Erratum: Chaos and entanglement spreading in a non-commutative gauge theory

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Erratum to: JHEP11(2018)072

Abstract: We point out minor mistakes appearing in the published version of our paper [1]. The main conclusions remain unaffected.

After the publication of the paper [1] minor errors were found, specifically, in the computation of the butterfly velocity.

First, in section 3 we forgot to take into account the effects of the compact space. To compensate for this fact, we can let the indices in equation (3.1) take values $i, j = 1, 2, \ldots 9$, where the additional coordinates, x^i with $i = 4, 5, \ldots, 9$, represent the coordinates on the S^5. We also need to take $k^i = 0$ for $i = 4, 5 \ldots, 9$, in equation (3.10), so the shockwave has momentum only along the boundary coordinates. With these changes, and noticing that the metric on the S^5 can be written as $G_{\theta\theta}(r) \, d\Omega_5^2$, we need to introduce the following changes:

- Equation (3.21) should now read:

$$M^2 = \left(\frac{2\pi}{\beta}\right)^2 \frac{G^{ii}(r_{\mathrm{H}})G'_{ii}(r_{\mathrm{H}})}{G'_{tt}(r_{\mathrm{H}})} = \left(\frac{2\pi}{\beta}\right)^2 \frac{1}{G'_{tt}(r_{\mathrm{H}})} \left[\frac{G'_{11}}{G_{11}} + 2\frac{G'_{22}}{G_{22}} + 5\frac{G'_{\theta\theta}}{G_{\theta\theta}}\right] . \quad (1)$$

- Equation (3.27) should be replaced by:

$$v_B^2(\phi) = \frac{G'_{tt}}{\left(2\frac{G'_{22}}{G_{22}} + \frac{G'_{11}}{G_{11}} + 5\frac{G'_{\theta\theta}}{G_{\theta\theta}}\right)} \frac{1}{G_{22} \sin^2 \phi + G_{11} \cos^2 \phi}\bigg|_{r=r_{\mathrm{H}}} . \quad (2)$$

Therefore, the correct version of equations (3.28) and (3.29) must be:

$$v_{B,x_1}^2 \equiv v_B^2(\phi = 0) = \frac{G'_{tt}}{G_{11}\left(2\frac{G'_{22}}{G_{22}} + \frac{G'_{11}}{G_{11}} + 5\frac{G'_{\theta\theta}}{G_{\theta\theta}}\right)}\bigg|_{r=r_{\mathrm{H}}} , \quad (3)$$

and

$$v_{B,x_2}^2 \equiv v_B^2(\phi = \pi/2) = \frac{G'_{tt}}{G_{22}\left(2\frac{G'_{22}}{G_{22}} + \frac{G'_{11}}{G_{11}} + 5\frac{G'_{\theta\theta}}{G_{\theta\theta}}\right)}\bigg|_{r=r_{\mathrm{H}}} , \quad (4)$$

respectively.

- The final result for the components of the butterfly velocity, equations (3.30) and (3.31) should now be:

$$v_B^2(\phi = 0) = v_{B,x_1}^2 = \frac{2}{3} , \quad (5)$$

and

$$v_B^2(\phi = \pi/2) = v_{B,x_2}^2 = \frac{2}{3}(1 + a^4 r_{\mathrm{H}}^4) . \quad (6)$$

In summary, the component of the butterfly velocity along the commutative x_1-direction turns out to be independent on ar_{H}, taking the universal conformal value, i.e., $v_{B,x_1}^2 = 2/3$. On the other hand, the components along the non-commutative plane approach the conformal value only in the IR, $v_{B,x_2}^2(ar_{\mathrm{H}} \to 0) \to 2/3$, and grow monotonically as ar_{H} is increased. The final formula for these components differs from the one previously reported, however, all of the qualitative features and main conclusions remain the same (for these components). We also note that figure 1 in the paper should be replaced by the figure below:
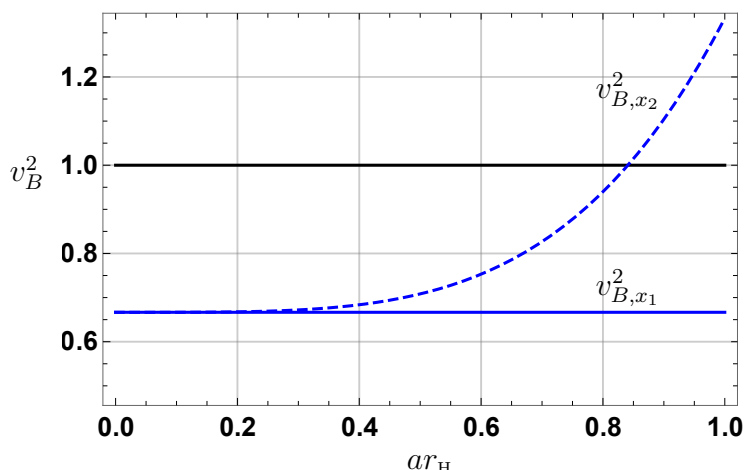
Figure 1. Butterfly velocity squared $v_{\rm B}^2$ versus the dimensionless parameter $ar_{\rm H}$. The horizontal blue line represents the butterfly velocity along the x_1-direction, which does not depend on $ar_{\rm H}$ and takes the conformal value $v_{\rm B}^2 = 2/3$, while the dashed blue curve represents the butterfly velocity along the x_2- and x_3-directions. The horizontal black line corresponds to the speed of light.

The section 5 requires similar modifications. In equation (5.1) we must let the indices take values $i, j = 1, 2, \ldots, D$, where the extra coordinates x^i with $i = d, d+1, \ldots, D$ describe the compact space, $G_{\theta\theta}(r)\, d\Omega_5^2$ in our case. With this in mind, equation (5.8) should now read:

$$M^2 = \left(\frac{2\pi}{\beta}\right)^2 \frac{G^{ii}(r_{\rm H})G'_{ii}(r_{\rm H})}{G'_{tt}(r_{\rm H})} = \left(\frac{2\pi}{\beta}\right)^2 \frac{1}{G'_{tt}(r_{\rm H})} \left[\frac{G'_{11}}{G_{11}} + 2\frac{G'_{22}}{G_{22}} + 5\frac{G'_{\theta\theta}}{G_{\theta\theta}}\right] . \tag{7}$$

The rest of the section is unchanged. In particular, equation (5.15) yields the right answer for the components of the butterfly velocity provided that the sum over k runs over all coordinates, including those in the compact space. Indeed, it can be checked that with this formula we can recover the results that reported above using the shockwave method, equations (5) and (6), respectively.

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References

[1] W. Fischler, V. Jahnke and J.F. Pedraza, *Chaos and entanglement spreading in a non-commutative gauge theory*, *JHEP* **11** (2018) 072 [arXiv:1808.10050] [INSPIRE].