambda)]_{sd\nu\bar{\nu}} \neq 0$. When running the RG equations given in appendix D from the NP scale Λ to the electroweak scale v and assuming that $\Lambda = 10 v$, one obtains

$$\begin{aligned} [\delta C_{\nu_\ell}^L(v) + C_{\nu_\ell}^R(v)]_{sd\nu\bar{\nu}} &= \frac{2\pi}{e^2\lambda_t} \frac{v^2}{\Lambda^2} [0.02C_{\ell q}^{(1)}(\Lambda)]_{sd\nu\bar{\nu}}, \\ [\delta C_{\nu_\ell}^L(v) - C_{\nu_\ell}^R(v)]_{sd\nu\bar{\nu}} &= \frac{2\pi}{e^2\lambda_t} \frac{v^2}{\Lambda^2} [2.02C_{\ell q}^{(1)}(\Lambda)]_{sd\nu\bar{\nu}}. \end{aligned} \quad (5.5)$$

In eq. (5.5), we see that the loop effects of the RGE generate a non-vanishing vectorial contribution at the scale v , which is about 1% of that of the axial-vectorial contribution. As a result, from the $K \rightarrow \pi\nu\bar{\nu}$ data, we obtain a constraint on $\delta C_{\nu_\ell}^L - C_{\nu_\ell}^R$ at the order of 10^2 , which is stronger than the direct bound of 10^3 that could be potentially obtained by BESIII from the hyperon modes shown in table 2.

Next, we investigate the $s \rightarrow d\nu\bar{\nu}$ and $s \rightarrow d\mu^+\mu^-$ processes. For this, we consider a vector leptoquark (LQ) $U_1 \sim (\mathbf{2}, \mathbf{1}, \frac{2}{3})$. In this model, we have the following relations at the NP scale Λ

$$[C_{\ell q}^{(1)}(\Lambda)]_{ij\alpha\beta} = [C_{\ell q}^{(3)}(\Lambda)]_{ij\alpha\beta}, \quad [C_{\ell d}(\Lambda)]_{ij\alpha\beta} = 0, \quad [C_{qe}(\Lambda)]_{ij\alpha\beta} = 0. \quad (5.6)$$

It means that U_1 contributes exclusively to charged-lepton modes but not to neutrino modes at tree level. However, the RGE mixes both types of leptons and as a result U_1 will also contribute to the neutrino modes at the electroweak scale. The RGE results for the relevant combinations of WCs assuming that $\Lambda = 10 v$ are as follows:

$$\begin{aligned} [\delta C_{\nu_\ell}^L(v) + C_{\nu_\ell}^R(v)]_{sd\mu\mu} &= \frac{2\pi}{e^2\lambda_t} \frac{v^2}{\Lambda^2} [-0.07C_{\ell q}^{(1)}(\Lambda) - 0.001C_{ed}(\Lambda)]_{sd\mu\mu}, \\ [\delta C_{\nu_\ell}^L(v) - C_{\nu_\ell}^R(v)]_{sd\mu\mu} &= \frac{2\pi}{e^2\lambda_t} \frac{v^2}{\Lambda^2} [-0.07C_{\ell q}^{(1)}(\Lambda) + 0.001C_{ed}(\Lambda)]_{sd\mu\mu}, \\ [\delta C_{10}(v) + C'_{10}(v)]_{sd\mu\mu} &= \frac{2\pi}{e^2\lambda_t} \frac{v^2}{\Lambda^2} [-1.96C_{\ell q}^{(1)}(\Lambda) + 0.99C_{ed}(\Lambda)]_{sd\mu\mu}, \\ [\delta C_{10}(v) - C'_{10}(v)]_{sd\mu\mu} &= \frac{2\pi}{e^2\lambda_t} \frac{v^2}{\Lambda^2} [-1.96C_{\ell q}^{(1)}(\Lambda) - 0.99C_{ed}(\Lambda)]_{sd\mu\mu}. \end{aligned} \quad (5.7)$$

From eq. (5.7), one can see that the loop effects of RGE from the NP scale Λ to the electroweak scale v is at the percent level. As a result, we obtain an indirect bound on the combination of WCs δC_{10} and C'_{10} at the order of $\sim 10^2$ from the $K \rightarrow \pi\nu\bar{\nu}$ data. The bound is weaker than that from the $s \rightarrow d\mu^+\mu^+$ kaon modes but stronger than the anticipated bound shown in table 5 from the hyperon modes.

Therefore, we conclude that loop effects from renormalization group evolution are important when connecting the low-energy EFT to new physics models. Although at present the strongest constraint on new physics come from the kaon modes, rare hyperon decays could provide better constraints on $\delta C_{\nu_\ell}^L - C_{\nu_\ell}^R$ than the kaon modes if the upper limits from future hyperon factories can reach a sensitivity to BRs of 10^{-8} or lower, e.g., for the Z' model considered above.

6 Conclusions and outlook

In this work, we first predicted the branching fractions of $B_1 \rightarrow B_2 \nu \bar{\nu}$ in the SM using model independent inputs. More specifically, form factors, up to next-to-leading order (NLO) in the SU(3)-breaking expansion, can be obtained in covariant baryon chiral perturbation theory. We discussed the constraints on $\delta C_{\nu_\ell}^L + C_{\nu_\ell}^R$ and $\delta C_{\nu_\ell}^L - C_{\nu_\ell}^R$ derived from the anticipated BESIII measurements in the near future, and studied the potential of the $B_1 \rightarrow B_2 \nu \bar{\nu}$ decays as probes of NP, in comparison with the kaon modes. We conclude that $\delta C_{\nu_\ell}^L + C_{\nu_\ell}^R$ can be well determined by the kaon mode but the $B_1 \rightarrow B_2 \nu \bar{\nu}$ decays from the BESIII measurements in the near future could be better than their kaon siblings in constraining $\delta C_{\nu_\ell}^L - C_{\nu_\ell}^R$.

In addition, the $s \rightarrow d \mu^+ \mu^-$ processes, which are dominated by long-distance contributions, were investigated. We provided predictions for the branching fraction of $K_L \rightarrow \mu^+ \mu^-$ and leptonic forward-backward asymmetry A_{FB} of $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ and $\Sigma^+ \rightarrow p \mu^+ \mu^-$ in the SM. Assuming that the experimental measurements of the leptonic forward-backward asymmetry A_{FB} of $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ and $\Sigma^+ \rightarrow p \mu^+ \mu^-$ are of the same precision, we studied the sensitivity of the $\Sigma^+ \rightarrow p \mu^+ \mu^-$ decay and corresponding kaon modes to different NP structures. We conclude that the current kaon bounds are a few orders of magnitude better than those of $\Sigma^+ \rightarrow p \mu^+ \mu^-$ except for the $\delta C_{10} + C'_{10}$ and $\delta C_9 - C'_9$ scenarios.

Finally, from an analysis of the renormalization group evolution, we showed that at present the strongest constraint on new physics parameters still come from the kaon modes. From the perspective of a UV theory, it is important to consider the loop effects from renormalization group evolution when connecting the low-energy EFT to new physics models. However, hyperons could still lead to the strongest constraints on some combinations of Wilson coefficients if a sensitivity of $\sim 10^{-8}$ for the branching fractions is achieved by hyperon factories in the future (e.g. at the planned super tau-charm factory [69, 70]).

In the present study, the long-distance form factors a and b from weak radiative hyperon decays were still poorly estimated, leading to large uncertainties to the predictions for the observables of the $\Sigma^+ \rightarrow p \mu^+ \mu^-$ decay. Therefore, further studies of weak radiative hyperon decays will be needed.

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A Form factors for the octet-to-octet transitions

The hadronic matrix elements for the octet-to-octet transitions are parameterized in terms of form factors as follows [42, 71]

$$\begin{aligned}
 \langle B_2(p_2) | \bar{d} \gamma_\mu s | B_1(p_1) \rangle &= \bar{u}_2(p_2) \left[f_1(q^2) \gamma_\mu + \frac{f_2(q^2)}{M_1} \sigma_{\mu\nu} q^\nu + \frac{f_3(q^2)}{M_1} q_\mu \right] u_1(p_1), \\
 \langle B_2(p_2) | \bar{d} \gamma_\mu \gamma_5 s | B_1(p_1) \rangle &= \bar{u}_2(p_2) \left[g_1(q^2) \gamma_\mu + \frac{g_2(q^2)}{M_1} \sigma_{\mu\nu} q^\nu + \frac{g_3(q^2)}{M_1} q_\mu \right] \gamma_5 u_1(p_1), \\
 \langle B_2(p_2) | \bar{d} s | B_1(p_1) \rangle &= f_S(q^2) \bar{u}_2(p_2) u_1(p_1), \\
 \langle B_2(p_2) | \bar{d} \gamma_5 s | B_1(p_1) \rangle &= g_P(q^2) \bar{u}_2(p_2) \gamma_5 u_1(p_1), \\
 \langle B_2(p_2) | \bar{d} \sigma_{\mu\nu} s | B_1(p_1) \rangle &= \bar{u}_2(p_2) \left[g_T(q^2) \sigma_{\mu\nu} + \frac{g_T^{(1)}(q^2)}{M_1} (q_\mu \gamma_\nu - q_\nu \gamma_\mu) \right. \\
 &\quad \left. + \frac{g_T^{(2)}(q^2)}{M_1^2} (q_\mu P_\nu - q_\nu P_\mu) + \frac{g_T^{(3)}(q^2)}{M_1} (\gamma_\mu \not{q} \gamma_\nu - \gamma_\nu \not{q} \gamma_\mu) \right] u_1(p_1), \\
 \langle B_2(p_2) | \bar{d} \sigma_{\mu\nu} \gamma_5 s | B_1(p_1) \rangle &= \bar{u}_2(p_2) \left[g_{T5}(q^2) \sigma_{\mu\nu} + \frac{g_{T5}^{(1)}(q^2)}{M_1} (q_\mu \gamma_\nu - q_\nu \gamma_\mu) \right. \\
 &\quad \left. + \frac{g_{T5}^{(2)}(q^2)}{M_1^2} (q_\mu P_\nu - q_\nu P_\mu) + \frac{g_{T5}^{(3)}(q^2)}{M_1} (\gamma_\mu \not{q} \gamma_\nu - \gamma_\nu \not{q} \gamma_\mu) \gamma_5 \right] u_1(p_1).
 \end{aligned} \tag{A.1}$$

B $K \rightarrow \pi \pi \nu \bar{\nu}$ decays in helicity basis

The differential decay rate for the four-body $K \rightarrow \pi \pi \nu \bar{\nu}$ decay as a function of five Cabibbo-Maksymowicz variables s_π , s_ℓ , θ_π , θ_ℓ and ϕ [72] has the following form

$$\frac{d^5 \Gamma}{ds_\pi ds_\ell d(\cos \theta_\pi) d(\cos \theta_\ell) d\phi} = \alpha^2 G_F^2 \mathcal{N}(s_\pi, s_\ell) J(s_\pi, s_\ell, \theta_\pi, \theta_\ell, \phi), \tag{B.1}$$

where $\mathcal{N}(s_\pi, s_\ell) = \sigma_\pi X / (2^{15} \pi^6 m_K^3)$, with $\sigma_\pi = \sqrt{1 - \frac{4m_\pi^2}{s_\pi}}$ and $X = \frac{1}{2} \lambda^{1/2}(m_K^2, s_\pi, s_\ell)$, and $s_\pi = (p_\pi + p_\pi)^2$, $s_\ell = (p_\nu + p_\nu)^2$ and $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$. The ranges of five Cabibbo-Maksymowicz variables are,

$$\begin{aligned}
 s_\pi : \quad & 4m_\pi^2 \leq s_\pi \leq m_K^2, \\
 s_\ell : \quad & 0 \leq s_\ell \leq (m_K - \sqrt{s_\pi})^2, \\
 \theta_\pi : \quad & 0 \leq \theta_\pi \leq \pi, \\
 \theta_\ell : \quad & 0 \leq \theta_\ell \leq \pi, \\
 \phi : \quad & 0 \leq \phi \leq 2\pi,
 \end{aligned} \tag{B.2}$$

and a useful form of J dependence on θ_ℓ and ϕ is [73]

$$\begin{aligned}
 J = & J_1 + J_2 \cos 2\theta_\ell + J_3 \sin^2 \theta_\ell \cos 2\phi + J_4 \sin 2\theta_\ell \cos \phi + J_5 \sin \theta_\ell \cos \phi \\
 & + J_6 \cos \theta_\ell + J_7 \sin \theta_\ell \sin \phi + J_8 \sin 2\theta_\ell \sin \phi + J_9 \sin^2 \theta_\ell \sin 2\phi,
 \end{aligned} \tag{B.3}$$

where the angular coefficients J_i are functions of s_π , s_ℓ and θ_π . In particular, after integrating out the angles θ_ℓ and ϕ , one notes that only J_1 and J_2 contribute to the total decay width, which is,

$$\Gamma = \alpha^2 G_F^2 \mathcal{N}(s_\pi, s_\ell) \cdot 4\pi \left(J_1 - \frac{1}{3} J_2 \right) \cdot \int_{4m_\pi^2}^{m_K^2} ds_\pi \int_0^{(m_K - \sqrt{s_\pi})^2} ds_\ell \int_{-1}^1 d(\cos \theta_\pi), \quad (\text{B.4})$$

where J_1 and J_2 can be explicitly expressed in terms of the helicity amplitudes as

$$\begin{aligned} J_1 &= \frac{s_\ell}{2} \left(3|H_+^V|^2 + 3|H_-^V|^2 + 2|H_0^V|^2 + 3|H_+^A|^2 + 3|H_-^A|^2 + 2|H_0^A|^2 \right), \\ J_2 &= \frac{s_\ell}{2} \left(|H_+^V|^2 + |H_-^V|^2 - 2|H_0^V|^2 + |H_+^A|^2 + |H_-^A|^2 - 2|H_0^A|^2 \right). \end{aligned} \quad (\text{B.5})$$

The explicit forms of these helicity amplitudes for the $K^+ \rightarrow \pi^+ \pi^0 \nu \bar{\nu}$ decay are

$$\begin{aligned} H_0^{V(A)} &= \frac{i\lambda_t^* (C_{\nu_\ell}^L - C_{\nu_\ell}^R)^* (4F \cdot m_K^2 X + G \cdot \sigma_\pi \cos \theta_\pi (- (s_l - s_\pi)^2 + m_K^4 + 4X^2))}{4m_K^3 \sqrt{s_l}}, \\ H_+^{V(A)} &= -\frac{i\lambda_t^* \sqrt{s_\pi} \sigma_\pi \sin \theta_\pi \left(G \cdot m_K^2 (C_{\nu_\ell}^L - C_{\nu_\ell}^R)^* + H \cdot X (C_{\nu_\ell}^L + C_{\nu_\ell}^R)^* \right)}{\sqrt{2} m_K^3}, \\ H_-^{V(A)} &= \frac{i\lambda_t^* \sqrt{s_\pi} \sigma_\pi \sin \theta_\pi \left(G \cdot m_K^2 (C_{\nu_\ell}^L - C_{\nu_\ell}^R)^* - H \cdot X (C_{\nu_\ell}^L + C_{\nu_\ell}^R)^* \right)}{\sqrt{2} m_K^3}, \end{aligned} \quad (\text{B.6})$$

and for the $K_L \rightarrow \pi^0 \pi^0 \nu \bar{\nu}$ decay, they are

$$\begin{aligned} H_0^{V(A)} &= \frac{i\sqrt{2} X \cdot F \cdot \text{Re} \left[(C_{\nu_\ell}^L - C_{\nu_\ell}^R) \lambda_t \right]}{m_K \sqrt{s_l}}, \\ H_+^{V(A)} &= 0, \\ H_-^{V(A)} &= 0, \end{aligned} \quad (\text{B.7})$$

where the complex form factors F , G and H stem from the following hadronic matrix elements [49]

$$\begin{aligned} \langle \pi(p_1) \pi(p_2) | \bar{s} \gamma_\mu d | K(p) \rangle &= -\frac{H}{m_K^3} \epsilon_{\mu\nu\rho\sigma} (p_{\bar{\ell}} + p_\ell)^\nu (p_1 + p_2)^\rho (p_1 - p_2)^\sigma, \\ \langle \pi(p_1) \pi(p_2) | \bar{s} \gamma_\mu \gamma_5 d | K(p) \rangle &= -\frac{i}{m_K} [F(p_1 + p_2)_\mu + G(p_1 - p_2)_\mu + R(p_{\bar{\ell}} + p_\ell)_\mu], \end{aligned} \quad (\text{B.8})$$

and can be expanded in partial waves in terms of the angle $\cos \theta_\pi$ [49],

$$\begin{aligned} F &= F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos \theta_\pi, \\ G &= G_p e^{i\delta_p}, \\ H &= H_p e^{i\delta_p}. \end{aligned} \quad (\text{B.9})$$

In the present work, we take $\epsilon_{0123} = -1$. The form factor R enters the rate proportional to the lepton masses squared and is vanishing for our purpose.

C $B_1 \rightarrow B_2 \ell^+ \ell^-$ decays in helicity basis

Neglecting the electromagnetic corrections, the $B_1 \rightarrow B_2 \ell^+ \ell^-$ decay amplitude is factorized as

$$\mathcal{M} = \frac{G_F \alpha}{\sqrt{2}} \lambda_t \left[H_\mu^V \langle \ell^+ \ell^- | \bar{\ell}^- \gamma^\mu \ell^+ | 0 \rangle + H_\mu^A \langle \ell^+ \ell^- | \bar{\ell}^- \gamma^\mu \gamma_5 \ell^+ | 0 \rangle + H^S \langle \ell^+ \ell^- | \bar{\ell}^- \ell^+ | 0 \rangle + H^P \langle \ell^+ \ell^- | \bar{\ell}^- \gamma_5 \ell^+ | 0 \rangle \right], \quad (\text{C.1})$$

with

$$\begin{aligned} H_\mu^V &= \frac{m_s}{2\pi q^2} C_7 \left(\langle B_2 | \bar{d} \sigma_{\mu\nu} q^\nu s | B_1 \rangle + \langle B_2 | \bar{d} \sigma_{\mu\nu} q^\nu \gamma_5 s | B_1 \rangle \right) + (C_9 + C_9') \langle B_2 | \bar{d} \gamma_\mu s | B_1 \rangle \\ &\quad - (C_9 - C_9') \langle B_2 | \bar{d} \gamma_\mu \gamma_5 s | B_1 \rangle + h_\lambda(q^2), \\ H_\mu^A &= (C_{10} + C_{10}') \langle B_2 | \bar{d} \gamma_\mu s | B_1 \rangle - (C_{10} - C_{10}') \langle B_2 | \bar{d} \gamma_\mu \gamma_5 s | B_1 \rangle, \\ H^S &= (C_S + C_S') \langle B_2 | \bar{d} s | B_1 \rangle + (C_S - C_S') \langle B_2 | \bar{d} \gamma_5 s | B_1 \rangle, \\ H^P &= (C_P + C_P') \langle B_2 | \bar{d} s | B_1 \rangle + (C_P - C_P') \langle B_2 | \bar{d} \gamma_5 s | B_1 \rangle, \\ h_\lambda(q^2) &= -\frac{4\sqrt{2}\pi}{\lambda_t} \left[\frac{1}{q^2} \bar{u}_2(k)(a + b\gamma_5) \sigma_{\mu\nu} q^\nu u_1(p) + \bar{u}_2(k) \gamma_\mu (c + d\gamma_5) u_1(p) \right], \end{aligned} \quad (\text{C.2})$$

The contributions to the amplitude (C.1) can be projected into different angular-momentum states of the dilepton pair, characterized by its polarization vectors $\epsilon^\mu(\lambda)$ and using the completeness relation $g^{\mu\nu} = \sum g^{mn} \epsilon^\mu(m) \epsilon^{\nu*}(n)$. Here, λ is the polarization state of the virtual gauge boson. The projections of H_μ^V and H_μ^A define the helicity amplitudes,

$$\begin{aligned} H_{\lambda_2, \lambda}^V &= H_\mu^V(\lambda_2) \epsilon^{\mu*}(\lambda), \\ H_{\lambda_2, \lambda}^A &= H_\mu^A(\lambda_2) \epsilon^{\mu*}(\lambda). \end{aligned} \quad (\text{C.3})$$

We do not explicitly denote the helicity of the parent hyperon λ_1 in the helicity amplitudes since λ_1 is fixed by the relation $\lambda_1 = \lambda_2 - \lambda$. H^S and H^P contributes only to the $\lambda = t$ component.

Therefore, the amplitude in helicity basis can be written as

$$\begin{aligned} \mathcal{M} &= \frac{G_F \alpha}{\sqrt{2}} \lambda_t \left\{ \left[(H_{\frac{1}{2}, t}^V + H_{-\frac{1}{2}, t}^V) \cdot L_t^V - H_{\frac{1}{2}, +}^V \cdot L_+^V - H_{-\frac{1}{2}, -}^V \cdot L_-^V - (H_{\frac{1}{2}, 0}^V + H_{-\frac{1}{2}, 0}^V) \cdot L_0^V \right] \right. \\ &\quad + \left[(H_{\frac{1}{2}, t}^A + H_{-\frac{1}{2}, t}^A) \cdot L_t^A - H_{\frac{1}{2}, +}^A \cdot L_+^A - H_{-\frac{1}{2}, -}^A \cdot L_-^A - (H_{\frac{1}{2}, 0}^A + H_{-\frac{1}{2}, 0}^A) \cdot L_0^A \right] \\ &\quad \left. + (H_{\frac{1}{2}, t}^S + H_{-\frac{1}{2}, t}^S) \cdot L^S + (H_{\frac{1}{2}, t}^P + H_{-\frac{1}{2}, t}^P) \cdot L^P \right\}, \end{aligned} \quad (\text{C.4})$$

where

$$g_{mn} = \begin{cases} -1, & m = n = \pm, 0, \\ 1, & m = n = t, \\ 0, & \text{otherwise.} \end{cases} \quad (\text{C.5})$$

Substituting the parameterized hadronic matrix elements in appendix A into eq. (C.4), we obtain the decay rate for $B_1(p_1) \rightarrow B_2(p_2)\ell^+\ell^-$ as a function of q^2 and $\cos\theta_\ell$:

$$\frac{d\Gamma}{dq^2 d\cos\theta_\ell} = \mathcal{N}_f(q^2) \left[I_1(q^2) + I_2(q^2) \cos\theta_\ell + I_3(q^2) \cos^2\theta_\ell \right], \quad (\text{C.6})$$

with

$$\begin{aligned} I_1(q^2) &= \left(|H_{\frac{1}{2},t}^A|^2 + |H_{-\frac{1}{2},t}^A|^2 \right) \cdot 8m_\ell^2 + \left(|H_{\frac{1}{2},t}^S|^2 + |H_{-\frac{1}{2},t}^S|^2 \right) \cdot (2q^2 - 5m_\ell^2) \\ &\quad + \left(|H_{\frac{1}{2},t}^P|^2 + |H_{-\frac{1}{2},t}^P|^2 \right) \cdot (2q^2 - 3m_\ell^2) - 8m_\ell \sqrt{q^2} \cdot \text{Re} \left[H_{\frac{1}{2},t}^{A*} H_{\frac{1}{2},t}^P + H_{-\frac{1}{2},t}^{A*} H_{-\frac{1}{2},t}^P \right] \\ &\quad + \left(|H_{\frac{1}{2},0}^V|^2 + |H_{-\frac{1}{2},0}^V|^2 \right) \cdot (8m_\ell^2 + 2q^2\beta^2) + \left(|H_{\frac{1}{2},0}^A|^2 + |H_{-\frac{1}{2},0}^A|^2 \right) \cdot 2q^2\beta^2 \\ &\quad + \left(|H_{\frac{1}{2},+}^V|^2 + |H_{-\frac{1}{2},-}^V|^2 \right) \cdot (8m_\ell^2 + q^2\beta^2) + \left(|H_{\frac{1}{2},+}^A|^2 + |H_{-\frac{1}{2},-}^A|^2 \right) \cdot q^2\beta^2, \\ I_2(q^2) &= -8m_\ell \sqrt{q^2} \beta \cdot \text{Re} \left[H_{\frac{1}{2},t}^S H_{\frac{1}{2},0}^{V*} + H_{-\frac{1}{2},t}^S H_{-\frac{1}{2},0}^{V*} \right] - 4q^2 \beta \cdot \text{Re} \left[H_{\frac{1}{2},+}^A H_{\frac{1}{2},+}^{V*} - H_{-\frac{1}{2},-}^A H_{-\frac{1}{2},-}^{V*} \right], \\ I_3(q^2) &= \left(|H_{\frac{1}{2},0}^V|^2 + |H_{-\frac{1}{2},0}^V|^2 + |H_{\frac{1}{2},0}^A|^2 + |H_{-\frac{1}{2},0}^A|^2 \right) \cdot (-2q^2\beta^2) \\ &\quad + \left(|H_{\frac{1}{2},+}^V|^2 + |H_{-\frac{1}{2},-}^V|^2 + |H_{\frac{1}{2},+}^A|^2 + |H_{-\frac{1}{2},-}^A|^2 \right) \cdot q^2\beta^2, \end{aligned} \quad (\text{C.7})$$

where $q = p_1 - p_2$, $\mathcal{N}_f(q^2) = \frac{G_F^2 \alpha^2 |\lambda_t|^2 \beta_\ell}{2048 \pi^3} \frac{\sqrt{Q_+ Q_-}}{M_1^3}$, $Q_\pm = (M_1 \pm M_2)^2 - q^2$, and $\beta_\ell = \sqrt{1 - \frac{4m_\ell^2}{q^2}}$. The θ_ℓ is the angle between the three-momenta of the final state particle B_2 and the emitting direction of ℓ^- . The hadronic amplitudes in eq. (C.7) are given as follows:

$$\begin{aligned} H_{\frac{1}{2},t}^V &= \frac{\sqrt{Q_+} (C_9 + C_9') \left[f_1(q^2)(M_1 - M_2) + f_3(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}} + \frac{\sqrt{Q_-} (C_9 - C_9') \left[g_1(q^2)(M_1 + M_2) - g_3(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}} \\ &\quad - \frac{4\sqrt{2}\pi \left[c \cdot \sqrt{Q_+}(M_1 - M_2) - d \cdot \sqrt{Q_-}(M_1 + M_2) \right]}{\sqrt{q^2} \lambda_t}, \\ H_{-\frac{1}{2},t}^V &= \frac{\sqrt{Q_+} (C_9 + C_9') \left[f_1(q^2)(M_1 - M_2) + f_3(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}} - \frac{\sqrt{Q_-} (C_9 - C_9') \left[g_1(q^2)(M_1 + M_2) - g_3(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}} \\ &\quad - \frac{4\sqrt{2}\pi \left[c \cdot \sqrt{Q_+}(M_1 - M_2) + d \cdot \sqrt{Q_-}(M_1 + M_2) \right]}{\sqrt{q^2} \lambda_t}, \\ H_{\frac{1}{2},0}^V &= \frac{C_7 m_s \left[\sqrt{Q_-} \left(g_T(q^2) - g_T^{(1)}(q^2) \frac{M_1 + M_2}{M_1} - g_T^{(2)}(q^2) \frac{Q_+}{M_1^2} \right) + \sqrt{Q_+} \left(g_{T5}(q^2) + g_{T5}^{(1)}(q^2) \frac{M_1 - M_2}{M_1} - g_T^{(2)}(q^2) \frac{Q_-}{M_1^2} \right) \right]}{2\pi \sqrt{q^2}} \\ &\quad + \frac{\sqrt{Q_-} (C_9 + C_9') \left[f_1(q^2)(M_1 + M_2) + f_2(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}} + \frac{\sqrt{Q_+} (C_9 - C_9') \left[g_1(q^2)(M_1 - M_2) - g_2(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}} \\ &\quad + \frac{4\sqrt{2}\pi \left[-a\sqrt{Q_-} - b\sqrt{Q_+} - c\sqrt{Q_-}(M_1 + M_2) + d\sqrt{Q_+}(M_1 - M_2) \right]}{\sqrt{q^2} \lambda_t}, \end{aligned}$$

$$\begin{aligned}
 H_{-\frac{1}{2},0}^V &= \frac{C_7 m_s \left[\sqrt{Q_-} \left(g_T(q^2) - g_T^{(1)}(q^2) \frac{M_1+M_2}{M_1} - g_T^{(2)}(q^2) \frac{Q_+}{M_1^2} \right) - \sqrt{Q_+} \left(g_{T5}(q^2) + g_{T5}^{(1)}(q^2) \frac{M_1-M_2}{M_1} - g_T^{(2)}(q^2) \frac{Q_-}{M_1^2} \right) \right]}{2\pi\sqrt{q^2}} \\
 &+ \frac{\sqrt{Q_-} (C_9 + C'_9) \left[f_1(q^2)(M_1+M_2) + f_2(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}} - \frac{\sqrt{Q_+} (C_9 - C'_9) \left[g_1(q^2)(M_1-M_2) - g_2(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}} \\
 &+ \frac{4\sqrt{2}\pi[-a\sqrt{Q_-} + b\sqrt{Q_+} - c\sqrt{Q_-}(M_1+M_2) - d\sqrt{Q_+}(M_1-M_2)]}{\sqrt{q^2}\lambda_t}, \\
 H_{\frac{1}{2},+}^V &= - \frac{C_7 m_s \left[g_T(q^2)\sqrt{Q_-}(M_1+M_2) - g_T^{(1)}(q^2)\sqrt{Q_-} \frac{q^2}{M_1} + g_{T5}(q^2)\sqrt{Q_+}(M_1-M_2) + g_{T5}^{(1)}(q^2)\sqrt{Q_+} \frac{q^2}{M_1} \right]}{2\pi q^2} \\
 &- \sqrt{2Q_-} (C_9 + C'_9) \left[f_1(q^2) + f_2(q^2) \frac{M_1+M_2}{M_1} \right] - \sqrt{2Q_+} (C_9 - C'_9) \left[g_1(q^2) - g_2(q^2) \frac{M_1-M_2}{M_1} \right] \\
 &- \frac{8\pi[-a\sqrt{Q_-}(M_1+M_2) - b\sqrt{Q_+}(M_1-M_2) - c\sqrt{Q_-}q^2 + d\sqrt{Q_+}q^2]}{q^2\lambda_t}, \\
 H_{-\frac{1}{2},-}^V &= - \frac{C_7 m_s \left[g_T(q^2)\sqrt{Q_-}(M_1+M_2) - g_T^{(1)}(q^2)\sqrt{Q_-} \frac{q^2}{M_1} - g_{T5}(q^2)\sqrt{Q_+}(M_1-M_2) - g_{T5}^{(1)}(q^2)\sqrt{Q_+} \frac{q^2}{M_1} \right]}{2\pi q^2} \\
 &- \sqrt{2Q_-} (C_9 + C'_9) \left[f_1(q^2) + f_2(q^2) \frac{M_1+M_2}{M_1} \right] + \sqrt{2Q_+} (C_9 - C'_9) \left[g_1(q^2) - g_2(q^2) \frac{M_1-M_2}{M_1} \right] \\
 &- \frac{8\pi[-a\sqrt{Q_-}(M_1+M_2) + b\sqrt{Q_+}(M_1-M_2) - c\sqrt{Q_-}q^2 - d\sqrt{Q_+}q^2]}{q^2\lambda_t}, \\
 H_{\frac{1}{2},t}^A &= \frac{\sqrt{Q_+}(C_{10}+C'_{10}) \left[f_1(q^2)(M_1-M_2) + f_2(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}} + \frac{\sqrt{Q_-}(C_{10}-C'_{10}) \left[g_1(q^2)(M_1+M_2) - g_2(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}}, \\
 H_{-\frac{1}{2},t}^A &= \frac{\sqrt{Q_+}(C_{10}+C'_{10}) \left[f_1(q^2)(M_1-M_2) + f_2(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}} - \frac{\sqrt{Q_-}(C_{10}-C'_{10}) \left[g_1(q^2)(M_1+M_2) - g_2(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}}, \\
 H_{\frac{1}{2},0}^A &= \frac{\sqrt{Q_-}(C_{10}+C'_{10}) \left[f_1(q^2)(M_1+M_2) + f_2(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}} + \frac{\sqrt{Q_+}(C_{10}-C'_{10}) \left[g_1(q^2)(M_1-M_2) - g_2(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}}, \\
 H_{-\frac{1}{2},0}^A &= \frac{\sqrt{Q_-}(C_{10}+C'_{10}) \left[f_1(q^2)(M_1+M_2) + f_2(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}} - \frac{\sqrt{Q_+}(C_{10}-C'_{10}) \left[g_1(q^2)(M_1-M_2) - g_2(q^2) \frac{q^2}{M_1} \right]}{\sqrt{q^2}}, \\
 H_{\frac{1}{2},1}^A &= -\sqrt{2Q_-} (C_{10} + C'_{10}) \left[f_1(q^2) + f_2(q^2) \frac{M_1+M_2}{M_1} \right] - \sqrt{2Q_+} (C_{10} - C'_{10}) \left[g_1(q^2) - g_2(q^2) \frac{M_1-M_2}{M_1} \right], \\
 H_{-\frac{1}{2},-1}^A &= -\sqrt{2Q_-} (C_{10} + C'_{10}) \left[f_1(q^2) + f_2(q^2) \frac{M_1+M_2}{M_1} \right] + \sqrt{2Q_+} (C_{10} - C'_{10}) \left[g_1(q^2) - g_2(q^2) \frac{M_1-M_2}{M_1} \right], \\
 H_{\frac{1}{2},t}^S &= \sqrt{Q_+} (C_S + C'_S) f_S + \sqrt{Q_-} (C_S - C'_S) g_P, \\
 H_{-\frac{1}{2},t}^S &= \sqrt{Q_+} (C_S + C'_S) f_S - \sqrt{Q_-} (C_S - C'_S) g_P, \\
 H_{\frac{1}{2},t}^P &= \sqrt{Q_+} (C_P + C'_P) f_S + \sqrt{Q_-} (C_P - C'_P) g_P, \\
 H_{-\frac{1}{2},t}^P &= \sqrt{Q_+} (C_P + C'_P) f_S - \sqrt{Q_-} (C_P - C'_P) g_P. \tag{C.8}
 \end{aligned}$$

D Renormalization group evolution of the SM dimension-six operator coefficients

The one-loop RG equations of the dimension-six operator coefficients are:

$$16\pi^2\mu\frac{dC_i(\mu)}{d\mu} = \gamma_{ij}C_j(\Lambda) \equiv \dot{C}_i(\Lambda), \quad (\text{D.1})$$

where $\dot{C}_i(\Lambda)$ can be found in refs. [74–76]. Therein, these coefficients $C_{Hq}^{(1)}$, $C_{H\ell}^{(1)}$, $C_{\ell\ell}$, $C_{qq}^{(1)}$, $C_{qq}^{(3)}$, $C_{qu}^{(1)}$ and $C_{qd}^{(1)}$ do not contribute to semi-leptonic decays. Furthermore, the indices ww in the terms like $[\cdots]_{ww\alpha\beta}\delta_{ij}$ denote two identical quarks. These terms do not contribute to FCNC transitions. Therefore, the $\dot{C}_i(\Lambda)$ contributing to down-quark FCNC semi-leptonic decays can be written as:

$$\begin{aligned} [\dot{C}_{\ell q}^{(1)}(\Lambda)]_{ij\alpha\beta} &= \frac{8}{3}g_1^2y_\ell^2[C_{\ell q}^{(1)}(\Lambda)]_{ijww}\delta_{\alpha\beta} + \frac{4}{3}g_1^2y_e y_\ell[C_{qe}(\Lambda)]_{ijww}\delta_{\alpha\beta} \\ &\quad + 12g_1^2y_\ell y_q[C_{\ell q}^{(1)}(\Lambda)]_{ij\alpha\beta} + 9g_2^2[C_{\ell q}^{(3)}(\Lambda)]_{ij\alpha\beta}, \\ [\dot{C}_{\ell q}^{(3)}(\Lambda)]_{ij\alpha\beta} &= \frac{2}{3}g_2^2[C_{\ell q}^{(3)}(\Lambda)]_{ijww}\delta_{\alpha\beta} + 3g_2^2[C_{\ell q}^{(1)}(\Lambda)]_{ij\alpha\beta} \\ &\quad - 6(g_2^2 - 2y_\ell y_q g_1^2)[C_{\ell q}^{(3)}(\Lambda)]_{ij\alpha\beta}, \\ [\dot{C}_{\ell d}(\Lambda)]_{ij\alpha\beta} &= \frac{8}{3}g_1^2y_\ell^2[C_{\ell d}(\Lambda)]_{ijww}\delta_{\alpha\beta} + \frac{4}{3}g_1^2y_e y_\ell[C_{ed}(\Lambda)]_{ijww}\delta_{\alpha\beta} \\ &\quad - 12y_\ell y_d g_1^2[C_{\ell d}(\Lambda)]_{ij\alpha\beta}, \\ [\dot{C}_{qe}(\Lambda)]_{ij\alpha\beta} &= \frac{8}{3}g_1^2N_c y_q^2[C_{qe}(\Lambda)]_{ijww}\delta_{\alpha\beta} + \frac{4}{3}g_1^2N_c y_q y_d[C_{ed}(\Lambda)]_{ijww}\delta_{\alpha\beta} \\ &\quad - 12y_q y_e g_1^2[C_{qe}(\Lambda)]_{ij\alpha\beta}, \\ [\dot{C}_{ed}(\Lambda)]_{ij\alpha\beta} &= \frac{8}{3}g_1^2y_e y_\ell[C_{\ell d}(\Lambda)]_{ijww}\delta_{\alpha\beta} + \frac{4}{3}g_1^2y_e^2[C_{ed}(\Lambda)]_{ijww}\delta_{\alpha\beta} \\ &\quad + 12y_e y_d g_1^2[C_{ed}(\Lambda)]_{ij\alpha\beta}, \\ [\dot{C}_{\ell edq}(\Lambda)]_{ij\alpha\beta} &= -\left(6(y_d(y_q - y_e) + y_e(y_e + y_q))g_1^2 + 3\left(N_c - \frac{1}{N_c}\right)g_3^2\right)[C_{\ell edq}(\Lambda)]_{ij\alpha\beta}, \end{aligned} \quad (\text{D.2})$$

where $y_q = \frac{1}{6}$, $y_d = -\frac{1}{3}$, $y_\ell = -\frac{1}{2}$ and $y_e = -1$ are hypercharge. Three gauge couplings g_1 , g_2 and g_3 are defined as $g_1 = \frac{e}{\cos\theta_W} = 0.36$, $g_2 = \frac{e}{\sin\theta_W} = 0.65$ and $g_3 = \sqrt{4\pi\alpha_s} = 1.22$, respectively, where α_s is the strong coupling constant and $N_c = 3$ is the number of colors.

When running the RG equations from the NP scale Λ to the electroweak scale v , the dimension-six operator coefficients for the case of $\alpha = \beta$ read:

$$\begin{aligned} [C_{\ell q}^{(1)}(v)]_{ij\alpha\alpha} &= \left[1 - \frac{1}{16\pi^2}\left(\frac{8}{3}g_1^2y_\ell^2 + 12g_1^2y_\ell y_q\right)\text{In}\frac{\Lambda}{v}\right][C_{\ell q}^{(1)}(\Lambda)]_{ij\alpha\alpha} \\ &\quad + \left[-\frac{9}{16\pi^2}g_2^2\text{In}\frac{\Lambda}{v}\right][C_{\ell q}^{(3)}(\Lambda)]_{ij\alpha\alpha} + \left[-\frac{1}{12\pi^2}g_1^2y_e y_\ell\text{In}\frac{\Lambda}{v}\right][C_{qe}(\Lambda)]_{ij\alpha\alpha}, \\ [C_{\ell q}^{(3)}(v)]_{ij\alpha\alpha} &= \left[1 - \frac{1}{16\pi^2}\left(-\frac{16}{3}g_2^2 + 12y_\ell y_q g_1^2\right)\text{In}\frac{\Lambda}{v}\right][C_{\ell q}^{(3)}(\Lambda)]_{ij\alpha\alpha} \\ &\quad + \left[-\frac{3}{16\pi^2}g_2^2\text{In}\frac{\Lambda}{v}\right][C_{\ell q}^{(1)}(\Lambda)]_{ij\alpha\alpha}, \end{aligned}$$

$$\begin{aligned}
 [C_{\ell d}(v)]_{ij\alpha\alpha} &= \left[1 - \frac{1}{16\pi^2} \left(\frac{8}{3} g_1^2 y_\ell^2 - 12 y_\ell y_d g_1^2 \right) \text{In} \frac{\Lambda}{v} \right] [C_{\ell d}(\Lambda)]_{ij\alpha\alpha} \\
 &\quad + \left[-\frac{1}{12\pi^2} g_1^2 y_e y_\ell \text{In} \frac{\Lambda}{v} \right] [C_{ed}(\Lambda)]_{ij\alpha\alpha}, \\
 [C_{qe}(v)]_{ij\alpha\alpha} &= \left[1 - \frac{1}{16\pi^2} \left(\frac{8}{3} g_1^2 N_c y_q^2 - 12 y_q y_e g_1^2 \right) \text{In} \frac{\Lambda}{v} \right] [C_{qe}(\Lambda)]_{ij\alpha\alpha} \\
 &\quad + \left[-\frac{1}{12\pi^2} g_1^2 N_c y_q y_d \text{In} \frac{\Lambda}{v} \right] [C_{ed}(\Lambda)]_{ij\alpha\alpha}, \\
 [C_{ed}(v)]_{ij\alpha\alpha} &= \left[1 - \frac{1}{16\pi^2} \left(\frac{4}{3} g_1^2 y_e^2 + 12 y_e y_d g_1^2 \right) \text{In} \frac{\Lambda}{v} \right] [C_{ed}(\Lambda)]_{ij\alpha\alpha} \\
 &\quad + \left[-\frac{1}{6\pi^2} g_1^2 y_e y_\ell \text{In} \frac{\Lambda}{v} \right] [C_{\ell d}(\Lambda)]_{ij\alpha\alpha}, \\
 [C_{\ell edq}(v)]_{ij\alpha\alpha} &= \left[1 + \frac{1}{16\pi^2} \left(6 (y_d (y_q - y_e) + y_e (y_e + y_q)) g_1^2 + 3 \left(N_c - \frac{1}{N_c} \right) g_3^2 \right) \text{In} \frac{\Lambda}{v} \right] \\
 &\quad \times [C_{\ell edq}(\Lambda)]_{ij\alpha\alpha}. \tag{D.3}
 \end{aligned}$$

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References

- [1] S.L. Glashow, J. Iliopoulos and L. Maiani, *Weak Interactions with Lepton-Hadron Symmetry*, *Phys. Rev. D* **2** (1970) 1285 [[INSPIRE](#)].
- [2] G. Buchalla and A.J. Buras, *The rare decays $K \rightarrow \pi \nu \bar{\nu}$, $B \rightarrow X \nu \bar{\nu}$ and $B \rightarrow l^+ l^-$: An Update*, *Nucl. Phys. B* **548** (1999) 309 [[hep-ph/9901288](#)] [[INSPIRE](#)].
- [3] V. Cirigliano, G. Ecker, H. Neufeld, A. Pich and J. Portoles, *Kaon Decays in the Standard Model*, *Rev. Mod. Phys.* **84** (2012) 399 [[arXiv:1107.6001](#)] [[INSPIRE](#)].
- [4] NA62 collaboration, *An investigation of the very rare $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay*, *JHEP* **11** (2020) 042 [[arXiv:2007.08218](#)] [[INSPIRE](#)].
- [5] NA62 collaboration, *Measurement of the very rare $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay*, *JHEP* **06** (2021) 093 [[arXiv:2103.15389](#)] [[INSPIRE](#)].
- [6] KOTO collaboration, *Search for the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 X^0$ decays at the J-PARC KOTO experiment*, *Phys. Rev. Lett.* **122** (2019) 021802 [[arXiv:1810.09655](#)] [[INSPIRE](#)].
- [7] KOTO collaboration, *Study of the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ Decay at the J-PARC KOTO Experiment*, *Phys. Rev. Lett.* **126** (2021) 121801 [[arXiv:2012.07571](#)] [[INSPIRE](#)].
- [8] A.J. Buras, D. Buttazzo, J. Girrbach-Noe and R. Knegjens, *$K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in the Standard Model: status and perspectives*, *JHEP* **11** (2015) 033 [[arXiv:1503.02693](#)] [[INSPIRE](#)].
- [9] J. Brod, M. Gorbahn and E. Stamou, *Updated Standard Model Prediction for $K \rightarrow \pi \nu \bar{\nu}$ and ϵ_K* , *PoS BEAUTY2020* (2021) 056 [[arXiv:2105.02868](#)] [[INSPIRE](#)].
- [10] E787 collaboration, *Search for the decay $K^+ \rightarrow \pi^+ \pi^0$ neutrino anti-neutrino*, *Phys. Rev. D* **63** (2001) 032004 [[hep-ex/0009055](#)] [[INSPIRE](#)].

- [11] E391A collaboration, *Study of the $K_L^0 \rightarrow \pi^0 \pi^0 \nu \bar{\nu}$ decay*, *Phys. Rev. D* **84** (2011) 052009 [[arXiv:1106.3404](#)] [[INSPIRE](#)].
- [12] L.S. Littenberg and G. Valencia, *The decays $K \rightarrow \pi\pi$ neutrino anti-neutrino within the standard model*, *Phys. Lett. B* **385** (1996) 379 [[hep-ph/9512413](#)] [[INSPIRE](#)].
- [13] C.-W. Chiang and F.J. Gilman, *$K(L, S) \rightarrow \pi\pi$ neutrino anti-neutrino decays within and beyond the standard model*, *Phys. Rev. D* **62** (2000) 094026 [[hep-ph/0007063](#)] [[INSPIRE](#)].
- [14] C.-Q. Geng and J. Tandean, *Probing new physics with the kaon decays $K \rightarrow \pi\pi \bar{E}$* , *Phys. Rev. D* **102** (2020) 115021 [[arXiv:2009.00608](#)] [[INSPIRE](#)].
- [15] H.-B. Li, *Prospects for rare and forbidden hyperon decays at BESIII*, *Front. Phys. (Beijing)* **12** (2017) 121301 [Erratum *ibid.* **14** (2019) 64001] [[arXiv:1612.01775](#)] [[INSPIRE](#)].
- [16] BESIII collaboration, *Polarization and Entanglement in Baryon-Antibaryon Pair Production in Electron-Positron Annihilation*, *Nature Phys.* **15** (2019) 631 [[arXiv:1808.08917](#)] [[INSPIRE](#)].
- [17] BESIII collaboration, *Complete Measurement of the Λ Electromagnetic Form Factors*, *Phys. Rev. Lett.* **123** (2019) 122003 [[arXiv:1903.09421](#)] [[INSPIRE](#)].
- [18] LHCb collaboration, *Evidence for the rare decay $\Sigma^+ \rightarrow p\mu^+\mu^-$* , *Phys. Rev. Lett.* **120** (2018) 221803 [[arXiv:1712.08606](#)] [[INSPIRE](#)].
- [19] HYPERCP collaboration, *Evidence for the decay $\Sigma^+ \rightarrow p\mu^+\mu^-$* , *Phys. Rev. Lett.* **94** (2005) 021801 [[hep-ex/0501014](#)] [[INSPIRE](#)].
- [20] A.A. Alves Junior et al., *Prospects for Measurements with Strange Hadrons at LHCb*, *JHEP* **05** (2019) 048 [[arXiv:1808.03477](#)] [[INSPIRE](#)].
- [21] A. Cerri et al., *Report from Working Group 4: Opportunities in Flavour Physics at the HL-LHC and HE-LHC*, *CERN Yellow Rep. Monogr.* **7** (2019) 867 [[arXiv:1812.07638](#)] [[INSPIRE](#)].
- [22] J. Tandean, *Rare hyperon decays with missing energy*, *JHEP* **04** (2019) 104 [[arXiv:1901.10447](#)] [[INSPIRE](#)].
- [23] X.-G. He, J. Tandean and G. Valencia, *The Decay $\Sigma^+ \rightarrow p\ell^+\ell^-$ within the standard model*, *Phys. Rev. D* **72** (2005) 074003 [[hep-ph/0506067](#)] [[INSPIRE](#)].
- [24] X.-G. He, J. Tandean and G. Valencia, *Decay rate and asymmetries of $\Sigma^+ \rightarrow p\mu^+\mu^-$* , *JHEP* **10** (2018) 040 [[arXiv:1806.08350](#)] [[INSPIRE](#)].
- [25] J.-Y. Su and J. Tandean, *Exploring leptoquark effects in hyperon and kaon decays with missing energy*, *Phys. Rev. D* **102** (2020) 075032 [[arXiv:1912.13507](#)] [[INSPIRE](#)].
- [26] G. Li, J.-Y. Su and J. Tandean, *Flavor-changing hyperon decays with light invisible bosons*, *Phys. Rev. D* **100** (2019) 075003 [[arXiv:1905.08759](#)] [[INSPIRE](#)].
- [27] X.-H. Hu and Z.-X. Zhao, *Study of the $s \rightarrow d\nu\bar{\nu}$ rare hyperon decays in the Standard Model and new physics*, *Chin. Phys. C* **43** (2019) 093104 [[arXiv:1811.01478](#)] [[INSPIRE](#)].
- [28] S. Meinel, *$\Lambda_c \rightarrow N$ form factors from lattice QCD and phenomenology of $\Lambda_c \rightarrow n\ell^+\nu_\ell$ and $\Lambda_c \rightarrow p\mu^+\mu^-$ decays*, *Phys. Rev. D* **97** (2018) 034511 [[arXiv:1712.05783](#)] [[INSPIRE](#)].
- [29] A. Buras, *Gauge Theory of Weak Decays*, Cambridge University Press, Cambridge U.K. (2020).

- [30] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak decays beyond leading logarithms*, *Rev. Mod. Phys.* **68** (1996) 1125 [[hep-ph/9512380](#)] [[INSPIRE](#)].
- [31] J. Brod, M. Gorbahn and E. Stamou, *Two-Loop Electroweak Corrections for the $K \rightarrow \pi \nu \bar{\nu}$ Decays*, *Phys. Rev. D* **83** (2011) 034030 [[arXiv:1009.0947](#)] [[INSPIRE](#)].
- [32] A.J. Buras, M. Gorbahn, U. Haisch and U. Nierste, *The Rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ at the next-to-next-to-leading order in QCD*, *Phys. Rev. Lett.* **95** (2005) 261805 [[hep-ph/0508165](#)] [[INSPIRE](#)].
- [33] A.J. Buras, M. Gorbahn, U. Haisch and U. Nierste, *Charm quark contribution to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ at next-to-next-to-leading order*, *JHEP* **11** (2006) 002 [Erratum *ibid.* **11** (2012) 167] [[hep-ph/0603079](#)] [[INSPIRE](#)].
- [34] J. Brod and M. Gorbahn, *Electroweak Corrections to the Charm Quark Contribution to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$* , *Phys. Rev. D* **78** (2008) 034006 [[arXiv:0805.4119](#)] [[INSPIRE](#)].
- [35] PARTICLE DATA GROUP collaboration, *Review of Particle Physics*, *PTEP* **2020** (2020) 083C01 [[INSPIRE](#)].
- [36] G. Buchalla and A.J. Buras, *Two loop large m_t electroweak corrections to $K \rightarrow \pi \nu \bar{\nu}$ for arbitrary Higgs boson mass*, *Phys. Rev. D* **57** (1998) 216 [[hep-ph/9707243](#)] [[INSPIRE](#)].
- [37] A. Kadeer, J.G. Korner and U. Moosbrugger, *Helicity analysis of semileptonic hyperon decays including lepton mass effects*, *Eur. Phys. J. C* **59** (2009) 27 [[hep-ph/0511019](#)] [[INSPIRE](#)].
- [38] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *On the Weak Radiative Decays (Effects of Strong Interactions at Short Distances)*, *Phys. Rev. D* **18** (1978) 2583 [Erratum *ibid.* **19** (1979) 2815] [[INSPIRE](#)].
- [39] R. Alonso, B. Grinstein and J. Martin Camalich, *SU(2) $\times$ U(1) gauge invariance and the shape of new physics in rare B decays*, *Phys. Rev. Lett.* **113** (2014) 241802 [[arXiv:1407.7044](#)] [[INSPIRE](#)].
- [40] O. Catà and M. Jung, *Signatures of a nonstandard Higgs boson from flavor physics*, *Phys. Rev. D* **92** (2015) 055018 [[arXiv:1505.05804](#)] [[INSPIRE](#)].
- [41] G.-S. Yang and H.-C. Kim, *Hyperon Semileptonic decay constants with flavor SU(3) symmetry breaking*, *Phys. Rev. C* **92** (2015) 035206 [[arXiv:1504.04453](#)] [[INSPIRE](#)].
- [42] H.-M. Chang, M. González-Alonso and J. Martin Camalich, *Nonstandard Semileptonic Hyperon Decays*, *Phys. Rev. Lett.* **114** (2015) 161802 [[arXiv:1412.8484](#)] [[INSPIRE](#)].
- [43] N. Cabibbo, E.C. Swallow and R. Winston, *Semileptonic hyperon decays*, *Ann. Rev. Nucl. Part. Sci.* **53** (2003) 39 [[hep-ph/0307298](#)] [[INSPIRE](#)].
- [44] G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, *Chiral Lagrangians for Massive Spin 1 Fields*, *Phys. Lett. B* **223** (1989) 425 [[INSPIRE](#)].
- [45] T. Ledwig, J. Martin Camalich, L.S. Geng and M.J. Vicente Vacas, *Octet-baryon axial-vector charges and SU(3)-breaking effects in the semileptonic hyperon decays*, *Phys. Rev. D* **90** (2014) 054502 [[arXiv:1405.5456](#)] [[INSPIRE](#)].
- [46] M. Ademollo and R. Gatto, *Nonrenormalization Theorem for the Strangeness Violating Vector Currents*, *Phys. Rev. Lett.* **13** (1964) 264 [[INSPIRE](#)].
- [47] S. Sasaki, *Continuum limit of hyperon vector coupling $f_1(0)$ from 2 + 1 flavor domain wall QCD*, *Phys. Rev. D* **96** (2017) 074509 [[arXiv:1708.04008](#)] [[INSPIRE](#)].

- [48] J. Martin Camalich, M. Pospelov, P.N.H. Vuong, R. Ziegler and J. Zupan, *Quark Flavor Phenomenology of the QCD Axion*, *Phys. Rev. D* **102** (2020) 015023 [[arXiv:2002.04623](#)] [[INSPIRE](#)].
- [49] NA48/2 collaboration, *Precise tests of low energy QCD from $K(e4)$ decay properties*, *Eur. Phys. J. C* **70** (2010) 635 [[INSPIRE](#)].
- [50] NA48/2 collaboration, *New measurement of the charged kaon semileptonic $K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu$ (K_{e4}) decay Branching Ratio and Hadronic Form Factors*, *Phys. Lett. B* **715** (2012) 105 [Addendum *ibid.* **740** (2015) 364] [[arXiv:1206.7065](#)] [[INSPIRE](#)].
- [51] NA48/2 collaboration, *Detailed study of the $K^\pm \rightarrow \pi^0 \pi^0 e^\pm \nu$ (K_{e4}^{00}) decay properties*, *JHEP* **08** (2014) 159 [[arXiv:1406.4749](#)] [[INSPIRE](#)].
- [52] Y. Grossman and Y. Nir, *$K_L \rightarrow \pi^0 \nu \nu$ beyond the standard model*, *Phys. Lett. B* **398** (1997) 163 [[hep-ph/9701313](#)] [[INSPIRE](#)].
- [53] S.Y. Hsueh et al., *A High Precision Measurement of Polarized Σ^- beta Decay*, *Phys. Rev. D* **38** (1988) 2056 [[INSPIRE](#)].
- [54] L.S. Geng, J. Martin Camalich, L. Álvarez-Ruso and M.J. Vicente Vacas, *Leading SU(3)-breaking corrections to the baryon magnetic moments in Chiral Perturbation Theory*, *Phys. Rev. Lett.* **101** (2008) 222002 [[arXiv:0805.1419](#)] [[INSPIRE](#)].
- [55] L.S. Geng, J. Martin Camalich and M.J. Vicente Vacas, *Leading-order decuplet contributions to the baryon magnetic moments in Chiral Perturbation Theory*, *Phys. Lett. B* **676** (2009) 63 [[arXiv:0903.0779](#)] [[INSPIRE](#)].
- [56] E.E. Jenkins, M.E. Luke, A.V. Manohar and M.J. Savage, *Weak radiative hyperon decays in chiral perturbation theory*, *Nucl. Phys. B* **397** (1993) 84 [[hep-ph/9210265](#)] [[INSPIRE](#)].
- [57] H. Neufeld, *Weak radiative baryon decays in chiral perturbation theory*, *Nucl. Phys. B* **402** (1993) 166 [[INSPIRE](#)].
- [58] F. Mescia, C. Smith and S. Trine, *$K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \mu^+ \mu^-$: A Binary star on the stage of flavor physics*, *JHEP* **08** (2006) 088 [[hep-ph/0606081](#)] [[INSPIRE](#)].
- [59] G. D'Ambrosio, G. Ecker, G. Isidori and J. Portoles, *The Decays $K \rightarrow \pi \ell^+ \ell^-$ beyond leading order in the chiral expansion*, *JHEP* **08** (1998) 004 [[hep-ph/9808289](#)] [[INSPIRE](#)].
- [60] NA48/2 collaboration, *New measurement of the $K^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ decay*, *Phys. Lett. B* **697** (2011) 107 [[arXiv:1011.4817](#)] [[INSPIRE](#)].
- [61] R.-X. Shi, L.-S. Geng, B. Grinstein, S. Jäger and J. Martin Camalich, *Revisiting the new-physics interpretation of the $b \rightarrow c \tau \nu$ data*, *JHEP* **12** (2019) 065 [[arXiv:1905.08498](#)] [[INSPIRE](#)].
- [62] FLAVOUR LATTICE AVERAGING GROUP collaboration, *FLAG Review 2019: Flavour Lattice Averaging Group (FLAG)*, *Eur. Phys. J. C* **80** (2020) 113 [[arXiv:1902.08191](#)] [[INSPIRE](#)].
- [63] I. Baum, V. Lubicz, G. Martinelli, L. Orifici and S. Simula, *Matrix elements of the electromagnetic operator between kaon and pion states*, *Phys. Rev. D* **84** (2011) 074503 [[arXiv:1108.1021](#)] [[INSPIRE](#)].
- [64] G. D'Ambrosio and T. Kitahara, *Direct CP Violation in $K \rightarrow \mu^+ \mu^-$* , *Phys. Rev. Lett.* **119** (2017) 201802 [[arXiv:1707.06999](#)] [[INSPIRE](#)].
- [65] A. Dery, M. Ghosh, Y. Grossman and S. Schacht, *$K \rightarrow \mu^+ \mu^-$ as a clean probe of short-distance physics*, *JHEP* **07** (2021) 103 [[arXiv:2104.06427](#)] [[INSPIRE](#)].

- [66] C.-H. Chen, C.Q. Geng and I.-L. Ho, *Forward backward asymmetry in $K^+ \rightarrow \pi^+ \ell^+ \ell^-$* , *Phys. Rev. D* **67** (2003) 074029 [[hep-ph/0302207](#)] [[INSPIRE](#)].
- [67] D.-N. Gao, *Long distance contribution to the forward backward asymmetry in decays $K^+ \rightarrow \pi^+ \ell^+ \ell^-$* , *Phys. Rev. D* **69** (2004) 094030 [[hep-ph/0311253](#)] [[INSPIRE](#)].
- [68] R. Alonso, B. Grinstein and J. Martin Camalich, *Lepton universality violation and lepton flavor conservation in B-meson decays*, *JHEP* **10** (2015) 184 [[arXiv:1505.05164](#)] [[INSPIRE](#)].
- [69] Q. Luo, W. Gao, J. Lan, W. Li and D. Xu, *Progress of Conceptual Study for the Accelerators of a 2–7 GeV Super Tau Charm Facility at China*, in *10th International Particle Accelerator Conference (IPAC2019)*, Melbourne Australia (2019), pg. 643.
- [70] H.P. Peng, Y.H. Zheng and X.R. Zhou, *Super Tau-Charm Facility of China*, *Physics* **49** (2020) 513.
- [71] T. Bhattacharya et al., *Probing Novel Scalar and Tensor Interactions from (Ultra)Cold Neutrons to the LHC*, *Phys. Rev. D* **85** (2012) 054512 [[arXiv:1110.6448](#)] [[INSPIRE](#)].
- [72] N. Cabibbo and A. Maksymowicz, *Angular Correlations in $K\pi$ Decays and Determination of Low-Energy $\pi\pi$ Phase Shifts*, *Phys. Rev.* **137** (1965) B438 [Erratum *ibid.* **168** (1968) 1926] [[INSPIRE](#)].
- [73] A. Pais and S.B. Treiman, *Pion Phase-Shift Information from $K_{\ell 4}$ Decays*, *Phys. Rev.* **168** (1968) 1858 [[INSPIRE](#)].
- [74] E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators I: Formalism and λ Dependence*, *JHEP* **10** (2013) 087 [[arXiv:1308.2627](#)] [[INSPIRE](#)].
- [75] E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators II: Yukawa Dependence*, *JHEP* **01** (2014) 035 [[arXiv:1310.4838](#)] [[INSPIRE](#)].
- [76] R. Alonso, E.E. Jenkins, A.V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators III: Gauge Coupling Dependence and Phenomenology*, *JHEP* **04** (2014) 159 [[arXiv:1312.2014](#)] [[INSPIRE](#)].