Magnetically charged AdS_5 black holes from class $\mathcal{S}$ theories on hyperbolic 3-manifolds

Jin-Beom Bae,^a Dongmin Gang^b and Kimyeong Lee^a

^a*Quantum Universe Center, Korea Institute for Advanced Study,
85 Hoegiro, Seoul 02445, Republic of Korea*

^b*School of Physics, Korea Institute for Advanced Study,
85 Hoegiro, Seoul 02455, Republic of Korea*

E-mail: bae@maths.ox.ac.uk, arima275@kias.re.kr, klee@kias.re.kr

ABSTRACT: We study the twisted index of 4d $\mathcal{N} = 2$ class $\mathcal{S}$ theories on a closed hyperbolic 3-manifold M_3 . Via 6d picture, the index can be written in terms of topological invariants called analytic torsions twisted by irreducible flat connections on the 3-manifold. Using the topological expression, we determine the full perturbative $1/N$ expansion of the twisted index. The leading part nicely matches the Bekestein-Hawking entropy of a magnetically charged black hole in the holographic dual AdS_5 with $AdS_2 \times M_3$ near-horizon.

KEYWORDS: AdS-CFT Correspondence, Black Holes in String Theory, Brane Dynamics in Gauge Theories

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1 Introduction and conclusion

Microscopic understanding of black hole entropy is one of the prominent success of string theory. Indeed, it is well known that string theory can provide microscopic interpretation to the Bekenstein-Hawking entropy of asymptotically flat black hole [1]. A lot of work has been done [2–4] in order to analyze black hole entropy involving quantum corrections, based on 2d field theory approach. Meanwhile, the entropy of black hole in AdS_3 has been analyzed via AdS/CFT correspondence [5], by counting microscopic states of 2d conformal field theory(CFT) [6].

It has been believed that entropy of higher-dimensional supersymmetric black hole in $AdS_d(d > 3)$ can be understood from boundary superconformal field theory(SCFT) using AdS/CFT. Recently, there has been remarkable progresses in this direction. In [7, 8], the entropy of static dyonic BPS black hole in $AdS_4 \times S^7$ is shown to agree with the topologically twisted index of 3d ABJM model [9] with $k = 1$. More recently, many works have been done regarding entropy of black holes in AdS_5 [10–17] using refined 4d superconformal index.

In this paper, our goal is to understand microscopic origin of a magnetically charged black hole [18] in AdS_5 using twisted index of 4d $\mathcal{N} = 2$ SCFT on $S^1 \times M_3$, where M_3 is a closed hyperbolic 3-manifold. The entropy of magnetically charged black holes of our interest is not easy to analyze quantitatively via localization technique, as we consider the 4d SCFT on closed hyperbolic 3-manifolds. To circumvent this technical difficulty, we suggest alternative way of computing twisted index of a certain class of 4d $\mathcal{N} = 2$ SCFTs on $S^1 \times M_3$. We start at 6d $(2, 0)$ theory on $S^1 \times M_3 \times \Sigma_g$, where Σ_g denote Riemann surface with genus g . As we shrinking the size of Σ_g , the 6d mother theory is reduced to 4d class $\mathcal{S}$ SCFT [19, 20] on $S^1 \times M_3$. On the other hand, one can arrive to a 3d $\mathcal{N} = 2$ class $\mathcal{R}$ SCFT [21, 22] on $S^1 \times \Sigma_g$ by reducing the size of M_3 in 6d theory. Since the 6d twisted index is invariant under continuous supersymmetry preserving deformations, we expect the equality between the 4d twisted index for class $\mathcal{S}$ theory and 3d twisted index for class $\mathcal{R}$ theory. Fortunately, the 3d twisted index was already studied in [23, 24] using

the 3d-3d relation. Thus our main claim in this paper is that one can utilize the twisted index computation to analyze the entropy of magnetically charged black hole in AdS_5 . At large- N limit, we checked our speculation using supergravity solution in [18, 25].

One interesting future problem is to widen our understanding of the higher-dimensional black hole entropy up to subleading order in the $1/N$ expansion. We believe that the magnetically charged black hole in AdS_5 is an excellent testground for it, because subleading correction of twisted index already computed in [23, 24]. It would be interesting to understand the meaning of those subleading correction at the M-theory side.

2 Class $\mathcal{S}$ theories on hyperbolic 3-manifold

2.1 Twisted index of class $\mathcal{S}$ theories on 3-manifold M_3

A 4d class $\mathcal{S}$ theory $\mathcal{T}_N[\Sigma_g]$ associated to a compact Riemann surface Σ_g of genus g is defined as [19, 20]

$$\mathcal{T}_N[\Sigma_g] := (4d \mathcal{N} = 2 \text{ SCFT at the infra-red (IR) fixed point of} \quad (2.1)$$

$$\text{a twisted compactification of 6d } A_{N-1} (2,0) \text{ theory along } \Sigma_g).$$

Using $\text{SO}(2)$ subgroup of $\text{SO}(5)$ R-symmetry of the 6d theory, we perform a partial twisting along Σ_g . It means that the following background gauge fields $A_{\text{SO}(2)}^{\text{background}}$ coupled to the $\text{SO}(2)$ R-symmetry are turned on

$$\text{topological twisting: } A_{\text{SO}(2)}^{\text{background}} = \omega(\Sigma_g), \quad (2.2)$$

where $\omega(\Sigma_g)$ is the spin connection on the Riemann surface. The topological twisting preserves 8 supercharges (16 supercharges) out of original 16 supercharges for $g > 1$ ($g = 1$). For $g \geq 1$, the system is believed to flow to a non-trivial SCFT under the renormalization group. Especially when $g > 1$, the $\text{SO}(5)$ R-symmetry of 6d theory is broken to $\text{SO}(2) \times \text{SO}(3)$ due to the topological twisting. The remaining symmetry can be identified with $u(1)_R \times su(2)_R$ R-symmetry of the 4d $\mathcal{T}_N[\Sigma_g]$ theory.

$$\begin{array}{c} \text{SO}(5) \text{ R-symmetry of 6d } A_{N-1} (2,0) \text{ theory} \\ \xrightarrow{\text{twisted compactification}} \text{SO}(2) \times \text{SO}(3) \text{ R-symmetry of 4d } \mathcal{T}_N[\Sigma_g] \text{ theory} \end{array} \quad (2.3)$$

For sufficiently large N , the system does not have any emergent IR symmetry and the 4d $\mathcal{T}_N[\Sigma_{g>1}]$ is a $\mathcal{N} = 2$ SCFT without any (non-R) flavor symmetry. Then, the superconformal $u(1)_R$ symmetry can be identified with the compact $\text{SO}(2) \subset \text{SO}(2) \times \text{SO}(3)$.¹

We consider following twisted index of the $\mathcal{T}_N[\Sigma_g]$ theory that is defined on closed hyperbolic 3-manifold $M_3 = \mathbb{H}^3/\Gamma$.

$$\begin{aligned} \mathcal{I}_{M_3}(\mathcal{T}_N[\Sigma_g]) &:= \mathcal{Z}_{BPS}(4d \mathcal{T}_N[\Sigma_g] \text{ on } S^1 \times M_3) \\ &= \text{Tr}(-1)^R \end{aligned} \quad (2.4)$$

¹For small N , there could be accidental IR symmetries in $\mathcal{T}_N[\Sigma_g]$ theory.

Here, the trace is taken over the Hilbert-space of $\mathcal{T}_N[\Sigma_g]$ on M_3 . $\mathcal{Z}_{BPS}(\mathcal{T} \text{ on } \mathbb{B})$ denotes the partition function of a theory $\mathcal{T}$ on a supersymmetric background $\mathbb{B}$ while R denote the charge of IR superconformal $u(1)_R$ R-symmetry, which is normalized as

$$R(Q) = \pm 1, \quad \text{for supercharge } Q. \quad (2.5)$$

Note that the topological twisting is performed along the M_3 using the $su(2)_R$ symmetry of the 4d SCFT, to preserve some supercharges.

The twisted index can be defined for arbitrary 4d $\mathcal{N} = 2$ SCFTs using $su(2)_R \times u(1)_R$ R-symmetry. For general 4d $\mathcal{N} = 2$ SCFTs, the charge R is not integer valued and thus the index is complex valued. For $\mathcal{T}_N[\Sigma_g]$ theory with sufficiently large N , on the other hand, the twisted index is an integer because the $u(1)_R$ symmetry comes from compact $SO(5)$ R-symmetry of 6d theory. The index is invariant under the continuous supersymmetric deformations of the 4d theory, thus it only depends on the topology (not on the metric choice) of M_3 .

$$\mathcal{I}_{M_3}(\mathcal{T}_N[\Sigma_g]) : \text{a topological invariant of } M_3 \quad (2.6)$$

In the next section, we express the twisted index $\mathcal{I}_{M_3}(\mathcal{T}_N[\Sigma_g])$ in terms of previously known topological invariants on 3-manifold, *analytic torsion* twisted by irreducible flat connections.

2.2 Twisted index computation using 6d picture

For a generic 4d $\mathcal{N} = 2$ SCFT, which we will denote as $\mathcal{T}_{4d, \mathcal{N}=2}$, the computation of the twisted index $\mathcal{I}_{M_3}(\mathcal{T}_{4d, \mathcal{N}=2})$ on a hyperbolic manifold M_3 is quite challenging. Unlike 3d cases [7, 26–29], there is no developed localization formula for the 4d twisted index on hyperbolic 3-manifolds. Since the landscape of hyperbolic 3-manifolds is much wilder than hyperbolic Riemann surface, obtaining localization formula for general M_3 might be very challenging task. For class $\mathcal{S}$ theories $\mathcal{T}_N[\Sigma_g]$, we can bypass these difficulties using 6d picture.

A 4d-3d relation. The twisted index for 4d class $\mathcal{S}$ theory $\mathcal{T}_N[\Sigma_g]$ on M_3 can be obtained from the supersymmetric partition function of 6d A_{N-1} (2,0) theory on $S^1 \times M_3 \times \Sigma_g$ by shrinking the size the Σ_g :

$$\begin{aligned} & \mathcal{Z}_{BPS}(\text{6d } A_{N-1} \text{ (2,0) theory on } S^1 \times M_3 \times \Sigma_g) \\ & \xrightarrow{\Sigma_g \rightarrow 0} \mathcal{Z}_{BPS}(\text{4d } \mathcal{T}_N[\Sigma_g] \text{ on } S^1 \times M_3). \end{aligned} \quad (2.7)$$

As usual Witten index, the 6d partition function is invariant under the continuous supersymmetric deformations. One possible supersymmetric deformation is the overall size change of the compact Riemann surface Σ_g . Thus, we expect

$$\begin{aligned} & \mathcal{Z}_{BPS}(\text{6d } A_{N-1} \text{ (2,0) theory on } S^1 \times M_3 \times \Sigma_g) \\ & = \mathcal{Z}_{BPS}(\text{4d } \mathcal{T}_N[\Sigma_g] \text{ on } S^1 \times M_3). \end{aligned} \quad (2.8)$$

On the other hand, one may consider a limit where the size of M_3 shrinks. In the case, from the same argument above we expect

$$\begin{aligned} & \mathcal{Z}_{BPS}(\text{6d } A_{N-1} \text{ (2,0) theory on } S^1 \times M_3 \times \Sigma_g) \\ & = \mathcal{Z}_{BPS}(\text{3d } \mathcal{T}_N[M_3] \text{ on } S^1 \times \Sigma_g). \end{aligned} \quad (2.9)$$

Here $\mathcal{T}_N[M_3]$ is a 3d class $\mathcal{R}$ theory associated to a hyperbolic 3-manifold M_3 :

$$\mathcal{T}_N[M_3] := (\text{3d } \mathcal{N} = 2 \text{ SCFT at the IR fixed point of a twisted compactification of 6d } A_{N-1} (2,0) \text{ theory along } M_3) . \quad (2.10)$$

In the partial topological twisting along M_3 , we use a $\text{SO}(3)$ subgroup of $\text{SO}(5)$ R-symmetry of the 6d theory and the twisting preserves $1/4$ supercharges. Therefore, the resulting low-energy theory is described by a 3d $\mathcal{N} = 2$ SCFT denoted as $\mathcal{T}_N[M_3]$. Combining (2.8) and (2.9), we have a following equality between the 4d/3d twisted indices

$$\text{‘4d-3d relation’}: \mathcal{I}_{M_3}(\mathcal{T}_N[\Sigma_g]) = \mathcal{I}_{\Sigma_g}(\mathcal{T}_N[M_3]) . \quad (2.11)$$

Here, the 3d twisted index $\mathcal{I}_{\Sigma_g}(\mathcal{T}_N[M_3])$ is defined as

$$\begin{aligned} \mathcal{I}_{\Sigma_g}(\mathcal{T}_N[M_3]) &:= \mathcal{Z}_{BPS}(\text{3d } \mathcal{T}_N[M] \text{ on } S^1 \times \Sigma_g) \\ &= \text{Tr}(-1)^R , \end{aligned} \quad (2.12)$$

where the trace is taken over the Hilbert-space of $\mathcal{T}_N[M_3]$ on Σ_g .

In summary, the 4d-3d relation (2.11) is depicted in the diagram below:

$$\begin{array}{ccc} \mathcal{Z}_{BPS}(\text{6d } A_{N-1} (2,0) \text{ theory on } S^1 \times M_3 \times \Sigma_g) & & \\ \downarrow M_3 \rightarrow 0 & \searrow \Sigma_g \rightarrow 0 & \\ \mathcal{Z}_{BPS}(\text{3d } \mathcal{T}_N[M_3] \text{ on } S^1 \times \Sigma_g) & & \mathcal{Z}_{BPS}(\text{4d } \mathcal{T}_N[\Sigma_g] \text{ on } S^1 \times M_3) \end{array}$$

See [30] for previous study on the relations among supersymmetric partition functions in various dimensions originated from the same 6d picture.

$\mathcal{I}_{\Sigma_g}(\mathcal{T}_N[M_3])$ from twisted analytic torsions on M_3 . The quantity on the r.h.s. of (2.11) is much easier to handle than the quantity on the l.h.s. First, we have an explicit field theoretic description for 3d theory $\mathcal{T}_N[M_3]$ for general $N \geq 2$ and closed hyperbolic 3-manifold M_3 [22, 31, 32]. Second, we have general localization formula for the 3d twisted index. Combining these developments, recently it was found that the twisted index $\mathcal{I}_{\Sigma_g}(\mathcal{T}_N[M_3])$ is simply given as [23, 24]:

$$\mathcal{I}_{\Sigma_g}(\mathcal{T}_N[M_3]) = \sum_{\mathcal{A}^\alpha \in \chi^{\text{irred}}(M_3; N)} (N \times \mathbf{Tor}_{M_3}^\alpha)^{g-1} . \quad (2.13)$$

For a technical reason, the above relation holds only for closed hyperbolic 3-manifold with vanishing $H_1(M_3, \mathbb{Z}_N)$ [24].² The summation is over irreducible $\text{PGL}(N, \mathbb{C})$ flat connections on M_3 :

$$\chi^{\text{irred}}(M_3; N) := \frac{\{\text{irreducible } \text{PGL}(N, \mathbb{C}) \text{ flat-connections on } M_3\}}{(\text{gauge quotient})} . \quad (2.14)$$

²Generalization to general M_3 is proposed in [33].

The set is finite for generic choice of M_3 . $\mathbf{Tor}_{M_3}^\alpha$ is a topological invariant called analytic (or Ray-Singer) torsion twisted by a flat connection $\mathcal{A}^\alpha$. The topological quantity is defined as follows [34, 35]

$$\mathbf{Tor}_{M_3}^\alpha := \frac{[\det' \Delta_1(\mathcal{A}^\alpha)]^{1/4}}{[\det' \Delta_0(\mathcal{A}^\alpha)]^{3/4}}. \quad (2.15)$$

where $\Delta_n(\mathcal{A}^\alpha)$ is a Laplacian acting on $pgl(N, \mathbb{C})$ -valued n -form twisted by a flat connection $\mathcal{A}^\alpha$:

$$\Delta_n(\mathcal{A}) = d_A * d_A * + * d_A * d_A, \quad d_A = d + \mathcal{A} \wedge. \quad (2.16)$$

One non-trivial prediction of the above relation is that the topological quantity on r.h.s. is an integer. The integrality has been checked for various examples in [24], which gives a strong consistency check for the 3d-3d relation (2.13).

From (2.11) and (2.13), we finally have following simple expression for the twisted index of 4d class S theories $\mathcal{T}_N[\Sigma_g]$ on closed hyperbolic 3-manifold M_3 with $H_1(M_3, \mathbb{Z}_N) = 0$:

$$\mathcal{I}_{M_3}(\mathcal{T}_N[\Sigma_g]) = \sum_{\mathcal{A}^\alpha \in \chi^{\text{irred}}(M_3; N)} (N \times \mathbf{Tor}_{M_3}^\alpha)^{g-1}. \quad (2.17)$$

3 Large N twisted index and a magnetically charged AdS_5 BH

Via AdS_5/CFT_4 , the twisted index is expected to count the microstates of a magnetically charged black hole in the holographic dual AdS_5 gravity with *signs*:

$$\mathcal{I}_{M_3}(\mathcal{T}_N[\Sigma_g]) = d_{\text{micro}}^+ - d_{\text{micro}}^-, \quad (3.1)$$

here d_{micro}^+ and d_{micro}^- stand for the number of microstates with even R -charge and odd R -charge, respectively. Unless there is highly fine-tuned cancellation between d_{micro}^+ and d_{micro}^- , the twisted index can see the Bekenstein-Hawking entropy S_{BH} of the black hole at large N :

$$\begin{aligned} \log \mathcal{I}_{M_3}(\mathcal{T}_N[\Sigma_g]) &= \log(d_{\text{micro}}^+ - d_{\text{micro}}^-) \simeq \log(d_{\text{micro}}^+ + d_{\text{micro}}^-) \simeq S_{BH}, \\ \text{unless } \left| \frac{d_{\text{micro}}^+ - d_{\text{micro}}^-}{d_{\text{micro}}^+ + d_{\text{micro}}^-} \right| &< e^{-\kappa N^3} \text{ at sufficiently large } N \text{ for some positive, finite } \kappa. \end{aligned} \quad (3.2)$$

Here the equivalence relation $\simeq$ is defined as

$$f(N) \simeq g(N) \text{ if } \lim_{N \rightarrow \infty} \frac{f - g}{N^3} = 0. \quad (3.3)$$

Since the black hole is made of N M5-branes, S_{BH} is expected to scale as $o(N^3)$.

In this section, we consider the large N limit of the 4d twisted index $\mathcal{I}_{M_3}(\mathcal{T}_N[\Sigma_g])$ using the relation in (2.17). For $g > 1$, indeed the twisted index (2.17) behaves as N^3 at large N limit. Furthermore, we will show that the leading term of $\log \mathcal{I}_{M_3}(\mathcal{T}_N[\Sigma_g])$ perfectly agree with the Bekenstein-Hawking entropy of the magnetically charged black hole in AdS_5 .

3.1 Full perturbative $1/N$ expansion of the index

The $1/N$ perturbative expansion of the 3d twisted index $\mathcal{I}_{\Sigma_g}(\mathcal{T}_N[M_3])$ is studied in [23, 24] using the 3d-3d relation. From the 4d-3d relation (2.11), the 4d twisted index $\mathcal{I}_{\Sigma_g}(\mathcal{T}_N[M_3])$ is expected to share the same $1/N$ expansion. Let us summarize the process of taking the large N limit to (2.13), in order to figure out leading large N terms of the $\mathcal{I}_{\Sigma_g}(\mathcal{T}_N[M_3])$.

In the large N limit, only two irreducible flat connections, $\mathcal{A}^{\text{geom}}$ and $\mathcal{A}^{\overline{\text{geom}}}$ are expected to give exponentially dominant contribution to the summation in (2.17).

$$\begin{aligned} \mathcal{I}_{M_3}(T_N[\Sigma_g]) &\xrightarrow{N \rightarrow \infty} (N \times \mathbf{Tor}_{M_3}^{\text{geom}})^{g-1} + (N \times \mathbf{Tor}_{M_3}^{\overline{\text{geom}}})^{g-1} \\ &\quad + (\text{exponentially smaller terms at large } N) . \end{aligned} \quad (3.4)$$

The two dominant flat-connections can be constructed from the unique unit hyperbolic structure on M_3 :

$$\mathcal{A}_N^{\text{geom}} := \rho_N(\omega + ie) , \quad \mathcal{A}_N^{\overline{\text{geom}}} := \rho_N(\omega - ie) . \quad (3.5)$$

Here ω and e are spin-connection and dreibein of the hyperbolic structure on M_3 . Both of them can be thought as $so(3)$ -valued 1-form on M_3 and they form two $\text{PGL}(2, \mathbb{C}) = \text{SO}(3)_{\mathbb{C}}$ flat connections $\mathcal{A}_{N=2}^{\text{geom}, \overline{\text{geom}}} = \omega \pm ie$ related to each other by complex conjugation. ρ_N is the principal embedding from $\text{PGL}(2, \mathbb{C})$ to $\text{PGL}(N, \mathbb{C})$.

Using mathematical results [36, 37], we obtain following asymptotic expansion of the twisted index at large N [23, 24]

$$\begin{aligned} \mathcal{I}_{M_3}(T_N[\Sigma_g]) &= \left(e^{i\theta(N, M_3)} + e^{-i\theta(N, M_3)} \right) \\ &\quad \times \exp \left[(g-1) \left(\frac{\text{vol}(M_3)}{6\pi} (2N^3 - N - 1) + \log N \right) \right] \\ &\quad \times \exp \left[(g-1) \left(\sum_{\gamma} \sum_{m=1}^{N-1} \sum_{k=m+1}^{\infty} \log |1 - e^{-k\ell_{\mathbb{C}}(\gamma)}| \right) \right] \\ &\quad + (\text{exponentially smaller terms at large } N) . \end{aligned} \quad (3.6)$$

The exponentially smaller terms come from contributions of irreducible flat connections other than $\mathcal{A}_N^{\text{geom}}$ and $\mathcal{A}_N^{\overline{\text{geom}}}$. Again, the above expansion only holds for closed hyperbolic 3-manifold with vanishing $H_1(M_3, \mathbb{Z}_N)$. $\theta(N, M_3)$ is an undetermined angle due to the relative phase differences of the contributions from two dominant flat-connections, $\mathcal{A}^{\text{geom}}$ and $\mathcal{A}^{\overline{\text{geom}}}$. $\sum_{\gamma}$ is summation over the nontrivial primitive conjugacy classes of the fundamental group $\pi_1(M_3)$. $\ell_{\mathbb{C}}(\gamma)$ is the complexified geodesic length of the γ , which is defined by following relation:

$$\text{Tr} P \exp \left(- \oint_{\gamma} \mathcal{A}_{N=2}^{\text{geom}} \right) = e^{\frac{1}{2}\ell_{\mathbb{C}}(\gamma)} + e^{-\frac{1}{2}\ell_{\mathbb{C}}(\gamma)} , \quad \Re[\ell_{\mathbb{C}}] > 0 . \quad (3.7)$$

The term $\sum_{\gamma}(\dots)$ in the above can be decomposed into two parts

$$\begin{aligned} &\sum_{\gamma} \sum_{m=1}^{N-1} \sum_{k=m+1}^{\infty} \log |1 - e^{-k\ell_{\mathbb{C}}(\gamma)}| \\ &= -\Re \sum_{\gamma} \sum_{s=1}^{\infty} \frac{1}{s} \left(\frac{e^{-s\ell_{\mathbb{C}}}}{1 - e^{-s\ell_{\mathbb{C}}}} \right)^2 + \Re \sum_{\gamma} \sum_{s=1}^{\infty} \frac{1}{s} \left(\frac{e^{-\frac{s(N+1)}{2}\ell_{\mathbb{C}}}}{1 - e^{-s\ell_{\mathbb{C}}}} \right)^2 \end{aligned} \quad (3.8)$$

The first term is N -independent while the second term is exponentially suppressed at large N . Note that the leading order behavior of the twisted index only depend on $\text{vol}(M_3)$ while subleadings depend on both $\text{vol}(M_3)$ and length spectrum $\{\ell_{\mathbb{C}}(\gamma)\}$.

3.2 A magnetically charged AdS_5 black hole

In [18], a magnetically charged black hole solution in 5d $\mathcal{N} = 4$ gauged supergravity is numerically constructed. The black hole interpolates

$$\begin{aligned} \text{UV: } & AdS_5 \text{ with } \mathbb{R} \times M_3 \text{ conformal boundary} \\ \text{IR: } & AdS_2 \times M_3 \text{ near-horizon} \end{aligned} \quad (3.9)$$

The 11d uplift of the solution is studied in [38]. Including the internal direction, the near horizon geometry is a warped product of $AdS_2 \times M_3 \times \Sigma_g \times S^4$ [39]. The Bekenstein-Hawking entropy of the AdS_5 black hole is universally given as [40]

$$S_{BH} = a_{4d} \frac{\text{vol}(M_3)}{\pi}. \quad (3.10)$$

Here a_{4d} is the a-anomaly coefficient of the dual 4d $\mathcal{N} = 2$ SCFT. For class $\mathcal{S}$ theory $\mathcal{T}_N[\Sigma_g]$, the anomaly coefficient is given by [41]

$$a_{4d}(\mathcal{T}_N[\Sigma_g]) = \frac{1}{3}(g-1)N^3 + \text{subleading}, \quad (3.11)$$

thus the Bekenstein-Hawking entropy for the magnetically charged black hole inside AdS_5 dual [25, 41] of 4d $\mathcal{T}_N[\Sigma_g]$ read

$$S_{BH} = \frac{(g-1)N^3 \text{vol}(M_3)}{3\pi}. \quad (3.12)$$

Here $\text{vol}(M_3)$ is the volume of the 3-manifold measured using unit hyperbolic metric normalized as

$$R_{\mu\nu} = -2g_{\mu\nu}. \quad (3.13)$$

Locally, the unit hyperbolic metric is given as

$$ds^2(\mathbb{H}^3) = d\phi^2 + \sinh^2 \phi (d\theta^2 + \sin^2 \theta d\nu^2). \quad (3.14)$$

According to Mostow's rigidity theorem [42], there is an unique unit hyperbolic metric on M_3 and the volume is actually a topological invariant.

It turns out that the large N leading part of the $\log \mathcal{I}_{M_3}(\mathcal{T}_N[\Sigma_g])$ nicely matches with the Bekenstein-Hawking entropy (3.12):

$$\log \mathcal{I}_{M_3}(\mathcal{T}_N[\Sigma_g]) = \frac{(g-1)N^3 \text{vol}(M_3)}{3\pi} + (\text{subleadings}) \quad (3.15)$$

From (3.6), the full perturbative $1/N$ corrections can be summarized as follows

$$\begin{aligned} & \text{a term with } \text{vol}(M_3): (g-1) \frac{\text{vol}(M_3)}{6\pi} (2N^3 - N - 1), \\ & \text{a term involving length spectrum: } (1-g) \times \Re \sum_{\gamma} \sum_{s=1}^{\infty} \frac{1}{s} \left(\frac{e^{-s\ell_{\mathbb{C}}}}{1 - e^{-s\ell_{\mathbb{C}}}} \right)^2, \\ & \text{logarithmic correction: } (g-1) \log N. \end{aligned} \quad (3.16)$$

Since we do not have understanding of contributions from other irreducible flat-connections, it is very difficult to determine the full non-perturbative corrections. From the above analysis, we could identify following non-perturbative correction:

$$\text{a non-perturbative correction: } (g-1)\Re \sum_{\gamma} \sum_{s=1}^{\infty} \frac{1}{s} \left(\frac{e^{-\frac{s(N+1)}{2}\ell_{\text{C}}}}{1-e^{-s\ell_{\text{C}}}} \right)^2. \quad (3.17)$$

It would be interesting future work to identify these subleading corrections in (3.16) and (3.17) from the quantum computation in the holographic dual M-theory. Especially, the logarithmic subleading correction should be reproduced from zero mode analysis on the 11d uplift of the AdS_5 black hole as done in [24, 43, 44] for AdS_4/CFT_3 examples.

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