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A study of the corrections to factorization in $\bar{B}^0 \rightarrow D^{*+} \omega \pi^-$

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ABSTRACT: A factorization hypothesis is tested by examining a form factor of the $\omega\pi$ production in hadronic $B^0 \rightarrow D^{*\pm} \omega \pi^\mp$ decays. The form factor is compared to that from available τ -lepton as well as e^+e^- data using the conserved vector current hypothesis. The difference of normalizations of form factor shapes from B and τ (e^+e^-) data indicates the important role of the large N_c limit in QCD. Moreover, the growth of the difference between the form factors with the $\omega\pi$ invariant mass is related to the perturbative QCD corrections of factorization. The current precision of B data does not allow one to find any evidence of corrections to factorization. A promising study could be performed with the Belle II and LHCb data sets.

KEYWORDS: B physics, e+-e- Experiments, QCD, Tau Physics

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1 Introduction

Non-leptonic decays of B mesons are usually considered through factorization approximation where decay amplitudes are factorized into products of hadronic matrix elements of color-singlet currents. Such approximation is not exact because only interaction between quarks in the same hadron is taken into account without consideration of gluon effects which redistribute the quarks. Corrections to factorization are, thus, important and could be extracted experimentally.

Hadronic decays of the B meson are dominated by $B \rightarrow D^{(*)}X$ transitions, where X denotes a system of one, two or more pions. An experimental factorization test suggested in ref. [2] studies the correction to factorization as a function of the invariant mass of the system X with two or more pions. In this paper we focus on the subset of the system X consisting of the ω and π mesons.

In 2015 the Belle collaboration reported a detailed amplitude analysis of the $\bar{B}^0 \rightarrow D^{*+}\omega\pi^-$ decays based on the full $B\bar{B}$ data sample at the $\Upsilon(4S)$ resonance [1]. This is a clean enough system to perform a factorization test because background associated with the ω meson emitted from the $(\bar{c}b)$ current is suppressed. The decay under consideration arises predominantly due to the weak interaction with color-favored and color-suppressed contributions shown in figure 1 taken from ref. [1].

The D^{**} and ρ -resonant contributions are fully extracted from amplitude analysis in ref. [1]. It allows us to test factorization separately in their production regions.

The D^{**} rate is found to be of the order of 15% [1]. A test of factorization in the D^{**} region is based on the polarization measurements of the D^{**} resonances. Observable significant transverse polarizations together with heavy quark symmetry can imply nonfactorizable effects in this channel. The factorization and structure of the $(V-A)$ weak current forbids production of resonances with spin > 1 . Observation of the $D_2^*(2460)$ resonance with spin of 2 reported in ref. [1] therefore directly violates factorization in this region.

Our goal is to test factorization in a color-favored channel with production of two vector resonances, off-shell $\rho(770)$ and $\rho(1450)$, which dominate in the total branching fraction. The influence of the model parameters used to describe the D^{**} states on the accuracy of the factorization test in the $\omega\pi$ resonant region is not significant. This is demonstrated by table III in ref. [1] where the dominant model uncertainties are shown.

Figure 1. (color online). (a) Color-suppressed and (b) color-favored quark diagrams taken from ref. [1] and showing D^{**} and $\omega\pi$ production in $\bar{B}^0 \rightarrow D^{*+}\omega\pi^-$ decays.

The last column in this table shows the uncertainty from the mixing of the D^{**} states. Variations of parameters of the ρ -like states, associated with this uncertainty, are small compared to other uncertainties.

A test of factorization can be performed in two ways. The first utilizes the fraction of the longitudinal polarization $\mathcal{P}_{D^*}$ of the D^* which should be the same as in the related semileptonic decay $\bar{B}^0 \rightarrow D^{*+}l^-\nu_l$ at squared four-momentum transfer equal to the mass squared of the intermediate ρ -like resonance [3]. Such a test has been performed in the CLEO [4] and BaBar [5] analyses and confirmed the factorization validity within their experimental accuracy. In the Belle amplitude analysis [1] the longitudinal polarization $\mathcal{P}_{D^*}$ is fixed in part from the factorization prediction. The relative normalizations of the helicity amplitudes are fixed at values measured in $\bar{B}^0 \rightarrow D^{*+}l^-\nu_l$ [6]. Free mass and width of the $\rho(1450)$ can slightly affect the $\mathcal{P}_{D^*}$ value but it agrees well with the factorization prediction. Such a test is sensitive only to the corrections affecting different partial waves. It is also a local test at a given resonance mass point not considering the dynamic behaviour of corrections to factorization.

In the other test a form factor of $\omega\pi$ production in the hadronic $\bar{B}^0 \rightarrow D^{*+}\omega\pi^-$ decay is studied. In the factorization approximation this form factor should be the same as in $\tau \rightarrow \omega\pi\nu_\tau$ decays. Assuming the vector current to be the same in electromagnetic and weak decays (conservation of vector current or CVC), similar correspondence should also exist for the $e^+e^- \rightarrow \omega\pi^0$ process. In such a case, the isovector part of the electromagnetic current J_μ^{el} matches the weak charged current:

$$\langle \rho^0 | J_\mu^{\text{el}} | 0 \rangle = \frac{1}{\sqrt{2}} \langle \rho^- | \bar{u} \gamma_\mu d | 0 \rangle. \quad (1.1)$$

Such a test uses the distribution of the invariant mass squared, $M^2(\omega\pi)$, and allows us to test factorization over the whole accessible kinematic range. Transition form factors $F_{\omega\pi}^\tau(q^2)$ in $\tau \rightarrow \omega\pi\nu_\tau$ decays and $F_{\omega\pi}^{e^+e^-}(q^2)$ in $e^+e^- \rightarrow \omega\pi^0$ processes can be measured directly from the differential width and the Born cross section. The transition form factor $F_{\omega\pi}^B(q^2)$ can be evaluated from the amplitude analysis of the $\bar{B}^0 \rightarrow D^{*+}\omega\pi^-$ decays.

The signal matrix element M_{sig} for a color-favored channel determines the $\omega\pi$ transition form factor $F_{\omega\pi}^B(q^2)$ as a function of $q^2 = M^2(\omega\pi)$:

$$M_{\text{sig}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* a_1 F_{\omega\pi}^B(q^2) \epsilon^{\mu\nu\alpha\beta} J_\mu^{(B \rightarrow D^*)} v_\nu^* q_\alpha p_\beta. \quad (1.2)$$

Here, a_1 is the relevant QCD coefficient, $J_\mu^{(B \rightarrow D^*)}$ describes a transition current of $B \rightarrow D^*$, v_μ is a four-vector of the ω meson polarization, q_μ is a four-momentum of the $\omega\pi$ pair and p_μ is a four-momentum of the ω . In eq. (1.2), the nonfactorizable corrections are encoded into the $a_1 F_{\omega\pi}^B(q^2)$ product as well as in the $J_\mu^{(B \rightarrow D^*)}$ current. If factorization is exact, we have for all available values of q^2 :

$$a_1 F_{\omega\pi}^B(q^2) = \left(c_1(\mu) + \frac{c_2(\mu)}{3} \right) F_{\omega\pi}^{\tau(e^+e^-)}(q^2), \quad (1.3)$$

where the Wilson coefficients $c_1(\mu)$ and $c_2(\mu)$ are renormalized at the scale of μ , and the current $J_\mu^{(B \rightarrow D^*)}$ is extracted from $B \rightarrow D^* l \nu_l$ data. Results of the amplitude analysis in ref. [1] are obtained under the assumption that corrections affecting the polarization of the D^* are absent. In such a case, the current $J_\mu^{(B \rightarrow D^*)}$ takes into account only nontrivial final-state interaction phases in helicity amplitudes (see appendix C in ref. [1]) which cancel after integration over angular variables. These phases have been measured in ref. [1], although uncertainties are large.

As a consequence, such a test is not exhaustive and should be considered as a complementary one to the first test discussed above. The corrections to factorization δ_{NF} in frame of the discussed test are determined as

$$1 + \delta_{\text{NF}} = \frac{|a_1 F_{\omega\pi}^B(q^2)|}{\left(c_1(\mu) + \frac{c_2(\mu)}{3} \right) |F_{\omega\pi}^{\tau(e^+e^-)}(q^2)|}. \quad (1.4)$$

The parameter δ_{NF} in eq. (1.4) describes nonfactorizable contributions to $B \rightarrow D^* \omega \pi$ decay appearing in different ways. The first way is related to the $1/N_c$ expansion in QCD where factorization does not depend on the mass of the produced $\omega\pi$ system [7]. As factorization ignores the color of the quarks produced by the virtual W, it is instructive to rewrite the B decay amplitude in a way that restores this dependence. In this case the effective coefficient a_1 in eq. (1.2) is written as:

$$a_1 = \left(c_1(\mu) + \frac{c_2(\mu)}{3} \right) [1 + \epsilon_1(\mu)] + c_2(\mu) \epsilon_8(\mu), \quad (1.5)$$

where hadronic parameters $\epsilon_1(\mu)$ and $\epsilon_8(\mu)$ determine the nonfactorizable contributions appearing from the color-singlet and color-octet current operators. In the large N_c limit, $|\epsilon_1| \ll 1$, whereas contributions from ϵ_8 can be more sizeable and $\epsilon_8 > 0$ [7]. In such a case,

$$a_1 = c_1(\mu) + \zeta c_2(\mu), \quad (1.6)$$

where $\zeta = 1/3 + \epsilon_8(\mu)$. Since $c_2(\mu) < 0$, the effective constant a_1 is expected to be slightly less than the naive factorization prediction.

The second way corresponds to the perturbative QCD [8]. This picture of factorization expands the B decay amplitude in powers of $M(\omega\pi)/m_b$. As the b-quark is heavy, the corrections to factorization are suppressed for a light $\omega\pi$ system. As the mass of the $\omega\pi$ system increases, the corrections become significant. It should be seen as a difference between form factor shapes extracted from B data and $\tau(e^+e^-)$ data when $M(\omega\pi)$ increases.

The factorization test could be applied above the $\omega\pi$ threshold. The form factor shape in the low-energy region is evaluated by extrapolation from the $\omega\pi$ production region and direct measurements in the ω conversion decay region. More precise data in the conversion region were obtained by the NA60 collaboration [9] from a study of the $\omega \rightarrow \mu^+\mu^-\pi^0$ decay. The NA60 data lie strongly above the prediction of the vector meson dominance (VMD) model which quite well describes the form factor in e^+e^- annihilation.

2 Analysis of the $F_{\omega\pi}(q^2)$ form factor

The form factor $F_{\omega\pi}^B(q^2)$ of $\omega\pi$ production in $B \rightarrow D^*\omega\pi$ decays can be defined as

$$F_{\omega\pi}^B(q^2) = \tilde{g}f_{\omega\pi}(q^2), \quad (2.1)$$

where $\tilde{g}$ is a coupling constant calculated from the combined fraction $f_{\rho+\rho'}$ of the ρ and ρ' in the total branching fraction $B \rightarrow D^*\omega\pi$ and

$$f_{\omega\pi}(q^2) = \sqrt{q^2} \left(\frac{F_\rho(q^2)}{D_\rho(q^2)} + A_{\rho'} e^{i\phi_{\rho'}} \frac{F_{\rho'}(q^2)}{D_{\rho'}(q^2)} \right). \quad (2.2)$$

In eq. (2.2), $F_\rho(q^2)$ ($F_{\rho'}(q^2)$) is the $\rho(770)$ ($\rho(1450)$) form factor in the decay to the $\omega\pi$ final state and $D_\rho(q^2)$ ($D_{\rho'}(q^2)$) is the Breit-Wigner denominator describing the $\rho(770)$ ($\rho(1450)$) shape. The form factors $F_\rho(q^2)$ and $F_{\rho'}(q^2)$ restrict a rapid growth of the B decay amplitude with p_ω which is the magnitude of the ω three-momentum in the $\omega\pi$ rest frame. The simple Blatt-Weisskopf parameterization is used for them:

$$F_\rho(q^2) = \frac{1}{1 + (rp_\omega)^2}, \quad F_{\rho'}(q^2) = \sqrt{\frac{1 + (rp_{0,\omega})^2}{1 + (rp_\omega)^2}}, \quad (2.3)$$

where $r = 1.6 \text{ GeV}^{-1}$ is a typical hadronic scale and $p_{0,\omega}$ is the ω three-momentum p_ω , when $q^2 = m_{\rho'}^2$. The functions $D_{\rho(\rho')}(q^2)$ are given by

$$D_{\rho(\rho')}(q^2) = q^2 - m_{\rho(\rho')}^2 + i\sqrt{q^2}\Gamma_{\rho(\rho')}(q^2), \quad (2.4)$$

where $\Gamma_\rho(q^2)$ and $\Gamma_{\rho'}(q^2)$ are the q^2 -dependent widths of the $\rho(770)$ and $\rho(1450)$ resonances. The width $\Gamma_\rho(q^2)$ ($\Gamma_{\rho'}(q^2)$) is defined in ref. [1] (eqs. (B5) and (B6)) with the additional factor of $m_\rho/\sqrt{q^2}$ ($m_{\rho'}/\sqrt{q^2}$) arising from the different definitions of $D_{\rho(\rho')}(q^2)$ in eq. (2.4) and in ref. [1] (eq. (B3)). In eq. (2.2), parameterization of the $f_{\omega\pi}(q^2)$ form factor is different from the VMD model. The difference is that the resonance masses m_ρ and $m_{\rho'}$ are replaced with the invariant mass $M(\omega\pi)$. It was found that a fit to the B data with the VMD function leads to the worse data description. It corresponds to the negative log-likelihood value lying about 3σ away from the global minimum obtained with the model in eq. (2.2). Therefore the model in eq. (2.2) is chosen as acceptable for the current B data description.

To obtain the form factor defined in eq. (2.1), a coupling constant $\tilde{g}$ should be determined but it could be extracted only combined with the coefficient a_1 . A product of $a_1\tilde{g}$

is determined by the color-favored branching fraction:

$$a_1 \tilde{g} = \frac{8\pi\sqrt{3\pi}m_B}{G_F\mathcal{F}(1)|V_{cb}||V_{ud}|} \sqrt{\frac{f_{\rho+\rho'}\Gamma(B \rightarrow D^*\omega\pi)}{J}}, \quad (2.5)$$

where

$$J = \int p_\omega^3 p_{D^*} \sqrt{q^2} |f_{\omega\pi}(q^2)|^2 (|f_S(q^2)|^2 + |f_P(q^2)|^2 + |f_D(q^2)|^2) dq^2. \quad (2.6)$$

In eq. (2.6), $f_S(q^2)$, $f_P(q^2)$ and $f_D(q^2)$ are the partial wave form factors describing a transition $B \rightarrow D^*$. In the frame of heavy quark effective theory they can be related to the Isgur-Wise function with a parameter ρ^2 , relative factors R_1 and R_2 and normalization factor $\mathcal{F}(1)$. The linear approximation of the Isgur-Wise function is used in ref. [1] with parameters ρ^2 , R_1 and R_2 measured in ref. [6]. This very simple parameterization was sufficient for amplitude analysis in ref. [1]. However, at the moment we have to know the product $\mathcal{F}(1) \times |V_{cb}|$ which has been measured with the best accuracy by the Belle collaboration [10] in the Caprini-Lellouch-Neubert (CLN) parameterization [11]. For consistency, we refit the $\bar{B}^0 \rightarrow D^{*+}\omega\pi^-$ data [1] with the CLN function where parameters ρ^2 , R_1 and R_2 are fixed at their values from ref. [10]. This fit results in a relative ρ' -strength $A_{\rho'} = 0.19 \pm 0.05$, relative ρ' -phase $\phi_{\rho'} = 2.52 \pm 0.11$ rad, ρ' -mass $m_{\rho'} = 1540 \pm 22$ MeV and ρ' -width $\Gamma_{\rho'} = 304 \pm 49$ MeV, where a statistical error only is shown. Finally, using the product $\mathcal{F}(1) \times |V_{cb}|$ fixed at the value $(35.06 \pm 0.58) \times 10^{-3}$ [10], $|V_{ud}| = 0.9742 \pm 0.0002$ [12] as well as the product $f_{\rho+\rho'} \times \mathcal{B}(B \rightarrow D^*\omega\pi) = (1.90 \pm 0.17) \times 10^{-3}$ and $\Gamma_B = 4.326 \times 10^{-13}$ GeV [12], we obtain $a_1 \tilde{g} = 2.66 \pm 0.28$. The error of $a_1 \tilde{g}$ is calculated from the error propagation formula. The main source of the error is related to the integral calculation J , where the covariance matrix of model parameters is applied.

In the factorization approximation, the effective coefficient a_1 at next-to-leading order was obtained in ref. [13]. It is renormalized at the scale of b -quark mass $\mu = m_b$ and leads to the value of $a_1(m_b) = 1.02$. The uncertainty of this value is related to the scale where Wilson coefficients are evaluated. Taking into account the values of a_1 calculated at the scales $\mu = 2m_b$ and $\mu = m_b/2$ (see [13]), we obtain $a_1 = 1.02 \pm 0.02$. In other words, $\tilde{g} = 2.61 \pm 0.28$. The coupling $\tilde{g}$ is the product of the ρ -meson weak decay constant f_ρ and $g_{\rho\omega\pi}$ coupling for the $\rho \rightarrow \omega\pi$ transition. The value for $\tilde{g}$ can be compared with that measured from the e^+e^- data. The combined SND2000 [14] and SND2016 [15] data are fit by the model in eq. (2.1), which is used for the B data. The mass and width of the ρ' resonance as well as the $\tilde{g}$ constant are free parameters in the fit which gives $\tilde{g} = 3.05 \pm 0.02$. Taking into account that $a_1 \tilde{g} = 2.66 \pm 0.28$, we obtain $a_1 = 0.87 \pm 0.09$. This value is less than $a_1(m_b) = 1.02 \pm 0.02$ obtained when the nonfactorizable corrections are neglected but they are consistent with each other within the statistical accuracy.

The $g_{\rho\omega\pi}$ value depends on parameterization of the $F_\rho(q^2)$ form factor in eq. (2.3) and should be determined from $\tilde{g}$ and f_ρ . We can estimate f_ρ experimentally from the decay width of $\rho \rightarrow e^+e^-$ using the CVC relation in eq. (1.1)

$$f_\rho = \sqrt{\frac{3m_\rho\Gamma(\rho \rightarrow e^+e^-)}{2\pi\alpha^2}}. \quad (2.7)$$

Figure 2. (color online). The $\omega\pi$ production form factor weighted by the value of a_1 with $a_1 = 1.02 \pm 0.02$ [13] and shown in a log scale. The points with error bars show e^+e^- data by the SND in ref. [14] (blue triangles, SND2000) and ref. [15] (magenta circles, SND2016), CMD-3 in ref. [19] (red circles), CMD-2 in ref. [18] (black triangles) and BaBar collaborations in ref. [20] (cyan squares) as well as ω conversion data by the NA60 in ref. [9] (open squares) and τ lepton data by the CLEO collaborations in ref. [17] (grey circles). The black solid line is the combined fit result to the SND2000 and SND2016 data taken from ref. [15]. The green dotted line corresponds to the function $a_1/\sqrt{2}|F_{\omega\pi}^B(q^2)|$. The green dashed area represents the 68% confidence level contour of statistical uncertainties of the model parameters in eq. (2.1). The zoomed-in plot in the low q^2 region is also shown.

It gives $f_\rho = (0.220 \pm 0.001) \text{ GeV}$. Finally, we obtain $g_{\rho\omega\pi} = (11.9 \pm 1.3) \text{ GeV}^{-1}$. This value can be compared with the SND value of $g_{\rho\omega\pi} = (13.9 \pm 0.1) \text{ GeV}^{-1}$ measured with the same $F_\rho(q^2)$ form factor. The difference between these values is not statistically significant. If the hadronic parameter $r \rightarrow 0$ in eq. (2.3), the SND measurement gives $g_{\rho\omega\pi} = (15.9 \pm 0.4) \text{ GeV}^{-1}$. This value is in good agreement with the prediction of QCD sum rules of 16 GeV^{-1} [16].

Now, when the form factor $F_{\omega\pi}^B(q^2)$ is fully extracted from the B data, it can be compared to τ and e^+e^- data in the frame of factorization. An additional factor $\sqrt{2}$ must be used to convert from the electromagnetic form factor to the weak one using the CVC relation in eq. (1.1).

Figure 2 demonstrates experimental data of $a_1|F_{\omega\pi}(q^2)|$ as a function of q^2 measured in τ lepton decays to $\omega\pi\nu_\tau$ by the CLEO collaboration (grey circles) [17] and $e^+e^- \rightarrow \omega\pi^0$ processes with the subsequent decay of the ω to $\pi^+\pi^-\pi^0$ or $\pi^0\gamma$.

The data from e^+e^- collisions were obtained either in direct e^+e^- annihilation by the CMD-2 (black triangles) [18], CMD-3 (red circles) [19] as well as SND (blue triangles [14] and magenta circles [15]) collaborations or using initial-state radiation (ISR) by the BaBar

Figure 3. (color online). The distribution of q^2 for $B \rightarrow D^* \omega \pi$ events measured by the Belle collaboration and taken from ref. [1]. Points with error bars show experimental data, histograms in color represent several resonant contributions and black histogram describes the signal fit. The region $4 \text{ GeV}^2 < q^2 < 5 \text{ GeV}^2$ is specially contoured.

collaboration (cyan squares) [20]. The NA60 data (open squares) [9] in the conversion $\omega \rightarrow \pi^0 \mu^+ \mu^-$ decays are also shown at low q^2 values. In figure 2, the green dotted line shows the product $a_1 |F_{\omega\pi}^B(q^2)|/\sqrt{2}$ obtained above from the B decay data. The dashed area corresponds to $\pm 1\sigma$ deviation from the line taking into account the statistical covariance matrix of parameters. The solid black line shows the fit result to the SND data ([14, 15]) by the VMD model. The VMD model cannot simultaneously describe the e^+e^- and ω conversion data.

The uncertainties in the parameterization of the form factor in eq. (2.1) give the model error which is not shown in figure 2. The reason is that the statistical uncertainties are still large. We believe that the model uncertainty distorting the form factor shape will decrease with a statistical error.

The $\omega\pi$ form factor extracted from the Belle data set in ref. [1] agrees well in shape with predictions obtained from τ and e^+e^- data in the region $q^2 < 4 \text{ GeV}^2$ but has a lower normalization. However, the difference between normalizations is not significant and both values are consistent with each other within the Belle form factor accuracy. More B data are needed to make a detailed comparison.

In the region above $q^2 = 4 \text{ GeV}^2$ the e^+e^- ISR data collected by the BaBar collaboration [20] are available in the e^+e^- sector. The clear bump is seen in the region between $q^2 = 4 \text{ GeV}^2$ and $q^2 = 5 \text{ GeV}^2$. This bump can be also seen in the Belle data. Figure 3 shows the overall $q^2 = M^2(\omega\pi)$ distribution for the $B \rightarrow D^* \omega \pi$ decays. The region of interest $4 \text{ GeV}^2 < q^2 < 5 \text{ GeV}^2$ is specially contoured. A similar structure is seen here but it is not statistically significant. The black histogram describes the data using the signal model in ref. [1]. The signal model is not sensitive to the bump and has a smooth shape. Note that the bump between $q^2 = 4 \text{ GeV}^2$ and 5 GeV^2 is most probably due to the $\rho(2150)$,

a broad structure around $2.15 - 2.2 \text{ GeV}$ observed in different final states [12]. It could be a reason of different shapes of the $\omega\pi$ form factor extracted in e^+e^- and B decay data shown in figure 2. The Belle sensitivity is not sufficient to identify the bump in figure 3. More B data are needed to make a definite conclusion about its origin.

3 Conclusion

Corrections to factorization are studied in $B \rightarrow D^*\omega\pi$ decays through the $\omega\pi$ production form factor extracted from the amplitude analysis of the Belle data [1]. The form factor is compared to available measurements of τ -lepton decays and e^+e^- annihilation.

The advantage of this approach is that the contributions of D^{**} states and ρ -like resonances can be exactly separated from each other. It is crucial to perform the factorization test in the ρ -like production region because the integrated contribution from the D^{**} states was found to be significant (about 15%). The $q^2 = M^2(\omega\pi)$ differential mass distribution in the ρ -like enriched region deteriorates by the D^{**} contribution and the factorization interpretation for the decay becomes less clear. It prevents from using the experimental q^2 distribution normalized to the semileptonic rate. Our approach is model-dependent but the model uncertainty will be controlled in a more rigorous way when the statistical accuracy is improved.

Figure 2 shows a product $a_1|F_{\omega\pi}(q^2)|$ of the $\omega\pi$ form factor and efficient constant a_1 obtained from the B data and predicted from factorization using the τ and e^+e^- data. The large uncertainties appearing when the $\omega\pi$ form factor is extracted from the B data do not allow us to observe corrections to factorization in a statistically significant manner. Experimental analysis of available data demonstrates that $\omega\pi$ production is similar in B decays and e^+e^-/τ -lepton processes. The uncertainty in the range below $q^2 = 1.2 \text{ GeV}$ (around 10%) is mainly due to the knowledge of the $a_1\tilde{g}$ product whereas parameters of the ρ' resonance contribute to the uncertainty at higher q^2 values (around 15%).

For a precision study of corrections to factorization in the $\bar{B}^0 \rightarrow D^{*+}\omega\pi^-$ decay it is necessary to perform a high-statistics analysis with a data set available at LHCb and, in future, Belle II detectors.

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