Cubic interaction vertices for massless higher spin supermultiplets in $d = 4$

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ABSTRACT: We construct a range of supersymmetric cubic vertices for three massless higher spin supermultiplets in the four-dimensional space. We use frame-like multispinor formalism, which allows to avoid most of the technical difficulties and provides a uniform description for bosons and fermions. Our work is based on the so-called Fradkin-Vasiliev formalism for construction of the cubic vertices, which requires the non-zero cosmological constant. Thus we first construct the vertices in AdS space and then consider the flat limit. We show that the AdS supersymmetric vertex is a sum of four elementary vertices for supermultiplet components, while one of the vertices vanishes in the flat limit in agreement with the Metsaev’s classification.

KEYWORDS: Gauge Symmetry, Higher Spin Gravity, Higher Spin Symmetry

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1 Introduction

Recently, a general classification of cubic interaction vertices for massless higher spin supermultiplets was developed in [1] for $N = 1$ supersymmetry and in [2] for extended $N > 1$ ones. In this work we deal with the case of $N = 1$ supersymmetry, so let us discuss the results of [1].

First of all, let us briefly remind the classification of cubic vertices for massless higher spin fields in $d = 4$ [3, 4]. There exist three types of such vertices. Vertices of the type I have $N = s_1 + s_2 + s_3$ derivatives and we call them trivially gauge invariant ones because they can be written as (schematically):

$$\mathcal{L}_1 \sim \mathcal{R}_1 \mathcal{R}_2 \mathcal{R}_3$$

where $\mathcal{R}_i$ — gauge invariant field strengths (curvatures) having s_i derivatives. In turn, the vertices of type II have $N = s_1 + s_2 - s_3$ derivatives in the bosonic case or $N = s_1 + s_2 - s_3 - 1$ in the fermionic one. They can be subdivided into two sub-types (here and in what follows we always assume that spins are ordered as $s_1 \geq s_2 \geq s_3$):

$$\begin{aligned} \mathcal{L}_1 &\sim \Phi_1 \mathcal{R}_2 \mathcal{R}_3 & s_1 &\geq s_2 + s_3 \\ \mathcal{L}_1 &\sim \Phi_1 \Phi_2 \mathcal{R}_3 & s_1 &< s_2 + s_3 \end{aligned}$$

We call the first sub-type abelian vertices, while the second sub-type (whose spins satisfy the so called strict triangular inequality) non-abelian ones. At last, the vertices of type III have $N = s_1 - s_2 - s_3$ derivatives and are known only in the light-cone formalism.

Type	Supermultiplets	Vertex
1a	$Y_1 = s_1 - \frac{1}{2}, Y_2 = s_2 - \frac{1}{2}, Y_3 = s_3$	$V_0 + V_1 + V_2$
1b	$Y_1 = s_1, Y_2 = s_2, Y_3 = s_3$	$V_1 + V_2 + V_3$
2a	$Y_1 = s_1 - \frac{1}{2}, Y_2 = s_2 - \frac{1}{2}, Y_3 = s_3 - \frac{1}{2}$	$V_0 + V_1 + V_2$
2b	$Y_1 = s_1, Y_2 = s_2 - \frac{1}{2}, Y_3 = s_3$	$V_0 + V_1 + V_3$
2c	$Y_1 + s_1, Y_2 = s_2, Y_3 = s_3 - \frac{1}{2}$	$V_1 + V_2 + V_3$
3a	$Y_1 = s_1 - \frac{1}{2}, Y_2 = s_2 - \frac{1}{2}, Y_3 = s_3$	$V_0 + V_1 + V_3$
3b	$Y_1 = s_1, Y_2 = s_2 - \frac{1}{2}, Y_3 = s_3 - \frac{1}{2}$	$V_1 + V_2 + V_3$
3c	$Y_1 = s_1, Y_2 = s_2, Y_3 = s_3$	$V_0 + V_2 + V_3$

Table 1. Different types of the massless flat supersymmetric cubic vertices

Now let us turn to the cubic vertices for the massless $N = 1$ supermultiplets. Having in our disposal three pairs of bosons and fermions (B_i, F_i) , $i = 1, 2, 3$, we can in general construct four independent vertices (we call them elementary), namely, one bosonic vertex and three fermionic ones:

$$V_0(B_1, B_2, B_3), \quad V_1(F_2, B_1, F_3), \quad V_2(F_1, B_2, F_3), \quad V_3(F_1, F_2, B_3).$$

A supersymmetric cubic vertex is a combination of such elementary ones (with their coupling constants appropriately adjusted) which is invariant under the supertransformations. Recall that $N = 1$ supermultiplets are characterized by superspin Y which for the massless ones coincides with the lowest spin. In table 1, based on the results of Metsaev [1], we provide all possible types of supersymmetric vertices and the combinations of the elementary ones they consist of (note that all $s_{1,2,3}$ in the table are integer).

As can be seen from the table 1, all supersymmetric vertices contain only three elementary ones. This fact can be easily understood as follows. In flat space the general structure of supertransformations for the massless supermultiplets has the form (schematically):

$$\delta B \sim F\zeta, \quad \delta F \sim dB\zeta$$

Then, for the supersymmetric vertex be invariant under the supertransformations, the number of derivatives in the bosonic and fermionic elementary vertices must satisfy

$$N_{BBB} = N_{BFF} + 1$$

Now let us consider three arbitrary supermultiplets $(s_i, s_i + \epsilon_i)$, $i = 1, 2, 3$, where $\epsilon_i = \pm \frac{1}{2}$. For simplicity we assume that the spins are all distinct: $s_1 > s_2 > s_3$. Here we restrict ourselves with the type 2 vertices we are mostly interested in this work. Then it is easy to calculate the number of derivatives in each elementary vertex:

$$\begin{aligned} N_0 &= s_1 + s_2 - s_3 \\ N_1 &= s_1 + s_2 - s_3 + \epsilon_2 - \epsilon_3 - 1 \\ N_2 &= s_1 + s_2 - s_3 + \epsilon_1 - \epsilon_3 - 1 \\ N_3 &= s_1 + s_2 - s_3 + \epsilon_1 + \epsilon_2 - 1 \end{aligned}$$

One can easily see that the relation $N_{BBB} = N_{BFF} + 1$ cannot be fulfilled by all four vertices simultaneously; hence, one of the vertices has the “wrong” number of derivatives and must be absent. There are four possible cases depending on which vertex is absent, and in each case, the parameters ϵ_i are fixed. We consider the cases in turn.

- (I) $V_0 + V_1 + V_2$: $\epsilon_{1,2} = -\frac{1}{2}$, $\epsilon_3 = \frac{1}{2}$. This corresponds to type 2a.
- (II) $V_0 + V_1 + V_3$: $\epsilon_{1,3} = \frac{1}{2}$, $\epsilon_2 = -\frac{1}{2}$. This corresponds to type 2b.
- (III) $V_0 + V_2 + V_3$: $\epsilon_1 = -\frac{1}{2}$, $\epsilon_{2,3} = \frac{1}{2}$. This also corresponds to type 2b, but with the roles of first and second supermultiplets interchanged.
- (IV) $V_1 + V_2 + V_3$: $\epsilon_{1,2} = \frac{1}{2}$, $\epsilon_3 = -\frac{1}{2}$. This corresponds to type 2c.

The classification of the supersymmetric vertices has been developed in [1] in the light-cone formalism. As for the Lorentz covariant formulation, till now just a few non-trivial examples were constructed using the superfield formalism. In two papers [5] and [6] the cubic interactions for one higher spin supermultiplets with two chiral supermultiplets $Y_2 = Y_3 = 0$ were constructed for half-integer and integer superspins, correspondingly. In [7] the cubic interaction for arbitrary half-integer superspin $Y = s_1 + \frac{1}{2}$ with two equal superspins $Y_2 = Y_3$ (which may be integer or half-integer) were constructed. The two lower superspin multiplets enter through their gauge invariant field strengths, so this gives examples of the type 2ab abelian vertices. The type 2c vertices are absent just because the authors considered only the case where lower superspins are equal, but, in-principle, they also can be constructed with the same technique. At last, in [8] the authors considered the case of arbitrary integer superspin $Y_1 = s_1$ and again two equal superspins $Y_2 = Y_3$ (which also may be integer or half-integer). From the number of derivatives it follows that these vertices belong to the class of trivially gauge invariant ones 1ab.

In this work we provide an explicit construction for the supersymmetric cubic vertices of type 2abc when all spins satisfy the strict triangular inequality (so our results are complementary to that of [7]). The construction is heavily based on our previous work [9] devoted to the construction of (what we now call) elementary bosonic and fermionic vertices using a so-called Fradkin-Vasiliev formalism [10–12]. Let us briefly remind the procedure.

Recall, that in a frame-like formalism each massless bosonic or fermionic higher spin particle is described by a set of gauge fields (one-forms) Φ (physical, auxiliary and extra ones). For each field, a gauge invariant curvature (two-form) $\mathcal{R}$ can be constructed. At last, in AdS space the free Lagrangian can be rewritten in the explicitly gauge invariant form as

$$\mathcal{L}_0 \sim \sum_k a_k \mathcal{R}_k \mathcal{R}_k$$

The first step in constructing the cubic vertex in the Fradkin-Vasiliev formalism is to find quadratic deformations for all curvatures $\Delta\mathcal{R} \sim \Phi\Phi$ such that the deformed curvatures $\hat{\mathcal{R}} = \mathcal{R} + \Delta\mathcal{R}$ transform covariantly under the gauge transformations $\delta\hat{\mathcal{R}} \sim \mathcal{R}\eta$. On the second step, one takes the sum of the three Lagrangians and replace the initial curvatures with the deformed ones, requiring that the result be gauge invariant (on-shell). The cubic

terms of the deformed Lagrangian constitutes the desired (on-shell) gauge invariant cubic vertex. As described, such formalism works only in AdS space with non-zero cosmological constant, but, as we have shown in [9], the resulting cubic vertices admits (on-shell) a non-singular flat limit, providing us with the flat space cubic vertices as well.

Our current work is a straightforward generalization of this procedure to the case of supersymmetric cubic vertices. The main idea is really very simple. We consider quadratic deformations for all bosonic and fermionic curvatures corresponding to all four elementary vertices simultaneously. Besides the usual requirement that the deformed curvatures must transform covariantly under the gauge transformations, we also require that under the supertransformations they transform exactly in the same way as the initial ones. This ensures that the deformed Lagrangian and hence a cubic vertex will be (on-shell) invariant under the supertransformations. Here we also begin with the AdS space with non-zero cosmological constant and then consider the flat limit. We will see, that in AdS all supersymmetric vertices necessarily contain all four elementary ones, while one of them always vanishes in the flat limit in complete agreement with Metsaev's results.

The paper is organized as follows. In section 2, we provide the necessary background information about the frame-like description of the massless higher spin fields and supermultiplets. In section 3, we construct the AdS vertices, while in section 4 we provide examples of the flat space supersymmetric vertices. Notation and conventions as well as some technical details are collected in a pair of appendices.

2 Free massless HS particles and supermultiplets

In this section we provide all necessary information on the massless higher spin bosons, fermions and supermultiplets in the frame-like multispinor formalism.

2.1 Boson

To describe the massless spin- $s \geq 2$ boson in the frame-like formalism, one needs the physical field $H^{\alpha(s-1)\dot{\alpha}(s-1)}$, a pair of auxiliary fields $\Omega^{\alpha(s)\dot{\alpha}(s-2)}$, $\Omega^{\alpha(s-2)\dot{\alpha}(s)}$ and (for $s \geq 3$) a set of so-called extra fields $\Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)}$, $2 \leq |m| \leq s-1$. All fields are gauge one-forms. If we iterate over the complete set of fields, we denote them as $\Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)}$, $|m| \leq s-1$ for brevity, assuming that for $m=0$ $\Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} \equiv H^{\alpha(s-1)\dot{\alpha}(s-1)}$.

The gauge transformations for the fields have the form:

$$\begin{aligned} \delta H^{\alpha(s-1)\dot{\alpha}(s-1)} &= D\eta^{\alpha(s-1)\dot{\alpha}(s-1)} + e^\alpha_{\dot{\beta}}\eta^{\alpha(s-2)\dot{\beta}\dot{\alpha}(s-1)} + e_\beta^{\dot{\alpha}}\eta^{\alpha(s-2)\beta\dot{\alpha}(s-1)}, \\ \delta \Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} &= D\eta^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} + \lambda^2 e^\alpha_{\dot{\beta}}\eta^{\alpha(s-2+m)\dot{\beta}\dot{\alpha}(s-1-m)} \\ &\quad + e_\beta^{\dot{\alpha}}\eta^{\alpha(s-1+m)\beta\dot{\alpha}(s-2-m)}, \quad 0 < m < s-1, \\ \delta \Omega^{\alpha(2s-2)} &= D\eta^{\alpha(2s-2)} + \lambda^2 e^\alpha_{\dot{\alpha}}\eta^{\alpha(2s-3)\dot{\alpha}}. \end{aligned} \tag{2.1}$$

For each field $\Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)}$, a gauge invariant two-form can be constructed:

$$\begin{aligned}\mathcal{T}^{\alpha(s-1)\dot{\alpha}(s-1)} &= DH^{\alpha(s-1)\dot{\alpha}(s-1)} + e^\alpha_{\dot{\beta}} \Omega^{\alpha(s-2)\dot{\beta}\dot{\alpha}(s-1)} + e_{\dot{\beta}}^\alpha \Omega^{\alpha(s-1)\dot{\beta}\dot{\alpha}(s-2)}, \\ \mathcal{R}^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} &= D\Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} + \lambda^2 e^\alpha_{\dot{\beta}} \Omega^{\alpha(s-2+m)\dot{\beta}\dot{\alpha}(s-1-m)} \\ &\quad + e_{\dot{\beta}}^\alpha \Omega^{\alpha(s-1+m)\dot{\beta}\dot{\alpha}(s-2-m)}, \quad 0 < m < s-1, \\ \mathcal{R}^{\alpha(2s-2)} &= D\Omega^{\alpha(2s-2)} + \lambda^2 e^\alpha_{\dot{\alpha}} \Omega^{\alpha(2s-3)\dot{\alpha}}.\end{aligned}\tag{2.2}$$

We refer to these two-forms as curvatures. Similarly, we assume that

$\mathcal{R}^{\alpha(s-1)\dot{\alpha}(s-1)} \equiv \mathcal{T}^{\alpha(s-1)\dot{\alpha}(s-1)}$ whenever the expression $\mathcal{R}^{\alpha(s-1+m)\dot{\alpha}(s-1-m)}$, $|m| \leq s-1$ is encountered. It is straightforward to check that these curvatures satisfy the following differential identities:

$$\begin{aligned}D\mathcal{T}^{\alpha(s-1)\dot{\alpha}(s-1)} &= -e^\alpha_{\dot{\beta}} \mathcal{R}^{\alpha(s-2)\dot{\beta}\dot{\alpha}(s-1)} - e_{\dot{\beta}}^\alpha \mathcal{R}^{\alpha(s-1)\dot{\beta}\dot{\alpha}(s-2)}, \\ D\mathcal{R}^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} &= -\lambda^2 e^\alpha_{\dot{\beta}} \mathcal{R}^{\alpha(s-2+m)\dot{\beta}\dot{\alpha}(s-1-m)} \\ &\quad - e_{\dot{\beta}}^\alpha \mathcal{R}^{\alpha(s-1+m)\dot{\beta}\dot{\alpha}(s-2-m)}, \quad 0 < m < s-1, \\ D\mathcal{R}^{\alpha(2s-2)} &= -\lambda^2 e^\alpha_{\dot{\alpha}} \mathcal{R}^{\alpha(2s-3)\dot{\alpha}}.\end{aligned}\tag{2.3}$$

On-shell all the curvatures, except for the highest ones, i.e. $\mathcal{R}^{\alpha(2s-2)}$, are zero, while the highest curvature can be parameterized by a gauge invariant zero-form $W^{\alpha(2s)}$:

$$\mathcal{R}^{\alpha(2s-2)} = -E_{\beta(2)} W^{\alpha(2s-2)\beta(2)}\tag{2.4}$$

In case of gravitation, $s = 2$, the equation (2.4) expresses a well-known fact that in absence of the matter, the Riemann tensor is traceless, i.e. is equal to the Weyl tensor. We thus refer to $W^{\alpha(2s)}$ as generalised Weyl tensor.

Let us rewrite the on-shell conditions in terms of the fields:

$$\begin{aligned}D\Omega^{\alpha(s-1)\dot{\alpha}(s-1)} &= -e_{\dot{\beta}}^\alpha \Omega^{\alpha(s-1)\dot{\beta}\dot{\alpha}(s-2)} - h.c., \\ D\Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} &= -e_{\dot{\beta}}^\alpha \Omega^{\alpha(s-1+m)\dot{\beta}\dot{\alpha}(s-2-m)} + O(\lambda^2) \\ D\Omega^{\alpha(2s-2)} &= -E_{\beta(2)} W^{\alpha(2s-2)\beta(2)} + O(\lambda^2)\end{aligned}$$

Hence, on-shell the auxiliary field expresses the non-zero derivatives of the physical field, the extra field $\Omega^{\alpha(s+1)\dot{\alpha}(s-3)}$, *h.c.* expresses the non-zero derivatives of the auxiliary field etc. The field $\Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)}$, *h.c.* thus expresses the m -th derivatives of the physical field which do not vanish on-shell. Finally, the Weyl tensor expresses the s -th nonvanishing derivatives. Whenever we talk about the number of derivatives, we imply the number of derivatives of the physical field and count the m -th field as an m -th derivative.

The free Lagrangian can be written in the explicitly gauge invariant form

$$\mathcal{L}_0 = i \sum_{m=1}^{s-1} \frac{(-1)^s (2s-2)!}{(s-1+m)!(s-1-m)! \lambda^{2m}} \mathcal{R}_{\alpha(s-1+m)\dot{\alpha}(s-1-m)} \mathcal{R}^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} + h.c.\tag{2.5}$$

This expression exists in the non-flat space only. Note that the torsion $\mathcal{T}^{\alpha(s-1)\dot{\alpha}(s-1)}$ is absent in this expression. The free Lagrangian terms have at most two derivatives of the physical field, and thus cannot contain the extra fields. This constitutes the so-called extra field decoupling condition. Together with the normalization condition, it uniquely determines the coefficients in the explicitly gauge invariant form of the Lagrangian.

2.2 Fermion

A massless half-integer spin- $(s + \frac{1}{2})$, $s \geq 1$, fermion is described by a set of multispinor gauge one-forms $\Psi^{\alpha(s+m)\dot{\alpha}(s-1-m)} + h.c.$, $0 \leq m \leq s-1$, where $m=0$ correspond to the physical field, all others being the extra ones. The general form of the gauge transformations is similar to the bosonic case. The only differences are the coefficients in the physical field transformation law:

$$\begin{aligned}\delta\Psi^{\alpha(s)\dot{\alpha}(s-1)} &= D\xi^{\alpha(s)\dot{\alpha}(s-1)} + e_{\beta}^{\dot{\alpha}}\xi^{\alpha(s)\beta\dot{\alpha}(s-2)} + \lambda e^{\alpha}_{\dot{\beta}}\xi^{\alpha(s-1)\dot{\alpha}(s-1)\dot{\beta}}, \\ \delta\Psi^{\alpha(s+m)\dot{\alpha}(s-1-m)} &= D\xi^{\alpha(s+m)\dot{\alpha}(s-1-m)} + e_{\beta}^{\dot{\alpha}}\xi^{\alpha(s+m)\beta\dot{\alpha}(s-2-m)} \\ &\quad + \lambda^2 e^{\alpha}_{\dot{\beta}}\xi^{\alpha(s-1+m)\dot{\alpha}(s-1-m)\dot{\beta}}, \quad 0 < m < s-1, \\ \delta\Psi^{\alpha(2s-1)} &= D\xi^{\alpha(2s-1)} + \lambda^2 e^{\alpha}_{\dot{\alpha}}\xi^{\alpha(2s-2)\dot{\alpha}}.\end{aligned}\tag{2.6}$$

Similarly, a set of the gauge invariant two-forms, curvatures, can be constructed:

$$\begin{aligned}\mathcal{F}^{\alpha(s)\dot{\alpha}(s-1)} &= D\Psi^{\alpha(s)\dot{\alpha}(s-1)} + e_{\beta}^{\dot{\alpha}}\Psi^{\alpha(s)\beta\dot{\alpha}(s-2)} + \lambda e^{\alpha}_{\dot{\beta}}\Phi^{\alpha(s-1)\dot{\alpha}(s-1)\dot{\beta}}, \\ \mathcal{F}^{\alpha(s+m)\dot{\alpha}(s-1-m)} &= D\Psi^{\alpha(s+m)\dot{\alpha}(s-1-m)} + e_{\beta}^{\dot{\alpha}}\Psi^{\alpha(s+m)\beta\dot{\alpha}(s-2-m)} \\ &\quad + \lambda^2 e^{\alpha}_{\dot{\beta}}\Psi^{\alpha(s-1+m)\dot{\alpha}(s-1-m)\dot{\beta}}, \quad 0 < m < s-1, \\ \mathcal{F}^{\alpha(2s-1)} &= D\Psi^{\alpha(2s-1)} + \lambda^2 e^{\alpha}_{\dot{\alpha}}\Psi^{\alpha(2s-1)\dot{\alpha}}.\end{aligned}\tag{2.7}$$

Again, on-shell all these curvatures, except for the highest ones, i.e. $\mathcal{F}^{\alpha(2s-1)}$, are zero, while the highest ones can be parameterized by a gauge-invariant zero-form $Y^{\alpha(2s+1)}$:

$$\mathcal{F}^{\alpha(2s-1)} = -E_{\alpha(2)}Y^{\alpha(2s+1)}\tag{2.8}$$

Again, the zero-curvature conditions imply that the field $\Phi^{\alpha(s+m)\dot{\alpha}(s-1-m)}$ expresses the m -th derivatives of the physical field $\Phi^{\alpha(s)\dot{\alpha}(s-1)}$ which do not vanish on-shell.

At last, the free Lagrangian can be written as

$$\mathcal{L}_0 = \sum_{m=0}^{s-1} \frac{(-1)^{s-1}(2s-1)!}{(s+m)!(s-1-m)!\lambda^{2m}} \mathcal{F}_{\alpha(s+m)\dot{\alpha}(s-1-m)} \mathcal{F}^{\alpha(s+m)\dot{\alpha}(s-1-m)} + h.c.\tag{2.9}$$

Again, this expression exists in non-flat space only.

2.3 Supermultiplets

A massless supermultiplet consists of a massless boson and a massless fermion whose spins differ by $\frac{1}{2}$. We call the lower one of the two spins the superspin. Then, there exist two different types of the massless supermultiplets, namely the integer superspin and the half-integer superspin massless supermultiplets. We first discuss the general properties of generic free massless supermultiplets, and then consider integer and half-integer superspin cases separately.

The most important property is the fact that the gauge invariant curvatures for the bosonic and fermionic components of the supermultiplets transform covariantly under the supertransformations, showing that the gauge transformations and supertransformations

agree. Moreover, the on-shell conditions (a union of the on-shell equations for the bosonic and fermionic components) are also consistent with the supertransformations.

We consider now the integer superspin- s . Its description requires bosonic gauge one-forms set $\Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)}$, $|m| \leq s-1$ and the fermionic ones $\Psi^{\alpha(s+m)\dot{\alpha}(s-1-m)}$, $-s \leq m \leq s-1$. The supertransformations have the form [13]:

$$\begin{aligned}
 \delta H^{\alpha(s-1)\dot{\alpha}(s-1)} &= iC\Psi^{\alpha(s-1)\dot{\beta}\dot{\alpha}(s-1)}\zeta_{\dot{\beta}} + iC\Psi^{\alpha(s-1)\beta\dot{\alpha}(s-1)}\zeta_{\beta}, \\
 \delta \Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} &= iC\Psi^{\alpha(s-1+m)\beta\dot{\alpha}(s-1-m)}\zeta_{\beta} \\
 &\quad + iC\lambda\Psi^{\alpha(s-1+m)\dot{\beta}\dot{\alpha}(s-1-m)}\zeta_{\dot{\beta}}, \quad 1 \leq m \leq s-1, \\
 \delta \Psi^{\alpha(s+m)\dot{\alpha}(s-1-m)} &= \tilde{C}\lambda\Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)}\zeta^{\alpha} \\
 &\quad + \tilde{C}\Omega^{\alpha(s+m)\dot{\alpha}(s-2-m)}\zeta^{\dot{\alpha}}, \quad 0 \leq m \leq s-2, \\
 \delta \Psi^{\alpha(2s-1)} &= \lambda\tilde{C}\Omega^{\alpha(2s-2)}\zeta^{\alpha} + \tilde{C}e_{\beta\dot{\alpha}}W^{\alpha(2s-1)\beta}\zeta^{\dot{\alpha}}.
 \end{aligned} \tag{2.10}$$

The generalized Weyl tensors $W^{\alpha(2s)}$, $Y^{\alpha(2s+1)}$ have their own transformation laws; however, we do not need them in the present work. Similarly, for the gauge invariant curvatures we obtain (note additional terms proportional to the Weyl tensor $W^{\alpha(2s)}$ for the fermionic curvatures $\mathcal{F}^{\alpha(2s-1)}$, $\mathcal{F}^{\alpha(2s-2)\dot{\alpha}}$, which will be important in what follows):

$$\begin{aligned}
 \delta \mathcal{T}^{\alpha(s-1)\dot{\alpha}(s-1)} &= iC\mathcal{F}^{\alpha(s-1)\beta\dot{\alpha}(s-1)}\zeta_{\beta} + iC\mathcal{F}^{\alpha(s-1)\dot{\alpha}(s-1)\dot{\beta}}\zeta_{\dot{\beta}}, \\
 \delta \mathcal{R}^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} &= iC\mathcal{F}^{\alpha(s-1+m)\beta\dot{\alpha}(s-1-m)}\zeta_{\beta} \\
 &\quad + iC\lambda\mathcal{F}^{\alpha(s-1+m)\dot{\beta}\dot{\alpha}(s-1-m)}\zeta_{\dot{\beta}}, \quad 1 \leq m \leq s-1, \\
 \delta \mathcal{F}^{\alpha(s+m)\dot{\alpha}(s-1-m)} &= \lambda\tilde{C}\mathcal{R}^{\alpha(s-1+m)\dot{\alpha}(s-1-m)}\zeta^{\alpha} \\
 &\quad + \tilde{C}\mathcal{R}^{\alpha(s+m)\dot{\alpha}(s-2-m)}\zeta^{\dot{\alpha}}, \quad 0 \leq m \leq s-3, \\
 \delta \mathcal{F}^{\alpha(2s-2)\dot{\alpha}} &= \lambda\tilde{C}\mathcal{R}^{\alpha(2s-3)\dot{\alpha}}\zeta^{\alpha} + \tilde{C}\left[\mathcal{R}^{\alpha(2s-2)} + E_{\alpha(2)}W^{\alpha(2s)}\right]\zeta^{\dot{\alpha}} \\
 \delta \mathcal{F}^{\alpha(2s-1)} &= \lambda\tilde{C}\mathcal{R}^{\alpha(2s-2)}\zeta^{\alpha} + \tilde{C}e_{\beta\dot{\alpha}}DW^{\alpha(2s-1)\beta}\zeta^{\dot{\alpha}}
 \end{aligned} \tag{2.11}$$

The requirement that the sum of the bosonic and fermionic Lagrangians be invariant under the supertransformations fixes the ratio $C/\tilde{C}$; under our choice of the normalization

$$C = (2s-1)\tilde{C} \tag{2.12}$$

The sign of the constants C , $\tilde{C}$ can be chosen arbitrarily; we choose $C, \tilde{C} > 0$.

In case of half-integer superspin- $(s - \frac{1}{2})$, the supertransformations have the form:

$$\begin{aligned}
 \delta H^{\alpha(s-1)\dot{\alpha}(s-1)} &= iC\Phi^{\alpha(s-1)\dot{\alpha}(s-2)}\zeta^{\dot{\alpha}} + iC\Phi^{\alpha(s-2)\dot{\alpha}(s-1)}\zeta^{\alpha}, \\
 \delta \Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} &= iC\Phi^{\alpha(s-1+m)\dot{\alpha}(s-2-m)}\zeta^{\dot{\alpha}} \\
 &\quad + iC\lambda\Phi^{\alpha(s-2+m)\dot{\alpha}(s-1-m)}\zeta^{\alpha}, \quad 1 \leq m \leq s-2, \\
 \delta \Omega^{\alpha(2s-2)} &= iC\lambda\Phi^{\alpha(2s-3)}\zeta^{\alpha} + iCe_{\beta\dot{\alpha}}Y^{\alpha(2s-2)\beta}\zeta^{\dot{\alpha}}, \\
 \delta \Phi^{\alpha(s+m-1)\dot{\alpha}(s-2-m)} &= \tilde{C}\lambda\Omega^{\alpha(s-1+m)\dot{\alpha}(s-2-m)\dot{\beta}}\zeta_{\dot{\beta}} \\
 &\quad + \tilde{C}\Omega^{\alpha(s-1+m)\beta\dot{\alpha}(s-2-m)}\zeta_{\beta}, \quad 0 \leq m \leq s-1.
 \end{aligned} \tag{2.13}$$

Again, we do not need the transformation laws of generalized Weyl tensors. For the gauge invariant curvatures we obtain (note again special cases for bosonic curvatures $R^{\alpha(2s-2)}$, $R^{\alpha(2s-3)\dot{\alpha}}$):

$$\begin{aligned}
 \delta\mathcal{T}^{\alpha(s-1)\dot{\alpha}(s-1)} &= iC\mathcal{F}^{\alpha(s-1)\dot{\alpha}(s-2)}\zeta^{\dot{\alpha}} + iC\mathcal{F}^{\alpha(s-2)\dot{\alpha}(s-1)}\zeta^{\alpha}, \\
 \delta\mathcal{R}^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} &= iC\mathcal{F}^{\alpha(s+m-1)\dot{\alpha}(s-2-m)}\zeta^{\dot{\alpha}} \\
 &\quad + iC\lambda\mathcal{F}^{\alpha(s-2+m)\dot{\alpha}(s-m-1)}\zeta^{\alpha}, \quad 1 \leq m \leq s-3, \\
 \delta\mathcal{R}^{\alpha(2s-3)\dot{\alpha}} &= iC[\mathcal{F}^{\alpha(2s-3)} + E_{\beta(2)}Y^{\alpha(2s-3)\beta(2)}]\zeta^{\dot{\alpha}}, \\
 \delta\mathcal{R}^{\alpha(2s-2)} &= iC\lambda\mathcal{F}^{\alpha(2s-3)}\zeta^{\alpha} + iCe_{\beta\dot{\alpha}}DY^{\alpha(2s-2)\beta}\zeta^{\dot{\alpha}}, \\
 \delta\mathcal{F}^{\alpha(s-1+m)\dot{\alpha}(s-2-m)} &= \tilde{C}\lambda\mathcal{R}^{\alpha(s-1+m)\dot{\alpha}(s-2-m)\dot{\beta}}\zeta_{\dot{\beta}} \\
 &\quad + \tilde{C}\mathcal{R}^{\alpha(s-1+m)\beta\dot{\alpha}(s-2-m)}\zeta_{\beta}, \quad 0 \leq m \leq s-1.
 \end{aligned} \tag{2.14}$$

The two factors $C, \tilde{C}$ are real; their ratio is fixed by requiring the sum of the bosonic and fermionic Lagrangians be invariant under the supertransformations:

$$(2s-2)C = \tilde{C} \tag{2.15}$$

Again, we fix their signs as $C, \tilde{C} > 0$.

The algebra of supertransformations is closed, i.e. the anticommutator of the two supertransformations with parameters ζ_1, ζ_2 gives (on-shell) a combination of Lorentz transformations and translations:

$$\begin{aligned}
 [\delta_1, \delta_2]\Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} &= iC\tilde{C}\left[\lambda\Omega^{\alpha(s-2+m)\beta\dot{\alpha}(s-1-m)}\eta_{\beta}^{\alpha} + \lambda\Omega^{\alpha(s-1+m)\dot{\alpha}(s-2-m)\dot{\beta}}\eta_{\dot{\beta}}^{\dot{\alpha}} \right. \\
 &\quad \left. + \lambda^2\Omega^{\alpha(s-1+m)\beta\dot{\alpha}(s-2-m)}\xi_{\beta}^{\dot{\alpha}} + \Omega^{\alpha(s-2+m)\dot{\alpha}(s-1-m)\dot{\beta}}\xi_{\dot{\beta}}^{\alpha}\right]
 \end{aligned} \tag{2.16}$$

where

$$\eta^{\alpha(2)} = 2\zeta_2^{\alpha}\zeta_1^{\alpha}, \quad \eta^{\dot{\alpha}(2)} = 2\zeta_1^{\dot{\alpha}}\zeta_2^{\dot{\alpha}}, \quad \xi^{\alpha\dot{\alpha}} = \zeta_2^{\alpha}\zeta_1^{\dot{\alpha}} - \zeta_1^{\alpha}\zeta_2^{\dot{\alpha}}$$

The expressions are the same for both supermultiplets; one can obtain similar ones for the fermions. In what follows we set $C\tilde{C} = 1$.

3 Supersymmetric vertices in AdS_4

In this section we consider cubic interactions for massless higher spin supermultiplets. Let us take three such supermultiplets (Ω_i, Φ_i) , $i = 1, 2, 3$. In what follows we assume that they are ordered by their superspins $Y_1 \geq Y_2 \geq Y_3$. With these fields four cubic vertices which we call elementary can be constructed, namely, one bosonic vertex and three fermionic ones:

$$V_0(\Omega_1, \Omega_2, \Omega_3), \quad V_1(\Omega_1, \Phi_2, \Phi_3), \quad V_2(\Phi_1, \Omega_2, \Phi_3), \quad V_3(\Phi_1, \Phi_2, \Omega_3).$$

A supersymmetric cubic vertex is a combination of the elementary ones (including appropriate relations on their coupling constants) invariant under the global supertransformations.

In [9], elementary cubic vertices for massless higher spin particles with spins satisfying the strict triangle inequality has been constructed. Let us briefly recall the main steps of such construction. For concreteness, we consider the bosonic vertex; the fermionic case is similar. We enumerate the particles with index $i = 1, 2, 3$ and assume that their spins s_i are ordered from highest to lowest: $s_1 \geq s_2 \geq s_3$. As we have seen in the previous section, each particle is described by the set of one-forms $\Omega_i^{\alpha(s_i-1+m)\dot{\alpha}(s_i-1-m)}$ and the corresponding gauge invariant two-forms $\mathcal{R}_i^{\alpha(s_i-1+m)\dot{\alpha}(s_i-1-m)}$.

The first step is to deform curvatures by adding quadratic terms. Without loss of generality, we consider the first particle. Its curvatures receive quadratic corrections $\Delta\mathcal{R}_1 \sim \Omega_2\Omega_3$ (see explicit expressions in appendix B). The corrections to the gauge transformations $\Delta\delta\Omega_1 \sim \eta_2\Omega_3 - \Omega_2\eta_3$ can be read immediately from the corrections to the curvatures. The main requirement at this step is that the deformed curvatures $\hat{\mathcal{R}}_1 = \mathcal{R}_1 + \Delta\mathcal{R}_1$ must transform covariantly under the gauge transformations: $\delta\hat{\mathcal{R}}_1 \sim \eta_2\mathcal{R}_3 + \eta_3\mathcal{R}_2$. The curvature deformation procedure is carried out for each of the three particles independently; each deformation is fixed up to a total factor, which we denote a , b and c for the first, the second and the third particle respectively.

The second step is to construct the interacting Lagrangian $\mathcal{L}$. It is built from the sum of three particles Lagrangians $\mathcal{L}_i$ expressed in curvatures, with linearized curvatures replaced by the deformed ones. The requirement that the Lagrangian be gauge invariant on-shell links the coefficients a, b, c up to a single total factor g (see appendix B).

We turn on now to the main objective of our paper — the cubic interaction of three massless supermultiplets. Again, the first step is to build quadratic deformations of the curvatures. Without loss of generality, we consider the first supermultiplet. The curvatures of the first supermultiplets receive the deformations of the following form:

$$\begin{aligned}\Delta\mathcal{R}_1 &= a_0\Delta\mathcal{R}_1(\Omega_2, \Omega_3) + a_1\Delta\mathcal{R}_1(\Phi_2, \Phi_3), \\ \Delta\mathcal{F}_1 &= a_2\Delta\mathcal{F}_1(\Omega_2, \Phi_3) + a_3\Delta\mathcal{F}_1(\Phi_2, \Omega_3).\end{aligned}\tag{3.1}$$

The total factors a_i of each elementary deformation are written out explicitly here. The structure of the deformations of the second and third supermultiplet curvatures is similar, with the coefficients b_i and c_i , $i = 0, 1, 2, 3$. Our main requirement here is that the deformed curvatures transform under the supertransformations exactly as the non-deformed ones:

$$\delta\hat{\mathcal{R}} = \hat{\mathcal{F}}\zeta, \quad \delta\hat{\mathcal{F}} = \hat{\mathcal{R}}\zeta.$$

This guarantees that the interacting Lagrangian (again constructed by the replacement of the initial curvatures by the deformed ones) be invariant under the supertransformations. As a result, we obtain the supersymmetric vertex as a linear combination of the four elementary ones:

$$\mathcal{L}_1 = g_0V(\Omega_1, \Omega_2, \Omega_3) + g_1V(\Omega_1, \Phi_2, \Phi_3) + g_2V(\Phi_1, \Omega_2, \Phi_3) + g_3V(\Phi_1, \Phi_2, \Omega_3).\tag{3.2}$$

This fact drastically simplifies the construction of the supersymmetric vertex, since most of the work has already been done in [9]. We consider now the cases 2a, 2b and 2c in turn.

Case 2a: corresponds to three half-integer superspin supermultiplets $(s_i, s_i - \frac{1}{2})$. Recall that all the curvatures except the highest ones vanish on-shell; so the cubic vertex is completely determined by the deformations of these highest curvatures. So in this section we consider their deformations only. The deformations for the highest curvatures for the bosonic and fermionic components of the first supermultiplet are:

$$\begin{aligned}
 \Delta \mathcal{R}_1^{\alpha(2s_1-2)} &= \sum_{k=0}^{\hat{s}_1} \frac{a_0 \lambda^{2k}}{k! (\hat{s}_1 - k)!} \Omega_2^{\alpha(\hat{s}_3) \beta(\hat{s}_1-k) \dot{\beta}(k)} \Omega_3^{\alpha(\hat{s}_2)}_{\beta(\hat{s}_1-k) \dot{\beta}(k)} \\
 &\quad + \sum_{k=0}^{\hat{s}_1-1} \frac{ia_1 \lambda^{2k}}{k! (\hat{s}_1 - k - 1)!} \Phi_2^{\alpha(\hat{s}_3) \beta(\hat{s}_1-k-1) \dot{\beta}(k)} \Phi_3^{\alpha(\hat{s}_2)}_{\beta(\hat{s}_1-k-1) \dot{\beta}(k)} \\
 \Delta \mathcal{F}_1^{\alpha(2s_1-3)} &= \sum_{k=0}^{\hat{s}_1} \frac{a_2 \lambda^{2k}}{k! (\hat{s}_1 - k)!} \Omega_2^{\alpha(\hat{s}_3) \beta(\hat{s}_1-k) \dot{\beta}(k)} \Phi_3^{\alpha(\hat{s}_2-1)}_{\beta(\hat{s}_1-k) \dot{\beta}(k)} \\
 &\quad + \sum_{k=0}^{\hat{s}_1} \frac{a_3 \lambda^{2k}}{k! (\hat{s}_1 - k)!} \Phi_2^{\alpha(\hat{s}_3-1) \beta(\hat{s}_1-k) \dot{\beta}(k)} \Omega_3^{\alpha(\hat{s}_2)}_{\beta(\hat{s}_1-k) \dot{\beta}(k)}
 \end{aligned} \tag{3.3}$$

Here and in what follows the parameters $\hat{s}_i$ are always determined by the spins of bosons:

$$\hat{s}_1 = s_2 + s_3 - s_1 - 1, \quad \hat{s}_2 = s_1 + s_3 - s_2 - 1, \quad \hat{s}_3 = s_1 + s_2 - s_3 - 1. \tag{3.4}$$

Now we have to adjust the parameters $a_{0,1,2,3}$ so that the deformed curvatures have correct supertransformations. Consider, for example, the ζ^α transformations for $\Delta \mathcal{R}_1^{\alpha(2s_1-2)}$. On the one hand, direct calculations give us:

$$\begin{aligned}
 \delta \Delta R^{\alpha(2s_1-2)} &= \sum_{k=0}^{\hat{s}_1} \frac{iC_2 \lambda^{2k+1} a_0}{k! (\hat{s}_1 - k)!} \Phi_2^{\alpha(\hat{s}_3-1) \beta(\hat{s}_1-k) \dot{\beta}(k)} \Omega_3^{\alpha(\hat{s}_2)}_{\beta(\hat{s}_1-k) \dot{\beta}(k)} \zeta^\alpha \\
 &\quad + \sum_{k=0}^{\hat{s}_1-1} \frac{i \left[C_2 \lambda a_0 - \tilde{C}_3 a_1 \right] \lambda^{2k}}{k! (\hat{s}_1 - k - 1)!} \Phi_2^{\alpha(\hat{s}_3) \beta(\hat{s}_1-k-1) \dot{\beta}(k)} \Omega_3^{\alpha(\hat{s}_2)}_{\beta(\hat{s}_1-k) \dot{\beta}(k)} \zeta^\beta \\
 &\quad + \sum_{k=0}^{\hat{s}_1} \frac{iC_3 \lambda^{2k+1} a_0}{k! (\hat{s}_1 - k)!} \Omega_2^{\alpha(\hat{s}_3) \beta(\hat{s}_1-k) \dot{\beta}(k)} \Phi_3^{\alpha(\hat{s}_2-1)}_{\beta(\hat{s}_1-k) \dot{\beta}(k)} \zeta^\alpha \\
 &\quad + \sum_{k=0}^{\hat{s}_1-1} \frac{i \left[C_3 \lambda a_0 - \tilde{C}_2 a_1 \right] \lambda^{2k}}{k! (\hat{s}_1 - k - 1)!} \Omega_2^{\alpha(\hat{s}_3) \beta(\hat{s}_1-k) \dot{\beta}(k)} \Phi_3^{\alpha(\hat{s}_2)}_{\beta(\hat{s}_1-k-1) \dot{\beta}(k)} \zeta_\beta
 \end{aligned} \tag{3.5}$$

On the other hand, we must have:

$$\begin{aligned}
 \delta \Delta R^{\alpha(2s_1-2)} &= iC_1 \Delta \mathcal{F}_1^{\alpha(2s_1-3)} \zeta^\alpha \\
 &= \sum_{k=0}^{\hat{s}_1} \frac{C_1 \lambda^{2k+1} a_3}{k! (\hat{s}_1 - k)!} \Phi_2^{\alpha(\hat{s}_3-1) \beta(\hat{s}_1-k) \dot{\beta}(k)} \Omega_3^{\alpha(\hat{s}_2)}_{\beta(\hat{s}_1-k) \dot{\beta}(k)} \zeta^\alpha \\
 &\quad + \sum_{k=0}^{\hat{s}_1} \frac{C_1 \lambda^{2k+1} a_2}{k! (\hat{s}_1 - k)!} \Omega_2^{\alpha(\hat{s}_3) \beta(\hat{s}_1-k) \dot{\beta}(k)} \Phi_3^{\alpha(\hat{s}_2-1)}_{\beta(\hat{s}_1-k) \dot{\beta}(k)} \zeta^\alpha
 \end{aligned} \tag{3.6}$$

One can easily see that the two expressions are equal only if:

$$\tilde{C}_2 a_1 = C_3 \lambda a_0, \quad C_1 a_2 = C_3 a_0, \quad C_1 a_3 = C_2 a_0. \tag{3.7}$$

Using the relations between the deformation parameters $a_{0,1,2,3}$ and the coupling constants $g_{0,1,2,3}$ (see appendix B):

$$\begin{aligned} g_0 &= \frac{(-1)^{s_1}(2s_1-2)!}{(\hat{s}_1)!(\hat{s}_2)!(\hat{s}_3)!} \frac{a_0}{\lambda^{2s_1-2s_3}} \\ g_1 &= \frac{(-1)^{s_1}(2s_1-2)!}{(\hat{s}_1-1)!(\hat{s}_2)!(\hat{s}_3)!} \frac{a_1}{\lambda^{2s_1-2s_3+1}} \\ g_2 &= \frac{(-1)^{s_1}(2s_1-3)!}{(\hat{s}_1)!(\hat{s}_2-1)!(\hat{s}_3)!} \frac{a_2}{\lambda^{2s_1-2s_3}} \\ g_3 &= -\frac{(-1)^{s_1}(2s_1-3)!}{(\hat{s}_1)!(\hat{s}_2)!(\hat{s}_3-1)!} \frac{a_3}{\lambda^{2s_1-2s_3-1}} \end{aligned}$$

we obtain finally

$$g_1 = \hat{s}_1 C_2 C_3 g_0, \quad g_2 = \hat{s}_2 C_1 C_3 g_0, \quad g_3 = -\hat{s}_3 C_1 C_2 \lambda g_0. \quad (3.8)$$

Thus the vertex V_3 vanishes in the flat limit $\lambda \rightarrow 0$ in complete agreement with the Metsaev's results. The consideration of generic curvature of any of the three supermultiplets is similar and yields the same relations.

Case 2b: is the case of two integer superspins and one half-integer superspin, with the lowest superspin being integer. Here we consider the case of highest-half-integer superspin, i.e. $(s_1, s_1 - \frac{1}{2})$, $(s_2, s_2 + \frac{1}{2})$ and $(s_3, s_3 + \frac{1}{2})$; the case of highest-integer superspin is similar. In this case, the highest curvatures deformations of the first supermultiplet have the following form:

$$\begin{aligned} \Delta \mathcal{R}_1^{\alpha(2s_1-2)} &= \sum_{k=0}^{\hat{s}_1} \frac{a_0 \lambda^{2k}}{k! (\hat{s}_1 - k)!} \Omega_2^{\alpha(\hat{s}_3)\beta(\hat{s}_1-k)\dot{\beta}(k)} \Omega_3^{\alpha(\hat{s}_2)}_{\beta(\hat{s}_1-k)\dot{\beta}(k)} \\ &\quad + \sum_{k=0}^{\hat{s}_1+1} \frac{i a_1 \lambda^{2k}}{k! (\hat{s}_1 - k + 1)!} \Psi_2^{\alpha(\hat{s}_3)\beta(\hat{s}_1-k+1)\dot{\beta}(k)} \Psi_3^{\alpha(\hat{s}_2)}_{\beta(\hat{s}_1-k+1)\dot{\beta}(k)} \\ \Delta \mathcal{F}_1^{\alpha(2s_1-3)} &= \sum_{k=0}^{\hat{s}_1+1} \frac{a_2 \lambda^{2k}}{k! (\hat{s}_1 - k + 1)!} \Omega_2^{\alpha(\hat{s}_3-1)\beta(\hat{s}_1+1-k)\dot{\beta}(k)} \Psi_3^{\alpha(\hat{s}_2)}_{\beta(\hat{s}_1+1-k)\dot{\beta}(k)} \\ &\quad + \sum_{k=0}^{\hat{s}_1+1} \frac{a_3 \lambda^{2k}}{k! (\hat{s}_1 - k + 1)!} \Psi_2^{\alpha(\hat{s}_3)\beta(\hat{s}_1-k+1)\dot{\beta}(k)} \Omega_3^{\alpha(\hat{s}_2-1)}_{\beta(\hat{s}_1-k+1)\dot{\beta}(k)} \end{aligned} \quad (3.9)$$

Again, the supercovariance conditions fix a_i , $i = 0, 1, 2, 3$ up to the total factor; their expressions are slightly different from the case 2a:

$$\tilde{C}_2 \lambda a_1 = -C_3 a_0, \quad \tilde{C}_2 a_1 = -C_1 a_2, \quad \tilde{C}_3 a_1 = C_1 a_3. \quad (3.10)$$

This leads to the following relations for the coupling constants:

$$(\hat{s}_1 + 1)g_1 = -C_2 C_3 g_0, \quad (\hat{s}_1 + 1)g_2 = \hat{s}_3 C_1 C_3 \lambda g_0, \quad (\hat{s}_1 + 1)g_3 = \hat{s}_2 C_1 C_2 g_0. \quad (3.11)$$

This time the vertex V_2 vanishes in the flat limit again in agreement with the Metsaev's results.

Case 2c: is the case of two integer superspins and one half-integer superspin, with the lowest spin being half-integer, i.e. $(s_1, s_1 + \frac{1}{2})$, $(s_2, s_2 + \frac{1}{2})$ and $(s_3, s_3 - \frac{1}{2})$. The expressions for the highest curvatures deformations of the first supermultiplets now have the following form:

$$\begin{aligned}
 \Delta \mathcal{R}_1^{\alpha(2s_1-2)} &= \sum_{k=0}^{\hat{s}_1} \frac{a_0 \lambda^{2k}}{k! (\hat{s}_1 - k)!} \Omega_2^{\alpha(\hat{s}_3) \beta(\hat{s}_1-k) \dot{\beta}(k)} \Omega_3^{\alpha(\hat{s}_2)}_{\beta(\hat{s}_1-k) \dot{\beta}(k)} \\
 &\quad + \sum_{k=0}^{\hat{s}_1} \frac{ia_1 \lambda^{2k}}{k! (\hat{s}_1 - k)!} \Psi_2^{\alpha(\hat{s}_3+1) \beta(\hat{s}_1-k) \dot{\beta}(k)} \Phi_3^{\alpha(\hat{s}_2-1)}_{\beta(\hat{s}_1-k) \dot{\beta}(k)} \\
 \Delta \mathcal{F}_1^{\alpha(2s_1-1)} &= \sum_{k=0}^{\hat{s}_1-1} \frac{a_2 \lambda^{2k}}{k! (\hat{s}_1 - k - 1)!} \Omega_2^{\alpha(\hat{s}_3+1) \beta(\hat{s}_1-k-1) \dot{\beta}(k)} \Phi_3^{\alpha(\hat{s}_2)}_{\beta(\hat{s}_1-k-1) \dot{\beta}(k)} \\
 &\quad + \sum_{k=0}^{\hat{s}_1} \frac{a_3 \lambda^{2k}}{k! (\hat{s}_1 - k)!} \Psi_2^{\alpha(\hat{s}_3+1) \beta(\hat{s}_1-k) \dot{\beta}(k)} \Omega_3^{\alpha(\hat{s}_2)}_{\beta(\hat{s}_1-k) \dot{\beta}(k)}
 \end{aligned} \tag{3.12}$$

In this case the supercovariance conditions are:

$$\begin{aligned}
 \delta \mathcal{F}_1^{\alpha(2s_1-1)} &= \tilde{C}_1 \mathcal{R}_1^{\alpha(2s_1-2)} \zeta^\alpha \\
 \delta \mathcal{R}_1^{\alpha(2s_1-2)} &= i C_1 \mathcal{F}_1^{\alpha(2s_1-2) \dot{\alpha}} \zeta_{\dot{\alpha}} + i C_1 \mathcal{F}_1^{\alpha(2s_1-2) \beta} \zeta_\beta.
 \end{aligned} \tag{3.13}$$

Again, the factors a_i , $i = 0, 1, 2, 3$ are fixed by the supercovariance conditions up to the total factor:

$$C_2 a_0 = C_1 a_3, \quad \tilde{C}_1 a_1 = C_3 a_3, \quad C_2 a_2 = C_3 \lambda a_3, \tag{3.14}$$

while for the coupling constants we obtain now:

$$g_0 = (\hat{s}_3 + 1) \tilde{C}_1 \tilde{C}_2 \lambda g_3, \quad g_1 = \hat{s}_2 \tilde{C}_1 C_3 g_3, \quad g_2 = -\hat{s}_1 \tilde{C}_2 C_3 g_3, \tag{3.15}$$

thus the bosonic vertex V_0 vanishes in the flat limit as it should be.

4 Supersymmetric vertices in the flat space

The deformation procedure we used in the previous section to construct cubic vertices in AdS_4 generates terms having up to $N_{\max} = s_1 + s_2 + s_3 - 2$ derivatives, while the corresponding flat vertex must have $N_b = s_1 + s_2 - s_3$ for the bosonic case and $N_f = s_1 + s_2 - s_3 - 1$ for the fermionic one. In space-time with $D > 4$ these higher derivatives terms reproduce the so-called abelian vertices [12], which are absent in $d = 4$. In [9] we have shown that all these higher derivatives terms can be combined in the total derivatives or vanish on-shell. This allowed us to obtain a non-singular flat limit.

In the same way we can consider the flat limit of the supersymmetric vertices. The supertransformations we used also admit a non-singular flat limit, so one can expect that the flat vertex also must be supersymmetric. However, as will be seen later on, it is important to use the full supertransformations (2.10) and (2.13), i.e. with generalized Weyl tensor terms. This is due to the fact that we use mass shell equations to simplify the vertex; the supertransformations have to preserve these equations, which is impossible

without Weyl tensor terms. In the previous section we have seen that in AdS_4 any such vertex consists of the four elementary ones, but one of them vanishes in the flat limit. As we have shown in [9], the flat vertices have the most simple form when all three spins are different. So in this section we provide as an illustration the examples of the supersymmetric vertices restricting ourselves with their simplest representatives.

Now we consider the cases 2a, 2b and 2c in turn.

Case 2a. In general such vertex has the following form:

$$V = ig_0 V_0(\Omega_1, \Omega_2, \Omega_3) + g_1 V_1(\Omega_1, \Phi_2, \Phi_3) + g_2 V_2(\Phi_1, \Omega_2, \Phi_3). \quad (4.1)$$

In case of sufficiently different spins, i.e. if $s_1 > s_2 > s_3$, the vertex can be written as:

$$\begin{aligned} V_{2a} = & ig_0 d\Omega_{3,\alpha(\hat{s}_2)\beta(\hat{s}_1)} \Omega_1^{\alpha(\hat{s}_2)\dot{\alpha}(\hat{s}_3)} \Omega_2^{\beta(\hat{s}_1)}_{\dot{\alpha}(\hat{s}_3)} \\ & + g_1 d\Phi_{3,\alpha(\hat{s}_2)\beta(\hat{s}_1-1)} \Omega_1^{\alpha(\hat{s}_2)\dot{\alpha}(\hat{s}_3)} \Phi_2^{\beta(\hat{s}_1-1)}_{\dot{\alpha}(\hat{s}_3)} \\ & + g_2 d\Phi_{3,\alpha(\hat{s}_2-1)\beta(\hat{s}_1)} \Phi_1^{\alpha(\hat{s}_2-1)\dot{\alpha}(\hat{s}_3)} \Omega_2^{\beta(\hat{s}_1)}_{\dot{\alpha}(\hat{s}_3)} + h.c. \end{aligned} \quad (4.2)$$

We pick this case to demonstrate how the invariance under the supertransformations is achieved. Let us begin with ζ^α -transformations:

$$\begin{aligned} \delta\Omega_1^{\alpha(\hat{s}_2)\dot{\alpha}(\hat{s}_3)} &= iC_1 \Phi_1^{\alpha(\hat{s}_2-1)\dot{\alpha}(\hat{s}_3)} \zeta^\alpha \\ \delta\Omega_2^{\beta(\hat{s}_1)}_{\dot{\alpha}(\hat{s}_3)} &= iC_2 \Phi_2^{\beta(\hat{s}_1-1)}_{\dot{\alpha}(\hat{s}_3)} \zeta^\beta \\ \delta\Phi_{3,\alpha(\hat{s}_2)\beta(\hat{s}_1-1)} &= -\tilde{C}_3 \Omega_{3,\alpha(\hat{s}_2)\beta(\hat{s}_1-1)} \gamma^\gamma \zeta^\gamma \\ \delta\Phi_{3,\alpha(\hat{s}_2-1)\beta(\hat{s}_1)} &= -\tilde{C}_3 \Omega_{3,\alpha(\hat{s}_2-1)\beta(\hat{s}_1)} \gamma^\gamma \zeta^\gamma \end{aligned}$$

Calculating the variation of the cubic vertex we obtain:

$$\begin{aligned} \delta V_{2a} = & [\tilde{C}_3 g_2 - \hat{s}_2 C_1 g_0] d\Omega_{3,\alpha(\hat{s}_2-1)\beta(\hat{s}_1)} \gamma^\gamma \Phi_1^{\alpha(\hat{s}_2-1)\dot{\alpha}(\hat{s}_3)} \Omega_2^{\beta(\hat{s}_1)}_{\dot{\alpha}(\hat{s}_3)} \zeta^\gamma \\ & + [\tilde{C}_3 g_1 - \hat{s}_1 C_2 g_0] d\Omega_{3,\alpha(\hat{s}_2)\beta(\hat{s}_1-1)} \gamma^\gamma \Omega_1^{\alpha(\hat{s}_2)\dot{\alpha}(\hat{s}_3)} \Phi_2^{\beta(\hat{s}_1-1)}_{\dot{\alpha}(\hat{s}_3)} \zeta^\gamma \\ & + i[\hat{s}_1 C_2 g_2 - \hat{s}_2 C_1 g_1] d\Phi_{3,\alpha(\hat{s}_2-1)\beta(\hat{s}_1-1)} \gamma^\gamma \Phi_1^{\alpha(\hat{s}_2-1)\dot{\alpha}(\hat{s}_3)} \Phi_2^{\beta(\hat{s}_1-1)}_{\dot{\alpha}(\hat{s}_3)} \zeta^\gamma \end{aligned}$$

This variation vanishes due to the relations (3.8). Now consider $\zeta^{\dot{\alpha}}$ -transformations:

$$\begin{aligned} \delta\Omega_{3,\alpha(\hat{s}_2)\beta(\hat{s}_1)} &= iC_3 e^{\gamma\dot{\alpha}} Y_{\alpha(\hat{s}_2)\beta(\hat{s}_1)} \gamma^\gamma \zeta_{\dot{\alpha}} \\ \delta\Phi_1^{\alpha(\hat{s}_1-1)\dot{\alpha}(\hat{s}_3)} &= \tilde{C}_1 \Omega_1^{\alpha(\hat{s}_1-1)\dot{\alpha}(\hat{s}_3)} \dot{\beta} \zeta_{\dot{\beta}} \\ \delta\Phi_2^{\beta(\hat{s}_1-1)}_{\dot{\alpha}(\hat{s}_3)} &= -\tilde{C}_2 \Omega_2^{\beta(\hat{s}_1-1)}_{\dot{\alpha}(\hat{s}_3)} \dot{\beta} \zeta^{\dot{\beta}} \end{aligned}$$

This produces:

$$\begin{aligned} \delta\mathcal{L}_1 = & -C_3 g_0 e^{\gamma\dot{\beta}} dY_{\alpha(\hat{s}_2)\beta(\hat{s}_1)} \gamma^\gamma \dot{\beta} \Omega_1^{\alpha(\hat{s}_2)\dot{\alpha}(\hat{s}_3)} \Omega_2^{\beta(\hat{s}_1)}_{\dot{\alpha}(\hat{s}_3)} \\ & + \tilde{C}_1 g_2 d\Phi_{3,\alpha(\hat{s}_2-1)\beta(\hat{s}_1)} \Omega_1^{\alpha(\hat{s}_2-1)\dot{\alpha}(\hat{s}_3)} \dot{\beta} \zeta_{\dot{\beta}} \Omega_2^{\beta(\hat{s}_1)}_{\dot{\alpha}(\hat{s}_3)} \\ & - \tilde{C}_2 g_1 d\Phi_{3,\alpha(\hat{s}_2)\beta(\hat{s}_1-1)} \Omega_1^{\alpha(\hat{s}_2)\dot{\alpha}(\hat{s}_3)} \Omega_2^{\beta(\hat{s}_1-1)}_{\dot{\alpha}(\hat{s}_3)} \dot{\beta} \zeta^{\dot{\beta}} \end{aligned}$$

Using the on-shell identities, in-particular

$$d\Phi_{3,\alpha(2s_3-3)} \approx -E^{\beta(2)} Y_{\alpha(2s_3-3)\beta(2)}$$

and the fact that the Lagrangian is defined up to total derivative, one can transform δV_{2a} to the following expression:

$$\begin{aligned} \delta V_{2a} = & \left[\hat{s}_2 C_3 g_0 - \tilde{C}_1 g_2 \right] E^{\gamma(2)} Y_{\alpha(\hat{s}_2-1)\beta(\hat{s}_1)\gamma(2)} \Omega_1^{\alpha(\hat{s}_2-1)\dot{\alpha}(\hat{s}_3)\dot{\beta}} \Omega_2^{\beta(\hat{s}_1)}{}_{\dot{\alpha}(\hat{s}_3)} \zeta_{\dot{\beta}} \\ & + \left[\tilde{C}_2 g_1 - \hat{s}_1 C_3 g_0 \right] E^{\gamma(2)} Y_{\alpha(\hat{s}_2)\beta(\hat{s}_1-1)\gamma(2)} \Omega_1^{\alpha(\hat{s}_2)\dot{\alpha}(\hat{s}_3)} \Omega_2^{\beta(\hat{s}_1-1)}{}_{\dot{\alpha}(\hat{s}_3)} \zeta^{\dot{\beta}} \end{aligned}$$

This expression also vanishes due to the relations (3.8).

Case 2b. We consider the case of superspins $Y_{s_1-1/2}, Y_{s_2}, Y_{s_3}$ with $s_2 > s_3$. The general form of the flat vertex is:

$$V_{2b} = ig_0 V_0(\Omega_1, \Omega_2, \Omega_3) + g_1 V_1(\Omega_1, \Psi_2, \Psi_3) + g_3 V_3(\Phi_1, \Psi_2, \Omega_3) \quad (4.3)$$

In case of sufficiently different spins it can be written as:

$$\begin{aligned} V_{2b} = & ig_0 D\Omega_{3,\alpha(\hat{s}_2)\beta(\hat{s}_1)} \Omega_1^{\alpha(\hat{s}_2)\dot{\alpha}(\hat{s}_3)} \Omega_2^{\beta(\hat{s}_1)}{}_{\dot{\alpha}(\hat{s}_3)} \\ & + g_1 D\Psi_{3,\alpha(\hat{s}_2)\beta(\hat{s}_1+1)} \Omega_1^{\alpha(\hat{s}_2)\dot{\alpha}(\hat{s}_3)} \Psi_2^{\beta(\hat{s}_1+1)}{}_{\dot{\alpha}(\hat{s}_3)} \\ & + g_3 D\Omega_{3,\alpha(\hat{s}_2-1)\beta(\hat{s}_1+1)} \Phi_1^{\alpha(\hat{s}_2-1)\dot{\alpha}(\hat{s}_3)} \Psi_2^{\beta(\hat{s}_1+1)}{}_{\dot{\alpha}(\hat{s}_3)} + h.c. \end{aligned} \quad (4.4)$$

Case 2c. This case corresponds to superspins $Y_{s_1}, Y_{s_2}, Y_{s_3-1/2}$. The general form of the flat vertex is:

$$V_{2c} = g_1 V_1(\Omega_1, \Psi_2, \Phi_3) + g_2 V_2(\Psi_1, \Omega_2, \Phi_3) + g_3 V_3(\Psi_1, \Psi_2, \Omega_3) \quad (4.5)$$

In case of sufficiently different spins, the vertex has the form:

$$\begin{aligned} V_{2c} = & g_1 D\Phi_{3,\alpha(\hat{s}_2-1)\beta(\hat{s}_1)} \Omega_1^{\alpha(\hat{s}_2-1)\dot{\alpha}(\hat{s}_3+1)} \Psi_2^{\beta(\hat{s}_1)}{}_{\dot{\alpha}(\hat{s}_3+1)} \\ & + g_2 D\Phi_{3,\alpha(\hat{s}_2)\beta(\hat{s}_1-1)} \Psi_1^{\alpha(\hat{s}_2)\dot{\alpha}(\hat{s}_3+1)} \Omega_2^{\beta(\hat{s}_1-1)}{}_{\dot{\alpha}(\hat{s}_3+1)} \\ & + g_3 D\Omega_{3,\alpha(\hat{s}_2)\beta(\hat{s}_1)} \Psi_1^{\alpha(\hat{s}_2)\dot{\alpha}(\hat{s}_3+1)} \Psi_2^{\beta(\hat{s}_1)}{}_{\dot{\alpha}(\hat{s}_3+1)} + h.c. \end{aligned} \quad (4.6)$$

5 Conclusion

We have constructed supersymmetric cubic vertices in the four-dimensional space for three massless supermultiplets with all their spins obeying the strict triangle inequality, $s_1 < s_2 + s_3$. Our procedure is a straightforward generalisation of the so-called Fradkin-Vasiliev formalism to the case of massless higher spin supermultiplets and it also based on our previous results in [9]. First of all, we construct supersymmetric vertices in AdS space and show that each such vertex necessarily contains all four (what we call) elementary vertices. At the same time, in the flat limit $\lambda \rightarrow 0$ one of the elementary vertices always vanishes in complete agreement with the Metsaev's classification [1]. We provide simple examples of the flat supersymmetric vertices and directly check that they are invariant under the supertransformations.

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A Notation and conventions

We use the same notations and conventions as in our previous works [9, 13]. First of all, all objects are the multispinors in their local indices $\Phi^{\alpha(k)\dot{\alpha}(l)}$, where $\alpha, \dot{\alpha} = 1, 2$ and k, l denote the number of completely symmetric undotted and dotted indices, respectively. All indices denoted by the same letter and placed on the same level are assumed to be symmetrized, where symmetrization is understood as the sum of the minimal number of terms necessary without normalization factor.

In the frame-like formalism, which we use, the fields are one-forms. We omit the wedge product signs for brevity. We work in AdS_4 space, characterized by a cosmological constant $\Lambda = -\lambda^2$ and its flat limit, described by the background frame $e^{\alpha\dot{\alpha}}$, which is one-form, and by the (exterior) background Lorentz covariant derivative D , for which the following identities hold:

$$De^{\alpha\dot{\alpha}} = 0, \quad D^2\Phi^{\alpha(k)\dot{\alpha}(l)} = -\lambda^2 \left[E^\alpha{}_\beta \Phi^{\alpha(k-1)\beta\dot{\alpha}(l)} + E^{\dot{\alpha}}{}_{\dot{\beta}} \Phi^{\alpha(k)\dot{\alpha}(l-1)\dot{\beta}} \right]$$

Here $E^{\alpha(2)}$ and $E^{\dot{\alpha}(2)}$ are the components of the background two-forms defined by the following identity:

$$e^{\alpha\dot{\alpha}} e^{\beta\dot{\beta}} = \epsilon^{\alpha\beta} E^{\dot{\alpha}\dot{\beta}} + \epsilon^{\dot{\alpha}\dot{\beta}} E^{\alpha\beta}$$

B Elementary cubic vertices

For concreteness we consider here the case of the bosonic vertices. As we have already mentioned, the first step in the construction is find such quadratic deformation of all gauge invariant curvatures $\Delta\mathcal{R} \sim \Omega\Omega$ that the deformed curvatures $\hat{\mathcal{R}} = \mathcal{R} + \Delta\mathcal{R}$ transform covariantly under the (deformed) gauge transformations: $\delta\hat{\mathcal{R}} \sim \mathcal{R}\eta$. This step is carried out for each three fields independently, so let us give here the result for the first one:

$$\begin{aligned} \Delta\mathcal{R}_1^{\alpha(2s_1-2-m)\dot{\alpha}(m)} &= \sum_{k=0}^{\hat{s}_1} \sum_{l=0}^{\min(m, \hat{s}_2)} \frac{a}{(\hat{s}_1 - k)!k!} \\ &\times \Omega_2^{\alpha(\hat{s}_3-m+l)\beta(\hat{s}_1-k)\dot{\alpha}(m-l)\dot{\beta}(k)} \Omega_3^{\alpha(\hat{s}_2-l)\dot{\alpha}(l)}_{\beta(\hat{s}_1-k)\dot{\beta}(k)} \end{aligned} \quad (\text{B.1})$$

Here we introduced convenient combinations

$$\hat{s}_1 = s_2 + s_3 - s_1 - 1, \quad \hat{s}_2 = s_1 + s_3 - s_2 - 1, \quad \hat{s}_3 = s_1 + s_2 - s_3 - 1.$$

Note that for spins $s_{1,2,3}$ satisfying the strict triangular inequality these combinations are non-negative. Note also that they are always integer even then two of the particles are fermions. As we see, the deformation is determined up to one arbitrary parameter a ; we denote b and c the corresponding parameters for the second and third particles.

The second step is to take the sum of the three Lagrangians with the initial curvatures replaced by the deformed ones and require it to be gauge invariant on-shell. This leads to the following relations:

$$\frac{(-1)^{s_1}(2s_1-2)!}{\lambda^{2s_1-2}}a = \frac{(-1)^{s_1+s_3}(2s_2-2)!}{\lambda^{2s_2-2}}b = \frac{(-1)^{s_3}(2s_3-2)!}{\lambda^{2s_3-2}}c \quad (\text{B.2})$$

Such procedure produces cubic terms with the number of derivatives up to $N_{\max} = s_1 + s_2 + s_3 - 2$, while the corresponding flat space analogue has only $N_0 = s_1 + s_2 - s_3$. In space-time with $d > 4$ these higher derivatives terms would reproduce the so-called abelian vertices [12], which are absent in $d = 4$. As we have shown in [9], all the terms with $N > N_0$ derivatives can be combined into total derivative or vanish on-shell. This allows us to get a non-singular flat limit $\lambda \rightarrow 0$ and obtain the corresponding flat versions. If g is the coupling constant for such vertex, then for the deformation parameters a, b, c we have, for example:

$$\frac{(-1)^{s_1}(2s_1-2)!}{(\hat{s}_1)!(\hat{s}_2)!(\hat{s}_3)!}a = \lambda^{2s_1-2s_3}g \quad (\text{B.3})$$

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