Erratum: $\mathcal{N} = 4$ supersymmetric Yang-Mills thermodynamics to order λ^2

Qianqian Du,^{a,b} Michael Strickland^b and Ubaid Tantary^b

^a*Institute of Particle Physics and Key Laboratory of Quark and Lepton Physics (MOS),
Central China Normal University,
Wuhan, 430079, China*

^b*Department of Physics, Kent State University,
Kent, OH 44242, U.S.A.*

E-mail: duqianqianstudent@mails.ccnu.edu.cn, mstrick6@kent.edu,
utantary@kent.edu

ERRATUM TO: [JHEP08\(2021\)064](#)

ABSTRACT: In this erratum we correct an error made in Q. Du, M. Strickland and U. Tantary, [JHEP08\(2021\)064](#). The end result is that one of the terms contributing at $\mathcal{O}(\lambda^2)$ changes from $-45/16$ to $-21/8$. This represents a small numerical correction to the originally reported result and none of our conclusions are changed. Due to this correction we also update the generalized Padé approximant that interpolates between the weak- and strong-coupling limits and the final figure comparing various approximations to the scaled entropy density.

ARXIV EPRINT: [2105.02101](#)

1 Corrections

In this erratum we correct an error made in ref. [1]. The expression for $\mathcal{F}_{3\text{-loop}}^{\text{bct}}$ contained in eq. (4.63) of the published version should read instead

$$\mathcal{F}_{3\text{-loop}}^{\text{bct}} = d_A \lambda^2 (d-2) [(D+4)b_1 - 8f_1] \left[\sum_P' \frac{\Pi^b(P)}{P^2} - 2 \sum_P' \frac{\Pi^f(P)}{P^2} - \frac{1}{8} (D+4) \frac{T^2}{(4\pi)^2} \right], \quad (1.1)$$

where the change is in the prefactor $D-2 \rightarrow d-2$. Relatedly, eqs. (4.59)–(4.61) are better written as

$$\Delta \Pi_{\mu\nu}(P) \equiv \Pi_{\mu\nu}(P) - m_D^2 \delta_{\mu 0} \delta_{\nu 0} \delta_{P_0}, \quad (1.2)$$

$$\Delta \mathcal{P}(P) \equiv \mathcal{P}(P) + M^2 \delta_{P_0}. \quad (1.3)$$

and

$$-\frac{1}{2} \not\sum_P \frac{1}{P^4} m_D^2 \Pi_{00}(P) \delta_{P_0}. \quad (1.4)$$

As a consequence of the correction to eq. (4.63), eq. (4.67) in the published version should read instead

$$\begin{aligned} F_{3\text{-loop}}^{\text{resum}} &= \mathcal{F}_{3\text{-loop}}^{\text{vacuum}} + \mathcal{F}_{3\text{-loop}}^{\text{sct}} + \mathcal{F}_{3\text{-loop}}^{\text{bct}} \\ &= -d_A \left(\frac{\pi^2 T^4}{6} \right) \frac{\lambda^2}{2\pi^4} \left[\frac{27}{8} + 3\gamma + 3 \frac{\zeta'(-1)}{\zeta(-1)} + 5 \log 2 - 6 \log \pi \right]. \end{aligned} \quad (1.5)$$

Taking this into account, our updated result for the $\text{SYM}_{4,4}$ free-energy through $\mathcal{O}(\lambda^2)$ which was eq. (5.1) in the published version, becomes

$$\begin{aligned} \mathcal{F} &= -d_A \left(\frac{\pi^2 T^4}{6} \right) \left\{ 1 - \frac{3}{2} \frac{\lambda}{\pi^2} + \left(3 + \sqrt{2} \right) \left(\frac{\lambda}{\pi^2} \right)^{3/2} \right. \\ &\quad \left. + \left[-\frac{21}{8} - \frac{9\sqrt{2}}{8} + \frac{3}{2} \gamma_E + \frac{3}{2} \frac{\zeta'(-1)}{\zeta(-1)} - \frac{25}{8} \log 2 + \frac{3}{2} \log \frac{\lambda}{\pi^2} \right] \left(\frac{\lambda}{\pi^2} \right)^2 \right\}. \end{aligned} \quad (1.6)$$

We have verified this result through an independent calculation of the soft contributions to the free energy using effective field theory methods [2].

As a result of this change, the coefficient b in the generalized Padé approximant presented in eq. (G2) of appendix G of the published version is corrected to

$$b = \frac{1}{\pi^2} \log \left(\frac{\lambda}{\pi^2} \right) + \frac{16\pi \left(45 \left(3 + \sqrt{2} \right) \zeta(3) + \pi^3 \right)}{18225 \zeta^2(3)} + \frac{36 \left[\frac{\zeta'(-1)}{\zeta(-1)} + \gamma_E \right] + 69\sqrt{2} + 59 - 75 \log 2}{36\pi^2}. \quad (1.7)$$

Finally, in figure 1 we present an updated plot comparing the various approximations. We note that the corrections are numerically small and, as a consequence, our conclusions are unchanged.

Acknowledgments

We thank Jens O. Andersen for calling this issue to our attention.

Figure 1. $\text{SYM}_{4,4}$ scaled entropy density $\mathcal{S}/\mathcal{S}_{\text{ideal}}$ as a function of λ . The green dotted, red dashed, and blue long-dashed curves correspond to the perturbative result truncated at $\mathcal{O}(\lambda)$, $\mathcal{O}(\lambda^{3/2})$, and $\mathcal{O}(\lambda^2)$, respectively. The purple dot-dashed curve corresponds to the large- N_c strong-coupling result truncated at $\mathcal{O}(\lambda^{-3/2})$. The solid gray line is a generalized Padé that interpolates between the known weak- and strong-coupling results.

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY 4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] Q. Du, M. Strickland and U. Tantary, $\mathcal{N} = 4$ supersymmetric Yang-Mills thermodynamics to order λ^2 , *JHEP* **08** (2021) 064 [[arXiv:2105.02101](https://arxiv.org/abs/2105.02101)] [[INSPIRE](#)].
- [2] J.O. Andersen, Q. Du, M. Strickland and U. Tantary, $N = 4$ supersymmetric Yang-Mills thermodynamics from effective field theory, *Phys. Rev. D* **105** (2022) 015006 [[arXiv:2111.12160](https://arxiv.org/abs/2111.12160)] [[INSPIRE](#)].