Erratum: General null asymptotics and superrotation-compatible configuration spaces in $d \geq 4$

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Hereby, the author corrects a mistake in section 5.3.2, paragraph **$d = 6$ and even**, p. 36, together with some other minor typos and also takes the opportunity to further clarify some core statements. The mistake does not affect the main results of the paper.

Corrections to section 5.3.2. The middle step of equation (5.93) contains a typo and the correct expression reads

$$J_{(1)B}^D = \frac{1}{2} \left(\mathcal{H}_{(\frac{5}{2})B}^D - l\tilde{K}_{(\frac{5}{2})} \right) = \frac{1}{2} \left(\mathcal{H}_{(\frac{5}{2})B}^D + \frac{3l}{4} h_{(\frac{3}{4})B}^D \right) \equiv 0 \quad (1)$$

Given (5.95) and (5.96), equations (5.97) and (5.98) should respectively contain the additional terms $-\Theta_{(\frac{3}{2})}\bar{L}_{(-\frac{3}{2})}$ and $J_{(-\frac{1}{2})}\Theta_{(\frac{3}{2})}$. Then the sentence “Both these terms can be seen to zero if $h_{(\frac{3}{2})AB} = 0$, thus completing the proof.” is to be modified to “Both these terms and $\Theta_{(\frac{3}{2})}$ can be seen to zero if $h_{(\frac{3}{2})AB} = 0$, thus completing the proof.”. Alternatively one can remove the terms $-\Theta_{(\frac{3}{2})}\bar{L}_{(-\frac{3}{2})}$ and $J_{(-\frac{1}{2})}\Theta_{(\frac{3}{2})}$ directly from (5.95) and (5.96) to match with (5.94).

The in-line equation between (5.100) and (5.101) contains a minor typo: the prefactor of $J_B^{D(tot)}$ is to be corrected to $\frac{2}{d-4}|_{d=6} = 1$. Equation (5.105) contains two obvious typos: $\Theta_{(0)}$ should be $\Theta_{(1)}$ and the indices on the second term of the equation should be swapped; also $[\Theta_{(0)}J_{(0)}^{(tot)}]_B^D$ in the sentence below is to be corrected to $[\Theta_{(1)}J_{(0)}^{(tot)}]_B^D$.

Equation (5.102) contains a typo and its correct form is

$$\frac{N_B^D}{2} = \bar{l}_{(2)B}^D + \frac{1}{2} \left(K_{(0)}^{DE} \mathcal{H}_{(2)EB} + \mathcal{H}_{(2)A}^D K_{(0)B}^A + lK_{(0)}^{DE} K_{(0)EB} \right) \quad (2)$$

where a factor of $1/2$ in the latter term here is removed with respect to (5.102). Explicit computation of its trace brings to

$$N = 0. \quad (3)$$

which corrects the statement in (5.103).¹ We can write eq. (1) in a manifestly trace-free form as

$$N_{AB} = \partial_u h_{(2)AB}^{(Tf)} - \frac{l}{2} h_{(2)AB}^{(Tf)} + 2K_{(0)(A}^E \mathcal{H}_{(2)B)E} + lK_{(0)(A}^E K_{(0)B)E} - \frac{h_{(0)AB}}{2} \left(\mathcal{H}_{(2)C}^D K_{(0)D}^C + \frac{l}{2} K_{(0)C}^D K_{(0)D}^C \right), \quad (4)$$

where $h_{(2)AB}^{(Tf)}$ denotes the trace-free part of $h_{(2)AB}$. The first line in eq. (4) represents the definition of news tensor within a radiative expansion of h_{AB} and time-dependent boundary metric. The second line captures the corrections induced by the generalised boundary conditions and their large gauge transformations coupled to the overleading terms of the asymptotic expansion.

Referring to the six-dimensional case, equation (5.104) is more correctly written as

$$N_{AB}^{(NR)} = N_{AB}^{(R)} + \text{non-radiative terms depending on leading orders} \quad (5)$$

such non-radiative terms may lead to non-linearities in higher dimensions, but this is not evident from the $d = 6$ example. The sentences “and vanishes if $l = 0$ ($h_{(0)AB}$ time-independent)” after (5.103), and “and its trace is non vanishing if $l \neq 0$ due to the non-linearities” after (5.104) should be removed. The latter sentence was followed by “which makes the identification of $N_{AB}^{(NR)}$ with a news tensor obscure”, which should be removed in favour of: “The identification of $N_{AB}^{(NR)}$ with a news tensor is obscure because of the coupling of the overleading terms with the boundary conditions/large gauge transformations. The relevant degrees of freedom (in the hard sector) should correspond to gauge-independent versions of this object.” In the sentence before (5.105) “if $\beta_{(0)} = 0$ and $h_{(0)AB}$ is Einstein, then only $\frac{l}{2} K_{(0)}^{DE} K_{(0)EB}$ remains to be equated to zero”, $\frac{l}{2} K_{(0)}^{DE} K_{(0)EB}$ should be modified to $[lK_{(0)}^{DE} K_{(0)EB}]^{(Tf)}$.

The comment in the third bullet point in section 3.2 starting with “In even dimensions this is no longer the case [...]” and ending with “(see equation (5.103) and subsection 5.3.2 for the details).” is to be changed to: “In even dimensions the free function N_{AB} is traceless and possess contributions from the radiative order and the overleading integer-power terms that couple with the generalised boundary conditions, as specified for $d = 6$ in (5.102). In particular, with a time-independent boundary metric and $\beta_{(0)} = 0$ these additional terms are associated to the large gauge transformations (superrotations in particular, compare with (5.102)).”

¹Independently of the computation, this is consistent because N_B^A is obtained as the integration function of the fourth main equation (4.6), which is the trace-free part of Einstein equations on the transverse directions. Writing it as (4.19), its trace is $\partial_r L = J$ where $J = 0$ at each order using the solutions of the previous three main equations. Thus the trace of L can only be in a r -independent term, which is in contrast with the definition of L_B^A (4.17) and (4.18). Furthermore $L = 0$ can be explicitly checked using its definition and as a consequence of the Bondi-gauge determinant condition that fixes the traces of $h_{(n)AB}$ $n \geq 2$.

Clarifications. Claim C.1 in section 3.2.1 page 15 concerns the way in which equation (2.14) can be generalised to (3.8) and is thus manifestly correct. A minus sign in front of equation (3.5) is missing; once accounted for it, (3.5) matches (3.8). It should not be intended, however, as the statement that the boundary metric $h_{(0)AB}$ is fixed to be non-Einstein. It rather means that when $\beta_{(0)}$ is fixed to zero and $h_{(0)AB}$ is time-independent, then general CL-superrotations imply that $h_{(0)AB}$ cannot be fixed to be Einstein because CL-superrotations act as a diffeomorphism and a generic Weyl rescaling, according to (7.9). Accordingly, the sentence in the abstract “One possibility requires the time-independent boundary metric on the cuts of $\mathcal{S}$ to be non-Einstein” can be changed to “One possibility requires the time-independent boundary metric on the cuts of $\mathcal{S}$ not to be fixed to an Einstein metric”. These match the comments on page 8 starting with “In fact, this feature” and ending with “the perturbation ϵ is such that $\gamma + \epsilon$ is not Einstein”.

Concerning the sentence below equation (5.109) “This analysis suggests that with the non-radiative falloff r^{-1} in h , a maximally polyhomogeneous expansion is to be considered in $d \geq 6$ even.”, it is meaningful to clarify that equation (5.105) brings to a differential constraint on $h_{(1)AB}$, which upon linearization around the round-sphere metric reduces to the constraint (3.32) of [70] (citation number follows main paper). This corresponds to the case mentioned on page 8 just before section 3. Notice that this directly relates to the comment on page 46 starting with “We have not discussed” and ending with “the Laplacian associated to this metric.” To better clarify this, it can be noted that the conclusions reached in [60,70,95] do depend on the (linearised) constraint (5.105) as well as on the time-independence of the boundary metric. With a time-dependent boundary metric one obtains a slight generalisation, but both (3.5) and (5.105) are needed to show that $h_{(1)AB}$ is pure gauge: without (5.105) a part of $h_{(1)AB}$ is left undetermined. However, the main text does not discuss further the constraint (5.105) because the scope is to address the construction of the most general solution space and to point out the differences between even and odd dimensions. It is thus interesting to explicitly note that in odd dimensions there are no constraints analogous to (5.105), unless the falloff conditions with half-integer powers before the radiative order (in addition to the integer ones) are assumed, as proved in pages 35, 36.

Concerning the identification of the news tensor, the author would like to remind the reader that in four dimensions a gauge-invariant definition of the news tensor is reached via the Geroch tensor [11, 110] that is added to the Bondi frame news tensor. Note that Φ_{AB} in (3.10) is formally the same as the trace-free part of the Geroch tensor, although its manifestation in the higher dimensional context via equation (3.5) is different from the four-dimensional case where (3.5) does not apply: namely, in higher dimensions Φ_{AB} appears in a dynamical equation above the radiative order while in four dimensions it is to be added by hand to build a gauge-invariant radiative news. The formal similarity is exact when (3.5) is integrated assuming the conditions in C.1 with the further restriction that $h_{(0)AB}$ is conformal to a sphere (this case is compatible with limits to i^0 , see the comment on page 16 starting with “We conclude that”). In such a case $h_{(1)AB}$ is linear in u and its part linear in u is pure gauge. Given the definition of $\mathcal{H}_{(2)AB}$ (see (3.12) and (3.9)), is

evident that Φ_{AB} appears in $N_{AB}^{(NR)}$ in even dimensions. Notice however that one probably would like to call “higher dimensional Geroch tensor” the object that is to be added to $N_{AB}^{(NR)}$ to define a gauge-invariant object, not Φ_{AB} itself. This is left for future analysis. Notice also from (5.95) that a similar discussion can be extended to odd dimensions greater than five as long as $h_{(\frac{3}{2})AB} \neq 0$, because it couples with $\mathcal{H}_{(2)AB}$. Otherwise the situation is somewhat more similar to the four-dimensional case in this respect and one still would have to find an appropriate definition of higher dimensional Geroch tensor.

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