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M2-doughnuts

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ABSTRACT: We present a family of new M2-brane solutions in $AdS_7 \times S^4$ that calculate toroidal BPS surface operators in the $\mathcal{N} = (2, 0)$ theory. These observables are conformally invariant and not subject to anomalies so we are able to evaluate their finite expectation values at leading order at large N . In the limit of a thin torus we find a cylinder, which is a natural surface generalization of both the circular and parallel lines Wilson loop. We study and comment on this limit in some detail.

KEYWORDS: AdS-CFT Correspondence, M-Theory

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1 Introduction and conclusions

The AdS/CFT correspondence [1] allows to calculate nonperturbative field theory quantities using classical geometry. A case in point are surface operators of the 6d $\mathcal{N} = (2, 0)$ theory, which are described at large N in terms of classical M2-branes in $AdS_7 \times S^4$ [2]. There is a clear analogy with the calculation of Wilson loops in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in 4d, but while there are plenty of examples of classical string solutions in $AdS_5 \times S^5$ (see for example [3–12]), explicit M2-brane solutions are very rare: the main examples are a pair of anti-parallel planes [2] and the sphere [4] (see also [13] for a generalisation of the anti-parallel planes to finite temperature).

For arbitrary surfaces, the asymptotic form of the solution near the boundary of AdS was calculated and this is sufficient to evaluate their anomaly [14, 15]. Recall that two dimensional surface observables generally suffer from conformal anomalies, which are generalizations of the usual central charge in 2d CFT and dimensions of local operators [15–19].

As usual, these anomalies arise from divergences in the evaluation of physical quantities and are local, which is why the asymptotic solution is enough to capture them. They also normally render the exact normalisation of a physical operator ambiguous. This is analogous to the prescription dependence in defining the normalisation of local operators as the number C in $\langle \mathcal{O}(x)\mathcal{O}(y) \rangle \sim C/|x - y|^{2\Delta}$. In recent years a lot of progress has been achieved in the exact evaluation of the coefficients entering the anomaly of surface operators of the $\mathcal{N} = (2, 0)$ theory [20–25], so the time is now ripe to turn to evaluating finite expectation values for “anomaless” surfaces, the cases where the anomaly vanishes.

The recent paper [26] presented a whole slew of surface operators in the $\mathcal{N} = (2, 0)$ theory that are BPS. In all of the classes of operators studied, invariance under supersymmetry implies that the corresponding M2-brane is calibrated with respect to appropriate calibration forms. This translates into first-order differential equations for their embedding in AdS and drastically simplifies the task of finding new classical solutions. Furthermore, there are quite a few examples of anomaless surfaces, so their expectation values should be well defined.

The purpose of this note is to present new nontrivial classical M2-brane solutions which also have a nontrivial expectation value. The surfaces we evaluate here are flat tori in $\mathbb{R}^4$ (so can also be thought of as Clifford tori in S^3) and break the R -symmetry $\text{SO}(5) \rightarrow \text{SO}(3)$ (they couple to 2 scalar fields in the abelian version of the theory). In the language of [26], they belong to type-III, or more specifically, to sub-type-L of Lagrangian surfaces in $\mathbb{R}^4$ and they preserve 4 supercharges.

In the following we present the classical solutions for any such torus with radii R_1 and R_2 . The final expression for the expectation value of the surface operator is (3.14)

$$\langle V_{R_1, R_2} \rangle = e^{-S_{\text{ren}}}, \quad S_{\text{ren}} = -\pi N \left(\frac{R_1}{R_2} + \frac{R_2}{R_1} \right). \quad (1.1)$$

As we argue below, this combination of the cross-ratio R_1/R_2 and a constant are the only allowed functions compatible with the symmetries of the torus. So what we really determined is the prefactor $-\pi$ and absence of constant piece. A simple calculation in the abelian theory, relying on the techniques of [15, 27–29] gives a trivial answer. Note that there is no Lagrangian description of the 6d theory for finite N , and no other calculation of this quantity. One approach to finite N would be to perform a semiclassical M-theory analysis beyond the leading large N limit considered here.

An even simpler version of this observable is the limit of $R_1 \rightarrow \infty$ with fixed R_2 , studied in section 5. The torus becomes a cylinder, where the invariant cross-ratio is now the ratio of its length $2\pi R_1$ to its radius R_2 , as in (5.6).

This observable shares some similarities to the quark-antiquark potential in a gauge theory. The latter is represented by the two antiparallel lines Wilson loop observable. One natural uplift to surface observables is to replace each of the lines with a plane, as was done in [2]. Another one is to replace the $\mathbb{R} \times \mathbb{Z}_2$ geometry with $\mathbb{R} \times S^1$, hence giving the cylinder.

What we study here is a version of this observable that preserves a fraction of the supercharges, so is BPS, yet like the circular Wilson loop in $\mathcal{N} = 4$ SYM, it has a nontrivial vacuum expectation value [30–32]. The factor $-N/2R_2$ in (5.6) is then one natural analog to the quark-antiquark potential in this theory at large N . The value we find here is very different from that arising in the two plane calculation in [2], as they are very distinct analogues to the quark-antiquark observable.

We hope to report on more finite results for BPS and non-BPS surface observables in the $\mathcal{N} = (2, 0)$ theory in the near future.

2 The tori

In the absence of a Lagrangian description, surface operators in the $\mathcal{N} = (2, 0)$ theory are defined indirectly by stating their properties: their geometry, the local breaking of R -symmetry (also referred to as “scalar coupling”, in analogy with Maldacena-Wilson loops, though there are no fields) and a representation of the ADE algebra underlying the theory. This is discussed in great detail in [26] which also analyses possible relationships between the geometry and scalar couplings that guarantee preserved supersymmetry. That analysis is similar to that of [33–35] in the case of Wilson loops in $\mathcal{N} = 4$ SYM.

One of the classes of BPS surface operators identified in [26], named type-III, consists of operators of arbitrary geometry in $\mathbb{R}^4 \subset \mathbb{R}^6$ and the supersymmetry condition is that the scalar coupling (or local $\text{SO}(5)$ vector) is

$$n^I = \frac{1}{2} \eta_{\mu\nu}^I \varepsilon^{ab} \partial_a x^\mu \partial_b x^\nu, \quad (2.1)$$

where $\eta_{\mu\nu}^I$ is the chiral 't Hooft symbol. A convenient choice is

$$\eta_{14}^1 = \eta_{23}^1 = \eta_{24}^2 = -\eta_{13}^2 = \eta_{12}^3 = \eta_{34}^3 = 1, \quad (2.2)$$

with the other components fixed by symmetry or zero. Note that ε^{ab} in (2.1) is normalized with a factor of the induced metric on the surface such that $\|n\| = 1$.

The surface operators we study here are particular examples of operators of type-III. They are flat tori in $\mathbb{R}^4$ and we parametrise their embedding by $\varphi_1, \varphi_2 \in [0, 2\pi)$ and fixed R_1, R_2 as

$$\begin{aligned} x^1 &= R_1 \cos \varphi_1, & x^2 &= R_1 \sin \varphi_1, \\ x^3 &= R_2 \cos \varphi_2, & x^4 &= R_2 \sin \varphi_2. \end{aligned} \quad (2.3)$$

Plugging this into (2.1) implies the scalar couplings should be

$$n^1 = -\sin(\varphi_1 + \varphi_2), \quad n^2 = \cos(\varphi_1 + \varphi_2). \quad (2.4)$$

Supersymmetry does not depend on the choice of representation, but we focus on operators in the fundamental representation of A_{N-1} , which by the AdS/CFT dictionary are dual to M2-branes at large N [2].

The analysis of [26] shows that generic surfaces of type-III preserve a single Poincaré supercharge Q (out of 16 Q 's and 16 S 's). It is also shown there that for Lagrangian surfaces in $\mathbb{R}^4$, where $n^3 = 0$, these supercharges are doubled. Furthermore, because the torus is in S^3 , the tori also preserve two 2 special supercharges S 's.

A concrete way to understand the supersymmetries is to notice that our ansatz has two $U(1)$ symmetries, respectively the shifts in φ_1 and φ_2 . The preserved supercharges are those invariant under the action of these two $U(1)$ symmetries (with $M_{\mu\nu}$ are rotations and R_{IJ} are R-symmetry transformations)

$$[M_{12} - R_{12}, Q] = [M_{34} - R_{12}, Q] = 0. \quad (2.5)$$

Using the 6d (anti)chiral gamma matrices γ_μ ($\bar{\gamma}_\mu$) and the 5d R-symmetry gamma matrices $\check{\gamma}_I$, the supercharges satisfying these constraints can be constructed explicitly, with

$$(1 + \gamma_1 \bar{\gamma}_2 \gamma_3 \bar{\gamma}_4)Q = (1 - i\gamma_1 \bar{\gamma}_3 \check{\gamma}_1)Q = (1 - i\gamma_1 \bar{\gamma}_4 \check{\gamma}_2)Q = 0. \quad (2.6)$$

These are 3 independent constraints, so two Q out of 16 satisfy them. The first says that Q is antichiral in 4d, which can also be seen by taking the difference of the equations (2.5). For more details and the analysis of special supersymmetries S , see [26].

Note that because the supercharges are antichiral, their anticommutator only sees a combination of the two $U(1)$ symmetries

$$\{Q, S\} \propto (M_{12} + M_{34} - 2R_{12}), \quad (2.7)$$

with all other commutators vanishing. This is compatible with the discrete symmetry of the tori where we exchange φ_1 with φ_2 and R_1 with R_2 , and in the following we assume without loss of generality that $R_1 \geq R_2$.

Beside their supersymmetry, another important property of these operators is that they do not suffer from conformal anomalies, so their expectation value is well-defined. The reason is that the anomalies of generic operators of type- $\mathbb{H}$ are related to the Euler class and self-intersection number of the surface (see details in [26]), but in the case of the torus those two topological invariants vanish.

Note that there are other BPS tori that belong to the type-S class in [26]. They have different n^I and are not studied here.

3 Doughnut solutions

We now find the classical solution of M2-branes in $AdS_7 \times S^4$ ending on the tori presented above. It is convenient to use the metric

$$ds^2 = 4L^2 y dx^\mu dx^\mu + \left(\frac{L}{y}\right)^2 dy^I dy^I, \quad (3.1)$$

with $\mu = 1, \dots, 6$, $I = 1, \dots, 5$ and $y = \|y^I\|$. Note also that compared to more common forms of this metric we absorbed a factor of L^3 into y^I . The background also has F_4 and F_7 fluxes, but those do not play a role in the calculation below.

We look for our M2-brane to have $U(1)^2$ isometry related to $M_{12} - R_{12}$ and $M_{34} - R_{12}$. Choosing as worldvolume coordinates the angles φ_1, φ_2 and $\rho = \sqrt{y_1^2 + y_2^2}$, the ansatz is¹

$$\begin{aligned} x^1 &= r_1(\rho) \cos \varphi_1, & x^2 &= r_1(\rho) \sin \varphi_1, \\ x^3 &= r_2(\rho) \cos \varphi_2, & x^4 &= r_2(\rho) \sin \varphi_2, \\ y_1 &= -\rho \sin(\varphi_1 + \varphi_2), & y_2 &= \rho \cos(\varphi_1 + \varphi_2), & y_3 &= \rho. \end{aligned} \quad (3.2)$$

This ansatz is clearly in an $AdS_5 \times S^2$ subspace, but in fact because the boundary surface is a Clifford torus in S^3 , there is a further restriction to AdS_4 and the solutions should satisfy the constraint

$$r_1^2 + r_2^2 + \frac{1}{\sqrt{\rho^2 + y_3^2}} = R_1^2 + R_2^2. \quad (3.3)$$

The boundary of space is at $y \rightarrow \infty$, and we expect $y_3/\rho \rightarrow 0$ there (because $n^3 = 0$ in (2.4)). Indeed we see that y_1 and y_2 in (3.2) match the expectation from (2.4). Likewise we should impose $r_1 \rightarrow R_1$ and $r_2 \rightarrow R_2$. At the interior of space the surface should end at $\rho = 0$, such that $y_1 = y_2 = 0$ and with either $r_1 = 0$ or $r_2 = 0$.

The Lagrangian for the M2-brane is

$$\mathcal{L} = 2L^3 \frac{\sqrt{[4((r'_1)^2 + (r'_2)^2)(\rho^2 + y_3^2)^{3/2} + 1 + (y'_3)^2][4r_1^2 r_2^2 (\rho^2 + y_3^2)^{3/2} + (r_1^2 + r_2^2)\rho^2]}}{(\rho^2 + y_3^2)^{3/4}}. \quad (3.4)$$

¹To avoid confusion with exponents we use subscripts y_I instead of superscripts to denote the components of y .

It is natural to assume that for the BPS solution the two terms in the square root are proportional to each other. This means either

$$\frac{4(r'_1)^2 + 4(r'_2)^2}{1 + (y'_3)^2} = \frac{4r_1^2 r_2^2}{(r_1^2 + r_2^2)\rho^2}, \quad \text{or} \quad \frac{4(r'_1)^2 + 4(r'_2)^2}{1 + (y'_3)^2} = \frac{r_1^2 + r_2^2}{4r_1^2 r_2^2} \frac{\rho^2}{(\rho^2 + y_3^2)^3}. \quad (3.5)$$

The second one looks more natural and can be rewritten as

$$4(r'_1)^2 + 4(r'_2)^2 = \left(\frac{1}{r_1^2} + \frac{1}{r_2^2} \right) \frac{\rho^2(1 + (y'_3)^2)}{4(\rho^2 + y_3^2)^3}. \quad (3.6)$$

This is easily solved with

$$2r'_i = \frac{1}{2r_i} \frac{\rho \sqrt{1 + (y'_3)^2}}{(\rho^2 + y_3^2)^{3/2}}. \quad (3.7)$$

We can get a third equation by differentiating the constraint (3.3). Substituting the equations above to eliminate r'_i , we find an equation for y_3 of the form

$$(\rho^2 - y_3^2)(y'_3)^2 = -2\rho y_3 y'_3. \quad (3.8)$$

The only solutions to this equation with $y_3/\rho \rightarrow 0$ at large ρ and no poles at finite ρ are with $y'_3 = 0$.

Plugging this into (3.7), we can immediately integrate to get

$$\begin{aligned} r_1^2(\rho) &= R_1^2 - \frac{1}{2\sqrt{\rho^2 + y_3^2}} = R_1^2 - \frac{1}{2y}, \\ r_2^2(\rho) &= R_2^2 - \frac{1}{2\sqrt{\rho^2 + y_3^2}} = R_2^2 - \frac{1}{2y}. \end{aligned} \quad (3.9)$$

The integration constants R_1^2 and R_2^2 are fixed from the boundary conditions at $y \rightarrow \infty$, where we recover $r_1 \rightarrow R_1$ and $r_2 \rightarrow R_2$. The only remaining unknown is the constant y_3 , and that can be determined from the minimum value of y at $\rho = 0$, where either $r_1(0) = 0$ or $r_2(0) = 0$. As we assume that $R_2 \leq R_1$, the first circle to shrink is r_2 , so this happens at $\rho = 0$ if $y_3 = \pm \frac{1}{2R_2^2}$. The different signs give two different solutions, where the M2-brane wraps the lower or upper hemisphere of the $S^2 \subset S^4$. In the following we use the positive choice, in order not to write $|y_3|$ everywhere.

These expressions indeed solve the equations of motion and are derived more rigorously in the next section from the calibration equations.

The classical value of the Lagrangian is now

$$\mathcal{L} = L^3 \left(4r_1 r_2 + \frac{r_1^2 + r_2^2}{r_1 r_2} \frac{\rho^2}{(\rho^2 + y_3^2)^{3/2}} \right). \quad (3.10)$$

It is convenient to change variables to y , where

$$\mathcal{L}' = \mathcal{L} \frac{d\rho}{dy} = 2L^3 \frac{(2y_3(R_1^2 + R_2^2) - 1)|y|^3 + (y_3(R_1^2 + R_2^2) - 1)(y + y_3)y_3 y - y_3^3}{y_3|y|^2 \sqrt{(y + y_3)((2y_3(R_1^2 + R_2^2) - 1)y - y_3)}}. \quad (3.11)$$

Integrating over y , φ_1 and φ_2 we find

$$\int \mathcal{L}' dy d\varphi_1 d\varphi_2 = 8\pi^2 L^3 \frac{(y - y_3) \sqrt{(y_3 + y)((2y_3(R_1^2 + R_2^2) - 1)y - y_3)}}{y_3 y} \quad (3.12)$$

This vanishes at $y = y_3$ and diverges for $y \rightarrow \infty$. Expanding in that limit and multiplying by the M2-brane tension

$$T_{M2} = \frac{1}{4\pi^2 \ell_P^3} = \frac{N}{4\pi L^3}, \quad (3.13)$$

gives

$$\begin{aligned} S &= 8\pi^2 L^3 T_{M2} y \frac{\sqrt{(2y_3(R_1^2 + R_2^2) - 1)}}{y_3} - 8\pi^2 L^3 T_{M2} \frac{y_3(R_1^2 + R_2^2)}{\sqrt{(2y_3(R_1^2 + R_2^2) - 1)}} \\ &= 4\pi N y R_1 R_2 - \pi N \left(\frac{R_1}{R_2} + \frac{R_2}{R_1} \right). \end{aligned} \quad (3.14)$$

The divergent term is $\frac{yN}{\pi} \text{Area}(\text{torus})$ and is cancelled by an appropriate counterterm. We are left with a finite conformally invariant answer (1.1).

4 BPS equations

The derivation above was rather intuitive, but involved guesswork. To put it on firmer footing we resort to the BPS equations presented in [26]. In the case of surfaces of type-III, the M2-branes are calibrated with respect to the 3-form

$$\phi = 2L^3 \eta_{\mu\nu}^I dx^\mu \wedge dx^\nu \wedge dy^I - \frac{L^3}{y^3} dy_1 \wedge dy_2 \wedge dy_3, \quad (4.1)$$

and obey the differential equations

$$\partial_\alpha X^M = \frac{1}{2} g_{\alpha\beta} \varepsilon^{\beta\gamma\delta} G^{ML} \phi_{LNP} \partial_\gamma X^N \partial_\delta X^P. \quad (4.2)$$

Here X^M are any of x^μ and y^I , g is the induced metric, ε is the Levi-Civita tensor (with $1/\sqrt{g}$) and G is the target space metric.

These equations are simple to solve. Let us examine the components of ϕ_{LNy_3} and restrict to our $AdS_4 \times S^2$ with coordinates as in (3.2): $r_1, r_2, \varphi_1, \varphi_2, \rho, y_3$. We have

$$\begin{aligned} \partial_{y_3} \cdot \phi &= 4L^3 (dx^1 \wedge dx^2 + dx^3 \wedge dx^4) - \frac{L^3}{y^3} dy_1 \wedge dy_2 \\ &= 4L^3 \left(r_1 dr_1 \wedge d\varphi_1 + r_2 dr_2 \wedge d\varphi_2 - \frac{\rho}{4(\rho^2 + y_3^2)^{3/2}} d\rho \wedge (d\varphi_1 + d\varphi_2) \right). \end{aligned} \quad (4.3)$$

Recall that our ansatz (3.2) assumes $\partial_{\varphi_1} y_3 = \partial_{\varphi_2} y_3 = 0$ (this also follows from the supersymmetry preserved). Taking X^M in (4.2) to be y_3 we find

$$\begin{aligned} \partial_{\varphi_1} y_3 &\propto r_2 r_2' - \frac{\rho}{4(\rho^2 + y_3^2)^{3/2}}, \\ \partial_{\varphi_2} y_3 &\propto r_1 r_1' - \frac{\rho}{4(\rho^2 + y_3^2)^{3/2}}. \end{aligned} \quad (4.4)$$

Setting those to zero exactly reproduces the equations (3.7) with $y'_3 = 0$. Indeed, we can also derive $y'_3 = 0$ from the fact that there is no $d\varphi_1 \wedge d\varphi_2$ terms on the second line on (4.3).

We thus rederived the equations in section 3 from the BPS conditions and consequently their solution (3.9).

5 Cylinder limit

In the discussion above the torus is defined for any two radii R_1 and R_2 . In the limit where $R_1 \rightarrow \infty$ and R_2 is kept fixed, the torus locally becomes a cylinder of radius R_2 . More precisely, redefining $v = R_1\varphi_1$ and shifting $x^1 \rightarrow x^1 - R_1$, the torus (2.3) becomes

$$\begin{aligned} x^1 &= 0, & x^2 &= v, \\ x^3 &= R_2 \cos \varphi_2, & x^4 &= R_2 \sin \varphi_2. \end{aligned} \quad (5.1)$$

This is a cylinder of radius R_2 extended along the x^2 direction. The scalar couplings (2.4) are

$$n^1 = -\sin \varphi_2, \quad n^2 = \cos \varphi_2. \quad (5.2)$$

A cylinder is clearly embedded in an $\mathbb{R}^3$ subspace of $\mathbb{R}^4$ so according to the classification of [26], is of type-N. Being extended uniformly in the x^2 direction, it is also a version of the type- $\mathbb{R}$ surfaces, which are the uplift to 6d of the BPS Wilson loops of [33]. Taking the limit does not change the number of preserved supercharges, but replaces the two special supersymmetries S with two further Q . To see that, note that in 3d there is no chirality condition (first condition of (2.6)) so the number of Q is doubled.

Taking the same limit on the M2-brane ansatz (3.2) and solution (3.9), we find the limiting solution

$$\begin{aligned} x^1 &= 0, & x^2 &= v, & x^3 &= r_2(\rho) \cos \varphi_2, & x^4 &= r_2(\rho) \sin \varphi_2, \\ y_3 &= \frac{1}{2R_2^2}, & y_1 &= -\rho \sin \varphi_2, & y_2 &= \rho \cos \varphi_2, \end{aligned} \quad (5.3)$$

with

$$r_2(y) = \sqrt{R_2^2 - \frac{1}{2y}}, \quad y = \sqrt{\rho^2 + \frac{1}{4R_2^4}}. \quad (5.4)$$

The action can also be derived as a limit of the action in (3.14). We find

$$S = 4\pi N y R_1 R_2 - \frac{\pi N R_1}{R_2}. \quad (5.5)$$

Identifying the first term with $\frac{Ny}{\pi} \text{Area}(\text{cylinder})$, we see that the length D of the cylinder is represented by $D = 2\pi R_1$. Thus we interpret the last term as

$$S_{\text{ren}} = -\frac{ND}{2R_2}. \quad (5.6)$$

This has the natural structure arising from a conformal potential, extensive in the length D and inversely proportional to the size $2R_2$.

This interpretation also places an interesting restriction on the expectation value of the torus (1.1). Using conformal symmetry and the discrete symmetry exchanging $R_1 \leftrightarrow R_2$, the expectation value can be an arbitrary function of $\frac{R_1}{R_2} + \frac{R_2}{R_1}$. But because it has to be extensive with R_1 in the limit $R_1 \rightarrow \infty$, the only possibilities are

$$S_{\text{torus}} = s_0 + s_1 \left(\frac{R_1}{R_2} + \frac{R_2}{R_1} \right), \quad (5.7)$$

with the same constant s_1 appearing in the expectation value of the cylinder.

As mentioned above, the cylinder reduces under compactification to $\mathcal{N} = 4$ SYM to the BPS Wilson loops of [33], which famously have vanishing expectation values [36–38]. Within the classification of [26], such surfaces in the $\mathcal{N} = (2, 0)$ theory are of type- $\mathbb{R}$. It was verified there that in the abelian theory they indeed have vanishing expectation values. Contrary to the expectation in that paper, we see here that this does not carry over to the holographic theory, as we found a nonzero answer.

In $\mathcal{N} = 4$ SYM the loops of [33] have trivial expectation value and the interesting observable is the 1/2 BPS circular Wilson loop with fixed scalar coupling [30–32]. The cylinder with fixed scalar coupling is not BPS in the $\mathcal{N} = (2, 0)$ theory. Instead, the BPS observables are the sphere with fixed coupling [4], or the cylinder studied here. The sphere is subject to conformal anomalies, but the cylinder does not, giving a new finite BPS observable in the theory.

Indeed, all the BPS tori discussed in this paper and their large N M2-doughnut avatars are finite BPS observables. It would be interesting to understand whether there are corrections subleading in N augmenting (3.14) and (5.6).

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