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Lepton flavor violation in the Littlest Higgs Model with T parity realizing an inverse seesaw

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ABSTRACT: We study lepton flavor violation (LFV) within the Littlest Higgs Model with T parity (LHT) realizing an inverse seesaw (ISS) mechanism of type I. With respect to the traditional LHT, there appear new $\mathcal{O}(10\text{ TeV})$ Majorana neutrinos, driving LFV. For $\tau \rightarrow \ell\ell'\ell''$ (including wrong-sign, $\ell = e, \mu$) decays and $\mu \rightarrow e$ conversion in Ti, we get typical rates only one order of magnitude below present bounds ($\ell \rightarrow \ell'\gamma$ can reach the current upper limit) and for $Z \rightarrow \bar{\tau}\ell$, $\mu \rightarrow ee\bar{e}$ and conversion in Au, results are within two orders of magnitude from present limits. Correlations among modes are drastically different to the traditional LHT and other models, which would ease the confrontation of this scenario to eventual measurements of LFV processes involving charged leptons.

KEYWORDS: Discrete Symmetries, Technicolor and Composite Models, Beyond Standard Model

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1 Introduction

The long-awaited discovery of the Higgs boson [1, 2] was the final milestone confirming [3–5] the Standard Model (SM) of the electroweak interactions [8–10]. Composite Higgs models [11–14] are among the most attractive candidates to solve the corresponding hierarchy problem, associated to the Higgs mass value and its stability against quantum corrections in presence of heavy new physics coupling to the Higgs proportionally to their masses. In this set of models, the Higgs boson is a pseudo-Nambu-Goldstone boson (pNGB) of a spontaneously broken global symmetry. Specifically, the Littlest Higgs model with T parity (LHT) [11, 15–20] is one of the most attractive such frameworks. LHT is based upon the spontaneous collective breaking of a global symmetry group $SU(5)$ down to $SO(5)$, by a vacuum expectation value at a scale of few TeV. The discrete T parity symmetry is possible since the coset space $SU(5)/SO(5)$ is invariant under it. This forbids singly-produced heavy particles (odd under T) and tree level corrections to observables with only SM particles. As a result, direct and indirect constraints on the LHT are significantly relaxed [21, 22]. Thus, the LHT remains phenomenologically viable and well-motivated [23–37, 39–46].

Understanding the tiny values of the neutrino masses is another puzzle that constitutes a very active area of research, as they are the first manifestation of beyond the SM physics. Within the LHT, it was shown recently [45] that the inverse see-saw of type I [47–49] is able to reproduce current data preserving the T symmetry. It is well known that the heavy Majorana masses thus introduced (in the 10 TeV scale) impact lepton flavor violating (LFV) processes [50–52]. Here we consider these effects in the following LFV processes: $\ell \rightarrow \ell' \gamma$ decays (which were first addressed in this context in ref. [45]), $Z \rightarrow \ell \bar{\ell}'$ decays, $\tau \rightarrow 3\ell$ decays, with all possible flavor combinations for the charged leptons ($\ell = e, \mu$) in the final state,¹ and $\mu \rightarrow e$ conversion in nuclei.² These will not only allow the wrong-sign decays ($\tau^- \rightarrow \ell^+ \ell'^- \ell'^-$) but also permit larger upper limits than those typically predicted in ref. [44] for the other processes. Noteworthy, correlation among the considered processes will be very different to the traditional LHT scenario (without heavy Majorana neutrinos) [45] and other models, which would ease the validation/falsification of this LHT realization, should LFV in the charged lepton sector be discovered and measured in different processes.

This article is structured as follows. In section 2 we review the generation of ISS neutrino masses within the LHT. All the new contributions that we study in this work arise from this implementation of neutrino masses in the LHT. Next, in section 3, these new contributions to the LFV processes that we study are presented. Then, their phenomenology is discussed in section 4. Finally, we present our conclusions in section 5. All necessary loop functions are given in the appendix.

2 Inverse seesaw neutrino masses in the LHT model

We will summarize next the main aspects first introduced in ref. [45] concerning the implementation of neutrino masses in the model (recovering the ISS scenario). The interested reader is addressed to this reference for further details and references of the topics discussed in this section. As we will concentrate on the corresponding new contributions to several LFV processes, we will not review here the bulk of the LHT, which is nicely and extensively explained in the reviews [12–14].

The scalar sector of the LHT is a non-linear σ model based on the coset space $SU(5)/SO(5)$, with the $SU(5)$ global symmetry spontaneously broken by the vacuum expectation value (vev) f (of order TeV, larger than the vev of the Higgs field, v) giving rise to 14 pseudo-Nambu-Goldstone bosons entering the matrix

$$\Pi = \begin{pmatrix} -\omega^0/2 - \eta/\sqrt{20} & -\omega^+/\sqrt{2} & -i\pi^+/\sqrt{2} & -i\Phi^{++} & -i\frac{\Phi^+}{\sqrt{2}} \\ -\omega^-/2 & \omega^0/2 - \eta/\sqrt{20} & \frac{v+h+i\pi^0}{2} & -i\Phi^+/\sqrt{2} & \frac{-i\Phi^0+\Phi^P}{\sqrt{2}} \\ -i\pi^-/\sqrt{2} & (v+h-i\pi^0)/2 & \sqrt{\frac{4}{5}}\eta & -i\pi^+/\sqrt{2} & (v+h+i\pi^0)/2 \\ i\Phi^{--} & i\frac{\Phi^-}{\sqrt{2}} & i\pi^-/\sqrt{2} & -\omega^0/2 - \eta/\sqrt{20} & -\omega^-/\sqrt{2} \\ i\frac{\Phi^-}{\sqrt{2}} & \frac{i\Phi^0+\Phi^P}{\sqrt{2}} & \frac{v+h-i\pi^0}{2} & -\omega^+/\sqrt{2} & \omega^0/2 - \eta/\sqrt{20} \end{pmatrix}. \quad (2.1)$$

¹Observation of these processes goes beyond the SM extended with massive light neutrinos [53–55].

²We do not consider $H \rightarrow \ell \bar{\ell}'$ as it does not enter as a relevant building block of the studied processes, and it is necessarily below current and near future sensitivities [56]. This is a general feature of LH models [41, 42, 57]. There are bright future prospects for ℓ to τ conversion in nuclei [58, 59], that we plan to study within the LHT elsewhere.

It includes the SM Higgs doublet (h and π fields), a complex weak isospin triplet Φ , and the longitudinal modes of the heavy $\mathcal{O}(\text{TeV})$ gauge fields $\omega^{\pm,0}$ and η . Although the fields in Π transform non-linearly under the symmetry, $\xi = e^{i\Pi/f}$ obeys a linear transformation under $\text{SU}(5)$. T parity is defined to make T -odd all but the SM Higgs doublet, so that the heavy states can only interact pairwise.

In the lepton sector, each SM doublet $l_L = (\nu_L \ell_L)^T$ is mirrored (with $_{1,2}$ subindexes, respectively) by introducing two incomplete quintuplets (σ^2 is the second Pauli matrix) as follows

$$\Psi_1 = \begin{pmatrix} -i\sigma^2 l_{1L} \\ 0 \\ 0 \end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} 0 \\ 0 \\ -i\sigma^2 l_{2L} \end{pmatrix}, \quad (2.2)$$

with Ψ_2 transforming with the fundamental $\text{SU}(5)$ representation V and Ψ_1 with its complex conjugated.

Then, T -parity is defined to act on the left-handed (LH) leptons as

$$\Psi_1 \longleftrightarrow \Omega \Sigma_0 \Psi_2, \quad (2.3)$$

with

$$\Omega = \text{diag}(-1, -1, 1, -1, -1), \quad \Sigma_0 = \begin{pmatrix} 0 & 0 & \mathbf{1}_{2 \times 2} \\ 0 & 1 & 0 \\ \mathbf{1}_{2 \times 2} & 0 & 0 \end{pmatrix}. \quad (2.4)$$

The SM doublet, $l_L = (l_{1L} - l_{2L})/\sqrt{2}$, will be T -even; while its heavy copy, $l_{HL} = (\nu_{HL} \ell_{HL})^T = (l_{1L} + l_{2L})/\sqrt{2}$, will be T -odd.³ This heavy doublet (one per family) will get its mass combining with a right-handed doublet l_{HR} in an $\text{SO}(5)$ multiplet Ψ_R ,

$$\Psi_R = \begin{pmatrix} \psi'_R \\ \chi_R \\ -i\sigma^2 l_{HR} \end{pmatrix}, \quad T : \Psi_R \leftrightarrow \Omega \Psi_R, \quad (2.5)$$

getting its large ($\sim f$) mass from the Yukawa Lagrangian

$$\mathcal{L}_{Y_H} = -\kappa f \left(\bar{\Psi}_2 \xi + \bar{\Psi}_1 \Sigma_0 \xi^\dagger \right) \Psi_R + h.c., \quad (2.6)$$

where the first term preserves the global symmetry for $\xi \rightarrow V \xi U^\dagger$ and the second one is its T -transformed for $U = \Omega$. Eq. (2.6) gives a vector-like mass $\sqrt{2}\kappa f$ to ν_H (κ is not a small parameter, as we mention below).

Symmetry allows a large vector-like mass for the lepton singlets χ_R as well, by combining directly with a LH singlet χ_L . This is

$$\mathcal{L}_M = -M \bar{\chi}_L \chi_R + h.c. \quad (2.7)$$

χ_L is $\text{SU}(5)$ singlet, so it is natural to include a small Majorana mass for it. We assume Lepton Number (LN) to be broken only by small Majorana masses μ in the heavy LH neutral sector. Then,

$$\mathcal{L}_\mu = -\frac{\mu}{2} \bar{\chi}_L^c \chi_L + h.c., \quad (2.8)$$

³They correspond to $(\Psi_1 \pm \Omega \Sigma_0 \Psi_2)/\sqrt{2}$, respectively.

and the resulting (T-even) neutrino mass matrix reduces to the inverse see-saw one:

$$\mathcal{L}_M^\nu = -\frac{1}{2} (\overline{\nu}_l^c \overline{\chi}_R \overline{\chi}_L^c) \mathcal{M}_\nu^{\text{T-even}} \begin{pmatrix} \nu_L \\ \chi_R^c \\ \chi_L \end{pmatrix} + h.c., \quad (2.9)$$

where

$$\mathcal{M}_\nu^{\text{T-even}} = \begin{pmatrix} 0 & i\kappa^* f \sin\left(\frac{v}{\sqrt{2}f}\right) & 0 \\ i\kappa^\dagger f \sin\left(\frac{v}{\sqrt{2}f}\right) & 0 & M^\dagger \\ 0 & M^* & \mu \end{pmatrix}, \quad (2.10)$$

with each entry standing for a 3×3 matrix accounting for the 3 lepton families. The κ entries are given by the Yukawa Lagrangian in eq. (2.6), M stands for the heavy Dirac mass matrix from eq. (2.7), and μ is the mass matrix of small Majorana masses in eq. (2.8). In the inverse see-saw, the hierarchy $\mu \ll \kappa \ll M$ holds, with $M \sim 4\pi f \sim 10 \text{ TeV}$ ($\kappa \sim \mathcal{O}(1)$ is assumed and μ needs to be much smaller than a GeV), according to electroweak precision data [45].

Let $\mathcal{U}$ be a unitary transformation that diagonalizes $\mathcal{M}$ and transforms the states $(\nu_L \ \Psi_L)$ in the gauge basis to the mass eigenstates $(\nu_L^l \ \Psi_L^h)$, light and heavy (quasi-Dirac) neutrinos, l and h , respectively

$$\mathcal{U}^\dagger \begin{pmatrix} \nu_L \\ \Psi_L \end{pmatrix} = \begin{pmatrix} \nu_L^l \\ \Psi_L^h \end{pmatrix}, \quad \text{with} \quad \Psi_L = \begin{pmatrix} \chi_R^c \\ \chi_L \end{pmatrix}. \quad (2.11)$$

The matrix $\mathcal{U}$ can be written as [60]

$$\mathcal{U} = \begin{pmatrix} \sqrt{1 - \mathcal{B}\mathcal{B}^\dagger} & \mathcal{B} \\ -\mathcal{B}^\dagger & \sqrt{1 - \mathcal{B}^\dagger\mathcal{B}} \end{pmatrix}, \quad (2.12)$$

such that $\mathcal{U}$ satisfies

$$\mathcal{U}^T \mathcal{M} \mathcal{U} = \begin{pmatrix} \mathcal{M}_\nu^l & 0_{3 \times 6} \\ 0_{6 \times 3} & \mathcal{M}_\chi^h \end{pmatrix}, \quad (2.13)$$

decoupling the heavy and light neutrino fields. $\mathcal{B}$ is a complex 3×3 matrix and $\sqrt{1 - \mathcal{B}\mathcal{B}^\dagger}$ shall be expanded for radicand close to one, keeping only order $\mathcal{B}\mathcal{B}^\dagger$ terms.

For diagonalizing $\mathcal{M}$, it is convenient to introduce

$$M_D = \begin{pmatrix} i\kappa^\dagger f \sin\left(\frac{v}{\sqrt{2}f}\right) \\ 0 \end{pmatrix}, \quad M_R = \begin{pmatrix} 0 & M^\dagger \\ M^* & \mu \end{pmatrix}, \quad (2.14)$$

hence,

$$\mathcal{M}_\nu^{\text{T-even}} = \begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix}. \quad (2.15)$$

A first approximation to $\mathcal{B}$ [61] is

$$\begin{aligned} \mathcal{B}^* &= M_D^T M_R^{-1} \rightarrow \mathcal{B} \\ &= M_D^\dagger (M_R^{-1})^* = \left(if \sin\left(\frac{v}{\sqrt{2}f}\right) \kappa M^{-1} \mu^* (M^T)^{-1} \quad -if \sin\left(\frac{v}{\sqrt{2}f}\right) \kappa M^{-1} \right). \end{aligned} \quad (2.16)$$

Therefore,

$$\sqrt{1 - \mathcal{B}\mathcal{B}^\dagger} \approx 1 - \frac{1}{2}\mathcal{B}\mathcal{B}^\dagger \approx 1 - \frac{1}{2}\theta\theta^\dagger, \quad (2.17)$$

where we have omitted terms of the order of μ because $\mu \ll \kappa \ll M$ and we redefined $\mathcal{B} \rightarrow \Theta$

$$\Theta = \begin{pmatrix} -\theta\mu^*(M^T)^{-1} & \theta \end{pmatrix}, \quad \text{with} \quad \theta = -if \sin\left(\frac{v}{\sqrt{2}f}\right) \kappa M^{-1}, \quad (2.18)$$

in which Θ is a 3×6 and θ a 3×3 matrix, as in refs. [45, 62]. In this way, the $\mathcal{U}$ matrix reads

$$\mathcal{U} = \begin{pmatrix} 1 - \frac{1}{2}\Theta\Theta^\dagger & \Theta \\ -\Theta^\dagger & 1 - \frac{1}{2}\Theta^\dagger\Theta \end{pmatrix}. \quad (2.19)$$

Then, the $\mathcal{M}_\nu^l$ and $\mathcal{M}_\chi^h$ matrices in the eq. (2.13) are given by [45, 60–62]

$$(\mathcal{M}_\nu^l)_{ij} = -(M_D^T M_R^{-1} M_D)_{ij} = \theta_{ik}^* \mu_{kl} \theta_{jl}^\dagger, \quad \mathcal{M}_\chi^h = M_R, \quad (2.20)$$

where we have assumed, without loss of generality, that the χ mass matrix, M , is diagonal and positive definite. The diagonalized (Majorana) mass terms of eq. (2.9) thus read

$$\mathcal{L}_M^\nu = -\frac{1}{2} \left(\sum_{i=1}^3 (\mathcal{M}_\nu^l)_i \overline{\nu_{Li}^l} \nu_{Li}^l + \sum_{j=4}^9 (\mathcal{M}_\chi^h)_j \overline{\Psi_{Lj}^h} \Psi_{Lj}^h \right). \quad (2.21)$$

We can work in the basis where the charged lepton mass matrix is diagonal

$$\mathcal{M}_\nu^l = U_{\text{PMNS}}^* \mathcal{D}_\nu^l U_{\text{PMNS}}^\dagger, \quad (2.22)$$

from eq. (2.20),

$$\mu = (\theta^*)^{-1} U_{\text{PMNS}}^* \mathcal{D}_\nu^l U_{\text{PMNS}}^\dagger (\theta^\dagger)^{-1}, \quad (2.23)$$

where U_{PMNS} is the Pontecorvo-Maki-Nakagawa-Sakata matrix [63, 64] (that we will denote simply U in the following) and $\mathcal{D}_\nu^l$ the diagonal neutrino mass matrix.

Applying explicitly $\mathcal{U}^\dagger$ to eq. (2.11)

$$\begin{pmatrix} 1 - \frac{1}{2}\Theta\Theta^\dagger & -\Theta \\ \Theta^\dagger & 1 - \frac{1}{2}\Theta^\dagger\Theta \end{pmatrix} \begin{pmatrix} \nu_L \\ \Psi_L \end{pmatrix} = \begin{pmatrix} \nu_L^l \\ \Psi_L^h \end{pmatrix}, \quad (2.24)$$

due to the eq. (2.22). Hence, the mixing relations between flavor and mass eigenstates are

$$\begin{aligned} \sum_{j=1}^3 U_{ij} \nu_{Lj}^l &= \sum_{j=1}^3 \left[\mathbf{1}_{3 \times 3} - \frac{1}{2}(\Theta\Theta^\dagger) \right]_{ij} \nu_{Lj}^l - \sum_{j=4}^9 \Theta_{ij} \Psi_{Lj}^h, \\ \Psi_{Li}^h &= \sum_{j=4}^9 \left[\mathbf{1}_{6 \times 6} - \frac{1}{2}(\Theta^\dagger\Theta) \right]_{ij} \Psi_{Lj}^h + \sum_{j=1}^3 \Theta_{ij}^\dagger \nu_{Lj}^l, \end{aligned} \quad (2.25)$$

where Θ matrix elements give the mixing between light and heavy (quasi-Dirac) neutrinos to leading order.

Let Φ be a flavor eigenstate composed by

$$\Phi_L = \mathcal{U} \begin{pmatrix} \nu_L^l \\ \Psi_L^h \end{pmatrix}, \quad (2.26)$$

thus, in terms of the mass eigenstates from the eq. (2.25) the SM charged current is modified as follows

$$\begin{aligned} \mathcal{L}_W &= \frac{g}{\sqrt{2}} W_\mu^+ \sum_{i=1}^9 \sum_{j=1}^3 \overline{\Phi_{Li}} \gamma^\mu \ell_{Lj} \\ &= \frac{g}{\sqrt{2}} W_\mu^+ \sum_{j=1}^3 \left(\sum_{i=1}^3 \left\{ U^\dagger \left[\mathbf{1}_{3 \times 3} - \frac{1}{2} (\Theta \Theta^\dagger) \right] \right\}_{ij} \overline{\nu_{Li}^l} + \sum_{i=4}^9 \Theta_{ij}^\dagger \overline{\Psi_{Li}^h} \right) \gamma^\mu \ell_{Lj}. \end{aligned} \quad (2.27)$$

We can split the Lagrangian above in two parts, each one fixing the coupling between the SM leptons and the light and heavy quasi-Dirac neutrinos, respectively,

$$\begin{aligned} \mathcal{L}_W^l &= \frac{g}{\sqrt{2}} W_\mu^+ \sum_{j=1}^3 \sum_{i=1}^3 \overline{\nu_i^l} W_{ij} \gamma^\mu P_L \ell_j + h.c., \quad \text{with} \quad W_{ij} = \left\{ U^\dagger \left[\mathbf{1}_{3 \times 3} - \frac{1}{2} (\Theta \Theta^\dagger) \right] \right\}_{ij}, \\ \mathcal{L}_W^{lh} &= \frac{g}{\sqrt{2}} W_\mu^+ \sum_{j=1}^3 \sum_{i=4}^9 \overline{\Psi_i^h} \Theta_{ij}^\dagger \gamma^\mu P_L \ell_j + h.c. \end{aligned} \quad (2.28)$$

Due to the presence of Θ , $\mathcal{L}_W^l$ ($\mathcal{L}_W^{lh}$) includes LFV transitions involving light (heavy) neutrinos.

The neutral current coupling to the Z^0 gauge boson is written as

$$\mathcal{L}_Z = \frac{g}{2 \cos \theta_W} Z_\mu \sum_{j=1}^9 \sum_{i=1}^9 \overline{\nu_{Li}} \gamma^\mu \nu_{Lj}. \quad (2.29)$$

We consider $\Theta \Theta^\dagger$ effects to leading order and write down the light and heavy neutral currents as

$$\begin{aligned} \mathcal{L}_Z^l &= \frac{g}{2 \cos \theta_W} Z_\mu \sum_{j=1}^3 \sum_{i=1}^3 \overline{\nu_i^l} X_{ij} \gamma^\mu P_L \nu_j^l, \quad \text{with} \quad X_{ij} = \{ U^\dagger [\mathbf{1}_{3 \times 3} - (\Theta \Theta^\dagger)] U \}_{ij}, \\ \mathcal{L}_Z^{lh} &= \frac{g}{2 \cos \theta_W} Z_\mu \sum_{j=1}^3 \sum_{i=4}^9 \overline{\Psi_i^h} (\Theta^\dagger U)_{ij} \gamma^\mu P_L \nu_j^l + h.c., \\ \mathcal{L}_Z^h &= \frac{g}{2 \cos \theta_W} Z_\mu \sum_{j=4}^9 \sum_{i=4}^9 \overline{\Psi_i^h} (\Theta^\dagger \Theta)_{ij} \gamma^\mu P_L \Psi_j^h. \end{aligned} \quad (2.30)$$

Noteworthy, $\mathcal{L}_Z^l$ includes LFV terms in a purely light neutrino's current. The flavor symmetry is also broken by $\mathcal{L}_Z^{lh}$ and $\mathcal{L}_Z^h$, including heavy neutrinos.

Now, we neglect the μ term ($\mu \ll \kappa \ll M$) in the Θ matrix, $\Theta = (0_{3 \times 3} \quad \theta)$. Therefore, the eigenstates in the eq. (2.25) transform as [45]

$$\begin{aligned} \sum_{j=1}^3 U_{ij} \nu_{Lj}^l &= \sum_{j=1}^3 \left[\mathbf{1}_{3 \times 3} - \frac{1}{2} (\theta \theta^\dagger) \right]_{ij} \nu_{Lj} - \sum_{j=7}^9 \theta_{ij} \chi_{Lj}, \\ \chi_{Li}^h &= \sum_{j=7}^9 \left[\mathbf{1}_{3 \times 3} - \frac{1}{2} (\theta^\dagger \theta) \right]_{ij} \chi_{Lj} + \sum_{j=1}^3 \theta_{ij}^\dagger \nu_{Lj}, \end{aligned} \quad (2.31)$$

hence, the Lagrangians from eqs. (2.28) and (2.30) read

$$\begin{aligned}\mathcal{L}_W^l &= \frac{g}{\sqrt{2}} W_\mu^+ \sum_{i,j=1}^3 \bar{\nu}_i^l W_{ij} \gamma^\mu P_L \ell_j + h.c., \quad \text{with} \quad W_{ij} = \sum_{k=1}^3 U_{ik}^\dagger \left[\mathbf{1}_{3 \times 3} - \frac{1}{2}(\theta \theta^\dagger) \right]_{kj}, \\ \mathcal{L}_W^{lh} &= \frac{g}{\sqrt{2}} W_\mu^+ \sum_{i=7}^9 \sum_{j=1}^3 \bar{\chi}_i^h \theta_{ij}^\dagger \gamma^\mu P_L \ell_j + h.c.,\end{aligned}\tag{2.32}$$

and

$$\begin{aligned}\mathcal{L}_Z^l &= \frac{g}{2 \cos \theta_W} Z_\mu \sum_{i,j=1}^3 \bar{\nu}_i^l \gamma^\mu (X_{ij} P_L - X_{ij}^\dagger P_R) \nu_j^l, \quad \text{with} \quad X_{ij} = \sum_{k=1}^3 \left(U^\dagger [\mathbf{1}_{3 \times 3} - (\theta \theta^\dagger)] \right)_{ik} U_{kj}, \\ \mathcal{L}_Z^{lh} &= \frac{g}{2 \cos \theta_W} Z_\mu \sum_{i=7}^9 \sum_{j=1}^3 \bar{\chi}_i^h \gamma^\mu (Y_{ij} P_L - Y_{ij}^\dagger P_R) \nu_j^l + h.c., \quad \text{with} \quad Y_{ij} = \sum_{k=1}^3 \theta_{ik}^\dagger U_{kj}, \\ \mathcal{L}_Z^h &= \frac{g}{2 \cos \theta_W} Z_\mu \sum_{i,j=7}^9 \bar{\chi}_i^h \gamma^\mu (S_{ij} P_L - S_{ij}^\dagger P_R) \chi_j^h, \quad \text{with} \quad S_{ij} = \sum_{k=1}^3 \theta_{ik}^\dagger \theta_{kj},\end{aligned}\tag{2.33}$$

where the dimension of the W and X square mixing matrices is 3×3 . Comparing our charged-current and neutral-current interactions from eqs. (2.32) and (2.33) with the SM ones, we observe that they differ by the presence of the θ matrix, which is a consequence of introducing Majorana neutrinos, that allows for both neutral and charged LFV transitions. We will focus on these new contributions in the remainder of this work.

We can define the B_{ij} and C_{ij} matrices according to SM charged and neutral currents (see eqs. (2.32) and (2.33)) [50, 51]

$$B_{ij} = \sum_{k=1}^3 U_{ik} \mathcal{U}_{kj}^\dagger \quad \text{and} \quad C_{ij} = \sum_{k=1}^3 \mathcal{U}_{ki} \mathcal{U}_{kj}^\dagger,\tag{2.34}$$

where B mixing matrix is 3×9 , whereas C is a 9×9 matrix. We are grouping both parts of light and heavy Majorana neutrinos. We need to recall that $\mathcal{U}_{ij}$ with $i, j = 4, 5, 6$ entries are suppressed by ISS hierarchy. Eqs. (3.17) give unitarity relations among these matrices, which are crucial to verify cancellation of ultraviolet divergences in loop diagrams within this setting.

3 New contributions to LFV processes

The relevant effective LFV $V \ell \ell'$ vertices ($V = \gamma, Z$), depicted in figure 1, can be written in terms of the allowed Lorentz structures accompanied by their corresponding form factors⁴

$$\Gamma_\mu^V(q^2) = e \left[\gamma_\mu \left(F_L^V(q^2) P_L + F_R^V(q^2) P_R \right) + 2 \left(i F_M^V(q^2) + F_E^V(q^2) \gamma_5 \right) \sigma_{\mu\nu} q^\nu \right],\tag{3.1}$$

with q the V boson momentum. $F_{L,R}^V(q^2)$ are the monopole form factors of given chirality and $F_{M,E}^V(q^2)$ are the magnetic and electric dipole form factors.

⁴We omit the pieces proportional to q^ν and $\gamma_5 q^\nu$, scalar and pseudoscalar form factors, as they do not contribute for real V and are negligible for virtual V [65] in the processes under study.

Figure 1. Topologies of the diagrams that contribute to the $(\gamma/Z) - \bar{\ell} - \ell'$ effective vertex.

3.1 $\ell \rightarrow \ell' \gamma$ decays

The $\ell \rightarrow \ell' \gamma$ vertex reduces to a dipole transition for an on-shell photon,

$$i\Gamma_{\gamma}^{\mu}(p_{\ell}, p_{\ell'}) = ie \left[iF_M^{\gamma}(Q^2) + F_E^{\gamma}(Q^2) \gamma_5 \right] \sigma^{\mu\nu} Q_{\nu}, \quad (3.2)$$

where $Q_{\nu} = (p_{\ell'} - p_{\ell})_{\nu}$. Neglecting $m_{\ell'} \ll m_{\ell}$,

$$\Gamma(\ell \rightarrow \ell' \gamma) = \frac{\alpha}{2} m_{\ell}^3 (|F_M^{\gamma}|^2 + |F_E^{\gamma}|^2). \quad (3.3)$$

Results below will be simplified using $F_M^{\gamma} = -iF_E^{\gamma}$, that holds for all contributions. All computations in this work were done in the 't Hooft-Feynman gauge.

The active (light) neutrino contribution is analogous to the SM one, just replacing U by W due to eq. (2.32).⁵ We note that the W matrix includes the SM contribution, given by U , and the new heavy neutrinos part given by the $\theta\theta^{\dagger}$ term. The corresponding Feynman diagrams are given by topologies II, IV, V and VI in figure 1.

The corresponding result is

$$F_M^{\gamma}(y_i) = \frac{\alpha_W}{16\pi} \frac{m_{\ell}}{M_W^2} \sum_i W^{i\ell'*} W^{i\ell} F_W(y_i), \quad (3.4)$$

with $y_i = \frac{m_{\nu_i}^2}{M_W^2} \sim 0$, $\alpha_W = \alpha/\sin^2\theta_W$ and⁶

$$F_W(x) = \frac{5}{6} - \frac{3x - 15x^2 - 6x^3}{12(1-x)^3} + \frac{3x^3}{2(1-x)^4} \ln x. \quad (3.5)$$

⁵Otherwise this branching ratio is unmeasurably small [53, 66, 67].

⁶All form factors reported in this subsection can be read from the quoted references. Therefore, we are not giving their expressions in terms of Passarino-Veltman functions, which can be consulted therein. This information will be provided in the next subsections, which are new results, to our knowledge.

Similarly (see eq. (2.32)), the $F_M^\gamma|_\chi(x)$ function turns out to be [52]

$$F_M^\gamma|_\chi(x) = \frac{1}{3} - \frac{2x^3 - 7x^2 + 11x}{4(1-x)^3} + \frac{3x}{2(1-x)^4} \ln x, \quad (3.6)$$

with $x_j = \frac{M_W^2}{M_j^2}$ and M_j the mass of the j -th χ_L^h state.

Finally, the ν_H states also contribute through the same topologies than active neutrinos, results are analogous by using $\{\nu_H, W_H, \omega\}$ instead of $\{\nu_l, W, \phi\}$:

$$F_M^\gamma|_{W_H}(x_i) = \frac{\alpha_W}{16\pi} \frac{m_\mu}{M_W^2} \frac{v^2}{4f^2} \sum_i V_{H\ell}^{ie*} V_{H\ell}^{i\mu} F_M^{\nu_H}(x_i), \quad (3.7)$$

with $x_i = \frac{m_{\nu_{H_i}}^2}{M_{W_H}^2}$.

Like the PMNS matrix, $V_{H\ell}$ are 3×3 unitary mixing matrices parametrizing the misalignment between the SM LH charged leptons ℓ and the heavy mirror ones ℓ_H . The observable rotations are

$$V_{H\nu} \equiv V_L^{H\dagger} V_L^\nu, \quad V_{H\ell} \equiv V_L^{H\dagger} V_L^\ell, \quad (3.8)$$

related by $V_{H\nu}^\dagger V_{H\ell} = V_{\text{PMNS}}^\dagger$ [32].

In this way, the new interactions derived in the previous section yield (we omit the upper-index γ of the F_M^γ form factors below and indicate explicitly instead the type of neutrino appearing in each contribution)⁷

$$\text{Br}(\mu \rightarrow e\gamma) = \frac{3\alpha}{2\pi} \left| W_{ej} W_{\mu j}^* F_M^\nu(y_j) + U_{ej} U_{\mu j}^* F_M^\chi(x_i) + \frac{v^2}{4f^2} V_{H\ell}^{ej*} V_{H\ell}^{\mu j} F_M^{\nu_H}(x_i) \right|^2, \quad (3.9)$$

where $x_i = \frac{M_W^2}{M_{N_j}^2} \ll 1$ and $y_j = \frac{m_{\nu_j}^2}{M_W^2} \sim 0$. We note that $F_M^\chi(x) = 1/3 - (11/4)x + \mathcal{O}(x^2)$ and $F_M^\nu(0) = 5/6 = F_M^{\nu_H}(0)$. The contribution from the third term is suppressed by $v^2/f^2 \ll 1$, so we will neglect it in the following. For analogous reasons we will disregard in the rest of this work the contributions involving T-odd particles, as all of them are suppressed by v^2/f^2 in the form factors (explicit expressions can be checked in refs. [32, 34, 44]).⁸

In this way, we get

$$\text{Br}(\mu \rightarrow e\gamma) = \frac{3\alpha}{2\pi} \left| \theta_{ej} \theta_{\mu j}^\dagger F_M^\chi(x) + \frac{5}{6} W_{ej} W_{\mu j}^\dagger \right|^2 \approx \frac{3\alpha}{8\pi} \left| \theta_{ej} \theta_{\mu j}^\dagger \right|^2. \quad (3.10)$$

We note that, even for heavy neutrinos as 'light' as 2 TeV, the correction induced by the $\mathcal{O}(x)$ term to $F_M^\chi(x)$ in observables is at the level of 1% and can be safely neglected.

⁷A similar expression holds for the $\tau \rightarrow \ell\gamma$ decays ($\ell = e, \mu$), only accounting for the hadronic tau decay width.

⁸We postpone to future work the complete analysis within the LHT keeping the $\mathcal{O}(v^2/f^2)$ contributions to the form factors. This shall be necessary, since the heavy Majorana neutrinos have to have $\mathcal{O}(\text{TeV})$ masses, as the T-odd particles, within a gauge invariant theory [46]. We will also need to study first semileptonic LFV tau decays within the LHT (following refs. [68–70]).

Figure 2. Z penguins diagrams that contribute to the $Z \rightarrow \bar{\ell}\ell$ decays. Diagrams corresponding to T-I and T-III allow to mix light and heavy Majorana neutrinos.

Then, the 90% C.L. limits $Br(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}$, $Br(\tau \rightarrow e\gamma) < 3.3 \times 10^{-8}$ and $Br(\tau \rightarrow \mu\gamma) < 4.2 \times 10^{-8}$ [6, 71] bind

$$|\theta_{ej}\theta_{\mu j}^\dagger| < 0.14 \times 10^{-4}, \quad |\theta_{ej}\theta_{\tau j}^\dagger| < 0.95 \times 10^{-2}, \quad |\theta_{\mu j}\theta_{\tau j}^\dagger| < 0.011. \quad (3.11)$$

As discussed in ref. [45], electroweak precision data [72] constrain $|\theta_{e1}| < 0.04$, $|\theta_{\mu 2}| < 0.03$ and $|\theta_{\tau 3}| < 0.09$, at 95% C.L., assuming that each heavy neutrino only mixes with one light neutrino of definite flavor and that only one mixing is non-vanishing at a time.

3.2 $Z \rightarrow \bar{\ell}\ell'$ decays

At leading order the $Z \rightarrow \bar{\ell}\ell'$ vertex reduces to

$$i\Gamma_Z^\mu(p_\ell, p_{\ell'}) = ieF_L^Z(Q^2)\gamma^\mu P_L. \quad (3.12)$$

We work in the approximation of zero light neutrino masses. Therefore, only diagrams with heavy neutrinos contribute to this process. In this type of decay we have that $Q^2 = M_Z^2$, so the Z width reads

$$\Gamma(Z \rightarrow \bar{\ell}\ell') = \frac{\alpha}{3}M_Z|F_L^Z(M_Z^2)|^2. \quad (3.13)$$

There are 10 contributions to F_L^Z , which are represented in figure 2. The result can be

written [50, 52, 73]

$$F_L^Z(M_Z^2) = \frac{\alpha_W}{8\pi c_W s_W} \sum_{i,j=1}^3 \left[\theta_{\ell i} \theta_{\ell i}^\dagger F^h(y_i; M_Z^2) + \theta_{\ell j} S_{ji} \theta_{\ell i}^\dagger \left(G^h(y_i, y_j; M_Z^2) + \frac{1}{\sqrt{y_i y_j}} H^h(y_i, y_j; M_Z^2) \right) \right], \quad (3.14)$$

where

$$\begin{aligned} F^h(y_i; M_Z^2) &= -2c_W^2 \left[M_Z^2 (\overline{C}_1 + \overline{C}_2 + \overline{C}_{12}) + 6\overline{C}_{00} - 1 \right] - (1 - 2s_W^2) \frac{1}{y_i} \overline{C}_{00} - 2s_W^2 \frac{1}{y_i} M_W^2 \overline{C}_0 \\ &\quad - \frac{1}{2} (1 - 2s_W^2) \left[\left(2 + \frac{1}{y_i} \right) \overline{B}_1 + 1 \right], \\ G^h(y_i, y_j; M_Z^2) &= -M_Z^2 (C_0 + C_1 + C_2 + C_{12}) + 2C_{00} - 1 - \frac{1}{2} \frac{1}{y_i y_j} M_W^2 C_0, \\ H^h(y_i, y_j; M_Z^2) &= M_W^2 C_0 + \frac{1}{2} M_Z^2 C_{12} - C_{00} + \frac{1}{4}, \end{aligned} \quad (3.15)$$

with $y_{i,j} = M_W^2/M_{i,j}^2$ and $M_{i,j}$ heavy neutrino masses.⁹ Analytic expressions for the functions F^h , G^h , and H^h at order M_Z^2 are written

$$\begin{aligned} F^h(y_i; M_Z^2) &= -\left(\frac{5}{2} - 2s_W^2 \right) \Delta_\epsilon - \frac{5 \ln y_i}{2(1-y_i)^2} - \frac{5}{2(1-y_i)} + \frac{1}{4} \\ &\quad + \frac{M_Z^2}{72M_W^2} \frac{1}{(1-y_i)^4} [6[24y_i^2(s_W^2 - 1) - 4y_i(5s_W^2 - 8) - (2s_W^2 - 1)] \ln y_i \\ &\quad - (1-y_i)[88y_i^3(s_W^2 - 1) - 2y_i^2(164s_W^2 - 171) - y_i(297 - 230s_W^2) - (2s_W^2 + 11)]], \\ G^h(y_i, y_j; M_Z^2) &= \frac{1}{2} \left(\Delta_\epsilon - \frac{1}{2} \right) - \frac{1}{2(y_i - y_j)} \left(-\frac{(1-y_j) \ln y_i}{(1-y_i)} + \frac{(1-y_i) \ln y_j}{(1-y_j)} \right) + \mathcal{O} \left(\frac{M_Z^2}{M_{i,j}^2} \right), \\ H^h(y_i, y_j; M_Z^2) &= -\frac{1}{4} \left(\Delta_\epsilon + \frac{1}{2} \right) - \frac{1}{4(y_i - y_j)} \left(-\frac{(1-4y_i)y_j \ln y_i}{(1-y_i)} + \frac{(1-4y_j)y_i \ln y_j}{(1-y_j)} \right) + \mathcal{O} \left(\frac{M_Z^2}{M_{i,j}^2} \right). \end{aligned} \quad (3.16)$$

We note that we preferred to write the variable $y_{i,j}$ so that it is small. Then, the neutrino masses are in the denominator, as opposed to the x_i variable employed for light neutrinos (and to previous literature). Taking this into account, our results reproduce those in refs. [50, 52, 73]. It is also worth to mention that the $\mathcal{O}(M_Z^2)$ terms are clearly negligible for the G^h and H^h functions. However, the second and third lines of F^h are $\mathcal{O}(M_Z^2/M_W^2)$, which is not small. Still, this correction turns out to be ~ 26 times smaller than the (convergent part of the) first line of F^h (and it can be checked that higher order terms in the M_Z^2 expansion are further suppressed).¹⁰ The (generalized) GIM mechanism that applies to the mixing matrices in eq. (3.14) [50, 51] cancels all UV divergences encoded in $\Delta_\epsilon = \frac{1}{\epsilon} - \gamma_E + \ln(4\pi) + \ln \left(\frac{\mu^2}{M_W^2} \right)$, regulating them in $4 - 2\epsilon$ dimensions. Specifically, this

⁹Loop functions are given in the appendix.

¹⁰We are not aware this issue was discussed previously.

happens thanks to the relations [50, 51]

$$\begin{aligned}
 \sum_{k=1}^9 B_{ik} B_{jk}^\dagger &= \delta_{ij}, & \sum_{k=1}^3 B_{ki}^\dagger B_{kj} &= \sum_{k=1}^9 C_{ik} C_{jk}^\dagger = C_{ij}, \\
 \sum_{k=1}^9 B_{ik} C_{kj} &= B_{ij}, \\
 \sum_{k=1}^9 m_{\Phi_k} C_{ik} C_{jk} &= \sum_{k=1}^9 m_{\Phi_k} B_{ik} C_{kj}^\dagger = \sum_{k=1}^3 m_{\Phi_k} B_{ik} B_{jk}^\dagger = 0,
 \end{aligned} \tag{3.17}$$

where $m_{\Phi_k} = m_k$ with $(k = 1, 2, 3)$ and $m_{\Phi_k} = M_k$ with $(k = 7, 8, 9)$ are the light and heavy masses of Majorana neutrinos, respectively.

3.3 $L \rightarrow 3\ell$ decays

We distinguish three types of three-lepton lepton decays, according to the notation $\ell \rightarrow \ell' \ell'' \ell'''$:

1. $\ell \neq \ell' = \ell'' = \ell'''$ (which contains the processes $\mu \rightarrow ee\bar{e}$, $\tau \rightarrow ee\bar{e}$ and $\tau \rightarrow \mu\mu\bar{\mu}$).
2. $\ell \neq \ell' \neq \ell'' = \ell'''$ (including the $\tau \rightarrow e\mu\bar{\mu}$ and $\tau \rightarrow \mu e\bar{e}$ decays).
3. $\ell \neq \ell' = \ell'' \neq \ell'''$ (constituted by the ‘wrong-sign’ processes: $\tau \rightarrow ee\bar{\mu}$ and $\tau \rightarrow \mu\mu\bar{e}$).

We will treat them in turn.

3.3.1 Type I: $\ell \rightarrow \ell' \ell'' \bar{\ell}'''$ with $\ell \neq \ell' = \ell'' = \ell'''$

The amplitude for Type I decays gets contributions from γ and Z penguin diagrams, as well as from boxes:

$$\mathcal{M}^{\text{Type I}} = \mathcal{M}_\gamma^{\text{Type I}} + \mathcal{M}_Z^{\text{Type I}} + \mathcal{M}_{\text{box}}^{\text{Type I}}, \tag{3.18}$$

where each amplitude is defined as follows [44]

$$\begin{aligned}
 \mathcal{M}_\gamma^{\text{Type I}} &= \bar{u}(p_1) e \left[i F_M^\gamma(0) 2 P_R \sigma^{\mu\nu} (p_1 - p_\ell)_\nu + F_L^\gamma((p_1 - p_\ell)^2) \gamma^\mu P_L \right] u(p_\ell) \\
 &\quad \times \frac{1}{(p_1 - p_\ell)^2} \bar{u}(p_3) \gamma_\mu e v(p_2) - (p_1 \leftrightarrow p_3), \\
 \mathcal{M}_Z^{\text{Type I}} &= \bar{u}(p_1) \left(-e F_L^Z(0) \right) \gamma^\mu P_L u(p_\ell) \frac{1}{M_Z^2} \bar{u}(p_3) \gamma_\mu \left(g_L^Z P_L + g_R^Z P_R \right) v(p_2) \\
 &\quad - (p_1 \leftrightarrow p_3), \\
 \mathcal{M}_{\text{box}}^{\text{Type I}} &= e^2 B_L(0) \bar{u}(p_1) \gamma^\mu P_L u(p_\ell) \bar{u}(p_3) \gamma_\mu P_L v(p_2),
 \end{aligned} \tag{3.19}$$

where again $F_E^\gamma = i F_M^\gamma$. The photon magnetic and Z left-handed vector form factors, $F_M^\gamma(0)$ and $F_L^Z(0)$ respectively, are evaluated at $Q^2 = (p_1 - p_\ell)^2 = 0$ because their leading terms are momentum independent for small momentum transfer $Q^2 \sim m_\ell^2$ whereas the photon left-handed vector form factor, $F_L^\gamma((p_1 - p_\ell)^2)$, is linear in Q^2 .

The complete F_M^γ is given by

$$F_M^\gamma = F_M^{\nu^l} + F_M^{\chi^h} = \frac{\alpha_W}{16\pi} \frac{m_\ell}{M_W^2} \sum_{j=1}^3 \left(W_{\ell'j} W_{\ell j}^\dagger F_M^{\nu^l}(x_j) + \theta_{\ell'j} \theta_{\ell j}^\dagger F_M^{\chi^h}(y_j) \right). \quad (3.20)$$

The F_L^γ form factor is obtained from topologies II, IV, V and VI in figure 1, and it is given by

$$F_L^\gamma = F_L^{\nu^l} + F_L^{\chi^h} = \frac{\alpha_W}{8\pi M_W^2} \sum_{j=1}^3 \left(W_{\ell'j} W_{\ell j}^\dagger F_L^{\nu^l}(x_j) + \theta_{\ell'j} \theta_{\ell j}^\dagger F_L^{\chi^h}(y_j) \right), \quad (3.21)$$

where

$$F_L^{\nu^l}(x) = 2M_W^2 \Delta_\epsilon + Q^2 \left(\frac{x^2(12 - 10x + x^2) \ln x}{6(1-x)^4} - \frac{7x^3 - x^2 - 12x}{12(1-x)^3} - \frac{5}{9} \right), \quad (3.22)$$

$$F_L^{\chi^h}(y) = 2M_W^2 \Delta_\epsilon + Q^2 \left(-\frac{(12y^2 - 10y + 1) \ln y}{6(1-y)^4} + \frac{20y^3 - 96y^2 + 57y + 1}{36(1-y)^3} \right).$$

We take in account the Z penguin diagrams that are shown in figure 2. These involve either purely light neutrinos, a mixing between light and heavy neutrinos, or diagrams in which only heavy neutrinos appear. The form factor from ν^l -diagrams in figure 2 is given by

$$F_L^{Z-\nu^l}(Q^2) = \frac{\alpha_W}{8\pi c_W s_W} \sum_{i,j=1}^3 \left[W_{\ell'i} W_{\ell i}^\dagger F^l(x_i; Q^2) + W_{\ell'j} X_{ji} W_{\ell i}^\dagger \left(G^l(x_i, x_j; Q^2) + \sqrt{x_i x_j} H^l(x_i, x_j; Q^2) \right) \right], \quad (3.23)$$

where

$$F^l(x_i; Q^2) = -2c_W^2 \left[Q^2(\bar{C}_1 + \bar{C}_2 + \bar{C}_{12}) + 6\bar{C}_{00} - 1 \right] - (1 - 2s_W^2) x_i \bar{C}_{00} - 2s_W^2 x_i M_W^2 \bar{C}_0 - \frac{1}{2}(1 - 2s_W^2) \left[(2 + x_i) \bar{B}_1 + 1 \right],$$

$$G^l(x_i, x_j; Q^2) = -Q^2(C_0 + C_1 + C_2 + C_{12}) + 2C_{00} - 1 - \frac{1}{2} x_i x_j M_W^2 C_0,$$

$$H^l(x_i, x_j; Q^2) = M_W^2 C_0 + \frac{1}{2} Q^2 C_{12} - C_{00} + \frac{1}{4}. \quad (3.24)$$

Analytic expressions for the above functions in the low Q^2 limit are

$$F^l(x_i; 0) = -\left(\frac{5}{2} - 2s_W^2 \right) \Delta_\epsilon + \frac{5x_i^2 \ln x_i}{2(x_i - 1)^2} - \frac{5x_i}{2(x_i - 1)} + \frac{1}{4}, \quad (3.25)$$

$$G^l(x_i, x_j; 0) = \frac{1}{2} \left(\Delta_\epsilon - \frac{1}{2} \right) + \frac{1}{2(x_i - x_j)} \left(\frac{(x_j - 1)x_i^2 \ln x_i}{x_i - 1} - \frac{(x_i - 1)x_j^2 \ln x_j}{x_j - 1} \right),$$

$$H^l(x_i, x_j; 0) = -\frac{1}{4} \left(\Delta_\epsilon + \frac{1}{2} \right) + \frac{1}{4(x_i - x_j)} \left(\frac{x_i(x_i - 4) \ln x_i}{x_i - 1} - \frac{x_j(x_j - 4) \ln x_j}{x_j - 1} \right).$$

The contribution from $\nu^l \chi^h$ -diagrams in figure 2 yields

$$F_L^{Z-\nu^l \chi^h}(Q^2) = \frac{\alpha_W}{8\pi c_W s_W} \sum_{i,j=1}^3 \left[\theta_{\ell'j} W_{\ell i}^\dagger \left(Y_{ji} G_1^{lh}(x_i, y_j; Q^2) + Y_{ji}^\dagger \sqrt{\frac{x_i}{y_j}} H_1^{lh}(x_i, y_j; Q^2) \right) \right. \\ \left. + W_{\ell'j} \theta_{\ell i}^\dagger \left(Y_{ji}^\dagger G_2^{lh}(x_j, y_i; Q^2) + Y_{ji} \sqrt{\frac{x_j}{y_i}} H_2^{lh}(x_j, y_i; Q^2) \right) \right], \quad (3.26)$$

where

$$G_1^{lh}(x_i, y_j; Q^2) = -Q^2(C_0 + C_1 + C_2 + C_{12}) + 2C_{00} - 1 - \frac{1}{2} \frac{x_i}{y_j} M_W^2 C_0, \\ H_1^{lh}(x_i, y_j; Q^2) = M_W^2 C_0 + \frac{1}{2} Q^2 C_{12} - C_{00} + \frac{1}{4}, \\ G_2^{lh}(x_j, y_i; Q^2) = -Q^2(C_0 + C_1 + C_2 + C_{12}) + 2C_{00} - 1 - \frac{1}{2} \frac{x_j}{y_i} M_W^2 C_0, \\ H_2^{lh}(x_j, y_i; Q^2) = M_W^2 C_0 + \frac{1}{2} Q^2 C_{12} - C_{00} + \frac{1}{4}. \quad (3.27)$$

These functions can be written in terms of those appearing for the light neutrino case previously:

$$\left\{ \{G, H\}_{1,2} \right\}^{lh}(x_i, y_j; 0) = \left\{ \{G, H\}_{1,2} \right\}^l(x_i, x_j; 0) \quad \text{with} \quad \left(x_j \rightarrow \frac{1}{y_j} \right).$$

The $F_L^{Z-\chi^h}$ form factor, which stands for the contribution from χ^h -diagrams, yields

$$F_L^{Z-\chi^h}(Q^2) = \frac{\alpha_W}{8\pi c_W s_W} \sum_{i,j=1}^3 \left[\theta_{\ell'j} \theta_{\ell i}^\dagger F^h(y_i; Q^2) \right. \\ \left. + \theta_{\ell'j} S_{ji} \theta_{\ell i}^\dagger \left(G^h(y_i, y_j; Q^2) + \frac{1}{\sqrt{y_i y_j}} H^h(y_i, y_j; Q^2) \right) \right], \quad (3.28)$$

including the functions

$$F^h(y_i; Q^2) = -2c_W^2 \left[Q^2(\bar{C}_1 + \bar{C}_2 + \bar{C}_{12}) + 6\bar{C}_{00} - 1 \right] - (1 - 2s_W^2) \frac{1}{y_i} \bar{C}_{00} \\ - 2s_W^2 \frac{1}{y_i} M_W^2 \bar{C}_0 - \frac{1}{2} (1 - 2s_W^2) \left[\left(2 + \frac{1}{y_i} \right) \bar{B}_1 + 1 \right], \\ G^h(y_i, y_j; Q^2) = -Q^2(C_0 + C_1 + C_2 + C_{12}) + 2C_{00} - 1 - \frac{1}{2} \frac{1}{y_i y_j} M_W^2 C_0, \\ H^h(y_i, y_j; Q^2) = M_W^2 C_0 + \frac{1}{2} Q^2 C_{12} - C_{00} + \frac{1}{4}. \quad (3.29)$$

Their analytic expressions, for low Q^2 , are

$$F^h(y_i; 0) = -\left(\frac{5}{2} - 2s_W^2 \right) \Delta_\epsilon - \frac{5 \ln y_i}{2(1-y_i)^2} - \frac{5}{2(1-y_i)} + \frac{1}{4}, \quad (3.30) \\ G^h(y_i, y_j; 0) = \frac{1}{2} \left(\Delta_\epsilon - \frac{1}{2} \right) + \frac{1}{2(y_j - y_i)} \left(-\frac{(1-y_j) \ln y_i}{(1-y_i)} + \frac{(1-y_i) \ln y_j}{(1-y_j)} \right), \\ H^h(y_i, y_j; 0) = -\frac{1}{4} \left(\Delta_\epsilon + \frac{1}{2} \right) + \frac{1}{4(y_j - y_i)} \left(-\frac{(1-4y_i)y_j \ln y_i}{(1-y_i)} + \frac{(1-4y_j)y_i \ln y_j}{(1-y_j)} \right).$$

Ultraviolet divergences cancel, thanks to the relations (3.17).

Figure 3. Box diagrams contributing to Type I and II $\ell \rightarrow \ell' \ell'' \bar{\ell}'''$ decays.

The box diagrams, represented in figure 3, yield

$$F_B^{\nu_i^l \nu_j^l} = \frac{\alpha_W}{16\pi M_W^2 s_W^2} \sum_{i,j=1}^3 W_{\ell i} W_{\ell' i}^\dagger W_{\ell' j}^\dagger W_{\ell j} f_B^l(y_i, y_j), \quad (3.31)$$

$$F_B^{\nu_i^l \chi_j^h} = \frac{\alpha_W}{16\pi M_W^2 s_W^2} \sum_{i,j=1}^3 W_{\ell i} W_{\ell' i}^\dagger \theta_{\ell' j}^\dagger \theta_{\ell j} f_B^{lh}(y_i, x_j), \quad (3.32)$$

$$F_B^{\chi_i^h \chi_j^h} = \frac{\alpha_W}{16\pi M_W^2 s_W^2} \sum_{i,j=1}^3 \theta_{\ell i}^\dagger \theta_{\ell' i} \theta_{\ell' j}^\dagger \theta_{\ell j} f_B^h(x_i, x_j), \quad (3.33)$$

with

$$\begin{aligned} f_B^l(y_i, y_j) &= \left(1 + \frac{1}{4} y_i y_j\right) \bar{d}_0(y_i, y_j) - 2 y_i y_j d_0(y_i, y_j), \\ f_B^{lh}(y_i, x_j) &= \left(1 + \frac{1}{4} \frac{y_i}{x_j}\right) \bar{d}_0^{lh}(y_i, x_j) - 2 \frac{y_i}{x_j} d_0^{lh}(y_i, x_j), \\ f_B^h(x_i, x_j) &= \left(1 + \frac{1}{4} \frac{1}{x_i x_j}\right) \bar{d}_0^h(x_i, x_j) - 2 \frac{1}{x_i x_j} d_0^h(x_i, x_j). \end{aligned} \quad (3.34)$$

After integrating the three-body phase space the decay width reads [44]

$$\begin{aligned} \Gamma(\ell \rightarrow \ell' \bar{\ell}'' \ell''') &= \frac{\alpha^2 m_\ell^5}{96\pi} \left[3|A_L|^2 + 2|A_R|^2 \left(8 \ln \frac{m_\ell}{m_{\ell'}} - 13 \right) + 2|F_{LL}|^2 + |F_{LR}|^2 + \frac{1}{2}|B_L|^2 \right. \\ &\quad \left. - (6A_L A_R^* - (A_L - 2A_R)(2F_{LL}^* + F_{LR}^* + B_L^*) - F_{LL} B_L^* + \text{h.c.}) \right], \end{aligned} \quad (3.35)$$

where we have defined

$$A_L = \frac{F_L^\gamma}{Q^2}, \quad A_R = \frac{2F_M^\gamma(0)}{m_\ell}, \quad F_{LL} = -\frac{g_L F_L^Z(0)}{e M_Z^2}, \quad F_{LR} = -\frac{g_R F_L^Z(0)}{e M_Z^2}, \quad B_L = B_L(0), \quad (3.36)$$

with $g_{L,R}$ the corresponding Z couplings to the charged lepton ℓ' .

3.3.2 Type II: $\ell \rightarrow \ell' \ell'' \bar{\ell}'''$ with $\ell \neq \ell' \neq \ell'' = \ell'''$

This type of decays can be related to the previous ones, although in this case there are no crossed penguin diagrams contributions. Similarly, there is no 1/2 factor in the phase

Figure 4. Box contributions to Type III $\ell \rightarrow \ell' \ell'' \bar{\ell}'''$ decays.

space integration, as all leptons are distinguishable. Instead, there are additional diagrams for the box contributions at this order, swapping ℓ' and ℓ'' .

With these comments in mind, its decay width reads

$$\Gamma(\ell \rightarrow \ell' \bar{\ell}'' \ell''') = \frac{\alpha^2 m_\ell^5}{96\pi} \left[2|A_L|^2 + 4|A_R|^2 \left(4 \ln \frac{m_\ell}{m_{\ell''}} - 7 \right) + |F_{LL}|^2 + |F_{LR}|^2 + |B_L|^2 \right. \\ \left. - \left(4A_L A_R^* - (A_L - 2A_R) \left(F_{LL}^* + F_{LR}^* + \frac{B_L^*}{2} \right) - F_{LL} \frac{B_L^*}{2} + \text{h.c.} \right) \right]. \quad (3.37)$$

3.3.3 Type III: $\ell \rightarrow \ell' \ell'' \bar{\ell}'''$ with $\ell \neq \ell' = \ell'' \neq \ell'''$

These processes only have box contributions. In addition to box diagrams in figure 3, there are contributions coming from box diagrams with LNV vertices shown in figure 4. They are indicated in the next equation and given in turn below:

$$F_B = F_B^{\nu_i^l \nu_j^l} + F_B^{\nu_i^l \chi_j^h} + F_B^{\chi_i^h \chi_j^h} + F_{B-\text{LNV}}^{\nu_i^l \nu_j^l} + F_{B-\text{LNV}}^{\nu_i^l \chi_j^h} + F_{B-\text{LNV}}^{\chi_i^h \chi_j^h}, \quad (3.38)$$

where [50]

$$F_B^{\nu_i^l \nu_j^l} = \frac{\alpha_W}{16\pi M_W^2 s_W^2} \sum_{i,j=1}^3 \{W_{\ell i} W_{\ell' i}^\dagger W_{\ell'' j} W_{\ell'' j}^\dagger + (\ell' \leftrightarrow \ell'')\} f_B^l(y_i, y_j), \\ F_B^{\nu_i^l \chi_j^h} = \frac{\alpha_W}{16\pi M_W^2 s_W^2} \sum_{i,j=1}^3 \{W_{\ell i} W_{\ell' i}^\dagger \theta_{\ell'' j}^\dagger \theta_{\ell'' j} + (\ell' \leftrightarrow \ell'')\} f_B^{lh}(y_i, x_j), \\ F_B^{\chi_i^h \chi_j^h} = \frac{\alpha_W}{16\pi M_W^2 s_W^2} \sum_{i,j=1}^3 \{\theta_{\ell i}^\dagger \theta_{\ell' i} \theta_{\ell'' j}^\dagger \theta_{\ell'' j} + (\ell' \leftrightarrow \ell'')\} f_B^h(x_i, x_j), \\ F_{B-\text{LNV}}^{\nu_i^l \nu_j^l} = \frac{\alpha_W}{16\pi M_W^2 s_W^2} \sum_{i,j}^3 W_{\ell i} W_{\ell' j}^\dagger W_{\ell'' i} W_{\ell'' j}^\dagger f_B^{l-\text{LNV}}(y_i, y_j), \\ F_{B-\text{LNV}}^{\nu_i^l \chi_j^h} = \frac{\alpha_W}{16\pi M_W^2 s_W^2} \sum_{i,j=1}^3 W_{\ell i} \theta_{\ell' j} W_{\ell'' i} \theta_{\ell'' j}^\dagger f_B^{lh-\text{LNV}}(y_i, x_j), \\ F_{B-\text{LNV}}^{\chi_i^h \chi_j^h} = \frac{\alpha_W}{16\pi M_W^2 s_W^2} \sum_{i,j=1}^3 \theta_{\ell i}^\dagger \theta_{\ell' j} \theta_{\ell'' i}^\dagger \theta_{\ell'' j} f_B^{h-\text{LNV}}(x_i, x_j). \quad (3.39)$$

For small (large) light (heavy) neutrino masses, i.e. $x_i, y_j \rightarrow 0$, the previous expressions simplify to

$$F_B^{\nu_i^l \nu_j^l} \approx -\frac{\alpha_W}{16\pi M_W^2 s_W^2} \sum_{i,j=1}^3 \{W_{\ell i} W_{\ell' i}^\dagger W_{\ell'' j} W_{\ell'' j}^\dagger + (\ell' \leftrightarrow \ell'')\} [1 + (y_j + y_i)(1 + \ln y_j)], \quad (3.40)$$

$$F_B^{\nu_i^l \chi_j^h} \approx \frac{\alpha_W}{16\pi M_W^2 s_W^2} \sum_{i,j=1}^3 \{W_{\ell i} W_{\ell' i}^* \theta_{\ell'' j}^\dagger \theta_{\ell'' j} + (\ell' \leftrightarrow \ell'')\} \left[x_j(1 + \ln x_j) + \frac{1}{4} y_i(\ln x_j - 7) \right], \quad (3.41)$$

$$F_{B-\text{LNV}}^{\nu_i^l \nu_j^l} = \frac{\alpha_W}{16\pi M_W^2 s_W^2} \sum_{i,j=1}^3 \{W_{\ell i} W_{\ell' j}^\dagger W_{\ell'' i} W_{\ell'' j}^\dagger\} [2\sqrt{y_i y_j}(1 + 2 \ln y_j)], \quad (3.42)$$

$$F_{B-\text{LNV}}^{\nu_i^l \chi_j^h} \approx \frac{\alpha_W}{32\pi M_W^2 s_W^2} \sum_{i,j=1}^3 \{W_{\ell i} \theta_{\ell' j} W_{\ell'' i} \theta_{\ell'' j}\} [2\sqrt{y_i x_j}(\ln x_j - 1)], \quad (3.43)$$

and

$$\begin{aligned} f_B^{h-\text{LNV}}(x_i, x_j) \Big|_{x_i, x_j \rightarrow 0} &\approx \sqrt{\frac{x_i}{x_j^3}} \ln \left(\frac{x_j}{x_i} \right) + \sqrt{\frac{x_i}{x_j}} (2 \ln x_i + 1) + 3\sqrt{x_i x_j} (\ln x_j + 1) \\ &\quad + \frac{1}{\sqrt{x_i x_j}} (\ln x_j + 1) + \sqrt{\frac{x_j}{x_i}} (2 \ln x_j + 1). \end{aligned} \quad (3.44)$$

We recall that the $y_j \rightarrow 0$ limit is not physical (as perturbative unitarity limits the maximum value of M_{N_j} to some tenths of TeVs), so that the previous expressions are of course free of infrared singularities. The corresponding total decay width is just given by

$$\Gamma(\ell \rightarrow \ell' \ell'' \bar{\ell}''') = \frac{\alpha^2 m_\ell^5}{192\pi} |F_B|^2. \quad (3.45)$$

3.4 $\mu \rightarrow e$ conversion in nuclei

The $\mu - e$ conversion in nuclei has penguin and box contributions as $\ell \rightarrow \ell' \ell'' \bar{\ell}'''$ decays, replacing the last two leptons by a $q = u$ or d quark. It has no crossed penguin diagrams because the lower fermionic line, where the gauge boson is attached, is now a coherent sum of quarks composing the probed nucleus. There is also no crossed box contribution due to the exchange of leptons.

The matrix element is

$$\mathcal{M}^{\mu q \rightarrow e q} = \mathcal{M}_\gamma^{\mu q \rightarrow e q} + \mathcal{M}_Z^{\mu q \rightarrow e q} + \mathcal{M}_{\text{box}}^{\mu q \rightarrow e q}, \quad (3.46)$$

with the amplitudes defined as [44]

$$\begin{aligned} \mathcal{M}_\gamma^{\mu q \rightarrow e q} &= \bar{u}(p_1) e \left[i F_M^\gamma(0) 2 P_R \sigma^{\mu\nu} (p_1 - p_\ell)_\nu + F_L^\gamma((p_1 - p_\ell)^2) \gamma^\mu P_L \right] u(p_\ell) \\ &\quad \times \frac{1}{(p_1 - p_\ell)^2} \bar{u}(p_3) \gamma_\mu (g_{Lq}^\gamma P_L + g_{Rq}^\gamma P_R) v(p_2), \\ \mathcal{M}_Z^{\mu q \rightarrow e q} &= \bar{u}(p_1) \left(-e F_L^Z(0) \right) \gamma^\mu P_L u(p_\ell) \frac{1}{M_Z^2} \bar{u}(p_3) \gamma_\mu (g_{Lq}^Z P_L + g_{Rq}^Z P_R) v(p_2), \\ \mathcal{M}_{\text{box}}^{\mu q \rightarrow e q} &= e^2 B_L^q(0) \bar{u}(p_1) \gamma^\mu P_L u(p_\ell) \bar{u}(p_3) \gamma_\mu P_L v(p_2), \end{aligned} \quad (3.47)$$

given in terms of the form factors

$$\begin{aligned}
 A_{1L} &= \frac{F_L^\gamma(Q^2)}{Q^2} = \frac{\alpha_W}{8\pi M_W^2} \sum_{i=1}^3 \theta_{ei} \theta_{\mu i}^\dagger \left(-\frac{(12y_i^2 - 10y_i + 1) \ln y_i}{6(1 - y_i)^4} + \frac{20y_i^3 - 96y_i^2 + 57y_i + 1}{36(1 - y_i)^3} \right), \\
 A_{2R} &= \frac{2F_M^\gamma(0)}{m_\mu} = \frac{\alpha_W}{8\pi M_W^2} \sum_{i=1}^3 \theta_{ei} \theta_{\mu i}^\dagger \left(-\frac{2y_i^3 - 7y_i^2 + 11y_i}{4(1 - y_i)^3} + \frac{3y_i \ln y_i}{2(1 - y_i)^4} \right), \\
 F_{LL}^u &= -\frac{F_L^Z(0)g_{Lu}^Z}{M_Z^2} = -\frac{\alpha_W}{16\pi M_W^2 s_W^2} \left(1 - \frac{4}{3}s_W^2 \right) \sum_{i,j=1}^3 \left[\theta_{ei} \theta_{\mu i}^\dagger \left(-\frac{5 \ln y_i}{2(1 - y_i)^2} - \frac{5}{2(1 - y_i)} \right) \right. \\
 &\quad + \theta_{ej} S_{ji} \theta_{\mu i}^\dagger \left(-\frac{1}{2(y_i - y_j)} \left(-\frac{(1 - y_j) \ln y_i}{(1 - y_i)} + \frac{(1 - y_i) \ln y_j}{(1 - y_j)} \right. \right. \\
 &\quad \left. \left. - \frac{1}{\sqrt{y_i y_j}} \frac{1}{4(y_i - y_j)} \left(-\frac{y_j(1 - 4y_i) \ln y_i}{1 - y_i} + \frac{y_i(1 - 4y_j) \ln y_j}{1 - y_j} \right) \right) \right], \\
 F_{LR}^u &= -\frac{F_L^Z(0)g_{Ru}^Z}{M_Z^2} = \frac{\alpha_W}{12\pi M_W^2} \sum_{i,j=1}^3 \left[\theta_{ei} \theta_{\mu i}^\dagger \left(-\frac{5 \ln y_i}{2(1 - y_i)^2} - \frac{5}{2(1 - y_i)} \right) \right. \\
 &\quad + \theta_{ej} S_{ji} \theta_{\mu i}^\dagger \left(-\frac{1}{2(y_i - y_j)} \left(-\frac{(1 - y_j) \ln y_i}{(1 - y_i)} + \frac{(1 - y_i) \ln y_j}{(1 - y_j)} \right) \right. \\
 &\quad \left. \left. - \frac{1}{\sqrt{y_i y_j}} \frac{1}{4(y_i - y_j)} \left(-\frac{y_j(1 - 4y_i) \ln y_i}{1 - y_i} + \frac{y_i(1 - 4y_j) \ln y_j}{1 - y_j} \right) \right) \right], \\
 F_{LL}^d &= -\frac{F_L^Z(0)g_{Lu}^Z}{M_Z^2} = -\frac{\alpha_W}{16\pi M_W^2 s_W^2} \left(-1 + \frac{2}{3}s_W^2 \right) \sum_{i,j=1}^3 \left[\theta_{ei} \theta_{\mu i}^\dagger \left(-\frac{5 \ln y_i}{2(1 - y_i)^2} - \frac{5}{2(1 - y_i)} \right) \right. \\
 &\quad + \theta_{ej} S_{ji} \theta_{\mu i}^\dagger \left(-\frac{1}{2(y_i - y_j)} \left(-\frac{(1 - y_j) \ln y_i}{(1 - y_i)} + \frac{(1 - y_i) \ln y_j}{(1 - y_j)} \right) \right. \\
 &\quad \left. \left. - \frac{1}{\sqrt{y_i y_j}} \frac{1}{4(y_i - y_j)} \left(-\frac{y_j(1 - 4y_i) \ln y_i}{1 - y_i} + \frac{y_i(1 - 4y_j) \ln y_j}{1 - y_j} \right) \right) \right], \\
 F_{LR}^d &= -\frac{F_L^Z(0)g_{Ru}^Z}{M_Z^2} = -\frac{\alpha_W}{24\pi M_W^2} \sum_{i,j=1}^3 \left[\theta_{ei} \theta_{\mu i}^\dagger \left(-\frac{5 \ln y_i}{2(1 - y_i)^2} - \frac{5}{2(1 - y_i)} \right) \right. \\
 &\quad + \theta_{ej} S_{ji} \theta_{\mu i}^\dagger \left(-\frac{1}{2(y_i - y_j)} \left(-\frac{(1 - y_j) \ln y_i}{(1 - y_i)} + \frac{(1 - y_i) \ln y_j}{(1 - y_j)} \right) \right. \\
 &\quad \left. \left. - \frac{1}{\sqrt{y_i y_j}} \frac{1}{4(y_i - y_j)} \left(-\frac{y_j(1 - 4y_i) \ln y_i}{1 - y_i} + \frac{y_i(1 - 4y_j) \ln y_j}{1 - y_j} \right) \right) \right], \\
 B_L^d &= \frac{\alpha_W}{16\pi M_W^2 s_W^2} \sum_{i=1}^3 \theta_{\mu i}^\dagger \theta_{ei} \left(|V_{td}|^2 [f_{B_d}(y_i, x_t) - f_{B_d}(y_i, 0)] - f_{B_d}(y_i, 0) \right), \\
 B_L^u &= \frac{\alpha_W}{16\pi M_W^2 s_W^2} \sum_{i=1}^3 \theta_{\mu i}^\dagger \theta_{ei} f_{B_u}(y_i, 0),
 \end{aligned} \tag{3.48}$$

Nucleus	N	Z	Z_{eff}	F_P	$\Gamma_{\text{Capt.}}[\text{GeV}]$
$^{27}_{13}\text{Al}$	14	13	11.5	0.64	4.6×10^{-19}
$^{48}_{22}\text{Ti}$	26	22	17.6	0.54	1.7×10^{-18}
$^{197}_{79}\text{Au}$	118	79	33.5	0.16	8.6×10^{-18}

Table 1. Input parameters for different nuclei.

where

$$\begin{aligned}
 f_{B_d}(y_i, x_t) &= \left(1 + \frac{1}{4} \frac{x_t}{y_i}\right) \left[\frac{y_i \ln y_i}{(1-y_i)^2(1-y_i x_t)} + \frac{y_i x_t^2 \ln x_t}{(1-x_t)^2(1-y_i x_t)} + \frac{y_i}{(1-y_i)(1-x_t)} \right] \\
 &\quad - 2 \frac{x_t}{y_i} \left[\frac{y_i^2 \ln y_i}{(1-y_i)^2(1-y_i x_t)} + \frac{x_t y_i \ln x_t}{(1-x_t)^2(1-y_i x_t)} + \frac{y_i}{(1-y_i)(1-x_t)} \right], \\
 f_{B_d}(y_i, 0) &= \bar{d}_0^h(y_i, 0) = \frac{y_i \ln y_i}{(1-y_i)^2} + \frac{y_i}{(1-y_i)}, \\
 f_{B_u}(y_i, 0) &= -4 \bar{d}_0^h(y_i, 0) = -4 \left(\frac{y_i \ln y_i}{(1-y_i)^2} + \frac{y_i}{(1-y_i)} \right), \tag{3.49}
 \end{aligned}$$

with $x_t = m_t^2/M_W^2$, being m_t the top quark mass.

The form factors $F_M^\gamma(0)$, F_L^γ and $F_L^Z(0)$ are given by (3.20), (3.21), and (3.28), respectively while the couplings $g_{L(R)q}^Z$ read [52]

$$g_{Lu}^Z = \frac{1 - \frac{4}{3}s_W^2}{2s_W c_W}, \quad g_{Ru}^Z = -\frac{2s_W}{3c_W}, \quad g_{Ld}^Z = \frac{-1 + \frac{2}{3}s_W^2}{2s_W c_W}, \quad g_{Rd}^Z = \frac{s_W}{3c_W}. \tag{3.50}$$

In terms of the latter, the $\mu - e$ conversion rate in a nucleus with Z protons and $N = A - Z$ neutrons yields [44, 52]

$$\begin{aligned}
 \mathcal{R} &= \frac{\alpha^5 Z_{\text{eff}}^4}{\Gamma_{\text{Capt}} Z} F_P^2 m_\mu^5 |2Z(A_{1L} + A_{2R}) - (2Z + N)(F_{LL}^u + F_{LR}^u + B_L^u) \\
 &\quad - (Z + 2N)(F_{LL}^d + F_{LR}^d + B_L^d)|^2, \tag{3.51}
 \end{aligned}$$

where Z_{eff} is the nucleus effective charge for the muon and F_P the associated form factor. In table 1 we gather the input parameters for Al, Ti and Au [44, 52, 74, 75].

4 Phenomenology

In this section we show the numerical analysis for each LFV processes described previously through Monte Carlo simulations. We will follow the light Majorana neutrinos massless approximation. Therefore, only diagrams involving heavy Majorana neutrinos contribute. We begin the discussion with a joint analysis considering $Z \rightarrow \bar{\ell}\ell'$, Type I and II $\ell \rightarrow \ell'\ell''\bar{\ell}'''$ decays and $\mu - e$ conversion rate in nuclei, as they share the same free parameters: three heavy neutrinos masses $\{M_i\}_{i=1,2,3}$ and neutral couplings given by the $(\theta S \theta^\dagger)$ entries. Afterwards, we focus in LFV Type III (well known as ‘wrong sign’ processes) that will

bind directly the LNV couplings. In the following subsections we are using the limits of $(\theta\theta^\dagger)_{\ell'\ell}$ previously obtained from the $\ell \rightarrow \ell'\gamma$ decays, (3.11).

All processes analyzed have 3 common free parameters which are the heavy neutrino masses M_i that will run from 15 to 20 TeV. We decided to take this interval based on the experience gained by doing simplified analysis for each process separately.¹¹ This range of M_i values corresponds to $f \in [1.2, 1.6]$ TeV, which is currently allowed (see e.g. ref. [44]).

We mention at this point that the LNV contributions that we are studying within the LHT also induce LNV semileptonic tau decays (analogously to neutrinoless double beta decays, but also with LFV). Of course their rates are very much suppressed as there is no resonant enhancement of the Majorana neutrino exchanges. Specifically, for typical values of the relevant parameters (that are allowed considering all other processes that we analyze in the remainder of this section) we get

$$\begin{aligned} BR(\tau^- \rightarrow e^+ \pi^- \pi^-) &\leq 1.6 \times 10^{-29}, & BR(\tau^- \rightarrow \mu^+ \pi^- \pi^-) &\leq 8.9 \times 10^{-29}, \\ BR(\tau^- \rightarrow e^+ K^- K^-) &\leq 2.5 \times 10^{-32}, & BR(\tau^- \rightarrow \mu^+ K^- K^-) &\leq 1.4 \times 10^{-31}, \\ BR(\tau^- \rightarrow e^+ \pi^- K^-) &\leq 6.7 \times 10^{-31}, & BR(\tau^- \rightarrow \mu^+ \pi^- K^-) &\leq 3.8 \times 10^{-30}, \end{aligned} \quad (4.1)$$

which are more than twenty orders of magnitude below current limits [6]. Much more interesting are the processes presented in the next subsections. For some of them, average values of the branching ratios or conversion rates are within one order of magnitude of current upper limits, as we will see.

4.1 Joint analysis for $Z \rightarrow \bar{\ell}\ell'$, type I and II $\ell \rightarrow \ell'\ell''\bar{\ell}'''$ decays and $\mu \rightarrow e$ conversion in nuclei

In this part we do a global analysis of the following 10 processes: LFV Z decays $Z \rightarrow \bar{\mu}e$, $Z \rightarrow \bar{\tau}e$, and $Z \rightarrow \bar{\tau}\mu$; LFV Type I $\mu \rightarrow ee\bar{e}$, $\tau \rightarrow ee\bar{e}$ and $\tau \rightarrow \mu\mu\bar{\mu}$; LFV Type II $\tau \rightarrow e\mu\bar{\mu}$ and $\tau \rightarrow \mu e\bar{e}$; $\mu - e$ conversion in nuclei $^{48}_{22}\text{Ti}$ and $^{197}_{79}\text{Au}$.

We do the analysis through a single Monte Carlo simulation where the 10 processes are run simultaneously. The peculiarity of all these LFV processes is that they share the same free parameters: three heavy neutrino masses $\{M_i\}_{i=1,2,3}$ and the neutral couplings given by $(\theta S\theta^\dagger)$ matrix.

Every process has to respect its own upper limit reported by PDG [6] (see also [7]), though the conditions on the heavy neutrinos masses and neutral couplings of heavy Majorana neutrinos are common to all.

These LFV processes receive two types of contributions: one is coming from charged couplings $(\theta\theta^\dagger)$ and the other one from neutral couplings $(\theta S\theta^\dagger)$. As a result, there is an interference between them. Therefore, we are able to determine the relative sign between the entries of the $(\theta\theta^\dagger)$ (which were bound in (3.11)) and $(\theta S\theta^\dagger)$ matrices, which turns out to be negative.

¹¹We do not present them here and only quote that the results on the individual processes agree with the joint analysis that we will discuss next.

LFV Z decays	Our mean values	Present limits [6]
$\text{Br}(Z \rightarrow \bar{\mu}e)$	1.20×10^{-14}	3.7×10^{-7}
$\text{Br}(Z \rightarrow \bar{\tau}e)$	1.46×10^{-8}	4.9×10^{-6}
$\text{Br}(Z \rightarrow \bar{\tau}\mu)$	1.09×10^{-8}	0.6×10^{-5}
LFV Type I		
$\text{Br}(\mu \rightarrow ee\bar{e})$	1.85×10^{-14}	1.0×10^{-12}
$\text{Br}(\tau \rightarrow ee\bar{e})$	4.16×10^{-9}	2.7×10^{-8}
$\text{Br}(\tau \rightarrow \mu\mu\bar{\mu})$	4.24×10^{-9}	2.1×10^{-8}
LFV Type II		
$\text{Br}(\tau \rightarrow e\mu\bar{\mu})$	3.60×10^{-9}	2.7×10^{-8}
$\text{Br}(\tau \rightarrow \mu e\bar{e})$	2.48×10^{-9}	1.8×10^{-8}
$\mu - e$ conversion rate		
$\mathcal{R}(\text{Ti})$	6.21×10^{-14}	4.3×10^{-13}
$\mathcal{R}(\text{Au})$	7.82×10^{-14}	7.0×10^{-12}
Heavy neutrino masses		
M_1 (TeV)	17.186	
M_2 (TeV)	17.185	
M_3 (TeV)	17.187	

Table 2. Mean values for branching ratios, conversion rates and three heavy neutrino masses compared to current upper limits (at 95% confidence level for the Z decays and at 90% for all other processes). Statistical errors are at the 1% level and order permille for the heavy neutrino masses.

The Monte Carlo simulation finds combinations of the free parameters values that return allowed results for each branching ratio and conversion rate [6]. In table 2 we show the mean values of our simulations that respect all experimental bounds. According to the current upper limits and our mean values, the $\tau \rightarrow ee\bar{e}$, $\tau \rightarrow \mu\mu\bar{\mu}$, $\tau \rightarrow e\mu\bar{\mu}$, $\tau \rightarrow \mu e\bar{e}$ and $\mu \rightarrow e$ conversion in Ti seem to be more promising in the near future than the LFV Z decays, the $\mu \rightarrow ee\bar{e}$ decays and $\mu \rightarrow e$ conversion in Au. However, prospects for future sensitivities on the latter processes (see e.g. table I in [52] and refs. therein and ref. [76] focusing on $Z \rightarrow \tau\ell$) all go below our mean values.

The modulus of the $(\theta S\theta^\dagger)_{e\mu}$ elements are all smaller than 7.5×10^{-10} , while for the other flavor combinations we get $|(\theta S\theta^\dagger)_{e\tau}| < 5.1 \times 10^{-7}$ and $|(\theta S\theta^\dagger)_{\mu\tau}| < 6.2 \times 10^{-7}$.

Figure 5. Heat map that stands for the correlation matrix among $(\theta S\theta^\dagger)_{e\mu}$ -processes: $Z \rightarrow \bar{\mu}e$, $\mu \rightarrow ee\bar{e}$, $\mu - e$ conversion in nuclei $^{48}_{22}\text{Ti}$ and $^{197}_{79}\text{Au}$, and their free parameters.

In order to find relations among the above processes we group them into 3 categories based on their neutral couplings: $(\theta S\theta^\dagger)_{e\mu}$, $(\theta S\theta^\dagger)_{e\tau}$, and $(\theta S\theta^\dagger)_{\mu\tau}$.

- $(\theta S\theta^\dagger)_{e\mu}$ -processes: $Z \rightarrow \bar{\mu}e$, $\mu \rightarrow ee\bar{e}$, and $\mu - e$ conversion in nuclei $^{48}_{22}\text{Ti}$ and $^{197}_{79}\text{Au}$.
- $(\theta S\theta^\dagger)_{e\tau}$ -processes: $Z \rightarrow \bar{\tau}e$, $\tau \rightarrow ee\bar{e}$, and $\tau \rightarrow e\mu\bar{\mu}$.
- $(\theta S\theta^\dagger)_{\mu\tau}$ -processes: $Z \rightarrow \bar{\tau}\mu$, $\tau \rightarrow \mu\mu\bar{\mu}$, and $\tau \rightarrow \mu e\bar{e}$.

In figure 5 a heat map shows the correlation matrix among $(\theta S\theta^\dagger)_{e\mu}$ -processes and their free parameters. First of all, we see that there is no sizeable correlation among any process probability and its free parameters. Second, the small correlations among every pair of $(\theta S\theta^\dagger)_{e\mu}$ matrix elements is negative. Furthermore, $Z \rightarrow \bar{\mu}e$ decay is strongly correlated with $\mu \rightarrow ee\bar{e}$. Similarly, the conversion rate in $^{48}_{22}\text{Ti}$ is highly correlated with the one in $^{197}_{79}\text{Au}$. In figure 6 (7) the correlations between $BR(Z \rightarrow \bar{\mu}e)$ and $BR(\mu \rightarrow ee\bar{e})$ ($\mathcal{R}(\text{Ti})$ and $\mathcal{R}(\text{Au})$) are shown in scatter plots. Although heavy neutrino masses are quite correlated, the solutions are mostly far from quasidegenerate scenarios, which is a natural solution.

Figure 6. Scatter plot $\text{Br}(Z \rightarrow \bar{\mu}e)$ vs. $\text{Br}(\mu \rightarrow ee\bar{e})$.

Figure 7. Scatter plot $\mathcal{R}(\text{Ti})$ vs. $\mathcal{R}(\text{Au})$.

In figure 8 the correlations among $(\theta S\theta^\dagger)_{e\tau}$ -processes and their free parameters are represented. The interpretation of this plot is very similar to figure 5. The branching ratios of these decays have a sizeable correlation to each other, but the predominant one is between $\text{Br}(Z \rightarrow \bar{\tau}e)$ and $\text{Br}(\tau \rightarrow ee\bar{e})$. We show all those behaviors in figures 9, 10 and 11.

For $(\theta S\theta^\dagger)_{\mu\tau}$ -processes their branching ratios are not correlated with any free parameter as we can observe in figure 12. Nevertheless, we can see sizeable correlations among branching ratios, where the largest one is between $\text{Br}(Z \rightarrow \bar{\tau}\mu)$ and $\text{Br}(\tau \rightarrow \mu e\bar{e})$. The correlations among decays are displayed in figures 13, 14 and 15.

In the three heat maps for the processes whose behavior involves neutral couplings given by $(\theta S\theta^\dagger)_{\ell'\ell}$ matrix, the three heavy masses are strongly correlated to each other. Still, solutions span the range [15, 20] TeV and do not favor (quasi)degenerate scenarios.

Finally, we display a heat map in figure 16 where only branching ratios and conversion rates are involved. This heat map, that stands for a correlation matrix, seems a block matrix where each block represents a category of $(\theta S\theta^\dagger)_{\ell'\ell}$ -processes, so that we can conclude that processes with different neutral coupling are mildly correlated, as expected.

The scatter plots among two pairs of heavy neutrino masses in figures 17 and 18 show neatly that solutions do not restrict to the nearly degenerate case.¹²

4.2 Numerical analysis for $\ell \rightarrow \ell'\ell''\bar{\ell}'''$ of type III: wrong sign decays

In this subsection we study two tau decays which are known as wrong-sign processes: $\tau \rightarrow ee\bar{\mu}$ and $\tau \rightarrow \mu\mu\bar{e}$. We analyze them assuming that the terms associated with LNV vertices are free parameters, thus we are able to bind these couplings. So the free parameters to each wrong sign processes are going to be

¹²This is of course independent of the mean values for these three masses being very approximately equal in all our runs.

Figure 8. Heat map that stands for the correlation matrix among $(\theta S \theta^\dagger)_{e\tau}$ -processes: $Z \rightarrow \bar{\tau}e$, $\tau \rightarrow ee\bar{e}$, $\tau \rightarrow e\mu\bar{\mu}$, and their free parameters.

Figure 9. Scatter plot $\text{Br}(Z \rightarrow \bar{\tau}e)$ vs. $\text{Br}(\tau \rightarrow ee\bar{e})$.

Figure 10. Scatter plot $\text{Br}(Z \rightarrow \bar{\tau}e)$ vs. $\text{Br}(\tau \rightarrow e\mu\bar{\mu})$.

Figure 11. Scatter plot $\text{Br}(Z \rightarrow ee\bar{e})$ vs. $\text{Br}(\tau \rightarrow e\mu\bar{\mu})$.

Figure 12. Heat map that stands for the correlation matrix among $(\theta S \theta^\dagger)_{\mu\tau}$ -processes: $Z \rightarrow \bar{\tau}\mu$, $\tau \rightarrow \mu\mu\bar{\mu}$, $\tau \rightarrow \mu e\bar{e}$, and their free parameters.

Figure 13. Scatter plot $\text{Br}(Z \rightarrow \bar{\tau}\mu)$ vs. $\text{Br}(\tau \rightarrow \mu\mu\bar{\mu})$.

Figure 14. $\text{Br}(Z \rightarrow \bar{\tau}\mu)$ vs. $\text{Br}(\tau \rightarrow \mu e\bar{e})$.

Figure 15. $\text{Br}(\tau \rightarrow \mu\mu\bar{\mu})$ vs. $\text{Br}(\tau \rightarrow \mu e\bar{e})$.

- $\tau \rightarrow ee\bar{\mu}$: the masses of heavy neutrinos: M_i ; LNV couplings: $(\theta_{\mu i}\theta_{\tau i})^\dagger$, and $\theta_{ei}\theta_{ei}$ with $i = 1, 2, 3$. We bind the couplings as follows (see also [77])

$$\begin{aligned} |\theta_{\mu 1}\theta_{\tau 1}| + |\theta_{\mu 2}\theta_{\tau 2}| + |\theta_{\mu 3}\theta_{\tau 3}| &< 0.32 \times 10^{-3}, \\ |\theta_{e 1}\theta_{e 1}| + |\theta_{e 2}\theta_{e 2}| + |\theta_{e 3}\theta_{e 3}| &< 0.01, \end{aligned} \quad (4.2)$$

and their product must satisfy (see also [77])

$$|\theta_{\mu i}\theta_{\tau i}||\theta_{e j}\theta_{e j}| < 0.32 \times 10^{-5}. \quad (4.3)$$

- $\tau \rightarrow \mu\mu\bar{e}$: the masses of heavy neutrinos: M_i ; LNV couplings: $(\theta_{ei}\theta_{\tau i})^\dagger$, and $(\theta_{\mu i}\theta_{\mu i})$ with $i = 1, 2, 3$. Bounds on the couplings are (see also [77])

$$\begin{aligned} |\theta_{e 1}\theta_{\tau 1}| + |\theta_{e 2}\theta_{\tau 2}| + |\theta_{e 3}\theta_{\tau 3}| &< 0.9 \times 10^{-3}, \\ |\theta_{\mu 1}\theta_{\mu 1}| + |\theta_{\mu 2}\theta_{\mu 2}| + |\theta_{\mu 3}\theta_{\mu 3}| &< 0.0075, \end{aligned} \quad (4.4)$$

Figure 16. Heat map that stands for the correlation matrix exclusively among the 10 processes analyzed in this section. We observe that this matrix seems a block matrix representation where each block corresponds to each neutral coupling category.

Figure 17. Scatter plot M_1 vs. M_2 .

Figure 18. Scatter plot M_2 vs. M_3 .

with their product fulfilling (see also [77])

$$|\theta_{ei}\theta_{\tau i}||\theta_{\mu j}\theta_{\mu j}| < 0.68 \times 10^{-5}. \quad (4.5)$$

The heavy neutrino masses M_i ($i = 1, 2, 3$) run from 15 to 20 TeV as the analysis above. In table 3 we show the mean values for branching ratios of wrong-sign processes, heavy neutrino masses and LNV couplings. Remarkably, these wrong-sign decays typically yield branching ratios only one order of magnitude below the current upper limits, as the pro-

Figure 19. Heat map that stands for a correlation matrix among wrong sign branching ratios and free parameters.

cesses with the brightest prospects presented in the previous section. For completeness, we quote here that neutrinoless double beta decays [6] bind $\sum_{i=1}^3 |\theta_{ei}|^2$ to be smaller than or, at most, of $\mathcal{O}(1 \times 10^{-5})$. As there can be cancellations among contributions in the previous sum, this limit is not in conflict with our results.

The heavy neutrino masses (M_i) present a sizeable correlation among them as in the previous analysis. Also, LNV couplings, $|\theta_{ei}\theta_{\tau i}|$ and $|\theta_{\mu i}\theta_{\mu i}|$, are moderately correlated with the heavy neutrino masses, while $|\theta_{\mu i}\theta_{\tau i}|$ and $|\theta_{ei}\theta_{ei}|$ have a minimum correlation with them.

LHT does not predict these “wrong sign” decays through T-odd leptons [44]. However, when we extend the LHT model involving Majorana neutrinos in the ISS, the branching ratios are predicted at similar rates, $\sim 10^{-9}$, than the other LFV three lepton tau decays. Within this setting, we can bind the LNV couplings shown in table 3, which were not restricted in ref. [77]. This behavior is described by the correlation matrix in figure 19.

The mean values for the heavy neutrino masses from the studies in the previous section differ only slightly from the ‘Wrong Sign’ analysis, $\sim 0.12\%$ in all cases.

Branching Ratios	Our mean values
$\text{Br}(\tau \rightarrow ee\bar{\mu})$	1.8×10^{-9}
$\text{Br}(\tau \rightarrow \mu\mu\bar{e})$	1.9×10^{-9}
Heavy neutrino masses	
M_1 (TeV)	17.170
M_2 (TeV)	17.166
M_3 (TeV)	17.166
LNV couplings	
$(\theta_{e1}\theta_{\tau1})^\dagger$	$(2.2 \pm 9.6) \times 10^{-7}$
$(\theta_{e2}\theta_{\tau2})^\dagger$	$(1.5 \pm 1.0) \times 10^{-6}$
$(\theta_{e3}\theta_{\tau3})^\dagger$	$(0.2 \pm 1.0) \times 10^{-6}$
$ \theta_{ei}\theta_{\tau i} $	2.76×10^{-4}
$(\theta_{\mu1}\theta_{\mu1})$	$(0.8 \pm 3.0) \times 10^{-5}$
$(\theta_{\mu2}\theta_{\mu2})$	$-(8.8 \pm 3.0) \times 10^{-5}$
$(\theta_{\mu3}\theta_{\mu3})$	$(1.0 \pm 3.0) \times 10^{-5}$
$ \theta_{\mu i}\theta_{\mu i} $	8.5×10^{-3}
$(\theta_{\mu1}\theta_{\tau1})^\dagger$	$(2.1 \pm 2.3) \times 10^{-6}$
$(\theta_{\mu2}\theta_{\tau2})^\dagger$	$-(0.4 \pm 2.3) \times 10^{-6}$
$(\theta_{\mu3}\theta_{\tau3})^\dagger$	$(0.6 \pm 2.3) \times 10^{-6}$
$ \theta_{\mu i}\theta_{\tau i} $	6.5×10^{-4}
$(\theta_{e1}\theta_{e1})$	$(3.9 \pm 2.0) \times 10^{-5}$
$(\theta_{e2}\theta_{e2})$	$(4.6 \pm 2.0) \times 10^{-5}$
$(\theta_{e3}\theta_{e3})$	$-(5.0 \pm 2.0) \times 10^{-5}$
$ \theta_{ei}\theta_{ei} $	5.7×10^{-3}

Table 3. Mean values for the free parameters and branching ratios in the wrong sign processes considering Majorana neutrinos in the LHT. Statistical errors which are not shown are smaller than the last significant figure. We recall the 90% C.L. limits [6]: 1.5×10^{-8} (on $\text{Br}(\tau \rightarrow ee\bar{\mu})$) and 1.7×10^{-8} (on $\text{Br}(\tau \rightarrow \mu\mu\bar{e})$).

5 Conclusions

The richness of the Littlest Higgs Model with T parity, LHT, allows for understanding light neutrino mass values via an ISS mechanism of Type I. As a consequence, there appears one heavy Majorana neutrino per family, with $\mathcal{O}(10 \text{ TeV})$ mass. In this way, this new mass scale is of the order of 4π times the vacuum expectation value associated to the collective spontaneous symmetry breakdown of the LHT, that produces T odd particles in the TeV region. We have focused in this work in the new contributions given by the heavy Majorana neutrinos to several LFV processes. Our results are encouraging:

- In all $\tau \rightarrow \ell\ell'\bar{\ell}''$ (including wrong-sign) decays and in $\mu \rightarrow e$ conversion in Ti, the mean values of our simulated events satisfying all present bounds are only one order of magnitude smaller than current limits. In $\mu \rightarrow ee\bar{e}$, $Z \rightarrow \bar{\tau}\ell$ and conversion in Au, our mean values are around two orders of magnitude smaller than current limits (only $Z \rightarrow \bar{\mu}e$ does not have the potential of probing our results in the near future).
- The pattern of correlations among processes is completely different to the ‘traditional’ LHT (without heavy Majorana neutrinos), where for instance wrong-sign decays are negligible. It should also be noted that the correlation between $L \rightarrow \ell\gamma$ and $L \rightarrow \ell\ell'\bar{\ell}''$ decays, which is a celebrated signature distinguishing underlying models producing the LFV, here is broken, as the former decays depend only on the charged current mixings $\theta\theta^\dagger$ and the neutral current ones also on the neutral current admixtures, $\theta S\theta^\dagger$, which reduces sizeably the correlation among both decay modes. Only within the LHT, upon eventual discovery of LFV in charged leptons in several processes, correlations among them would immediately distinguish the usual scenario [44] from the one studied here. If both heavy Majorana neutrinos and T-odd particles (both with $\mathcal{O}(\text{TeV})$ masses) contributed commensurately, a general analysis (that we plan to undertake in future work) would be needed.

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A Loop functions

Two-point functions. Considering a diagram with two legs, the general form of the loop integral is [32]

$$\frac{i}{16\pi^2} \{B_0, B^\mu\} = \mu^{4-D} \int \frac{d^D q}{(2\pi)^D} \frac{\{1, q^\mu\}}{(q^2 - m_0^2)[(q+p)^2 - m_1^2]}, \quad (\text{A.1})$$

where m_0 and m_1 are the internal masses. The corresponding tensor coefficients are functions of the invariant quantities $(args) = (p^2, m_0^2, m_1^2)$, where p is the momentum of the particle. The functions $B \equiv B(0; M_1^2, M_2^2)$ and $\bar{B} \equiv B(0; M_2^2, M_1^2)$ read

$$B_0 = \bar{B}_0 = \Delta_\epsilon + 1 - \frac{M_1^2 \ln \frac{M_1^2}{\mu^2} - M_2^2 \ln \frac{M_2^2}{\mu^2}}{M_1^2 - M_2^2}, \quad (\text{A.2})$$

$$\begin{aligned} B_1 &= -\frac{\Delta_\epsilon}{2} + \frac{4M_1^2 M_2^2 - 3M_1^4 - M_2^4 + 2M_1^4 \ln \frac{M_1^2}{\mu^2} + 2M_2^2 (M_2^2 - 2M_1^2) \ln \frac{M_2^2}{\mu^2}}{4(M_1^2 - M_2^2)^2} \\ &= -\bar{B}_0 - \bar{B}_1, \end{aligned} \quad (\text{A.3})$$

with $\Delta_\epsilon \equiv \frac{2}{\epsilon} - \gamma + \ln 4\pi$. These functions are ultraviolet divergent in $D = 4$ dimensions.

Three-point functions. Appendix C of [32] shows the three-point functions that we used. The function's arguments are $(args) = (p_1^2, Q^2, p_2^2; m_0^2, m_1^2, m_2^2)$, with p_1 and p_2 the external momenta, m_0 the mass propagator, M_1 and M_2 the masses of particles within the loop and $Q \equiv p_2 - p_1$. Thus,

$$\frac{i}{16\pi^2} = \{C_0, C^\mu, C^{\mu\nu}\}(args) = \mu^{4-D} \int \frac{d^D q}{(2\pi)^D} \frac{\{1, q^\mu, q^\mu q^\nu\}}{(q^2 - m_0^2)[(q + p_1)^2 - m_1^2][(q + p_2)^2 - m_2^2]}. \quad (\text{A.4})$$

The functions $C \equiv C(0, Q^2, 0; M_1^2, M_2^2, M_2^2)$ with $x \equiv M_2^2/M_1^2$ are given by

$$C_0 = \frac{1}{M_2^2} \left[\frac{1 - x + \ln x}{(1 - x)^2} + \frac{Q^2}{M_1^2} \frac{-2 - 3x + 6x^2 - x^3 - 6x \ln x}{12x(1 - x)^4} \right], \quad (\text{A.5})$$

$$C_1 = C_2 = \frac{1}{M_1^2} \frac{-3 + 4x - x^2 - 2 \ln x}{4(1 - x)^3} + \mathcal{O}(Q^4), \quad (\text{A.6})$$

$$C_{11} = C_{22} = 2C_{12} = \frac{1}{M_1^2} \frac{11 - 18x + 9x^2 - 2x^3 + 6 \ln x}{18(1 - x)^4} + \mathcal{O}(Q^4), \quad (\text{A.7})$$

$$C_{00} = -\frac{1}{2}B_1 - \frac{Q^2}{M_1^2} \frac{11 - 18x + 9x^2 - 2x^3 + 6 \ln x}{72(1 - x)^4} + \mathcal{O}(Q^4). \quad (\text{A.8})$$

Defining $\bar{C} \equiv C(0, Q^2, 0; M_2^2, M_1^2, M_1^2)$,

$$\bar{C}_0 = \frac{1}{M_2^2} \left[\frac{-1 + x - \ln x}{(1 - x)^2} + \frac{Q^2}{M_1^2} \frac{-1 + x - 3x^2 - 2x^3 + 6x^2 \ln x}{12x(1 - x)^4} \right] + \mathcal{O}(Q^4), \quad (\text{A.9})$$

$$\bar{C}_1 = \bar{C}_2 = \frac{1}{M_1^2} \frac{1 - 4x + 3x^2 - 2x^2 \ln x}{4(1 - x)^3}, \quad (\text{A.10})$$

$$\bar{C}_{11} = \bar{C}_{22} = 2\bar{C}_{12} = \frac{1}{M_1^2} \frac{-2 + 9x - 18x^2 + 11x^3 - 6x^3 \ln x}{18(1 - x)^4}, \quad (\text{A.11})$$

$$\bar{C}_{00} = -\frac{1}{2}\bar{B}_1 - \frac{Q^2}{M_1^2} \frac{-2 + 9x - 18x^2 + 11x^3 + 6x^3 \ln x}{72(1 - x)^4} + \mathcal{O}(Q^4). \quad (\text{A.12})$$

It is important to note that functions C_{00} y $\bar{C}_{00}$ are ultraviolet divergent in $D = 4$ dimensions.

In the limit $Q^2 = 0$ the following useful relations among two and three point functions hold

$$\overline{B}_1 + 2\overline{C}_{00} = 0, \quad (\text{A.13})$$

$$-\frac{1}{4} + \frac{1}{2}\overline{B}_1 + C_{00} - \frac{x}{2}M_1^2 C_0 = 0, \quad (\text{A.14})$$

$$-\frac{1}{2} + \overline{B}_1 + 6\overline{C}_{00} - xM_1^2 \overline{C}_0 = \Delta_\epsilon - \ln \frac{M_1^2}{\mu^2}. \quad (\text{A.15})$$

Four-point functions. The functions that we used in our development are all ultraviolet finite

$$\frac{i}{16\pi^2} \{D_0, D^\mu, D^{\mu\nu}\}(args) = \int \frac{d^4 q}{(2\pi)^4} \frac{\{1, q^\mu, q^\mu q^\nu\}}{(q^2 - m_0^2)[(q + k_1)^2 - m_1^2][(q + k_2)^2 - m_2^2][(q + k_3)^2 - m_3^2]}, \quad (\text{A.16})$$

with $k_j = \sum_i^j p_i$ and $(args) = (p_1^2, p_2^2, p_3^2, p_4^2, (p_1 + p_2)^2, (p_2 + p_3)^2; m_0^2, m_1^2, m_2^2, m_3^2)$. In the limit of zero external momenta, only the following integrals are relevant

$$\frac{i}{16\pi^2} D_0 = \int \frac{d^4 q}{(2\pi)^2} \frac{1}{(q^2 - m_0^2)(q^2 - m_1^2)(q^2 - m_2^2)(q^2 - m_3^2)}, \quad (\text{A.17})$$

$$\frac{i}{16\pi^2} D_{00} = \frac{1}{4} \int \frac{d^4 q}{(2\pi)^2} \frac{q^2}{(q^2 - m_0^2)(q^2 - m_1^2)(q^2 - m_2^2)(q^2 - m_3^2)}. \quad (\text{A.18})$$

In terms of the mass ratios $x = m_1^2/m_0^2$, $y = m_2^2/m_0^2$, $z = m_3^2/m_0^2$ the integrals above can be written as [32, 44]

$$d_0(x, y, z) \equiv m_0^4 D_0 = \frac{x \ln x}{(1-x)(x-y)(x-z)} - \frac{y \ln y}{(1-y)(x-y)(y-z)} + \frac{z \ln z}{(1-z)(x-z)(y-z)}, \quad (\text{A.19})$$

$$\tilde{d}_0(x, y, z) \equiv 4m_0^2 D_{00} = \frac{x^2 \ln x}{(1-x)(x-y)(x-z)} - \frac{y^2 \ln y}{(1-y)(x-y)(y-z)} + \frac{z^2 \ln z}{(1-z)(x-z)(y-z)}, \quad (\text{A.20})$$

$$\tilde{d}'_0(x, y, z) = \frac{x^2 \ln x}{(1-x)(x-y)(z-x)} + \frac{y^2 \ln y}{(1-y)(x-y)(z-y)} + \frac{z^2 \ln z}{(1-z)(x-z)(y-z)}, \quad (\text{A.21})$$

with $\tilde{d}(x, y) = \tilde{d}'(x, y, 1)$. For two equal masses ($m_0 = m_3$) we get

$$d_0(x, y) = - \left[\frac{x \ln x}{(1-x)^2(x-y)} - \frac{y \ln y}{(1-y)^2(x-y)} + \frac{1}{(1-x)(1-y)} \right], \quad (\text{A.22})$$

$$\tilde{d}_0(x, y) = - \left[\frac{x^2 \ln x}{(1-x)^2(x-y)} - \frac{y^2 \ln y}{(1-y)^2(x-y)} + \frac{1}{(1-x)(1-y)} \right]. \quad (\text{A.23})$$

Light-heavy four-point functions. The form factors involved in the $\ell \rightarrow \ell' \ell'' \bar{\ell}'''$ decay are given by the eqs. (3.32) and (3.33). Since the masses of light neutrinos satisfy $m_i \ll M_W$, we find convenient to define the y_i variable as $y_i = \frac{m_i^2}{M_W^2}$, so that $y_i \rightarrow 0$. On the other hand, heavy neutrino masses satisfy $M_W \ll M_j$, hence it is natural to define $x_j = \frac{M_W^2}{M_j^2}$, for the x_j variable to fulfill $x_j \rightarrow 0$.

The $f_B^l(y_i, y_j)$ function is formed by the d_0 and $\bar{d}_0$ functions. As just light neutrinos are considered in the f_B function, the d_0 and $\bar{d}_0$ ones have $y_{i,j}$ as variables. Therefore, d_0 and $\bar{d}_0$ functions are given from the eqs. (A.22) and (A.23). Thus, the $f_B^l(y_i, y_j)$ function can be written

$$f_B^l(y_i, y_j) = \left(1 + \frac{1}{4} y_i y_j\right) \bar{d}_0(y_i, y_j) - 2 y_i y_j d_0(y_i, y_j). \quad (\text{A.24})$$

The $f_B^{lh}(y_i, x_j)$ function mixes light and heavy neutrinos, then it has y_i and x_j as variables. The d_0 and $\bar{d}_0$ functions defined in the previous section, have variables which behave as $\frac{m_i^2}{M_W^2}$, while for heavy neutrinos variables we have $x_j = \frac{M_W^2}{M_j^2}$, though. We have to refactor them considering the y_i and x_j variables for light and heavy neutrinos respectively,

$$d_0^{lh}(y_i, x_j) = \frac{y_i x_j \ln y_i}{(1 - y_i)^2 (1 - y_i x_j)} + \frac{x_j^2 \ln x_j}{(1 - x_j)^2 (1 - y_i x_j)} + \frac{x_j}{(1 - y_i)(1 - x_j)}, \quad (\text{A.25})$$

$$\bar{d}_0^{lh}(y_i, x_j) = \frac{y_i^2 x_j \ln y_i}{(1 - y_i)^2 (1 - y_i x_j)} + \frac{x_j \ln x_j}{(1 - x_j)^2 (1 - y_i x_j)} + \frac{x_j}{(1 - y_i)(1 - x_j)}, \quad (\text{A.26})$$

where $y_i = m_i^2/M_W^2$ ($i = 1, 2, 3$) and $x_j = M_W^2/M_j^2$ ($j = 1, 2, 3$). From the equations above the $f_B^{lh}(y_i, x_j)$ function reads

$$f_B^{lh}(y_i, x_j) = \left(1 + \frac{1}{4} \frac{y_i}{x_j}\right) \bar{d}_0^{lh}(y_i, x_j) - 2 \frac{y_i}{x_j} d_0^{lh}(y_i, x_j). \quad (\text{A.27})$$

Finally, the $f_B^h(x_i, x_j)$ function just has heavy neutrino variables $x_{i,j} = M_W^2/M_j^2$, hence, we need to refactor the d_0 and $\bar{d}_0$ functions as

$$d_0^h(x_i, x_j) = - \left[\frac{x_i^2 x_j \ln x_i}{(1 - x_i)^2 (x_i - x_j)} - \frac{x_i x_j^2 \ln x_j}{(1 - x_j)^2 (x_i - x_j)} + \frac{x_i x_j}{(1 - x_i)(1 - x_j)} \right], \quad (\text{A.28})$$

$$\bar{d}_0^h(x_i, x_j) = - \left[\frac{x_i x_j \ln x_i}{(1 - x_i)^2 (x_i - x_j)} - \frac{x_i x_j \ln x_j}{(1 - x_j)^2 (x_i - x_j)} + \frac{x_i x_j}{(1 - x_i)(1 - x_j)} \right], \quad (\text{A.29})$$

with $i, j = 1, 2, 3$. Therefore, the $f_B^h(x_i, x_j)$ function is given by

$$f_B^h(x_i, x_j) = \left(1 + \frac{1}{4} \frac{1}{x_i x_j}\right) \bar{d}_0^h(x_i, x_j) - 2 \frac{1}{x_i x_j} d_0^h(x_i, x_j). \quad (\text{A.30})$$

For the functions with a pair of LNV vertices $f_B^{(l, lh, h)-\text{LNV}}(z_i, z_j)$, we can apply the same arguments as the previous $f_B^{l, lh, h}(z_i, z_j)$. Therefore

$$\begin{aligned} f_B^{l-\text{LNV}}(y_i, y_j) &= \sqrt{y_i y_j} \left(2\bar{d}_0(y_i, y_j) - (4 + y_i y_j) d_0(y_i, y_j) \right), \\ f_B^{lh-\text{LNV}}(y_i, x_j) &= \sqrt{\frac{y_i}{x_j}} \left(2\bar{d}_0^{lh}(y_i, x_j) - \left(4 + \frac{y_i}{x_j} \right) d_0^{lh}(y_i, x_j) \right), \\ f_B^{h-\text{LNV}}(x_i, x_j) &= \frac{1}{\sqrt{x_i x_j}} \left(2\bar{d}_0^h(x_i, x_j) - \left(4 + \frac{1}{x_i x_j} \right) d_0^h(x_i, x_j) \right). \end{aligned} \quad (\text{A.31})$$

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