

SUSY breaking constraints on modular flavor S_3 invariant SU(5) GUT model

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ABSTRACT: Modular flavor symmetry can be used to explain the quark and lepton flavor structures. The SUSY partners of quarks and leptons, which share the same superpotential with the quarks and leptons, will also be constrained by the modular flavor structure and show a different flavor(mixing) pattern at the GUT scale. So, in realistic modular flavor models with SUSY completion, constraints from the collider and DM constraints can also be used to constrain the possible values of the modulus parameter. In the first part of this work, we discuss the possibility that the S_3 modular symmetry can be preserved by the fixed points of T^2/Z_N orbifold, especially from T^2/Z_2 . To illustrate the additional constraints from collider etc on modular flavor symmetry models, we take the simplest UV SUSY-completion S_3 modular invariance SU(5) GUT model as an example with generalized gravity mediation SUSY breaking mechanism. We find that such constraints can indeed be useful to rule out a large portion of the modulus parameters. Our numerical results show that the UV-completed model can account for both the SM (plus neutrino) flavor structure and the collider, DM constraints. Such discussions can also be applied straightforwardly to other modular flavor symmetry models, such as A_4 or S_4 models.

KEYWORDS: Beyond Standard Model, Supersymmetry Breaking, GUT, Quark Masses and SM Parameters

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1 Introduction

The discovery of the Higgs boson by the ATLAS and CMS collaborations of LHC fills the last missing piece of the standard model(SM). Although SM is very successful in describing the interactions up to the electroweak scale, the origin of the mass hierarchies and mixing patterns among quarks and lepton is still unexplained. One of the most interesting solutions to such difficulties is the non-abelian discrete flavor symmetry group approach, which can lead to fairly predictive models. This approach, however, has several drawbacks. The flavor symmetry group is usually broken down to different subgroups by the vacuum expectation values (VEVs) of certain flavon fields. So, additional flavor symmetry breaking sector are generally needed to obtain the desired vacuum alignment, complicated the model buildings. Moreover, large corrections from higher dimensional operators to the leading order predictions of discrete flavor symmetry models can spoil their predictability. Regarding the previous drawbacks, alternative approaches to explain the flavor structures are always welcome.

A new direction to flavor symmetry, the modular flavor symmetry, was proposed recently [1, 2] to solve the flavor problem of SM. The discrete groups S_{3,A_4} , S_4 and A_5 are isomorphic to the finite modular groups Γ_N for $N = 2, 3, 4, 5$, respectively [3]. The Yukawa couplings are modular forms that can transform non-trivially under the finite modular group Γ_N . In ref. [2], the observed mixing and mass patterns in the neutrino sector can be explained by a flavon-free supersymmetric modular flavor symmetry model, whose superpotential can be completely determined by modular invariance. Modular form Yukawa structures transform under the finite modular groups S_3 [4, 5], A_4 [2, 6–8], S_4 [9, 10] and A_5 [11, 12] have been considered in the literatures.

Unfortunately, most of the economic realizations concentrate on the lepton sector and do not explain the quark flavor structure. The GUT framework, which can fit the leptons and quarks in certain GUT group multiplets, are known to be very predictive. It

is therefore very interesting to survey if the modular flavor symmetry can be adopted in the GUT framework. Several works had been proposed to combine GUT with modular flavor symmetry with additional flavon fields. The interesting work of [13], which is based on SU(5) GUT and adopt the minimal non-abelian discrete symmetry $\Gamma_2 \simeq S_3$ for quark and lepton flavors, can reproduce the Cabibbo-Kobayashi-Maskawa (CKM) and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrices successfully as well as predict the leptonic CP violation phase and the effective mass of the neutrinoless double beta decay with a common modulus parameter. Although reintroducing the flavon fields seems to lost the minimality, such models can still be very predictive.

In previous modular flavor symmetry models, supersymmetry are always adopted. The non-renormalization theorem of SUSY, which states that no new term in the superpotential will be generated in perturbation theory radiatively, can preserve the constraints of modular flavor symmetry on the superpotential in the SUSY limit. Besides, the gauge couplings unification, which can not be exact in SM, can be successfully realized in its SUSY extensions. The SUSY partners of quarks and leptons, which share the same superpotential with the quarks and leptons, can also be constrained by the modular flavor structure and show a different flavor(mixing) pattern at the GUT scale, unlike the ordinary “SUSY breaking universality” inputs at the GUT scale adopted to evade the flavor-changing and CP-violating constraints in the MSSM. However, supersymmetry must be broken in realistic models. Predictions of previous mentioned modular flavor symmetry models are valid only if the SUSY breaking corrections are negligible [2]. So, we need to survey the SUSY breaking effects in modular flavor symmetry models. As the soft SUSY breaking parameters are determined by SUSY breaking mechanism, it is therefore important to combine the SUSY breaking mechanism with modular flavor symmetry models and survey their low energy consequences. Depending on the way the visible sector ‘*feels*’ the SUSY breaking effects in the hidden sector, the SUSY breaking mechanisms can be classified into gravity mediation [14–25], gauge mediation [26–32], anomaly mediation [33, 34] scenarios, etc. Soft SUSY breaking parameters from economical generalized gravity mediation with non-renormalizable Kahler potential and superpotential involving high-representation Higgs fields of various GUT group were discussed in [35]. To illustrate the important constraints of SUSY breaking effects on modular GUT models, we UV-complete the modular S_3 SU(5) GUT model [13] as an example with the most economical generalized gravity mediation mechanism, with which the **24** representation Higgs of SU(5) fields in non-renormalizable superpotential trigger both GUT and SUSY breaking.

This paper is organized as follows. In section 2, we discuss the possibility to preserve S_3 modular flavor symmetry by the fixed points of T^2/Z_2 . In section 3, we UV-complete the modular S_3 SU(5) GUT model with the most economical generalized SUGRA models and obtain the resulting soft SUSY breaking parameters. In section 4, we give our numerical results to show the additional constraints on modular flavor symmetry from SUSY breaking. Section 5 contains our conclusions.

2 S_3 modular flavor group preserved by the fixed points of T^2/Z_N orbifold

Discrete family symmetry might come from extra dimensions. It can arise as an accidental symmetry of the orbifold fixed points or be broken by boundary conditions, see [36–38]. Specific values for the modulus could also be obtained in extra dimensions through orbifolding [39]. We will show that S_3 modular group can be preserved by some of the fixed points from T^2/Z_2 orbifolding.

We consider the $M_4 \times T^2/Z_2$ orbifold. The extra dimensional coordinates x_5, x_6 are combined to be $z = x_5 + ix_6$. The torus T^2 is defined by the complex plane \mathbb{C} modulo out the lattice

$$z \sim z + \omega_1, \quad z \sim z + \omega_2, \tag{2.1}$$

with ω_1, ω_2 the lattice vectors. Consistency of the discrete symmetry on the orbifold T^2/Z_N constrains $N = 2, 3, 4, 6$ [40–43].

The inhomogeneous modular group $\bar{\Gamma}$ can be generated by

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}. \tag{2.2}$$

which satisfy

$$S^2 = \mathbb{1}, \quad (ST)^3 = \mathbb{1}. \tag{2.3}$$

The action of S, T on the basis vectors are given as

$$S \begin{pmatrix} \omega_2 \\ \omega_1 \end{pmatrix} = \begin{pmatrix} \omega_1 \\ -\omega_2 \end{pmatrix}, \quad T \begin{pmatrix} \omega_2 \\ \omega_1 \end{pmatrix} = \begin{pmatrix} \omega_1 + \omega_2 \\ \omega_1 \end{pmatrix}. \tag{2.4}$$

If the action of S, T on the orbifold fixed points permute the fixed points, the fixed points can form a finite representation of typical subgroup of the modular group. We should check the symmetry preserved by the orbifold fixed point. The presentation of the finite modular group $\Gamma_2 \simeq \bar{\Gamma}/\bar{\Gamma}(2)$ is given as

$$\Gamma_2 \simeq \{S, T | S^2 = T^2 = (ST)^3 = 1\}. \tag{2.5}$$

The generator in (2.2) can also be used here. The requirement $T^2 = 1$ can be imposed as an constraints on the fixed points.

For T^2/Z_2 orbifold with basis vector ω_1, ω_2 , the fix points can be found to be

$$x_1 = 0, \quad x_2 = \frac{\omega_1}{2}, \quad x_3 = \frac{\omega_2}{2}, \quad x_4 = \frac{\omega_1 + \omega_2}{2}. \tag{2.6}$$

Orbifold consistency conditions $z \sim -z$ for Z_2 would not give any constraint on the values of ω_2/ω_1 . The action of S changes the fixed points as

$$x'_1 = 0, \quad x'_2 = -\frac{\omega_2}{2} \sim \frac{\omega_2}{2}, \quad x'_3 = \frac{\omega_1}{2}, \quad x'_4 = \frac{\omega_1 - \omega_2}{2} \sim \frac{\omega_1 + \omega_2}{2}. \tag{2.7}$$

Such an action amounts to the permutation of the fix points

$$(2\ 3) : (x_1, x_2, x_3, x_4) \mapsto (x_1, x_3, x_2, x_4) \tag{2.8}$$

by the (2 3) element of permutation group. The action of T changes the fixed points as

$$x_1'' = 0, \quad x_2'' = \frac{\omega_1}{2}, \quad x_3'' = \frac{\omega_1 + \omega_2}{2}, \quad x_4'' = \frac{2\omega_1 + \omega_2}{2} \sim \frac{\omega_2}{2}. \tag{2.9}$$

Such an action amounts to the permutation of the fix points

$$(x_1, x_2, x_3, x_4) \mapsto (x_1, x_2, x_4, x_3) \tag{2.10}$$

by the (3 4) element of permutation group. It is obvious that the action of T is non-trivial and it satisfies $T^2 = 1$. In fact, it can be checked that under T^2

$$(T \circ T) \equiv T \cdot T \begin{pmatrix} \omega_2 \\ \omega_1 \end{pmatrix} = \begin{pmatrix} 2\omega_1 + \omega_2 \\ \omega_1 \end{pmatrix}, \tag{2.11}$$

the fixed points are indeed unchanged

$$x_0''' = 0, \quad x_1''' = \frac{\omega_1}{2}, \quad x_2''' = \frac{\omega_2}{2} + \omega_1 \sim \frac{\omega_2}{2}, \quad x_3''' = \frac{\omega_1 + \omega_2}{2} + \omega_1 \sim \frac{\omega_1 + \omega_2}{2}. \tag{2.12}$$

We also have

$$(S \circ T) \equiv T \cdot S : (x_1, x_2, x_3, x_4) \mapsto (x_1, x_3, x_4, x_2), \tag{2.13}$$

which amounts to the (2 3 4) element of permutation group. We have the following relations for the fixed points

$$S \circ S = 1, \quad T \circ T = 1, \quad (S \circ T)^3 = 1, \tag{2.14}$$

which is just the $S_3 \simeq \Gamma_2$ permutation group. So, it can be seen that the fixed points of T^2/Z_2 can keep the S_3 permutation group.

For T^2/Z_3 orbifold under the action of Z_3

$$z \sim e^{i\frac{2\pi m}{3}} z, \tag{2.15}$$

for $m = 0, 1, 2$, the orbifold consistency conditions will fix $\omega_2/\omega_1 = e^{i\pi/3}, e^{i2\pi/3}, \sqrt{3}e^{i5\pi/6}$. There are three fix points under the action of Z_3 . Obviously, the residual symmetry preserved by the fixed points is at most Z_2 because the fix point $z = 0$ will not change under the action of generators on the basis vectors. So we can arrive at the conclusion that the finite modular group $S_3 \simeq \bar{\Gamma}_2$ group cannot survive for fixed points set of T^2/Z_3 .

For T^2/Z_4 orbifold, which fixed $\omega_2/\omega_1 = e^{i\pi/2}$ by orbifold consistency conditions, can have the same type of fixed points as T^2/Z_2 case because all fixed points of the T^2/Z_4 orbifold can only appear in the second twisted sector. Similar discussions can be given and the fixed points can be compatible with the $\bar{\Gamma}_2 \simeq S_3$ symmetry for $\omega_2/\omega_1 = e^{i\pi/2}$.

There are in total six fixed points for T^2/Z_6 orbifold. Fixed points from the second twisted sector are the same as that in T^2/Z_3 while the fixed points from the third twisted

sector are the same as that in T^2/Z_2 . It can be shown that the residual symmetry for the fixed points can at most be $S_3 \times Z_2$ for T^2/Z_6 orbifold with $\omega_2/\omega_1 = e^{i\pi/3}, e^{i2\pi/3}, \sqrt{3}e^{i5\pi/6}$.

The transformation of superfields on orbifold for $M_4 \times T^2/Z_2$ can be give by

$$\phi(x, z + \omega_1) = T_5\phi(x, z), \quad \phi(x, z + \omega_2) = T_6\phi(x, z), \quad \phi(x, -z) = P_{R_\phi}\phi(x, z), \quad (2.16)$$

with the consistency conditions

$$P_{R_\phi}^2 = 1, \quad T_5T_6 = T_6T_5, \quad T_5P_{R_\phi} = P_{R_\phi}T_5^{-1}, \quad T_6P_{R_\phi} = P_{R_\phi}T_6^{-1}. \quad (2.17)$$

The $N = 2$ gauge vector supermultiplets are place in the bulk with opposite parity choices for its ($N=1$) vector and chiral supermultiplets components to reduce the $N = 1$ SUSY in 6D(amount to $N = 2$ in 4D) to $N = 1$ SUSY in 4D. Unlike ordinary orbifold GUT models which are adopted to solve the doublet-triplet splitting problem, the $SU(5)$ gauge symmetry in our setting is preserved both in the bulk and in the fixed point branes so that the zero modes are still $SU(5)$ symmetric.¹ The supermultiplets of three generation matter contents in $SU(5)$ $\mathbf{10}$ and $\bar{\mathbf{5}}$ representation are placed on three different fix points of T^2/Z_2 (except $z = 0$). The three generations, located in various fixed points, can keep the S_3 modular flavor symmetry.

3 Soft SUSY breaking parameters from S_3 modular invariance in $SU(5)$ GUT

It is possible to explain both the quark and lepton flavors by a finite modular group with a common modulus parameter τ , for example in [13]. The choices of τ will be constrained by the flavor structure of the quark and lepton sectors. On the other hand, as supersymmetry is necessary to keep the form of the superpotential by the non-renormalizable theorem, the SUSY breaking effects should also be taken into account in realistic modular flavor models. Predictions of the modular flavor symmetry models are valid only if the SUSY breaking corrections are negligible [2]. With proper SUSY breaking mechanism, the SUSY partners of quarks and leptons, which share the same superpotential with the quarks and leptons, could also be constrained by the modular flavor structure. Therefore, some regions of the allowed τ by flavor consideration can be favored or disfavored by additional LHC or DM constraints. We will take the simplest realistic S_3 modular flavor model mentioned above as an example to illustrate the main features. Besides, to generate the soft SUSY breaking spectrum in this model, the economical generalized gravity mediation SUSY breaking mechanism [35] is adopted so that no additional SUSY breaking spurions are needed.

Under the modular transformation

$$\tau \mapsto \gamma(\tau) \equiv \frac{a\tau + b}{c\tau + d}, \quad \text{with} \quad ad - bc = 1, \quad (3.1)$$

the superfield $\phi^{(I)}$ with modular weight $-n_{I,\phi}$ transform in a representation $\rho^{(I)}$ of the quotient group Γ_N as follow

$$\phi^{(I)} \mapsto (c\tau + d)^{-n_{I,\phi}} \rho^{(I)}(\gamma)\phi^{(I)}. \quad (3.2)$$

¹Our setting also allow the possibility to break $SU(5)$ into $SU(3)_c \times SU(2)_L \times U(1)_Y$ by non-trivial boundary condition at the fixed point $z = 0$.

	$T_{1,2}$	T_3	$F_{1,2}$	F_3	$N_{1,2}^c$	N_3^c	H_5	$H_{\bar{5}}$	H_{24}	$Y_2^{(2)}$	$Y_1^{(4)}, Y_2^{(4)}$
SU(5)	10	10	$\bar{5}$	$\bar{5}$	1	1	5	$\bar{5}$	24	1	1
S_3	2	1'	2	1'	1	1'	1	1	1	2	1, 2
weight	-2	0	-2	0	0	0	0	0	0	2	4

Table 1. In order to compare the constraints from SUSY and modular flavor symmetry, we choose the same charge assignments of SU(5), S_3 and modular weight for superfields (and modular forms) as that from ref.([13]). The subscript i of F_i and T_i denotes the i -th family.

So the modular invariant Kahler potential can take the following form

$$K \supseteq (-i\tau + i\bar{\tau})^{-n_{I,\phi}} \left(\phi^{(I)\dagger} \phi^{(I)} \right)_1 + h \log(-i\tau + i\bar{\tau}) . \tag{3.3}$$

Based on the modular weight choices of the various SU(5) multiplets, which are given in table 1 with the third generation being special, the Kahler potential takes the following form²

$$K_0 \supseteq \sum_{i=1,2} (-i\tau + i\bar{\tau})^2 \mathbf{10}_i^\dagger \mathbf{10}_i + \sum_{i=1,2} (-i\tau + i\bar{\tau})^2 \bar{\mathbf{5}}_i^\dagger \bar{\mathbf{5}}_i + \mathbf{10}_3^\dagger \mathbf{10}_3 + \bar{\mathbf{5}}_3^\dagger \bar{\mathbf{5}}_3, \\ + H_5^\dagger H_5 + \bar{H}_{\bar{5}}^\dagger \bar{H}_{\bar{5}} + \Phi_{24}^\dagger \Phi_{24}. \tag{3.4}$$

It is obvious from the Kahler potential that the kinetic terms are not canonical. So we need to rescale the fields by a wavefunction normalization factor Z_ϕ to obtain the proper forms of the physical spectrums. From the Kahler potential, we can obtain the normalization factor

$$Z_{Q_{L;1,2}} = Z_{U_{L;1,2}} = Z_{E_{L;1,2}} = Z_{D_{L;1,2}} = Z_{L_{L;1,2}} = 4(\text{Im } \tau)^2, \\ Z_{Q_{L;3}} = Z_{U_{L;3}} = Z_{E_{L;3}} = Z_{D_{L;3}} = Z_{L_{L;3}} = 1, \\ Z_{H_u} = Z_{H_d} = 1 . \tag{3.5}$$

The value of v_{24}/Λ was set to be 0.3 in [13] to generate additional contributions to the flavor structure. Here we neglect the subleading terms proportional to $v_{24}^2/\Lambda^2 \sim 0.1$ in the normalization factors.

To adopt the generalized gravity mediation scenario, the **24** representation Higgs of SU(5) is assumed to trigger both the SU(5) gauge symmetry breaking and the SUSY breaking. The lowest and F-component VEVs of the adjoint Higgs of SU(5) can take the following form

$$\langle \Phi_{24} \rangle = (v_{24} + \theta^a F_{24}) \sqrt{\frac{3}{5}} \begin{pmatrix} \frac{1}{3} & & & & \\ & \frac{1}{3} & & & \\ & & \frac{1}{3} & & \\ & & & -\frac{1}{2} & \\ & & & & -\frac{1}{2} \end{pmatrix}. \tag{3.6}$$

²We do not include the heavy right-handed neutrinos $N_{L;1,2,3}^c$ superfields in the Kahler potential because they will be integrated out at the low energy effective theory. They will not affect the low energy soft SUSY breaking spectrum.

Modular invariance also constrain to some extent the non-renormalizable terms involving the **24** representation Higgs in the Kahler potential

$$K \supseteq K_0 \left(1 + c_0 (-i\tau + i\bar{\tau})^{-n_{24}} \frac{\text{Tr}(\Phi_{24}^\dagger \Phi_{24})}{\Lambda^2} \right), \quad (3.7)$$

although additional terms involving the $Y_i^{(2)}, Y_i^{(4)}$ etc can still be consistent with the modular flavor symmetry of the model. As noted in [44], the presence of such additional terms will reduce the predictive power of these modular group constructions. So we simply assume that proper mechanism will forbidden other terms to appear in the non-renormalizable Kahler potential (3.7). We should note that the normalization factors will eventually be canceled out in the final expressions although the kinetic terms are not canonical. So the (universal) soft SUSY breaking masses for the sfermions (at the GUT scale) are given as

$$m_\phi^2 = \frac{c_0 |F_{24}|^2}{2 \Lambda^2}. \quad (3.8)$$

To introduce the higher-dimensional operators involving **24** representation Higgs fields, we need to know the decompositions of the tensor products of various SU(5) representations for the Yukawa coupling terms [35]

$$\begin{aligned} \mathbf{10} \otimes \mathbf{10} \otimes \mathbf{5} &= (\bar{\mathbf{5}} \oplus \bar{\mathbf{45}} \oplus \bar{\mathbf{50}}) \otimes \mathbf{5} \\ &= (\mathbf{1} \oplus \mathbf{24}) \oplus (\mathbf{24} \oplus \mathbf{75} \oplus \mathbf{126}) \oplus (\mathbf{75} \oplus \mathbf{175}'), \end{aligned} \quad (3.9)$$

$$\mathbf{10} \otimes \bar{\mathbf{5}} \otimes \bar{\mathbf{5}} = \mathbf{10} \otimes (\bar{\mathbf{10}} \oplus \bar{\mathbf{15}}) = (\mathbf{1} \oplus \mathbf{24} \oplus \mathbf{75}) \oplus (\mathbf{24} \oplus \bar{\mathbf{126}}). \quad (3.10)$$

We can see that there are two types of contraction of Yukawa with adjoint representation Higgs Φ_{24} for both Yukawa couplings. The most general discussions on soft SUSY parameters related to the introduction of higher-dimensional operators can be seen in our previous works. Although not shown explicitly, the contraction rules within [13] adopt the following contractions

$$W \supseteq \mathbf{10}_{ij} \mathbf{10}_{mn} \mathbf{5}_k \frac{\mathbf{24}_l^k}{\Lambda} \epsilon^{ijmnl} + \mathbf{10}_{ij} \bar{\mathbf{5}}^k \bar{\mathbf{5}}_H^j \frac{\mathbf{24}_k^i}{\Lambda}. \quad (3.11)$$

Modular invariance also constrain stringently the form of the superpotential. The modular weight choices and the modular flavor group representation choices can fix the form of the superpotential except the non-renormalizable terms involving Φ_{24} . To compare the constraints from SUSY and constraints from the SM flavor structures, we adopt the same form of non-renormalizable terms involving Φ_{24} as that in [13].

The A_u trilinear couplings can be obtained by substituting the F-component VEV of Φ_{24} into the superpotential involving $10 - 10 - 5_H$ couplings of SU(5)³

$$W_1 \supseteq \left(\alpha'_1 Y_1^{(4)} + \alpha'_2 Y_2^{(4)} \right) T_{1,2} T_{1,2} H_5 k'_1 \frac{\Phi_{24}}{\Lambda} + \beta'_1 Y_2^{(2)} T_{1,2} T_3 H_5 k'_2 \frac{\Phi_{24}}{\Lambda} + \gamma' T_3 T_3 H_5 k'_4 \frac{\Phi_{24}}{\Lambda}, \quad (3.12)$$

³The mass matrix for up-type quarks can take an asymmetric form with additional **45** representation Higgs fields.

with the production rule of **2** representation of S_3 given as

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = (x_1 y_1 + x_2 y_2)_{\mathbf{1}} \oplus (x_1 y_2 - x_2 y_1)_{\mathbf{1}'} \oplus \begin{pmatrix} x_1 y_1 - x_2 y_2 \\ -x_1 y_2 - x_2 y_1 \end{pmatrix}_{\mathbf{2}}, \quad (3.13)$$

and the expressions of Y_1, Y_2 etc given in the appendix. The trilinear couplings are therefore given by

$$(\tilde{A}_u)_{ij} = -2\sqrt{\frac{3}{5}} \frac{F_{\mathbf{24}}}{\Lambda} \begin{pmatrix} [\alpha'_1(Y_1^2 + Y_2^2) + \alpha'_2(Y_1^2 - Y_2^2)] k'_1 & 2\alpha'_2 Y_1 Y_2 k'_1 & -k'_2 \beta'_1 Y_2 \\ 2\alpha'_2 Y_1 Y_2 k'_1 & [\alpha'_1(Y_1^2 + Y_2^2) - \alpha'_2(Y_1^2 - Y_2^2)] k'_1 & k'_2 \beta'_1 Y_1 \\ -k'_2 \beta'_1 Y_2 & k'_2 \beta'_1 Y_1 & k'_4 \gamma' \end{pmatrix} \quad (3.14)$$

The \tilde{A}_d trilinear couplings can be obtained from the $10 - \bar{5} - \bar{5}_H$ coupling

$$W_2 \supseteq (\alpha_1 Y_1^{(4)} + \alpha_2 Y_2^{(4)}) T_{1,2} F_{1,2} H_{\bar{5}} k_1 \frac{\Phi_{\mathbf{24}}}{\Lambda} + \beta_1 Y_2^{(2)} T_{1,2} F_3 H_{\bar{5}} k_2 \frac{\Phi_{\mathbf{24}}}{\Lambda} + \beta_2 Y_2^{(2)} T_3 F_{1,2} H_{\bar{5}} k_3 \frac{\Phi_{\mathbf{24}}}{\Lambda} + \gamma T_3 F_3 H_{\bar{5}} k_4 \frac{\Phi_{\mathbf{24}}}{\Lambda}, \quad (3.15)$$

after substituting the F-term VEV of $\Phi_{\mathbf{24}}$ with the contraction given in (3.11). It reads

$$(\tilde{A}_d)_{ij} = \frac{1}{3} \sqrt{\frac{3}{5}} \frac{F_{\mathbf{24}}}{\Lambda} \begin{pmatrix} [\alpha_1(Y_1^2 + Y_2^2) + \alpha_2(Y_1^2 - Y_2^2)] k_1 & 2\alpha_2 Y_1 Y_2 k_1 & -k_2 \beta_1 Y_2 \\ 2\alpha_2 Y_1 Y_2 k_1 & [\alpha_1(Y_1^2 + Y_2^2) - \alpha_2(Y_1^2 - Y_2^2)] k_1 & k_2 \beta_1 Y_1 \\ -k_3 \beta_2 Y_2 & k_3 \beta_2 Y_1 & \gamma k_4 \end{pmatrix}. \quad (3.16)$$

The trilinear couplings for the lepton Yukawa couplings can be similarly obtained to be

$$(\tilde{A}_e)_{ij}^T = -\frac{3}{2} (\tilde{A}_d)_{ij}. \quad (3.17)$$

Due to the fact that the superfields in the superpotential are not canonically normalized, proper wavefunction factors should be divided from the previous expressions of trilinear terms to get the physical ones. So, the genuine trilinear soft terms are given as

$$A_{ijk}^{u,d,e} = \frac{\tilde{A}_{ijk}^{u,d,e}}{\sqrt{Z_i Z_j Z_k}}, \quad (3.18)$$

with Z_ϕ given in (3.5).

Unlike the KKL'T realization, in which gauge fields origin from the D3/D7 brane, the gauge fields here need not transform under modular transformations. The gaugino masses are generated by non-renormalizable term involving the $\Phi_{\mathbf{24}}$ Higgs.

$$\mathcal{L} \supseteq \int d^2\theta \frac{1}{4} W^a W^a - \frac{c_1}{4\Lambda} W_\mu^a \Phi_a^b W_\mu^b. \quad (3.19)$$

After the **24** representation Higgs Φ acquires VEV, the gaugino masses are given as

$$M_i = -\frac{c_1}{Z_i} a_i \frac{F_{\mathbf{24}}}{12\Lambda} \sqrt{\frac{3}{5}}, \quad (3.20)$$

with $a_i = (1, 3, -2)$ for $U(1)_Y, SU(2)_L, SU(3)_c$ and the wavefunction normalization factor

$$Z_i = 1 - a_i \frac{c_1}{12\Lambda} \sqrt{\frac{3}{5}} v_{24} \approx 1. \quad (3.21)$$

4 Numerical results

This minimal UV-completion S_3 modular flavor SUSY GUT model has three additional SUSY breaking related free parameters $c_0, c_1, F_{24}/\Lambda$ in addition to the modulus parameter τ and other parameters chosen in the S_3 modular SU(5) GUT model [13] to generate the flavor structure for quarks and leptons. As noted previously, constraints on the superpotential by the modular flavor structure will lead to “non-universality” of the SUSY breaking parameters for the SUSY partners of quarks and leptons at the input scale, which share the same superpotential with the quarks and the leptons. Therefore, a different flavor pattern at the EW scale (in contrast to “universality” soft SUSY breaking inputs) will be generated and low energy flavor constraints etc will impose non-trivial constraints on the input parameters. So, it is interesting to survey if this SUSY completion model can survive the low energy (SUSY) flavor, collider as well as the DM constraints.

The package SPheno4.0.4 [45, 46] is used to scan the parameters spaces of

$$\tau, c_0, c_1, \frac{F_{24}}{\Lambda}, \quad (4.1)$$

with the model file generated by SARAH4.14.3 [47–49]. The DM relic density and other collider constraints are obtained with the package micrOMEGAS5.2.4 [50–53]. We use the package HiggsBounds5.3.2 [54] to check whether or not the predictions of the survived points can pass the exclusion bounds obtained from the Higgs searches by the colliders.

We impose the following constraints in our numerical studies:

- (I) The conservative lower bounds from current LHC constraints on SUSY particles [55–58], the Z-boson invisible decay and the LEP search for sparticles [59] as well as Higgs mass

$$m_{\tilde{t}_1} \gtrsim 1.2\text{TeV}, \quad m_{\tilde{g}} \gtrsim 2.2\text{TeV}, \quad m_{\tilde{b}_1} \gtrsim 0.95\text{TeV}, \quad m_{\tilde{u}_1}, m_{\tilde{d}_1} \gtrsim 1.37\text{TeV}, \\ m_h \in 125 \pm 2\text{GeV}, \quad m_{\tilde{\chi}^\pm} > 103.5\text{GeV}, \quad \Gamma(Z \rightarrow \tilde{\chi}_0 \tilde{\chi}_0) < 1.71 \text{ MeV}. \quad (4.2)$$

- (II) Constraints from various flavor constraints, such as the rare B mesons decay [60] and lepton flavor violation(LFV) bounds [61–64]:

$$0.85 \times 10^{-4} < Br(B^+ \rightarrow \tau^+ \nu) < 2.89 \times 10^{-4}, \\ 1.7 \times 10^{-9} < Br(B_s \rightarrow \mu^+ \mu^-) < 4.5 \times 10^{-9}, \\ 2.99 \times 10^{-4} < Br(B_S \rightarrow X_s \gamma) < 3.87 \times 10^{-4}, \\ Br(\mu \rightarrow e \gamma) < 4.2 \times 10^{-13}, \quad Br(\tau \rightarrow e \gamma) < 3.3 \times 10^{-8}, \\ Br(\tau \rightarrow \mu \gamma) < 4.4 \times 10^{-8}, \quad Br(\mu \rightarrow 3e) < 1.0 \times 10^{-12}, \\ Br(\tau \rightarrow 3e) < 2.7 \times 10^{-8}, \quad Br(\tau \rightarrow 3\mu) < 2.1 \times 10^{-8}. \quad (4.3)$$

- (III) The relic density of dark matter(DM) should satisfy the Planck data $\Omega_{DM} = 0.1199 \pm 0.0027$ [65] in combination with the WMAP data [66](with a 10% theoretical uncertainty).

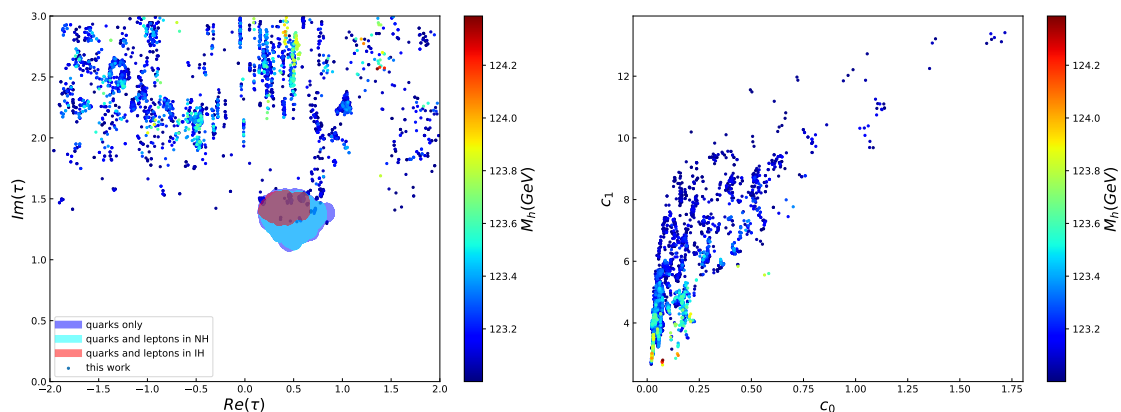


Figure 1. In the left panel, we show the allowed range of $Im(\tau)$ versus $Re(\tau)$ that can satisfy the SM flavor structure, the collider and DM constraints. The best-fit region of quark flavor structure in [13] is labeled with blue color and the best-fit regions for both quark and lepton flavor structures with neutrino mass pattern either in IH (NH) is labeled with red color (cyan color), respectively. In the right panel, we show the survived range of c_0, c_1 parameter with the corresponding Higgs masses.

We have the following discussions related to our numerical results:

- It can be seen from the left-panel of figure 1 that the constraints from (I) to (III) can rule out much of the allowed parameter space of $\Im(\tau)$ versus $\Re(\tau)$, especially a large portion of the best-fit region of the SM (plus neutrino) flavor structure in [13]. On the other hand, it is interesting to note that the survived region of $\Im(\tau)$ versus $\Re(\tau)$ can still have a overlap with such best-fit region, which means that the UV-completed model can account for both the SM (plus neutrino) flavor structure and the collider, DM constraints.

A tiny fraction of allowed parameter space can lie within the best-fit region for quark flavor structure, which is label with blue color. The best-fit regions for both quarks and leptons flavor structures with neutrino mass pattern either in invert hierarchy(IH) (label with red color) or normal hierarchy(NH) (label with cyan color) can also be consistent with the collider and DM constraints. As the best-fit region for IH case has large overlap with the best-fit region for NH case, constraints from the collider and DM still not show any preference between the two cases. If more experimental data for the SM (plus neutrino) flavor structure can be released in the future to separate the best-fit region for NH case and IH case, SUSY breaking consideration can possibly show which type of neutrino mass hierarchy is more preferable.

- The allowed range of c_0, c_1 is shown in the right-panel of figure 1 with the corresponding Higgs mass. It is obvious from the figure that the discovered 125 GeV Higgs can also be accommodated in this SUSY UV-completion framework. From the right panel of figure 1, we can see that the predicted Higgs mass is always below 125 GeV. Taking into account the theoretical uncertainty in calculating the Higgs mass, which

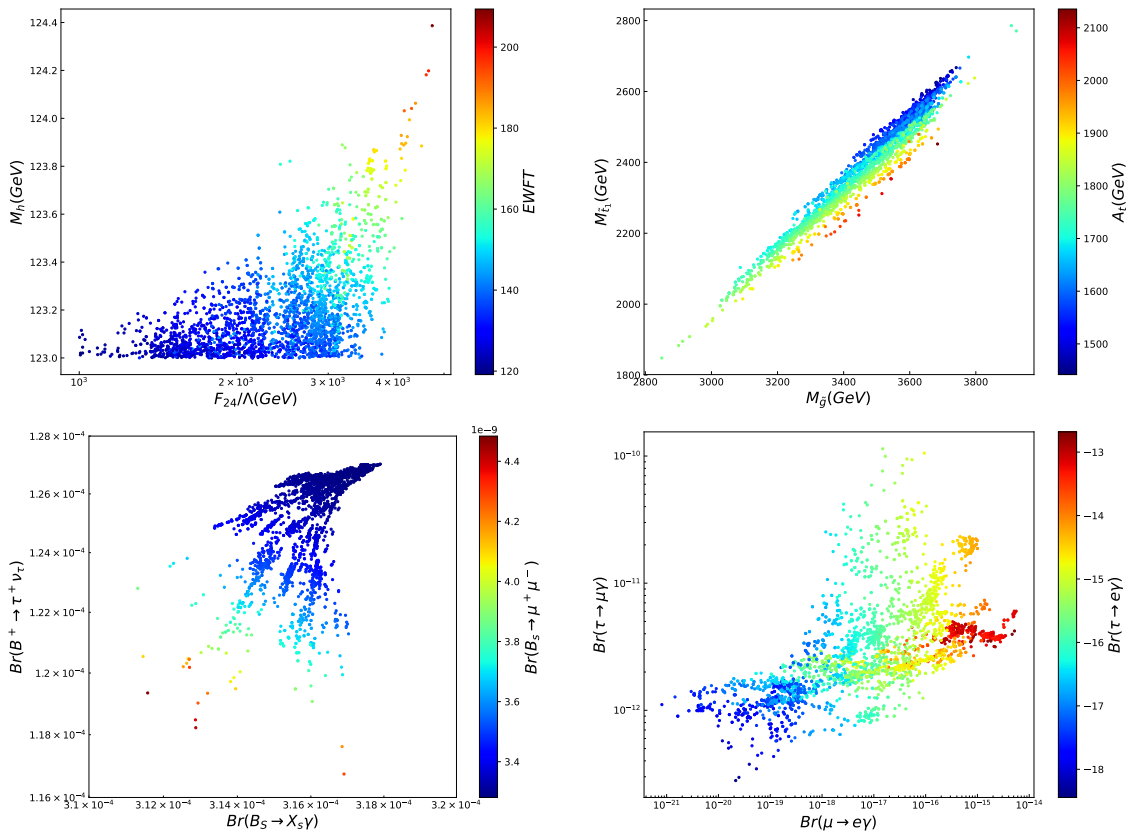


Figure 2. The survived regions for the mass parameter F_{24}/Λ versus the Higgs mass are shown in the upper left panel. Survived regions for gluino masses versus the lightest stop masses are shown in the upper right panel. Branch ratios of various new physics sensitive B meson and LFV decay modes are shown in the lower left and right panel, respectively.

is about $0.5 \sim 2$ GeV, the best-fit region of $\Im(\tau)$ versus $\Re(\tau)$ for flavor structures can also marginally accommodate the 125 GeV Higgs mass.

- The survived region for the mass parameter F_{24}/Λ , which is the only new mass parameter that determines the soft SUSY breaking scale at the GUT scale, is constrained to lie almost between 1.0 TeV to 4.8 TeV (see the upper-left panel of figure 2). The values of F_{24}/Λ and c_0, c_1 can determine the whole soft SUSY spectrum. Our numerical results indicate that the gluino masses are predicted to lie between 2.85 TeV to 3.92 TeV. The lighter stop, \tilde{t}_1 , are predicted to lie between 1.85 TeV to 2.78 TeV. Both of them can be possibly tested on the near future upgraded LHC. Because new flavor mixing and CP violating structures will appear in the squark and slepton sector (in contrast to the “universality “ soft SUSY breaking inputs), some of the low energy flavor constraints will become important. The branch ratios of typical SUSY-sensitive B meson and LFV decay modes are shown in the lower panels of figure 2, which again can be checked in the near-future collider experiments.

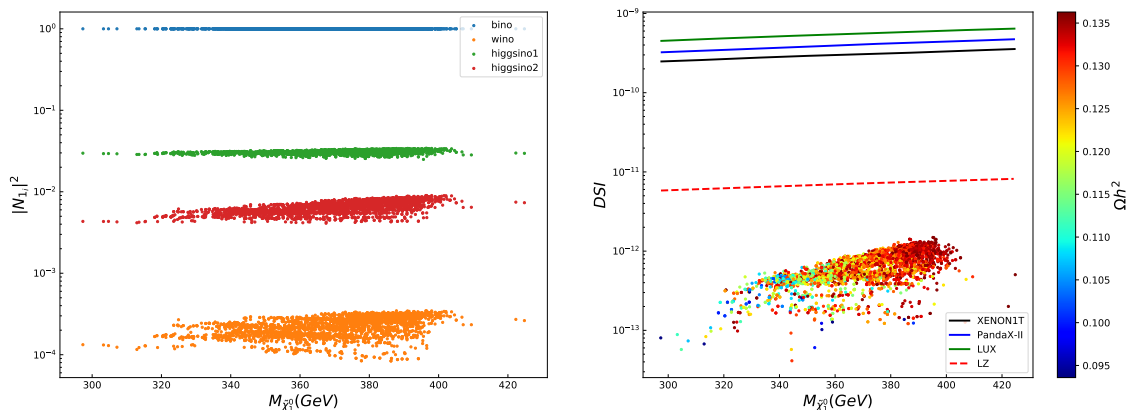


Figure 3. In the left panel, relative components of the LSP are shown. In the right panel, the SI DM-nucleon scattering cross section for DM direct detection is shown. Exclusion bounds of various direct detection experiments, such as XENON1T, PANDAX and LZ, are also given.

- From our numerical results, it can also be seen that correct DM relic abundance can be obtained and the DM candidate is always the lightest neutralino, which is mostly bino-like(see the left-panel of figure 3). As pure bino-like DM can not annihilate into W or Z gauge bosons, they can only annihilate into standard model fermions via t-channel sfermions exchange. So, unless bino is lighter than 300 GeV, pure bino-like DM will always lead to overabundance of DM relic density. On the other hand, coannihilation with stau can reduce the DM abundance efficiently. We checked that the bino-stau coannihilation effect is very crucial to obtain correct DM relic density. The spin-independent(SI) DM-nucleon cross section for DM direct detection is shown in the right-panel of figure 3. We can see that most of the allowed parameter space will survived the current DM direct detection bounds, such as LUX [67],XENON1T [68] and PANDAX [69]. Besides, they can even survive the near-future LUX-ZEPLIN [70] experimental bounds. So, no DM signal is anticipated to be found in the near-future DM direct detection experiments.

5 Conclusions

Modular flavor symmetry can be used to explain the quark and lepton flavor structure. The SUSY partners of quarks and leptons, which share the same superpotential with the quarks and leptons, will also be constrained by the modular flavor structure and show a different flavor(mixing) pattern at the GUT scale. So, in realistic modular flavor models with SUSY completion, constraints from the collider and DM constraints can also be used to constrain the possible values of the modulus parameter. In the first part of this work, we discuss the possibility that the S_3 modular symmetry can be preserved by the fixed points of T^2/Z_N orbifold, especially from T^2/Z_2 . To illustrate the additional constraints from collider etc on modular flavor symmetry models, we take the simplest UV SUSY-completion S_3 modular invariance SU(5) GUT model as an example with generalized gravity mediation SUSY breaking mechanism. We find that such constraints can indeed be useful to rule out a large

portion of the modulus parameters. Our numerical results show that the UV-completed model can account for both the SM (plus neutrino) flavor structure and the collider, DM constraints. Such discussions can also be applied straightforwardly to other modular flavor symmetry models, such as A_4 or S_4 models.

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A Expressions of the modular forms for S_3 modular group

For completeness, we collect the expressions of modular forms with different modular weight from [13]. The modular forms of weight 2 corresponding to the S_3 doublet are written as

$$\begin{aligned}
 Y_1(\tau) &= \frac{i}{4\pi} \left(\frac{\eta'(\tau/2)}{\eta(\tau/2)} + \frac{\eta'((\tau+1)/2)}{\eta((\tau+1)/2)} - \frac{8\eta'(2\tau)}{\eta(2\tau)} \right), \\
 Y_2(\tau) &= \frac{\sqrt{3}i}{4\pi} \left(\frac{\eta'(\tau/2)}{\eta(\tau/2)} - \frac{\eta'((\tau+1)/2)}{\eta((\tau+1)/2)} \right),
 \end{aligned}$$

with $\eta(\tau)$ the Dedekind eta-function defined by

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \tag{A.1}$$

where $q = e^{2\pi i\tau}$. This doublet modular forms have the following q -expansions:

$$Y_{\mathbf{2}}^{(2)} = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \end{pmatrix}_{\mathbf{2}} = \begin{pmatrix} \frac{1}{8} + 3q + 3q^2 + 12q^3 + 3q^4 + \dots \\ \sqrt{3}q^{1/2}(1 + 4q + 6q^2 + 8q^3 + \dots) \end{pmatrix}_{\mathbf{2}}. \tag{A.2}$$

The modular forms of weight 4, that is $Y^{(4)}$, can be constructed by using the tensor product of the two doublets $(Y_1(\tau), Y_2(\tau))^T$

$$\mathbf{1} : Y_{\mathbf{1}}^{(4)} = \left(Y_1(\tau)^2 + Y_2(\tau)^2 \right)_{\mathbf{1}}, \tag{A.3}$$

$$\mathbf{2} : Y_{\mathbf{2}}^{(4)} = \begin{pmatrix} Y_1(\tau)^2 - Y_2(\tau)^2 \\ -2Y_1(\tau)Y_2(\tau) \end{pmatrix}_{\mathbf{2}}. \tag{A.4}$$

The S_3 singlet $\mathbf{1}'$ modular form of the weight 4 vanishes.

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