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Erratum: Anomaly free Froggatt-Nielsen models of flavor

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ABSTRACT: We provide the constraints from rare meson decays, which were neglected in the original manuscript. These constraints give the most stringent bounds for light Z' , but can be neglected for heavy Z' .

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1 Corrections to section 5.1

The paragraph *Rare meson decays (RMD)* in the original manuscript [1] should be replaced with the text below (figures 1 and 2 replace figures 13 and 14 in [1]):

Rare meson decays (RMD). The tree level Z' exchanges induce rare FCNC meson decays such as $M_1 \rightarrow M_2 \ell_i^+ \ell_j^-$, where $M_1(M_2)$ is the initial (final) state meson. An example of such an exotic decay is $K^+ \rightarrow \pi^+ \mu^+ e^-$ which arises at one loop in the SM but with a tremendous suppression, with a matrix element proportional to the neutrino masses, and are thus vanishingly small in practice. The tree level transition matrix element due to a Z' exchange instead has no suppression due to neutrino masses. For heavy Z' the off-shell contribution is proportional to a product of two off-diagonal couplings, one for the quark flavor changing current and one for the lepton current. The resulting bounds are therefore expected to be weaker than the bounds that follow from the lepton FCNC transitions or from meson mixings. In contrast, when the Z' is light enough to be produced on-shell, the $M_1 \rightarrow M_2 Z'$ decays depend only on quark current off-diagonal couplings. In this case the bounds from rare meson decays in general become the most stringent ones.

For the $M_1 \rightarrow M_2$ transitions the strongest experimental bounds follow from the $M_1 \rightarrow M_2 Z'$ decays with $M_{1,2}$ both pseudoscalars, for which the matrix element is

$$\langle M_2 | J_\mu | M_1 \rangle = g' c_{qV}^{kl} [f_+(q^2)(p_1 + p_2)_\mu + f_-(q^2)q_\mu], \quad (1.1)$$

with J^μ defined in (5.10) of ref. [1], and $p_{1,2}$ the momenta of mesons $M_{1,2}$, such that $p_{1,2}^2 = m_{1,2}^2$, with $m_{1,2}$ the corresponding meson masses. The quark flavor-violating coupling c_{qV}^{kl} encodes the strength of the $q_l \rightarrow q_k$ transition, e.g., the c_{dV}^{23} coupling is obtained for the $B \rightarrow K$ and c_{dV}^{13} for the $B \rightarrow \pi$ matrix elements, respectively. The form factors $f_{+,-}(q^2)$ depend on the momentum exchange squared, q^2 , with $q_\mu = (p_1 - p_2)_\mu$. We follow the common practice and trade the form factor $f_-(q^2)$ for

$$f_0(q^2) = f_+(q^2) + f_-(q^2) \frac{q^2}{m_1^2 - m_2^2}. \quad (1.2)$$

We use the z -expansion based parametrization of the form factors [2],

$$f_i(q^2) = \frac{1}{1 - q^2/m_{\text{pole}}^2} \sum_n^{N-1} a_{i,n} \left[z^n(q^2) - (-1)^{n-N} \frac{n}{N} z^N(q^2) \right], \quad i = +, 0, \quad (1.3)$$

where

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad (1.4)$$

with $t_+ = (m_1 + m_2)^2$, $t_0 = (m_1 + m_2)(\sqrt{m_1} - \sqrt{m_2})^2$.

For light Z' , with a mass below the lepton pair production threshold, $m_{Z'} \leq m_{\ell_i} + m_{\ell_j}$, the $M_1 \rightarrow M_2 \ell_i^- \ell_j^+$ transition is a genuine 3-body decay, mediated by an off-shell Z' . The longitudinal Z' contribution dominates, leading to a decay width proportional to $\propto g'^4/m_{Z'}^4 \simeq 1/(4\langle\phi\rangle^4)$. The expression for the differential width is well approximated by taking the $m_{Z'} \rightarrow 0$ limit, giving

$$\begin{aligned} \frac{d\Gamma(M_1 \rightarrow M_2 \ell_i^- \ell_j^+)}{dm_{M_1 \ell_i}^2 dm_{\ell_i \ell_j}^2} &\simeq \frac{g'^4 |c_{qV}^{kl}|^2 f_0^2(m_{\ell_i \ell_j}^2)}{128\pi^3 m_{\ell_i \ell_j}^4 m_1^3 m_{Z'}^4} \left[|c_{\ell V}^{ij}|^2 (m_{\ell_i} - m_{\ell_j})^2 (m_{\ell_i \ell_j}^2 - (m_{\ell_i} + m_{\ell_j})^2) \right. \\ &\quad \left. + |c_{\ell A}^{ij}|^2 (m_{\ell_i} + m_{\ell_j})^2 (m_{\ell_i \ell_j}^2 - (m_{\ell_i} - m_{\ell_j})^2) \right] (m_1^2 - m_2^2)^2, \end{aligned} \quad (1.5)$$

where $m_{\ell_i \ell_j}^2 = (p_{\ell_i} + p_{\ell_j})^2$ is the invariant mass squared of the leptonic pair, and similarly $m_{M_1 \ell_i}^2 = (p_1 + p_{\ell_i})^2$.

For heavy Z' , with a mass $m_{Z'} \gg m_1$, the Z' can be integrated out. This also gives a decay width proportional to $\propto g'^4/m_{Z'}^4 \simeq 1/(4\langle\phi\rangle^4)$. In the limit of massless final states, $m_2, m_{\ell_i}, m_{\ell_j} \rightarrow 0$, the differential rate is given by a rather compact expression,

$$\frac{d\Gamma(M_1 \rightarrow M_2 \ell_i^- \ell_j^+)}{dm_{M_1 \ell_i}^2 dm_{\ell_i \ell_j}^2} \simeq \frac{g'^4 |c_{qV}^{kl}|^2 (|c_{\ell V}^{ij}|^2 + |c_{\ell A}^{ij}|^2) f_+^2(m_{\ell_i \ell_j}^2)}{32\pi^3 m_1^3 m_{Z'}^4} m_{M_2 \ell_i}^2 (m_1^2 - m_{M_2 \ell_i}^2 - m_{\ell_i \ell_j}^2). \quad (1.6)$$

In the numerical analysis we work with the full dependence on $m_2, m_{\ell_i}, m_{\ell_j}$.

Phenomenologically, the most relevant is the intermediate mass regime, $m_{\ell_i} + m_{\ell_j} < m'_{Z'} < m_1 - m_2$. In this case the Z' is produced on-shell in the 2-body decay, $M_1 \rightarrow M_2 Z'$, followed by a decay into two leptons, $Z' \rightarrow \ell_i \bar{\ell}_j$. The total decay width is given by

$$\Gamma(M_1 \rightarrow M_2 \ell_i^+ \ell_j^-) = \Gamma(M_1 \rightarrow M_2 Z') \times \text{Br}(Z' \rightarrow \ell_i^+ \ell_j^-). \quad (1.7)$$

The two-body decay width for $M_1 \rightarrow M_2 Z'$ transition depends only on the quark current matrix element, and is given by

$$\Gamma(M_1 \rightarrow M_2 Z') = g'^2 |c_{qV}^{kl}|^2 f_+(m_{Z'}^2)^2 \frac{m_1}{16\pi} \frac{(y_2^4 + (y_{Z'}^2 - 1)^2 - 2y_2^2(1 + y_{Z'}^2))^{3/2}}{y_{Z'}^2}, \quad (1.8)$$

where $y_i = m_i/m_1$. The branching fractions for $Z' \rightarrow \ell_i \bar{\ell}_j$ are easily calculated. Ignoring the kinematical factors for final state particles they are given by $\text{Br}(Z' \rightarrow \ell_i \bar{\ell}_j) = (|c_{\ell V}^{ij}|^2 + |c_{\ell A}^{ij}|^2)/(\sum_{ff'} |c_{\ell V}^{ff'}|^2 + |c_{\ell A}^{ff'}|^2)$. For our benchmark we have $\text{Br}(Z' \rightarrow ee, \mu\mu) \simeq 0.1$ and $\text{Br}(Z' \rightarrow \sum_{\alpha} \nu_{\alpha} \nu_{\alpha}) \simeq 0.4$ for the flavor conserving channels, and $\text{Br}(Z' \rightarrow e\mu) \sim 10^{-5}$ for the flavor violating decay.

In the numerical analysis we use the PDG values for the measurements of the branching ratios and the upper bounds [3], while the numerical inputs for the form factors are taken from ref. [4] (that collects the original determinations from refs. [5–13]). The envelope

Figure 1. The strongest bounds on the Z' mass as a function of gauge coupling g' of the horizontal $G_{\text{FN}} = \text{U}(1)_{\text{FN}}$ group, setting kinetic mixing to zero. *Region I (II)* collects bounds due to the direct production of Z' in proton and electron beam dumps (at e^+e^- colliders), while *Region III* shows the exclusion from SN1987A. The solid (dashed) dark green line depicts the present (future) $\mu \rightarrow e$ bound, the solid black line the bound from $K - \bar{K}$ mixing, the solid blue line the limits from rare meson decays (RMD), see text for details.

of constraints on g' as a function of Z' mass due to rare meson decays (RMD) is shown as a blue solid line in figures 1 and 2. We see that the RMD constraints are the most stringent ones up to several GeV Z' masses, i.e., as long as the $B \rightarrow \pi Z'$ decays are still kinematically allowed.

The kinematical thresholds from K, D, B decays are clearly visible in figures 1 and 2. At the lowest Z' masses the $K^+ \rightarrow \pi^+ Z'$, $Z' \rightarrow \nu \bar{\nu}$ decay leads to the strongest constraints, a result of $K \rightarrow \pi + \text{inv}$ being well constrained experimentally, as well as the large $Z' \rightarrow \nu \bar{\nu}$ branching ratio. Conservatively, we use the integrated branching ratio $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) < 2.3 \times 10^{-10}$ to set the limit on g' in figures 1 and 2. A more careful analysis of missing mass differential rates, taking care of spill-overs between different bins, would further improve these bounds.

The bounds on $K^+ \rightarrow \pi^+ \mu^+ e^-$ decays, $\text{Br}(K^+ \rightarrow \pi^+ \mu^+ e^-) < 1.3 \times 10^{-11}$, $\text{Br}(K^+ \rightarrow \pi^+ \mu^- e^+) < 6.6 \times 10^{-11}$ lead to less stringent constraints on g' due to small $\text{Br}(Z' \rightarrow e \mu)$. The related decay, $K_L \rightarrow \mu e$, with the decay width given by

$$\Gamma(K_L \rightarrow e^- \mu^+) = |\text{Re}(g_{sd}^A)|^2 (|g_{e\mu}^V|^2 + |g_{e\mu}^A|^2) f_K^2 \frac{m_\mu^2 (m_K^2 - m_\mu^2)^2}{8\pi m_{Z'}^4 m_K^3}, \quad (1.9)$$

where $f_K = 155 \text{ MeV}$ is the kaon decay constant, is also always less constraining. When $K \rightarrow \pi Z'$ decays are kinematically forbidden the decays of heavier mesons become relevant. In figures 1 and 2. we show the limits that follow from the bounds on the decays, $\text{Br}(D^+ \rightarrow \pi^+ \mu \mu) < 7.3 \times 10^{-8}$, $\text{Br}(B^+ \rightarrow \pi^+ \mu \mu) < 6.9 \times 10^{-8}$, and $\text{Br}(B^+ \rightarrow K^+ \mu \mu) < 4.8 \times 10^{-7}$.

Figure 2. The top panels and bottom left (right) panel show bounds on the Z' mass and the g' gauge coupling constant for the $G_{\text{FN}} = \text{U}(1)'_{\text{FN}}$ model benchmark in the limit of no couplings to quarks (leptons). Direct Z' production searches exclude orange and red regions, cf. figure 1. *Top left:* bounds involving electrons, from the electron EDM ($\bar{\nu}_e - e$ scattering, $(g-2)_e$, white dwarf cooling), shown as black line (green line, blue line, purple excluded region). *Top right:* bounds involving muons, from $\mu \rightarrow e$ conversion, ($\mu \rightarrow 3e$, $\mu \rightarrow e\gamma$, neutrino trident production, $\nu_\mu - e$ scattering), shown as green (purple, blue, black, cyan) lines, while the gray region is consistent with $(g-2)_\mu$. *Bottom left:* bounds involving taus, from $\tau \rightarrow 3\mu$ ($\tau \rightarrow 3e$, $\tau \rightarrow \mu\gamma$, $\tau \rightarrow e\gamma$) shown with blue (purple, brown, black) lines. *Bottom right:* bounds involving quarks only, from $K - \bar{K}$ mixing ($D - \bar{D}$, $B_s - \bar{B}_s$, $B_d - \bar{B}_d$ mixing, neutron EDM, APV, RMD), shown as black (red, green, brown, blue, purple, light blue) lines, while the gray region is excluded by SN1987A. Solid (dashed) lines give present (future) bounds, see text for details. Note that the RMD bounds involve both lepton and quark couplings.

2 Corrections to section 5.2

Figure 3 replaces figure 18 in [1]. The sentence “figure 18 also gives the FCNC constraints from $D - \bar{D}$, $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing (red, brown, blue lines).” is replaced by: “figure 3 also gives the FCNC constraints from rare meson decays, $D - \bar{D}$, $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing (blue, red, brown, cyan lines).”

3 Corrections to section 6

The last sentence of the next to last paragraph in section 6 of [1] should read:

For light Z' , with masses from several 10s MeV to several 100s MeV, the rare meson decays and beam dump searches are the most constraining, see figures 1, 2 and 3 for the bounds on two representative benchmarks.

Figure 3. The strongest bounds on the parameter space for the Z'_1 (top left), Z'_2 (top right), and Z'_3 (bottom) gauge bosons in the $U(1)^3_{\text{FN}}$ model. The color coding is the same as in figure 1.

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References

- [1] A. Smolkovič, M. Tamararo and J. Zupan, *Anomaly free Froggatt-Nielsen models of flavor*, *JHEP* **10** (2019) 188 [[arXiv:1907.10063](https://arxiv.org/abs/1907.10063)] [[INSPIRE](#)].
- [2] C. Bourrely, I. Caprini and L. Lellouch, *Model-independent description of $B \rightarrow \pi l \nu$ decays and a determination of $|V_{ub}|$* , *Phys. Rev. D* **79** (2009) 013008 [Erratum *ibid.* **82** (2010) 099902] [[arXiv:0807.2722](https://arxiv.org/abs/0807.2722)] [[INSPIRE](#)].
- [3] PARTICLE DATA GROUP collaboration, *Review of Particle Physics*, *PTEP* **2020** (2020) 083C01 [[INSPIRE](#)].
- [4] Y. Aoki et al., *FLAG Review 2021*, [arXiv:2111.09849](https://arxiv.org/abs/2111.09849) [[INSPIRE](#)].
- [5] H. Na, C.T.H. Davies, E. Follana, G.P. Lepage and J. Shigemitsu, *The $D \rightarrow K, l \nu$ Semileptonic Decay Scalar Form Factor and $|V_{cs}|$ from Lattice QCD*, *Phys. Rev. D* **82** (2010) 114506 [[arXiv:1008.4562](https://arxiv.org/abs/1008.4562)] [[INSPIRE](#)].
- [6] H. Na, C.T.H. Davies, E. Follana, J. Koponen, G.P. Lepage and J. Shigemitsu, *$D \rightarrow \pi, l \nu$ Semileptonic Decays, $|V_{cd}|$ and 2^{nd} Row Unitarity from Lattice QCD*, *Phys. Rev. D* **84** (2011) 114505 [[arXiv:1109.1501](https://arxiv.org/abs/1109.1501)] [[INSPIRE](#)].

- [7] ETM collaboration, *Scalar and vector form factors of $D \rightarrow \pi(K)\ell\nu$ decays with $N_f = 2 + 1 + 1$ twisted fermions*, *Phys. Rev. D* **96** (2017) 054514 [Erratum *ibid.* **99** (2019) 099902] [*Erratum ibid.* **100** (2019) 079901] [[arXiv:1706.03017](#)] [[INSPIRE](#)].
- [8] HPQCD and HPQCD collaborations, *Improved V_{cs} determination using precise lattice QCD form factors for $D \rightarrow K\ell\nu$* , *Phys. Rev. D* **104** (2021) 034505 [[arXiv:2104.09883](#)] [[INSPIRE](#)].
- [9] E. Dalgic, A. Gray, M. Wingate, C.T.H. Davies, G.P. Lepage and J. Shigemitsu, *B meson semileptonic form-factors from unquenched lattice QCD*, *Phys. Rev. D* **73** (2006) 074502 [*Erratum ibid.* **75** (2007) 119906] [[hep-lat/0601021](#)] [[INSPIRE](#)].
- [10] FERMILAB LATTICE and MILC collaborations, *$|V_{ub}|$ from $B \rightarrow \pi\ell\nu$ decays and $(2+1)$ -flavor lattice QCD*, *Phys. Rev. D* **92** (2015) 014024 [[arXiv:1503.07839](#)] [[INSPIRE](#)].
- [11] J.M. Flynn et al., *$B \rightarrow \pi\ell\nu$ and $B_s \rightarrow K\ell\nu$ form factors and $|V_{ub}|$ from $2+1$ -flavor lattice QCD with domain-wall light quarks and relativistic heavy quarks*, *Phys. Rev. D* **91** (2015) 074510 [[arXiv:1501.05373](#)] [[INSPIRE](#)].
- [12] HPQCD collaboration, *Rare decay $B \rightarrow K\ell^+\ell^-$ form factors from lattice QCD*, *Phys. Rev. D* **88** (2013) 054509 [*Erratum ibid.* **88** (2013) 079901] [[arXiv:1306.2384](#)] [[INSPIRE](#)].
- [13] J.A. Bailey et al., *$B \rightarrow Kl^+l^-$ Decay Form Factors from Three-Flavor Lattice QCD*, *Phys. Rev. D* **93** (2016) 025026 [[arXiv:1509.06235](#)] [[INSPIRE](#)].